In order to compensate for the scarcity of events at very high energy the LHC has to provide a luminosity of $10^{34} \text{cm}^{-2} \text{s}^{-1}$. This is obtained with a large beam current distributed over 2835 particle bunches, and a large transverse bunch density so as to operate close to the beam-beam limit. The beam-beam interaction has two components, the head-on interaction as in previous colliders and the long range interaction due to multiple unwanted crossings which occur before the beam separation is sufficient. The single bunch and multibunch collective instabilities are kept under control by a proper design of the beam enclosure and by feedback systems. The unavoidable imperfections of the high field superconducting magnets create non-linear field errors which limit the useful range of particle betatron amplitudes where the motion is stable, the so-called Dynamic Aperture. An extended set of corrector magnets is foreseen to compensate for the effects of the strongest multipoles of low order. The machine lattice is designed with the aim of leaving sufficient freedom in the choice of the operating conditions to optimize performance.
ACCELERATOR PHYSICS ISSUES OF THE LHC
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Abstract

In order to compensate for the scarcity of events at very high energy the LHC has to provide a luminosity of \(10^{34} \text{cm}^{-2} \text{s}^{-1}\). This is obtained with a large beam current distributed over 2835 particle bunches, and a large transverse bunch density so as to operate close to the beam-beam limit. The beam-beam interaction has two components, the head-on interaction as in previous colliders and the long range interaction due to multiple unwanted crossings which occur before the beam separation is sufficient. The single bunch and multibunch collective instabilities are kept under control by a proper design of the beam enclosure and by feedback systems. The unavoidable imperfections of the high field superconducting magnets create non-linear field errors which limit the useful range of particle betatron amplitudes where the motion is stable, the so-called Dynamic Aperture. An extended set of corrector magnets is foreseen to compensate for the effects of the strongest multipoles of low order. The machine lattice is designed with the aim of leaving sufficient freedom in the choice of the operating conditions to optimize performance.

1. INTRODUCTION

The LHC luminosity is given by the formula

\[
L = \frac{1}{4\pi} \frac{\gamma}{\beta^*} \left[ N \varepsilon_n \right] [N_{bf}] F
\]

where \(\gamma\) is the energy of the protons divided by their rest energy, \(\beta^*\) is the value of the betatron function at the collision point, \(N\) is the number of protons in each of the \(k\) bunches, \(\varepsilon_n\) is the invariant transverse emittance, \(f\) is the revolution frequency and \(F\) is a reduction factor due to the finite crossing angle of the beams which is 0.9 in the LHC.

In order to compensate for the very low cross section in quark and gluon collisions in the TeV energy range, the LHC must provide a very large luminosity, of the order of \(10^{34} \text{cm}^{-2} \text{s}^{-1}\), that is 50 times the present world record. In formula 1, \(\gamma\) is limited by the bending magnet field and \(\beta^*\) is similarly largely determined by the available technology of high gradient quadrupole lenses. The first bracket is proportional to the beam-beam parameter which is limited by the electromagnetic interaction of colliding bunches. The second bracket is proportional to the beam current which has to be increased to reach the required luminosity.

In order to provide enough flexibility in machine operation.

2. BEAM-BEAM EFFECTS

When two opposite beams cross each other, the particle trajectories in one beam are perturbed by the electromagnetic field of the other beam.

2.1 Head-on and long range effects

The head-on crossing of two counter-rotating bunches has two effects: it excites betatron resonances and changes the tunes of the particles in a way which depends on their oscillation amplitudes, thus generating a tune spread in the beams. Experience at the SPS has shown that the beam lifetime is strongly reduced when particles straddle resonances of order less than 12. Therefore the tune “footprint” (Fig. 1), which is the image of the beam in the tune diagram, should be small enough to be lodged in between these resonances. This limits the intensity of the bunches which can be collided, and therefore the luminosity.

The LHC is operated with a large number of closely spaced bunches in order to reach the required high luminosity. As a consequence the beams must collide at a small angle to prevent unwanted collisions in the region around the experiment where they travel in the same vacuum chamber. However, the so-called long range interactions of the separated bunches when they pass close to each other in that part common to the two beams cannot be suppressed.

These interactions are non-linear and generate a tune spread which adds to that of the head-on collision. The contribution of each long range interaction is small but there are many of them (12 in the case of the LHC on either side of each interaction point). They are responsible for a significant enlargement of the tune footprint as displayed in Fig. 1.

The strength of the head-on interaction, usually indicated by the beam-beam parameter \(\xi\), is independent of \(\beta^*\) and can be written
where \( r_p \) is the classical proton radius and \( \epsilon_n \) the invariant transverse emittance.

The long range interaction, on the contrary, is strongly related to \( \beta^* \). To ensure separation the crossing angle has to be larger than the divergence of the beam at the collision point, which is inversely proportional to \( \beta^* \). At the same time it is limited by the aperture of the final focus quadrupoles, which have a single channel common to both beams. As a consequence the long range interactions play an important role in the optimization of the insertions for high luminosity experiments in which one tries to reduce \( \beta^* \) as much as possible.

2.2 The LHC working point

The LHC working point in the tune diagram can be safely chosen in areas close to the diagonal between 3rd and 10th order resonances or between 4th and 9th order resonances, provided the largest horizontal or vertical dimension of the tune footprint stays below 0.01. This allows sufficient safety margins around the coupling resonance and the 3rd and 4th order resonances, which are expected to be wider in the LHC than in previous machines, owing to the larger multipolar errors in the magnetic field of the superconducting magnets and the larger machine size. In the case of the LHC operating with two high luminosity experiments, the optimum crossing angle is 200 \( \mu \text{rad} \), providing a separation of about 7 \( \sigma \) (\( \sigma \) is the RMS beam size). The effect of the long-range interactions is minimized by letting the beams cross in the horizontal plane in one experiment and in the vertical plane in the other, or even better at 45° in all experiments [2]. In this case, about 30% of the total tune spread of 0.01 is produced by the spurious long range interactions, while 70% comes from the wanted head-on collisions. The value of the beam-beam parameter is \( \xi = 0.0034 \), very close to that achieved routinely in the SPS proton antiproton collider. Adding other experiments will reduce the maximum achievable luminosity since it is the total tune spread which is limited.

3. COLLECTIVE EFFECTS

3.1 How to treat them at the design stage

After more than thirty years of experimental investigation and theoretical synthesis the evaluation of collective effects in accelerators is now a well mastered subject, although new phenomena emerge from time to time. At the design stage all elements surrounding the beam are checked for their coupling impedance. This information is entered into a data base from which one can extract at each moment a precise model of the machine coupling impedance. Using this, a sophisticated computer program, integrating all presently known phenomena, calculates tune shifts, instability growth rates and energy lost by the beam to its surroundings. If some of these results are not acceptable, the elements responsible are identified and modified until the design is globally optimized.

3.2 Different sorts of collective effects

Single-beam collective effects include incoherent and coherent phenomena.

The main examples of incoherent effects are synchrotron radiation losses, direct space charge and Laslett tune shifts due to image currents, as well as intra-beam scattering.

Coherent effects include parasitic losses, associated with the real part of the longitudinal coupling impedance \( Z_L \), and complex tune shifts of the beam oscillation modes. The imaginary part of the tune shifts give the growth rate of instabilities. The real part determines whether the coherent modes are Landau damped as a consequence of the tune spread present in the beam. The modes which are not Landau damped and have a positive growth rate require damping by an active feedback system.

A modern way of treating feedback systems is to consider them as additional synthetic coupling impedances and introduce them as such in the impedance model. Their effect on the beam is then computed self-consistently and globally by the beam dynamics module of the computer program. In this way their effects on all modes are automatically taken into account.

3.3 Incoherent effects

The direct space charge tune shift amounts to 1.2 \( 10^{-3} \) at injection in the LHC. This value is small enough not to pose any problem for the tuning of the machine, yet it is large enough to produce Landau damping for mode numbers higher than \( m=0 \), the rigid dipole mode. However it decreases like \( \gamma^{-2} \) and therefore becomes negligible at high energy where Landau damping has to be provided by octupoles.
Owing to image currents induced on the flattened beam pipe all particles suffer another incoherent detuning proportional to the total beam current. This amounts to about $10^{-2}$ at injection, a large value which has to be corrected progressively at each step of the injection process by retuning the machine quadrupoles. Since the beam fills only part of the LHC circumference, especially during injection, the image currents present low frequency components which can leak out of the beam pipe and produce a modulation of the tune shift along the beam. This effect cannot be compensated by quadrupoles and studies are engaged to calculate its magnitude. Another complicated effect under investigation concerns the magnetic images induced in the ferromagnetic yoke of the two in one LHC magnet.

Intrabeam scattering produces a growth of horizontal and longitudinal emittances in the LHC if the 6-dimensional phase space density of the beam is too large. High luminosity imposes a large density in the transverse coordinates, but leaves open the possibility of longitudinally diluting the beam to reduce the intrabeam scattering growth rates. This sets the requirements for the LHC RF system: since the bunch must be short to limit the loss in luminosity due to the finite crossing angle, the momentum spread has to be increased to reduce the intrabeam scattering growth rates. This fixes the minimum RF voltage, both at injection and at high energy.

### 3.4 Single bunch effects - broad band impedance

The transverse broad band impedance of the LHC is shown in Fig. 2. One can distinguish the broad peak due to the bellows around 3 GHz and the capacitive impedance due to space charge which dominates at very high frequency. The peak at low frequency corresponds to the abort kicker and the wriggles are due to the strip lines of the beam position monitors.

The transverse broad band impedance is responsible for the head-tail instability which is suppressed for the rigid dipole mode $m = 0$ by operating with positive chromaticity, and for the transverse mode coupling instability which cannot be suppressed. The threshold for this instability is twice the nominal bunch current, a safe situation provided the coupling impedance is kept at the level estimated at present.

The longitudinal broad band impedance is responsible for microwave instabilities and tune shifts of the longitudinal single bunch modes which may lead to suppression of Landau damping. The LHC operates below thresholds for both of these effects.

### 3.5 Multibunch effects - narrow band impedance

High $Q$ cavity modes can couple bunches together both in transverse and longitudinal planes and lead to coupled bunch instabilities. To minimize these effects the most dangerous resonant modes of the accelerating cavities and of the feedback cavities are damped to reduce their $Q$ factors, and the inevitable cross section variations of the experiments vacuum chambers are equipped with smooth transitions to avoid trapped modes.

The most important multibunch effect in the LHC is the transverse resistive wall instability. Its growth rate is proportional to the square root of the resistivity of the beam pipe. It is proportional to the machine radius and to the inverse cube of the beam pipe height, which explains why it is dominant in large high energy machines which tend to have a small beam pipe. The instability is minimized in the LHC by coating the inside of the beam screen, which is cooled down to 20K, with 50 $\mu$m of pure copper. With this measure the $e$-folding time of the most dangerous mode at a frequency of a few kHz exceeds 100 turns, which makes it easy to damp with feedback.

Fig 3 shows the calculated growth rates of multibunch modes. The peak on the right for mode $m=0$ is due to the resistive wall. The other sharp peaks, which mostly affect head-tail modes $m = 1$ are due to partially damped cavity modes.

### 3.6 The beam screen

Each LHC beam emits 3.6 kW of synchrotron radiation at 7 TeV, and the return currents induced by the beam on the inside surface of the beam pipe contribute another 2 kW to the heat load. Since it would be too expensive to absorb this at 1.9 K, a beam screen, independently cooled at about 20 K, is introduced inside the vacuum chamber. However, to restore cryopumping on the vacuum chamber at 1.9 K the beam screen must have millions of holes and this is a nightmare for the accelerator physicists. The beam electromagnetic fields can leak out of these holes and build up coherently a TEM wave which propagates at the speed of light in the coaxial structure formed by the beam screen inside the vacuum chamber. This would heat up the vacuum chamber, which we wanted to avoid in the first place, and destroy the beam by retroaction. The solution is to make holes smaller than the beam screen wall thickness to reduce leakage and to elongate them in the direction of the beam current to reduce their coupling impedance to the beam. Distributing holes of different lengths in a semi-random fashion will further reduce the danger of coherent modes and RF absorbers on the outer surface of the screen will damp coaxial waves.
4. DYNAMIC APERTURE

4.1 Magnetic errors limit the dynamic aperture

In superconducting magnets it is more difficult to provide a field of the required quality than in classical magnets with iron pole faces. Multipole errors are introduced by magnetization of the superconducting filaments (persistent currents), by tiny displacement of coil conductors (geometry), and by redistribution of currents between strands during ramping of the field. The multipole coefficients $b_n$ and $a_n$ are defined by the formula

$$B_y + iB_x = B_1 \sum_n (b_n + ia_n) \left( \frac{z}{R} \right)^{n+1}$$

where $B_1$ is the nominal vertical magnetic field, $B_y$ and $B_x$ are the actual components of the field in the vertical and horizontal planes, $R_r = 1\,\text{cm}$ is the reference radius, and $z = x + iy$. For each category of errors we distinguish those which affect all magnets equally, those which vary from magnet to magnet and those which may affect equally all the magnets of a production line but vary randomly from one production line to another.

Non-linear magnetic fields perturb the particle trajectories and lead to particle loss at large amplitudes. The beams must survive for more than $10^7$ turns at injection energy (this corresponds to about 15 min. for injection and the first phase of the ramp) and $4 \times 10^8$ turns at high energy (about 10h). The dynamic aperture is the largest amplitude below which all particles survive for the relevant number of turns. It is generally expressed in units of $\sigma$, the RMS beam size at the maximum $B$ in the arcs. Experience with previous machines suggests that a dynamic aperture of $6\sigma$ is adequate for a safe operation.

The dominant mechanisms which limit the dynamic aperture are, as in the case of the beam-beam effect, tune spread and excitation of resonances. The tune spread is generated essentially by the average value of the multipoles around the machine, while resonances are excited by all harmonics.

4.2 Evaluation of the dynamic aperture

There is at present no reliable way to evaluate the dynamic aperture other than computer simulation and experiments in existing machines. For the LHC, particles are tracked element by element in a computer model simulating as closely as possible the real machine. For a rough estimate $10^3$ to $10^4$ turns are sufficient. This allows a very accurate evaluation of the tune spread as a function of betatron amplitude and momentum deviation, and the identification of the border between regular motion at small amplitudes and chaotic motion at large amplitudes. Below the chaotic border particles are supposed to survive indefinitely. For a more refined estimate one tracks for up to $10^6$ turns, and extrapolates the "survival plot" (Fig. 4) to $10^7$ turns. Usually the dynamic aperture obtained in this way lies just above the chaotic border. For heavy simulation campaigns to study the effect of varying parameters one tracks for each case 300 particles, each with different starting amplitudes, for $10^5$ turns [4]. With the computer system available at present for LHC design, this takes about 1 hour. The process is repeated for 60 imperfect machines corresponding to 60 different realizations of the random and systematic per arc errors. In a reduced number of cases the tracking is extended to 5 different values of the ratio of horizontal and vertical amplitudes. In order to test whether the computer models used are sufficiently realistic to give reliable results, extensive experimental campaigns have been launched in the CERN SPS and DESY HERA [5]. They have shown that, although reality is always worse than the model predicts, the difference can be reduced to around 20% if all known details, like residual closed orbit or coupling and power supply ripple are taken into account. They have confirmed that minimizing tune spread with multipole correctors increases the dynamic aperture.

As a result of all these studies the LHC dynamic aperture is now close to $6\sigma$, which is considered as a minimum value for safe operation. We aim at increasing it by some 10 to 15% to provide a further safety margin for unpredictable effects.

4.3 Optimization of the dynamic aperture

This process involves several steps. In a first step one uses a table of errors evaluated by the magnet designers and vary the machine parameters (cell length and phase advance, magnet aperture) to find a realistic optimum. Then using the machine obtained in this way one identifies the dominant multipole errors and sees with the magnet designers whether they can be reduced. When increasing pressure on the magnet designers begins to give diminishing returns, one starts considering the introduction of corrector systems in the machine.

Apart from the classical orbit correcting dipoles and chromaticity correcting sextupoles, the LHC is equipped at present with small 10 cm long coils at the ends of each bending magnet to partially correct the effect of the systematic part of $b_5$ and $b_7$, which are mainly due to persistent currents. Fig. 5 shows how the correction of $b_5$ is effective in reducing the tune spread for particles with a large momentum deviation. Such a correction increases significantly the...
dynamic aperture [6]. Octupoles are foreseen close to the main quadrupoles to allow the possibility of increasing Landau damping of transverse instabilities. Small additional coils could be installed in the magnet ends to correct $a_5$, $b_4$, and $a_3$ depending on the needs.

5. LATTICE DESIGN

The parameters of the arc FODO cells were chosen to give the best compromise between dynamic aperture and maximum attainable energy for a given set of multipole errors and for a given magnetic field. Work is now in progress to better control the magnetic errors and to refine the correction system in order to ensure a sufficient safety margin for operation.

The LHC insertions are heavily constrained by the LEP tunnel which was not conceived for a 7 TeV machine. In order to introduce flexibility in the design of the insertions and of the dispersion suppressor small quadrupole correctors are introduced close to the main quadrupoles at both ends of each arc. They will also be used for fast and accurate tune adjustments during operation.

The original antisymmetry between ring 1 and ring 2 and the left and right sides of interaction points, which seems natural in a machine with two-in-one magnets, was partially abandoned in order to allow separating the vertical and horizontal tunes by several units. For this, quadrupoles on the left and right sides of interaction points have to be powered independently. With this measure the large coupling effect originating from the magnet $a_2$ is reduced and can be well corrected by 2 pairs of skew quadrupoles per arc.

6. CONCLUSIONS

The accelerator physics in the LHC benefits from the vast amount of knowledge accumulated in the operation of its predecessors. However the combination of high density, large current beams with the most sophisticated superconducting magnet technology poses interesting challenges. Developments at the frontier of present knowledge in collective effects and single particle non-linear dynamics are being pursued, often in worldwide collaborations. Further detailed calculations concerning beam-beam effects in the special LHC environment as well as refinements of the machine lattice are in progress to finalise the design.

REFERENCES


![Fig. 1: Tune footprint for alternate crossings in ATLAS and CMS](image)
Fig. 2: Calculated LHC broad-band transverse coupling impedance

Fig. 3: Growth rates of transverse multibunch modes

Fig. 4: Survival Plot for LHC Version 4
Fig. 5: Horizontal tune shift versus amplitude due to $b_5$, with and without correction by spool pieces in magnet ends. Each line corresponds to a different ratio of horizontal to vertical amplitude.