ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

PHYSICS AT LEP2

Editors: G. Altarelli, T. Sjostrand and F. Zwirner

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ABSTRACT

This is the final report of the Workshop on Physics at LEP2, held at CERN during 1995. The first part of vol. 1 is devoted to aspects of machine physics of particular relevance to experiments, including the energy, luminosity and interaction regions, as well as the measurement of beam energy. The second part of vol. 1 is a relatively concise, but fairly complete, handbook on the physics of $e^+e^-$ annihilation above the WW threshold and up to $\sqrt{s} \approx 200$ GeV. It contains discussions on WW cross-sections and distributions, W mass determination, Standard Model processes, QCD and gamma–gamma physics, as well as aspects of discovery physics, such as Higgs, new particle searches, triple gauge boson couplings and $Z'$. The second volume contains a review of the existing Monte Carlo generators for LEP2 physics. These include generators for WW physics, QCD and gamma–gamma processes, Bhabha scattering and discovery physics. A special effort was made to co-ordinate the different parts, with a view to achieving a systematic and balanced review of the subject, rather than just publishing a collection of separate contributions.
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INTRODUCTION
Guido Altarelli

The present Workshop on Physics at LEP2 was promoted jointly by the Particle Physics Experiments and the Theory Divisions of CERN. Given the success of the 1989 Workshop on Z Physics at LEP, it was considered natural to produce a similar effort in the preparation of the LEP2 phase. In 1993 the Division Leaders (at the time) J. Allaby and J. Ellis, requested me to chair the committee responsible for organizing this Workshop. At the same time they invited the spokesmen of the four LEP experiments to nominate two members each to form the Organizing Committee. Three more members, two theorists and one machine physicist agreed to join this Committee.

The Organizing Committee was appointed by October 1993. The structure and the goals of the Workshop were discussed at the first meetings of this body, held on 9 November and 2 December 1993. The working groups were defined and most of the conveners were soon designated. The purpose of the Workshop was to encourage experimentalists and theorists to make a joint effort to update, to better organize and to improve still further the vast amount of knowledge and of software in the area of LEP2 physics. Considering that the start of LEP2 was finally planned for 1996, it was decided to run most of the Workshop in 1995 and have its complete results by the end of the same year. Accordingly, the Organizing Committee met again starting from September 1994 and the activity of the working groups began towards the end of 1994.

Four sectors were defined: i) Machine–Physics Interface; ii) Standard Physics; iii) Discovery Physics; iv) Event Generators. For each sector there were one or two co-ordinators and four or eight contact persons, designated by the four LEP experiments, to promote the Workshop activities within the collaborations. In each sector different working groups were formed under the responsibility of two (exceptionally one or three) conveners, normally theorists. The final structure of the Workshop and the appointed persons are shown in p. 3.

At the first general meeting, which was held at CERN on 2–3 February 1995, the conveners presented the status of each subject, detailed programmes were set up, and the formation of the working groups was nearly completed. By the time of the second general meeting, which took place at CERN on 15–16 June 1995, many interesting results were already available. In particular, as requested by L. Foà, Director of Research, a detailed report was presented on the physics interest of the phase IV of LEP2, at the time not yet approved, consisting in the energy upgrade of the machine up to about 192 GeV with an estimated integrated luminosity per year of 170 pb$^{-1}$. The ‘Interim Report on the Physics Motivations for an Energy Upgrade of LEP2’ was published already in June as a joint PPE and TH report (CERN-TH/95-151, CERN-PPE/95-78). The proposed upgrade was finally approved by the CERN Council in December 1995. In the last phase of the Workshop a closer interaction within and between the different working groups was focused to the final presentation of the results, which took place at the third and final general meeting, held at CERN on 2–3 November 1995. Nearly complete preliminary
drafts of the articles for the present report started to be circulated soon after. By the end of the year all the final versions of the chapters of these two volumes could be delivered for printing.

I think that the physics output of the Workshop has been quite satisfactory, even beyond the initial expectations. The main reasons for this success were the experience gained during the first Workshop on LEP in 1989 and the opportunity of bringing together the most prominent experts in the field. The latter move was made possible by a special programme of invitations set up by the Theory Division, which provided most of the funding, but received a very welcome help from the Experimental Physics Division. We thank for that support the Division Leaders G. Veneziano and G. Goggi. But what mostly determined the great achievements of the Workshop was the beautiful response and the enthusiastic drive of the people concerned. I am especially satisfied of the collaboration of the members of the Organizing Committee and of the other co-ordinators. We were all impressed by the intelligent and dedicated leadership of the conveners and the contact persons. But the bulk of the work was done by the nearly four hundred physicists at the highest professional level in this domain of research who participated in this Workshop. Last but not least, Mrs. Suzy Vascotto deserves a special mention for her invaluable help in the organization and in the running of the Workshop.

I am sure that the LEP community will find the present work useful during the forthcoming years. I also hope that the beautiful co-operative spirit of this Workshop, in particular between experimentalists and theorists, will continue throughout the period of LEP2 operation.
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ALEPH Jean-François Grivaz, Francesco Ragusa
DELPHI Giorgio Matthiae, François Richard
L3 Marta Falcini, Martin W. Grünewald
       (at first Felicitas Pauss)
OPAL Giorga Mikenberg, Terry Wyatt
Theory Torbjörn Sjöstrand, Fabio Zwirner

PHYSICS GROUPS

Standard physics

Co-ordinators: Guido Altarelli, Fabio Zwirner

Conveners:
- WW cross-sections and distributions: Wim Beenakker, Frits A. Berends
- W mass: Zoltan Kunszt, W. James Stirling
- Standard Model processes: Fawzi Boudjema, Barbara Mele
- QCD: Paolo Nason, Bryan Webber
- Gamma–gamma physics: Patrick Aurenche, Gerhard Schuler

Contact persons:\n- Ramon Miquel, Patrice Perez,
- Clara Matteuzzi, Klaus Möning,
- Martin Grünewald, Martin Pohl,
- Dorothee Schaile, David Charlton

1) The contact persons are listed in alphabetical order of their experiment, i.e. ALEPH, DELPHI, L3, OPAL.
New physics

Co-ordinators: Guido Altarelli, Fabio Zwirner

Conveners:
- Triple gauge coupling: George J. Gounaris, Jean-Loïc Kneur, Dieter Zeppenfeld
- Higgs: SM and SUSY: Marcela Carena, Peter Zerwas
- New particles: real production and virtual effects: Gian F. Giudice, Reinhold Rückl
- Z prime: Pierre Chiappetta, Claudio Verzegnassi

Contact persons:
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- Klas Hultqvist, Stavros Katsanevas,
- Paolo Bagnaia, Marta Felcini,
- Peter Igo-Kemenes, Sachio Komamiya

Event generators

Co-ordinator: Torbjörn Sjöstrand

Conveners:
- WW and Standard Model processes: Dima Bardin, Ronald Kleiss
- Gamma-gamma processes: Leif Lömblad, Michael Seymour
- Discovery physics: Michelangelo Mangano, Giovanni Ridolfi
- Bhabha scattering: Stanislaw Jadach, Oreste Nicrosini
- QCD processes: Ian Knowles, Torbjörn Sjöstrand

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- Manel Martinez, Boleslaw Pietrzyk,
- Robert Sekulin, Catherine Vander Velde,
- Fritz Erné, Francesca Nessi,
- Ehud Duchovni, John W. Gary

Physics/machine interface

Co-ordinators: Stephen Myers, François Richard

Conveners:
- Absolute energy measurements and polarization: Massimo Placidid
- Interaction point: Georg von Holtey
- Integrated luminosity versus energy: Stephen Myers

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PHYSICS/MACHINE INTERFACE
PROSPECTS FOR ENERGY AND LUMINOSITY AT LEP2

S. Myers, C. Wyss

1 Introduction

This paper is a shortened and updated version of Ref. [1] where the status and scope of the LEP Energy Upgrade Programme as per June 1995 are also reported. Its aim is to provide concise information about further steps towards higher energies, as discussed at the 1995 Chamonix LEP Performance Workshop [2] and by the LEP2 Physics Workshop [3]. The steps are determined by discrete layout modifications and equipment upgrades creating space and capacity for additional sets of superconducting cavities. LEP layouts, expected energies, peak luminosities, schedules and global costs estimates are given for each step considered. After a discussion of limitations in Section 2, steps to upgrade the LEP2 beam energy up to 96 GeV, which are within the cooling power of the present cryogenic plants, are presented in detail in Sections 3 to 5. Further steps, up to an ultimate beam energy of 104 GeV, which were studied in the framework of the LEP2 Physics Workshop and require an upgrade of the cryoplants, are outlined in Appendix A.

2 On the Path to Higher Energies

2.1 Energy and Luminosity

2.1.1 Computing Energy and Peak Luminosity

The calculations for energy, peak luminosity, and HOM power, leading to the figures shown in the various tables (see e.g. Table 2b, Section 3) were performed with the following assumptions:

- The beam energy for a given voltage has been computed for a quantum lifetime of 15 hours, for the $108^\circ$ phase advance lattice [4].

- A horizontal emittance of 30 nm at 90 GeV scaled with $\gamma^2$. This corresponds to the value anticipated for the $108^\circ$ phase advance lattice.

- A vertical to horizontal emittance ratio (4%) equal to the $\beta$ ratio (which ensures that the beam-beam tune shifts in the horizontal and vertical planes are equal)

- Operation with 8 bunches per beam except in the scenarios ‘Y’ (see Appendix A) where the electro-static separators are removed in order to make space for additional sc cavities.
- Bunch currents limited by the Transverse Mode Coupling Instability (at injection energy) to 0.75 mA with 100% of the copper cavities remaining in the tunnel and 1.0 mA with 50% or more removed.

- Total beam current limited by the amount of installed RF power, assuming 1 MW available for the beam from each installed klystron and one klystron per 8 cavities. Note that the situation which is considered foresees that all cavities are available but are being operated below their maximum gradient by an amount (corresponding to an accelerating voltage of about 160 MV) which would allow them to be driven rapidly to the nominal gradient in the event of two groups of eight cavities tripping simultaneously. This is necessary to avoid total beam loss each time a group of cavities trips.

- The copper RF system is capable of providing 2.8 MV per installed cavity (340 MV for 120 cavities).

- The HOM power per cavity is calculated by adding the powers associated with each bunch and the fields associated with each counter-rotating beam. Each cavity is equipped with two HOM couplers allowing to carry to room temperature loads a total power of 1600 W.

2.1.2 Integrated Luminosity

The aim for LEP2 integrated luminosity was and still is 500 pb⁻¹ in three years. From the achieved results on LEP1 in 1993 and 1994 this yearly integrated luminosity aim of 170 pb⁻¹/year would require a peak luminosity of around 9 × 10³¹ cm⁻² s⁻¹, for a yearly net physics time of 100 days and an overall efficiency equal to the average of that achieved in 1993 and 1994. It can be seen from the tables given for the different phases that this value is exceeded in all scenarios with 8 bunch operation.

However there are several reasons why extrapolation from LEP1 may be somewhat optimistic and operation at LEP2 energies may be different from that of LEP1.

- The technical efficiency of the machine may be reduced due to the large number of superconducting cavities and the dependence on the simultaneous availability of four cryogenic systems.

- The intensity lifetime at higher energies is known [5] to be less than at 45 GeV.

- It is rather unlikely that LEP2 can be operated at the beam–beam limit throughout the coast as is the case for LEP1. This means that the luminosity may decrease as I², rather than I for LEP1. Against this, operation in the energy range 65 to 70 GeV at the end of 1995 showed that very small emittance ratios of less than 0.5% could be routinely achieved [6].
With these considerations it would be more reasonable to aim for a peak luminosity of around \(1 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}\), to be assured of the target integrated yearly luminosity. It can be seen from the tables given for the different phases described later that this value appears within reach for all scenarios with 8 bunch operation.

### 2.2 Equipment limits

#### 2.2.1 Accelerating Gradient

When considering the likelihood of reaching higher accelerating gradients, we should consider the following.

**Radiation**

The experience gathered so far [7] shows that when increasing the gradient from 6 to 7 MV/m, radiation increases from a few Gy/h to nearly 100 Gy/h. Besides damaging the organic seals of the cryostats (max. dose \(10^5\) to \(10^6\) Gy), it has also been ascertained that radiation enhances electron multipactoring in the main couplers. The 2.5 kV dc bias [8] in the main couplers has proved to suppress multipactoring, but extended operational experience must still be gathered. Should multipactoring occur, the RF power is shut down for equipment safety. High radiation levels might therefore reduce the availability of the sc accelerating system to unacceptable levels.

Further, intense radiation may constitute a source of background for the experiments.

**Field Strength in the Main Couplers**

When going for example from 6 to 7 MV/m, the equivalent coupler power in the fixed main couplers increases to \((7/6)^2\), or 36\%. As a consequence, the main coupler might be driven at a regime where multipactoring may occur. As discussed above, this would entail a reduced system availability.

**Impact of Synchrotron Radiation**

Following the experience from KEK, dedicated collimators have been installed in LEP to protect the sc cavities from the synchrotron radiation (SR) created in the arcs. With the sc cavity modules installed in LEP so far, no difficulties linked to SR have been encountered. However, this has to be confirmed at higher energies.
Experience with other accelerators

Superconducting cavities have been operated in TRISTAN [9] (KEK) and are in operation at HERA [10] (DESY) and CEBAF [11]. All the cavities in these accelerators are Nb-sheet cavities, which are subject to thermal quenches, contrary to the Nb-film cavities of LEP.

At TRISTAN (f = 500 MHz, T = 4.2 K), the average accelerating gradient in 1994 was 3.8 MV/m, the maximum achieved in operation being 4.7 MV/m. Without beam an average of 7 MV/m was measured in 1994. Thermal quenches, multipacting at the input couplers and discharges due to SR stimulated gas desorption are quoted as limiting factors.

At HERA (f = 500 MHz, T = 4.2 K), the sc cavities were tested to 5 MV/m before installation, have run at a maximum of 4 MV/m with beams, and are routinely operated at about 2.6 MV/m. The relatively low maximum gradient in operation is due to a Q degradation because of hydrogen contamination of the Nb sheet. Multipacting in the input couplers (no dc bias) was the major reason of faults in the RF system.

At CEBAF (f = 1.5 GHz, T = 2 K), the nominal value of 5 MV/m has already been exceeded in the first period of operation, an average of 6.2 MV/m has been reached and 7.3 MV/m are expected in the future, though the average value achieved during the tests in the vertical cryostat has been 9.5 MV/m. The main limitation is field emission.

Given the very limited experience with low-frequency SC cavities above 4 MV/m, it is believed that for the LEP2 sc cavities the nominal value of 6 MV/m should be considered as the maximum possible gradient in operation.

2.2.2 Number of SC Cavities

The layout of the straight sections at Points 2 and 6 was originally optimized for the Cu accelerating system (64 Cu cavities at each Point) and subsequently partially modified to accommodate sc cavities as well.

The layout at Points 4 and 8 has been completely redesigned for the LEP2 Programme, to allow the installation of maximum 96 sc cavities at each of these Points.

By removing all the Cu cavities and making the layout of Points 2 and 6 identical to that of Points 4 and 8, a total of 384 sc cavities could be installed in LEP, provided that the 16 separators for the Bunch Trains Scheme are removed from LEP as well (each separator occupies the location of a four-cavity module), precluding operation with more than four bunches per beam.
2.2.3 The Cryogenic Limits

Present Cryoplants

The LEP2 cryoplants are designed to deliver an ultimate equivalent refrigeration power of 18 kW at 4.5 K. In their present configuration, they deliver an equivalent power close to 11.5 kW at 4.5 K, both suppliers being at the lower end of the 5% contractual tolerance admitted for the specified 12 kW.

The cryoplants have to cope with two basic load types, one that is independent of the accelerating field in the cavities and one that is field dependent, proportional to $E_{acc}^2/Q$, where $Q$ is the cavity quality factor. Figure 1 shows the contribution of the various loads to the overall cryogenic budget, for the cases where 64, 72 or 80 sc cavities are installed and the nominal $Q(E)$ acceptance curve shown in Fig. 2 is considered. The constant losses are a function of the length of the transfer lines and of the number of cavities. Their cumulated value is shown in Fig. 1 for 64 cavities. It can be computed that, with the given assumptions, the cryogenic power limit of 11.5 kW at 4.5 K would be reached by operating 64 sc cavities at about 6.9 MV/m, 72 sc cavities at 6.5 MV/m or 80 sc cavities at 6.1 MV/m.

![Cryogenic Load vs Accelerating Gradient](image)

Figure 1: Cryogenic Load (64 to 80 SC Cavities) as Function of the Accelerating Gradient

At the time of writing, no experience is available with the operation of a large number of sc cavity modules and some prudence should be applied when discussing the optimum use of the spare cryogenic power that appears to be available. In fact, it could be used to:
i) cope temporarily with accidental higher losses in some modules (due e.g. to a leak of cryostats' vacua or to a contamination of cavities),

ii) add more modules and operate them at the nominal 6 MV/m,

iii) operate the nominal number of modules at fields higher than 6 MV/m,

iv) cover possible HOM losses reaching the liquid Helium bath.

![Graph](image)

**Figure 2: Quality Factor Q vs Accelerating Gradient**

In order to establish some reference figures and sound limitations, the computations of the overall cryogenic load were made also for the case that the average effective Q(E) values of the modules would be 20% below (see Fig. 2) the Q(E) curve admitted for the acceptance of the modules not yet equipped with couplers.

Under these conditions, the cryogenic power limit of 11.5 kW at 4.5 K would be reached by operating 64 sc cavities at about 6.4 MV/m, 72 sc cavities at 6.0 MV/m or 80 sc cavities at 5.6 MV/m. By comparing the total accelerating voltages achievable with these parameters, it can be seen that an increase from 72 to 80 sc cavities would bring a gain of only 27 MV (instead of nearly 81 MV if they could be operated at the nominal 6 MV/m), making the additional investment useless.

Until experience has been gathered with the operation of large numbers of cavities, possibly by end 1996, we therefore do not recommend to install more than 72 sc cavities at any LEP.
Point, unless the cryopants are appropriately upgraded to keep a reserve for coping with unforeseen difficulties or driving cavities at higher gradients.

**Partial Upgrading of the Cryopants**

Figure 3 shows the estimated cryogenic budgets for the cases where 80, 88 or 96 sc cavities are to be cooled by a cryopant.

![Cryogenic Load (80 to 96 SC Cavities) as Function of the Accelerating Gradient](chart.png)

It can be computed that a partial upgrade to an equivalent cryogenic power of 15 kW would be enough for allowing to drive 96 sc cavities at an accelerating gradient of about 6.5 MV/m, provided that the average Q values follow the nominal acceptance curve. For the pessimistic case where the effective Q(E) values are 20% below the acceptance curve of Fig. 2, a gradient of 6.1 MV/m could still be sustained with 96 sc cavities.

Studies are in progress [12] about the cryopant upgrades necessary for the LHC; as a result of a possible gradual implementation of these upgrades, at least the reliability of the LEP2 cryopants could be increased in the next years by the installation of additional Helium compressors.
2.2.4 Limitations from the Magnet System and its Power Converters

Energy limitations due to the magnets’ design were reviewed [13] at the 1995 Workshop on LEP Performance. For the sake of completeness, they are summarized in the Table 1, with additional information about the capabilities of the magnets’ power converters (the 108°/60° optics is assumed). The nominal power converters’ ratings can be found in Ref. [14]; the figures between brackets are achievable with limited effort [15].

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</table>

* The figures for the MQA quadrupoles are dependent on the final optimization of the optics parameters for the Bunch Trains Scheme, which is not yet complete for LEP2.

3 Increasing the LEP Energy within the Cryogenic Limit

Two new LEP2 phases, named Phase IIIb and IV, respectively, were discussed at the 1995 Chamonix Workshop on LEP Performance; Tables 2a and 2b summarize cavity number and distribution, expected energies and luminosities, respectively.
### Table 2a: Cavity distribution

<table>
<thead>
<tr>
<th>Phase (Cavity type)</th>
<th>Point 2</th>
<th>Pt 4</th>
<th>Pt 6</th>
<th>Pt 8</th>
<th>Totals</th>
<th>Total MV max</th>
</tr>
</thead>
<tbody>
<tr>
<td>IIIb</td>
<td>Cu 26</td>
<td>24</td>
<td>40</td>
<td>56</td>
<td>26</td>
<td>64 56 52 24 216</td>
</tr>
<tr>
<td>IV</td>
<td>26</td>
<td>24</td>
<td>40</td>
<td>72</td>
<td>26</td>
<td>64 72 52 24 248</td>
</tr>
</tbody>
</table>

Use of 16 active spares and replacement of 8 prototypes

Maximum energy upgrade with present cryoplants

### Table 2b: Energy and luminosity (limiting parameter underlined)

<table>
<thead>
<tr>
<th>Phase</th>
<th>MV max (MV)</th>
<th>E max (GeV)</th>
<th>MV oper. (MV)</th>
<th>E oper. (GeV)</th>
<th>U max/beam (mA)</th>
<th>Beam Power (MW)</th>
<th>ξbb</th>
<th>L max (10^31 cm^-2 s^-1)</th>
<th>HOM Power/cavity (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IIIb</td>
<td>2557</td>
<td>94.7</td>
<td>2394</td>
<td>93.1</td>
<td>7.19</td>
<td>30.5</td>
<td>0.0383</td>
<td>11.1</td>
<td>1010</td>
</tr>
<tr>
<td>IV</td>
<td>2884</td>
<td>97.6</td>
<td>2721</td>
<td>96.2</td>
<td>7.13</td>
<td>34.5</td>
<td>0.0344</td>
<td>10.3</td>
<td>995</td>
</tr>
</tbody>
</table>

1) Figures for an accelerating gradient reduced by 0.5 MV/m.

### 3.1 Making spare modules active (Phase IIIb)

Four modules have been ordered as spares; space for their installation can be efficiently made available at Points 2 and 6, the schematic layout is shown in Fig. 4. Phase IIIb requires the procurement of four klystrons, four circulators, waveguides and RF controls for four modules, vacuum and cryogenic equipment for module installation and operation. Additional high pressure storage tanks for He gas are also necessary. The extension of the cryogenic transfer lines was already included in the original LEP2 Programme.
3.2 Adding 32 sc cavities (Phase IV)

At each of Points 4 and 8, 16 cavities can be added, reaching thus the cryogenic limit of 72 sc cavities established under Section 2.2.3. Their installation would take place between the quadrupoles QS9 and QS10 (see the schematic layout of Fig. 4). In addition to the procurement of 32 sc cavities, Phase IV requires the procurement of two new RF units (each consisting of a klystron power converter, HV cabling, a HV filter capacitor, the klystrons protection system, two klystrons, two circulators, waveguides and controls for eight modules), the extension of the cryogenic transfer lines as shown in Fig. 4, and the vacuum and cryogenic equipment necessary for module installation and operation. Additional high pressure storage tanks for He gas are also necessary.

An examination of Table 2b shows that the Phase IV configuration allows to reach 95 GeV per beam with a reasonable confidence.

4 Schedules

4.1 Boundary Conditions

Schedules have to cope with the following:

- tendering procedures requiring the adjudications' approval by the Finance Committee, require a 6 to 9 month's period;

- deliveries must occur at the beginning of the winter shutdowns (SD) at latest;

- sc cavities: provided that basic materials are procured by CERN before the order is placed, bare cavity delivery can start 6 months after contract adjudication. Orders for sc cavities are to be placed so as not to have large time gaps in their manufacture. The average sc cavity production rate achieved so far with three firms is 60 sc cavities/year; module delivery followed with a time lag of 9 months;

- RF power and controls: delivery within two years after contract adjudication;

- cabling of RF controls: for a set of 8 modules, a total of 6 months is necessary (two months of racks precabling before the winter shutdown, 4 months underground installation). Because of CERN staff availability, parallel work is limited at $2 \times 8$ modules;

- cryogenics: the extensions of cryogenic transfer lines require one year after contract adjudication. Transfer lines should be installed and tested one winter shutdown before that for module installation, because of incompatible activities. The delivery of additional high pressure storage tanks for He gas occurs 18 months after contract adjudication.
4.2 Tentative Schedule

Taking into account the above boundaries, a tentative installation schedule has been worked out [16], which includes in a global approach the realisation of Phases III [1], IIIb and IV. A favourable decision for Phases IIIb and IV is foreseen for December 1995 which will allow us to complete the LEP2 upgrade by May 1998.

The optimum installation schedule foresees the following major milestones:
1995–1996 SD: extension of cryogenic transfer lines at Points 2, 4, 6 and 8, cabling of RF controls at Points 4 and 8.
1996–1997 SD: installation at Point 2 of the eight sc cavity modules ordered for Phase III, installation at Points 4 and 8 of the four spare modules and of the four modules driven out of the original layout because of the Bunch Trains separators.
1997–1998 SD: installation at Point 6 of the eight modules to be ordered for Phase IV.

Tables 3a and 3b summarize cavity number and distribution, and expected energies for the years 1996 to 1998. The schematic layouts corresponding to this schedule are shown in Fig. 4.

5 Cost estimates for Phases IIIb and IV

The cost estimates for Phases IIIb and IV are of 7 and 29 MCHF, respectively. These estimates are based on the unit prices paid so far. The high pressure He storage tanks, estimated at 3 MCHF are not included in the above sums; they are foreseen in the LHC budget, as they are needed anyhow for LHC and would constitute a pilot production for the full LHC needs.

6 Acknowledgements

The authors would like to thank G. Bressani for the computations of the average Q(E) curves, H. Gaillard for the schematic layout figures, D. Gusewell and K. Hübner for many helpful comments. Thanks are also due to E. Chiaveri, G. Geschonke and to the colleagues who have contributed with them, for the detailed cost estimates for Phases IIIb and IV.
Table 3a: Cavity Distribution vs Time (Assuming an approval of Phases IIIb and IV by December 1995)

<table>
<thead>
<tr>
<th>Year</th>
<th>Point 2</th>
<th>Pt 4</th>
<th>Pt 6</th>
<th>Pt 8</th>
<th>Totals</th>
<th>Total MV max</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Cavity type)</td>
<td>Cu</td>
<td>Nb &amp; prot.</td>
<td>NbCu</td>
<td>NbCu</td>
<td>Cu</td>
<td>NbCu</td>
</tr>
<tr>
<td>Oct 1996</td>
<td>60</td>
<td>16</td>
<td>16</td>
<td>56</td>
<td>60</td>
<td>32</td>
</tr>
<tr>
<td>May 1997</td>
<td>26</td>
<td>24</td>
<td>40</td>
<td>72</td>
<td>60</td>
<td>32</td>
</tr>
<tr>
<td>May 1998</td>
<td>26</td>
<td>24</td>
<td>40</td>
<td>72</td>
<td>26</td>
<td>64</td>
</tr>
</tbody>
</table>

Phase IV

Table 3b: Energy and Luminosity vs Time (limiting parameter underlined) (Assuming an approval of Phases IIIb and IV by December 1995)

<table>
<thead>
<tr>
<th>Year</th>
<th>MV max (MV)</th>
<th>E max (GeV)</th>
<th>MV oper. (MV)</th>
<th>E oper. (GeV)</th>
<th>I_{max}/beam (mA)</th>
<th>Beam Power (MW)</th>
<th>\xi_{bb}</th>
<th>L_{max} \times 10^{31} cm^{-2} s^{-1}</th>
<th>HOM Power/cavity (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct 96 1)</td>
<td>2110</td>
<td>90.0</td>
<td>1946 1810</td>
<td>88.0</td>
<td>\textbf{6.0}</td>
<td>20.3</td>
<td>0.0379</td>
<td>8.7</td>
<td>704</td>
</tr>
<tr>
<td>May 97 1)</td>
<td>2654</td>
<td>95.6</td>
<td>2490 2300</td>
<td>94.1</td>
<td>\textbf{6.0}</td>
<td>26.6</td>
<td>0.0310</td>
<td>7.6</td>
<td>704</td>
</tr>
<tr>
<td>May 98 1)</td>
<td>2884</td>
<td>97.6</td>
<td>2721 2503</td>
<td>96.2</td>
<td>7.13</td>
<td>34.5</td>
<td>0.0344</td>
<td>10.3</td>
<td>995</td>
</tr>
</tbody>
</table>

1) Figures for an accelerating gradient reduced by 0.5 MV/m.
Figure 4: Schematic layout of the RF sections in the years 1997 and 1998
References


APPENDIX A

A.1 Increasing the LEP Energy beyond the present Cryogenic Limit

In the following, information is given about the LEP2 potential, should it be decided to make a step in energy requiring an upgrade of the cryoplants. Different scenarios can be envisaged, by increasing importance of modification. Essentially two families of options can be considered, the first (X label) conserves the Bunch Trains separators in LEP to maximize luminosity, the second (Y label) does away with Bunch Trains to maximize energy.

A.2 Keeping the Bunch Trains Separators (X Phases)

The phases described in the following are successive increments from Phase IV. Tables A1a and A1b summarize cavity number and distribution, expected energies and luminosities for the Phases X1, X2 and X3 described below.

A.2.1 Addition of a Set of 32 SC Cavities (Phase X1)

We can see from Fig. A1 that at Points 4 and 8, space is still available for the installation of 2 modules between the quadrupoles QS10 and QS11; the corresponding cryoplants must be upgraded.

A.2.2 Replacement of the Remaining Cu Cavities by 32 SC Ones (Phase X2)

Progressing in the stepwise approach to increase the beam energy, one could consider the replacement at Points 2 and 6 of the remaining 52 Cu cavities, which still provide some 150 MV. From the schematic layout given in Fig. A2, it can be seen that each group of Cu cavities can be replaced by a four-cavity module; another set of 32 SC cavities could so be installed, providing a net increase of about 180 MV in accelerating voltage. The cryoplants at Points 2 and 6 will need to be upgraded.
A.2.3 Identical RF Layouts (Phase X3)

The next step would then be to make the layout of Points 2 and 6 identical to that of Points 4 and 8 (see Fig. A3). This will require a complete rearrangement of the quadrupole magnets, the sc cavity modules, the vacuum system, the cryogenic transfer lines, and of the RF power distribution system. This intervention will certainly need a prolonged shutdown; no planning study has been made so far. After rearrangement, it would be possible to install 16 additional sc cavities, with a net gain of about 163 MV.

Table A1a: Cavity distribution

<table>
<thead>
<tr>
<th>Phase (Cavity type)</th>
<th>Point 2</th>
<th>Pt 4</th>
<th>Pt 6</th>
<th>Pt 8</th>
<th>Totals</th>
<th>Total MV max</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>Cu 26</td>
<td>Nb &amp; prot. 24</td>
<td>NbCu 40</td>
<td>NbCu 88</td>
<td>Cu 26</td>
<td>NbCu 64</td>
</tr>
<tr>
<td>X2</td>
<td>0</td>
<td>24</td>
<td>56</td>
<td>88</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>X3</td>
<td>0</td>
<td>24</td>
<td>64</td>
<td>88</td>
<td>0</td>
<td>88</td>
</tr>
</tbody>
</table>

Symmetrical LEP, all even Points identical

Table A1b: Energy and luminosity (limiting parameter underlined)

<table>
<thead>
<tr>
<th>Phase</th>
<th>MV max (MV)</th>
<th>E max (GeV)</th>
<th>MV oper. (MV)</th>
<th>E oper. (GeV)</th>
<th>I_{max}/beam (mA)</th>
<th>Beam Power (MW)</th>
<th>( \xi_{bb} )</th>
<th>L_{max} ( \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1} )</th>
<th>HOM Power/cavity (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1 1)</td>
<td>3211</td>
<td>100.2</td>
<td>3047</td>
<td>2802</td>
<td>98.9</td>
<td>96.9</td>
<td>7.12</td>
<td>38.5</td>
<td>0.0316</td>
</tr>
<tr>
<td>X2 1)</td>
<td>3390</td>
<td>101.6</td>
<td>3227</td>
<td>2955</td>
<td>100.3</td>
<td>98.2</td>
<td>7.34</td>
<td>42.0</td>
<td>0.0313</td>
</tr>
<tr>
<td>X3 1)</td>
<td>3554</td>
<td>102.8</td>
<td>3390</td>
<td>3104</td>
<td>101.6</td>
<td>99.4</td>
<td>7.32</td>
<td>44.0</td>
<td>0.0300</td>
</tr>
</tbody>
</table>

1) Figures for an accelerating gradient reduced by 0.5 MV/m.
LEP 2 LAYOUT PHASE X1

Fig. A1

LEP 2 LAYOUT PHASE X2

Fig. A2

LEP 2 LAYOUT PHASE X3

Fig. A3
A.3 Taking out Bunch Trains separators to make Room for Cavities (Y Phases)

The phases described in the following are additions relative to Phase IV. Tables A2a and A2b summarize cavity number and distribution, expected energies and luminosities for the phases Y1 to Y3 described below.

A.3.1 Addition of a Set of 48 SC Cavities (Points 4 and 8, Phase Y1)

The replacement of the separators with sc cavity modules is a relatively simple operation at Points 4 and 8, as the cryogenic transfer lines were foreseen for feeding those modules and the RF cabling for them was already made before the choice of installing separators. Klystrons and circulators were also already foreseen there. As originally foreseen, waveguides and controls can be installed there during a winter shutdown. Phase Y1 corresponds to Phase X1 plus 16 sc cavities, the corresponding layout is shown in Fig. A4.

Table A2a: Cavity distribution

<table>
<thead>
<tr>
<th>Phase</th>
<th>Point 2</th>
<th>Pt 4</th>
<th>Pt 6</th>
<th>Pt 8</th>
<th>Totals</th>
<th>Total MV max</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Cavity type)</td>
<td>Cu</td>
<td>Nb &amp; prot.</td>
<td>NbCu</td>
<td>NbCu</td>
<td>Cu</td>
<td>NbCu</td>
</tr>
<tr>
<td>Y1</td>
<td>26</td>
<td>24</td>
<td>40</td>
<td>96</td>
<td>26</td>
<td>64</td>
</tr>
<tr>
<td>Y2</td>
<td>0</td>
<td>24</td>
<td>64</td>
<td>96</td>
<td>0</td>
<td>88</td>
</tr>
<tr>
<td>Y3</td>
<td>0</td>
<td>24</td>
<td>72</td>
<td>96</td>
<td>0</td>
<td>96</td>
</tr>
</tbody>
</table>

ZLs removed, filling Points 4 and 8 with SCCs

ZLs removed, filling also Points 2 and 6 with SCCs

All-out Maximum Energy configuration
Table A2b: Energy and luminosity (limiting parameter underlined)

<table>
<thead>
<tr>
<th>Phase</th>
<th>MV max (MV)</th>
<th>E max (GeV)</th>
<th>MV oper. (MV)</th>
<th>E oper. (GeV)</th>
<th>I_max / beam (mA)</th>
<th>Beam Power (MV)</th>
<th>( \xi_{bb} )</th>
<th>I_max ( 10^{31} \text{ cm}^{-2} \text{ s}^{-1} )</th>
<th>HOM Power/cavity (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1 1)</td>
<td>3374</td>
<td>101.4</td>
<td>3211</td>
<td>100.2</td>
<td>4.00</td>
<td>22.8</td>
<td>0.0342</td>
<td>5.9</td>
<td>626</td>
</tr>
<tr>
<td>Y2 1)</td>
<td>3717</td>
<td>104.0</td>
<td>3554</td>
<td>102.8</td>
<td>4.00</td>
<td>25.2</td>
<td>0.0317</td>
<td>5.6</td>
<td>626</td>
</tr>
<tr>
<td>Y3 1)</td>
<td>3880</td>
<td>105.3</td>
<td>3717</td>
<td>104</td>
<td>4.00</td>
<td>26.5</td>
<td>0.0305</td>
<td>5.5</td>
<td>626</td>
</tr>
</tbody>
</table>

1) Figures for an accelerating gradient reduced by 0.5 MV/m.

A.3.2 Addition of a second set of 48 SC cavities (Points 2 and 6, Phase Y2)

At Points 2 and 6, the simple section of the cryogenic transfer line between QS6 and QS7 can be replaced with a new section equipped for feeding a module. The RF power could be provided by the klystrons driving a neighbouring group of modules, considering that with only four bunches only half of the previously provided RF power will still be needed. The 52 Cu cavities would be replaced by 32 sc ones as for Phase X2, the corresponding layout is shown in Fig. A5.

A.3.3 Identical RF Layouts (Phase Y3)

The reach the maximum number of 384 cavities that can be installed in LEP, one has to make all accelerating sections identical, allowing thus to add a final set of 16 sc cavities, the corresponding layout is shown in Fig. A6.
A.4 Schedules and cost estimates for the X and Y Phases

From the considerations given in Chapter 4 of this note, it can be inferred that about 30 to 36 months (a cryoplants upgrade requires two years after contract adjudication), depending on the phase which would be retained, are necessary for the realization of a possible next step in energy. The complete reshuffling of the straight sections at Points 2 and 6 is guessed to add at least 6 months to the above quoted figures. Better estimates would require at least a preliminary planning study. The above quoted 30 to 36 months are to be understood as the time necessary from taking the formal decision to equipment commissioning.

Concerning costs, Fig. A7 shows in graphical form the outcome of crude estimates for the various phases. Although the cost of modules, RF power and controls, cryogenic and vacuum equipment for the modules is known, the cost estimate for other items requires more work. Among the latter, one can quote the cost (estimated here at 3.5 MCHF per plant) for the partial upgrading of the cryoplants, the cost of industrial support and the cost of making the layouts of all even Points identical.

Figure A7: Upgrade Costs vs Operational Energy. Phases IIIb and IV: Final Estimates. Phases X, Y: Preliminary Estimates.


INTERACTION REGIONS

Convener: G. von Holtey


1 Beam Induced Particle Backgrounds

From the two main types of beam induced particle backgrounds to the LEP experiments—off-energy electrons and synchrotron radiation (SR) photons—only the latter is expected to increase with beam energy. The rate of off-energy electrons produced by beam-gas Bremsstrahlung is to first order independent of beam energy. The second source for off-energy particles, scattering of beam particles from thermal photons, is energy dependent but does not add to the off-energy background to the experiments. The number of radiated SR-photons in magnetic quadrupolar fields, as well as the critical energy of the spectra, increases sharply with beam energy and would lead to unacceptably high photon background levels at 90 GeV [1]. To cope with this situation, the background protection system had to be upgraded for LEP2 by adding collimators, enlarged vacuum chambers, photon absorbers and in particular synchrotron radiation masks close to the IPs [2]. All these additional elements will be in place for 1996 [3].

The SR-photon background rate is difficult to calculate as it is strongly dependent on several optics and beam parameters, which are not all known. The estimates given below must therefore be used with great caution. Apart from the beam energy, the most sensitive parameter is the beam size in the horizontal plane and the particle density distribution far into the horizontal tails. The photon rate at the detectors increases typically by a factor of ten when the beam distribution is changed from a gaussian to an exponential density distribution with constant RMS beam size (see Fig. 1). For small emittances and high energies this factor can be as large as 40. It is this effect, the build-up of non-gaussian tails with high currents near the beam-beam limit, that can make the SR-photon background increase faster than linear with beam current.

The exponential increase of the photon rate with increasing beam emittance, mainly due to small angle back scattering, made the introduction of SR-masks for LEP2 necessary, as the nominal horizontal beam emittance for LEP2 is $\varepsilon_x = 50\text{nm}$ (for the 90° optics). The effect of the SR-masks is seen in the simulation results of Fig. 1b, where the sharp exponential rise of the photon rate is much reduced for emittances larger than 35nm. However, in spite of the
Figure 1: Simulated photon background rates as a function of the beam energy (a) and the horizontal beam emittance (b). The results are valid for IP6, equipped with all additional LEP2 background protection elements, including SR-masks.

With constant emittance the SR-photon rate at the detectors is expected to increase by about a factor of ten when doubling the beam energy from 45 to 90 GeV (Fig. 1a). This factor can be compensated for if, as it is now planned, the 108° optics is used at LEP2.

The above arguments hold strictly only for a machine without bunch trains. The vertical separation bumps needed for bunch trains are a source of additional particle backgrounds [4]. However, with reduced beam-beam effects at high energy, smaller separation can be tolerated, thus keeping the separation bump amplitudes well below the threshold value above which the photon background rate has been seen to rise very rapidly. The main effect of bunch trains on the background situation at LEP2 is therefore to increase the sensitivity of the machine to beam instabilities, which lead to blow-up of the beam size, and consequently to increased background rates.

The expected SR-photon background rate, per unit beam current, at the W-pair energy is more than one order of magnitude higher than at 45 GeV with the 90° optics and about the same if the low emittance 108° optics can be used. Therefore the expected particle background is a strong argument in favour of the 108° optics. Running LEP at the energy limit (corresponding to zero ‘missing cavities’) may produce unstable beam conditions with small beam lifetimes and occasional significant beam losses with the consequence of large beam backgrounds in the detectors.
2 Luminosity Lifetimes at LEP2 Energies

Beam lifetimes in LEP1 are well understood, they are limited by beam-beam collisions [5]. For single or separated beams the dominant lifetime limitation is due to scattering on thermal photons and only in second place due to beam-gas scattering.

Losses from scattering on thermal photons will increase with beam energy. The black body photon radiation is effectively boosted twice by the Lorentz factor gamma of order 10⁵ in the Compton scattering process. The mean energy loss induced by this process increases from 1.1% at LEP1 to 2.2% at LEP2, thus resulting in a shorter lifetime [6].

Beam lifetimes reduce significantly when beams are brought into collision. The dominant process for beam lifetimes is Bhabha scattering with very small scattering angle but non-negligible energy loss by radiation of a photon in the initial state. The process of radiative Bhabha scattering as lifetime limitation is often referred to as beam-beam Bremsstrahlung. The full kinematics of the process and the introduction of a cutoff parameter corresponding to the mean half-distance between particles in the bunch is discussed in [7]. The cross section for LEP is approximately independent of energy and about 0.21 barn.

For fixed beam sizes, luminosity increases quadratically with beam currents. Above a certain current per bunch, beam sizes will start to increase by the beam-beam effect. This corresponds to operation at the beam-beam limit, characterized by a constant beam-beam tuneshift and a linear increase of luminosity with current. The relative strength of the beam-beam effect decreases with beam energy, therefore more collisions should be possible at higher energies for the same beam current. This results in lower lifetime from collisions at the IPs at higher energies. The typical contribution to beam lifetimes for LEP1 and LEP2 is given in Table 1.

<table>
<thead>
<tr>
<th>Process</th>
<th>LEP1</th>
<th>LEP2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal photons</td>
<td>88 h</td>
<td>50 h</td>
</tr>
<tr>
<td>Beam gas</td>
<td>200 h</td>
<td>200 h</td>
</tr>
<tr>
<td>B.B.Bremsstrahlung</td>
<td>25 h</td>
<td>12.5 h</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>17 h</td>
<td>9.5 h</td>
</tr>
</tbody>
</table>

Beam lifetime and luminosity lifetime are equal for operation in the beam-beam limit. This was the case for LEP1. At high energies, the beam-beam limit will be at higher bunch currents and the luminosity lifetime is expected to become 1/2 of the current lifetime towards the end of coast.

The optimum time in coast has been studied with a program that simulates beam lifetimes
and the beam-beam effect for various single beam sizes [7]. The result is typically a 6 hour coast for LEP2, compared with 10–12 hours for LEP1. However the maximum is rather flat and the integrated luminosity decreases only slowly if coasts are kept longer.

3 Beam Spot Position Measurements

A knowledge of the transverse position of the LEP luminous region (beam spot) at the interaction points is useful in identifying long lived particles (primarily $b$ quarks) that decay some distance from their production point. At LEP1, the high rate of tracks from $Z^0$ decay allows the beam spot position to be determined with high accuracy ($\sigma_x \sim 20\mu m$, $\sigma_y \sim 10\mu m$) typically every few minutes. Movements of up to 100 $\mu m$ or more are observed during the course of some fills.

At LEP2, the event rate will be much lower, and such an accurate beamspot determination will not be possible using tracks. The track rate will be dominated by two-photon ($\gamma\gamma$) events, which will provide a limited beamspot measurement. However the LEP beam orbit monitor (BOM) system [9], [10] provides an alternative method to monitor short term movements, and this has been extensively tested using LEP1 data. Unfortunately, displacements of the superconducting low-beta quadrupole magnets (QS0s) either side of each interaction point can cause movement of the beam spot not seen by the BOM system, so the QS0 positions must also be monitored to produce an accurate measurement.

3.1 Beam Spot Position Requirements

Tagging of $b$ quarks at LEP2 is primarily required in the search for $H \rightarrow b\bar{b}$. The primary vertex position is constrained by the beam spot, but the size of the luminous region (approximately 150 $\mu m$ in $x$ and 5 $\mu m$ in $y$) must also be taken into account. Thus an accurate beam spot position is much more important in the vertical ($y$) than the horizontal ($x$) direction. Knowledge of the beam spot position improves the performance (efficiency vs. purity) of $b$-tagging algorithms, and translates into an increased Higgs sensitivity for a given integrated luminosity.

The requirements have been studied in detail by the LEP experiments. The ALEPH study [11] simulated the process $e^+e^- \rightarrow H\nu\bar{\nu}$, $H \rightarrow b\bar{b}$ together with appropriate backgrounds. An impact parameter based $b$-tag was used together with appropriate kinematic cuts. The beam spot position resolution was varied between 10 $\mu m$ and 1 cm. A resolution of 100 $\mu m$ in $x$ and $y$ was found to give a 10% reduction in the integrated luminosity needed to discover an 80 GeV Higgs at $\sqrt{s} = 175$ GeV, compared to the situation with no beam spot measurements. Improving the beam spot resolution below 100 $\mu m$ brought little further gain in integrated luminosity.

A similar study performed by L3 [12], found that the absence of beam spot information
would lead to a 20% increase in the integrated luminosity required to discover an 80 GeV Higgs at $\sqrt{s} = 190$ GeV. Full $b$-tagging performance was achieved with a resolution of 150 $\mu$m in $x$ and 50 $\mu$m in $y$. In DELPHI [13], the effect of removing the beam spot information in $Z \rightarrow b\bar{b}$ events was found to reduce the event tagging efficiency by 10%, and no improvement was found for resolutions below 20 $\mu$m.

These studies show that extremely accurate beam spot measurements are not required, and the gain in $b$-tagging power is not very large. However, being conservative, and bearing in mind the likely small number of candidate events, a target of 100 $\mu$m in $x$ and 20 $\mu$m in $y$ has been set and agreed upon by all the experiments.

### 3.2 Measurement from Two-Photon Events

The cross section for $\gamma\gamma$ events at LEP2 is much higher than at LEP1, and is the dominant source of tracks, which can be used for measuring the beam spot position. If the rate is high enough, they may be used to follow movements during a fill, otherwise they will be useful in providing an absolute position measurement on a fill by fill basis.

The tracks from $\gamma\gamma$ events have an angular distribution peaked in the forward direction, and a steeply falling momentum spectrum. Hence the rate seen in detectors depends critically on the acceptance and trigger momentum thresholds. In ALEPH [14], $\gamma\gamma$ events were simulated at $\sqrt{s} = 175$ GeV, and the beam spot determined using all events with at least one track with momentum greater than 0.3 GeV in the vertex detector. Chunks of 3.6 nb$^{-1}$ gave a beam spot position accurate to 42 $\mu$m in $x$ and 32 $\mu$m in $y$, corresponding to a 1% loss in effective luminosity in the Higgs search. Chunks of 60 nb$^{-1}$ gave a measurement accurate to around 10 $\mu$m in $x$ and $y$, corresponding to around 30 minutes running at a luminosity of $3 \times 10^{31}$ cm$^{-2}$s$^{-1}$. The Monte Carlo was cross-checked with $\gamma\gamma$ data taken at $\sqrt{s} = 89.4$ GeV, and found to be in reasonable agreement [15].

In OPAL, simulated $\gamma\gamma$ events tagged in the forward and luminosity calorimeters were studied [16], using a more restrictive event selection requiring at least 2 charged tracks with $p_T > 0.25$ GeV in the central jet chamber acceptance. This yielded a measurement of 15 $\mu$m in $x$ and 10 $\mu$m in $y$ for 1 pb$^{-1}$ (a 9 hour fill at $3 \times 10^{31}$ cm$^{-2}$s$^{-1}$). Using untagged $\gamma\gamma$ events and a $p_T > 1$ GeV requirement on the tracks gave a similar resolution for just 150 nb$^{-1}$ [17]. This is similar to the ALEPH result taking into account the tighter tracking cuts used.

These results show that a reasonable beam spot measurement should be possible from $\gamma\gamma$ events, tracking movements over the time scale of 1 hour to better than the required accuracy. However, the event selection may be vulnerable to background contamination, and the beam moves around on shorter time scales, so it is worth using the BOMs to improve the accuracy of the beam spot measurements.
3.3 Measurement using BOM pickups and QS0 monitoring

Several sets of wide band BOM pickups, measuring the beam $x$ and $y$ positions, are installed on each side of each interaction point (see Fig. 2). Knowledge of the LEP beam optics and corrector magnet strengths allows the beam positions to be extrapolated to the interaction points themselves [18]. These extrapolations were performed for a few selected fills in 1994 data, and the results compared with beam spots measured from tracks by the four LEP experiments [13], [16], [19], [20]. While it was found that the BOM system had the potential to perform a measurement to the required accuracy, there were severe systematic problems associated with movements of the QS0 magnets over time. If the QS0s on either side of the IP move symmetrically, a 'π-bump' is created in the machine which changes the IP beam spot position, but is not seen by the BOMs which are outside the QS0s.

Two major improvements were implemented in 1995. Firstly, the BOM extrapolation was implemented online, with the results being stored in a database accessible to the experiments [23]. This calculation was active for the majority of the 1995 $Z^0$ scan period. Four independent measurements of each coordinate were calculated, using BOM data for the incoming and outgoing beam on each side of the experiment. Secondly, the QS0 magnets at IP2 and IP8 were instrumented with hydrostatic level measuring equipment, and ALEPH and DELPHI also installed their own monitoring of the QS0 positions relative to the experiments.

The hydrostatic level systems consist of capacitative liquid level measuring sensors mounted on the tunnel floor, the QS0 support girders and the magnets themselves [24]. These are currently installed in IP2 and IP8, and will be installed in the other two interaction regions during the 1995-1996 shutdown. The DELPHI luminosity calorimeter (STIC) is attached to their QS0 support structures and movements relative to the main detector are measured by several pin potentiometer probes [22]. This system was installed in 1994, and has an intrinsic resolution of a few microns. For the 1995 run, a similar system was installed in ALEPH, monitoring movements of the QS0 supports relative to reference surfaces attached to their main detector [21].

Movements of the order of 50 $\mu$m in the QS0 vertical position have been seen in all these...
systems, and correlated with the currents in the QS0 magnets. Slow drifts are observed when the currents are changed, consistent with thermal effects due to the currents in the QS0s and bus bars. Sudden jumps and movements in the horizontal direction are also observed, and are not yet fully understood.

Consideration of the LEP beam optics shows that the beam spot position at the IP $y_{IP}$ can be estimated from the BOM extrapolation from the left side of the IP, $y_{BOM,L}$, and from the position of the left QS0, $y_{Q,L}$:

$$y_{IP} = y_{BOM,L} + \alpha y_{Q,L}$$

and similarly on the right side. The best estimate is obtained by averaging the BOM ($y_{BOM}$) and QS0 ($y_{Q}$) data on both sides:

$$y_{IP} = y_{BOM} + \alpha y_{Q}$$

Thus asymmetric movement of the QS0s has no effect at the IP, whilst symmetric motion causes the largest effect. The parameter $\alpha$ is expected to have the value 1.4 [18].

The correlations between beam spot measurements from tracks, those from the improved BOM extrapolations and QS0 movements were studied by ALEPH [21] and DELPHI [22]. In the ALEPH study, the data were normalised by assuming a constant offset between the vertex and BOM measurements throughout the 1 month data taking period. The RMS of the difference between vertex and BOM measurements was then taken as a measure of the BOM measurement resolution, after subtracting off the component from the vertex beam spot resolution. Using BOMs alone gave a resolution of 42 $\mu$m in $x$ and 14 $\mu$m in $y$, a clear improvement on the 74 $\mu$m and 27 $\mu$m obtained assuming a constant beam spot position instead of the BOM measurements. Correcting the BOM measurements using the measured QS0 displacements improves the resolution in $y$. The QS0 movements on each side contribute to the improvement in a consistent manner and the best resolution of 7 $\mu$m is obtained using both sides with $\alpha = 1.06$. The potentiometers actually measure movements at the end of a cantilevered structure which may magnify the QS0 movements, so this measured value is not incompatible with the expected 1.4. A typical example fill is shown in Fig. 3, clearly showing the agreement between both sides and the improvement that can be gained in the vertical plane using the QS0 information. During the first few hours of this fill, the QS0s on either side of ALEPH moved by 40 and 70 $\mu$m respectively. Movements in the horizontal plane are less well understood, and no significant improvement was obtained when applying a QS0 correction.

The DELPHI study considered movements of the beam spot during fills, the data being normalised to the difference in average BOM and vertex detector beam spots in each fill. Using the STIC probes to correct the BOM measurements, an improvement was seen in the $y$ direction, the BOM resolution being reduced from 8 to 6 $\mu$m with $\alpha = 1.4$. A similar improvement was also found using the hydrostatic level measuring system to monitor the QS0 positions. The beam spot movement within a fill has an RMS of 9 $\mu$m in $y$, so the improvement using BOMs and QS0 monitoring appears modest. However, vertex movements not seen by the BOMs of up to 80 $\mu$m have been seen in some fills, and these are nicely tracked by the QS0 monitoring. A nice example using the STIC probe monitoring is shown in Fig. 4. The beam spot movement
in $x$ within fills has an RMS of 25 $\mu$m, and this was not improved by using BOM and QS0 measurements.

In OPAL, measurements of the QS0 position were not available from either hydrostatic or potentiometer systems. An analysis was therefore performed using BOM information alone, giving a beam spot resolution of 19 $\mu$m in $x$ and 6 $\mu$m in $y$ [17], again normalising the average positions from BOMs and vertex detector measurements in each fill. This resolution is similar to that obtained by DELPHI after correcting for QS0 motion, and suggests that the latter effect is much smaller in OPAL. Examining the residual differences between vertex and BOM measurements shows some evidence for systematic effects attributable to QS0 movements, but only on a scale of 10–15 $\mu$m, smaller than those seen in ALEPH and DELPHI. The reason for this smaller effect is not yet understood.

The OPAL analysis also considered the problem of establishing the absolute calibration between the BOM coordinate system and the detector. This was done using the fill averaged beam spot measured from tracks. By accurately determining the latter quantity using all the tracks from $Z^0$ events, the fill to fill variation in BOM to vertex detector offsets was studied, and is shown for some of the BOMs in Fig. 5. Variations of up to 100 $\mu$m in individual BOM offsets were observed. Since no QS0 corrections were applied, most of these variations may be explained by QS0 current cycling between fills. An appropriate fraction of the $Z^0$ event tracks was then used to simulate the lower track rate from $\gamma\gamma$ events expected at LEP2. By combining compatible offsets from adjacent runs, a resolution of 23 $\mu$m in $x$ and 9 $\mu$m in $y$ was achieved, not much worse than that obtained by removing fill to fill offsets. Hence the effects from BOM
offset changes (probably due to QS0 movement) can be partially compensated for.

The resolutions in $x$ and $y$ measured by the experiments in 1995 LEP1 data under various conditions are summarised in Table 2, showing that the target resolution of 100$\mu$m in $x$ and 20$\mu$m in $y$ can be met. Large movements are seen in the beam spot positions during a data taking period, though less so during the course of individual fills. These movements can be tracked using the BOM system corrected by QS0 position monitoring, which follow both the in-fill and fill to fill variations well. It should be stressed that large deviations due to QS0 movement are sometimes seen, and that these are expected to be more important at the higher magnet currents required for LEP2. Hence it will be important to monitor the QS0 positions continuously during machine operation.
Figure 5: BOM measurement offsets with respect to vertex position measurements vs fill in 1995 OPAL data.

Table 2: Summary of beamspot requirements and resolutions measured by the experiments (see text)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Measurement</th>
<th>Offset per fill</th>
<th>Resolution x (μm)</th>
<th>y (μm)</th>
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<tr>
<td>All</td>
<td>Requirement</td>
<td>–</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>ALEPH</td>
<td>Constant assumption</td>
<td>no</td>
<td>74</td>
<td>27</td>
</tr>
<tr>
<td>ALEPH</td>
<td>BOM</td>
<td>no</td>
<td>42</td>
<td>14</td>
</tr>
<tr>
<td>ALEPH</td>
<td>BOM+QS0</td>
<td>no</td>
<td>41</td>
<td>7</td>
</tr>
<tr>
<td>DELPHI</td>
<td>Constant assumption</td>
<td>yes</td>
<td>25</td>
<td>9</td>
</tr>
<tr>
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<td>BOM</td>
<td>yes</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>DELPHI</td>
<td>BOM+QS0</td>
<td>yes</td>
<td>27</td>
<td>6</td>
</tr>
<tr>
<td>OPAL</td>
<td>Constant assumption</td>
<td>no</td>
<td>79</td>
<td>32</td>
</tr>
<tr>
<td>OPAL</td>
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<td>26</td>
<td>10</td>
</tr>
<tr>
<td>OPAL</td>
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<td>yes</td>
<td>19</td>
<td>6</td>
</tr>
<tr>
<td>OPAL</td>
<td>BOM</td>
<td>sim γγ</td>
<td>23</td>
<td>9</td>
</tr>
</tbody>
</table>
3.4 Conclusion

The beam spot position requirements for LEP2 and methods of meeting them have been extensively studied. A knowledge of the beam spot position helps in $b$-tagging, which has been evaluated in the context of the $H \rightarrow b \bar{b}$ search. A position resolution of 100 $\mu$m in $x$ and 20 $\mu$m in $y$ is adequate, and higher precisions do not bring significant gains in physics performance.

Measuring the beam spot position from both $\gamma \gamma$ events and the BOM system has been studied. The former may provide enough resolution by itself to monitor slow variations, and the latter should do considerably better than the requirements when corrected for the movements of the QS0 magnets. Hence the physics requirements should be met. It is also worth emphasising that the BOM measurements are very useful for understanding the behaviour of the LEP machine itself.

4 Acknowledgements

Contributions to the discussions during the ten meetings of the Working Group from E. Brambilla, M. Hublin, M. Koratzinos, E. Martin, S. Myers, M. Placidi, D. Plane, F. Richard, T. Taylor and D. Treille are gratefully acknowledged.
References


[23] M. Lamont and J. Wenninger, Status of ‘TURBOIP’ program to measure positions and angles of beams at IPs, transparencies shown at the September 14th, 1995 meeting of the Interaction Region subgroup of WG4.

BEAM ENERGY MEASUREMENTS AT LEP2

Convener: M. Placidi


1 Introduction

The requirements on the accuracy in the measurement of \( M_W \) [1] based on the hypothesis of collecting an Integrated Luminosity of about 500 pb\(^{-1} \) (\( \approx 8000 W^+ W^- \) pairs) in each experiment imply statistical and systematic precisions [2]:

\[
(\Delta M_W) = \left( 25 \text{ MeV} / c^2 \right)_{\text{stat}} \oplus \left( 25 \text{ MeV} / c^2 \right)_{\text{syst}}
\]

(1)

To cope with the requirements (1) the systematic error induced by the knowledge of the absolute average beam energy should be of the order of

\[
\delta E_{\text{beam}} \leq 15 \text{ MeV} \quad \text{or} \quad \frac{\delta E_{\text{beam}}}{E_{\text{beam}}} \leq 1.7 \times 10^{-4}
\]

(2)

The accuracy in the beam energy determination highly depends on the availability of transverse polarization \( (P_L) \) to apply the Resonant Depolarization (RD) method for energy measurement[3]. Application of the RD method provides an instantaneous precision of \( \leq \pm 1 \text{ MeV} \) with \( \geq 5\% \) transverse polarization.

Prospects for transverse polarization at energies higher than LEP1 are discussed in Section 2 where a Maximum Polarizable Energy \( E_{\text{Pol}}^{\max} \geq 60 \text{ GeV} \) is anticipated.

The problematics of beam energy measurement in conjunction with the experimentation at LEP2 has been examined in a twofold approach.

Two methods whose accuracy requires cross-calibration with precise energy measurements with the RD method are discussed in Sections 3 and 6 together with considerations on attainable accuracies, feasibility and technical implications for the machine.

Alternative scenarios have been considered and conclusions are reported in Sections 4 and 5.

2 Transverse Polarization beyond \( Z^0 \)

Prospects for transverse polarization beyond \( Z^0 \) energy are based on assessment of the strength of depolarization effects at higher energies. The asymptotic polarization level is

\[
P_\infty = \frac{8}{5\sqrt{3}} \frac{1}{1 + \tau_p/\tau_d}
\]

(3)
where $1/\tau_p$ and $1/\tau_d$ are the rates of self-polarization and depolarization of the beam. Although the rate of radiative self-polarization increases with $E^6$, the depolarization effects due to resonances driven by machine imperfections also increase rapidly with energy. In the simplest model of resonant depolarization the net result is that the ratio $\tau_p/\tau_d \approx a \cdot E^2$. The parameter $a$ can be chosen to reproduce the polarization level attainable at the energy of the $Z$. It can be controlled by the application of techniques of orbit control and spin harmonic compensation.

It is known however that depolarizing resonances are further enhanced by the increasing energy spread in the beam. Despite some lack of confidence in the calculations of these effects in the past, experimental tests with wigglers at LEP1 have given results consistent with theory [4].

Recently the theory has been used [5] to compute optimum combinations of beam energy and RF voltage (controlling the synchrotron tune). By using more RF voltage than normally necessary at a given energy, these could provide conditions of maximum polarization. First indications are that polarization levels of 10% should not be out of reach at energies of 60 GeV or beyond. Calculations along these lines and simulation work will continue.

3 Energy calibration using present techniques

The default scenario for beam energy calibration at LEP2 is as follows:

- Precise calibration will be performed with resonant depolarization at energies where it has proven to work reliably. A first choice is an energy corresponding to LEP1 (around 45 GeV). A second one, $E_{\text{beam}} = E_{\text{max}}^{\text{Pol}}$, will be the highest where polarization in excess of at least 5% can be reliably obtained. This second energy, expected to lie in the range 60-75 GeV, will be left as parameter in the following discussion.

- Extrapolation to the higher energy will be made in two ways:
  i) using the flux-loop which provides a calibration of the non-linearity of the ensemble of the LEP dipoles and
  ii) using a set of NMR probes which will provide a precise ($10^{-6}$) measurement of the local magnetic field in a sample of LEP magnets.

- On-line monitoring of the energy will be provided by
  i) the field information from the in-situ NMR probes;
  ii) recording of the horizontal orbit and
  iii) recording of various parameters of the RF system and its asymmetries.

This is a straight extrapolation of the techniques used so far for the successful scan of the $Z^0$ line shape.
3.1 Present experience on the LEP Energy

The LEP beam energy is affected by a multiplicity of effects, which induce variability in time and deviation of the center-of-mass energy from $E_{CM} = E_{e^+} + E_{e^-}$. Some of these effects are well known. They include

- variations of the LEP circumference due to ground motion. This category includes earth tides, lake level, and other ground swelling. These effects can be corrected using measured orbits and the known momentum compaction factor, down to a precision of a 1–2 MeV. It should be born in mind that they vary with the horizontal tune so that if it were to be changed from the present 90 degrees phase advance per cell, a new measurement of the compaction factor should be foreseen. This is not expected to cause a problem or a substantial error. Also, this means that regular calibrations at a suitable energy should be performed to avoid systematic drifts in the orbit offsets.

- Variations due to characteristics of the LEP dipoles. It is known that the magnetic field in the LEP dipoles increases slowly during a fill. This effect will be less severe at LEP2 due to shorter fills. Furthermore, the rise can be monitored by placing NMR probes inside some of the LEP magnets. In 1995 two such probes were installed, and were most useful in understanding the LEP energy, which they seem to track with a precision of better than 3 MeV. It is foreseen that a larger number of such probes will be present in the future, allowing better sampling, and hopefully smaller errors.

- Temperature variations are no longer a problem since temperature regulation of the cooling water has been implemented.

- Variation in the RF voltage distribution around the ring. A good understanding of these effects requires logging the status of all RF stations, (voltage and power). In addition measurement of the phase of the cavities should be performed a few times a year. Control of these effects can be obtained by measurements of the longitudinal position of $e^+e^-$ interactions vertices in the LEP detectors, as well as with measurements of the difference in horizontal orbit between electrons and positrons in the arcs (saw-toothing monitor). The size of effects is expected to grow at LEP2 according to the large RF power in operation. The precision of the monitoring should remain similar.

- Systematic effects on the CM Energy at each Interaction Point caused by residual vertical dispersion and small collision offsets [6] when operating LEP in bunch-train configuration have been simulated [7] and found to be in good agreement with the measurements. The strategy proposed to minimize the CM energy shifts [8] was applied with appropriate use of the Vernier Scan technique [9] during the 1995 Physics Run [10].

Once these effects are corrected for, an r.m.s. scatter of $<10^{-4}$ on the beam energy is to be expected. To know the average of the two reference polarization-calibrated energies with a precision of the order of 2 MeV it can be envisaged to perform, once every two weeks or so, a calibration at 45 GeV and at $E_{Pol}^{\text{max}}$ during a dedicated ramp.
3.2 The Flux-Loop

It is expected that the main uncertainty will then come from having to extrapolate from these energies up to the operating higher energy. The error comes primarily from the uncertainty in the knowledge of the linearity of the magnets. In absence of a direct measurement of the beam energy, this can be obtained using the flux-loop technique.

At the construction of LEP, an electrical loop was inserted around the pole tips of every magnet in the ring. The voltage induced in the loop when the whole dipole system undergoes a magnetic cycle of a given excursion allows a measurement of the variation of the magnetic flux up to beam energies of 100 GeV. The response curve of the magnets can be obtained to an absolute precision only at the level of \(~5\text{ MeV}\). However the linearity can be rather well determined.

Better measurements might be obtained by comparing with the NMR probes in the reference magnet and in the dipoles in the tunnel. The process of cycling the magnets is repeated several times, providing an estimate of the short term reproducibility which is found to be at the level of a few \(10^{-5}\). The long term reproducibility was found to be more at the level of \(2 \times 10^{-4}\), however the reproducibility of the derivative from 40 to 65 GeV is much better.

In the LEP2 era, the flux-loop calibration will be extended up to 100 GeV. Regular flux-loop measurements should be performed. The accuracy on the extrapolation has been estimated as follows:

- First, the uncertainty stemming from the resonant depolarization results was estimated using linear extrapolation error formulae. This would be the case if one could trust with nearly infinite precision the non-linearity curves from the flux-loops. The resulting curves are shown in Fig. 1 for two values of the energy to which one extrapolates, as a function of the maximum polarizable energy \(E_{\text{max}}^{\text{Pol}}\).

- Second, the uncertainty coming from uncertainties in the non-linearity coefficient. Here a worst case estimate can be obtained by assuming that the non-linearity coefficient is totally unknown, and only inferred from the resonant depolarization energies themselves. This is obviously too pessimistic in view of the above discussion. The resulting dependence on the maximum polarizable energy is shown in the curves labeled ‘quadratic’.

- Finally, the flux-loop information is added assuming a conservative \(2 \times 10^{-4}\) precision. The results are shown in Fig. 2 and summarized in Table 1.
Figure 1: Accuracy $\Delta E_{\text{beam}}$ from linear and quadratic extrapolations from an intermediate Energy point to $E_{\text{beam}}=80.5$ GeV and 95 GeV.

Figure 2: Accuracy $\Delta E_{\text{beam}}$ for quadratic extrapolations from an intermediate Energy point to $E_{\text{beam}}=80.5$ GeV and 95 GeV, including flux-loop information assumed a $10^{-4}$ conservative precision.
Table 1: Uncertainties (MeV) from Flux-Loop Extrapolation to two operating Energies and for two values of $E_{Pol_{\text{max}}}$ in the conservative assumption of using ‘quadratic’ extrapolation and a precision of $10^{-4}$ on the flux-loop measurements.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{beam}}=80.5$ GeV</td>
<td>$E_{\text{beam}}=95.0$ GeV</td>
</tr>
<tr>
<td>$E_{\text{Pol}_{\text{max}}}$</td>
<td>60 GeV 65 GeV 60 GeV 65 GeV</td>
</tr>
<tr>
<td>$FLE$ extrapolation</td>
<td>$\leq \pm 12 \leq \pm 7 \leq \pm 18 \leq \pm 14$</td>
</tr>
<tr>
<td>$RD$ calibration</td>
<td>$\pm 2 \pm 2$</td>
</tr>
<tr>
<td>Central Orbit Uncertainty</td>
<td>$\pm 3 \pm 4$</td>
</tr>
<tr>
<td>Dipole temperature correction</td>
<td>$\pm 2 \pm 2$</td>
</tr>
<tr>
<td>RF and collision offsets</td>
<td>$\pm 2 \pm 3$</td>
</tr>
<tr>
<td>Total</td>
<td>$\leq \pm 13 \leq \pm 8.5 \leq \pm 19 \leq \pm 15$</td>
</tr>
</tbody>
</table>

3.3 The NMR probes

A possibility to estimate the operating energy is to make use of the NMR probes in a sample of LEP dipoles in the tunnel. The limitation is not in the accuracy in the measurement of the magnetic field, good to $\sim 10^{-6}$, but in the fact that the NMR’s sample a very small fraction of the guiding field. However, it is expected that the behavior of the LEP dipoles is sufficiently homogeneous to warrant that the 8 NMR probes foreseen to be installed will provide adequate sampling. The calibration of the absolute scale will be provided by comparing with the $RD$ measurements. How well the NMR’s sample the non-linear behavior of the LEP dipoles will be determined using the two available polarizable energies. At LEP1 in 1995, with two NMR’s, the energies were tracked with a precision of $\sim 3$ MeV. This figure should be even better at LEP2 with a larger number of probes. The associated error will be measured directly using data, but it seems likely it will be less than 10 MeV.

3.3.1 Summary

It is seen that polarization at 60 GeV would be sufficient to derive the beam energy at the $W^{\pm}$ pair threshold (80.5 GeV per beam) with sufficient accuracy. Polarization at 65 GeV should be enough to safely infer the beam energy with a precision of better than 15 MeV over the whole energy range of LEP2. This conservative estimate should be reduced to 10 MeV or so when several flux-loops will have been performed and compared with the results of the in-situ NMR probes.
4 Heavy Ion Acceleration

The possibility of injecting and accelerating He or Pb Ions, less relativistic than protons, (Heavy Ion Acceleration, HIA) has been proposed [11] inspired from an original experiment [12] where the speed $c\beta_p$ of protons circulating on the same orbit of electrons was measured to determine the common value of the momentum $p_e \equiv p_p = m_p c \beta_p \gamma_p$.

The attainable accuracy at the LEP2 energies is indeed very good for Helium and Lead Ions:

$$\frac{\Delta p}{p} = \gamma^2 \frac{\Delta \beta}{\beta} \sim 1.5 \times 10^{-5}$$

(4)

The implementation of the method would require important investments to equip LEP with a variable frequency RF system to track the speed of ions during the acceleration phase, and the experiment could probably be performed only once a year.

5 Visible Synchrotron Radiation

The use of Synchrotron Radiation in the visible range has been investigated [13] as a possible method to achieve high precision absolute beam energy measurements with accuracies in principle higher than magnetic spectrometers. The method is based on the comparison of the photon flux transmitted by high precision slits intercepting two well defined fractions of the S.R. cone of emission. The use of visible light is suggested to allow the adoption of standard optical components as an advantage w.r.t the X-ray techniques to amplify the narrow emission angles of radiation to a magnification M=100. The proposed technique has not yet been investigated experimentally as it makes use of selecting slits requiring high precision tolerances both in the machining process and in the positioning in the ring. Besides these implications the accuracy of the method relies on the precise knowledge of the magnetic field at the emission point selected by the optical setup. Calculations performed at the CEBAF energies (0.5 to 4 GeV) anticipate a $10^{-4}$ absolute accuracy. Application of the method mainly addresses to Experimental halls or to energy-feedback applications for Linear Accelerators.

6 Möller scattering

Another possibility to directly measure the beam energy at LEP2 is offered by the Möller scattering technique (MS) [14]. The absolute energy scale would be established by cross-calibration with RD at any convenient low energy (say $E_{beam} = 50$ GeV), and the MS technique used at higher energies. By use of a symmetric detector it is, in principle, also possible to measure the energy of the $e^+$ beam, using Bhabha scattering. However space limitation in LEP may preclude this possibility.

The Möller scattering provides a continuous measurement of the beam energy at the
gas Target location. The knowledge of the beam energy at the four experiments must be derived from this measurement with reasonable monitoring of the time evolution, during the measuring time, of the horizontal orbit as well as of the RF parameters.

6.1 Basic Method

The energy $E_{\text{beam}}$ is determined by measuring $\theta$ and $\kappa$ in the two body kinematic relation for elastic scattering on a target electron at rest:

$$
E_{\text{beam}} = \frac{8m_e}{\theta^2} \frac{1}{1 - \kappa^2} - m_e
$$

(5)

where $\theta$ is the ‘opening angle’ between the two scattered electrons (Fig.3):

$$
\theta = \tan \theta_1 + \tan \theta_2
$$

(6)

The scattered electrons are detected in a near-symmetric configuration (i.e. $E_{1,2} \simeq E_{\text{beam}}/2$ and $\theta_1 \simeq \theta_2$) and the opening angle $\theta$ has a minimum value for $\theta_1 = \theta_2$:

$$
\theta_{\text{min}} = \sqrt{\frac{8}{1 + E_{\text{beam}}/m_e}}
$$

(7)

The value of $\theta_{\text{min}}$ ranges from 9.53 to 6.73 mrad at $E_{\text{beam}} =$45 GeV to 90 GeV respectively. The parameter $\kappa = \cos \theta^*$ can be determined from the quantities in the laboratory system:

$$
\kappa = \frac{\tan \theta_1 - \tan \theta_2}{\tan \theta_1 + \tan \theta_2} \quad \text{or} \quad \kappa = \frac{E_{\text{beam}} + m_e}{E_{\text{beam}} - m_e} \frac{E_1 - E_2}{E_1 + E_2}
$$

(8)

The proposed detector is shown schematically in Fig. 3. It consists of the following elements:

- A hydrogen gas jet target (GJT) similar to that used in the UA6 experiment [15]. The bound electrons of the hydrogen atoms serve as target.

- A silicon micro strip detector (SMD) with full azimuthal acceptance and a polar angle acceptance from 2 to 10 mrad measuring $(r, \phi)$ co-ordinates in the transverse plane.

- A high resolution electromagnetic calorimeter (ECAL) with a similar angular coverage.

- Small silicon micro strip detector planes in vacuum, close to the GJT, to detect recoil protons from elastic e-p scattering.
Two different determinations of $E_{\text{beam}}$, with very different systematic errors are possible depending on how the parameter $\kappa$ in eqn. (5) is measured:

$$E_{\text{beam}} = E_A : \quad \kappa_A \equiv (E_1 - E_2)/(E_1 + E_2)$$

$$E_{\text{beam}} = E_B : \quad \kappa_B \equiv (\tan \theta_1 - \tan \theta_2)/\theta$$

$E_A$ uses both the SMD and the ECAL but is independent of the beam position, whereas $E_B$ uses only the SMD and requires the beam position to be known.

To study the required precision for the quantities $\theta$, $\kappa_A$, $\kappa_B$ we derive from eqn. (5):

$$\frac{\delta E_{\text{beam}}}{E_{\text{beam}}} = -2 \frac{\delta \theta}{\theta} + 2 \kappa_{\text{obs}} \cdot \delta \kappa - (\delta \kappa)^2$$

here $\kappa_{\text{obs}} = \kappa + \delta \kappa$ is the measured $\kappa$ ($\kappa_A$ or $\kappa_B$). Eqn. (11) indicates that 1 MeV precision on $E_{\text{beam}}$ at 90 GeV requires knowing $\theta$ to $5 \times 10^{-8}$. The corresponding tolerances on $L$ and $l$ (30 m and 20 cm for $\theta = \theta_{\text{min}}$) are 150 $\mu$m and 1 $\mu$m respectively. This level of survey accuracy is challenging but not unrealistic. Compared to this absolute accuracy, the requirement on the resolution of $l$ is much looser. The beam energy has an intrinsic spread of $\sigma_E/E = 1.5 \times 10^{-3}$, (hence $\sigma_l/l = 0.75 \times 10^{-3}$). There is a similar size effect from Fermi motion of the target electrons. Thus the position resolution of SMD can be at a level of 50 $\mu$m, easily achieved by the standard silicon micro strip detector technology.

To estimate the statistical error on the $E_{\text{beam}}$ measurement we consider the centre of mass cross section for Møller ($e^- e^-$) scattering, integrated over the angular range $-c_0 < \cos \theta^* < c_0$:

$$\sigma = \frac{4\pi \alpha^2}{s} \frac{c_0(9 - c_0^2)}{(1 - c_0^2)}$$

The 2 mrad minimum scattering angle accepted by the detector corresponds to the center of mass acceptance of $c_0 = 0.479$ at 90 GeV. The accepted Møller cross sections is 15.5 $\mu$b. With
a luminosity on the target electrons of $4 \times 10^{31}$ cm$^{-2}$ sec$^{-1}$ [14] the Møller event rate is

$$r_M = 620 \text{ Hz} \ (2.2 \times 10^6 \text{ events/hour}).$$

(13)

The statistical accuracy estimated from the Monte Carlo simulation described below is typically 2 MeV for a sample of $10^6$ events at 90 GeV, which can be collected in about 30 min. The statistical error after a measuring time $t_m$ is then:

$$\Delta E_{0}^{\text{stat}} \simeq \frac{1.4}{\sqrt{t_m}} \text{ MeV hr}^{1/2} \quad (\simeq 2 \text{ MeV in 30 min data taking}).$$

(14)

In the case of Bhabha ($e^+e^-$) the scattering cross section is about 1/4 of that for Møller scattering and the statistical error two times larger. The statistical error is thus sufficiently small compared to the goal of 1–2 MeV overall accuracy.

### 6.2 Simulation of Systematic Effects

In order to assess the systematic effects, a Monte Carlo study was made with realistic assumptions. Radiative corrections, Fermi motion and ECAL resolution are considered together with beam energy spread, size and angular divergence [16]. The effect of each individual contribution was studied with the following parameters for our model:

- $L$ : 30 m
- SMD acceptance : 2.00 – 6.00 mrad
- ECAL acceptance : 1.67 – 6.33 mrad
- ECAL resolution : $\sigma_E/E = 3.37/E(\text{GeV})^{1/4}\%$
- Beam size : $\sigma_x = 1.54 \quad \sigma_y = 0.54 \quad (\text{mm})$
- Divergence : $\sigma_{x'} = 28.5 \quad \sigma_{y'} = 5.0 \quad \mu\text{rad}$
- Energy spread : $\sigma_E = 125 \text{ MeV} \quad (E_0 = 90 \text{ GeV})$

All of these effects were incorporated in a Monte Carlo event generator. The results presented below used a modified version of the Bhabha event generator BHAGENE3 [17]. The radiative corrections were also checked using both the Møller scattering generator BMOLLR [18] and a simple code based on a factorised soft photon approach [14].

### 6.2.1 Binding Effects

The kinematical effects due to atomic binding of the target electrons [19] are similar to those produced by initial state radiation in that the $e^-e^-$ CM energy and the Lorentz boost between
the e⁻e⁻ CM and the Lab systems are modified. If a bound electron has Fermi momentum $p_f$ then the CM energy $\sqrt{s}$ is modified according to the relation:

$$s' = s\left[1 - \frac{p_f}{m_e} \cos \chi \right]$$  \hspace{1cm} (15)

where $\chi$ is the angle between $\vec{p}_f$ and the beam direction. The probability distribution for $p_f$ found by taking the Fourier transform of the ground state hydrogen wave function is:

$$\frac{dP}{dp_f} = 4\pi \frac{p_f^2}{[1 + (p_f/p_f^0)^2]^4}$$  \hspace{1cm} (16)

where $p_f^0 = 3.73$ keV.

The modification of the centre of mass energy $s$ to $s'$ modifies the opening angle $\theta$ according to $\delta \theta/\theta = (p_f/m_e) \cdot \cos \chi$. For a typical value of $p_f = p_f^0$ and $\cos \chi = \pm 1$, we find $\delta E_{\text{beam}}/E_{\text{beam}} = \pm 3.6 \times 10^{-3}$. This causes a significant increase in the width of the reconstructed $E_{\text{beam}}$ distribution (larger than the LEP energy spread). However, this shift averages to zero when integrated over $\cos \chi$ and the net shift (due to higher order terms) is small.

### 6.2.2 Radiative Corrections

The main effect of photon radiation is due to initial state radiation. The CM energy and the Lorentz boost between the e⁻e⁻ CM system and the LAB are modified. This effect modifies $\theta$ rather than $\kappa$. The consequence is a loss of statistics from the narrow central peak of $E_{\text{beam}}$ distribution due to highly radiative events rather than its significant broadening.

### 6.2.3 Beam Size and Divergence

In the $E_B$ method the parameter $\kappa$ is defined by (6) and (7). The main sources for $\delta \kappa$ are the uncertainties in the beam position and the finite transverse beam size. In the worst case, when the scattering plane is horizontal, the scatter of collision point from the center has the r.m.s of 1.54 mm. This corresponds to the shift in $\kappa$ of 0.015 r.m.s. From eqn. (11), the shift in the measured $E_0$ is estimated to be 20 MeV at $E_0 = 90$ GeV.

In order to make a correction for this shift to an accuracy of 1 MeV, the beam size should be known to a level of 5% relative precision. Possible ways to achieve this precision are discussed in Section 6.3.

### 6.2.4 ECAL Resolution and Gain Stability

In the $E_A$ method, The ECAL resolution $\sigma_E$ propagates to the resolution of $\kappa_A$ (when $\kappa \simeq 0$) as $\sigma_\kappa/\kappa = (1/\sqrt{2}) \cdot \sigma_E/E$. Similarly the relative gain calibration error between different calorimeter modules leads to a shift in $\kappa$. As indicated in eqn. (11) a high resolution calorimeter is important.
to reduce the contribution from $\overline{\delta \kappa}^2$. A resolution of $\approx 1\%$ may be achieved at $E_e = 45$ GeV using existing techniques (crystal or lead glass calorimeter). Using such a calorimeter the shift due to the quadratic term in $\delta \kappa$ is about 4.5 MeV at $E_0 = 90$ GeV. To make a correction for this shift with an accuracy of 1 MeV, the ECAL resolution should be known to a relative accuracy of 10–20%. The resolution can be continuously monitored using electrons from $e$-$p$ elastic scattering (see Section 6.3 below). Gain calibration can be maintained by using a suitable gain monitoring system. A precision of 0.1% level for short/medium term has been already achieved in existing large scale calorimeters [20]. The Gain of the calorimeter can also be checked using the electrons from $e$-$p$ elastic scattering. Within the acceptance of ECAL, the energy of the electron is almost the full beam energy and varies only by 0.1% over the range of accepted scattering angles. Thus the ECAL gain may be controlled to $\approx 0.1\%$.

### 6.2.5 Simulation Results

The reconstructed $E_{\text{beam}}$ with method B with all effects considered is shown in Fig. 4. The systematic bias due to individual contributions as well as the overall systematic shift for both methods A and B are summarized in Table 2. For method A the most important systematic shifts are due to ECAL resolution and Fermi motion, whereas for method B the finite beam size effect dominates.

![Figure 4: The reconstructed $E_{\text{beam}}$ distribution.](image-url)
Table 2: Systematic shifts (MeV) of the energy measurement for $E_0=50$ and 90 GeV.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$E_0 = 50$ GeV</th>
<th></th>
<th>$E_0 = 90$ GeV</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_A$</td>
<td>$E_B$</td>
<td>$E_A$</td>
<td>$E_B$</td>
</tr>
<tr>
<td>Radiative effects</td>
<td>0.1 ± 0.2</td>
<td>0.2 ± 0.2</td>
<td>-0.4 ± 0.7</td>
<td>-0.4 ± 0.7</td>
</tr>
<tr>
<td>Fermi motion</td>
<td>-1.1 ± 0.7</td>
<td>-1.1 ± 0.7</td>
<td>-2.0 ± 1.3</td>
<td>-2.0 ± 1.3</td>
</tr>
<tr>
<td>Beam size</td>
<td>0.2 ± 0.1</td>
<td>-2.0 ± 0.2</td>
<td>0.4 ± 0.4</td>
<td>-8.1 ± 0.4</td>
</tr>
<tr>
<td>Beam divergence</td>
<td>0.1 ± 0.1</td>
<td>-0.7 ± 0.2</td>
<td>0.8 ± 0.4</td>
<td>-2.1 ± 0.4</td>
</tr>
<tr>
<td>ECAL resolution</td>
<td>-2.9 ± 0.2</td>
<td>0.2 ± 0.2</td>
<td>-7.2 ± 0.4</td>
<td>0.7 ± 0.4</td>
</tr>
<tr>
<td>All effects</td>
<td>-8.9 ± 0.8</td>
<td>-6.3 ± 0.8</td>
<td>-9.4 ± 1.5</td>
<td>-16.7 ± 1.5</td>
</tr>
</tbody>
</table>

6.3 Elastic e-p Scattering to monitor the Target-Detector distance

As mentioned in Section 6.1, a 1 MeV uncertainty on $E_{\text{beam}}$ corresponds to a relative uncertainty in $\theta$ of 5 parts in $10^6$. This implies, from (4), that the distance $L$ from the target to the detector plane must be controlled to the same accuracy, i.e., to $\pm 150 \mu$m for $L = 30$ m. The position and size (a few mm) of the gas jet are not stable. It is proposed to monitor continuously the mean value of $L$, $\langle L \rangle$, by detecting and analyzing elastic e-p scattering events. The electron and proton scattering angles are related by:

$$\tan \theta_p = 2/[\theta_e(1 + E_{\text{beam}}/m_p)].$$

(17)

For the SMD acceptance region: $2 < \theta_e < 7$ mrad and $50 < E_{\text{beam}} < 90$ GeV, $\theta_p$ is limited to the range: $71^\circ < \theta_p < 87^\circ$. The recoil proton is detected, in coincidence with the scattered electron, in the Recoil Proton Tracker (RPT) a small silicon strip detector in vacuum, built into the support of the GJT. A similar arrangement was used in the UA6 experiment [15]. If $r$ is the radial distance of the e-p collision point from the RPT and, $z$ and $z_D$ are the coordinates, parallel to the beam direction, of the collision point and the proton hit in the RPT respectively, then the $z$ coordinate of the collision point is determined to be:

$$z = z_D - \frac{r \theta_e}{2}(1 + \frac{E_{\text{beam}}}{m_p}).$$

(18)

The resolution in $z$ is about 500 $\mu$m, mainly due to finite beam size. The accuracy with which the mean value of $z$ for an ensemble of events can be determined is then completely dominated by the statistical error $\sigma_{z_{\text{tar}}}^2/\sqrt{N}$ where $N$ is the number of recorded events and $\sigma_{z_{\text{tar}}}$ is the length of the luminous region of the target, typically $\simeq 5$ mm.

The cross section for ep scattering is 46 $\mu$b for electrons detected between 2 and 7 mrad. If the azimuthal acceptance is 16%, the accepted cross section is 7.4 $\mu$b, or 74 events/second with a luminosity of $10^{31}$ cm$^{-2}$ sec$^{-1}$. In a 1 hour of data collection $2.7 \times 10^8$ events will be
recorded and the mean longitudinal target position determined with a statistical accuracy of 10 μm. This is more than an order of magnitude better than the tolerance of ±150μm in L required for 1 MeV precision on $E_{\text{beam}}$.

6.4 Monitoring the transverse beam profile using Møller events

We discuss two methods to monitor the transverse beam size and its position using the Møller events. Since the majority of the electron pairs are coplanar, the vertex position of an event lies on a straight line between the positions of the electrons in the SMD plane.

The beam position and profile can be measured assuming that the intersection of the line between the two hit positions of the electrons with the nominal Y (X)-axis is, to a good approximation, the Y (X) coordinate of the vertex position if the line is nearly parallel to the X (Y)-axis.

The second method uses a maximum likelihood fit. The probability $P$ to find two electrons on SMD at $(x_1, y_1)$ and $(x_2, y_2)$ is calculated event by event for a certain model of the beam profile function $B(x, y; x_0, y_0, \sigma_x, \sigma_y)$, where $(x_0, y_0)$ and $(\sigma_x, \sigma_y)$ are the position and spread of the beam respectively. For a given ensemble of Møller events, the likelihood $\ln L = \sum \ln P$ is maximized to obtain the beam transverse parameters.

The two methods have been tested by Monte Carlo simulation. 5000 detected Møller events determine the horizontal beam size to a relative precision of ~5% and the mean beam position to ~100 μm. This number of events can be collected in about 10 seconds (eqn. 13) hence the beam geometry can be monitored several times per minute.

6.4.1 Summary

Two complementary methods, with different dominant systematic errors, can measure the LEP beam energy using Møller scattering. The calibration of all relevant parameters (calorimeter resolution and gain, beam size and position, target detector distance) can be determined either directly from the Møller scattering data itself, or from concurrently detected e-p elastic scattering events. A ~2 MeV statistical error is obtainable in 30 minutes data taking time at 90 GeV. Detailed simulations anticipate a ~2 MeV intrinsic systematic error when cross calibrating with the resonant depolarization. Systematic errors from the extrapolation to the IP's of $E_{\text{beam}}$ measured at the Gas Target location remain to be investigated.
7 Conclusions

The problem of the measurement of the beam energy in LEP2 has been addressed and a selection of possible alternatives to a direct application of the Resonant Depolarization Method has been considered, namely extrapolation of Resonant Depolarization measurements based on magnetic field information from flux-loop and NMR probes, Heavy Ion Acceleration, Visible Synchrotron Radiation and Möller Scattering technique.

Use of the flux-loop up to a magnetic field corresponding to a 100 GeV beam energy would provide a calibration of the non-linearity of each octant of the ring magnetic structure.

A reasonable set of NMR probes (one per octant) will provide a very precise \(10^{-6}\) and continuous measurement of the local magnetic field, **on-line with the operation of the machine**. Polarization at 60 GeV is shown to be sufficient to derive the beam energy at the \(W^\pm\) pair threshold within the required accuracy (eqn. 2). Polarization at 65 GeV would provide a 15 MeV (or better) precision over the whole LEP2 energy range.

Future experience with regular Flux-loop measurements and their comparison with in-situ NMR probes should reduce to about 10 MeV the above conservative estimates. Both methods make use of existing technologies and little additional equipment (NMR) is needed.

The Heavy Ion Acceleration method would provide a very high precision, but its implementation would also require important modifications in the Machine at the RF level.

The use of Visible Synchrotron Radiation seems more adapted to beam energies lower than those available at LEP2 and to layouts, like those encountered in the Experimental Halls, offering easier ways for light extraction.

The Möller Scattering method allows for continuous **on-line energy measurements** over time intervals of about 30 min with an intrinsic precision of about 2 MeV. Its implementation requires important implications for the Machine. A reconfiguration of the magnetic structure in the LSS of interest would be needed to provide adequate spacing between target and detector. The method also involves the construction, installation and operation of a Gas Target whose technical realization is estimated to be of the order of 18 months [21]. Further investigation is needed to evaluate the systematic errors from extrapolating the local beam energy information to the four IP's, as it will be affected by the changes in the machine conditions during the measurement. The order of magnitude of the extrapolation errors should be similar to that from the average Resonant Depolarization method.

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References

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1 Introduction

1.1 Goals of W-pair production

The measurements performed at LEP1 have provided us with an extremely accurate knowledge of the parameters of the Z gauge boson: its mass, partial widths, and total width. Except perhaps for the b¯b and c¯c decay widths all data are in perfect agreement with the Standard-Model (SM) predictions [2, 3]. There even is first evidence that the contributions of gauge-boson loops to the gauge-boson self-energies are indeed required [4]. Thus an indirect confirmation of the existence of the triple gauge-boson couplings (TGC's) has been obtained.

The future measurements of W-pair production at LEP2 will add two important pieces of information to our knowledge of the SM. One is a determination of the W mass, which at present is only directly measured at hadron colliders. The envisaged precision of 40–50 MeV gives a significant improvement on the present Tevatron measurements [5]. Since the W mass is one of the key parameters of the electroweak theory, such an improved accuracy makes the tests on the SM more stringent.

The second piece of information that W-pair production can provide is the structure of the triple gauge-boson couplings. These couplings now play a role in the tree-level cross-section, in contrast to LEP1 physics where they only enter through loop corrections. So the Yang-Mills character of the TGC's can be established by studying W-pair differential cross-sections at LEP2 energies. Since these couplings are at the heart of the non-abelian gauge theories, this information is essential for a direct confirmation of the SM.

1.1.1 Measurement of the mass of the W boson

With the precise measurement of the mass of the W boson at LEP2 the situation for the electroweak input parameters changes with respect to LEP1. The common practice at LEP1 [6] is to use for \( M_W \) a value derived from the Fermi constant \( G_\mu \), which is accurately known from muon decay. The relation to obtain \( M_W \) follows from the SM prediction for muon decay

\[
G_\mu = \frac{\alpha \pi}{\sqrt{2} M_W^2 (1 - M_W^2 / M_Z^2)} \left( \frac{1}{1 - \Delta r} \right),
\]

where \( \Delta r = 0 \) at tree level and where \( \Delta r \) is \( m_t \)- and \( M_H \)-dependent when loop corrections are included. Thus, \( M_W \) in LEP1 calculations is \( m_t \)- and \( M_H \)-dependent through the above procedure. At LEP2, where one wants to measure \( M_W \) and, hence, wants to treat \( M_W \) as a fit parameter, the above relation now primarily acts as a test of the SM. The above relation predicts for any chosen \( M_H \) and measured \( M_W \) a value for \( m_t \) that can be used as input for LEP2 loop calculations and can also be compared with the directly observed top-quark mass from the Tevatron [7].
As to the actual procedure to measure $M_W$ from W-pair production, two methods are advocated [8]. A third one, involving the measurement of the endpoint energy of a charged lepton originating from a decaying W boson, was suggested a few years ago but turns out to be less promising than other methods.

One procedure requires a measurement of the total W-pair cross-section close to threshold, where the size of $\sigma_{\text{tot}}$ is most sensitive to the W mass. The energy proposed is 161 GeV. For this method the theory should obviously predict $\sigma_{\text{tot}}$ with a sufficient accuracy, to wit $\sim 2\%$.\(^a\) Therefore the radiative corrections (RC’s) to the total cross-section should be under control.

The other method looks at the decay products of the W bosons, in particular at the four- and two-jet production. From the measured momenta of the decay products one tries to reconstruct the W mass. In this reconstruction a good knowledge of the W-pair centre-of-mass energy is essential. Since there will be an energy loss due to initial-state radiation (ISR) of primarily photons, the W-pair energy will be different from the laboratory energy of the incoming electrons and positrons. So, for this method the ISR should be well under control, i.e. the theoretical error on the average energy loss, $\langle E_\gamma \rangle = \frac{\int dE_\gamma (d\sigma/dE_\gamma) E_\gamma}{\sigma_{\text{tot}}}$, should be less than 30 MeV, which translates into a theoretical error of less than 15 MeV on the reconstructed W mass. This again is an aspect of radiative corrections.

1.1.2 Test of non-abelian couplings

Within the SM the triple gauge-boson couplings have the Yang-Mills (YM) form. Amongst others, this specific form for the TGC’s leads to a proper high-energy behaviour of the W-pair cross-section and is a requirement for having a renormalizable theory. Couplings different from the YM form, called anomalous or non-standard couplings, will in general lead to a high-energy behaviour of cross-sections increasing with energy and thereby violating unitarity. At LEP2 the energy is too low to see such effects and in order to establish the presence of anomalous couplings one therefore has to study in detail the angular distributions of the W bosons and their decay products. In particular, the angular distribution in the W-production angle $\theta$ is sensitive to non-standard couplings. Again, the knowledge of RC’s to the tree level SM predictions is required, since they affect the Born-level angular distributions [9].

As elsewhere in this volume a detailed report on non-standard TGC’s is given [10], only a few comments will be made here.

When one considers the most general coupling between three gauge bosons allowed for by Lorentz invariance, assuming the gauge bosons to be coupled to conserved currents, one ends up with seven possible couplings for the ZWW and $\gamma$WW interaction. Of these seven there are three which are CP violating and one which is CP conserving but C and P violating [11].

\(^a\)Throughout this report the required theoretical accuracy is taken to be half the expected statistical error.
In practice it will be impossible to set limits on all these couplings. Therefore usually some assumptions are made to reduce the number of 14 couplings [11, 12]. For instance, one may restrict oneself to CP-conserving non-standard couplings so that 8 couplings are left. Of these the electromagnetic ones can be reduced further by omitting the C and P violating one and requiring the strength of the electromagnetic coupling to be determined by the charge. Two possible anomalous electromagnetic couplings remain and four ZWW anomalous couplings. Even with this reduced number it will be impossible to set experimental limits on all of them simultaneously.

However, there are theoretical arguments that such a purely phenomenological approach is also not required. First of all one might use symmetry arguments, motivated by specific models for the non-standard physics, to find relations between the anomalous couplings [12]. Alternatively, when one considers the electroweak theory as an effective theory originating from a field theory that manifests itself at higher energies, then also some small anomalous couplings may be present at lower energies. In such a $SU(2) \times U(1)$ gauge-invariant framework the non-standard physics, situated at an energy scale $\Lambda$, decouples at low energies and the anomalous TGC's are suppressed by factors $(E/\Lambda)^{d-4}$, according to the dimension $(d)$ of the corresponding operators. This naturally introduces a hierarchy amongst the anomalous TGC's based on the dimension of the corresponding operators [13].

From the perspective of this report the origin of the non-standard couplings is not so important, but the fact that they often modify angular distributions is relevant. Also SM effects – like RC's, the finite decay width of the W bosons, and background contributions – provide deviations from the tree-level distributions [9, 10].

1.2 How to obtain accurate predictions?

In order to extract the wanted information from W-pair production it is clear from the remarks above that RC's are needed for total and differential cross-sections and, moreover, for the determination of the energy loss. Anticipating $\sim 10^4$ W-pair events, the theoretical accuracy that should be targeted for the SM predictions is $\sim 0.5\%$, although specific final states, distributions, or observables in fact often do not require such a precision (like, e.g., $\sigma_{\text{tot}}$ at 161 GeV or the energy loss).

Ideally one would like to have the full RC's to the final state of four fermions, which originate from the two decaying vector bosons. In practice such a very involved calculation does not exist and is hopefully not required in its complete form for the present accuracy. For the discussion of the LEP2 situation and strategy it is useful to distinguish three levels of sophistication in the description of W-pair production.

The first level is to consider on-shell W-pair production, $e^+e^- \rightarrow W^+W^-$, which at tree level is described by three diagrams: neutrino exchange in the $t$ channel, and $\gamma$ and $Z$ exchange in the $s$ channel. Here the complete $O(\alpha)$ RC's are known, comprising the virtual one-loop
corrections and the real-photon bremsstrahlung [14, 15]. When one wants to divide the $O(\alpha)$ corrections into different parts the situation differs from LEP1 [9].

As to the bremsstrahlung, a gauge-invariant separation into initial-state radiation, final-state radiation (FSR), and its interference is not possible like, e.g., in $\mu$-pair production at LEP1. The reason is that the photon should couple to all charged particles in a line of the Feynman diagram. The $t$-channel diagram then makes a separation into ISR and FSR impossible. However, the leading logarithmic (LL) part of ISR is in itself gauge-invariant. This can be combined with the LL QED virtual corrections so that a LL description with structure functions for ISR can be given [16].

A separation of the virtual corrections into a photonic and weak part is also not possible since charged vector bosons are already present at Born level, necessitating an interplay between $\gamma$- and $Z$-exchange diagrams in order to preserve $SU(2)$ gauge invariance.

Once one has a description of on-shell $W$-pair production one can attach to it the on-shell $W$ decay. Again, the RC's to this decay are known [17]–[20].

The next level of sophistication is to consider off-shell production of $W$ pairs, which then decay into four fermions [21]. The $W$ propagators with energy-dependent widths can be taken into account. Although this description of four-fermion production through virtual $W$ bosons is a natural extension of the on-shell evaluation, it is not a gauge-invariant treatment. In fact there are more diagrams needed to calculate such a four-fermion process. As to RC's, the ISR and FSR can be implemented in LL approximation, but the full set of virtual corrections have not yet been calculated.

The final level for the study of $W$-pair production would be a full $O(\alpha)$-corrected evaluation of all possible four-fermion final states. At tree level there now exist evaluations where, besides the three off-shell $W$-pair diagrams, all other diagrams for a specific four-fermion final state have been included [22]. On top of that ISR can be taken into account. Again, one has to be aware of possible gauge-invariance problems. In particular, the introduction of energy-dependent widths in the $W$-propagators will destroy electromagnetic gauge invariance and may introduce dramatically wrong cross-sections in certain regions of phase space.

As long as a full $O(\alpha)$-corrected evaluation of four-fermion production is not available, certain approximative schemes, like for instance the 'pole scheme' [23]–[25], may be useful. This goes beyond the treatments where only ISR through structure functions is taken into account. Actual numerical results from a complete 'pole-scheme' evaluation are not yet available.

2 On-Shell $W$-Pair Production and $W$ Decay

Although the actual process that will be probed at LEP2 is $e^+e^- \rightarrow 4f(\gamma,g)$, we first focus on the production and subsequent decay of on-shell $W$ bosons, being basic building blocks in some of the schemes for handling off-shell $W$ bosons. In contrast to off-shell $W$-pair production the
on-shell processes are not plagued by the problem of gauge invariance for unstable particles and a complete set of $\mathcal{O}(\alpha)$ radiative corrections is available. Consequently, they are well suited for studying the typical sizes of various radiative corrections. Moreover, many of the important features of the production and decay of off-shell W bosons are already contained in the on-shell limit. We indicate explicitly where the width of the W bosons radically changes the on-shell predictions.

2.1 Notation and conventions

We use the Björken–Drell metric $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ and fix the totally antisymmetric tensor by $\epsilon^{0123} = +1$. The matrix $\gamma_5$ is defined as $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and the helicity projectors $\omega_\pm$, which are used to project on right- and left-handed massless fermions, as

$$\omega_\pm = \frac{1}{2}(1 \pm \gamma_5).$$

First we set the conventions for the process

$$\text{e}^+(p_+, \kappa_+) + \text{e}^-(p_-, \kappa_-) \rightarrow \text{W}^+(k_+, \lambda_+) + \text{W}^-(k_-, \lambda_-),$$

where the arguments indicate the momenta and helicities of the incoming fermions and outgoing bosons ($\kappa_i = \pm \frac{1}{2}$, $\lambda_i = 1, 0, -1$). Note that we sometimes use the shorthand version $\kappa_i = \pm$ in certain sub- and superscripts. In the centre-of-mass (CM) system of the $\text{e}^+\text{e}^-$ pair, which we will refer to as the laboratory (LAB) system in the following, the momenta read

$$p_\pm^\mu = E(1, 0, 0, \mp 1), \quad k_\pm^\mu = E(1, \mp \beta \sin \theta, 0, \mp \beta \cos \theta),$$

with $E$ denoting the beam energy, $\theta$ the scattering angle between the $\text{e}^+$ and the $\text{W}^+$, and $\beta = 1 - M_W^2/E^2$ the velocity of the W bosons. The Mandelstam variables used in the following are given by

$$s = (p_+ + p_-)^2 = (k_+ + k_-)^2 = 4E^2,$$
$$t = (p_+ - k_+)^2 = (p_- - k_-)^2 = -E^2(1 + \beta^2 - 2\beta \cos \theta),$$
$$u = (p_+ - k_-)^2 = (p_- - k_+)^2 = -E^2(1 + \beta^2 + 2\beta \cos \theta).$$

In order to define helicity amplitudes we need to introduce the corresponding polarization vectors for the $\text{W}^+$ and $\text{W}^-$ boson

$$\varepsilon_\pm^\mu(k_\pm, +1) = \frac{1}{\sqrt{2}}(0, \mp \cos \theta, -i, \pm \sin \theta),$$

$^b$Note that the helicity of the massive W bosons is not Lorentz-invariant. We define it in the LAB system.
\[ \varepsilon_{\pm}^{\mu}(k_{\pm}, -1) = \frac{1}{\sqrt{2}} (0, \mp \cos \theta, \mp i, \pm \sin \theta), \]
\[ \varepsilon_{\pm}^{\mu}(k_{\pm}, 0) = \frac{E}{M_W}(\beta, \mp \sin \theta, 0, \pm \cos \theta). \] (6)

Because we are neglecting the electron mass, the helicity of the positron is opposite to the helicity of the electron
\[ \kappa_\pm = -\kappa_+ = \kappa. \] (7)

Henceforth we refer to the helicity amplitudes for \( W \)-pair production as \( \mathcal{M}(\kappa, \lambda_+, \lambda_-, s, t) \). CP invariance implies the relation:
\[ \mathcal{M}(\kappa, \lambda_+, \lambda_-, s, t) = \mathcal{M}(\kappa, -\lambda_-, -\lambda_+, s, t). \] (8)

This holds in the SM if we neglect the CP-violating phase in the quark-mixing matrix. Even in the presence of this CP-violating phase, the CP breaking occurs at \( \mathcal{O}(\alpha^3) \) in SM \( W \)-pair production and is additionally suppressed by the smallness of the mixing angles between the quarks. Consequently in the SM one effectively has only 12 independent helicity-matrix elements instead of 36. In the presence of substantial (non-standard) CP violation this number should be increased to 18. This allows for a decomposition of the matrix elements in terms of an explicit set of 12 (18) independent basic matrix elements multiplied by purely kinematical invariant functions (coefficients) [9, 14, 15].

From the helicity amplitudes the differential cross-sections for explicit \( W \)-boson polarization and various degrees of initial-state polarization can be constructed. For example the differential cross-section for unpolarized electrons, positrons, and \( W \) bosons is given by
\[ \frac{d\sigma}{d\Omega} = \frac{\beta}{64\pi^2 s_{\kappa, \lambda_+, \lambda_-}} \frac{1}{4} |\mathcal{M}(\kappa, \lambda_+, \lambda_-, s, t)|^2. \] (9)

For the numerical evaluations we have to fix the input parameters. We use the following default set [2, 26]:
\[ \alpha \equiv \alpha(0) = 1/137.0359895, \quad G_\mu = 1.16639 \times 10^{-8} \text{ GeV}^{-2}, \]
\[ M_Z = 91.1884 \text{ GeV}, \quad M_H = 300 \text{ GeV}, \]
\[ m_e = 0.51099906 \text{ MeV}, \quad m_\mu = 105.658389 \text{ MeV}, \quad m_\tau = 1.7771 \text{ GeV}, \]
\[ m_u = 47.0 \text{ MeV}, \quad m_c = 1.55 \text{ GeV}, \]
\[ m_d = 47.0 \text{ MeV}, \quad m_s = 150 \text{ MeV}, \quad m_b = 4.7 \text{ GeV}. \]

The masses of the light quarks are adjusted in such a way that the experimentally measured hadronic vacuum polarization [27] is reproduced. In the actual calculations either these light quark masses are used or the dispersion-integral result for the hadronic vacuum polarization
Table 1: Calculated $m_t$ for $\alpha_s(M_Z^2) = 0.123$ and different Higgs- and W-boson masses, using the state-of-the-art calculation described in section 2.3.3. The theoretical error in $m_t$ is roughly 1–2 GeV.

<table>
<thead>
<tr>
<th>$M_H$ [GeV]</th>
<th>60</th>
<th>300</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$M_W$ [GeV]</td>
<td>$m_t$ [GeV]</td>
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<td></td>
</tr>
<tr>
<td>80.10</td>
<td>119.1</td>
<td>137.3</td>
<td>154.2</td>
</tr>
<tr>
<td>80.18</td>
<td>133.9</td>
<td>151.6</td>
<td>168.0</td>
</tr>
<tr>
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<td>181.2</td>
</tr>
<tr>
<td>80.34</td>
<td>161.7</td>
<td>178.3</td>
<td>193.9</td>
</tr>
<tr>
<td>80.42</td>
<td>174.4</td>
<td>190.7</td>
<td>206.1</td>
</tr>
</tbody>
</table>

Of [27]. The strong coupling constant is calculated according to the parametrization of [28], using $\alpha_s(M_Z^2) = 0.123$ as input. The W-pair threshold region is very sensitive to $M_W$ and consequently, if $M_W$ were to be calculated from the other parameters using the muon decay width including radiative corrections (like at LEP1), to the masses of the top quark and Higgs boson. This is of course not very natural, since we want to use $M_W$ as a model-independent fit parameter. In view of this we use $\alpha, G_\mu, M_Z$ as input\(^a\), treat $M_W$ as free fit parameter, and calculate $m_t$ from muon decay\(^d\). This calculated value of $m_t$ can then be confronted with the direct bounds from Fermilab (weighted average $m_t = 180 \pm 12$ GeV [7]) and the indirect ones from the precision measurements at LEP1/SLC ($m_t = 180_{-8}^{+17}$ GeV [2]). In this scheme $m_t$ hence plays the role played by $M_W$ at LEP1. The above set of parameters and the default W mass $M_W=80.26$ GeV [2, 5] yield

\[
m_t = 165.26 \text{ GeV}.
\]

At present the error on this value as a result of the 160 MeV error on $M_W$ is roughly 27 GeV. The precise $M_W$ measurement at LEP2 will lead to a reduction of this error by a factor of four. Of course the so-obtained top-quark mass will become $\alpha_s$- and $M_H$-dependent through the RC's associated with the muon decay. We come back to that point in section 2.3.3 and only mention here that the $M_H$ dependence of $m_t$ amounts to roughly 33 GeV in the range $60 \text{ GeV} < M_H < 1 \text{ TeV}$, which clearly exceeds the above-mentioned expected error from $M_W$ (see table 1).

Finally, the sine and cosine of the weak mixing angle are defined by

\[
c_W = \cos \theta_W = \frac{M_W}{M_Z}, \quad s_W = \sin \theta_W = \sqrt{1 - c_W^2}. \tag{10}
\]

\(^a\)We do not eliminate $M_Z$ or $G_\mu$, as $M_Z$ is needed in the RC’s and $G_\mu$ reduces the size of the RC’s to the production and decay of the W bosons.

\(^d\)It should be noted that the RC’s associated with the muon decay make that this parameter set is not overcomplete.
After inserting the explicit form of the Z/fermion couplings that is suppressed by a factor $m_e/M_W$ and thus completely negligible. The $t$-channel diagram involving the $\nu_e$ exchange only contributes for left-handed electrons. The $s$-channel diagrams, containing the non-abelian triple gauge-boson couplings, contribute for both helicities of the electron. The corresponding matrix element reads

$$M_{\text{Born}}(\kappa, \lambda_+ , \lambda_-, s, t) = \frac{e^2}{2s_W^2} \left( \frac{1}{t} M_1^{\kappa} \delta_{\kappa-} + e^2 \frac{1}{s} \frac{c_w}{s_W} g_{eeZ} \frac{1}{s - M_Z^2} 2 (M_3^{\kappa} - M_2^{\kappa}) \right), \quad (11)$$

with $\delta_{\kappa-} = 1$ for left-handed electrons and $\delta_{\kappa-} = 0$ for right-handed electrons, and

$$M_1^{\kappa} = v(p_+) \bar{\sigma}_+ (k_+, \lambda_+) (\bar{k}_+ - \bar{\sigma}_+) \bar{\sigma}_-(k_-, \lambda_-) \omega_\kappa u(p_-),$$

$$M_2^{\kappa} = v(p_+) \frac{\bar{k}_+ - \bar{k}_-}{2} [\epsilon_+(k_+, \lambda_+) \cdot \epsilon_-(k_-, \lambda_-)] \omega_\kappa u(p_-),$$

$$M_3^{\kappa} = v(p_+) (\bar{\sigma}_+(k_+, \lambda_+) [\epsilon_-(k_-, \lambda_-) \cdot k_+] - \bar{\sigma}_-(k_-, \lambda_-) [\epsilon_+(k_+, \lambda_+) \cdot k_+]) \omega_\kappa u(p_-). \quad (12)$$

After inserting the explicit form of the Z-boson–fermion couplings

$$g_{eeZ} = \frac{s_W}{c_W} - \frac{1}{2s_W c_W}, \quad (13)$$

we can organize the lowest-order amplitude into two gauge-invariant subsets:

$$M_{\text{Born}}(\kappa, \lambda_+ , \lambda_-, s, t) = \frac{e^2}{2s_W^2} M_I(\kappa, \lambda_+ , \lambda_-, s, t) \delta_{\kappa-} + e^2 M_Q(\kappa, \lambda_+ , \lambda_-, s, t), \quad (14)$$

where

$$M_I(\kappa, \lambda_+ , \lambda_-, s, t) = \frac{1}{t} M_1^{\kappa} + \frac{1}{s - M_Z^2} 2 (M_3^{\kappa} - M_2^{\kappa}),$$

$$M_Q(\kappa, \lambda_+ , \lambda_-, s, t) = \frac{1}{s} \frac{c_w}{s_W} \frac{1}{s - M_Z^2} 2 (M_3^{\kappa} - M_2^{\kappa}). \quad (15)$$

The gauge invariance of the two contributions $M_I$ and $M_Q$ can be simply inferred from the fact that they are accompanied by different coupling constants, one of which involving the
electromagnetic coupling constant $e$, the other the charged-current coupling constant $e/\sqrt{2}s_W$. Whereas $M_I$ corresponds to the pure $SU(2)$ contribution, the parity-conserving contribution $M_Q$ is a result of the symmetry-breaking mechanism.

The lowest-order cross-section determines the essential features of $W$-pair production. The threshold behaviour is important for the determination of the $W$ mass from the measurement of the cross-section in a single energy point very close to threshold [8], i.e. at $\sqrt{s} = 161$ GeV. For small $\beta$ the matrix elements behave as

$$M_2^\alpha, M_3^\alpha \propto \beta, \quad M_1^\alpha \propto 1$$

for fixed scattering angles. Consequently, the $s$-channel matrix elements vanish at threshold and the $t$-channel graph dominates in the threshold region. For $\beta \ll 1$ the differential cross-section for unpolarized beams and $W$ bosons is given by [9]

$$\frac{d\sigma}{d\Omega} \approx \frac{\alpha^2}{s} \frac{1}{4s_W^4} \beta \left( 1 + 4\beta \cos \theta \frac{3c_W^2 - 1}{4c_W^2 - 1} + O(\beta^2) \right),$$

where the leading term $\propto \beta$ originates from the $t$-channel diagram only. Note that the leading term is angular-independent. In the total cross-section

$$\sigma_{\text{Born}} \approx \frac{\pi \alpha^2}{s} \frac{1}{4s_W^4} 4\beta + O(\beta^3),$$

all terms $\propto \beta^2$ drop out and the $s$-channel and the $s$--$t$-interference contributions are proportional to $\beta^3$. This is the consequence of CP conservation, fermion-helicity conservation in the initial state, and the orthogonality of different partial waves [9]. Hence in the threshold region the $t$ channel is dominant and the cross-section for $e^+e^- \to W^+W^-$ is not very sensitive to the triple gauge-boson couplings.

In table 2 we give the integrated cross-section for different centre-of-mass energies and different polarizations of the electrons ($+,-$). Positrons are assumed to be unpolarized. Using right-handed electrons one could study a pure triple-gauge-coupling process, but this would require longitudinally polarized electron beams, the prospect of which looks rather unfavourable for LEP2. Furthermore, for all energies the cross-section for right-handed electrons is suppressed by two orders of magnitude compared with the dominant left-handed mode, mainly because there is no $t$-channel contribution. Therefore essentially only the latter can be investigated at LEP2. With transverse beam polarization, however, one could obtain information on the right-handed matrix element via its interference with the left-handed one.

As stated in section 2.1, for on-shell $W$-pair production and unpolarized electrons and positrons there are nine independent helicity-matrix elements or six if CP is conserved. These yield nine or six independent observables. Taking into account the decay of the $W$ bosons there are many more observables: 81 or, if CP is conserved, 36 products of the various helicity-matrix

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*This holds for arbitrary CP-conserving $s$-channel contributions in the limit of vanishing electron mass [11].
Table 2: Integrated lowest-order cross-section in pb for different polarizations of the electrons and different centre-of-mass energies.

<table>
<thead>
<tr>
<th>√s [GeV]</th>
<th>σ_{Bom}</th>
<th>σ'_{Bom}</th>
<th>σ''_{Bom}</th>
</tr>
</thead>
<tbody>
<tr>
<td>161.0</td>
<td>3.812</td>
<td>7.622</td>
<td>0.002</td>
</tr>
<tr>
<td>175.0</td>
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<td>0.202</td>
</tr>
<tr>
<td>184.0</td>
<td>17.427</td>
<td>34.567</td>
<td>0.287</td>
</tr>
<tr>
<td>190.0</td>
<td>17.762</td>
<td>35.203</td>
<td>0.321</td>
</tr>
<tr>
<td>205.0</td>
<td>17.609</td>
<td>34.867</td>
<td>0.350</td>
</tr>
</tbody>
</table>

Table 2: Integrated lowest-order cross-section in pb for different polarizations of the electrons and different centre-of-mass energies.

elements [11]. Because the $V-A$ structure of the W decays is well established, these observables can be extracted in a model-independent way from the data for the five-fold differential cross-section $d\sigma/(d\cos \theta_1 d\cos \theta_1 d\phi_1 d\cos \theta_2 d\phi_2)$, where $\theta_{1,2}$ and $\phi_{1,2}$ represent the decay angles of the two W bosons. These observables may serve to put limits on anomalous couplings [10, 12]. Note that the five-fold differential cross-section requires the identification of the charge of at least one of the decaying W bosons, as otherwise the information on the sign of $\cos \theta$ would be lost. This is possible for hadronic-leptonic or leptonic-leptonic events. If no charge identification is possible only production-forward-backward-symmetric observables would be left. As argued above, in those observables for instance the $s$-channel contribution is suppressed by $\beta^2$ in the threshold region$^5$. Consequently, if one cannot identify the jet charges the purely hadronic events will not be of much use for studies of the gauge couplings at LEP2. Of course this does not concern the W-mass determination as it does not rely on $s$-channel contributions.

2.3 Radiative corrections

As has been argued in section 1, the SM theoretical predictions for W-pair production should have an uncertainty of about 0.5% (2% at $\sqrt{s} = 161$ GeV) in order to obtain reasonable limits on the structure of the gauge-boson self-couplings, and to determine the W-boson mass with the envisaged precision of roughly 40–50 MeV. In this context radiative corrections are indispensable.

Much effort has been made in recent years to obtain such precise theoretical predictions for W-pair production. In the following we discuss the existing results for the virtual and real electroweak corrections in the on-shell case. In addition we discuss the quality of an improved Born approximation (IBA) that contains all familiar, LEP1-like leading corrections for the W-pair production cross-section at LEP2 energies. Such a discussion is in particular relevant as the present off-shell LEP2 Monte Carlos often make use of such an approximation.

$^5$If CP is violated there exists an anomalous gauge-boson coupling that does not yield a suppressed $s$-channel contribution [9, 11].
2.3.1 Radiative electroweak $\mathcal{O}(\alpha)$ corrections

The $\mathcal{O}(\alpha)$ radiative corrections can be naturally divided into three classes, the virtual, soft-photonic, and hard-photonic contributions. Since the process $e^+e^- \to W^+W^-$ involves the charged current in lowest order, the corresponding radiative corrections cannot be separated on the basis of Feynman diagrams into electromagnetic and weak contributions in a gauge-invariant way. Like we have already observed in equation (15), $SU(2)$ gauge invariance requires an interplay between the $\gamma$- and Z-exchange diagrams.

The complete radiative electroweak $\mathcal{O}(\alpha)$ corrections have been calculated independently by two groups [14, 15]. For the unpolarized case they have been checked and found to agree within a couple of per-mil (i.e. within the integration error).

The virtual corrections contain infra-red (IR) divergences, which result from virtual photons exchanged between external charged particles. They are compensated for by adding the cross-section for the process $e^+e^- \to W^+W^-\gamma$. If the energy of the emitted photon is small compared with the detector resolution (soft photons), this process cannot be distinguished experimentally from the non-radiative W-pair production process. In practical experiments the soft-photon approximation is in general not sufficient and one has to include the radiation of hard photons, too. When adding these real-photon effects to the contribution of the virtual corrections, not only the IR singularities but also the large Sudakov double logarithms $\log^2(s/m_e^2)$ drop out.

Still there are various sources of potentially large $\mathcal{O}(\alpha)$ corrections left at LEP2 energies. First of all there are large QED corrections of the form $(\alpha/\pi)\log(Q^2/m_\nu^2)$ with $Q^2 \gg m_\nu^2$, originating from collinear photon radiation off the electron or positron (see appendix A). They form a gauge-invariant subset of QED corrections and amount to roughly 6\% at LEP2, not taking into account possible enhancements from the corresponding coefficients (like, e.g., close to thresholds).

From the renormalization two sets of potentially large fermionic (formally weak) corrections arise. The first set is associated with the charge renormalization at zero momentum transfer, where the relevant scale is set by the fermion masses entering the vacuum polarization. In high-energy experiments, however, the running charge should be evaluated at scales much larger than the masses of the light fermions $f_4 = \{e, \mu, \tau, u, d, c, s, b\}$. This leads to large logarithmic ('mass singular') contributions of the form $(\alpha/\pi)\log(Q^2/m_\nu^2)$ with $Q^2 \gg m_\nu^2$, which can amount to a shift in $\alpha$ of 8\% at LEP2 energies. Related to the top quark, corrections $\propto m_t^2/M_W^2$ will occur. They show up as universal corrections via the renormalization of the W and Z masses (or equivalently $s_W^2$) if the corresponding renormalization scales are small compared with the mass splitting in the $(t, b)$ isospin doublet.

Finally the long-range electromagnetic interaction between the slowly moving W bosons leads to the so-called Coulomb singularity [30]. This singularity yields an $\mathcal{O}(\alpha)$ correction factor $\alpha\pi/(2\beta)$, resulting in an $\mathcal{O}(\alpha)$-corrected cross-section that does not vanish at threshold for left-handed fermions.

\footnote{The process $e^+e^- \to W^+W^-\gamma$ has also been calculated in [29].}
<table>
<thead>
<tr>
<th>$M_H$ [GeV]</th>
<th>300</th>
<th>300</th>
<th>300</th>
<th>60</th>
<th>300</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_s$ =</td>
<td>0.117</td>
<td>0.123</td>
<td>0.129</td>
<td>0.123</td>
<td>0.123</td>
<td>0.123</td>
</tr>
<tr>
<td>$m_t$ [GeV] =</td>
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<td>165.73</td>
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</tr>
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<td>$\sqrt{s}$ [GeV]</td>
<td>$\sigma$ [pb]</td>
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<td></td>
</tr>
<tr>
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<td>17.671</td>
<td>17.698</td>
<td>17.677</td>
<td>17.637</td>
</tr>
</tbody>
</table>

Table 3: Integrated unpolarized cross-section including radiative electroweak $\mathcal{O}(\alpha)$ corrections for different values for the Higgs-boson mass and $\alpha_s$, at various centre-of-mass energies. The theoretical error in $m_t$ is roughly 1–2 GeV.

handed electrons. The right-handed cross-section remains suppressed by at least $\beta^2$. This correction factor exhibits the fact that the free-particle approximation is inadequate near the $W$-pair production threshold in the presence of the long-range Coulomb interaction. We want to stress here that the Coulomb singularity is changed substantially by effects that effectively truncate the range of the interaction, like the off-shellness and the decay of the $W$ bosons. This will be treated in section 3.4.2 and will lead to the conclusion that higher-order Coulombic corrections to the total cross-section are not important.

The sensitivity of the total unpolarized cross-section to $\alpha_s$ and the unknown mass of the Higgs boson is illustrated in table 3. The dependence on $\alpha_s$, originating from the calculation of $m_t$, is completely negligible ($\lesssim 0.1\%$). Compared with the lowest-order cross-sections of table 2, a variation of $M_H$ between 60 and 1000 GeV, however, influences the total cross-section by around 0.5% at the three highest energy points, by about 0.8% at $\sqrt{s} = 175\text{ GeV}$, and by 1.4% at threshold. These numbers are lowered by about 0.5% if the radiative corrections are calculated in the so-called $G_\mu$-representation, which absorbs the universal $m_t$ and $M_H$ effects present in the $W$ wave-function factors (see section 2.4.2). In this representation a variation of $M_H$ between 300 and 1000 GeV has a negligible impact on the cross-section. Keeping in mind that we would like to reach a theoretical accuracy of 2% ($0.5\%$) at $\sqrt{s} = 161\text{ GeV}$ ($\sqrt{s} \geq 175\text{ GeV}$), it should be clear that the Higgs-mass dependence constitutes a major uncertainty. A more detailed investigation [31] revealed that this Higgs-boson-mass dependence is the result of the Yukawa interaction between the two slowly moving $W$ bosons (mediated by the Higgs boson). As such the effect is largest close to threshold and for the lightest Higgs masses, which
Figure 2: Unpolarized total cross-section in the threshold region. The dotted curve corresponds to Born, the dashed one to $G_\mu$-Born, and the solid one to the results including full radiative electroweak $O(\alpha)$ corrections.

yield the shortest range of the interaction. For instance, upon increasing the lower Higgs-mass bound from 60 to 90 GeV, the resulting uncertainty is reduced by roughly a factor of two.

In figure 2 we display the influence of the full $O(\alpha)$ corrections on the total unpolarized cross-section near the W-pair production threshold.

As has been stated before, it should be noted that, unlike at LEP1, the presence of the charged current at lowest order in $e^+e^- \rightarrow W^+W^-$ complicates the separation between QED and weak corrections. So, in order to have a cleaner look at the QED corrections we present in addition to the Born cross-section also the lowest-order cross-section in the so-called $G_\mu$ representation, i.e. the Born cross-section with $\alpha$ replaced by $\sqrt{2} G_\mu M_W^2 \delta_W^2 / \pi$, which already contains an important part of the leading weak effects discussed above (see also section 2.3.3). We will refer to this cross-section as $G_\mu$-Born. As a result of the steep drop with decreasing centre-of-mass energy of the W-pair cross-section close to threshold, large and predominantly

\begin{footnote}
At the start of the 161 GeV run no significant change in the $M_H$ bound is expected. So, only after the higher-energy LEP2 runs have taken place the improved knowledge on $M_H$ can be used for an a posteriori reduction of the $M_H$ dependence of the 161 GeV run.
\end{footnote}

\begin{footnote}
As the $t$ channel is dominant at LEP2 energies, the $G_\mu$-Born describes the leading weak corrections reasonably well for the default Higgs mass $M_H=300$ GeV.
\end{footnote}
soft QED effects can be observed. Compared with $G_\mu$-Born the $\mathcal{O}(\alpha)$ corrections amount to more than $-25\%$ in the direct vicinity of the threshold ($\sqrt{s} < 165\,\text{GeV}$), apart from the region very close to threshold where the positive Coulomb-singularity contribution takes over (see close-up in figure 2), and still $-17\%$ ($-11\%$) at $\sqrt{s} = 175\,\text{GeV}$ (190 GeV). The size of these effects necessitates the inclusion of higher-order QED corrections in order to end up with an acceptable theoretical uncertainty. A discussion of these higher-order QED corrections can be found in section 2.3.3. The finite W width has a drastic impact on the effects related to the Coulomb singularity (see section 3.4.2), but the large soft QED effects are merely smoothened and stay sizeable.

In figure 3 the effect of the $\mathcal{O}(\alpha)$ corrections on the unpolarized differential cross-section $d\sigma/(d\cos \theta_\pm)$ is shown for $\sqrt{s} = 190\,\text{GeV}$. Here $\theta_\pm$ stands for the polar angle of the $W^+$ boson with respect to the incoming positron$^3$. Apart from the expected normalization effects that are already observed in the total cross-section, also a distortion of the distribution occurs. As this is exactly the type of signature one might expect from anomalous gauge-boson couplings, this underlines again the importance of having a profound knowledge of the SM radiative corrections. The origin of the distortion can be traced back to hard initial-state photonic corrections. Hard-photon emissions boost the centre-of-mass system of the $W^+W^-$ pair. As a

\[ ^3 \text{In the presence of hard-photon radiation in general } \theta_+ \neq \theta_. \]
result of that, events that are forward in the centre-of-mass system of the produced W bosons can show up as backward events in the LAB system and vice versa. Since the cross-section in the backward direction is substantially lower than in the forward direction, the net effect of this redistribution (migration) of events will be a distortion of the differential distribution with respect to the lowest-order one. Of course these boost effects are much more pronounced at high energies [9].

2.3.2 Approximations in the LEP2 region

The complete analytic results for the electroweak $O(\alpha)$ corrections are very lengthy and complicated, resulting in huge and rather slow computer programs. Moreover, the formulae are completely untransparent. In view of this, simple approximative expressions are desirable. Apart from providing fast computer programs, which are useful for many applications, simple transparent formulae should reveal the physical content and the origin of the dominant radiative corrections. Furthermore, if these approximative expressions represent the exact corrections adequately, one might consider implementing them in the existing LEP2 Monte Carlos.

Owing to the lack of a calculation of the complete $O(\alpha)$ corrections, the present LEP2 Monte Carlos for off-shell W-pair production include only the known leading universal corrections. In order to assess the theoretical uncertainty inherent in this approach, the on-shell case can be used as guideline. The size of the non-leading $O(\alpha)$ corrections in this case should provide a reasonable estimate for the corresponding left-out non-leading corrections in the off-shell case.

We start out by investigating the structure of the matrix element for $e^+e^- \to W^+W^-$. Whereas it involves only three different tensor structures in lowest order, at $O(\alpha)$ twelve independent tensor structures occur, each of which is associated with an independent invariant function, which can be considered as an s- and t-dependent effective coupling. The dominant radiative corrections, as e.g. those that are related to UV, IR, or mass singularities, in general have factorization properties and are at $O(\alpha)$ restricted to those invariant functions that appear already at lowest order. Therefore the contributions of the other invariant functions should be relatively small. Indeed, a numerical analysis reveals that in a suitably chosen representation for the basic set of independent matrix elements only the three Born-like invariant functions plus one extra right-handed piece, related to $M_f^+$, are relevant for a sufficiently good approximation [32, 33]. This results in the following approximation for the matrix element:

$$M^{app} = M^{\alpha} F^\alpha + M^{\alpha}_Q F^\alpha,$$

with $M_f$ and $M_Q$ defined in equation (15). The term involving $F^\alpha$ is needed for right-handed electrons because of contributions $\propto \beta^2 \cos \theta$, originating from the interference of $M_{\text{Born}}$ with particular 1-loop matrix elements, which are not present at lowest order. Neglecting the other invariant functions in the basis given in [32] introduces errors well below the per-cent level (see table 4). This is of course only true for observables where $M^{app}$ is not suppressed or absent. All this demonstrates that improved Born approximations are possible. Note, however,
that in contrast to the situation at LEP1 the invariant functions $F_{lQ}^\kappa$ are both energy- and angular-dependent.

In order to construct an improved Born approximation (IBA) one has to specify simple expressions for the invariant functions $F_{lQ}^\kappa$, which reproduce the corresponding exact expressions with sufficient accuracy. In the LEP2 energy region the following expressions can be used as a reasonable ansatz \cite{32}

$$
F_{l}^\kappa = 2\sqrt{2} G_\mu M_W^2 + \frac{4\pi\alpha}{2s_W^2} \frac{\pi\alpha}{4\beta} (1 - \beta^2)^2 \delta_\kappa, \nonumber
$$

$$
F_{Q}^\kappa = 4\pi\alpha(s) + 4\pi\alpha \frac{\pi\alpha}{4\beta} (1 - \beta^2)^2 . \nonumber
$$

The terms containing $G_\mu$ and $\alpha(s)$ incorporate all the leading universal corrections associated with the running of $\alpha$ and the corrections $\propto m_t^2/M_W^2$ associated with the $\rho$ parameter (see also section 2.3.3). As these are linked to the renormalization of the electric charge at zero momentum transfer and of the weak mixing angle, they only contribute to the structures already present at lowest order. The corresponding leading $O(\alpha)$ contributions can be recovered from equation (20) by substituting

$$
\alpha(s) \to \alpha[1 + \Delta\alpha(s)], \quad 2\sqrt{2} G_\mu M_W^2 \to \frac{4\pi\alpha}{2s_W^2} 1 + \Delta\alpha(M_W^2) - \frac{s_W^2}{s_W^2} \Delta\rho , \nonumber
$$

with

$$
\Delta\alpha(s) = \frac{\alpha}{3\pi} \sum_{f \neq t} N_C^f Q_f^2 \log \frac{s}{m_f^2} , \quad \Delta\rho = \frac{3\alpha m_t^2}{16\pi s_W^2 M_W^2} . \nonumber
$$

The $1/\beta$ term describes the effect of the Coulomb singularity, which for $\beta \ll 1$ yields a simple correction factor to the lowest-order cross-section:

$$
\delta\sigma_{\text{Coul}} = \frac{\pi\alpha}{2\beta} \sigma_{\text{Born}} . \nonumber
$$

The factor $(1 - \beta^2)^2$ is introduced by hand to switch off the Coulomb singularity fast enough. In this way one avoids corrections that are too large away from threshold, where the Coulomb singularity should not play a role anymore. As has been studied in \cite{32}, heavy-mass contributions $\propto \log(m_t)$ and $\log(M_H)$ that are not covered by equation (20) have a negligible impact on the approximation. Since the above IBA analysis has been performed for the default Higgs-boson mass $M_H = 300$ GeV, the large light-Higgs-boson corrections close to threshold, shown in table 3, are absent. By adding a simple approximation for these corrections \cite{31} to the IBA, however, the full $M_H$ dependence of the exact $O(\alpha)$ corrections can be reproduced.

In addition to the contributions described so far, one has to include the leading logarithmic QED corrections. These can be calculated using the structure-function method as described in appendix A. They comprise all contributions $\propto (\alpha/\pi)\log(m_e^2/Q^2)$. In the following numerical
analysis the scale $Q^2 = s$ has been used and, in order to extract the effect of the non-leading virtual corrections, hard-photon radiation is left out.

As we want to compare the approximation with an $O(\alpha)$ calculation we do not use the square of the matrix element defined in equations (19) and (20) for the numerical analysis, but only the square of the terms involving $G_\mu$ and $\alpha(s)$ and the interference of these terms with the others. Moreover, in this interference $G_\mu$ and $\alpha(s)$ are replaced by the corresponding lowest-order expressions in terms of $s_W^2$ and $\alpha(0)$. Nevertheless, the so-obtained approximation still contains higher-order contributions through the squares of $\alpha(s)$ and $G_\mu$. To allow for a meaningful comparison these have been included in the numbers for the exact one-loop results given in table 4 in the same way (see [32] for more details).

In table 4 we show the difference between the exact and approximated virtual and soft $O(\alpha)$ radiative corrections to the total and differential cross-section for right-handed, left-handed, and unpolarized electrons. The positrons are assumed to be unpolarized. In the LEP2 energy region the relative difference between the exact result and the approximation can be as large as 1–2% for left-handed or unpolarized electrons, and reach 3–8% for right-handed electrons. As there is no obvious reason why these remaining non-leading corrections should be smaller in the case of off-shell W bosons or $e^+e^- \rightarrow 4f$, their inclusion in the LEP2 data analysis seems to be unavoidable.

### 2.3.3 Higher-order corrections

In section 2.3.1 we have encountered large $O(\alpha)$ corrections. A short description of the way to include the corresponding higher-order corrections is hence in place.

In two distinct ways the higher-order corrections enter the calculation of the distributions for W-pair production. First of all there is the calculation of $m_t$ from $M_W$ and the input parameters, either to be used in the calculation of the W-pair RC's or to be confronted with the Tevatron data. As $m_t$ enters the relation between $\alpha$, $G_\mu$, $M_Z$, and $M_W$, resulting from the muon decay width, at the 1-loop level, the highest precision possible is required for this relation. As this relation relies on calculations performed at $Q^2 \approx 0$ for the muon decay width and at the subtraction points $Q^2 = M_W^2, M_Z^2$ for the renormalization procedure, we can use the state-of-the-art calculation developed for the LEP1 analysis [6]. This yields

$$G_\mu = \frac{\alpha\pi}{\sqrt{2}s_W^2 M_W^2} \rho_c.$$  \hspace{1cm} (24)

Usually $\rho_c$ is written in the form

$$\rho_c = \frac{1}{1 - \Delta r},$$  \hspace{1cm} (25)
<table>
<thead>
<tr>
<th></th>
<th>unpolarized</th>
<th>right-handed</th>
<th>left-handed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s} = 161$ GeV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>1.45 0.00</td>
<td>-1.56 -0.01</td>
<td>1.45 0.00</td>
</tr>
<tr>
<td>10°</td>
<td>1.63 0.00</td>
<td>4.41 0.00</td>
<td>1.63 0.00</td>
</tr>
<tr>
<td>90°</td>
<td>1.44 0.00</td>
<td>-1.57 -0.01</td>
<td>1.44 0.00</td>
</tr>
<tr>
<td>170°</td>
<td>1.26 0.00</td>
<td>-7.52 0.00</td>
<td>1.26 0.00</td>
</tr>
<tr>
<td>$\sqrt{s} = 165$ GeV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
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<td>-2.09 -0.01</td>
<td>1.28 0.02</td>
</tr>
<tr>
<td>10°</td>
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<td>0.49 0.00</td>
<td>1.67 0.00</td>
</tr>
<tr>
<td>90°</td>
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<td>-2.09 -0.02</td>
<td>1.18 0.02</td>
</tr>
<tr>
<td>170°</td>
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<td>-4.64 0.00</td>
<td>0.77 0.00</td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
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<td>1.28 0.03</td>
</tr>
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<tr>
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<td>-2.59 -0.03</td>
<td>1.06 0.05</td>
</tr>
<tr>
<td>170°</td>
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<td>-5.30 0.00</td>
<td>0.69 0.00</td>
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<tr>
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<td></td>
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<td>1.57 0.00</td>
</tr>
<tr>
<td>90°</td>
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<td>-2.82 -0.05</td>
<td>0.72 0.08</td>
</tr>
<tr>
<td>170°</td>
<td>0.10 0.00</td>
<td>-7.72 0.01</td>
<td>0.32 0.00</td>
</tr>
<tr>
<td>$\sqrt{s} = 190$ GeV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
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<td>-2.91 -0.04</td>
<td>1.28 0.03</td>
</tr>
<tr>
<td>10°</td>
<td>1.67 0.00</td>
<td>0.59 -0.01</td>
<td>1.67 0.00</td>
</tr>
<tr>
<td>90°</td>
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<td>-2.92 -0.06</td>
<td>1.01 0.06</td>
</tr>
<tr>
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</tr>
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<td></td>
</tr>
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<td>1.65 0.00</td>
</tr>
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<td>-1.68 -0.01</td>
<td>1.77 0.00</td>
</tr>
<tr>
<td>90°</td>
<td>1.55 0.00</td>
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<td>1.64 0.00</td>
</tr>
<tr>
<td>170°</td>
<td>1.61 0.00</td>
<td>-4.37 0.00</td>
<td>1.94 0.00</td>
</tr>
</tbody>
</table>

Table 4: Quality of the IBA (first column per polarization) and the form-factor approximation (19) (second column per polarization), given in per cent relative to Born. Results are given for the total cross-section (total) and the differential cross-section at 10, 90, and 170°.
where $\Delta r$ contains all the one-loop corrections to the muon decay width with the inclusion and the proper arrangement of the higher-order terms. Next we subdivide $\rho_c$ as introduced in equation (25) into a leading term, $\Delta r_L$, and remainder terms, $\Delta r_{\text{rem}}$, as follows:

$$\rho_c = \frac{1}{1 - \Delta r} = \frac{1}{1 - \Delta r_L - \Delta r_{\text{rem}}} = \frac{1}{(1 - \Delta \alpha)(1 + \frac{c_W^2}{s_W^2} \Delta \rho_X - \Delta r_{\text{rem}})},$$

with

$$\Delta r_{\text{rem}} = \Delta r^\alpha + \Delta r^{\alpha \alpha_s} + \frac{c_W^2}{s_W^2} \Delta \rho_X - \Delta \alpha.$$  

(27)

This contains all the terms known at present: the complete one-loop $O(\alpha)$ corrections $\Delta r^\alpha$ (two-, three-, four-point functions) and complete two-loop $O(\alpha \alpha_s)$ insertions $\Delta r^{\alpha \alpha_s}$ to two-point functions, from which the leading $O(\alpha)$ and $O(\alpha \alpha_s)$ terms are subtracted. The Born version of equation (24), i.e. $\rho_c = 1$, relates $G_\mu$ directly to $\alpha$, $M_W$, and $M_Z$, hence leaving no room for using $G_\mu$ as additional independent input parameter. However, having introduced radiative corrections to the Born relation we can solve equation (24) in terms of the top quark mass and, in turn, this particular value for $m_t$ will be used throughout the rest of the calculation. One should notice that this procedure is not free of ambiguities since $\Delta r$ is also a function of $M_H$, but with much smaller sensitivity due to the well known screening. So in the end we should also allow for some variation in $M_H$. Moreover, $\Delta r$ is also a function of $\alpha$, through higher-order corrections, for instance $O(\alpha \alpha_s)$, and, as a consequence, also some variation in the strong coupling constant should be allowed.

The leading terms $\Delta \alpha$ and $\Delta \rho_X$ appearing in equation (27) are given by

$$\Delta \rho_X = \Delta \rho^\alpha + \Delta \rho^{\alpha \alpha_s} + \tilde{X}$$

$$= \frac{3\alpha}{16\pi s_W^2 c_W^2 M_Z^2} \frac{m_t^2}{1 - \frac{2}{3} \frac{\pi^2}{3} \frac{\alpha_s(m_t^2)}{\pi}} + \tilde{X},$$

$$\Delta \alpha = 1 - \frac{\alpha}{\alpha(M_H^2)},$$

$$\alpha(s) = \frac{\alpha}{1 + \Sigma^{\mu \neq t}(s)/s},$$

(28)

where $\Sigma^{\mu \neq t}(s)$ is the renormalized $O(\alpha)$ transverse photon self-energy originating from fermion loops, excluding top-quark loops.

The term $\tilde{X}$ in equation (28) is a next-to-leading order term, whose proper treatment is rather important:

$$\tilde{X} = \text{Re} \left( \frac{\Pi_2(M_Z^2)}{M_Z^2} - \frac{\Pi_W(M_Z^2)}{M_W^2} \frac{1^{\text{loop}}_{\overline{\text{MS}}}}{M_Z^2} - \Delta \rho^\alpha, \right)$$

(29)

where $\Pi_V$ denotes the unrenormalized transverse self-energy of the $V$ gauge boson. The ultraviolet (UV) divergences are removed according to the $\overline{\text{MS}}$ renormalization scheme with $\mu = M_Z$.
In contrast to $\Delta \rho_X$, the leading contribution $\Delta \rho_X$ appearing in equation (26) is normalized by $G_\mu$ rather than by $\alpha/(s_W^2 M_W^2)$, as is required by the resummation proposed in [34]:

$$\Delta \rho_X = \Delta \rho^\alpha + \Delta \rho^{\alpha^2} + \Delta \rho^{\alpha^3} + \Delta \rho^{\alpha^4} + X$$

$$= N_C t x_t 1 + x_t \Delta \rho^{(2)} \frac{m_t^2}{M_H^2} + c_1 \frac{\alpha_s(m_t^2)}{\pi} + c_2 \frac{\alpha_s(m_t^2)}{\pi}^2 + X,$$

(30)

where $N_C = 3$ and

$$x_t = \frac{G_\mu}{\sqrt{2}} \frac{m_t^2}{8\pi^2},$$

(31)

$$X = 2s_W^2 c_W^2 \frac{G_\mu M_H^2}{\sqrt{2\pi\alpha}} \hat{X}.$$

(32)

The coefficients $c_1$ and $c_2$ describe the first- and second-order QCD corrections for the leading $x_t$ contribution to $\Delta \rho$, calculated in [35, 36]. Correspondingly:

$$c_1 = -\frac{2}{3} + \frac{\pi^2}{3},$$

(33)

$$c_2 = -\pi^2 (2.564571 - 0.180981 n_f),$$

(34)

with $n_f$ the total number of flavours ($n_f = 6$). The function $\Delta \rho^{(2)}(m_t^2/M_H^2)$ describes the leading two-loop electroweak $x_t$ correction to $\Delta \rho$, calculated first in the $M_H = 0$ approximation in [37] and later in [38] for an arbitrary relation between $M_H$ and $m_t$.

It should be noted that the higher-order radiative corrections discussed above are usually calculated in the limit of a heavy top mass, i.e. usually only the leading part of the corrections is under control. This often raises the question of what effect can be estimated from the sub-leading terms, since $m_t \approx 2 M_Z$. As far as QCD $O(\alpha_s)$ and $O(\alpha_s^2)$ corrections are concerned the sub-leading terms are well under control. In the pure electroweak sector, however, there has been, so far, no calculation of higher-order sub-leading terms at an arbitrary scale. The only available piece of calculation concerns the $\rho$ parameter at $Q^2 = 0$ [39], therefore relevant only for $\nu_\mu-\nu_e$ scattering. If however one is willing to extrapolate the $Q^2 = 0$ result to a higher scale, by assuming that the ratio of leading to next-to-leading corrections is representative for the corresponding theoretical uncertainty, then the answer is next-to-leading $\approx$ leading.

Even before considering the actual impact of the higher-order terms on $m_t$, we should mention at this point that the way in which the non-leading terms can be treated and the exact form of the *leading-remainder* splitting give rise to several possible options in the actual implementation of radiative corrections. This in turn becomes a source of theoretical uncertainty. For instance, for $\Delta r$ we can introduce the decomposition into leading and remainder. Since we know how to proceed with all objects in the leading approximation, the only ambiguity is due to the treatment of the remainders. Clearly, after the splitting $\Delta r = \Delta r_L + \Delta r_{\text{rem}}$ there are in principle several possible ways of handling the remainder:

$$\frac{1}{1 - \Delta r} = \frac{1}{1 - \Delta r_L - \Delta r_{\text{rem}}}, \quad \frac{1}{1 - \Delta r_L} 1 + \frac{\Delta r_{\text{rem}}}{1 - \Delta r_L}, \quad \frac{1}{1 - \Delta r_L} 1 + \frac{\Delta r_{\text{rem}}}{1 - \Delta r_L}, \quad \frac{1}{1 - \Delta r_L} + \Delta r_{\text{rem}}.$$

(35)
Actually, these options differ among themselves, but the difference can be related to the choice of the scale in the remainder term. A complete evaluation of the sub-leading $O(G^2 M_Z^2 m_t^2)$ corrections would greatly reduce the associated uncertainty. In conclusion, we observe that a natural and familiar language for the basic ingredients of the physical observables is that of effective couplings. However, it should be stressed that the above formulae belong to a specific realization of this language and other realizations could also be used.

Now we can assess the influence of the higher-order corrections on the calculation of $m_t$ from equation (24). Using for instance $M_W = 80.26$ GeV and $M_Z = 91.1844$ GeV we find

$$m_t = 165^{+16}_{-18} (M_H, \alpha_s) \text{ GeV},$$

the central value of which corresponds to $M_H = 300$ GeV and $\alpha_s(M_Z^2) = 0.123$. The errors are derived by varying $M_H$ and $\alpha_s$ in the range $60 \text{ GeV} < M_H < 1 \text{ TeV}$ and $\alpha_s = 0.123 \pm 0.006$. It should be noted in this respect that the total variation induced by $M_H$ alone (at $\alpha_s = 0.123$) is about $33 \text{ GeV}$. The following is observed for the higher-order corrections:

- by neglecting the $O(\alpha^2)$ term in $\Delta r$ we find for the same input parameters (and $M_H = 300$ GeV, $\alpha_s = 0.123$) a shift in $m_t$ of $-1.9 \text{ GeV}$.
- If instead we switch off the $O(\alpha \alpha_s^2)$ correction the corresponding shift will be $-1.5 \text{ GeV}$.
- If finally both the $O(\alpha \alpha_s)$ and $O(\alpha \alpha_s^2)$ corrections are neglected we find a shift in $m_t$ of $-10.5 \text{ GeV}$. Here by $O(\alpha \alpha_s)$ the full result is implied and not only the leading part of it.

Based on the above observations, the remaining theoretical uncertainty in the calculation of $m_t$ from missing higher-order corrections and sub-leading $O(\alpha^2)$ corrections to $\Delta r$ is estimated to be roughly $1-2 \text{ GeV}$.

The second way the higher-order corrections enter the calculations for $W$-pair production is through the process itself, so through self-energies, vertices, etc. The known weak higher-order effects comprise the running of $\alpha$ [see equation (28)] and the complete $O(\alpha \alpha_s)$ corrections to the gauge-boson self-energies [35]. Other higher-order calculations, as those for $\Delta \rho$ at $O(\alpha^2)$ or $O(\alpha \alpha_s^2)$, have been performed in the limit where $m_t$ is the largest scale. The QCD corrections associated with the gauge-boson vertex corrections have, to our knowledge, not been calculated yet, but they are at most logarithmic in the top mass. If one takes into account the leading weak effects by replacing $e^2/(2s_W^2)$ by $2\sqrt{2} G_\mu M_W^2$ in the $M_I$ part of equation (15) and $\alpha$ by $\alpha(s)$ in the $M_Q$ part, the remaining unknown higher-order weak effects are expected to be negligible compared with the required theoretical accuracy.

As pointed out in the previous subsection, the virtual and real corrections reveal the presence of large logarithmic QED effects of the form $\alpha L/\pi = (\alpha/\pi) \log(Q^2/m_e^2)$ with $Q^2 \gg m_e^2$. They arise when photons or light fermions are radiated off in the direction of incoming or outgoing light particles. In the case of $W$-pair production the only terms of this sort originate from initial-state photon emission. Radiation of photons from the final-state W bosons can only lead to
Table 5: Unpolarized total cross-section, given in pb, including radiative electroweak $O(\alpha)$ corrections and higher-order terms in the leading-log approximation. These higher-order terms are given with and without soft-photon exponentiation for two different ‘natural’ scale choices.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>Born $+ O(\alpha)$</th>
<th>+ h.o.t. for $Q^2 = s$</th>
<th>+ h.o.t. for $Q^2 = s \frac{1-\beta}{1+\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$O(\alpha)^2$ LL</td>
<td>exp. LL</td>
<td>$O(\alpha)^2$ LL</td>
</tr>
<tr>
<td>161.0</td>
<td>2.472 ± 0.001</td>
<td>3.103</td>
<td>3.003</td>
</tr>
<tr>
<td>165.0</td>
<td>8.581 ± 0.003</td>
<td>9.079</td>
<td>9.049</td>
</tr>
<tr>
<td>170.0</td>
<td>12.270 ± 0.004</td>
<td>12.583</td>
<td>12.585</td>
</tr>
<tr>
<td>175.0</td>
<td>14.465 ± 0.004</td>
<td>14.654</td>
<td>14.670</td>
</tr>
<tr>
<td>184.0</td>
<td>16.613 ± 0.005</td>
<td>16.668</td>
<td>16.693</td>
</tr>
<tr>
<td>190.0</td>
<td>17.257 ± 0.006</td>
<td>17.259</td>
<td>17.286</td>
</tr>
<tr>
<td>205.0</td>
<td>17.677 ± 0.006</td>
<td>17.613</td>
<td>17.638</td>
</tr>
</tbody>
</table>

Sizeable corrections if the energy of the W bosons is much larger than their mass. The leading large logarithmic corrections can be calculated by using the so-called structure-function method [16] in leading-log (LL) approximation, i.e. only taking along the terms $\propto (\alpha L/\pi)^n$. This procedure also allows the inclusion of soft-photon effects to all orders by means of exponentiation and is discussed in detail in appendix A.

In table 5 the higher-order effects related to the large logarithmic QED corrections are displayed. The $O(\alpha)^2$ LL entry contains, in addition to the full $O(\alpha)$ result, the contribution from $O(\alpha)^2$ LL corrections using $\hat{\sigma}_0 = \sigma_{\text{Born}}$ in the convolution (68). The exp. LL entry contains on top of that the exponentiation of soft-photon effects. Compared with the full $O(\alpha)$ results we observe large $O(\alpha)^2$ LL effects near threshold, e.g. 25% at $\sqrt{s} = 161$ GeV, and moderate ones when sufficiently far above threshold, i.e. < 1% for energies above roughly 175 GeV. The additional soft-photon exponentiation is only sizeable near threshold, e.g. −4% at $\sqrt{s} = 161$ GeV. Note that finite-W-width effects will smoothen the threshold behaviour and hence reduce the size of the $O(\alpha)^2$ LL effects, nevertheless they will stay sizeable near threshold. The dependence of the higher-order LL corrections [beyond $O(\alpha)$] on the scale choice $Q^2$ is negligible, since all natural scales are roughly equal close to threshold. When comparing the popular scale choice $Q^2 = s$ with $Q^2 = s(1-\beta)/(1+\beta)$, motivated by the behaviour of the total cross-section near threshold and at high energies [9], the differences are at the 0.1% level (0.2–0.3% at $\sqrt{s} = 161$ GeV).
An additional improvement of the theoretical predictions can be obtained by using an improved Born cross-section in the convolution (68), taking into account corrections related to \( G_\mu, \alpha(s), M_R \) etc.\(^k\)

### 2.4 The width of the W boson

Evidently the width of the W boson is a crucial ingredient for the (off-shell) W-pair production cross-section, especially in the threshold region. Moreover, the branching ratios enter the cross-sections for definite fermions in the final state. As at present the width of the W boson is experimentally poorly known, we need a precise theoretical calculation instead in order to obtain adequate theoretical predictions for off-shell W-pair production.

#### 2.4.1 The W-boson width in lowest order

The W width is dominated by decays into fermion-antifermion pairs. In lowest order the partial width for the decay of a W boson into two fermions with masses \( m_{f_i} \) and \( m_{f_j} \) (\( i, j \) denote the generation index and \( f, f' \) stand for \( u, d \) or \( \nu, \bar{\nu} \)) is given by

\[
\Gamma_{W_{f_i f_j}}^{\text{Born}} = N_C^f \frac{\alpha}{6} \frac{M_W}{2s_W^2} |V_{ij}|^2 \left( 1 - \frac{m_{f_i}^2 + m_{f_j}^2}{2M_W^2} - \frac{(m_{f_i}^2 - m_{f_j}^2)^2}{2M_W^4} \right) \times \frac{M_W^2 - (m_{f_i} + m_{f_j})^2}{M_W^2 - (m_{f_i} - m_{f_j})^2}.
\]

(37)

For leptonic decays the mixing matrix is diagonal (\( V_{ij} = \delta_{ij} \)) and the colour factor \( N_C^f \) equals one. For decays into quarks there is a non-trivial quark mixing matrix and \( N_C^f = 3 \). For the quark mixing matrix we have used \( s_{12} = 0.221, s_{23} = 0.04 \), and \( s_{13} = 0.004 \) [26]. The total width is obtained as a sum over the partial fermionic decay widths with \( m_{f_i} + m_{f_j} < M_W \)

\[
\Gamma_W^{\text{Born}} = \sum_{i,j} \Gamma_{W_{u_i d_j}}^{\text{Born}} + \Gamma_{W_{s_i \nu_i}}^{\text{Born}}.
\]

(38)

As the quark masses are small compared with \( M_W \), the fermion-mass effects are small for the W decay. If we neglect them we obtain the simple formulae

\[
\Gamma_{W_{f_i f_j}}^{\text{Born}} = N_C^f \frac{\alpha}{6} \frac{M_W}{2s_W^2} |V_{ij}|^2, \quad \Gamma_W^{\text{Born}} \approx \frac{3\alpha}{2} \frac{M_W}{2s_W^2}.
\]

(39)

\(^k\)As the finite decay width of the W bosons will have a substantial impact on the Coulomb singularity, a LL analysis involving this Coulomb singularity only makes sense for off-shell W bosons.
<table>
<thead>
<tr>
<th>( M_H ) [GeV]</th>
<th>300</th>
<th>300</th>
<th>300</th>
<th>60</th>
<th>300</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_s )</td>
<td>0.117</td>
<td>0.123</td>
<td>0.129</td>
<td>0.123</td>
<td>0.123</td>
<td>0.123</td>
</tr>
<tr>
<td>( m_t ) [GeV]</td>
<td>164.80</td>
<td>165.26</td>
<td>165.73</td>
<td>148.14</td>
<td>165.26</td>
<td>181.20</td>
</tr>
<tr>
<td>( \Gamma_W^{\text{Born}} ) [GeV]</td>
<td>1.9490</td>
<td>1.9490</td>
<td>1.9490</td>
<td>1.9490</td>
<td>1.9490</td>
<td>1.9490</td>
</tr>
<tr>
<td>( \bar{\Gamma}_W^{\text{Born}} ) [GeV]</td>
<td>2.0354</td>
<td>2.0354</td>
<td>2.0354</td>
<td>2.0354</td>
<td>2.0354</td>
<td>2.0354</td>
</tr>
<tr>
<td>( \Gamma_W ) [GeV]</td>
<td>2.0642</td>
<td>2.0663</td>
<td>2.0684</td>
<td>2.0681</td>
<td>2.0663</td>
<td>2.0639</td>
</tr>
<tr>
<td>( \bar{\Gamma}_W ) [GeV]</td>
<td>2.0791</td>
<td>2.0817</td>
<td>2.0844</td>
<td>2.0813</td>
<td>2.0817</td>
<td>2.0819</td>
</tr>
<tr>
<td>( \delta_{\text{ew}} )</td>
<td>0.03416</td>
<td>0.03398</td>
<td>0.03380</td>
<td>0.03491</td>
<td>0.03398</td>
<td>0.03275</td>
</tr>
<tr>
<td>( \delta_{\text{ew}} )</td>
<td>-0.00347</td>
<td>-0.00347</td>
<td>-0.00347</td>
<td>-0.00369</td>
<td>-0.00347</td>
<td>-0.00341</td>
</tr>
<tr>
<td>( \delta_{\text{QCD}} )</td>
<td>0.02495</td>
<td>0.02623</td>
<td>0.02751</td>
<td>0.02623</td>
<td>0.02623</td>
<td>0.02623</td>
</tr>
</tbody>
</table>

Table 6: Higgs-mass and \( \alpha_s \), dependence of the W-boson width

### 2.4.2 Higher-order corrections to the W-boson width

The electroweak and QCD radiative corrections for decays into massless fermions \( (m_f \ll M_W) \) have been calculated in [17]-[19]. The full one-loop electroweak and QCD corrections, together with the complete photonic and gluonic bremsstrahlung, were evaluated for arbitrary fermion masses in [20]. The various calculations are in good agreement. The relative corrections to the total W-boson decay width are given in table 6. The electroweak corrections in the on-shell scheme \( \delta_{\text{ew}} \equiv \Gamma_W/\Gamma_W^{\text{Born}} - 1 - \delta_{\text{QCD}} \) are predominantly originating from the running of \( \alpha \) and the corrections \( \propto \alpha m_f^2/M_W^2 \) to the \( \rho \) parameter. These corrections can be easily accounted for by parametrizing the lowest-order width in terms of \( G_\mu \) and \( M_W \) instead of \( \alpha \) and \( \delta_W \). The width in this \( G_\mu \) parametrization \( \bar{\Gamma}_W \) is related to the width in the on-shell parametrization \( \Gamma_W \) by

\[
\Gamma_W = \frac{\Gamma_W - \Gamma_W^{\text{Born}} \Delta r^{1-\text{loop}}}{1 - \Delta r} = \frac{\Gamma_W^{\text{Born}} 1 + \delta_{\text{ew}} + \delta_{\text{QCD}} - \Delta r^{1-\text{loop}}}{1 - \Delta r} = \bar{\Gamma}_W^{\text{Born}} (1 + \delta_{\text{ew}} + \delta_{\text{QCD}} - \Delta r^{1-\text{loop}}) \equiv \bar{\Gamma}_W^{\text{Born}} (1 + \delta_{\text{ew}} + \delta_{\text{QCD}}). \tag{40}
\]

As can be seen from table 6 the electroweak corrections with respect to the parametrization with \( G_\mu \), \( \delta_{\text{ew}} \), depend in a negligible way on \( M_H \), and remain below 0.5\% for the total width. The QCD corrections \( \delta_{\text{QCD}} \) are practically constant and equal to \( 2\alpha_s(M_W^2)/(3\pi) \), their value for zero fermion masses. For the numerical evaluation we use \( \alpha_s(M_W^2) = 0.123 \) (i.e. equal to the default input value). The difference between \( \Gamma_W \) and the more precise \( \bar{\Gamma}_W \) is caused by missing higher-order terms related to \( \Delta r \).

We obtain the following improved Born approximation for the total and partial widths [40]

\[
\Gamma_{W\ell}\ell \approx \Gamma_{W\ell}\ell^{\text{Born}} = \frac{G_\mu M_W^3}{\sqrt{2} \pi},
\]

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<table>
<thead>
<tr>
<th></th>
<th>Born $m_f \neq 0$</th>
<th>complete $m_f \neq 0$</th>
<th>complete $m_f = 0$</th>
<th>IBA $(m_f = 0)$</th>
<th>Branching ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(W \rightarrow e\nu_e)$</td>
<td>0.2262</td>
<td>0.2255</td>
<td>0.2255</td>
<td>0.2262</td>
<td>0.1083</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \mu\nu_{\mu})$</td>
<td>0.2262</td>
<td>0.2255</td>
<td>0.2255</td>
<td>0.2262</td>
<td>0.1083</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \tau\nu_{\tau})$</td>
<td>0.2261</td>
<td>0.2255</td>
<td>0.2255</td>
<td>0.2262</td>
<td>0.1082</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{lep.})$</td>
<td>0.6785</td>
<td>0.6763</td>
<td>0.6765</td>
<td>0.6787</td>
<td>0.3249</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{ud})$</td>
<td>0.6455</td>
<td>0.6684</td>
<td>0.6684</td>
<td>0.6708</td>
<td>0.3211</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{us}) \times 10^4$</td>
<td>0.3315</td>
<td>0.3432</td>
<td>0.3432</td>
<td>0.3444</td>
<td>0.0165</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{ub}) \times 10^4$</td>
<td>0.1080</td>
<td>0.1122</td>
<td>0.1124</td>
<td>0.1128</td>
<td>0.000005</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{cd}) \times 10^2$</td>
<td>0.3312</td>
<td>0.3431</td>
<td>0.3432</td>
<td>0.3444</td>
<td>0.0165</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{cs})$</td>
<td>0.6441</td>
<td>0.6672</td>
<td>0.6673</td>
<td>0.6697</td>
<td>0.3205</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{cb}) \times 10^2$</td>
<td>0.1080</td>
<td>0.1121</td>
<td>0.1124</td>
<td>0.1128</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{had.})$</td>
<td>1.3569</td>
<td>1.4054</td>
<td>1.4055</td>
<td>1.4104</td>
<td>0.6751</td>
</tr>
<tr>
<td>$\Gamma(W \rightarrow \text{all})$</td>
<td>2.0354</td>
<td>2.0817</td>
<td>2.0820</td>
<td>2.0891</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Partial and total W-decay widths $\Gamma_W$ in different approximations given in GeV.

$$
\Gamma_{W_{ud},j} \approx \Gamma_{W_{ud},j}^{\text{Born}} \left( 1 + \frac{\alpha_s(M_W^2)}{\pi} + \frac{G_{\mu}M_W^2}{2\sqrt{2}\pi} |V_{ij}|^2 \right) = \Gamma_{W_{ud},j}^{\text{Born}} \left( 1 + \frac{\alpha_s(M_W^2)}{\pi} \right),
$$

$$
\Gamma_W \approx \Gamma_W^{\text{Born}} \left( 1 + \frac{2\alpha_s(M_W^2)}{3\pi} \right) = \Gamma_W^{\text{Born}} \left( 1 + \frac{2\alpha_s(M_W^2)}{3\pi} \right). \tag{41}
$$

In Table 7 we compare the improved Born approximation (IBA) for the partial and total widths, given by equation (41), with the lowest-order widths, the widths including the complete first-order and leading higher-order corrections for finite fermion masses, and the same for vanishing fermion masses, all in the $G_{\mu}$ parametrization. The effects of the fermion masses, which are of the order $m_f^2/M_W^2$, are below 0.3%. Consequently the exact numerical values for the masses of the external fermions are irrelevant. The IBA reproduces the exact results within 0.4% (0.6% for the decays into a b-quark). The branching ratios for the individual decay channels derived from equation (41), which depend only on $\alpha_s$ and $V_{ij}$, agree numerically within 0.1% with those obtained from the full one-loop results.
Table 8: Number of Feynman diagrams for W-pair produced four-fermion final states.

3 Off-Shell W-Pair Production

3.1 Lowest order: an introduction

So far we have only considered the production of stable W bosons. This is, however, only an approximation and in particular in the threshold region it is not sufficient. Rather, one has to describe the W bosons as resonances, with a finite width so as to avoid singularities inside the physical phase space, and analyse their presence through their decay products:

$$e^+ + e^- \rightarrow W^+ + W^- \rightarrow f_1 + \bar{f}_2 + f_3 + \bar{f}_4.$$  \hspace{1cm} (42)

Process (42) involves two resonant W bosons (doubly-resonant) and can be viewed as a very natural first step beyond the on-shell limit. In lowest order this process is represented by the three Feynman diagrams shown in figure 4. However, the full four-fermion process does not only proceed through the three doubly-resonant diagrams. There are also contributions from other diagrams with the same initial and final states, but different intermediate states. Classifications of four-fermion production processes and of the contributing diagrams are given in [41, 42]. In table 8 we give the number of diagrams contributing for final states that can be reached by W-pair intermediate states. These so-called charged-current processes are sometimes referred to as CCn, with n denoting the number of contributing diagrams [e.g. CC3 denotes process (42)]. The simplest case (boldface numbers in table 8) is fully covered by doubly- and singly-resonant diagrams as given in figures 4 and 5. Additional graphs of the types shown in figure 6.
must be taken into account if electrons or electron-neutrinos are produced (roman numbers in table 8). If the produced final state consists of particle–antiparticle pairs, the final state can also be obtained through intermediate Z-pair production, leading to extra Feynman diagrams (italic numbers in table 8). Finally, it should be mentioned that QCD diagrams, involving an intermediate gluon, contribute in the case of final states consisting of two quark–antiquark pairs.

3.2 Semi-analytical approach

We will now, in a first step of sophistication beyond on-shell W-pair production, introduce a finite W width and perform the CC3 calculation. In the following a semi-analytical method will be emphasized, whereas Monte Carlo methods are treated elsewhere [22]. The starting point is

\[
\sigma^{\text{CC3}}(s) = \int ds_+ \rho_W(s_+) \int ds_- \rho_W(s_-) \sigma_0^{\text{CC3}}(s; s_+, s_-),
\]

with \(s_+ = k_+^2\) and \(s_- = k_-^2\) the virtualities of the internal W bosons. The Breit-Wigner densities

\[
\rho_W(s_{\pm}) = \frac{1}{\pi} \frac{M_W \Gamma_W}{|s_{\pm} - M_W^2 + iM_W \Gamma_W|^2} \times \text{BR}
\]

Figure 5: Example of a singly-resonant diagram.

Figure 6: Examples of additional diagrams for final states with electrons and electron-neutrinos.
contain the finite width of the W boson, the coupling constants of its decay to fermions, and
the corresponding branching ratio. In the limit of stable W bosons, the on-shell cross-section
is recovered via
\[
\rho_W(s_\pm) \xrightarrow{W \rightarrow 0} \delta(s_\pm - M_W^2) \times \text{BR}.
\]  
(45)
The two-fold differential cross-section contains terms corresponding to the Feynman diagrams
of figure 4 and their interferences. In the notation of [43] it may be described by three terms,
\[
\sigma_0^{CC_3}(s_1; s_+, s_-) = \left(\frac{G_\mu M_W^2}{8\pi s}\right)^2 C_{CC_3} G_{CC_3}^{33} + C_{CC_3}^{st} G_{CC_3}^{3f} + C_{CC_3}^{tt} G_{CC_3}^{ff}.
\]  
(46)
The couplings \(C_{CC_3}^{st,t}\) contain the weak mixing angle, the Z–e vector and axial-vector coupling,
the triple-gauge-boson couplings, and the s-channel Z and γ propagators. The kinematical
functions \(G_{CC_3}^{33,3f,ff}\) are known analytically [21]:
\[
G_{CC_3}^{ff}(s_1; s_+, s_-) = \frac{1}{48} \lambda(s, s_+, s_-) + 12 s (s_+ + s_-) - 48 s_+ s_-
+ 24 s_+ (s - s_+ - s_-) \mathcal{L}(s; s_+, s_-),
\]  
\[
G_{CC_3}^{33}(s_1; s_+, s_-) = \frac{\lambda(s, s_+, s_-)}{192} \lambda(s, s_+, s_-) + 12 \left( s s_+ + s s_- + s_+ s_- \right),
\]  
\[
G_{CC_3}^{3f}(s_1; s_+, s_-) = \frac{1}{48} \left( s - s_+ - s_- \right) \lambda(s, s_+, s_-) + 12 s \left( s s_+ + s s_- + s_+ s_- \right)
- 24 s_+ s_- \left( s s_+ + s s_- + s_+ s_- \right) \mathcal{L}(s; s_+, s_-),
\]  
where
\[
\lambda(s, s_+, s_-) = s^2 + s_+^2 + s_-^2 - 2 s s_+ - 2 s s_- - 2 s_+ s_-,
\]  
\[
\mathcal{L}(s; s_+, s_-) = \frac{1}{\lambda(s, s_+, s_-)} \log \frac{s - s_+ - s_-}{s - s_+ - s_-}. \]  
(51)
The numerical importance of the W-boson off-shellness in equation (43) is displayed in figure 7.
Clearly, the effect of the finite W width is comparable to the one from universal initial-state
radiation, defined in section 3.4.1.

The second level of sophistication of W-pair production is reached by performing calculations
for complete sets of Feynman diagrams for specific final states. The simplest case, the CC11
class of processes, corresponds to four-fermion final states without electrons or electron-neutrinos
and without particle–antiparticle pairs. The corresponding gauge-invariant set of Feynman
diagrams consists of the three doubly-resonant diagrams of figure 4 plus at most eight singly-
resonant diagrams of the type shown in figure 5. This class of processes is still calculable in

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the semi-analytical approach and is of specific interest for the W-mass determination and the TGC studies. The corresponding cross-section is given by [43]

\[
\sigma^{\text{CC3}}(s) = \int ds_+ \, ds_- \frac{\lambda(s, s_+, s_-)}{\pi s^2} \frac{\lambda_{ij}(s, s_+, s_-)}{\pi s^2} \frac{d^2\mathcal{G}_k(s, s_+, s_-)}{ds_+ \, ds_-} \]

(52)

with

\[
\frac{d^2\mathcal{G}_k}{ds_+ \, ds_-} = C_k \mathcal{G}_k(s, s_+, s_-). \quad (53)
\]

Initial- and final-state couplings, as well as intermediate W, Z, and γ propagators are collected in the \(C_k\)'s. The kinematical details are contained in the \(\mathcal{G}_k\)'s. Because of symmetry properties, under the exchanges of the three virtualities, only six of these kinematical functions are independent. Apart from the three kinematical functions of the CC3 process, three more enter as a result of the singly-resonant diagrams. Explicit expressions can be found in [43].

A similar semi-analytical analysis of the remaining four-fermion final states has not been performed so far, although this would be desirable.
3.3 Lowest order: the gauge-invariance issue

The discussion in the previous subsections has avoided the question of gauge invariance which arises when going from on-shell W-pair production to the off-shell case. There are two sources of gauge non-invariance one has to be aware of.

First of all there is the issue of gauge non-invariance as a result of incomplete sets of contributions. Consider to this end again the process

\[ e^+ + e^- \rightarrow W^+ + W^- \rightarrow f_1 + f_2 + f_3 + f_4, \]  

(54)

represented by the three Feynman diagrams shown in figure 4. Forgetting about the finite W width the corresponding matrix element can be written in the following form:

\[
\mathcal{M}^\kappa_{\text{Born}} = \delta_{\kappa-} \frac{e^2}{2s_W^2} \mathcal{M}^{\kappa, \rho\sigma}_{I} + e^2 \mathcal{M}^{\kappa, \rho\sigma}_{Q}
\]

\[
\times \frac{1}{k_+^2 - M_W^2} \frac{1}{\sqrt{2s_W}} \tilde{u}_1 \gamma_\rho \omega_- v_2 \times \frac{1}{k_-^2 - M_W^2} \frac{e}{\sqrt{2s_W}} \tilde{u}_3 \gamma_\sigma \omega_- v_4,
\]

(55)

where

\[
\mathcal{M}^{\kappa, \rho\sigma}_{I} = \frac{2M_Z^2}{s(s-M_Z^2)} \tilde{v}_+ \left( \frac{k_+ + g^{\rho\sigma}}{s-M_Z^2} - \gamma^\rho k_+^\sigma + \gamma^\sigma k_-^\rho \right) \omega_\kappa u_-,
\]

\[
\mathcal{M}^{\kappa, \rho\sigma}_{Q} = \frac{1}{t} \tilde{v}_+ \gamma^\rho \left( \frac{k_+ + \gamma}{s} \right) \gamma^\sigma \omega_\kappa u_- - \frac{s}{M_Z^2} \mathcal{M}^{\kappa, \rho\sigma}_{Q}
\]

(56)

are directly related to the corresponding on-shell quantities \( \mathcal{M}_I \) and \( \mathcal{M}_Q \) defined in equation (15). Here \( k_\pm \) are the momenta of the \( W^\pm \) bosons as reconstructed from the decay products, and \( s \) and \( t \) the usual Mandelstam variables for \( e^+e^- \rightarrow W^+W^- \). We have assumed that all fermion masses are negligible. In a renormalizable gauge such as a \( R_\xi \) gauge one would otherwise have to consider in addition diagrams involving one or two unphysical charged Higgs bosons instead of the W bosons.

Whereas for on-shell W bosons the contributions of \( \mathcal{M}_I \) and \( \mathcal{M}_Q \) are separately gauge-invariant, for off-shell W bosons not even \( \mathcal{M}_{\text{Born}} \) is gauge-invariant. This can be illustrated by considering the doubly-resonant diagrams in an axial gauge [9]. To this end we replace the conventional \( \text{t'} \) Hooft–Feynman gauge-fixing term for the W boson by \( -(n^\mu W^+_{\mu})(n^\nu W^-_{\nu}) \). This modifies the W-boson propagators in the diagrams of figure 4 and, since this gauge-fixing term does not eliminate the mixing between the W-boson field and the corresponding unphysical Higgs field, leads to mixing propagators between those fields, giving rise to additional Feynman diagrams. Combining all relevant diagrams yields the following extra contribution to \( \mathcal{M}^\kappa_{\text{Born}} \) in the axial gauge:

\[
\mathcal{M}^\kappa_{\text{Born, axial gauge}} - \mathcal{M}^\kappa_{\text{Born, \text{t'} Hooft–Feynman gauge}} = e^2 \delta_{\kappa-} \frac{1}{2s_W^2} - \frac{M_Z^2}{s} \times \frac{1}{k_-^2 - M_W^2} \frac{e}{\sqrt{2s_W}} \tilde{u}_3 \gamma_\sigma \omega_- v_4.
\]

(57)
Note that these gauge-dependent terms involve either a pole in \( (k_+^2 - M_W^2) \) or \( (k_0^2 - M_W^2) \) but not both, i.e. they are only singly-resonant. These terms are exactly cancelled by the gauge-dependent contributions of eight singly-resonant diagrams contributing to the same final state with the topology shown in figure 5.

The sum of the doubly-resonant diagrams and the singly-resonant ones with the topology shown in figure 5 is in general gauge-invariant. This follows directly from the fact that for final states with four different fermions and no electrons or positrons those diagrams are the only ones that contribute. As a consequence the non-resonant diagrams are not needed to cancel the gauge-dependent terms of the doubly-resonant diagrams. Among the non-resonant diagrams and the singly-resonant ones that do not have the topology shown in figure 5 further gauge-invariant subsets can be identified by considering other four-fermion final states.

Thus, in general all graphs that contribute to a given final state have to be taken into account and one is lead to consider the complete process \( e^+e^- \rightarrow 4f \), including all resonant and non-resonant graphs, in order to obtain a manifestly gauge-independent result.

Simple estimates indicate that all non-doubly-resonant contributions are typically suppressed by a factor \( \Gamma_W/M_W \approx 2.5\% \) on the cross-section level for each non-resonant W propagator\(^4\). This is confirmed by explicit calculations \([22]\). For instance, using covariant (\( R^\perp \)) gauges the universal non-doubly-resonant graphs that occur for all final states (see figure 5) contribute less than 0.15\% to the total cross-section for \( 175 \text{ GeV} < \sqrt{s} < 205 \text{ GeV} \), and 0.3\% at 161 GeV. The contribution of the non-doubly-resonant \( t \)-channel photon-exchange graphs, which only occur when there are electrons in the final state, depends very much on the angular cut imposed on the outgoing electrons; for \( \theta^0 \) they contribute at the per-cent level, e.g. \( \sim 4\% \) at \( \sqrt{s} = 190 \text{ GeV} \). Although all these non-doubly-resonant contributions are suppressed, they should nevertheless be taken into account to reach a theoretical accuracy of \( \sim 0.5\% \). Finally, the QCD graphs have been shown not to interfere with the electroweak graphs to any sizable extent, and can thus be computed using standard (Monte Carlo) QCD programs.

Even when considering the complete set of graphs contributing to a given final state, there is still a more fundamental gauge-invariance problem to be solved. The resonant graphs discussed above involve poles at \( k_+^2 = M_W^2 \). These have to be cured by introducing the finite width in one way or another, while at the same time preserving gauge independence and, through a proper energy dependence, unitarity. In field theory, such widths arise naturally from the imaginary parts of higher-order diagrams describing the boson self-energies, resummed to all orders. This procedure has been used with great success in the past: indeed, the Z resonance can be described to very high numerical accuracy. However, in doing a Dyson summation of self-energy graphs, we are singling out only a very limited subset of all the possible higher-order

\(^4\)For differential distributions that do not involve an explicit phase, like e.g. \( \sigma_{\text{tot}} \) or \( d\sigma/(d\cos \theta) \), there will be no interference between doubly- and singly-resonant diagrams. Consequently the non-doubly-resonant contributions are suppressed by \( \Gamma_W^2/M_W^2 \approx 0.1\% \) in those cases \([44]\).

\(^m\)There are similar poles associated with diagrams containing internal Z propagators. These correspond to Z-pair production and are here considered as background to W-pair production.
diagrams. It is therefore not surprising that one often ends up with a result that retains some
gauge dependence.

For example it is very tempting to systematically replace $1/(q^2 - M^2)$ by $1/(q^2 - M^2 + iMT)$,
also for $q^2 < 0$. Here $\Gamma$ denotes the physical width of the particle with mass $M$ and momentum $q$.
This is the so-called ‘fixed-width scheme’. As in general the resonant diagrams are not
gauge-invariant by themselves, this substitution will again destroy gauge invariance. Moreover,
the ‘fixed-width scheme’ has no physical motivation. In perturbation theory the propagator for
space-like momenta does not develop an imaginary part. Consequently, unitarity is violated
in this scheme. To improve on the latter the constant width could be replaced by a running one.
This can, however, not cure the gauge non-invariance problem. At this point one might
ask oneself the legitimate question whether the gauge breaking occurring in the ‘fixed-width
scheme’ is numerically relevant or, like the gauge breaking in the LEP1 analyses, negligible for
all practical purposes. Of course, such a statement can only be made on the basis of an explicit
comparison with truly gauge-invariant schemes in the full LEP2 energy range. We will come
back to that point later on, after having defined a scheme that is gauge-invariant and reliable
at LEP2.

Below we will list a few ways to come to a gauge-invariant result and discuss their validity.

One way to sidestep the gauge non-invariance problem is by simply multiplying the full
matrix element by $[q^2 - M^2]/[q^2 - M^2 + iMT(q^2)]$, which is evidently gauge-invariant [45, 46].
In this way the pole at $q^2 = M^2$ is softened into a resonance, at the expense of mistreating
the non-resonant parts. It should be noted that when the doubly-resonant diagrams are not
dominant, like at energies at and below the $W$-pair production threshold, this so-called ‘fudge-
factor scheme’ can lead to large deviations [47].

The second possibility is the so-called ‘pole scheme’ [23]-[25]. In this scheme one decomposes
the complete amplitude, consisting of contributions from doubly-resonant diagrams $R_{+-}$
(corresponding in lowest order to $\mathcal{M}_{\text{Born}}$), singly-resonant diagrams $R_+, R_-$, and non-resonant
diagrams $N$, according to their poles as follows:

$$
\mathcal{M} = \frac{R_{+-}(k_+^2, k_-^2, \theta)}{(k_+^2 - M_W^2)(k_-^2 - M_W^2)} + \frac{R_+(k_+^2, k_-^2, \theta)}{k_+^2 - M_W^2} + \frac{R_-(k_+^2, k_-^2, \theta)}{k_-^2 - M_W^2} + N(k_+^2, k_-^2, \theta)
$$

$$
= \frac{R_{+-}(M_W^2, k_+^2, \theta) - R_{+-}(M_W^2, M_W^2, \theta)}{k_+^2 - M_W^2} + \frac{R_+(M_W^2, k_-^2, \theta)}{k_+^2 - M_W^2} + \frac{R_-(M_W^2, k_+^2, \theta) - R_-(M_W^2, M_W^2, \theta)}{k_-^2 - M_W^2}
$$

$$
+ \frac{R_{+-}(k_+^2, k_-^2, \theta) + R_{+-}(M_W^2, M_W^2, \theta) - R_{+-}(M_W^2, k_-^2, \theta) - R_{+-}(k_+^2, M_W^2, \theta)}{(k_+^2 - M_W^2)(k_-^2 - M_W^2)}
$$

$$
+ \frac{R_+(k_+^2, k_-^2, \theta) - R_+(M_W^2, k_-^2, \theta)}{k_+^2 - M_W^2}
$$

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would introduce an additional dependence on their integration boundaries are independent of $k_+^2$ and $k_-^2$. Otherwise these angular variables would introduce an additional dependence on $k_+^2$ and $k_-^2$, and the correct pole terms could only be extracted after the angular integrations had been performed. This would complicate the pole decomposition and, in particular, would not be suited for a Monte-Carlo generator. Appropriate variables are e.g. the angles in the $e^+e^-$-CM system but not the Mandelstam variables. In equation (58) $M$ is decomposed into gauge-invariant subsets originating from double-pole terms, single-pole terms, and non-pole terms (with respect to $M_W$). Introducing now the finite width only in the pole factors and not in the finite constant residues in brackets does not destroy gauge invariance. We note that different ways of introducing the finite width, e.g. constant or running, differ only by terms that are of higher-order and/or that are not of the double-pole type.

The same holds for different choices of the angular variables $\theta$ (see section 3.4.4).

A drawback of the ‘pole scheme’ is the fact that it is not defined below the $W$-pair production threshold and that it yields unreliable results just above this threshold (see section 3.4.4).

Apart from yielding gauge-invariant results, the ‘pole-scheme’ decomposition also constitutes a systematic expansion according to the degree of resonance, i.e. in powers of $\Gamma_W/(M_W\beta)$. Here the enhancement factor $1/\beta$ represents the influence of the nearby threshold on the expansion. Sufficiently far above the $W$-pair threshold and after imposing appropriate angular and invariant mass cuts, in order to reduce $Z$-pair and $t$-channel photon-exchange backgrounds, the cross-section for off-shell $W$-pair production is dominated by (or may even be defined by) the double-pole terms $R_{+-}(M_W^2, M_W^2, \theta)$. At least at lowest order these may be related to on-shell $W$-pair production in the following way:

$$R_{+-}(M_W^2, M_W^2, \theta) = \frac{R_{+-}(k_+^2, k_-^2, \theta) - R_{+-}(k_+^2, M_W^2, \theta)}{k_+^2 - M_W^2} + N(k_+^2, k_-^2, \theta)$$  \hspace{1cm} (58)

where $\lambda_{++,\lambda_{--}}$, $\lambda_{+\lambda_+}$, $\lambda_{-\lambda_-}$ denote the matrix elements for the production of two on-shell $W$ bosons and their subsequent decay into fermion–antifermion pairs. As such the ‘pole scheme’ is a natural starting point for the systematic evaluation of higher-order corrections [9, 25] (see also section 3.4.4).

As a third method, one may determine the minimal set of Feynman diagrams that is necessary to compensate for the gauge violation caused by the self-energy graphs, and try to include these [48, 49]. This is obviously the theoretically most satisfying solution, but it may cause

\[ \text{There is no unique prescription for the propagator including the finite width. Instead of the constant term } M_W \Gamma_W \text{ one can also use a width depending on the invariant mass } k_+^2. \text{ This corresponds to different definitions of the (renormalized) } W \text{ mass, which is fixed by the pole of the propagator. A popular choice is } k^2 \Gamma_W/M_W, \text{ which amounts to a shift in } M_W \text{ by } \Gamma_W^2/(2M_W) \approx 26 \text{ MeV relative to the propagator with the constant-width term.} \]
an increase in the complexity of the matrix elements and a consequent slowing down of the numerical calculations. For the vector bosons, the lowest-order widths are given by the imaginary parts of the fermion loops in the one-loop self-energies. It is therefore natural to include the other possible one-particle-irreducible fermionic one-loop corrections [47, 50, 51]. For the process $e^+e^- \rightarrow 4f$ this amounts to adding the fermionic triple-gauge-boson vertex corrections. The complete set of fermionic contributions form a gauge-independent subset and obey all Ward identities exactly, even with resummed propagators [52]. This implies that the high-energy and collinear limits are properly behaved. In contrast to all other schemes mentioned above, the ‘fermion-loop scheme’ recommended here does not modify the theory by hand but selects an appropriate set of higher-order contributions to restore gauge invariance. To solve the problem of gauge invariance related to the width, we in fact only have to consider the imaginary parts of these fermionic contributions.

The ‘fermion-loop scheme’ should work properly for all tree-level calculations involving resonant W bosons and Z bosons or other particles decaying exclusively into fermions. This also includes, for instance, the hard-photon process $e^+e^- \rightarrow 4f + \gamma$, which requires in addition to fermionic vertex corrections also fermionic box corrections. For resonating particles decaying also into bosons, such as the top quark, or for calculating RC’s to $e^+e^- \rightarrow 4f$, which also involves bosonic corrections, the ‘fermion-loop scheme’ is not really suited.

Although the latter scheme is well-justified in standard perturbation theory, it should be stressed that any working scheme is arbitrary to a greater or lesser extent: since the Dyson summation must necessarily be taken to all orders of perturbation theory, and we are not able to compute the complete set of all Feynman diagrams to all orders, the various schemes differ even if they lead to formally gauge-invariant results. In [53] another technique, the so-called ‘pinch technique’, has been introduced in order to construct a gauge-parameter-independent Dyson summation. Even if the ‘pinch technique’ yields gauge-independent results, it still contains some arbitrariness in the sense that one still has the freedom to shift gauge-independent parts that fulfill the Ward identities from the vertex corrections to the self-energies. For instance, it has been demonstrated in [54] that the ‘background-field method’ can be used to construct an infinite variety of such shifts, all representing (theoretically) equally well-justified schemes for resumming self-energies.

Now it is a numerical question how much the predictions of different schemes differ. In [51] a detailed study has been given for the process $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$, a process that is highly sensitive to $U(1)$ electromagnetic gauge violation. In this process the electron may emit a virtual photon, whose $k^2$ can be as small as $m_e^2$: with a total centre-of-mass energy of $\sqrt{s}$ available, we have a mass ratio of $s/m_e^2 = O(10^{11})$, large enough to amplify even a tiny gauge violation in a disastrous way. In table 9 we give the cross-section corresponding to the $t$-channel photon-exchange diagrams, responsible for the amplification of the gauge-breaking terms in the collinear limit. The results are given for two values of the minimum electron scattering angle

---

As the Ward identities are linear, we can separate the real and imaginary parts.

This was noted already in [55], and investigated further in [46].
\( \theta_{\text{min}}, \) displaying the effect of cutting away the dangerous collinear limit. It is clear that a naive introduction of a running width without a proper inclusion of fermionic corrections to the three-vector-boson vertex, which breaks \( U(1) \) electromagnetic gauge invariance, leads to completely unreliable results. The above-described gauge-invariant methods as well as the \( U(1) \)-preserving 'fixed-width scheme' numerically deviate by much less than \( \Gamma_W/M_W \). Hence, a naive running width is not suited for LEP2, whereas a constant width, although \( SU(2) \times U(1) \) gauge breaking, might constitute a workable approach. As in the collinear limit \( k^2 \rightarrow 0 \) the gauge-breaking terms originating from a naive running width are proportional to the dominant lowest-order graphs, it is possible to multiply the \( \gamma WW \) Yang-Mills vertex with a simple factor to successfully restore the \( U(1) \) gauge invariance. This factor is, however, certainly not universal. It will depend on the way the running width is introduced and on the process under investigation. Moreover, such a simple factor breaks unitarity and at high energies the full expression from the fermion loops is required for having a proper energy dependence.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>( \sigma \text{ [pb]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{\text{min}} = 0^\circ )</td>
<td>( \theta_{\text{min}} = 10^\circ )</td>
</tr>
<tr>
<td>Fixed width</td>
<td>0.0887(8)</td>
</tr>
<tr>
<td>Running width, no correction</td>
<td>60738(176)</td>
</tr>
<tr>
<td>Fudge factor, with running width</td>
<td>0.08892(8)</td>
</tr>
<tr>
<td>Pole scheme, with running width</td>
<td>0.08921(8)</td>
</tr>
<tr>
<td>Fermion-loop scheme</td>
<td>0.08968(8)</td>
</tr>
</tbody>
</table>

Table 9: Cross-section in different schemes for the \( t \)-channel photon-exchange diagrams of \( e^+e^- \rightarrow e^-\nu_\mu u\bar{d} \). All schemes were computed using the same sample, so the differences are much more significant than the integration error suggests.

### 3.4 Radiative corrections

So far no complete treatment for the \( \mathcal{O}(\alpha) \) corrections to off-shell \( W \)-pair production is available. Essentially only the initial-state photonic corrections, the final-state Coulomb correction, and the full hard process \( e^+e^- \rightarrow 4f+\gamma \) have been treated so far. These are discussed in the following. The leading weak effects are normally taken into account through dressed lowest-order matrix elements, using \( G_\mu \) and \( \alpha(s) \). In addition we describe a general strategy for the calculation of corrections beyond the lowest order using the 'pole scheme'.

---

\( ^9 \) An evaluation of all resummed one-particle-irreducible fermionic \( \mathcal{O}(\alpha) \) corrections, in the context of the 'fermion-loop scheme', is in progress [52].

\( ^* \) The uncertainty associated with different theoretical definitions of \( s_W^2 \), i.e. \( s_W^2 = 1 - M_W^2/M_Z^2 \) or \( s_W^2 = \pi \alpha(4 M_W^2) / (\sqrt{2} G_\mu M_W^2) \), are found to be below 0.1%.
3.4.1 Initial-state radiation

Most of the published calculations for corrections to off-shell W-pair production that have been done so far cover only part of the photonic corrections, mainly because these are easily treatable and constitute a large part of the RC's. From the discussion of the on-shell process we know that initial-state radiation (ISR) yields large corrections originating from the leading collinear logarithms. As will be discussed in appendix A these can be easily obtained by applying the structure-function method. Just in the same way as in the on-shell case the corrections to the off-shell cross-section are calculated by convoluting the lowest-order cross-section (in a certain gauge-invariant scheme) with the appropriate structure functions. In appendix A a detailed analysis is given of the theoretical uncertainties associated with the leading-log procedure.

In the present-day Monte Carlos different ways of implementing the leading logarithmic corrections have been adopted [22]. One method involves solving the evolution equations for the structure functions numerically using techniques known from parton-shower algorithms. In this way soft-photon exponentiation and resummation of the leading logarithms from multiple hard-photon emission are automatically taken into account. Photons are generated according to the matrix elements in the collinear limit, in this way allowing for a finite $p_T$ kick to the photon. The second method involves a fully inclusive treatment of the radiated photons by folding the improved lowest-order cross-section with leading-log structure functions. So, no explicit photons are generated. The most recent development is a kind of merger of an explicit $e^+e^- \rightarrow 4f\gamma$ Monte Carlo folded with structure functions, allowing for a consistent definition of observable and unobservable photon radiation (see section 3.4.3).

Another approach is to include the complete ISR. To this end one has to define it in a way that preserves the $U(1)$ electromagnetic gauge invariance. This is non-trivial because of the presence of the $t$-channel diagram which involves a non-conserved charge flow in the initial state. One technique to circumvent this problem is the so-called current-splitting technique [56], which amounts to splitting the electrically neutral neutrino in the $t$-channel diagram into two oppositely flowing leptons each with charge one. One of them is attributed to the initial state to build a continuous flow of charge, the other is attributed to the final state to do the same there. The modified $\nu_e$ propagator leads to additional real and virtual initial-state photonic diagrams which render the ISR gauge-invariant. In this way one obtains the usual universal $s$-channel ISR known from LEP1 plus additional non-universal contributions arising from the $t$-channel diagrams. The latter are non-factorizing with respect to the lowest-order cross-section, but are screened by a factor $k_1^2 k_2^2 / s^2$, which automatically guarantees a unitary behaviour at high energies. Moreover they turn out to be numerically small at LEP2 energies. The universal ISR contains all leading logarithms and can be supplemented by the known universal higher-order terms [57].

It should be noted that both the leading-log and the current-splitting method leave out non-leading photonic corrections, for instance associated with radiation of photons off the intermediate W bosons. For on-shell W bosons we give in table 10 a comparison of exact and leading-log evaluations of the quantity $\frac{\Delta E/\gamma}{E/\gamma} (d\sigma/dE/\gamma)$, needed for the average energy loss
Table 10: The quantity $dE_\gamma E_\gamma (d\sigma/dE_\gamma)$ needed for the average energy loss. The exact on-shell result from hard-photon radiation is given as well as the corresponding leading-log approximation, using $L = \log(Q^2/m_\gamma^2)$ or $L - 1$.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>$dE_\gamma E_\gamma \frac{d\sigma}{dE_\gamma}$ [GeV · pb]</th>
<th>leading-log results $Q^2 = s$</th>
<th>$Q^2 = s$</th>
<th>$Q^2 = Q_0^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>161.0</td>
<td>0.1377 ± 0.0001</td>
<td>0.1436</td>
<td>0.1379</td>
<td>0.1375</td>
</tr>
<tr>
<td>165.0</td>
<td>3.619 ± 0.002</td>
<td>3.792</td>
<td>3.642</td>
<td>3.607</td>
</tr>
<tr>
<td>170.0</td>
<td>10.120 ± 0.006</td>
<td>10.651</td>
<td>10.232</td>
<td>10.089</td>
</tr>
<tr>
<td>175.0</td>
<td>17.437 ± 0.011</td>
<td>18.431</td>
<td>17.708</td>
<td>17.403</td>
</tr>
<tr>
<td>184.0</td>
<td>30.882 ± 0.029</td>
<td>32.873</td>
<td>31.589</td>
<td>30.902</td>
</tr>
<tr>
<td>190.0</td>
<td>39.562 ± 0.037</td>
<td>42.224</td>
<td>40.578</td>
<td>39.595</td>
</tr>
<tr>
<td>205.0</td>
<td>59.126 ± 0.076</td>
<td>63.581</td>
<td>61.117</td>
<td>59.322</td>
</tr>
</tbody>
</table>

$\langle E_\gamma \rangle$. It is clear that a sufficiently accurate leading-log evaluation should be based on $L - 1$ rather than $L$. Furthermore, the often-used scale choice $Q^2 = s$ leads to deviations of about 3% at 190 GeV, which would translate into an error of $\sim 60$ MeV on $\langle E_\gamma \rangle$. The scale choice $Q_0^2 = 4M_W E/(1 + \beta)$, however, agrees with the exact result at the 0.3% level for all LEP2 energies, leading to errors below 10 MeV in $\langle E_\gamma \rangle$. As there is no obvious reason why these non-leading terms should be smaller in the off-shell case, some care has to be taken with the choice of a suitable leading-log scale.

3.4.2 The Coulomb singularity

Another potentially large photonic correction, not associated with the initial state, is due to the Coulomb singularity in the threshold region, which has been discussed for off-shell W-pair production in [58]-[60]. It originates from the IR limit and at one-loop it emerges from a single IR-singular scalar three-point function and a related IR-singular scalar four-point function, the IR-singular part of which is just a scaled version of the one contained in the three-point function (see figure 8). The gauge invariance of the coefficients of these two scalar functions allows us

$^*$The (lowest-order) average energy loss $\langle E_\gamma \rangle$ can be obtained by normalizing to the lowest-order cross-section of table 2. Dressing this lowest-order cross-section by LL structure functions, running couplings etc., only makes sense when the energy-weighted cross-section appearing in the numerator is treated in the same way!
Figure 8: Diagrams that contribute to the Coulomb singularity.

To include the width in a gauge-invariant way. Thus one obtains a gauge-invariant correction factor to the lowest-order cross-section resulting from the doubly-resonant diagrams:

\[
\sigma_{\text{Coul}} = \sigma_{\text{Born}} \frac{\alpha \pi}{2 \beta} \left( 1 - \frac{2}{\pi} \arctan \frac{|\beta_M + \Delta|^2 - \beta^2}{2 \beta \Im \beta_M} \right),
\]

with

\[
\beta = \frac{1}{s} \frac{s^2 - 2s(k_+^2 + k^-_2) + (k_+^2 - k_-^2)^2}{s^2 - 2s(k_+^2 + k_-^2) + (k_+^2 - k_-^2)^2},
\]

\[
\beta_M = \frac{1 - 4 M^2}{s}, \quad M^2 = M_W^2 - i M_W \Gamma_W - i \epsilon,
\]

\[
\Delta = \frac{|k_+^2 - k_-^2|}{s},
\]

and \(-\pi/2 < \arctan y < \pi/2\). Here \(\bar{\beta}\) is the average velocity of the W bosons in their centre-of-mass system. Equation (60) only refers to the Coulomb singularity; the finite remnants of the above mentioned three-point function, containing also the IR divergence, have been left out. From equation (60) we obtain the usual on-shell Coulomb singularity for stable W bosons and \(\bar{\beta} \neq 0\) by first going on-shell, i.e. \(\Delta = 0\) and \(\bar{\beta}^2 = \Re \beta_M^2 = \beta^2\), and subsequently taking the limit \(\Gamma_W \rightarrow 0\). Note that in this limit the otherwise negligible \(i \epsilon\) in \(M^2\) becomes relevant.

In contrast to the on-shell case there are various effects present in equation (60) that effectively truncate the range of the Coulomb interaction. The presence of a Coulomb singularity requires that \(\bar{\beta}\) be at least of the same order of magnitude as \(\Delta\) and \(|\beta_M|\), which is bounded by the W width according to \(|\beta_M| \lesssim \Gamma_W/M_W\). This confirms the intuitive argument [58] that the Coulomb singularity is modified substantially by finite-width effects if the characteristic time of the Coulomb interaction \((t_{\text{Coul}} \sim 1/|\beta^2 M_W|)\) is of the same order as or larger than the typical decay time \((t_r \sim 1/\Gamma_W)\) of the W bosons. While the effect of the finite W width is contained in \(\beta_M\), the off-shellness shows up in \(\Delta\) and in the difference \(|\beta_M^2 - \bar{\beta}^2|\), which effectively involves \(|k_+^2 + k_-^2 - 2M_W^2|/s\). This is in agreement with the argument [59] that the off-shellness of the virtual W bosons will affect the Coulomb singularity, if the typical times \((t_{\text{off}}^\pm \sim 1/|k_{\pm|0} - M_W^2 + \bar{k}_\pm^2| \sim 1/|k_{\pm|0} - M_W|)\) for which off-shell \(W^\pm\) bosons with four-momenta \(k_\pm\) exist is of the same order as or smaller than \(t_{\text{Coul}}\). It should be noted that as a
result of the convolution with the Breit-Wigner functions from the W resonances, the quantity \( \Delta \) plays a negligible role in an adequate description of the Coulomb phenomenon \([59]\). Consequently the at first sight deviating formulae of \([59]\) and \([60]\) constitute equally well-justified representations of this phenomenon.

In the case of the total CC3 cross-section, the Coulomb singularity gives rise to a correction factor which reaches its maximal value of \( \sim 5.7\% \) at the nominal threshold and drops smoothly below and above threshold. While it amounts to \( 2.4\% \) at \( \sqrt{s} = 176 \text{ GeV} \) it is only \( 1.8\% \) at \( 190 \text{ GeV} \). The corresponding effect on the reconstructed W mass, resulting from the pronounced energy dependence, is a shift at the level of \( 5-10 \text{ MeV} \). Because of the fact that the Coulomb singularity is screened by the finite width of the W bosons and the off-shell effects, higher-order Coulomb corrections are not important for off-shell W-pair production \([59]\)–\([61]\). Moreover, bound states of the two W bosons do not have the time to form because of the finite-width effects. The typical time scale needed for the formation of a bound state \( t_{\text{form}} \approx 1/(\alpha^2 M_W) \approx 234 \text{ GeV}^{-1} \) is much larger than the typical decay time \( t_r = 1/T_W \approx 0.5 \text{ GeV}^{-1} \).

### 3.4.3 The hard-photon process

As mentioned before, in W-pair production the distinction between initial- and final-state radiation is not unique, unlike in the case of neutral particles such as the Z boson. In the matrix element, the universal leading logarithmic parts are easily separable, but the non-universal finite terms do not split naturally. An accurate calculation of the photon spectrum, beyond the leading logarithmic approximation, thus has to take into account initial-state radiation, final-state radiation, radiation off the W bosons, and various interference effects. These calculations have been performed for the final state consisting of 4 fermions and one photon \([62]\)–\([64]\). However, the forward emission of many photons, described well by the structure functions and parton-shower algorithms described in section 3.4.1 and in appendix A, is not included in these calculations. Recently efforts have been made to combine the two approaches \([65, 66]\). We give here the most salient points of these papers.

First we discuss the one-photon matrix element. This contains all graphs with two resonant W propagators, including radiation off the W bosons and the four-vector-boson interaction. The resulting non-leading terms are negative and tend to decrease the energy lost in initial-state radiation. On top of that one can also include the non-doubly-resonant graphs. The universal ones that contribute for all channels are totally negligible \([62, 63]\), just as in the non-radiative process, whereas the \( t \)-channel graphs show the expected collinear log(1 – \( \cos \theta \)) behaviour.

The next step is to combine this hard matrix element, which is needed for large angles, with the resummed leading-log structure function \([65]\) or parton shower \([66]\), which gives a good description of (multiple-) photon radiation at small angles. This is done by using the exact matrix element (convoluted with initial-state radiation) outside a cone \([65]\) around the incoming
Figure 9: Photon energy spectrum in the one-photon approximation (1γ) and the same combined with resummed structure functions (full) for the reaction e+e− → μ+νμτ−ντ.

electrons and positrons⁴. Inside this cone one subtracts the leading logarithmic contribution from the hard matrix element, adds the tree-level matrix element, and subsequently convolutes everything with initial-state radiation. Of course the structure function or parton shower has to be restricted to radiate within the cone or virtuality cut-off. In the former case the scale $Q^2 = s (1-\cos \theta_c)/2$ is used, with $\theta_c$ defining the cone.

In order to compare this approach with the one-photon matrix element on the one hand, and a purely leading-log description on the other, we have to define some kinematical observables. The most relevant ones for the W-mass measurement are the observable and unobservable photon energies. We define these with respect to the ADLO/TH set of canonical cuts defined in [22]. A photon which passes all cuts is called ‘observable’, and one that is combined with one of the beams is ‘unobservable’. Photons close to final-state particles are not counted either way. The unobservable photon energy is the quantity most relevant to the W-mass reconstruction without explicit photons, whereas the observable photon energy gives an indication how well the resummed leading-log and one-photon matrix elements describe large-angle radiation.

In order to get sensible results for these photon energies we will have to include an estimate of the unresummed soft and virtual corrections, which are not yet fully known. These corrections are only needed inside the cone as a weight of the structure function. As such the corresponding uncertainty partially cancels out in the average photon energies $\langle E_\gamma \rangle = \int \frac{dE_\gamma}{dE_\gamma} \frac{d\sigma}{dE_\gamma} E_\gamma / \sigma$. In the actual analysis presented below we circumvent the problem by leaving out the virtual corrections and tuning the non-leading-log part of the soft corrections (through the IR regulator

⁴Instead of a cone also a cut-off on the $e^\pm_\text{in}−\gamma$ virtuality can be used to define the region of multiple-photon radiation [66].
Table 11: Cross-section for observable photons and the energy lost to observable and unobservable photons at $\sqrt{s} = 175$ GeV.

mass $\lambda_{IR})$ in such a way that it cancels in the total cross-section against the non-leading-log part of the hard-photon corrections. Consequently, the resulting total cross-section is simply given by the lowest-order cross-section convoluted with leading logarithmic structure functions. As the initial-final and final-final interference is expected to be of order $\alpha \Gamma_W/M_W$ (see section 3.4.4), this is a reasonable approximation.

The results for the structure-function algorithm of [65] are given in table 11 for leptonic, semi-leptonic, and hadronic final states\(^a\). For the cone $\theta_c = 10^\circ$ was chosen, but the results do not depend strongly on this parameter ($\theta_c = 5^\circ$ only shifts the values by about 1%). In table 11 we first give the observable tree-level non-radiative doubly-resonant cross-section ($\sigma_0$), the same convoluted with leading-log structure functions ($\sigma_0^{\text{ir}}$), and with in addition the universal non-doubly-resonant (background) graphs ($\sigma_0^{\text{ir+bkg}}$). For the next entries we consider the

\(^a\)It should be noted that only the universal background diagrams were taken into account.
full calculation described above, the exact one-photon matrix element (1γ), and the leading-log result (LL). We also give the full result including the universal non-doubly-resonant diagrams (full+backgr). The statistical errors on the cross-sections are $O(0.1\%)$, but the differences (LL – full) and ((res + backgr) – res) were computed directly in this form and have relative errors of a few per cent and a few tens of per cent, respectively, on these differences. The statistical errors on the average photon energies are slightly larger, 0.3–0.5%. Note that we introduced an upper cut-off on the photon energy to avoid the Z peak. This influences only the observable average energy.

One can see that an appreciable fraction (around one quarter) of the events will be accompanied by photons observable in the ADLO/TH set of cuts. This is due to the excellent forward coverage ($\theta_\gamma > 1^\circ$) and electromagnetic calorimeter ($E_\gamma > 0.1$ GeV) assumed in the canonical cuts. Reducing the angle to $10^\circ$ this fraction still is around 20%, half of which also has an $E_\gamma > 1$ GeV. Neither the 1γ matrix element nor the leading-log approximation give a satisfactory description of the observable photons. By radiating only one photon one misses the additional convolution with the structure functions, which scales down the one-photon cross-section as a result of the large, negative soft-photon effects. On the other hand, the leading-log approximation misses the negative effects from initial-final state interference and the radiation off the intermediate W bosons. Finally, given that the un-exponentiated large-angle contribution of the cross-section still is 20% of the total cross-section, the $E_\gamma$ spectrum in the region $E_\gamma < 1$ GeV should not be trusted to 1%, even not in the full calculation. Most of the cross-section here is, however, associated with the final state and hence does not influence the W-mass measurement.

The observable photon energy is dominated by final-state radiation and hence not very interesting. The unobservable energy spectrum is much more independent of the specific final state. It is not completely independent due to the possibility of observable jets in the beam pipe (there is no angular cut on jets in the canonical cuts). The initial-state radiation associated with these events is sometimes combined with the final state by the canonical cuts, thus lowering the average energy. As the radiation off jets is not modelled correctly anyway, a jet-angle cut will have to be imposed to exclude this contamination. One sees that analysing observable photons separately reduces the average energy loss, and hence the size of the theoretical corrections to be applied to the fitted W-mass. For the unobservable radiation the leading-log approximation is, as expected, quite good. Like in the total cross-section, the contribution of the universal non-doubly-resonant graphs is negligible.

We conclude that the one-photon matrix element does not describe observable photons or unobservable photons well, whereas a leading logarithmic description is not accurate enough for large-angle photons, as it does not include the negative contributions associated with interference terms and radiation off the W bosons. Furthermore, a separate analysis of the events with an observable photon (around one quarter of all events with the canonical cuts) reduces the average energy loss considerably, thus reducing the uncertainties coming from this theoretical input.
Finally, we discuss the way in which a reasonable approximation to the full radiative corrections to the process $e^+e^- \rightarrow 4f$ can be computed. We do not consider QCD corrections. The perturbative QCD corrections to the final state are well-understood, as the main part is confined to the decay of a single $W$, which should be similar to the hadronic $Z$ decays studied at LEP1. Perturbative QCD interference effects between the decay products of different $W$ bosons are suppressed by a factor $\alpha_s^2/(N_C^2-1)$. The related non-perturbative interference effects associated with the hadronization are discussed in section 3.5.

An electroweak $O(\alpha)$ calculation would consist of the traditional three parts: one-loop graphs, soft-photon bremsstrahlung, and hard-photon bremsstrahlung. Only the sum is IR finite. The separation between hard and soft radiation is, as usual, defined with respect to a photon-energy cut-off $E_{\text{min}}$. Because of the finite $W$ width one either has to use $E_{\text{min}} \ll \Gamma_W$ to ensure the validity of the soft-photon approximation in the soft-bremsstrahlung integrals (i.e., neglecting the effect of the photon momentum on the phase space and on the non-IR-singular parts of the matrix element), or one has to take into account finite photon energies in the Breit-Wigner resonances. Close to threshold, moreover, the limited amount of available phase space demands $E_{\text{min}} \ll M_W\beta^2$.

The soft-bremsstrahlung integrals factorize into a simple multiplicative factor and the Born amplitude, and can easily be added, either using a 'fixed-width scheme' or the 'fermion-loop scheme'. Hard-photon radiation (for an arbitrary cut-off) has already been addressed in section 3.4.3.

What remains are the virtual corrections. However, the full one-loop calculation of the virtual diagrams appears daunting. For the most simple final state, $e^+e^- \rightarrow \mu^+\nu_\mu\bar{u}\bar{d}$, there are 3579 Standard-Model Feynman diagrams for massless fermions. This increases to 7158 when one electron is included in the final state, and reaches 15948 for the most complicated final state $e^+e^-\nu_\mu\bar{\nu}_e$. However, not all contributions are equally important. For instance the numerical significance of the non-doubly-resonant (background) diagrams is small in the Born and hard-photon calculations, especially if one requires that the event resembles $W$-pair production. So, we can assume that the one-loop corrections to these diagrams are even smaller. The problem then amounts to achieve a clean separation of the radiative corrections to $W$-pair production.

One way to achieve this separation is the 'pole scheme' introduced in section 3.3 for tree-level matrix elements. There have been various attempts to define one-loop corrections in this scheme [23]–[25], which differ considerably. Since sofar no actual calculation for $W$-pair production has been completed in such a scheme, we restrict ourselves to a few comments. The idea behind the 'pole scheme' is to include a minimal part of the higher-order corrections needed to generate a finite width. By making a systematic expansion both in the coupling parameter $\alpha$ and in the width $\Gamma$ ($\propto \alpha$), one can identify gauge-invariant contributions of progressively smaller influence on the final result. We will roughly sketch the method for a single, neutral particle. For factorizable diagrams, i.e. diagrams that factorize into corrections...
to the production, propagation, and decay of the unstable particles, the amplitude to all orders can be Dyson resummed as follows [24, 67]

$$\mathcal{M}_{\text{fact}}(p^2, \theta) = \sum_{n=0}^{\infty} \frac{W(p^2, \theta) - W(M^2, \theta)}{p^2 - M^2} \frac{1}{1 - \Sigma'(M^2)} + \frac{W(M^2, \theta)}{p^2 - M^2} \frac{1}{1 - \Sigma(M^2)},$$

(62)

where $m$ denotes the real mass, and $M$ the complex mass given by $M^2 - m^2 - \Sigma(M^2) = 0$. The corrections to production and decay are contained in the function $W$, and $\Sigma(p^2)$ is the one-particle-irreducible self-energy. The variable $\theta$ in the vertex-correction function $W$ stands for other variables, which have to be chosen sensibly (see section 3.3). The first term of equation (62) does not have a pole and the residue at the pole $p^2 = M^2$ of the last term is gauge-invariant. In [25] it has been shown how to derive the single-pole residue $W(M^2, \theta)/[1 - \Sigma'(M^2)]$ and the non-pole terms. The former consists essentially of the on-shell amplitude plus some terms of $O(m^2)$. It should, however, be noted that this type of decomposition only works when the on-shell limit exists. So, for instance below the W-pair production threshold the 'pole-scheme' method makes no sense, and as a result of that the procedure is not reliable just above threshold.

When one turns to the production of charged unstable particles, additional problems arise related to the infra-red divergences that occur in the limit $\Gamma \to 0$. Again these problems can be overcome [25, 9].

So far the above discussion only took into account factorizable diagrams. There are also non-factorizable diagrams, e.g. diagrams in which a photon connects the initial and final state or the decay products of different unstable particles. Those non-factorizable diagrams can give rise to double-pole contributions when the virtual photon becomes soft. Combined with the related soft bremsstrahlung it has been shown that the non-factorizable double-pole contributions cancel up to order $\alpha \Gamma / m$ in the fully inclusive total cross-section [69, 70]. For sufficiently exclusive distributions this is in general not the case [9, 70, 71].

### 3.5 Reconnection effects

Nearly half of all W pairs decay hadronically. In the LEP2 energy range the average space-time distance between the $W^+$ and $W^-$ decay vertices is smaller than 0.1 fm, i.e. less than a typical hadronic size of 1 fm. Therefore the fragmentation of the $W^+$ and the $W^-$ may not be independent, and this could influence the W-mass reconstruction. Here a short summary of the problem is given in a historical order. More can be found in the report of the W-mass group [8].

The problem is related to two different physical effects: colour reconnections and Bose-Einstein effects.
The colour-reconnection problem was first pointed out in [72] using the string-model approximation. Assuming that the W pair decays into two quark-antiquark pairs $q\bar{q}$ and $Q\bar{Q}$, respectively, it can either fragment into two strings stretched between $q\bar{q}$ and $Q\bar{Q}$, or into two strings stretched between $q\bar{Q}$ and $\bar{q}Q$. In the three-colour world the probability for the second configuration was taken to be $1/9$ in a very crude approximation. The properties of these two configurations are very different. While the first one gives ‘classical’ event multiplicities and a flat rapidity distribution, the second one has a small multiplicity and the particles are grouped at large rapidity values [72].

The fragmentation of the initial $q\bar{q}$ pairs into hadrons is conventionally described in terms of a perturbative parton cascade followed, at a later stage, by a non-perturbative hadronization phase. In perturbative QCD (PQCD) the influence of one W fragmentation on the other one is called ‘interconnection’. Here the timing is very important [8, 73]: the gluon-emission time of high-energy gluons, $\tau_g \sim 1/E_g$, is shorter than the W lifetime, $\tau_W \sim 1/\Gamma_W \approx 0.5\text{GeV}^{-1}$. Therefore the gluons with $E_g \gg \Gamma_W$ are emitted independently from the $q\bar{q}$ and $Q\bar{Q}$ systems, while gluons with $E_g \leq \Gamma_W$ may feel four colour charges ($\Rightarrow$ PQCD interference) [73]. The non-perturbative hadronization (at distances $\sim 1\text{fm}$) is NOT independent. The influence of PQCD interference is shown to be not very important [73, 74]. It is suppressed by a colour factor $1/(N_C^2-1) = 1/8$ as compared to the total rate of double primary gluon emission and, in addition, only gluons with $E_g \leq \Gamma_W$ are concerned; thus the total suppression factor is about 100 compared to a naive instantaneous reconnection scenario with all events reconnected. It is much more complicated to estimate the influence of the fragmentation effects. In the Lund string model analogies to two types of superconductors have been used [73]. The reconnection probability is taken to be either proportional to the space–time volume over which the $W^+$ and $W^-$ strings overlap (type I superconductor) or to the probability that vortex lines cross (type II superconductor). Using these models the fragmentation error was estimated to be 30 MeV on the W-mass measurement. This error is increased to 40 MeV by adding the estimated error coming from the perturbative effects as well as from the interplay between perturbative and non-perturbative effects. Also an extended version of this approach has been investigated [8], where additionally the space–time evolution of the parton cascade, multiple reconnections, and a finite vortex-core radius have been taken into account.

In [75] the space–time picture is not used, except for the timing of the gluon emission. Rather it is assumed that reconnections reducing the total string length are preferred, as indicated by PQCD experience. During fragmentation, in addition to the standard string connections between quarks and gluons emitted from each W, also any other possible interconnection (recoupling) between partons emitted from different W bosons is possible with a probability that may be very different from $1/9$. The reduced string length leads to a reduction in multiplicity in the central rapidity region. In this model the shift in the reconstructed W-mass varies between 6 and 60 MeV (‘shortest-string’ version) or between 13 and 130 MeV (‘random-reconnection’ version) if the recoupling probability varies between 10 and 100%. Methods to measure this probability using data from LEP2 have been suggested [8, 75]. A similar model has been introduced in the ARIADNE program leading to similar mass shifts [76].

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The preliminary estimate of the colour reconnection in HERWIG, based on a reduction of
the space–time extent of clusters, gives mass shifts of the same order as the ones mentioned
above [8].

The dependence of the shift in the reconstructed W-mass on the parameters of different
models has been studied by the LEP experiments in more details [8]. It appears that uncer-
tainties using experimental procedures are not smaller than the ones obtained above. The issue
whether diagnostic signals for reconnection can be found is still open.

Thus the colour-reconnection effect may add a big systematic error to the W-mass mea-
surement in the four-quark channel. The interesting aspect is that different approaches give
uncertainties of the same order.

It may be worthwhile to note that similar (but not identical) ‘unconventional’ colour con-
nections can also appear inside a single Z system, as is discussed in [8]. The presence/absence
of a signal in Z decays at LEP1 could give an indication of the expected effects in the W^+W^-
system.

Bose–Einstein correlations have been observed experimentally as an enhancement in the two-
particle correlation function for identical bosons. In the case of a W pair decaying hadronically
a possibility exists of Bose–Einstein correlations between particles that come from different W
bosons [77]. A test of this effect has been made in a model based on PYTHIA and JETSET [77].
The model gives large reconstructed positive mass shifts of the order of 100 MeV, rising with
increasing c.m.s. energy and with decreasing source radius. The real shift may be smaller due to
various damping factors not included in [77] (which is intended as a ‘worst-case’ scenario), but
effects of the order of 50 MeV could be expected within a large class of possible Bose–Einstein
models.

In the coming years both colour (re)arrangement and Bose–Einstein effects can and should
be studied both theoretically and experimentally. Also the large statistics collected already at
LEP1 may help to study both these effects.

4 Concluding Remarks

In this report the present status of the theoretical knowledge of W-pair production and the
related process of four-fermion production is reviewed.

In lowest-order complete evaluations for all four-fermion final states exist, taking into ac-
count all Feynman diagrams. The theoretical problem how to incorporate an energy-dependent
width in the W and Z propagators without spoiling gauge invariance has been solved for the
lowest-order cross-section. The numerical relevance of imposing electromagnetic gauge invar-
ance is explicitly demonstrated.
As to the radiative corrections, the state of the art depends on whether on- or off-shell W-pair production, or four-fermion production is considered. Complete $\mathcal{O}(\alpha)$ RC's only exist for on-shell W-pair production. The dominant RC's that can be taken into account properly for all three types of processes comprise leading-log ISR, the leading weak effects related to $G_\mu$ and $\alpha(s)$, and the Coulomb singularity. The $\mathcal{O}(\alpha)$ RC's for four-fermion production obviously do not exist, and can at best be calculated using approximative techniques. Only taking along the above dominant effects, the total and differential cross-sections at 175 and 190 GeV are estimated to be known at the 1–2% level (based on on-shell experience). For the total cross-section at 161 GeV an uncertainty of about 2% is expected. Part of the latter uncertainty is due to the possible variation of the Higgs mass. This Higgs-mass dependence is largest for light Higgs bosons and is most pronounced at threshold.

Another question, relevant for the reconstruction of the W mass, is the accuracy of the average emitted photon energy $\langle E_\gamma \rangle$. At 175 and 190 GeV a precision of 10–20 MeV seems feasible, provided a proper scale is used in the structure-function part of the calculation.

### A Various Methods of Calculating QED Corrections

#### A.1 The structure-function method

As pointed out in the previous sections, the virtual and real corrections reveal the presence of large logarithmic QED effects of the form $\alpha L/\pi \equiv (\alpha/\pi) \log(Q^2/m^2)$ with $Q^2 \gg m^2$. They arise when photons or light fermions are radiated off in the direction of incoming or outgoing light particles [78, 79], provided the momentum of the latter is kept fixed (exclusive) [79]. In a massless theory these large logarithms would show up as collinear divergences (like in QCD), but in the SM the masses of the particles act as natural cut-offs. The fact that they can nevertheless give rise to sizeable corrections is due to the difference in scale between the mass of the radiating particle and its energy. This is illustrated by considering the propagator $P$ of a light particle after photon emission

$$P = \frac{1}{(q-k)^2-m^2} \frac{q_0 \gg m}{2q_0 k_0} \left[ 1 - \frac{1}{1 - m^2/2q_0^2} \cos \theta \right].$$

In the limit $m \to 0$ this propagator gives rise to a pole at $\cos \theta = 1$. For finite $m$ it yields large logarithmic terms of the form $\log(q_0^2/m^2)$ when the photon momentum is integrated over (inclusive photon). A consequence of the direct relation between the large QED logarithms and collinear divergences is that they are controlled by renormalization group equations and that they are universal, i.e. they are process-independent. They can be calculated using the so-called structure-function method [16], taken over from QCD. This procedure also allows the inclusion of soft-photon effects to all orders by means of exponentiation.
Figure 10: Initial-state collinear radiation in $e^+e^- \rightarrow W^+W^-$. The unlabeled external lines represent an arbitrary number of undetected (inclusive) particles.

The structure-function method is based on the mass-factorization theorem (see figure 10):

$$s^4 \frac{d^4 \sigma_{e^+e^-\rightarrow W^+W^-X}}{dt_1 du_1 dt_2 du_2} = 0 \frac{dx_+}{x_+^2} 0 \frac{dx_-}{x_-^2} 0 \frac{\Gamma_{ie^+}(x_+, Q^2)}{\Gamma_{je^-}(x_-, Q^2)} s^4 \frac{d^4 \hat{\sigma}_{ij\rightarrow W^+W^-X'}}{(dt_1 du_1 dt_2 du_2)(Q^2)},$$  \hspace{1cm} (63)

which links the cross-section $\sigma_{e^+e^-\rightarrow W^+W^-X}$, where X indicates an arbitrary number of undetected particles, to the hard scattering cross-section $\hat{\sigma}_{ij\rightarrow W^+W^-X'}$, which is free of the large collinear logarithms\footnote{If the final state consists of exclusively treated light particles (e.g. $e^+e^- \rightarrow 4f$), large collinear final-state QED logarithms appear. These large logs can be treated in a way similar to the initial-state ones, provided proper account is taken of the fact that now $\hat{p} = p/z$.}. The indices $i, j$ represent all light-particle transitions allowed by QED (e.g. $\gamma, e^\pm, \mu^\pm, \cdots$). Furthermore we have defined

$$t_1 \equiv (p_+ - k_-)^2 - M_W^2, \hspace{1cm} u_1 \equiv (p_+ - k_-)^2 - M_W^2,$$

$$t_2 \equiv (p_+ - k_+)^2 - M_W^2, \hspace{1cm} u_2 \equiv (p_+ - k_+)^2 - M_W^2.$$  \hspace{1cm} (64)

Analogously their hatted counterparts are given by the same expressions with $p_\pm$ replaced by $\hat{p}_\pm = x_\pm p_\pm$. It should be noted that (63) allows the implementation of various cuts on the energies and angles of the produced W bosons, and that it can be used to extract more commonly used distributions [like $d\sigma/(d\cos \theta_\pm)$] by supplying the appropriate Jacobians [80]. The
structure functions $\Gamma_{ij}$ describe ‘mass singular’ initial-state collinear radiation and represent the probability of finding in the parent particle $j$ at the scale $Q^2$ a particle $i$ with fraction $x$ of longitudinal momentum. So $(1-x)$ denotes the fraction of energy carried away by collinear radiation. All large collinear logarithms are contained in the structure functions, which depend, just like the hard scattering cross-section, on the mass-factorization scale $Q^2$. These structure functions can be decomposed into a part containing the collinear logarithms and a non-log part according to

$$\Gamma_{ij}^{\text{log}}(x, Q^2) = \delta_{ij} \delta(1-x) + \sum_{n=1}^{\infty} \frac{\alpha}{\pi} \frac{n}{m} a_{mn}^{ij}(x) L^n,$$

$$\Gamma_{ij}^{\text{non-log}}(x, Q^2) = \sum_{n=1}^{\infty} \frac{\alpha}{\pi} \frac{n}{m} b_{ij}^{n}(x).$$

(65)

Analogously the hard scattering cross-section can be decomposed into the scale-independent Born cross-section plus non-log higher-order terms

$$d\sigma_{ij}(Q^2) = d\sigma_{ij}^{\text{Born}} + \sum_{n=1}^{\infty} \frac{\alpha}{\pi} c_{ij}^{n}(Q^2).$$

(66)

For convenience we have dropped the explicit dependence on $x_\pm$ in this decomposition. This dependence enters via the reduced momenta $\hat{p}_\pm$.

The so-called leading-log (LL) approximation consists in only taking along the terms $\propto (\alpha L/\pi)^n$, which automatically means that the hard scattering cross-section $\hat{\sigma}$ should be identical to the scale-independent Born cross-section. In addition this Born cross-section could be dressed by the leading corrections that are not of the LL type, e.g. running couplings or a Coulomb factor, in order to improve the approximation. As these LL contributions constitute the most important higher-order QED effects, it is on most occasions sufficient to add the $\mathcal{O}(\alpha^2)$ LL contributions to the full $\mathcal{O}(\alpha)$ results and to exponentiate the soft-photon distribution. In this way the resulting cross-section becomes scale-dependent. A further simplification can be achieved by realizing that in $e^+e^-$ collisions the bulk of the QED corrections frequently originates from pure photon radiation, especially when the soft-photon contributions are dominant$^w$.

After these simplifications the relevant structure function takes on the form $[81, 82]$

$$\Gamma_{ee}^{\text{LL,exp}}(x, Q^2) = \phi(\alpha, x, Q^2) = \exp\left(-\frac{1}{2} \frac{\gamma_E}{\beta_{\text{exp}}} + \frac{3}{8} \frac{\beta_S}{\beta_{\text{exp}}} \right) \frac{1}{2} \left(1-x\right)^{\beta_{\text{exp}}/2-1} - \frac{1}{4} \beta_{\text{H}} (1+x)$$

$$- \frac{1}{4 \beta_{\text{H}}^2} (1+3x^2) \log(x) + 4(1+x) \log(1-x) + 5 + x$$

$$- \frac{1}{4 \beta_{\text{H}}^3} \left(1+x\right) 6 \text{Li}_2(x) + 12 \log^2(1-x) - 3\pi^2$$

$^w$For more details concerning polarized structure functions and QED corrections that are not of the pure-photon-radiation type we refer to [9].

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\[ + \frac{1}{1 - x} \frac{3}{2} (1 + 8x + 3x^2) \log(x) + 6(x + 5)(1 - x) \log(1 - x) \]
\[ + 12(1 + x^2) \log(x) \log(1 - x) - \frac{1}{2} (1 + 7x^2) \log^2(x) + \frac{1}{4} (39 - 24x - 15x^2) \]

Here \( Li_2(y) \) is the dilogarithm, \( \gamma_E \) the Euler constant, and \( \Gamma(y) \) the Gamma function (not to be confused with the structure functions). In terms of this structure function the total W-pair production cross-section including exponentiated LL QED corrections can for example be written as

\[ \sigma_{\text{LL,exp}}(s, Q^2) = \frac{1}{4M_W^2/s} \int \frac{dz}{z} \phi(2\alpha, z, Q^2) \hat{\sigma}_0(zs), \]

where \( \hat{\sigma}_0(zs) \) denotes the (possibly improved) Born cross-section at the reduced CM energy squared \( zs \).

Note that some non-leading terms can be incorporated, taking into account the fact that the residue of the soft-photon pole is proportional to \( L - 1 \) rather than \( L \) for the initial-state photon radiation. Using

\[ \eta = \frac{2\alpha}{\pi} L \quad \text{and} \quad \beta = \frac{2\alpha}{\pi}(L - 1) \]

we can identify a couple of popular options for including non-leading terms:

- \( \beta_{\text{exp}} = \beta, \beta_s = \beta_H = \eta [9, 83] \) (\textsc{mixed}' choice)
- \( \beta_{\text{exp}} = \beta_s = \beta, \beta_H = \eta [84] \) (\textsc{eta}' choice); the original Gribov–Lipatov form factor [85]
- \( \beta_{\text{exp}} = \beta_s = \beta_H = \beta [56, 86] \) (\textsc{beta}' choice).

The differences between these options are in the coefficient in front of the exponentiated soft-photon term and in the use of \( \eta \) or \( \beta \) in the hard parts.

In table 12 we quantify for the CC3 process \( e^+e^- \rightarrow W^+W^- \rightarrow 4f \) the uncertainty associated with these three different structure functions. We give the total cross-section without cuts, the radiative energy loss \( \langle E_\gamma \rangle \), and the invariant-mass loss \( \langle M_\gamma \rangle \). The radiative energy loss is defined as

\[ \langle E_\gamma \rangle = \frac{1}{\sigma} (2 - x_+ - x_-)E \, d\sigma \]

and the radiative invariant-mass loss as

\[ \langle M_\gamma \rangle = \frac{1}{\sigma} (1 - x_+ x_-)E \, d\sigma. \]

As can be seen from the comparison between the third and fourth column of table 12, the effect on the total cross-section due to the different coefficient in front of the exponentiated soft-photon term in the \textsc{mixed} and \textsc{eta} structure functions is of the order of 0.3–0.4%. This
Table 12: Effects of different structure functions on $\sigma$ (in pb), $\langle E_\gamma \rangle$ (in GeV), and $\langle M_\gamma \rangle$ (in GeV), for the CC3 process $e^+e^- \rightarrow W^+W^- \rightarrow 4f$.

different choice in the coefficient does not affect $\langle E_\gamma \rangle$ and $\langle M_\gamma \rangle$, since it is a factorized soft-photon contribution which largely cancels out in the ratios. By comparing the fourth and fifth column of table 12, one can estimate the amount of the so-called $\eta \rightarrow \beta$ effect in the hard part of the structure function. The impact on the cross-section is small, i.e. of the order of 0.1% (depending on the energy). The effect on $\langle E_\gamma \rangle$ and $\langle M_\gamma \rangle$ is 3, 10, 19 MeV (at $\sqrt{s} = 175, 190, 205$ GeV, respectively) and therefore not significant in view of the expected experimental accuracy at LEP2. From the last two columns of table 12, finally, we can infer that the effects of the $\mathcal{O}(\beta^3)$ hard part of the structure function is completely negligible.

At this point we would like to note that the leading effects related to the production of undetected $e^+e^-$ pairs from conversion of a virtual photon emitted from the initial state (+ corresponding loop corrections) can be accounted for by replacing $\alpha$ in $\phi(\alpha, x, Q^2)$ by $\alpha[1+\alpha L/(6\pi)]$. This takes into account the QED $\beta$-function contributions appearing in the renormalization group equations. Also other fermion pairs can be included, but care has to be taken with the fact that the real pair-production process requires the virtual photon to be sufficiently hard.

Another source of uncertainties is related to the fact that the scale $Q^2$ is a free parameter\(^*\). As mentioned already in section 2.3.3, all ‘natural’ scale choices are roughly equal close to threshold. Hence, when using the structure-function method to calculate higher-order corrections [beyond $\mathcal{O}(\alpha)$] the $Q^2$ dependence is negligible. This is, of course, not the case for LL $\mathcal{O}(\alpha)$ corrections, as they are larger. So, in situations where the exact $\mathcal{O}(\alpha)$ corrections are not known and one has to resort to a LL approximation instead, as is the case for $e^+e^- \rightarrow 4f$, the scale dependence is larger.

\(^*\)For a discussion of ‘appropriate’ scale choices we refer to [9].
A.2 The parton-shower method

The basic assumption of the QED parton-shower method is that the structure function of an electron (or positron) obeys the Altarelli–Parisi equation [87], which can be expressed by the integral equation

$$\Gamma_{ee}(x,Q^2) = \Pi_{ee}(Q^2,Q') \Gamma_{ee}(x,Q^2) + \frac{\alpha}{2\pi} Q^2 \frac{dK^2}{K^2} \Pi_{ee}(Q^2,K^2) \int_{0}^{1} \frac{dy}{y} P_{ee}(y) \Gamma_{ee}(x/y,K^2)$$

(72)

in the leading-log approximation [88]. Here $\epsilon$ is a small quantity specified later and $P_{ee}(x) = (1 + x^2)/(1 - x)$. The function $\Pi_{ee}$, which is nothing but the Sudakov factor, is given by

$$\Pi_{ee}(Q^2,Q'^2) = \exp \left( -\frac{\alpha}{2\pi} Q^2 \frac{dK^2}{K^2} \int_{0}^{1} dx P_{ee}(x) \right),$$

(73)

and denotes the probability that the electron (or positron) evolves from $Q'^2$ to $Q^2$ without emitting a hard photon. The scale $Q^2_s$ is, like in appendix A.1, a free parameter (of order $m^2$). Often it is chosen such that, when the $K^2$ dependence is integrated out, the factor $\beta$ emerges rather than $\eta$.

Equation (72) can be solved by iterating the right-hand side in a successive way. Hence it is clear that the emission of $n$ photons corresponds to the $n$-th iteration. As such it is possible to regard the process as a stochastic one, suggesting the following algorithm for the photon shower [89]: (a) Set $x_b = 1$. The variable $x_b$ is the fraction of the light-cone momentum of the virtual electron (or positron) that annihilates. (b) Choose a random number $\xi$. If it is smaller than $\Pi_{ee}(Q^2,Q^2)$, then the evolution stops. If not, one proceeds by finding the virtuality $K^2$ that satisfies $\xi = \Pi_{ee}(K^2,Q^2)$. With this virtuality a branching takes place. (c) Fix $z$ according to the probability $P_{ee}(x)$ between 0 and $1 - \epsilon$. Then $x_b$ is replaced by $x_b z$. Subsequently one should go to (b), replacing $Q^2_s$ by $K^2$, and repeat until the evolution stops.

Once an exclusive process is fixed by this algorithm, each branching of a photon in the process is dealt with as a true process, that is, an electron with $x$, $K^2$ decays as $e^-(x,-K^2) \rightarrow e^-(xy,-K^2)+\gamma(x[1-y],Q^2_0)$. Here $Q^2_0$ is a cut-off to avoid the IR divergence. It is unphysical, so any physical observable should not depend on it. The momentum conservation at the branching gives $-K^2 = -K'^2/y + Q^2_0/(1-y) + k^2_f/(y[1-y])$, which in turn determines $k^2_f$ from $y$, $K^2$, $K'^2$. Hence the $k^2_f$ distribution can be taken into account in the simulation as well as the shape of $x$. This feature represents the essential difference between the structure-function method, which treats the photons inclusively, and the QED parton-shower method. Keeping in mind that $y \leq 1 - \epsilon$, the kinematical boundary $y(K^2 + Q^2_0/[1-y]) \leq K'^2$, equivalent to $k^2_f > 0$, fixes $\epsilon$ to $\epsilon = Q^2_0/K'^2$, since strong ordering ($K^2 \ll K'^2$) is expected.

The above description of the algorithm represents the “single-cascade scheme”. This implies that only the $e^-$ or the $e^+$ is able to radiate photons when the axial gauge vector is chosen along the momentum of the other initial-state particle, namely $e^+$ or $e^-$. For programming purposes, however, it is convenient to use a symmetrization procedure (the so-called “double-cascade
scheme”) to ensure the symmetry of the radiation with respect to the electron and positron [90, 91]. The so-obtained QED parton shower can then be combined with the matrix element of any hard process initiated by $e^+e^-$ annihilation.

The Altarelli–Parisi equations can also be solved by a different algorithm [92], which yields equivalent results for electromagnetic radiative corrections.

Starting from the observation that the most important corrections come from multiple soft-photon emission, the photon-number distribution can be approximated by a Poisson distribution with average photon number

$$\tilde{n}_\gamma = \frac{\alpha}{2\pi} \log \frac{Q^2}{m^2_e} \int_0^{1-\epsilon} dx P_{ee}(x). \quad (74)$$

The technical infra-red cut-off $\epsilon$ will drop out of the physical sum over soft photons. After the number of photons has been chosen from the Poisson distribution, their transverse and longitudinal momenta can be generated properly, ordered according to the $1/(p_\perp \cdot k)$ poles and splitting functions, respectively.

Using momentum conservation at each branching, the correct energy loss and $p_T$ boost for the hard scattering is obtained. This algorithm has been shown to yield results that are consistent with the structure-function formalism and provides a realistic approximation for the photonic $k_T^2$ distributions.

### A.3 YFS exponentiation

Let us first answer the basic question: what the exclusive Yennie–Frautschi–Suura (YFS) exponentiation is and what it is not? YFS exponentiation is, in one word, a technique of summing up all IR singularities for a given arbitrary process to infinite order. The formal proof in [68] is based on Feynman-diagram techniques and goes ‘order by order’. The YFS technique is not by any means bound to the leading-log summation technique and/or approximations. It is applicable to arbitrary stable particles (with arbitrary mass and spin) in the initial and final state. The YFS summation is inherently exclusive, i.e. all subtractions/summations of IR-singular contributions are done before any phase-space integrations over virtual- or real-photon four-momenta are performed. This means that the Monte Carlo technique can be used to integrate over the multiple real-photon phase space. The first practical solution of this kind was given in [93]. Here the QED matrix element that enters the YFS exponentiation consists of two parts. The $O(\alpha)$ part is taken to be exact, i.e. from Feynman diagrams, whereas a LL approximation is used to write down an economic ansatz for the $O(\alpha^2)$ part. The YFS-exponentiation procedure, which involves a subtraction of the IR part of the matrix element, knows nothing about the origin of the matrix element (ansatz or Feynman rules), and it will fail if the ansatz has the wrong IR limit. The above subtraction procedure is done on the fully differential distribution before phase-space integration. Hence it should be clear that a LL ansatz for the matrix element in which the emitted photons have zero transverse momenta is not possible.
Contrary to the LL techniques described in appendices A.1 and A.2, in the YFS approach the parameter \( \beta = 2(\alpha/\pi)\log(s/m^2_e) - 1 \) results from the integration over phase space and there is no freedom to adjust it. There is no discussion “do we have \(-1\) in the definition of the leading logarithm or not”. The \(-1\) is mandatory, because otherwise the IR singularities do not cancel. The orthodox YFS exponentiation technique [68] is essentially ‘order by order’ and as such it does not sum up all the LL corrections to infinite order, something which is in principle possible for the LL techniques described in appendices A.1 and A.2.

As the exact \( \mathcal{O}(\alpha) \) matrix element for \( e^+e^- \to 4f \) is not yet known, the present implementation of YFS exponentiation for this process relies on a pure LL ansatz [94]. But this will, of course, change as soon as an adequate approximative \( \mathcal{O}(\alpha) \) calculation becomes available.

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DETERMINATION OF THE MASS OF THE W BOSON

Conveners: Z. Kunszt and W. J. Stirling


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1 Introduction and Overview

Previous studies of the physics potential of LEP2 indicated that with the design luminosity of 500 pb\(^{-1}\) one may get a direct measurement of the W mass with a precision in the range 30 – 50 MeV. This report presents an updated evaluation of the estimated error on \(M_W\) based on recent simulation work and improved theoretical input. The most efficient experimental methods which will be used are also described.

1.1 Machine parameters

The LEP2 machine parameters are by now largely determined. Collider energy values and time-scales for the various runs, expected luminosities and errors on the beam energy and luminosity are discussed and summarized elsewhere in this report [2, 3]. Here we note that (i) collider energies in the range 161 – 192 GeV will be available, and (ii) the total luminosity is expected to be approximately 500 pb\(^{-1}\) per experiment. It is likely that the bulk of the luminosity will be delivered at high energy (\(\sqrt{s} \gtrsim 175\) GeV). The beam energy will be known to within an uncertainty of 12 MeV, and the luminosity is expected to be measured with a precision better than 1%.

1.2 Present status of \(M_W\) measurements

Precise measurements of the masses of the heavy gauge W and Z bosons are of fundamental physical importance. The current precision from direct measurements is \(\Delta M_Z = \pm 2.2\) MeV and \(\Delta M_W = \pm 160\) MeV [4]. So far, \(M_W\) has been measured at the CERN [5] and Fermilab Tevatron [6, 7, 8] pp colliders. The present measurements are summarized in Fig. 1. In calculating the world average, a common systematic error of \(\pm 85\) MeV arising from uncertainties in the parton distributions functions is taken into account. The current world average value is

\[
M_W = 80.26 \pm 0.16 \text{ GeV}.
\]

An indirect determination of \(M_W\) from a global Standard Model (SM) fit to electroweak data from LEP1 and SLC [4] gives the more accurate value

\[
M_W = 80.359 \pm 0.051 \pm 0.013 \text{ GeV}.
\]

In Fig. 1 this range is indicated by dashed vertical lines. Note that the central value in (2) corresponds to \(M_H = 300\) GeV and the second error indicates the change in \(M_W\) when \(M_H\) is varied between 60 GeV and 1000 GeV – increasing \(M_H\) decreases \(M_W\). The direct measurement of \(M_W\) becomes particularly interesting if its error can be made comparable to, or smaller than, the error of the indirectly measurement, i.e. \(< O(50)\) MeV. In particular, a precise value of \(M_W\)

\(^{1}\)prepared by F. Jegerlehner, Z. Kunszt, G.J. van Oldenborgh, P.B. Renton, T. Riemann, W.J. Stirling
obtained from direct measurement could contradict the value determined indirectly from the global fit, thus indicating a breakdown of the Standard Model. An improvement in the precision of the \( M_W \) measurement can be used to further constrain the allowed values of the Higgs boson mass in the Standard Model, or the parameter space of the Minimal Supersymmetric Standard Model (MSSM).

Standard Model fits to electroweak data determine values for \( M_W \) (or \( M_H \)), \( M_t \), and \( \alpha_s(M_Z) \). The direct determinations of the top quark mass [9, 10] give an average value of \( M_t = 180 \pm 12 \) GeV. Fig. 2 compares the direct determinations of \( M_t \) and \( M_W \) with the indirect determinations obtained from fits to electroweak data [4]. Note the correlation between the two masses in the latter. Within the current accuracy, the direct and indirect measurements are in approximate agreement. The central values of \( M_W \) and their errors, determined in several ways from indirect electroweak fits, are given in Table 1. The results are evidently somewhat sensitive to the inclusion (or not) of data on the Z partial width ratios \( R_b \) and \( R_c \) and the SLD/SLC measurement of \( A_{LR} \), all of which differ by 2.5 standard deviations or more from the Standard Model values. However, the conclusion on the agreement of the direct and indirect determinations is unchanged. As we shall see in the following sections, a significant reduction in the error on \( M_W \) is expected from both LEP2 and the Tevatron.
1.3 Improved precision on $M_W$ from the Tevatron

The Tevatron data so far analysed, and shown in Fig. 1, come from the 1992/3 data-taking (Run 1a). The results from CDF [7] are based on approximately 19 pb$^{-1}$ and are final, whereas those from D0 [8] are based on approximately 13 pb$^{-1}$ and are still preliminary. It is to be expected that the final result will have a smaller error. In addition, there will be a significantly larger data sample from the 1994/6 data-taking (Run 1b). This should amount to more than 100 pb$^{-1}$ of useful data for each experiment. When these data are analysed it is envisaged that the total combined error on $M_W$ will be reduced to about 70 MeV. In particular, the combined CDF/D0 result will depend on the common systematic error arising from uncertainties in the parton distribution functions. Thus when the first $M_W$ measurements emerge from LEP one may assume that the world average error will have approximately this value. For more details see Ref. [11].

After 1996 there will be a significant break in the Tevatron programme. Data-taking will start again in 1999 with a much higher luminosity (due to the main injector and other improvements). Estimating the error on $M_W$ which will ultimately be achievable (with several fb$^{-1}$ of total luminosity) is clearly more difficult. If one assumes that an increase in the size of the data sample leads to a steady reduction in the systematic errors, one might optimistically envisage that the combined precision from the Tevatron experiments will eventually be in the

Figure 2: A comparison of direct versus indirect determinations of $M_t$ and $M_W$. The contour for the indirect determination corresponds to a 70\% confidence level. The dark shaded region within the contour is that compatible with the direct Higgs search limit, $M_H > 65$ GeV.
Table 1: Results of Standard Model fits to $M_t$, $M_H$ and $\alpha_s(M_Z)$. The fits are to LEP1 and SLC data. An upper limit $M_H = 1000$ GeV has been imposed. The electromagnetic coupling constant $\alpha_s(M_Z)$ is also determined in these fits. Also given is the result of the fit given in terms of $M_W$. The results are given for all data, as well as for excluding $R_b$ and $R_c$ and also $A_{LR}$.

$$\Delta M_W = 30 - 40 \text{ MeV}$$ range, assuming a common systematic error of about 25 MeV [12]. However it is important to remember that these improved values will be obtained after the LEP2 measurements.

### 1.4 Impact of a precision measurement of $M_W$

Within the Standard Model, the value of $M_W$ is sensitive to both $M_t$ and $M_H$. For example, for a fixed value of $M_H = 300$ GeV, a precision of $\Delta M_W = \pm 25$ MeV translates to a precision on $M_t$ of $\pm 3.9$ GeV. The impact of a precise measurement of $M_W$ on the indirect determination of $M_H$ is shown in Table 2.

<table>
<thead>
<tr>
<th>$M_H$ (GeV)</th>
<th>$M_t = 180$ GeV (fixed)</th>
<th>$M_t = 180 \pm 5$ GeV</th>
<th>$M_t = 180 \pm 5$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta M_W = 25$ MeV</td>
<td>$\Delta M_W = 50$ MeV</td>
<td>$\Delta M_W = 25$ MeV</td>
</tr>
<tr>
<td>100</td>
<td>+48, -36</td>
<td>+112, -63</td>
<td>+86, -54</td>
</tr>
<tr>
<td>300</td>
<td>+111, -84</td>
<td>+259, -148</td>
<td>+196, -126</td>
</tr>
<tr>
<td>800</td>
<td>+297, -212</td>
<td>+740, -367</td>
<td>+538, -310</td>
</tr>
</tbody>
</table>

Table 2: Estimated error on $M_H$ (in GeV) for several values of $M_H$ and for $\Delta M_W = 25$ and 50 MeV. The estimates are given for both $M_t = 180$ GeV (fixed) and for $M_t = 180 \pm 5$ GeV. All other Standard Model parameters are fixed.

In order to assess the impact of a precise measurement of $M_W$ it is necessary to make an estimate of the improvements which will be made on the electroweak data from LEP1 and SLC. Details of the improvements which are assumed here are discussed in [13]. The importance of
a precise measurement of $M_W$ can perhaps best be appreciated by considering the (almost) model independent $\epsilon$ parameters [14]. The parameter $\epsilon_1 (= \Delta \rho)$ is sensitive mainly to the $Z$ partial and total widths. The parameter $\epsilon_3$ depends linearly on both $\Delta \rho$ and $\Delta \kappa$, where $\Delta \kappa$ is determined from $\sin^2 \theta_{\text{eff}}$. The parameter $\epsilon_2$ depends linearly on $\Delta \rho$, $\Delta \kappa$ and $\Delta r_W$. This latter quantity is determined essentially from $M_W$, and so improvements in the precision of $\epsilon_2$ depend directly on improving the error on $M_W$. This is illustrated in Fig. 3, which shows the 70% confidence level contours for fits to projected global electroweak data. The different contours correspond to different values of $\Delta M_W$. In these fits all electroweak data measurements have been set to correspond to the Standard Model values $M_t = 180$ GeV, $M_H = 100$ GeV and $\alpha_s(M_Z) = 0.123$. The $\epsilon$ variables are constructed to be sensitive to vector boson propagator effects, from both physics within the Standard Model and beyond.

![Figure 3: 70% confidence level contour plots for fits to $\epsilon$'s in units of $10^{-3}$. The outer contour is for a fit with the current error, $\Delta M_W = \pm 160$ MeV, whereas the inner contours are for $\Delta M_W = \pm 50$ MeV and $\Delta M_W = \pm 25$ MeV respectively.](image.png)

Numerically, the projected data give a precision

$$\Delta \epsilon_1 = \pm 1.1 \times 10^{-3} \quad \Delta \epsilon_3 = \pm 1.0 \times 10^{-3}.$$  \hspace{1cm} (3)

For $\Delta M_W = 160$ MeV, the error $\Delta \epsilon_2 = \pm 3.7 \times 10^{-3}$ is obtained, whereas for the projected errors on $M_W$ one obtains

$$\Delta \epsilon_2 = \pm 1.0 (\pm 1.6) \times 10^{-3} \quad \text{for} \quad \Delta M_W = 25 (50) \text{ MeV}.$$ \hspace{1cm} (4)

The smaller the volume in $\epsilon$ space allowed by the precision electroweak measurements, the greater the constraint on physics beyond the Standard Model.

The MSSM is arguably the most promising new-physics candidate. It is therefore especially important to consider the MSSM prediction for $M_W$. Figure 4 [15] shows $M_W$ as a function of $M_t$ in the SM (solid lines) and in the MSSM (dashed lines). In each case the prediction is a band of values, corresponding to a variation of the model parameters (dominantly $M_H$ in the
SM case, with $90 \text{ GeV} < M_H < 1000 \text{ GeV}$ chosen here) consistent with current measurements and limits. An additional constraint of ‘no SUSY particles at LEP2’ is imposed in the MSSM calculation.

![Graph showing $M_W$ as a function of $M_t$ in the SM (solid lines) and in the MSSM (dashed lines), from Ref. [15].](image)

**Figure 4:** Predictions for $M_W$ as a function of $M_t$ in the SM (solid lines) and in the MSSM (dashed lines), from Ref. [15]. In each case the prediction is a band of values, corresponding to a variation of the model parameters consistent with current measurements and limits. An additional constraint of ‘no SUSY particles at LEP2’ is imposed in the MSSM calculation.

### 1.5 Methods for measuring $M_W$

Precise measurements of $M_W$ can in principle be obtained using the enhanced statistical power of the rapidly varying total cross-section at threshold, the sharp (Breit-Wigner) peaking behaviour of the invariant-mass distribution of the $W^\pm$ decay products and the sharp end-point spectrum of the lepton energy in $W^\pm$ decay. One can obtain a rough idea of the relative power of these methods by estimating their statistical precision assuming 100% efficiency, perfect detectors and no background. More complete discussions are given in Sections 2 and 3.

**A) Threshold cross-section measurement** of the process $e^+e^- \rightarrow W^+W^-$. The statistical power of this method, assuming 100% signal efficiency and no background, is

$$\Delta M_W \geq 91 \text{ MeV} \times \frac{100 \text{ pb}^{-1}}{L},$$

where the minimum value is attained at $\sqrt{s} \simeq 161 \text{ GeV}$. Here $L$ denotes the total integrated luminosity.
B) Direct reconstruction methods, which reconstruct the Breit-Wigner resonant shape from the $W^{\pm}$ final states using kinematic fitting techniques to improve the mass resolution. The statistical power of this method, again assuming 100% efficiency, perfect detector resolution and no background, can be estimated as

$$\Delta M_W \sim \frac{\Gamma_W}{\sqrt{N}} \approx 50 \text{ MeV} \frac{100 \text{ pb}^{-1}}{L},$$

approximately independent of the collider energy. This order of magnitude estimate is confirmed by more detailed studies, see below.

C) Determination of $M_W$ from the lepton end-point energy. The end-points of the lepton spectrum in $W^+W^- \rightarrow q\bar{q}l\nu$ depend quite sensitively on the $W$ mass. For on-shell $W$ bosons at leading order:

$$E_- \leq E_l \leq E_+,$$

$$E_\pm = \frac{\sqrt{s}}{4} \left( 1 \pm \sqrt{1 - 4M_W^2/s} \right).$$

In this case the statistical error on $M_W$ is determined by the statistical error on the measurement of the lepton end-point energy,

$$\Delta M_W = \frac{s - 4M_W^2}{M_W E_\pm}. \quad (8)$$

In practice, however, the end-points of the distribution are considerably smeared by finite width effects and by initial state radiation, and only a fraction of events close to the end-points are sensitive to $M_W$. This significantly weakens the statistical power of this method from what the naive estimate (8) would predict.

The detailed studies described in the following sections show that the errors which can realistically be achieved in practice are somewhat larger than the above estimates for Methods A and B. The statistical precisions of the two methods are in fact more comparable (for the same integrated luminosity) than the factor 2 difference suggested by the naive estimates (5) and (6). The overall statistical error for Method C has been estimated at $\mathcal{O}(300 \text{ MeV})$ [1] for $L = 500 \text{ pb}^{-1}$, significantly larger than that of the other two methods. It will not therefore be considered further here, although it is still a valid measurement for cross-checking the other results.

It is envisaged that most of the LEP2 data will be collected at energies well above threshold, and so the statistically most precise determination of $M_W$ will come from Method B. However with a relatively modest amount of luminosity spent at the $W^+W^-$ threshold (for example 50 pb$^{-1}$ per experiment), Method A can provide a statistical error of order 100 MeV, not significantly worse than Method B and with very different systematics. The two methods can therefore be regarded as complementary tools, and both should be used to provide an internal cross-check on the measurement of the $W$ mass at LEP2. This constitutes the main motivation for spending some luminosity in the threshold region.
The threshold cross-section method is also of interest because it appears to fit very well into the expected schedule for LEP2 operation in 1996. It is anticipated that the maximum beam energy at LEP2 will increase in steps, with the progressive installation of more superconducting RF cavities, in such a way that a centre-of-mass energy of 161 GeV will indeed be achievable during the first running period of 1996. This would then be the ideal time to perform such a threshold measurement. The achievable statistical error on \( M_W \) depends of course critically on the available luminosity at the threshold energy. In Section 2 we present quantitative estimates based on integrated luminosities of 25, 50 and 100 \( pb^{-1} \) per experiment.

1.6 Theoretical input information

1.6.1 Cross-sections for the \( W^+W^- \) signal and backgrounds

Methods (A) and (B) for measuring \( M_W \) described above require rather different theoretical input. The threshold method relies on the comparison of an absolute cross-section measurement with a theoretical calculation which has \( M_W \) as a free parameter. The smallness of the cross-section near threshold is compensated by the enhanced sensitivity to \( M_W \) in this region. In contrast, the direct reconstruction method makes use of the large \( W^+W^- \) statistics at the higher LEP2 energies, \( \sqrt{s} \simeq 175 \text{ GeV} \). Here the more important issue is the accurate modeling of the \( W^\pm \) line-shape, i.e. the distribution in the invariant mass of the \( W^\pm \) decay products.

In this section we describe some of the important features of the theoretical cross-sections which are relevant for the \( M_W \) measurements. A more complete discussion can be found in the contribution of the WW and Event Generators Working Group to this Report [18].

We begin by writing the cross-section for \( e^+e^- \rightarrow 4f \ (+\gamma, g, \ldots) \), schematically, as

\[
\sigma = \sigma_{WW} + \sigma_{bkl} , \\
\sigma_{WW} = \sigma_{0_{WW}} (1 + \delta_{EW} + \delta_{QCD}) ,
\]

We note that this decomposition of the cross-section into ‘signal’ and ‘background’ contributions is practical rather than theoretically rigorous, since neither contribution is separately exactly gauge invariant nor experimentally distinguishable in general. The various terms in (9) correspond to

(i) \( \sigma_{0_{WW}} \): the Born contribution from the 3 ‘CC03’ leading-order diagrams for \( e^+e^- \rightarrow W^+W^- \) involving \( t \)-channel \( \nu \) exchange and \( s \)-channel \( \gamma \) and \( Z^0 \) exchange, calculated using off-shell \( W \) propagators.

(ii) \( \delta_{EW} \): higher-order electroweak radiative corrections, including loop corrections, real photon emission, etc.

(iii) \( \delta_{QCD} \): higher-order QCD corrections to \( W^+W^- \) final states containing \( q\bar{q} \) pairs. For the threshold measurement, where in principle only the total cross-section is of primary
interest, the effect of these is to generate small corrections to the hadronic branching ratios which are entirely straightforward to calculate and take into account. More generally, such QCD corrections can lead to additional jets in the final state, e.g. $W^+W^- \rightarrow q\bar{q}q\bar{q}g$ from one hard gluon emission. This affects the direct reconstruction method, insofar as the kinematic fits to reconstruct $M_W$ assume a four-jet final state, and both methods insofar as cuts have to be imposed in order to suppress the QCD background (see Sections 2,3 below). Perturbative QCD corrections, real gluon emission to $O(\alpha_s^2)$ and real plus virtual emission to $O(\alpha_s^3)$, have been recently discussed in Refs. [19, 20] respectively, together with their impact on the measurement of $M_W$.

(iv) $\sigma_{\text{bkd}}$: 'background' contributions, for example from non-resonant diagrams (e.g. $e^+e^- \rightarrow \mu^+\mu^-W^-\gamma$) and QCD contributions $e^+e^- \rightarrow q\bar{q}gg(\gamma)$, $q\bar{q}Q\bar{Q}(\gamma)$ to the four-jet final state. All of the important backgrounds have been calculated, see Table 6 below. At threshold, the QCD four-jet background is particularly large in comparison to the signal.

In what follows we consider (i) and (ii) in some detail. Background contributions and how to suppress them are considered in later sections.

1.6.2 The $W^+W^-$ off-shell cross-section

The leading-order cross-section for off-shell $W^+W^-$ production was first presented in Ref. [21]:

$$\sigma(s) = \int_0^s ds_1 \left( \sqrt{s} - \sqrt{s_1} \right)^2 ds_2 \rho(s_1)\rho(s_2) \sigma_0(s, s_1, s_2),$$

where

$$\rho(s) = \frac{1}{\pi M_W^2} \frac{\Gamma_W}{(s - M_W^2)^2 + s^2 \Gamma_W^2 / M_W^2}.$$  

The cross-section $\sigma_0$ can be written in terms of the $\nu, \gamma$ and $Z$ exchange contributions and their interferences:

$$\sigma_0(s, s_1, s_2) = \frac{\sqrt{s}}{256\pi s_1 s_2} \left[a_{\gamma\gamma} + a_{ZZ} + a_{\gamma Z} + a_{\nu\nu} + a_{\nu Z} + a_{\nu\gamma}\right],$$

where $g^4 = e^4 / \sin^4 \theta_W$. Explicit expressions for the various contributions can be found in Ref. [21] for example. The stable (on-shell) $W^+W^-$ cross-section is simply

$$\sigma^{\text{on}}(s) = \sigma_0(s, M_W^2, M_W^2).$$

A theoretical ansatz of this kind will be the basis of any experimental determination of the mass and width of the W boson. The reason for this is the large effect of the virtuality of the W bosons produced around the nominal threshold. An immediate conclusion from Eq. (10) may be drawn: the W mass influences the cross sections exclusively through the off-shell W propagators; all the other parts are independent of $M_W$ and $\Gamma_W$ (neglecting for the moment
the relatively minor dependence due to radiative corrections. It will be an important factor in the discussion which follows that near threshold the (unpolarized) cross-section is completely dominated by the $t$-channel neutrino exchange diagram. This leads to an $S$-wave threshold behaviour $\sigma_t \sim \beta$, whereas the $s$-channel vector boson exchange diagrams give the characteristic $P$-wave behaviour $\sigma_s \sim \beta^3$.

By tradition (for example at LEP1 with the Z boson), the virtual $W$ propagator in Eq. (11) uses an $s$-dependent width,

$$\Gamma_W(s) = \frac{s}{M_W^2} \Gamma_W,$$

where $\Gamma_W \equiv \Gamma_W(M_W^2)$. Another choice, equally well justified from a theoretical point of view, would be to use a constant width in the $W$ propagator (for a discussion see Ref. [22]):

$$\bar{\rho} = \frac{1}{\pi} \frac{\overline{M}_W \Gamma_W}{s - M_W^2 + \overline{M}_W \Gamma_W^2}.$$

The numerical values of the width and mass in the two expressions are related [23]:

$$\overline{M}_W = M_W - \frac{1}{2} \frac{\Gamma_W^2}{M_W} = M_W - 26.9 \text{ MeV},$$

$$\Gamma_W = \Gamma_W - \frac{1}{2} \frac{\Gamma_W^2}{M_W^2} = \Gamma_W - 0.7 \text{ MeV}.$$

These relations may be derived from the following identity: $(s - \overline{M}_W^2 + i \overline{M}_W \Gamma_W) = (s - M_W^2 + i \Gamma_W/M_W)/(1 + i \Gamma_W/M_W)$. Numerically, the consequences are below the anticipated experimental accuracy.

### 1.6.3 Higher-order electroweak corrections

The complete set of $O(\alpha)$ next-to-leading order corrections to $W^+W^-$ production has been calculated by several groups [24, 25], for the on-shell case only, see for example Refs. [26, 18] and references therein. There has been some progress with the off-shell (i.e. four fermion production) corrections but the calculation is not yet complete. However using the on-shell calculations as a guide, it is already possible to predict some of the largest effects. For example, it has been shown that close to threshold the dominant contribution comes from the Coulomb correction, i.e. the long-range electromagnetic interaction between almost stationary heavy particles. Also important is the emission of photons collinear with the initial state $e^\pm$ ('initial state radiation') which gives rise to logarithmic corrections $\sim \alpha \ln(s/m_e^2)$. These leading logarithms can be resummed to all orders, and incorporated for example using a 'structure function' formalism. In this case, the generalization from on-shell to off-shell $W$'s appears to be straightforward. For the Coulomb corrections, however, one has to be much more careful, since in this case the inclusion of the finite $W$ decay width has a dramatic effect. Finally, one can
incorporate certain important higher-order fermion and boson loop corrections by a judicious choice of electroweak coupling constant. Each of these effects will be discussed in turn below.

In summary, certain $O(\alpha)$ corrections are already known to be large because their coefficients involve large factors like $\ln(s/m_c^2)$, $M_W/\Gamma_W$, $M_t^2/M_W$, etc. Once these are taken into account, one can expect that the remaining corrections are no larger than $O(\alpha)$. When estimating the theoretical systematic uncertainty on the $W$ mass in Section 2 below, we shall therefore assume a conservative overall uncertainty on the cross-section of $\pm 2\%$ from the as yet uncalculated $O(\alpha)$ and higher-order corrections.

1.6.4 Coulomb corrections

The Coulomb corrections for on-shell and off-shell $W^+W^-$ production have been discussed in detail in Refs. [27, 28, 29], where a complete set of references to earlier studies can also be found.

The result for on-shell $W^+W^-$ production is well-known [30] — the $O(\alpha)$ correction diverges as $\alpha \pi/v_0$ as the relative velocity $v_0 = 2[1 - 4M_W^2/s]^{1/2}$ of the $W$ bosons approaches zero at threshold. Note that $\sigma_0 \sim v_0$ near threshold and so the Coulomb-corrected cross-section is formally non-vanishing when $\sqrt{s} = 2M_W$.

For unstable $W^+W^-$ production the finite decay width $\Gamma_W$ screens the Coulomb singularity [27], so that very close to threshold the perturbative expansion in $\alpha \pi/v_0$ is effectively replaced by an expansion in $\alpha \pi M_W/\Gamma_W$ [28]. In the calculations which follow we use the expressions for the $O(\alpha)$ correction given in Ref. [28]. The net effect is a correction which reaches a maximum of approximately $+6\%$ in the threshold region. Although this does not appear to be large, we will see below that it changes the threshold cross-section by an amount equivalent to a shift in $M_W$ of order $100$ MeV. In Ref. [28] the $O(\alpha)$ result is generalized to all orders. However the contributions from second order and above change the cross-section by $\ll 1\%$ in the threshold region [31] (see also [29]) and can therefore be safely neglected.

Note also that the Coulomb correction to the off-shell $W^+W^-$ cross-section

$$\sigma(s) = \int_0^{(\sqrt{s} - \sqrt{s_1})^2} ds_1 \int_0^{(\sqrt{s} - \sqrt{s_2})^2} ds_2 \rho(s_1)\rho(s_2) \sigma_0(s, s_1, s_2)[1 + \delta_C(s, s_1, s_2)]$$

provides an example of a (QED) interconnection effect between the two $W$ bosons: the exchange of a soft photon distorts the line shape $(d\sigma/d\sqrt{s_1})$ of the $W^\pm$ and therefore, at least in principle, affects the direct reconstruction method [32, 33, 34]. In Ref. [32], for example, it is shown that the Coulomb interaction between the $W$ bosons causes a downwards shift in the average reconstructed mass of $O(\alpha \Gamma_W) \sim O(20 \text{ MeV})$. Selecting events close to the Breit-Wigner peak reduces the effect somewhat. However the calculations are not yet complete, in that QED interactions between the decay products of the two $W$ bosons are not yet fully included.
1.6.5 Initial state radiation

Another important class of electroweak radiative corrections comes from the emission of photons from the incoming $e^+$ and $e^-$. In particular, the emission of virtual and soft real photons with energy $E < \omega$ gives rise to doubly logarithmic contributions $\sim \alpha \ln(s/m_e^2) \ln(s/\omega^2)$ at each order in perturbation theory. The infra-red ($\ln \omega$) logarithms cancel when hard photon contributions are added, and the remaining collinear ($\ln(s/m_e^2)$) logarithms can be resummed and incorporated in the cross-section using a ‘flux function’ or a ‘structure function’ [35] (see also Refs. [36, 37, 38, 39, 18]).

The ISR corrected cross section in the flux function (FF) approach is

$$\frac{d\sigma_{\text{ISR}}(s)}{ds_1 ds_2} = \frac{s'}{s} \int_{s_{\text{min}}}^{s} ds' F(x, s) \sigma_{\text{CC3}}(s'/s_1, s'/s_2) + \delta_C \sigma_{\text{CC3}}(s'/s_1, s'/s_2) + \sigma_{\text{ISR - non-univ}}, \tag{19}$$

where $x = 1 - s'/s$ and

$$F(x, s) = tx^{t-1}(1 + S) + H(s', s), \tag{20}$$

with

$$t = \frac{2\alpha}{\pi} \ln \frac{s}{m_e^2} - 1. \tag{21}$$

The $S$ term comes from soft and virtual photon emission, the $H$ term comes from hard photon emission, $\delta_C$ contains the Coulomb correction (18), $\sigma_{\text{CC3}}$ is the doubly resonating Born cross section, and $\sigma_{\text{CCn}}$ the background contributions. Explicit expressions can be found in the above references. The additional term $\sigma_{\text{ISR - non-univ}}$ is discussed in Refs. [38] and [18]. The invariant mass lost to photon radiation may be calculated as

$$\langle m_\gamma \rangle = \frac{1}{\sigma} ds_1 ds_2 \frac{ds'}{s} \frac{\sqrt{s}}{2} 1 - \frac{s'}{s} \frac{d\sigma}{ds_1 ds_2 ds'} \tag{22}$$

where $d\sigma/ds_1 ds_2 ds'$ is the contents of the curly brackets in Eq. (19).

Alternatively, the structure function (SF) approach may be used:

$$\frac{d\sigma_{\text{QED}}(s)}{ds_1 ds_2} = \int_{x_1^{\text{min}}}^{1} dx_1 \int_{x_2^{\text{min}}}^{1} dx_2 D(x_1, s)D(x_2, s) \sigma_{\text{CC3}}(x_1 x_2 s, s_1, s_2) + \delta_C \sigma_{\text{CC3}}, \tag{23}$$

where

$$D(x, s) = \frac{t}{2}(1 - x)^{t-1/2}(1 + S) + H(x, s). \tag{24}$$

Here, the invariant mass loss is

$$\langle m_\gamma \rangle = \frac{1}{\sigma} ds_1 ds_2 dx_1 dx_2 D(x_1, s)D(x_2, s) \frac{\sqrt{s}}{2} 1 - x_1 x_2 \frac{d\sigma(x_1 x_2 s, s_1, s_2)}{ds_1 ds_2 dx_1 dx_2}. \tag{25}$$
In addition, the radiative energy loss may be determined,

$$
\langle E_\gamma \rangle = \frac{1}{\sigma} \int ds_1 ds_2 \int dx_1 dx_2 \; D(x_1, s) D(x_2, s) \sqrt{s} \left(2 - x_1 - x_2\right) \frac{d\sigma}{ds_1 ds_2 dx_1 dx_2}.
$$

(26)

Initial state radiation affects the W mass measurement in two ways. Close to threshold the cross-section is smeared out, thus reducing the sensitivity to $M_W$ (see Fig. 5 below). For the direct reconstruction method, the relatively large average energy carried away by the radiated photons leads to a large positive mass-shift if it is not taken into account in the rescaling of the final-state momenta to the beam energy (see Section 3 below). By rescaling to the nominal beam energy we obtain for the mass-shift $\Delta M_W = \langle E_\gamma \rangle M_W / \sqrt{s}$. Note however that a fit to the mass distribution gives more weight to the peak, and therefore in practice the effective value of $\langle E_\gamma \rangle$ or $\langle m_\gamma \rangle$ is less than that given by Eqs. (22,25,26) (see Section 3.4). Table 3 shows the influence of the various cross-section contributions on the average energy and invariant mass losses. The invariant mass loss may be calculated both in the SF and FF approaches. A comparison shows that the predictions in both schemes differ only slightly, which allows us to use the numerically faster FF approach for the numerical estimates. At the lower LEP2 collider energies, the energy and invariant mass losses are nearly equal, while at higher energies their difference is non-negligible. Note also that the inclusion of the non-universal ISR corrections and background terms is of minor influence. The latter has been studied only for CC11 processes; for reactions of the CC20 type the background is larger and the numerical estimates are not yet available. The Coulomb correction is numerically important and cannot be neglected [29]. The dependence of the predictions on the details of the treatment of QED is discussed in detail in Ref. [18] and will not be repeated here.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>175</th>
<th>192</th>
<th>205</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF, CC3</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\sigma$ (pb)</td>
<td>13.182</td>
<td>16.488</td>
<td>17.077</td>
</tr>
<tr>
<td>$\langle E_\gamma \rangle$</td>
<td>1115</td>
<td>2280</td>
<td>3202</td>
</tr>
<tr>
<td>$\langle m_\gamma \rangle$</td>
<td>1112</td>
<td>2271</td>
<td>3185</td>
</tr>
<tr>
<td>$\langle E_\gamma \rangle - \langle m_\gamma \rangle$</td>
<td>3</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>change to FF</td>
<td>$\Delta \langle m_\gamma \rangle$</td>
<td>0.5</td>
<td>−0.8</td>
</tr>
<tr>
<td>add $\delta_C$</td>
<td>$\Delta \langle m_\gamma \rangle$</td>
<td>11.8</td>
<td>16.3</td>
</tr>
<tr>
<td>add $\sigma_{\text{ISR}}$</td>
<td>$\Delta \langle m_\gamma \rangle$</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>add $\sigma_{\text{CC20}}$</td>
<td>$\Delta \langle m_\gamma \rangle$</td>
<td>$\leq 0.1$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 3: Influence of different cross-section treatments on the average energy loss $\langle E_\gamma \rangle$ and invariant mass loss $\langle m_\gamma \rangle$ in MeV.
### Table 4: Parameter values used in the numerical calculations in Table 5 and Fig. 5. Masses and widths are given in GeV.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$</td>
<td>91.1888</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.23</td>
</tr>
<tr>
<td>$\Gamma_Z$</td>
<td>2.4974</td>
</tr>
<tr>
<td>$\Gamma_W$</td>
<td>2.078</td>
</tr>
<tr>
<td>$\alpha^{-1}$</td>
<td>137.0359895</td>
</tr>
<tr>
<td>$G_\mu$</td>
<td>$1.16639 \times 10^{-5}$ GeV$^{-2}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_W \equiv \sin^2 \theta_W^{(\ell)\text{eff}}$</td>
<td>0.2320</td>
</tr>
<tr>
<td>$m_e$</td>
<td>$5.1099906 \times 10^{-4}$</td>
</tr>
<tr>
<td>$(\hbar c)^2$</td>
<td>$3.8937966 \times 10^8$ pb GeV$^2$</td>
</tr>
</tbody>
</table>

#### 1.6.6 Improved Born approximation

In the Standard Model, three parameters are sufficient to parametrise the electroweak interactions, and the conventional choice is $\{\alpha, G_\mu, M_Z\}$ since these are the three which are measured most accurately. In this case the value of $M_W$ is a prediction of the model. Radiative corrections to the expression for $M_W$ in terms of these parameters introduce non-trivial dependences on $M_t$ and $M_H$, and so a measurement of $M_W$ provides a constraint on these masses. However the choice $\{\alpha, G_\mu, M_Z\}$ does not appear to be well suited to $W^+W^-$ production. The reason is that a variation of the parameter $M_W$, which appears explicitly in the phase space and in the matrix element, has to be accompanied by an adjustment of the charged and neutral weak couplings. Beyond leading order this is a complicated procedure.

It has been argued [40] that a more appropriate choice of parameters for LEP2 is the set $\{M_W, G_\mu, M_Z\}$ (the so-called $G_\mu$-scheme), since in this case the quantity of prime interest is one of the parameters of the model. Using the tree-level relation

$$g^2 = e^2 / \sin^2 \theta_W = 4\sqrt{2}G_\mu M_W^2$$  \hspace{1cm} (27)

we see that the dominant $t$-channel neutrino exchange amplitude, and hence the corresponding contribution to the cross-section, depends only on the parameters $M_W$ and $G_\mu$. It has also been shown [40] that in the $G_\mu$-scheme there are no large next-to-leading order contributions to the cross-section which depend on $M_t$, either quadratically or logarithmically. One can go further and choose the couplings which appear in the other terms in the Born cross-section such that all large corrections at next-to-leading order are absorbed, see for example Ref. [41]. However for the threshold cross-section, which is dominated by the $t$-channel exchange amplitude, one can simply use combinations of $e^2$ and $g^2$ defined by Eq. (27) for the neutral and charged weak
couplings which appear in the Born cross-section, Eq. (12).

In summary, the most model-independent approach when defining the parameters for computing the $e^+e^- \rightarrow W^+W^-$ cross-section appears to be the $G_\mu$-scheme, in which $M_W$ appears explicitly as a parameter of the model. Although this makes a non-negligible difference when calculating the Born cross-section, compared to using $\alpha$ and $\sin^2 \theta_W$ to define the weak couplings (see Table 5 below), a full next-to-leading-order calculation will remove much of this scheme dependence [40].

<table>
<thead>
<tr>
<th>$\sigma_{WW}$</th>
<th>$\sqrt{s} = 161$ GeV</th>
<th>$\sqrt{s} = 175$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$ (on-shell, $\alpha$)</td>
<td>3.813</td>
<td>15.092</td>
</tr>
<tr>
<td>$\sigma_0$ (on-shell, $G_\mu$)</td>
<td>4.402</td>
<td>17.425</td>
</tr>
<tr>
<td>$\sigma_0$ (off-shell with $\Gamma_W(M_W^2)$, $G_\mu$)</td>
<td>4.747</td>
<td>15.873</td>
</tr>
<tr>
<td>$\sigma_0$ (off-shell with $\Gamma_W(s)$, $G_\mu$)</td>
<td>4.823</td>
<td>15.882</td>
</tr>
<tr>
<td>... + $O(\alpha)$ Coulomb</td>
<td>5.122</td>
<td>16.311</td>
</tr>
<tr>
<td>... + $O(\alpha)$ Coulomb + ISR</td>
<td>3.666</td>
<td>13.620</td>
</tr>
</tbody>
</table>

Table 5: Decomposition of the theoretical $e^+e^- \rightarrow W^+W^-$ cross-section (in picobarns) as defined and discussed in the text, at two LEP2 collider energies.

### 1.6.7 Numerical evaluation of the cross-section

Figure 5 shows the $e^+e^- \rightarrow W^+W^-$ cross-section at LEP2 energies. The different curves correspond to the sequential inclusion of the different effects discussed above. The parameters used in the calculation are listed in Table 4. Note that both the initial state radiation and the finite width smear the sharp threshold behaviour at $\sqrt{s} = 2M_W$ of the on-shell cross-section. The different contributions are quantified in Table 5, which gives the values of the cross-section in different approximations just above threshold ($\sqrt{s} = 161$ GeV) and at the standard LEP2 energy of $\sqrt{s} = 175$ GeV. At threshold we see that the effects of initial state radiation and the finite W width are large and comparable in magnitude. For the threshold method, the primary interest is the dependence of the cross-section on $M_W$. This will be quantified in Section 2 below. For both methods, the size of the background cross-sections is important. For completeness, therefore, we list in Table 6 some relevant cross-section values obtained using the PYTHIA Monte Carlo. This includes finite-width effects, initial state radiation and Coulomb corrections. Notice that the values for $\sigma_{WW}$ agree to within about 1% accuracy with those given in the last row of Table 5.
Cross-sections (in picobarns) for signal and background channels as given by PYTHIA. W+W−, (Z/γ)(Z/γ), Weν and Zee refer to four-fermion production with intermediate formation of either one or two vector bosons; in the case of Z pair production, the generator includes the photon contribution. The process Z/γ → all refers to the production of a fermion pair via a Z boson or photon. All interactions include initial state radiation, and therefore may contain additional photons in the final state. The final row lists cross-sections generated for the Bhabha scattering process (including t-channel exchange) using the BABAMC program.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Cross-section (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>at 161 GeV</td>
</tr>
<tr>
<td>e⁺e⁻ → W⁺W⁻ → all</td>
<td>3.64</td>
</tr>
<tr>
<td>e⁺e⁻ → W⁺W⁻→q̅q̅q̅</td>
<td>1.67</td>
</tr>
<tr>
<td>e⁺e⁻ → W⁺W⁻→q̅lν</td>
<td>1.59</td>
</tr>
<tr>
<td>e⁺e⁻ → W⁺W⁻→lνlν</td>
<td>0.38</td>
</tr>
<tr>
<td>e⁺e⁻ → Z/γ → all</td>
<td>221.</td>
</tr>
<tr>
<td>e⁺e⁻ → Z/γ → q̅q̅</td>
<td>151.</td>
</tr>
<tr>
<td>e⁺e⁻ → Z/γ → l⁺l⁻, νν</td>
<td>70.</td>
</tr>
<tr>
<td>e⁺e⁻ → (Z/γ)(Z/γ) → all</td>
<td>0.46</td>
</tr>
<tr>
<td>e⁺e⁻ → Zee → all</td>
<td>2.53</td>
</tr>
<tr>
<td>e⁺e⁻ → Weν → all</td>
<td>0.37</td>
</tr>
<tr>
<td>e⁺e⁻ → e⁺e⁻,</td>
<td>cos θ</td>
</tr>
</tbody>
</table>
Figure 5: The cross-section for $e^+e^- \to W^+W^-$ in various approximations: (i) Born (on-shell) cross-section, (ii) Born (off-shell) cross-section, (iii) with first order Coulomb corrections, and (iv) with initial state radiation. The parameter values are listed in Table 4.

2 Measurement of $M_W$ from the $W^+W^-$ Threshold Cross-Section

As discussed in Section 1.5, one can exploit the rapid increase of the $W^+W^-$ production cross-section at $\sqrt{s} \sim 2M_W$ to measure the W mass. In the following, we briefly discuss the basic features of this method, suggest an optimal collider strategy for data-taking, and estimate the statistical and systematic errors. The intrinsic statistical limit to the resolution on $M_W$ is shown to be energy-dependent: in particular, arguments are presented in favour of a single cross-section measurement at a fixed energy $\sqrt{s} \sim 161$ GeV.

\footnote{preparing by D. Gelé, T.G. Shears, W.J. Stirling, A. Valassi, M.F. Watson}
2.1 Collider strategy

The cross-section for $W^+W^-$ production increases very rapidly near the nominal kinematic threshold $\sqrt{s} = 2M_W$, although the finite W width and ISR smear out the abrupt rise of the Born on-shell cross-section. This means that for a given $\sqrt{s}$ near threshold, the value of the cross-section is very sensitive to $M_W$. This is illustrated in Fig. 6, where the $W^+W^-$ excitation curve is plotted for various values of the W mass. The calculation is the same as that discussed in Section 1.6, and includes finite W width effects, ISR and QED Coulomb corrections. A measurement of the cross-section in this region therefore directly yields a measurement of $M_W$.

\[ \Delta \sigma_{WW} = \frac{\sigma_{WW}}{\sqrt{N}} = \frac{\sqrt{\sigma_{WW}}}{\sqrt{\epsilon_{WW} L}}, \]  

FIGURE 6: The cross-section for $W^+W^-$ production as a function of $\sqrt{s}$ in the threshold region, for various values of $M_W$. Finite-width effects, the QED Coulomb correction and Initial State Radiation are included.

For an integrated luminosity $L$ and an overall signal efficiency $\epsilon_{WW} = \sum \epsilon_i \text{BR}_i$ (where the sum extends over the various channels selected, with branching ratios $\text{BR}_i$ and efficiencies $\epsilon_i$), the error on the $W^+W^-$ cross-section due to signal statistics is given by
where $N = \epsilon_{WW} \sigma_{WW} \mathcal{L}$ is the number of selected signal events. The corresponding error on the W mass is

$$\Delta M_{WW} = \sqrt{\sigma_{WW}} \frac{dM_{W}}{d\sigma_{WW}} \frac{1}{\epsilon_{WW} \mathcal{L}}.$$  \hspace{1cm} (29)

The sensitivity factor $\sqrt{\sigma_{WW}} |dM_{W}/d\sigma_{WW}|$ is plotted in Fig. 7 as a function of $\sqrt{s} - 2M_{W}$. There is a minimum at

$$(\sqrt{s})^{\text{opt}} \approx 2M_{W} + 0.5 \text{ GeV},$$  \hspace{1cm} (30)

corresponding to a minimum value of approximately $0.91 \text{ GeV pb}^{-1/2}$. Note that the 0.5 GeV offset of the minimum of the sensitivity above the nominal threshold is insensitive to the actual value of $M_{W}$, since in the threshold region the cross-section is to a first approximation a function of $\sqrt{s} - 2M_{W}$ only.

As discussed below, the statistical uncertainty is expected to be the most important source of error for the threshold measurement of $M_{W}$: the optimal strategy for data-taking consists therefore in operating at the collider energy $(\sqrt{s})^{\text{opt}}$ in order to minimize the statistical error on $M_{W}$. The statistical sensitivity factor is essentially flat within $(\sqrt{s})^{\text{opt}} \pm 0.65 \text{ GeV}$, where it increases at most to 0.95 GeV pb$^{-1/2}$ (+4%); bearing in mind that the present uncertainty on $M_{W}$ from direct measurements is 160 MeV (and is expected to decrease further in the coming years), this corresponds to $\pm 2\sigma$ on the current world average $M_{W}$ value. In other words, $M_{W}$ is already known to a level of precision good enough to choose, a priori, one optimal energy for the measurement of the $W^{+}W^{-}$ cross-section at the threshold. Using the latest world average value $M_{W} = 80.26 \pm 0.16 \text{ GeV}$ (see Eq. (1)) gives an optimal collider energy of $(\sqrt{s})^{\text{opt}} \approx 161.0 \text{ GeV}$.

## 2.2 Event selection and statistical errors

The error on the W mass due to the statistics of $W^{+}W^{-}$ events collected has been given in the previous section. Background contamination with an effective cross-section $\sigma_{bkg}$ introduces an additional statistical error. The overall effect is that the statistical error on $M_{W}$ is modified according to

$$\Delta M_{W} \rightarrow \Delta M_{W} \frac{\sigma_{bkg}}{\epsilon_{WW} \sigma_{WW}}.$$  \hspace{1cm} (31)

In the following subsections we present estimates of this statistical error for realistic event selections, for an integrated luminosity of 100 pb$^{-1}$ at $\sqrt{s} = 161 \text{ GeV}$. Tight selection cuts are required to reduce the background contamination while retaining a high efficiency for the signal, especially as the signal cross-section is a factor of 4–5 lower at threshold than at higher centre-of-mass energies.

The studies are based on samples of signal and background events generated by means of Monte Carlo programs (mainly PYTHIA [43]) tuned to LEP1 data. These events were run through the complete simulation program giving a realistic detector response, and passed through the full reconstruction code for the pattern recognition.
Figure 7: The sensitivity of the $W^+W^-$ cross-section to the $W$ mass, plotted as a function of $\sqrt{s} - 2M_W$. The significance of the three curves to the $W$ mass measurement is discussed in the text. A value of $M_W = 80.26$ GeV has been used in the calculations.

2.2.1 Fully hadronic channel, $W^+W^- \rightarrow q\bar{q}q\bar{q}$.

The pure four quark $W^+W^-$ decay mode benefits from a substantial branching ratio (46%) corresponding to a cross-section $\sigma(W^+W^- \rightarrow q\bar{q}q\bar{q}; \sqrt{s} = 161$ GeV) = 1.67 pb. Obviously, the typical topology of such events consists of four or more energetic jets in the final state. Due to its large cross-section (see Table 6), the main natural background to this four-jet topology comes from $e^+e^- \rightarrow Z(\rightarrow q\bar{q}(+n\gamma))\gamma$ events which can be separated into two classes depending on the virtuality of the $Z$: (i) the production of an on-mass-shell $Z$ accompanied by a radiative photon of nearly 55 GeV (at $\sqrt{s} = 161$ GeV), which is experimentally characterised by missing momentum carried by the photon escaping inside the beam pipe (typically 70% of the time), and (ii) events with a soft ISR $\gamma$ and a large total visible energy, which potentially constitute the most dangerous QCD background contribution.

Note that a semi-analytical calculation of the genuine four-fermion background cross-section [16] for a wide range of four-fermion final states (with non-identical fermions) shows that in the threshold region $\sigma_{\text{bgd}}(4f)/\sigma_{WW} \ll 1\%$, and therefore the effect on the $M_W$ determination
from these final states is negligible.

Although the effective four-jet-like event selection depends somewhat on the specific detector under consideration, a general and realistic guideline selection can be described. The most relevant conditions to be fulfilled by the selected events can be summarised as follows:

- A minimum number of reconstructed tracks of charged and neutral particles is required. A typical value is 15. This cut removes nearly all low multiplicity reactions, such as dilepton production \((e^+e^- \rightarrow ll, l = e, \mu, \tau)\) and two-photon processes.

- A veto criterion against hard ISR photons from \(q\bar{q}\gamma\) events in the detector acceptance can be implemented by rejecting events with an isolated cluster with significant electromagnetic energy (larger than 10 GeV for example).

- A large visible energy, estimated using the information from tracks of charged particles and from the electromagnetic and hadron calorimeters. For example, a minimum energy cut value of 130 GeV reduces by a factor of 2 the number of \(q\bar{q}\gamma\) events with a photon collinear to the beam axis.

- A minimum number (typically 5) of reconstructed tracks per jet. This criterion acts on the low multiplicity jets from \(\tau\) decays as well as from \(\gamma\) conversions or \(\gamma\) interactions with the detector material.

- A minimum jet polar angle. The actual cut value depends on the detector setup, but is likely to be around \(10^\circ - 15^\circ\). This cut is mainly needed to eliminate poorly measured jets in the very forward region, where the experiments are generally less well instrumented.

These selection criteria almost completely remove the harmless background sources \((e^+e^- \rightarrow ZZ, Zee\) and \(W\nu\)), but there is still an unacceptable level of \(e^+e^- \rightarrow \gamma\) contamination (about three times higher than the signal). The second step is to suppress the remaining QCD background \((e^+e^- \rightarrow q\bar{q}g\gamma_{soft}\) events) by performing a \(W^\pm\) mass reconstruction based on a constrained kinematic fit. The following additional criteria can then be imposed:

- A \(\chi^2\) probability cut associated with a minimum constrained dijet mass requirement – a typical choice of standard values of 1% and 70 GeV respectively is used here. This procedure appears to be an efficient tool to improve the mass resolution and therefore to reduce the final background, see Fig. 8.

In summary, a reasonable signal detection efficiency in excess of 50% is achievable for \(W^+W^-\rightarrow q\bar{q}q\bar{q}\) events. Although the final rejection factor of QCD events is approximately 500, a substantial residual four-jet background still remains, giving a purity of around 70%. The contributions from other backgrounds (\(ZZ, Zee, W\nu\) and two-photon events) are negligible for the four-jet analysis.
Figure 8: Distributions of the fitted dijet mass using a constrained kinematic fit as described in the text. The solid histogram shows the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ signal and the hatched histogram represents the $e^+e^-\rightarrow q\bar{q}\gamma$ background events.

The signal and background efficiencies for the typical event selection described above are given in Table 7, assuming a total integrated luminosity of 100 pb$^{-1}$ (i.e. 25 pb$^{-1}$ per interaction point).

<table>
<thead>
<tr>
<th></th>
<th>$W^+W^-\rightarrow q\bar{q}q\bar{q}$</th>
<th>$W^+W^-\rightarrow q\bar{q}l\nu$</th>
<th>$W^+W^-\rightarrow l\nu l\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal cross-section</td>
<td>0.94 pb</td>
<td>0.76 pb</td>
<td>0.23 pb</td>
</tr>
<tr>
<td>Signal efficiency</td>
<td>55%</td>
<td>47%</td>
<td>60%</td>
</tr>
<tr>
<td>$\Delta M_W$ (stat.) for signal</td>
<td>180 MeV</td>
<td>197 MeV</td>
<td>354 MeV</td>
</tr>
<tr>
<td>Background cross-section</td>
<td>0.39 pb</td>
<td>0.03 pb</td>
<td>0.01 pb</td>
</tr>
<tr>
<td>$\Delta M_W$ (stat.) for background</td>
<td>106 MeV</td>
<td>37 MeV</td>
<td>74 MeV</td>
</tr>
<tr>
<td>Total $\Delta M_W$ (stat.)</td>
<td>209 MeV</td>
<td>200 MeV</td>
<td>360 MeV</td>
</tr>
</tbody>
</table>

Table 7: Typical accepted cross-sections at $\sqrt{s} = 161$ GeV and corresponding statistical uncertainty on $M_W$, assuming an integrated luminosity of 100 pb$^{-1}$. Note that $l = e, \mu, \tau$.

2.2.2 Semileptonic channel, $W^+W^-\rightarrow q\bar{q}l\nu$.

The decay channel $W^+W^-\rightarrow q\bar{q}l\nu$ is characterised by the presence of two or more hadronic jets, an isolated, energetic lepton or a narrow jet in the case of hadronic $\tau$ decays, and missing energy and momentum due to the undetected neutrino. Since the $W^+W^-$ cross-section is small
at threshold, processes such as $e^+e^- \rightarrow Z/\gamma$ and $e^+e^- \rightarrow Zee$ constitute important backgrounds. Their cross-sections at 161 GeV are listed in Table 6. Typical experimental selection cuts for the $W^+W^- \rightarrow q\bar{q}e\nu$ and $W^+W^- \rightarrow q\bar{q}\mu\nu$ channels include:

- A multiplicity cut to reject purely leptonic events. By requiring at least 6 charged tracks in the event, backgrounds from the $W^+W^- \rightarrow l\nu\nu$, $Z/\gamma \rightarrow l^+l^-$ and leptonic two-photon channels can be removed.

- Identification of an electron or muon using standard experimental cuts. The lepton is required to have a high momentum and to be isolated from the hadronic jets. This isolation can be achieved by restricting the energy in a cone around the lepton candidate, or by requiring a minimum transverse momentum relative to the nearest jet. An example of such a cut is to require less than 2.5 GeV of energy (charged and neutral) inside a 200 mrad cone around the track. This suppresses hadronic background, much of which originates from the semi-leptonic decays of heavy quarks inside jets.

- Cuts on missing momentum and its direction. The neutrino carries away momentum, leaving the event with a net momentum imbalance. In order to distinguish the signal events from $Z^0/\gamma$ or two photon backgrounds, in which the missing momentum tends to be along the beam direction, the event is required either to have a large transverse momentum, or to have missing momentum pointing away from the beam direction. These quantities are illustrated in Fig. 9(a) and (b) and show clear differences between signal and background events. Effective experimental cuts are $|\cos \theta_{\text{miss}}| < 0.95$ and $p_{\text{miss}}/E_{\text{cm}} > 0.07$.

- Cuts on the corrected, visible energy in the event. Requiring $E_{\text{vis}}/E_{\text{cm}} < 1$ removes some residual background from $Z/\gamma \rightarrow$ hadrons. Similar results can be achieved by cutting on the energy or mass of the hadronic system.

Additional improvements to the selection may be obtained by applying a kinematic fit, in which energy-momentum constraints are applied in conjunction with assumptions about the W masses. The background events tend to give poor probabilities in the fit.

Selection cuts based on these quantities give efficiencies of 70–75% for the $q\bar{q}e\nu$ and $q\bar{q}\mu\nu$ channels, but only about 5% for the $q\bar{q}\tau\nu$ decays. The purity of the selected sample is 90–95%, with typical accepted cross-sections as listed in Table 7. The corresponding statistical uncertainty on $M_W$ is approximately 200 MeV for 100 pb$^{-1}$. More work will be needed to improve the efficiency for selecting $\tau$ decays, and hence to enhance the statistical precision of the cross-section measurement. Hadronic $\tau$ decays can be identified as a third jet with low multiplicity and a small opening angle.

### 2.2.3 Fully leptonic channel, $W^+W^- \rightarrow l\nu\nu$.

The fully leptonic channel is not used by the direct reconstruction method due to the lack of sufficient constraints on the kinematics of the event, but it can be used for event-counting in
the threshold measurement of $M_W$. It has, however, a branching ratio of only 11% ($l = e, \mu, \tau$), a factor 4 lower than the other two channels. This is reflected in the relative weight of this channel to the overall precision on $M_W$ using the threshold method.

The $W^+W^- \rightarrow \ell\nu\ell\nu$ channel is characterised by two acoplanar, energetic leptons and large missing momentum. In $4/9$ ($1/9$) of the cases, however, one (both) of these leptons is a $\tau$, which typically decays to a narrow hadronic jet. Typical experimental selections for all $W^+W^- \rightarrow \ell\nu\ell\nu$ channels require:

- Low charged multiplicity (typically not greater than 6), which allows the rejection of most of the hadronic backgrounds. The most important background to this channel is then given by dilepton ($Z/\gamma$, two-photon or Bhabha) events. At least two good charged tracks (with a typical minimum energy of 1 GeV) are required in any case.

- One identified electron or muon (requiring two would lose most of the events with one $W \rightarrow \tau\nu_\tau$ decay); this lepton is required to have an angle of typically at least 25° to the beam axis, as both Bhabhas and two-photon events are concentrated at low angles. Since the leptons from $W^+W^-$ decays at threshold have approximately half the beam energy, an energy window may be imposed on the lepton, centered around this value (such as [28–55] GeV); this is effective both against low-energy two-photon events (which can be further reduced by requiring a large total visible mass), and against di-muons and
Bhabhas, where the energy of each lepton is typically equal to the beam energy.

- Explicit tagging of events with isolated, energetic photons or luminosity clusters allows one to reject radiative Z events with hard, detected ISR.

- Cuts on the missing momentum and its direction can also be used: large transverse missing momentum (typically more than 4 GeV) is required to discriminate against two-photon events, while combined cuts on the angle between the jets and on the missing momentum out of the beam-thrust plane are very effective against di-tau Z/γ events.

- To reconstruct the two original lepton directions, the event is forced into two jets, which must be of low invariant mass and well contained in the detector. A cut on the acoplanarity $\Delta\phi$ between these two jets (such as $\Delta\phi < 174^\circ$) is effective against all residual backgrounds. The distribution of this variable for signal and background events is shown in Fig. 10.

Selection cuts of this kind give overall efficiencies of approximately 60% (60% to 70% in all individual channels except the $\tau\nu_\tau\tau\nu_\tau$ channel, where it is only of the order of 30%), with a purity close to 95%. Typical accepted cross-sections are listed in Table 7. The corresponding statistical error on $M_W$ is approximately 360 MeV (354 MeV from signal statistics plus 74 MeV from background statistics) for a total integrated luminosity of 100 pb$^{-1}$.

![Figure 10: Distribution of the jet acoplanarity $\Delta\phi$, in the selection of $W^+W^-\rightarrow l\nu l\nu$ events at threshold. The solid histogram includes the events selected by all cuts (except that on $\Delta\phi$). The shaded area corresponds to the background from the Z/γ and γγ dilepton events.](image)
2.3 Systematic errors

Uncertainties in the \( W^+W^- \) production cross-section translate directly into systematic errors on the W mass. The uncertainties fall into three categories: multiplicative uncertainties in the cross-section; an additive uncertainty due to background subtraction; other sources, for example beam energy and W width.

2.3.1 Luminosity, higher-order corrections and selection efficiency

Quantities which enter multiplicatively in the calculation or measurement of the cross-section contribute an error to the W mass which can be expressed as

\[
\Delta M_W = \frac{d\sigma}{dM_W} \sigma^{-1} \frac{\Delta C}{C},
\]

where \( \Delta C \) is the uncertainty on the multiplicative quantity \( C \), and the cross-section \( \sigma \) includes contributions from signal and background weighted by their relative efficiencies. The quantity \( \sigma |d\sigma/dM_W|^{-1} \) is shown in Fig. 7 as a function of \( \sqrt{s} \). Like the statistical sensitivity factor, it also has a minimum near the nominal threshold and has the value of approximately 1.7 GeV at \( (\sqrt{s})_{\text{opt}} \sim 161 \) GeV.

The three most important uncertainties of this type are:

- The luminosity, which is expected to be known to about 0.5%, including both the known theoretical error on the Bhabha cross-section and the expected experimental error, thus contributing about 8 MeV to \( \Delta M_W \).

- Unknown higher-order corrections to the theoretical cross-section (see Section 1.1.6 and Ref. [18]), which if we conservatively assume a value of \( \pm 2\% \), contribute about 34 MeV to \( \Delta M_W \).

- Uncertainties in the signal efficiency, described in Section 2.3.4 below, which depend on the particular decay channel under consideration.

2.3.2 Background subtraction

An uncertainty \( \Delta \sigma_{\text{bkgd}} \) on the residual background cross-section predicted by Monte Carlo propagates as an additive uncertainty to the measured \( W^+W^- \) cross-section, from which the background has to be subtracted. It contributes an error

\[
\Delta M_W = \frac{d\sigma}{dM_W} \sigma^{-1} \frac{\Delta \sigma_{\text{bkgd}}}{\epsilon_{WW}}
\]
where \( \epsilon_{WW} \) is the signal efficiency, which is found by multiplying the selection efficiency for a given channel by the appropriate branching ratio. The quantity \( |d\sigma/dM_W|^{-1} \) is shown in Fig. 7 as a function of \( \sqrt{s} \), and is approximately 470 MeV pb\(^{-1} \) at \( (\sqrt{s})_{\text{opt}} \approx 161 \) GeV. Experimental methods for determining the uncertainty in the background are described in Section 2.3.4 below. A systematic contribution to \( \Delta M_W \) of about 59 MeV (32 MeV) in the \( W^+W^- \rightarrow q\bar{q}q\bar{q} \) (\( W^+W^- \rightarrow q\bar{q}l\nu \), \( W^+W^- \rightarrow l\nu\nu \)) channels is expected.

### 2.3.3 Beam energy and W width

The error introduced by an uncertainty in the beam energy \( E_{\text{beam}} \) to \( M_W \) is

\[
\Delta M_W = \frac{d\sigma}{d M_W} \frac{d\sigma}{d \Delta E_{\text{beam}}} \Delta E_{\text{beam}}. \tag{34}
\]

In the threshold region the cross-section \( \sigma_{WW} \) is essentially a function of the single variable \( \sqrt{s} - 2M_W \) only (see Fig. 6), and hence the ratio of derivatives in (34) is approximately unity, i.e. \( \Delta M_W \approx 1.0 \Delta E_{\text{beam}} \). It is estimated that the beam energy will be known to an accuracy of 12 MeV.

The error on \( M_W \) introduced by an uncertainty in the W width \( \Gamma_W \) is

\[
\Delta M_W = \frac{d\sigma}{d M_W} \frac{d\sigma}{d \Gamma_W} \Delta \Gamma_W \approx 0.16 \Delta \Gamma_W, \tag{35}
\]

where the value of the ratio of the derivatives corresponds to \( \sqrt{s} = 161 \) GeV. In the Standard Model, \( \Gamma_W \) is proportional to the third power of \( M_W \),

\[
\Gamma_W = 2.080 \left( \frac{M_W}{80.26 \text{ GeV}} \right)^3. \tag{36}
\]

If the current world average value of \( M_W = 80.26 \pm 0.16 \) GeV is used then \( \Delta \Gamma_W = 12 \) MeV. In contrast, a (combined) measurement of \( \Gamma_W \) by the CDF and D0 collaborations at the Tevatron \( p\bar{p} \) collider [42] gives \( \Gamma_W = 2.051 \pm 0.061 \) GeV, consistent with the Standard Model calculation, but with a much larger error. If we use the Standard Model width, then the contribution (2 MeV) to \( \Delta M_W \) is negligible. See Ref. [17] for a further discussion.

### 2.3.4 Experimental determination of systematic uncertainties

Two methods have been proposed to determine the uncertainty in the signal efficiency and background cross-section. The first method examines the sensitivity of the signal efficiencies and background cross-section to uncertainties in fragmentation. Some preliminary studies have been performed in the \( W^+W^- \rightarrow q\bar{q}q\bar{q} \) channel by varying the fragmentation parameters \( Q_0, \sigma_q, b \) and \( \Lambda_{QCD} \) [43] within one standard error bounds, and noting the effect on the signal efficiency and background cross-section in PYTHIA [43] generated events.
The second method uses data and Monte Carlo event samples from LEP1 to determine the uncertainty in the background cross-section. The selections described in Section 2.2 have been scaled to the LEP1 centre-of-mass energy and applied to both real and simulated data. The difference between data and Monte Carlo gives the error on the background cross-section at $\sqrt{s} \approx M_Z$. The results are then rescaled to $\sqrt{s} = 161$ GeV. It is assumed that the fractional uncertainty on the selection efficiencies for the background is the same at the two energies.

The uncertainty on the signal efficiency appears small from the results of the fragmentation test, within the low statistics of the event samples tested. Further studies are needed to quantify any effects. Therefore a conservative estimate of the signal efficiency error of 2% is used at present, contributing an error of 34 MeV to $M_W$. The uncertainty on the background cross-section in the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ channel (8%) is estimated to contribute about 59 MeV to the error on $M_W$. The uncertainty in the $W^+W^-\rightarrow q\bar{q}l\nu$ and $W^+W^-\rightarrow l\nu l\nu$ channels, estimated to be 50% and 100% respectively, both give an error of about 32 MeV. These errors are expected to decrease with further study.

### 2.3.5 Conclusions

Table 8 summarizes the estimated systematic errors for the $W^+W^-\rightarrow q\bar{q}q\bar{q}$, $W^+W^-\rightarrow q\bar{q}l\nu$ and $W^+W^-\rightarrow l\nu l\nu$ channels.

<table>
<thead>
<tr>
<th>Source</th>
<th>$W^+W^-\rightarrow q\bar{q}q\bar{q}$</th>
<th>$W^+W^-\rightarrow q\bar{q}l\nu$</th>
<th>$W^+W^-\rightarrow l\nu l\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity (*)</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>HO corrections (*)</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>Beam energy (*)</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>W width (*)</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Signal efficiency</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>Background cross-section</td>
<td>59</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>Total (MeV)</td>
<td>77</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 8: Summary of systematic error contributions, in MeV, to the threshold measurement in the $W^+W^-\rightarrow q\bar{q}q\bar{q}$, $W^+W^-\rightarrow q\bar{q}l\nu$ and $W^+W^-\rightarrow l\nu l\nu$ channels. The quantities denoted (*) are common to all channels. The total for each channel is found by combining the sources in quadrature.

### 2.4 Summary

Table 9 summarizes the results presented above for the estimated statistical, systematic and total errors on $M_W$ (for all decay channels combined) using the threshold method, i.e. by
measuring the $W^+W^-$ cross-section at the optimal collider energy of 161 GeV. Our estimates for some of the systematic errors, for example the unknown higher-order theoretical corrections, are probably too conservative, and others, for example the uncertainty in the estimates of the various background cross-sections, will almost certainly decrease with more study. Nevertheless, for the amount of luminosity likely to be available for the threshold measurement the overall error is dominated by statistics.

<table>
<thead>
<tr>
<th>Total luminosity</th>
<th>$\Delta M_W$ (stat)</th>
<th>$\Delta M_W$ (stat+sys)</th>
<th>$\Delta M_W$ (total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 \times 25 \text{ pb}^{-1} = 100 \text{ pb}^{-1}$</td>
<td>134</td>
<td>139</td>
<td>144</td>
</tr>
<tr>
<td>$4 \times 50 \text{ pb}^{-1} = 200 \text{ pb}^{-1}$</td>
<td>95</td>
<td>101</td>
<td>108</td>
</tr>
<tr>
<td>$4 \times 100 \text{ pb}^{-1} = 400 \text{ pb}^{-1}$</td>
<td>67</td>
<td>76</td>
<td>84</td>
</tr>
</tbody>
</table>

Table 9: Summary of statistical and systematic errors (in MeV) on $M_W$ from a cross-section measurement (all channels) at $\sqrt{s} = 161$ GeV. The total luminosity refers to four experiments combined. The third column includes the channel-dependent systematic errors only (see Table 8), and the fourth column includes the overall common systematic error.

### 3 Direct Reconstruction of $M_W$

In this section, we discuss the measurement of the W mass by kinematic reconstruction of the invariant mass of the W decay products. The statistical precision of this method which could be obtained by combining four experiments each with 500 pb$^{-1}$ at $\sqrt{s} = 175$ GeV, assuming 100% efficiency and perfect detector resolution, is about 10 MeV, limited by the finite width of the W. In practice, this ideal precision will be degraded, partly through loss of statistics, but mainly because detector effects limit the resolution on the reconstructed mass. This has been studied in detail by the four experiments, using Monte Carlo events with full detector simulation. We discuss methods of improving the mass resolution over that obtained by simple calculation of invariant masses. In particular, a kinematic fit using the constraints of energy and momentum conservation, together with the equality of the two W masses in an event, proves to be a very powerful technique for improving the mass resolution, and also turns out to be a useful background rejection criterion. For this reason, we concentrate on the channels $W^+W^-\rightarrow q\bar{q}q\bar{q}$ and $W^+W^-\rightarrow q\bar{q}l\nu$ where the lepton is an electron or muon, for which constrained fits are most useful.

We start by discussing the basic selection of W-pair events in these two channels, and the reconstruction of jets. We discuss the techniques of the constrained fit in some detail, followed by the determination of $M_W$ from the distribution of reconstructed masses, indicating the

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statistical error which may be expected. Finally we describe the main sources of systematic error pertaining to this measurement.

3.1 Event selection and jet reconstruction

The criteria used to select W-pair events are essentially the same as those described in Section 2, but at the higher energies used for direct reconstruction the background from \(Z/\gamma \rightarrow q\bar{q}\) is lower, so looser cuts can be used.

3.1.1 \(W^+W^- \rightarrow q\bar{q}q\bar{q}\)

\(W^+W^- \rightarrow q\bar{q}q\bar{q}\) events are characterised by high multiplicity (about twice that of a \(Z/\gamma \rightarrow q\bar{q}\) event at LEP1), high visible energy, and exhibit a four-jet structure. The main background comes from \((Z^0/\gamma)^{*} \rightarrow q\bar{q}\) events, for which the cross-section is much higher, as shown in Table 6. Typical selection cuts for \(W^+W^- \rightarrow q\bar{q}q\bar{q}\) events include:

- high multiplicity of tracks and calorimeter clusters to remove purely leptonic events; a significant fraction of \(W^+W^- \rightarrow q\bar{q}l\nu\) and \((Z^0/\gamma)^{*} \rightarrow q\bar{q}\) events can also be removed.

- high visible energy and low missing momentum, to remove \(W^+W^- \rightarrow q\bar{q}l\nu\) and radiative \(Z/\gamma \rightarrow q\bar{q}\) events.

- explicit removal of events with an isolated electromagnetic cluster consistent with being an initial state photon to remove radiative \(Z/\gamma \rightarrow q\bar{q}\) events.

- event shape variables to separate the four-jet \(W^+W^- \rightarrow q\bar{q}q\bar{q}\) events from the remaining background, almost entirely composed of non-radiative \(Z/\gamma \rightarrow q\bar{q}\) events. For example, Fig. 11 shows the distribution of \(y_{34}^D\), the value of \(y_{cut}\) in the Durham (\(k_T\)) jet-finding scheme at which events change from three jets to four jets, after cuts on the above quantities have been applied. The \(W^+W^- \rightarrow q\bar{q}q\bar{q}\) events tend to have larger values of this variable than the background. Other event shape variables, such as jet broadening measures can also be used.

In Table 10 we show values of efficiency, purity, accepted cross-sections and numbers of events produced by typical cuts on these variables. The efficiency of selection cuts tends to fall slightly with energy because as \(\sqrt{s}\) is increased the W's are more boosted and event shape variables have less separating power. The purity can be further enhanced by using kinematic fits, as described below, though at a cost in efficiency.

In order to reconstruct the W mass, a jet-finder is used to force the selected events to contain four jets. Jets are usually reconstructed using both tracks and calorimeter information combined to give the best resolution. The typical jet energy resolution is around 20% for
Figure 11: Distribution of $y_{34}^\sigma$, the value of $y_{cut}$ in the Durham ($k_t$) jet-finding scheme at which events change from three jets to four jets, after cuts on multiplicity, visible energy and missing momentum have been applied to events at 175 GeV. The solid histogram shows $W^+W^-\rightarrow q\bar{q}q\bar{q}$ events, the open histogram background, mainly non-radiative $(Z^0/\gamma)^*\rightarrow q\bar{q}$ events. The dotted line indicates a typical cut value.
Table 10: Typical accepted cross-sections and numbers of events for the signal and main backgrounds for the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ channel for two values of c.m. energy, determined from Monte Carlo including full detector simulation. Values are given both for basic selection cuts, and after demanding a good kinematic fit. The efficiency, purity and expected statistical error on $M_W$ for an integrated luminosity of 500 pb$^{-1}$ are also given.
a jet energy of 20 GeV, improving to 15% at an energy of 60 GeV; over this same energy range the angular resolution improves from 4° to 1.3° for jets at 90° to the beam direction. Studies of various jet finders comparing the reconstructed jets with the initial quark directions show no major differences among the commonly used schemes. The W mass may then be reconstructed by forming the invariant mass of pairs of jets. For each event, there are three possible combinations; Monte Carlo studies show that the combination where the two highest energy jets are combined together is rarely the correct one, and combinatorial background can be reduced by discarding this combination. The mass resolution can be improved by using beam energy constraints or kinematic fits, as described in the next section.

3.1.2 W⁺W⁻→q̅q′ν

The distinguishing feature of W⁺W⁻→q̅q′ν and W⁺W⁻→q̅qμν events is the presence of a high momentum, isolated lepton. Typical selection cuts for W⁺W⁻→q̅q′ν and W⁺W⁻→q̅qμν events include:

- high multiplicity of tracks and calorimeter clusters to remove purely leptonic events;
  - the multiplicity of W⁺W⁻→q̅q′ν events is lower than that of W⁺W⁻→q̅q̅q̅ events, so suitable cuts do not remove (Z⁰/γ)⁺→q̅q background in this case.
  - the presence of a high momentum, isolated, lepton.

An isolated lepton can be identified as an electron or muon with fairly loose, standard experimental cuts with high efficiency (∼95%), though only in a limited acceptance, typically |cos(θ)| < 0.93. For example electrons can be identified using the match between track momentum and energy deposited in the electromagnetic calorimeter, where the pattern of energy deposition is consistent with an electromagnetic shower. Muon identification uses matching between a track in a central tracking chamber and one in outer muon detectors. The lepton momentum spectrum and its degree of isolation, as measured by the total scalar sum of charged particle momentum plus electromagnetic energy in a 200mrad cone around the lepton, are shown in Fig. 12. Typical efficiencies, purities, accepted cross-sections and numbers of events are indicated in Table 11. W⁺W⁻→q̅qτν events can be selected as above, but instead of requiring an identified electron or muon, searching for a narrow, low multiplicity jet isolated from the other jets.

The W mass can be estimated from these events by simply forming the invariant mass of the hadronic system, scaling to the beam energy, or preferably using the full information in a kinematic fit as described below. In this case, the hadronic system is forced to be two jets, which are reconstructed as in the W⁺W⁻→q̅q̅q̅ case. The lepton energy resolution is much better than that for a reconstructed jet, as long as care is taken to include all the electromagnetic energy associated with an electron.
Figure 12: (a) Momentum spectrum of particles identified as leptons and passing isolation cuts at 161 GeV. (b) Scalar sum of charged particle momentum and electromagnetic calorimeter energy in a 200 mrad cone around high momentum identified electrons and muons at 161 GeV. (c) and (d) as (a) and (b) for $\sqrt{s} = 175$ GeV. In each case the solid histogram shows the contribution from leptons in $W^+W^-\rightarrow q\bar{q}l\nu$ events, the open histogram the background. The dashed lines show possible cuts.
<table>
<thead>
<tr>
<th></th>
<th>$\sqrt{s} = 175$ GeV</th>
<th></th>
<th>$\sqrt{s} = 192$ GeV</th>
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<tbody>
<tr>
<td></td>
<td>Accepted cross-section (pb)</td>
<td>Events for 500pb$^{-1}$</td>
<td>Accepted cross-section (pb)</td>
</tr>
<tr>
<td>After basic selection cuts:</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$W^+W^-\rightarrow q\bar{q}\ell\nu$ ($l = e$ or $\mu$)</td>
<td>3.1</td>
<td>1550</td>
<td>3.8</td>
</tr>
<tr>
<td>$W^+W^-\rightarrow q\bar{q}\tau\nu$</td>
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<td>100</td>
<td>0.4</td>
</tr>
<tr>
<td>$(Z^0/\gamma)^*\rightarrow q\bar{q}$</td>
<td>0.2</td>
<td>100</td>
<td>0.2</td>
</tr>
<tr>
<td>$Z^0$ ee</td>
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<td>100</td>
<td>0.3</td>
</tr>
<tr>
<td>efficiency</td>
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<td></td>
<td>74%</td>
</tr>
<tr>
<td>purity</td>
<td>83%</td>
<td></td>
<td>80%</td>
</tr>
<tr>
<td>After kinematic fit:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W^+W^-\rightarrow q\bar{q}\ell\nu$ ($l = e$ or $\mu$)</td>
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<td>1500</td>
<td>3.4</td>
</tr>
<tr>
<td>$W^+W^-\rightarrow q\bar{q}\tau\nu$</td>
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<td>25</td>
<td>0.06</td>
</tr>
<tr>
<td>$(Z^0/\gamma)^*\rightarrow q\bar{q}$</td>
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<td>0.05</td>
</tr>
<tr>
<td>$Z^0$ ee</td>
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</tr>
<tr>
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<td></td>
<td>68%</td>
</tr>
<tr>
<td>purity</td>
<td>96%</td>
<td></td>
<td>95%</td>
</tr>
<tr>
<td>$\Delta(M_W)$ (stat)</td>
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<td></td>
<td>93 MeV</td>
</tr>
</tbody>
</table>

Table 11: Typical accepted cross-sections and numbers of events for the signal and main backgrounds for the $W^+W^-\rightarrow q\bar{q}\ell\nu$ ($l = e$ or $\mu$) channel for two values of c.m. energy, determined from Monte Carlo including full detector simulation. Values are given both for basic selection cuts, and after demanding a good kinematic fit. The efficiency, purity and expected statistical error on $M_W$ for an integrated luminosity of 500 pb$^{-1}$ are also given.
3.2 Constrained fit

After the event has been reconstructed as a number of jets and a number of leptons we next turn to the reconstruction of the W mass from these four fermions, which are treated as individual objects with measurable quantities. We try to reconstruct the best estimator for the W mass on an event by event basis from the measured quantities and the constraints imposed by energy and momentum conservation, and the possible additional constraint that the masses of the two W’s are equal.

Without imposing any constraints the direct reconstruction of the di-jet mass gives:

\[
m_{ij} = \frac{2E_i E_j (1 - \cos \theta_{ij})}{E_i + E_j}
\]

(37)

where \(E_i\) and \(E_j\) are the jet energies, \(\theta_{ij}\) the jet-jet opening angle and where the jet masses have been neglected. Taking only the errors on the jet energies into account, as these are much larger than the errors on the angle measurements, this then gives:

\[
\frac{\sigma(m_{ij})}{m_{ij}} = \frac{\sigma(E_i)}{2E_i} + \frac{\sigma(E_j)}{2E_j}
\]

(38)

Typical jet energy measurement errors of 15% at 45 GeV lead to a relative uncertainty of 10% on \(M_W\), and give distributions of reconstructed mass as shown in the top half of Fig. 13. To make a precision measurement of \(M_W\) it is necessary to improve this resolution by making use of the knowledge of the total energy and momentum which is given in an \(e^+e^-\) collider.

3.2.1 Rescaling methods

These methods are especially useful for the analysis of the semi-leptonic channels and most have been described previously [1]. The basic principle is to write the momentum, energy, and equal mass constraints as functions of the measured fermion momenta and solve for those variables which have the largest measurement uncertainties, generally jet energies.

The first step is to include the beam energy constraint, which is equivalent to the constraint that the two masses are equal:

\[
E_i + E_j = E_b.
\]

(39)

This leads to a determination of the reconstructed \(M_W\):

\[
m_{ij} = \frac{E_b}{E_i + E_j} \frac{2E_i E_j (1 - \cos \theta_{ij})}{E_i + E_j}.
\]

(40)

Assuming that only the errors on the jet energies are important the error on the mass becomes:

\[
\frac{\sigma(m_{ij})}{m_{ij}} \sim \frac{|E_i - E_j|}{E_i + E_j} \frac{\sigma(E_i)}{E_i} + \frac{\sigma(E_j)}{E_j}.
\]

(41)
Together with the other errors which were neglected, this leads to a relative uncertainty on
the reconstructed mass of typically 5%. This method is especially suited to the semi-leptonic
channel in which the leptonic W decay into $\tau \nu_\tau$ does not allow advantage to be taken of the
measured lepton energy.

In the case of semi-leptonic decays to an electron or muon, we can include the parameters
of the measured lepton by writing:

\[
\vec{p}_\nu = -E_i \vec{e}_i - E_j \vec{e}_j,
\]

\[
E_i + E_j = E_b,
\]

\[
E_\ell + E_\nu = E_b ,
\]

where $\vec{e}$ are unit vectors in the direction of the particles. These five equations with five
unknowns, $\vec{p}_\nu$, $E_i$, and $E_j$, can be solved, and yield two distinct solutions. This ambiguity leads
to a problem if the two solutions are close to each other. Taking the solution which is closest
to the measured jet energies leads to a relative error on $M_W$ of about 4%.

The two exact solutions of Eq. (42) will give two minima also for the constrained fit in the
$\chi^2(M_W)$ distribution when the two solutions are close. Monte Carlo studies suggest that about
40% of the events are afflicted by this problem. Current analyses have not yet included this
effect in the determination of $M_W$ and one might therefore expect an improvement in resolution
if this effect is taken correctly into account.

3.2.2 The constrained fit

The most effective way to use all the information available in an event is to perform a constrained
kinematic fit. In such a fit, the measured parameters are varied until a solution is found which
satisfies the constraints imposed and also minimises the $\chi^2$ difference between the measured
and fitted values. Several methods exist to perform this minimization. A traditional one solves
the problem using Lagrange multipliers, minimizing

\[
S(\vec{y}, \lambda) = (\vec{y} - \vec{y}_0)^\top V^{-1} (\vec{y} - \vec{y}_0) + 2\lambda \cdot \vec{f}(\vec{y}),
\]

where $V$ is the error matrix, $\vec{y}$ the fitted variables, $\vec{y}_0$ the measured values, $\lambda$ Lagrange multipliers,
and $\vec{f}(\vec{y})$ the constraints written as functions which must vanish. An equivalent method
is to use penalty functions where terms of the type $\vec{f}^2/\sigma^2$ are added to the $\chi^2$. The procedure
then minimizes the total $\chi^2$ in an iterative way, for each step decreasing the $\sigma$ of the penalties.
Results are in general the same but the convergence is slightly slower.

The inputs to the fit are measurements of the energy and angles, of the four jets in the
hadronic final state or of the two jets and lepton in the semi-leptonic final state, together with
their error matrix. Errors on jet parameters can be extracted from data or Monte Carlo, and
may be functions of both energy and position measured in the detector. In practice most of
the jet measurement errors are nearly uncorrelated, making the error matrix diagonal. For the
jet masses two different strategies are used. Either the jets are assumed to be massless and the measured jet energy is used as the measured jet momentum, or one includes the reconstructed jet masses in the fit. (Note that the $\tau$ lepton can be treated in nearly the same way as jets, except that the errors on the transverse momenta are given by the missing mass and that the $\tau$ mass is used explicitly in the fit. This has not yet been studied in detail.)

The fit can be performed using only the constraints of energy and momentum conservation, or also including the constraint that the masses of the two reconstructed W's be equal. In the hadronic channel, this gives a 4C or 5C fit respectively. In the semi-leptonic channel, the number of constraints is reduced by three because the parameters of the neutrino are unmeasured, resulting in a 1C or 2C fit. The equal mass constraint can be included exactly, or the width of the W can be taken into account by adding a term to the $\chi^2$ proportional to the difference in mass of the two W's. In practice, because the mass resolution is larger than the W width, both methods give almost the same results. If an equal mass constraint is not applied, the reconstructed masses of the two di-fermion systems are strongly anticorrelated. Thus only the average invariant mass can usefully be extracted per event. The inclusion of an equal mass constraint is preferred over a fit using only energy and momentum conservation because it gives improved mass resolution and superior background rejection.

In the 4-jet channel we do not know which jets are to be combined to produce the heavy particles we are interested in reconstructing. Therefore the constrained fit is performed for all three possible combinations. In the case with no equal mass constraint there are three different mass solutions with the same $\chi^2$. When an equal mass constraint is imposed the three different solutions will in general have different $\chi^2$ and one can therefore use this information to distinguish between the solutions. The procedure employed by most analyses is to first define a mass window, wide enough not to bias the mass measurement, and if more than one solution exists inside this window choose the solution with the lowest $\chi^2$. This procedure is however not perfect and one is left with a fraction of wrong combinations as shown in Fig. 13.

### 3.2.3 Results of the fit

In Fig. 13 we show invariant mass distributions before and after the fit, in the latter case taking only those events which give a good fit. Before the fit, the mass distributions are very broad. After the fit the mass resolutions obtained are typically 3.5% for the $W^+W^-\rightarrow q\bar{q}l\nu$ channel and around 2.5% for the 4-jet channel. It is also clear from this figure that the $\chi^2$ of the constrained fit can be used to eliminate possible background which does not comply with the $W^+W^-$ hypothesis: the level of background is much lower in both channels after demanding a good fit. However, a fraction of $W^+W^-$ events, varying from about 10% to 30% depending on the channel, also fails to give a good fit. These events are discussed below. Typical values of efficiency and purity after the fit are given in Tables 10 and 11.

From the constrained fit we can calculate the error on the fitted mass. This error is highly correlated to the actual value of the fitted mass, so selecting events with a simple cut on this
Figure 13: Distributions of the invariant mass for selected events for the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ channel at $\sqrt{s} = 192$ GeV (left) and the $W^+W^-\rightarrow q\bar{q}l\nu$ channel at $\sqrt{s} = 175$ GeV (right). The top plots are before the kinematic fit, the bottom ones after a five-constraint ($W^+W^-\rightarrow q\bar{q}q\bar{q}$) or two-constraint ($W^+W^-\rightarrow q\bar{q}l\nu$) kinematic fit. For the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ channel, the open histograms show the correctly found jet-jet combinations, the singly hatched areas correspond to the incorrectly found jet-jet combinations and the cross-hatched parts indicate the background from other sources (mainly $q\bar{q}(\gamma)$). For the $W^+W^-\rightarrow q\bar{q}l\nu$ channel the open histograms show signal events, the cross-hatched areas background including the $W^+W^-\rightarrow q\bar{q}\tau\nu$ contribution. The statistics correspond to an integrated luminosity of 500 pb$^{-1}$. 

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quantity would seriously bias the measurement. The reason for this effect is related to the
kinematics of the events. When the mass approaches the kinematic limit the precision on the
sum of the unknown masses will be better and better while the precision on the difference be-
tween the masses will deteriorate. This leads to a mass resolution that broadens with increasing
energy. Going from 175 GeV to 192 GeV typically increases the measurement errors by 25%
on an event by event basis. The effect of this on the final statistical error on $M_W$ is diluted by
the fixed $\Gamma_W$ and compensated by an increase of the $W^+W^-$ cross-section.

The constrained fit assumes that the errors on the measured quantities are Gaussian and
uncorrelated between jets. Several effects lead to non-Gaussian errors and correlations. Each of
these will lead to tails in the distributions and hence to a peak at low probability for the fitted
$\chi^2$. The most important is gluon radiation, but also overlapping jets, initial state radiation, $\Gamma_W$,
and acceptance effects play a role. The hard gluon radiation is of course in direct disagreement
with the treatment of jets as independent objects. Even rather soft gluon radiation lead to jets
being broadened in a specific direction, giving correlations that are not included in the current
implementations of the constrained fit. Studies have been performed to try to recover some of
the 4-jet events which fail to give a good fit by treating them as 5-jet events. However, although
some fraction of these events then give a good fit, the combinatorial problem is severe, and it
appears that their inclusion has little effect on the ultimate mass resolution.

### 3.2.4 Inclusion of initial state radiation

As will be seen below, energy lost in initial state radiation biases the fitted mass if it is not
included in the fit. Initial state radiation can be included in the constrained fit using the
following procedure. We know that there is a large probability that the photon is produced
close to the collision axis and hence not detected. We can therefore as a good assumption take
the momentum to be collinear with the $z$-axis. We also know to a high precision the expected
distribution of the photon energy. In a simple approximation this reads:

$$p(x)dx = \beta x^{\beta-1}dx,$$

where $x = E_\gamma / \sqrt{s}$ and $\beta$ is a parameter which is smaller than unity and depends on $\sqrt{s}$. Since
this is non-Gaussian we cannot take this term directly into the $\chi^2$ expression. Instead we
introduce the likelihood concept and use the standard $-2 \ln(\text{likelihood})$ as the term to add to
the $\chi^2$. If we just use the probability as the likelihood this approach will not work since the
distribution has a pole for $x \to 0$. Instead one can choose to use the confidence limit as an
estimator for the likelihood:

$$C(x) = \int_{-\infty}^{x} p(y)dy$$

With this formulation the constrained fit works but its implementation is rather difficult since
one has to divide the fit into two parts: one where one assumes the photon goes in the forward
direction and one where one assumes the opposite. When the fitted $E_\gamma$ approaches zero the
first order derivative of $\chi^2$ will still diverge, but this only means that the fit prefers the zero
solution, since the measurement does not have sufficient resolution to distinguish between no
and a $\gamma SR$ with a small energy. Monte Carlo studies show that photons below about 3 GeV cannot be resolved and when photons are above typically 8 GeV the W bosons can not be on mass-shell. The final improvement in the $M_W$ measurement is therefore limited.

### 3.3 Determination of the mass and width of the W

In this section we describe several strategies for extracting $M_W$ from distributions of reconstructed invariant masses such as those show in Fig. 13. As we aim for a precision measurement of $M_W$ with sub-permille accuracy, this is a non-trivial task, because we have to control any systematic effect to an accuracy of a few MeV. The total width of the W boson, $\Gamma_W$, may either be extracted simultaneously with the mass, or the functional dependence $\Gamma_W = \Gamma_W(M_W)$ of the Standard Model may be imposed as a constraint for increased accuracy on $M_W$. The methods to analyse the data in terms of $M_W$ and $\Gamma_W$ fall into four groups: (1) Monte Carlo calibration of simple function; (2) (De-) Convolution of underlying physics function; (3) Monte Carlo interpolation; (4) Reweighting of Monte Carlo events.

In general, all methods make use of Monte Carlo event generators \[18\] and detector simulation to determine the effects of the detector such as resolution. Thus any method has to be checked for possible systematic biases introduced by using Monte Carlo event samples generated with certain input values for $M_W$ and $\Gamma_W$. In addition, systematic errors may arise due to deficiencies in the Monte Carlo simulation describing the detector and/or the data. Other systematic errors arise from the technical implementation of the fitting methods, such as fit range, bin width in case of binned data, choice of functions to describe signal and background etc.

#### 3.3.1 Monte Carlo calibration

To fit the invariant mass distribution, this method uses a simple, ad-hoc function, e.g., a double Gaussian or a Breit-Wigner convoluted with a Gaussian, to describe the signal peak and another simple function to describe the background. One of the fit parameters is used as an estimator, $M$, for the W mass, e.g., the mean of the Gaussian or the Breit-Wigner. The same fitting procedure is applied to both data and Monte Carlo events. Since for the latter the input W mass, $M_{W}^{MC}$, is known, the Monte Carlo result is used to evaluate the bias $\Delta$ of this method, $\Delta \equiv M_{W}^{MC} - M_{W}^{MC}$, where this bias may depend on the final state analysed. The mass of the W measured in the data is now simply given by the estimator $M_{\text{data}}^{\text{MC}}$ derived from fitting the data distribution, corrected for the bias $\Delta$ evaluated from Monte Carlo events: $M_W = M_{\text{data}}^{\text{MC}} - \Delta$.

This procedure automatically takes into account all corrections for all biases as long as they are implemented in the Monte Carlo simulation, such as initial-state radiation, background contributions, detector resolutions and efficiencies, selection cuts etc. The knowledge of how well the Monte Carlo describes the underlying physics and the detector response enters in the systematic error on the bias $\Delta$. 

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The fundamental drawback of this method is that the simple function used in the fit is not unique. Depending on its choice, even different statistical errors on \( M_W \) can be obtained. In addition, the estimate of the bias correction \( \Delta \) depends itself to some extent on the Monte Carlo parameters \( M_W^{MC} \) and \( \Gamma_W^{MC} \), or on the centre-of-mass energy, \( \sqrt{s} \), \( \Delta = \Delta(s; M_W^{MC}, \Gamma_W^{MC}) \). Such a dependence, however, can be corrected for by iteration.

### 3.3.2 Convolution

The drawbacks listed above are alleviated in the convolution method. Here, the correct function, i.e., the underlying physics function, is used as a fitting function. This function is simply the differential cross-section in the two invariant masses (denoted by \( m_1, m_2 \)). Note that this function is not a simple Breit-Wigner distribution in \( m_1 \) and \( m_2 \) due to phase space effects and radiative corrections. Several analytical codes exist (e.g., GENTLE etc. [18]), which calculate the differential cross-section including QED corrections, \( \frac{d^2 \sigma(s; M_W, \Gamma_W)}{d m_1 \cdot d m_2} \), as a function of the centre-of-mass energy, \( \sqrt{s} \), and the Breit-Wigner mass and total width of the W boson, \( M_W \) and \( \Gamma_W \).

The effects of the detector are included by convolution. The prediction for the distribution of the reconstructed invariant masses (denoted by \( \overline{m}_1, \overline{m}_2 \)) is thus given by:

\[
\frac{d^2 \sigma(s; M_W, \Gamma_W)}{d \overline{m}_1 \cdot d \overline{m}_2} = \int d m_1 \cdot d m_2 \ G(s; \overline{m}_1, \overline{m}_2, m_1, m_2) \cdot \frac{d^2 \sigma(s; M_W, \Gamma_W)}{d m_1 \cdot d m_2}.
\]

(46)

The transfer or Green's function \( G \), which depends on the final state analysed, can be interpreted as the probability of reconstructing the pair of invariant masses \( (\overline{m}_1, \overline{m}_2) \) given the event contained the pair of true invariant masses \( (m_1, m_2) \). Several simplifications for \( G \) are possible, down to a 1-dimensional resolution function \( G = G([\overline{m}_1 + \overline{m}_2]/2 - [m_1 + m_2]/2) \).

The actual fitting of invariant mass distributions can be performed either on the measured or on unfolded distributions. In the former case, the underlying physics function, \( \frac{d^2 \sigma(s; M_W, \Gamma_W)}{d m_1 \cdot d m_2} \), is convoluted with \( G \), and the result is fitted to the data with \( M_W \) and \( \Gamma_W \) as fit parameters. In the latter case, the measured distribution, \( \frac{d^2 \sigma(s; M_W, \Gamma_W)}{d \overline{m}_1 \cdot d \overline{m}_2} \), is first unfolded for detector effects, by applying the "inverse" of \( G \). The underlying physics function, \( \frac{d^2 \sigma(s; M_W, \Gamma_W)}{d m_1 \cdot d m_2} \), can now be fitted directly to the unfolded distribution. From a purely mathematical point of view, both methods are equivalent. In practice, however, \( G \) is determined only up to a certain statistical accuracy. Since folding is an intrinsically stable procedure in contrast to unfolding, which is unstable, the former method is preferred. Reference [49] gives more details on the features of unfolding procedures. Backgrounds are described by simple functions, derived from data and from Monte Carlo simulations, which are added to the signal.

This method allows cuts on generated invariant masses \( (m_1, m_2) \) and ISR energy loss, since these are the only variables used in most semianalytical codes. In addition, cuts on the reconstructed invariant masses \( (\overline{m}_1, \overline{m}_2) \) are possible. Since cuts on other variables cannot be
applied, it must be checked whether selection cuts bias the invariant mass distribution, for example by testing the method with Monte Carlo events (cf. method (1)).

3.3.3 Other Monte Carlo based methods

The problems of the previous two methods can be solved by a Monte Carlo interpolation technique. Several samples of Monte Carlo events corresponding to different input values of \( M_W^{MC} \) and \( \Gamma_W^{MC} \) are simulated, e.g., in a grid around the current central values for \((M_W, \Gamma_W)\) extending a few times the total (expected) error in all directions. The Monte Carlo samples of the accepted events are compared to the accepted data events, thereby taking the influence of event selection cuts into account. Backgrounds are included by adding the corresponding Monte Carlo events. The compatibility of the invariant mass distributions is calculated, e.g., in terms of a \( \chi^2 \) quantity. Interpolation of the \( \chi^2 \) within the generated \((M_W^{MC}, \Gamma_W^{MC})\) grid allows to find the values \( M_W \) and \( \Gamma_W \) which minimise the \( \chi^2 \). Like method (1), this method corrects automatically all possible biases due to all effects considered in the Monte Carlo simulation. The only drawback is that a rather large amount of Monte Carlo events must be simulated.

This problem, however, can be solved by a reweighting procedure. In that case only one sample of Monte Carlo events is needed, which has been generated with fixed values \( M_W^{MC} \) and \( \Gamma_W^{MC} \). Event-by-event reweighting in the generated invariant masses \((m_1, m_2)\) is performed to construct the prediction for the invariant mass distributions corresponding to values \( M_W^{gen} \) and \( \Gamma_W^{gen} \). These distributions are then fitted to the data distributions. Since individual Monte Carlo events are reweighted, it is straightforward to implement the effects of selection cuts. Moreover, using a Monte Carlo generator also in the calculation of the event weights, it is even possible to extend further the set of variables on which the event weights depend to include any kinematic variable describing the four-fermion final state, such as the reconstructed fermion energies and angles.

3.3.4 Expected statistical error on \( M_W \)

So far the experiments have studied methods (1), (2) and (3) [44, 45, 46, 47]. For the statistical errors on \( M_W \) quoted in Tables 10 and 11, the experiments have mainly used method (1), which is adequate for this purpose. It should be noted that the more involved analyses (2), (3) and (4) do not aim for a reduction in the statistical error on \( M_W \). This error is fixed by the number of selected events, the natural width of the W boson and the detector resolution in invariant masses. Instead the advantage of these methods lies in the fact that they allow a thorough investigation of the various systematic biases arising in the determination of the W mass. It is possible to disentangle systematic effects due to background diagrams, initial-state radiation, event selection, detector calibration and resolution etc. Thus, in order to have a better control of systematic effects, it is envisaged that the final analyses will use the more involved strategies (2), (3), (4) or a combination thereof.
As shown in Table 10, the statistical error on $M_W$ from the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ channel is expected to be about 73 MeV, roughly independent of $\sqrt{s}$, for an integrated luminosity of 500 pb$^{-1}$. For the $W^+W^-\rightarrow q\bar{q}\ell\nu$ channel a similar value is expected at 175 GeV, but at higher energies the worsening resolution on $M_W$ causes this value to increase somewhat.

3.4 Systematic errors

Three classes of systematic error have been envisaged: errors coming from the accelerator, from the knowledge of the underlying physics phenomena, and from the detector. The last two classes are sometimes related as some physics parameters may affect the detector response. In contrast with the threshold cross-section method, effects which distort the mass distribution must be considered here. Most of the following error estimates have been obtained using simulations. The models used will be checked against LEP2 data when available and represent today’s state of the art.

3.4.1 Error from the LEP beam

The direct mass reconstruction method relies on constraining or rescaling the energies of the reconstructed W’s to the beam energy. The error on the beam energy is foreseen to be less than 12 MeV (see Section 1.1), translating to $12 \text{ MeV} \times \frac{M_W}{E_{\text{beam}}}$ on the mass. The beam energy spread will be $\approx 40 \text{ MeV} \times s/(91 \text{ GeV})^2$ with a negligible effect on the statistical error. The operation with bunch trains should not significantly affect the beam energy. Experience gained at LEP1 will guarantee a very good follow up of the behaviour of the beam even at short time scales.

3.4.2 Error from the theoretical description

The most important distortion of the mass distribution comes from initial state radiation (ISR). The average energy carried by radiated ISR photons is $\langle E_{\text{ISR}} \rangle = 1.2 \text{ GeV}$ per event at $\sqrt{s} = 175 \text{ GeV}$ and rises linearly up to $2.2 \text{ GeV}$ at $\sqrt{s} = 200 \text{ GeV}$. From the beam energy constraint, one expects an average mass shift of the order of $\langle E_{\text{ISR}} \rangle M_W/\sqrt{s}$, i.e. about 500 MeV at 175 GeV. A fit to the mass distribution gives more weight to the peak yielding a shift of only 200 MeV at this energy. As a direct measurement of the ISR spectrum seems impossible to an accuracy relevant for the W mass measurement, we will rely on the theoretical calculations as described in Section 1.6.5 which present an error of 10 MeV on $\langle E_{\text{ISR}} \rangle$. Together with uncertainties on the shape of the spectrum, this translates into an error smaller than 10 MeV on the W mass.

Background events contained in the final sample will also distort the mass distribution (Fig. 13). These backgrounds can be monitored with the data themselves under a dedicated analysis. The most important source of background comes from $(Z^0/\gamma)^*\rightarrow q\bar{q}$ events which has
a flat mass distribution in the fit area. The effects of an error on the level or in the shape of this background have been estimated by changing its production cross-section by 20% and by shifting the background mass distribution by 0.5 GeV resulting in an error of 12 MeV for the \( W^+W^-\rightarrow q\bar{q}q\bar{q} \) channel and 6 MeV for the \( W^+W^-\rightarrow q\bar{q}l\nu \) channel. The background four jet rate as measured at LEP1 is not perfectly reproduced by the simulations [50] and will have to be better determined.

The fragmentation model for \( W \) events is the same as was fitted to \( Z \) events at LEP1. To estimate errors from this model, each of its parameters has been varied and the error is derived according to a 1 standard deviation from the best fit value. The total error is 16 MeV, which should be taken as an upper limit as the parameters have been varied separately without taking into account correlations.

Colour reconnection and Bose-Einstein correlations may seriously affect the mass distribution in the \( W^+W^-\rightarrow q\bar{q}q\bar{q} \) channel. These are discussed in detail in Section 4.

### 3.4.3 Error from the detector

Effects of miscalibration appear to be very small. This is due to the use of the beam energy which gives a precise scale. For instance, a conservative shift of 2% in the momentum of charged tracks, and miscalibration of 5% for electromagnetic and hadronic calorimeters produces a shift of 10 MeV on the \( W \) mass.

During the LEP2 era, it is foreseen to run for short periods at \( \sqrt{s} = M_Z \) in order to calibrate and determine the efficiencies of the detectors with high statistics. Genuine LEP2 events like \( e^+e^-\rightarrow Z^0\gamma \) will also be used to monitor the hadronic recoil mass to the photon. Two \( Z \) events produced at LEP1 can be mixed and boosted to a LEP2 energy to study the adequacy of the mass reconstruction in comparison with the simulations.

In the \( W^+W^-\rightarrow q\bar{q}q\bar{q} \) channel, the wrong assignment of a jet to a \( W \) distorts the mass distribution giving rise to a low mass tail. The amount of such misassignments depends on the algorithms used. By changing parameters in such algorithms we estimate an error of up to 5 MeV.

Several methods to extract the \( W \) mass from the measurements have been studied (see Section 3.3). If a simple function is fitted to the measured mass distribution, errors from the choice of the mass fit range are to be expected, and are typically 10 MeV. A convolution of the physics function with (or a deconvolution from) detector effects should largely reduce such errors. However this convolution depends on the accuracy of the simulation of the event structure, for instance on the fragmentation model used (see above).

In the present simulation studies, limited statistics have been generated. The statistical error on the mass shift with 500k events simulated is of the order of 10 MeV.
<table>
<thead>
<tr>
<th>Source</th>
<th>$W^+W^+\rightarrow q\bar{q}q\bar{q}$</th>
<th>$W^+W^-\rightarrow q\bar{q}l\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{beam}}$</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>ISR</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>fragmentation</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>backgrounds</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>calibration</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>MC statistics</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>mass fit</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>jet assignment</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>interconnection</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td><strong>31</strong></td>
<td><strong>29</strong></td>
</tr>
</tbody>
</table>

Table 12: *Estimated systematic errors on $M_W$ per experiment in MeV. Colour reconnection and Bose-Einstein effects are not included in the total.*

### 3.4.4 Common errors

The effects mentioned above are summarized in Table 12 and total to about 30 MeV for each channel at $\sqrt{s} = 175$ GeV. Although errors from the beam energy, ISR and background will vary with energy, we do not expect large variations over the 165 to 190 GeV range.

Errors on the beam energy and ISR are common to all experiments. Errors from fragmentation and background are partially common as far as the physical parameters are concerned, but a particular analysis will retain a level of background or be sensitive to fragmentation tails in a different way from another analysis, hence with a slightly different error. If these errors were considered common to the four experiments, they would amount to a total of 25 MeV (23 MeV) in the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ ($W^+W^-\rightarrow q\bar{q}l\nu$) channel.

### 3.5 Summary

In Table 13 we show the total error on $M_W$ which might be expected by combining four experiments each with an integrated luminosity of 500 pb$^{-1}$ at $\sqrt{s} = 175$ GeV. As discussed in the previous section, systematic errors from the beam energy measurement and initial state radiation, fragmentation and backgrounds are considered common to both channels and to all experiments. The total expected error is around 34 MeV, with roughly equal contributions from statistics and systematics. This value is expected to rise, but only very slowly, with $\sqrt{s}$ because of the worsening mass resolution.
Table 13: Estimated total error on $M_W$, in MeV, which could be obtained by combining four experiments each with an integrated luminosity of 500 pb$^{-1}$ at $\sqrt{s} = 175$ GeV. Colour reconnection and Bose-Einstein effects are not included in the systematic error.

<table>
<thead>
<tr>
<th>Source</th>
<th>$W^+W^- \rightarrow q\bar{q}q\bar{q}$</th>
<th>$W^+W^- \rightarrow q\bar{q}/\nu$</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>36</td>
<td>36</td>
<td>25</td>
</tr>
<tr>
<td>Common systematic</td>
<td>25</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>Uncorr. systematic</td>
<td>9</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>total</td>
<td>45</td>
<td>44</td>
<td>34</td>
</tr>
</tbody>
</table>

4 Interconnection Effects

4.1 Introduction

The success of the precision measurements of the W boson mass $M_W$ strongly relies on accurate theoretical knowledge of the dynamics of the production and decay stages in $e^+e^- \rightarrow W^+W^- \rightarrow 4$ fermions. Owing to the large W width, $\Gamma_W$, these stages are not independent but may be interconnected by QCD (and electroweak) interference effects, which must be kept under theoretical control. Interconnection phenomena may obscure the separate identities of the two W bosons, so that the final state may no longer be considered as a superposition of two separate W decays [51, 52]. Thus the “direct reconstruction” method of measuring $M_W$ at LEP2 using the hadronic $(q_1\bar{q}_2 q_3\bar{q}_4)$ channel has an important caveat — the colour reconnection effects [51, 52, 53] — which may distort the mass determination [52]. Another delicate question is whether Bose-Einstein effects could induce a further uncertainty in the mass determination [52, 54].

The hadronic effects mentioned above are all well-known from other processes. Interferences in the production and decay of unstable objects are observed e.g. in multipion final states at low invariant masses [55]. QCD interconnection could be exemplified by $J/\psi$ production in B decay: in the weak decay $b \rightarrow cW^- \rightarrow c\bar{s}$ the $c\bar{s} \rightarrow J/\psi$ formation requires a “cross-talk” between the $c\bar{s}$s and the c+spectator colour singlets. Bose–Einstein effects are readily visible in high-energy multiparticle production, including LEP1. In view of the precedents, the working hypothesis must be that interconnection effects are at play also for $W^+W^-$ events.

The space–time picture of hadroproduction in hadronic $W^+W^-$ decays plays a very important rôle in understanding the physics of QCD interference phenomena [52, 56]. Consider, for instance, a centre of mass energy of 175 GeV where the typical separation of the two decay vertices in space and in time is of the order of 0.1 fm. A gluon with an energy $\omega \gg \Gamma_W$ therefore has a wavelength much smaller than the separation between the $W^+$ and $W^-$ decay vertices, and is emitted almost incoherently either by the $q_1\bar{q}_2$ system from one W or by the $q_3\bar{q}_4$ one

---

from the other. Only fairly soft gluons, $\omega \lesssim \Gamma_W$, feel the joint action of all four quark colour charges. On the other hand, the typical distance scale of hadronization is about 1 fm, i.e. much larger than the decay vertex separation. Similarly, observed Bose–Einstein radii are of this order. As a result, the hadronization phase may induce sizeable interference effects.

As one could anticipate from the above, the perturbative QCD interconnection effects appear to be very small [52]. In examining the size of the non-perturbative W mass distortions one has to rely on existing QCD Monte Carlo models rather than on exact calculations. These models have done a very good job in describing a large part of the experimental data at the $Z^0$ resonance, so it seems plausible that they (after the appropriate extensions) could provide a reliable estimate of the magnitude of the interconnection-induced shift in the W mass. However, one has to bear in mind that there is a true limit to our current understanding of the physics of hadronization [52, 57], so the actual value of the shift may be beyond our current reach.

One of the achievements of the activity of our working group is that the colour reconnection physics has been tested in several approaches [52, 53, 57, 58, 59]. The Bose–Einstein studies are somewhat lagging behind, but progress is visible also here [54, 60, 61].

There is another challenging reason to study the phenomenon of colour recoupling in hadronic $W^+W^-$ events: it could provide a new laboratory for probing non-perturbative QCD dynamics [51]. The very fact that different assumptions about the confinement forces may give different predictions for various final-state characteristics means that it might be possible to learn from experiment about the structure of the QCD vacuum [51, 52, 53].

Finally, note that there are other effects — this time originating in purely QED radiative phenomena — which, in principle, prevent the final state from being treated as two separate W decays. For instance, final-state QED interactions, for the threshold region exemplified by the Coulomb forces between two unstable W bosons, induce non-factorizable corrections to the final-state mass distributions in $e^+e^- \rightarrow W^+W^- \rightarrow 4$ fermions [32, 33, 34]. Of course, there is no reason why all such QED effects cannot, in principle, be computed to arbitrary accuracy, and taken into account in the mass determination. However, at the moment no complete formulae are available which could be relevant for the whole LEP2 energy range. This topic certainly needs further detailed studies and a comprehensive activity here is in progress. For further comments see Section 1.

4.2 Colour reconnection

Several quite different colour reconnection models have been presented by now. A crude survey of features is found in Table 14. The listing is by no means complete; several simpler toy models have also been proposed [52]. The three main philosophies are the following:

- A colour-confinement string is created in each W decay, spanned between the decay-product partons. These strings expand out from the respective decay vertex and eventually fragment to hadrons. Before that time, a reconnection can occur by a space–time
encounter between sections of the two strings. In one extreme (QCD vacuum structure analogous to a type I superconductor) the reconnection probability is related to the integrated space–time overlap of the extended strings, in another (likewise, with type II superconductor) to the crossing of two string cores (vortex lines). Many subvariants are conceivable. The models in [59] represent further refinements of the ones presented in [52].

- Perturbative QCD prefers configurations with minimal string length in $Z^0$ decays. Here length is defined in terms of the $\lambda$ measure, which may be viewed as the rapidity range along the string: $\lambda \approx i \ln(m_i^2/m_\rho^2)$, where $m_i$ is the invariant mass of the $i$'th string piece and $m_\rho$ sets a typical hadronic mass scale. It is plausible that, when the partons of a $W$ pair are separating and strings formed between them, such configurations are favoured which correspond to a reduced total string length. Therefore a reasonable criterion is to allow $\lambda$-reducing reconnections at the scale of $\Gamma_W$, within the limits of what is given by colour algebra factors. The models in [53] and [57] are variants on this theme.

- In a cluster approach to fragmentation, clusters are formed by the recombination of a quark from one gluon branching with an antiquark from an “adjacent” branching (alternatively primary quark flavours or diquarks). Normally “adjacent” is defined in terms of the shower history, but another reasonable criterion is the space–time size of the formed clusters. Therefore reconnection relative to the ordinary picture could be allowed anytime the sum of the squared sizes of the formed clusters is reduced. Restrictions are imposed, so that e.g. the quark and antiquark from a gluon splitting cannot form a cluster together. This approach is found in [58].

The models need not be viewed as mutually contradictory. Rather, each may represent some aspect of the true nature of the process.

The main philosophies can be varied in a multitude of ways — Table 14 is to be viewed as an appetizer to a proper appreciation of this. Program details can be found in the QCD event generators section and in the original literature. The overall reconnection probability depends on free parameters in all the models, since we do not understand the non-perturbative dynamics; colour suppression factors like $1/N_C^2$ may be present, as in the perturbative phase, but are multiplied by unknown functions of the space–time and momentum variables. Furthermore, the models can only be incomplete representations of the physics, e.g., the effects of “negative antennae/dipoles” [52] and of virtual-gluon corrections remain to be addressed.

Several studies have been performed to estimate the effect (bias) of colour reconnections on the $W$ mass measurement, and to evaluate the possibility of determining experimentally whether such phenomena are taking place in the $W^+W^-$ system. Reconnection effects could also give visible signals at LEP1. To allow unbiased comparisons, each reconnection model should be retuned to general event-shape data with the same care as for the “no reconnection” scenarios. This process is now under way. The ARIADNE studies make use of a retuning of the model with reconnection effects, by its author [57, 62].
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>based on</td>
<td>PYTHIA</td>
<td>ARIADNE</td>
<td>HERWIG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>reconnection</td>
<td>space–time overlap (I) or crossing (II) of strings</td>
<td>string length reduced</td>
<td>cluster space–time sizes reduced</td>
<td></td>
<td></td>
</tr>
<tr>
<td>criterion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reconnection</td>
<td>I: free parameter</td>
<td>free parameter</td>
<td>partly predicted</td>
<td>free parameter</td>
<td></td>
</tr>
<tr>
<td>probability</td>
<td>II: partly predicted</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>model of all events</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>space–time picture implemented for</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>W vertices</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>parton shower</td>
<td>no</td>
<td>yes</td>
<td>—</td>
<td>—</td>
<td>yes</td>
</tr>
<tr>
<td>fragmentation</td>
<td>yes</td>
<td>yes</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>multiple reconnections</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>reconnection</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>inside W/Z</td>
<td>almost invisible</td>
<td>small but visible</td>
<td>visible, needs retuning</td>
<td>large, needs retuning</td>
<td></td>
</tr>
<tr>
<td>change of event</td>
<td>invisible</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>properties</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 14: Survey of colour reconnection models. The information should be taken as indicative; only the original literature gives the ideological motivation behind the choices.
In order to set limits on possible W mass shifts, the experimental sensitivity with which reconnection may be observed has to be evaluated for each model (as a function of the model parameters, where appropriate). If no effects are observed in the LEP2 data, this sets a limit on the reconnection probability, which can be turned into an estimate of the systematic uncertainty in the W mass. To complicate matters, different models will in general predict different mass shifts for the same probability of reconnection. This complication will also appear if reconnections are observed in the data and one would like to correct the observed W mass accordingly.

Predictions for the systematic error in W mass measurement are presented in Table 15. It is important to understand that the value of the W mass shift depends very strongly on the definition of this shift, as well as on the method used to reconstruct the jets, the fitting function used, and the tuning of the models. These factors need to be controlled strictly to allow meaningful comparisons. Three estimates of the mass shift have been studied:

- **Averaging** Some Monte Carlo authors [52, 57] form a distribution (difference between reconstructed and generated W mass) event-by-event, and use the difference in the means of two such distributions (with and without reconnection) as an estimator of the systematic shift.

- **Fitting** Two mass distributions are formed, one with and one without reconnection, and the W mass is obtained by fitting each separately. The function used was a Breit-Wigner, modulated by phase space factors. The difference between the fit results gives the estimated mass shift. The definition of jets and correspondence between jets and W’s followed method 3 of [52].

- **Detector** The analysis was performed as if using real data. Mass distributions are obtained after the events (with initial state radiation) have been passed through a LEP detector simulation program, thereby including typical experimental acceptance and quality cuts. The W mass is determined for each event using a 5 constraint kinematic fit (see, e.g., Section 3). The shift is taken as the difference between two fitted mass distributions as above, although the definition of jets, the association between jets and W’s and the fitting function were different. The experimental results presented here were obtained using a simulation of the OPAL experiment [63].

Table 15 contains an (incomplete) comparison of the shifts observed using all three definitions of the shift in the W mass, where the errors are from finite Monte Carlo statistics. As indicated, the three are not expected to agree. It is noted that there is an upwards shift after detector simulation. The study of [63], including the same kinematic fitting, jet and W reconstruction, was repeated using events without detector simulation. Mass shifts which are 1–2σ (statistical) lower than the detector simulation fits were observed; they were stable against changes in fit range, and cuts on $P_t > 150$ MeV/c and kinematic fit probability (>1%). They are also consistent with the shifts in the ‘fitting’ column in Table 15.
Table 15: Summary of effects on $M_W$ at 175 GeV. See the text for details. The $\rho$ parameter in Type I models relates the string overlap integral to the reconnection probability. The $d$ parameter gives the vortex line core diameter in Type II models. The numbers for [53] can be rescaled by the reconnection probability, which here is a free parameter. The sensitivities in the last two columns include detector simulation [63]. Several of the numbers in the table are preliminary and further studies are underway to understand them.

If the shift is genuinely caused by specific experimental cuts, it may be possible to redesign those cuts to reduce $\Delta M_W$ to the level seen without detector simulation. If so, we might also construct a related observable for the detection of reconnection.

Two related experimental methods of detecting the presence of reconnection phenomena at LEP2 have been studied, including OPAL simulation [63]: multiplicity in a central rapidity bin [53] and interjet multiplicity [63].

In ref. [53] a significant decrease was found in the central rapidity region for events with a thrust cut $T > 0.76$. (A larger difference but poorer statistical significance was obtained for more restrictive thrust cuts.) An independent, PYTHIA-based implementation of the GH model [63] showed a much reduced signal. It was here pointed out that one reason for the difference is the effect of $W$ polarization, which was neglected in ref. [53]. Recent studies [65] confirm that the polarization is important; it reduces the signal by roughly a factor of 2 compared to the original results. However, although significantly reduced, a clear signal for reconnection effects is still visible in [65]; this appears to be larger than the result obtained in ref. [63].

Following the prescription of [53], the central multiplicity is defined as the number of particles, $n$, observed within a given rapidity interval. In order to quantify the level at which colour
reconnection could be observed, the sensitivity at a given multiplicity, \( n \), is defined as [63]:

\[
\frac{\sum_{i=1}^{n} N_{\text{rec}}(i) - \sum_{i=1}^{n} N_{\text{norm}}(i)}{1 + \sum_{i=1}^{n} N_{\text{norm}}(i)}
\]

(47)

where \( N_{\text{rec}}(i) \) and \( N_{\text{norm}}(i) \) are the number of events in the reconnected and normal event samples having a central multiplicity \( i \), respectively. The sensitivities given in Table 15 are obtained assuming that 3000 four quark events have been observed, that is about what is expected from the nominal luminosity of 500 pb\(^{-1}\), by scaling the sensitivities from samples of 50000 generated events by \( 3000/50000 \). If the results of the four experiments are combined, the statistical significance will double. With such statistics a tentative conclusion from the models so far studied is that reconnection effects could be observed down to 25–30\%, corresponding to a 3\( \sigma \) effect. A smaller integrated luminosity is not very encouraging at an energy of 175 GeV. It should also be remembered that the significance level is based on an assumed perfect knowledge on the no-reconnection multiplicity distribution; an entirely experimental procedure based on a comparison with mixed leptonic–hadronic events would have a smaller sensitivity.

A reconnection probability of 30\% could induce a systematic effect of about 50 MeV on the W mass measurement using the four-jet method, which is of the same order as the total measurement error expected in this mode for all four LEP experiments combined. This mass shift is found to depend upon the model used and may be larger than estimated above. The uncertainty is further increased by other reconnection sources, including perturbative interconnection, interplay between perturbative and non-perturbative effects [52], virtual effects, and so on. The 50 MeV number above is therefore to be viewed as a realistic uncertainty.

It is also interesting to study event topologies at LEP1. For instance, a \( qgq\bar{q} \) event is normally expected to have the partons ordered along a single string, but an alternative subdivision into a \( q\bar{q} \) and a \( gg \) singlet is possible. Such “reconnection” phenomena (though not quite of the same character as the ones we worry about for the W mass issue) could give rapidity gaps in events where the quark and antiquark are tagged to lie in the same hemisphere with respect to the thrust axis [66, 63]. Generic event shapes presumably cannot lead to any definite conclusions; in part, the information on event shapes is used to tune generators in the first place. If a properly tuned generator with “reconnection” included could perform as well as the conventional ones, it would provide indications that a given approach is not unreasonable. Models including “reconnection” which do not, even with appropriate retuning, describe the data are less useful. A recent retuning of ARIADNE with “reconnection” by DELPHI [67] shows that the quality of the description of LEP1 data can even be improved. However, it may be entirely fortuitous.

4.3 Bose–Einstein Effects

Since the hadronization regions of the W\(^+\) and W\(^-\) overlap, it is natural to assume that some coherence effects are present between identical low-momentum bosons stemming from different
W's due to Bose–Einstein (BE) correlations. How much such effects would influence the W mass measurement is difficult to answer. Intuitively, since the BE effect favours production of identical bosons close in phase-space, one would expect the softest particles from each W to be “dragged” closer to each other. This reduces the momentum of the W’s and thus increases the measured W mass. However, most of our understanding of the details of multi-particle production comes from probabilistic hadronization models where BE correlations are absent. Only a few attempts have been made to include such effects and to investigate their influence on the W mass measurement.

One such attempt [54] used an algorithm (LUBOEI) implemented in the JETSET. This algorithm is based on the assumptions that BE effects are local in phase space and do not alter the event multiplicity. (Notice that any effects on the multiplicity are already, at least partially, accounted for by the tuning of the generators to data.) A re-weighting of events with a BE factor is then equivalent, in some approximation, to letting the momenta of the bosons produced in the hadronization be shifted somewhat, e.g. to reproduce the difference in two-particle correlation functions expected for a source with Gaussian distribution in space-time,

\[
\frac{C_{\text{BE}}(Q)}{C_{\text{noBE}}(Q)} = 1 + \lambda \exp(-Q^2 R^2).
\]

Here \( R \) is the source radius and \( \lambda \) is an incoherence parameter. \( C_{\text{BE}}(Q) \) and \( C_{\text{noBE}}(Q) \) are the two-particle correlations as functions of relative four-momenta in a world with and one without BE effects, respectively. This is a well-defined generator procedure but, since it is not possible to switch off BE in the data, the main challenge of experimental BE studies is to define an appropriate no-BE reference sample. With this algorithm, the observed correlations at LEP are reproduced using \( \lambda \approx 1 \) and \( R \approx 0.5 \text{ fm} \) [68]. One problem with this algorithm is that it does not inherently conserve energy and momentum. This has to be done separately by rescaling all momenta in an event. Therefore, when this algorithm is used on \( W^+W^- \) events, identical bosons within each W are shifted closer to each other in phase space resulting in an artificial negative shift in the measured W mass even if there is no cross-talk between the W’s. In Ref. [54] this artificial shift was corrected for and an estimate for the true shift in the measured W mass was obtained. The result was \( \langle \Delta M_W \rangle \approx 100 \text{ MeV} \) for 170 GeV center of mass energy. This shift increases with energy and decreasing source radius.

A variation of this algorithm has been investigated [60], where not only the momenta of identical particles are shifted closer to each other, but also non-identical particles are shifted away from each other, so that the ratio of correlation functions of identical and non-identical particles is the same as with the original algorithm. In this way, the energy-momentum non-conservation can be made much smaller, minimizing possible dependence on the correction procedure. The shift in the W mass is in this case smaller, around 30 MeV. But modifying the algorithm slightly, not allowing the shifting of non-identical bosons from different W’s, again gives a mass shift around 100 MeV.

In both these cases the shift was calculated on the generator level, assuming that the origin of each particle was known. It is clear that an experimentally feasible reconstruction algorithm may be more or less sensitive to these BE effects.
Another attempt to estimate the influence of BE correlations on the W mass measurement was based on a simplified toy model of W decays, in which only decays into two or three particles were considered \[61\]. Each of the two W's was treated as a scalar, decaying into a heavy particle and one or two pions, according to a relativistic Breit-Wigner. The decay amplitudes can thus be written down explicitly, with and without symmetrization with respect to exchange of the pions. This allowed the generation of such low-multiplicity events, each with two weights corresponding to the symmetrized and non-symmetrized cases, respectively, and thus the study of the effect on the reconstructed W mass. The mass of the heavy particle in the model was chosen to correspond roughly to the total mass of the hadronic decay products of a W boson with one or two pions removed, i.e. 75–79 GeV.

The model calculations confirm that substantial shifts of the W mass due to the BE symmetrization can occur, and also suggests that the magnitude of the effect can depend significantly on the precise way in which the W mass is reconstructed from the data. In the particular case studied, the peak position of the mass distribution was less sensitive to the BE effect than the average mass. The interpretation was hampered by severe threshold effects, however, and it is in any case not clear how to draw quantitative conclusions valid for the multi-particle decays that form the bulk of real hadronic W decays.

At our present level of understanding, effects of BE correlations on the W-mass measurement of the order of 100 MeV cannot be excluded, though this number may be viewed as a “worst case” estimate. Much more work is needed to develop realistic models and to study the sensitivity of different reconstruction algorithms. It may also be possible to find signals, e.g. by studying the two-particle correlation function using only pairs where the particles come from different reconstructed W’s, that could enable us to deduce from data the magnitude of the effect.

### 4.4 Summary and Outlook

Several activities are under way, especially on the experimental front, so it would be premature to draw any definite conclusions. However, it is not excluded that interconnection and Bose–Einstein effects could each contribute a 50 MeV mass shift in the hadronic W⁺W⁻ decay channel. Most interconnection models predict a positive mass shift, but we know of no general argument why it would have to be so. Furthermore, not all possible interconnection effects have been studied so far. In contrast, a positive mass shift from BE effects appears plausible on general grounds.

In spite of the bias towards positive shifts, the conservative approach would be to quote a symmetric “theory uncertainty” of the order of ±100 MeV. This is to be viewed as some sort of theorist’s 1σ error, that is, larger numbers can not be excluded but are less likely. The value above refers to studies at 175 GeV. The energy dependence has not yet been well studied, but indications are that the uncertainty is not smaller even at the maximum LEP2 energy. We note here that the interconnection phenomena are not expected to die out rapidly with increasing
energy [52], since the W resonance is so wide that the two W decays appear "on top of each other", in terms of hadronic distance scales, over the full LEP2 energy range.

Interconnection effects might be found in the data itself, at LEP2 or even by a careful analysis of LEP1 data. If so, it would be very interesting. However, a non-discovery would not prove non-existence: so far, we have failed to find realistic signals for some of the current approaches. We could only hope to exclude some extreme scenarios with large values of the reconnection probability. Conversely, at our present level of understanding, a discovery would not guarantee a unique recipe for how to "correct" the data. This is illustrated by BE effects, where it should be fairly straightforward to observe cross-talk at LEP2 in principle, but where statistics presumably will not allow a sufficiently precise investigation for the effects on the measured W mass to be well defined.

At the end of this workshop, another model of interconnections appeared [69], with which its authors find mass shifts of up to several hundred MeV, i.e. more than observed in previous studies. This exemplifies the need for continued theoretical and experimental activity in the field to better understand the issues. However, in view of the well-known complexity of hadronic final states, it would be unrealistic to expect these studies to give any simple answers. Monte Carlo models are all we have so far, and may be the best we can get in the foreseeable future. Therefore the main conclusion is clear: one cannot blindly average W mass results in the hadronic W⁺W⁻ channel with numbers obtained elsewhere.

5 Conclusions

The goal of ΔMW < O(50) GeV appears to be achievable at LEP2. The 'worst-case' scenario, in which only the q̅q′l channel is used in the direct reconstruction method because of colour reconnection in the q̅q′q̅q′ channel (see below), gives a total estimated error of ΔMW = 44 MeV, assuming four experiments each collecting 500 pb⁻¹ at √s = 175 GeV.

Two methods have been studied in detail. A precise measurement of the e⁺e⁻ → W⁺W⁻ threshold cross section can be compared to the theoretical prediction as a function of MW, see Fig. 6. The maximum statistical power is obtained by running the collider at an optimal energy (√s)opt ≈ 2MW + 0.5 GeV ∼ 161 GeV. All three decay channels (q̅q′q̅q′, q̅q′lν and lνlν) can be used, and the total estimated errors are listed in Table 9. Some marginal improvement in the systematic error can be expected, but ultimately the measurement is statistics limited. An integrated luminosity of at least 50 pb⁻¹ per experiment is required to obtain a precision comparable to that expected from the combined Fermilab Tevatron experiments. The total error using the threshold cross-section method for four experiments each with 50 pb⁻¹ is estimated to be 108 MeV.

One very attractive feature of the threshold measurement is that it appears to fit in very well with the anticipated LEP2 machine schedule, viz. a run of at least 25 pb⁻¹ luminosity at √s = 161 GeV in 1996 followed by the bulk of the luminosity at higher energy.
The **direct reconstruction method**, in which $M_W$ is reconstructed from the invariant mass of the $\text{W}^\pm$ decay products, appears to offer the highest precision. The statistical and systematic errors are summarised in Table 13. More work is needed to investigate different fitting procedures, different fragmentation parameters etc., and this may lead to small but important reductions in the errors. The major outstanding problem concerns the issues of **colour reconnection**, i.e. non-perturbative strong interactions between the decay products of two hadronically decaying $\text{W}$ bosons which may significantly distort the invariant mass distributions. This question is currently receiving much attention, but at this time it is impossible to say whether the problem can be completely overcome. At best, one may eventually hope to demonstrate that the effect on the reconstructed $M_W$ value is either small or under theoretical control, so that the results from the $q\bar{q}q\bar{q}$ and $q\bar{q}\nu$ channels can be combined. At worst, the precise magnitude of the colour reconnection effect may remain unknown, with different models giving different mass shifts larger or comparable to the remaining systematic and statistical errors. We have therefore chosen to present results both for the $q\bar{q}\nu$ channel only (where colour reconnection is absent) and for the two channels combined. The estimated total errors are 44 MeV and 34 MeV respectively, for 500 pb$^{-1}$ luminosity for each experiment at $\sqrt{s} = 175$ GeV.

---

Figure 14: **Collider energy dependence of the statistical errors.**
The collider energy dependence of the direct reconstruction measurement has also been studied. The results are summarised in Fig. 14. There is clearly a ‘blind window’ between 161 GeV and 170 GeV where a precision measurement of $M_W$ is impossible. In this window the threshold cross section method loses sensitivity to $M_W$, and the direct reconstruction method has insufficient statistics.

Increasing the LEP2 energy from 175 GeV to 192 GeV has an essentially neutral effect on the measurement of $M_W$. Most of the anticipated errors, in both the $q\bar{q}q\bar{q}$ and $q\bar{q}l\nu$ channels, appear to be approximately energy independent.\(^5\) Two important energy-dependent systematic errors, which are common to all experiments and both decay channels, are from initial state radiation and from beam energy calibration. Both are expected to increase slightly with energy, although the overall effect is marginal.

References


\(^5\)Note that the statistical errors in Fig. 14 show less energy dependence for the $q\bar{q}l\nu$ channel than those in Table 11. The former correspond to an average of the ALEPH and OPAL studies while the latter correspond to the OPAL study only.


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1 Introduction

While the energy increase of LEP will enable pair production of $W$'s and might open up the threshold for new particles, notably the Higgs boson, a host of standard model processes will also show up, as shown in Fig. 1.

\[ \sigma(e^+e^- \rightarrow \chi) \ (pb) \]

\[ \Sigma q\bar{q} \ (ISR) \]

\[ Z\gamma \]

\[ \gamma\gamma \]

\[ e^+e^-\tau^+\tau^- \]

\[ \nu_\mu\nu_\mu\gamma \]

\[ e\nu W \]

\[ Z\gamma\gamma \]

\[ ZZ \]

\[ WW\gamma \]

\[ \nu_e\nu_e Z \]

\[ ZZ\gamma \]

\[ \sqrt{s} \ (GeV) \]

\[ \sigma (pb) \]

Figure 1: Cross sections for some typical standard model processes. For $\gamma \rightarrow + -$ only the dominant $\gamma$-channel contribution is shown. The photons in and are such that $|\cos \gamma| < 0.9$. For $\mu^-\mu^+$ there is the additional cut $10 GeV$. In $\gamma \rightarrow + -$ and the photon cut is $\gamma > 10 GeV$ and all particles are separated with opening angles: $15^0$, $10^0$.

Some of these processes can be considered as potential backgrounds to those most interesting signals LEP2 intends to investigate. For instance, there are four-fermion processes that cannot
be associated with the doubly-resonant production or with the Higgs-boson production. Therefore, it is essential to know as precisely as possible the expected yield for these processes. Quark- and lepton-pair production will be dominant reactions at LEP2 and can be exploited for precision tests in this new energy range. Moreover, starting below the threshold for pair production one sees that other processes will take place, like for instance single and production. Beyond the threshold, one can envisage pair production or even triple vector-boson production, which involves the quartic vector couplings.

The aim of the Working Group has therefore been to provide as precise an evaluation as possible of all those processes that were not investigated within the cross sections and distributions, the or the groups in this Workshop, and which did not deal specifically with QCD issues. The other objective was to indicate, like in the case of two-fermion and single-photon processes, which interesting physics issues could be investigated.

2 Two-Fermion Production

2.1 General considerations, LEP2 vs LEP1

Quark- and lepton-pair production at LEP1 (and SLC) has provided one of the most stringent tests on the Standard Model (SM) of electroweak interactions. It has also either constrained or ruled out some alternative models, especially through their virtual indirect effects. At LEP2 energies, this process still remains one of the dominant processes. For instance, as evidenced from Fig. 1, quark pair production has a larger cross section than the process, the bread and butter of LEP2, and even larger after the inclusion of the initial state radiation (ISR).

In view of this expected wealth of events, it is worth enquiring if the characteristics of the two-fermion observables will continue to be conducive to further tests of the SM and beyond. One important observation, however, is that, as one moves away from the -peak, not only fermion-pair production drops precipitously, but also the photon exchange becomes very important. The latter dominates the cross section for up-type quarks and even more so for , see Fig. 2. In particular, at , one has for and hadrons respectively. Another critical fact is the very large "correction" due to initial-state radiation (QED). Above this more than doubles the muon Born cross section, as displayed in Fig. 2. Therefore, it is essential that these corrections be controlled very precisely.

At LEP1 the latter were also very important, leading to a reduction factor of the peak cross section, and were essentially due to soft-photon emission, while hard radiation (energetic collinear photons) was inhibited. Indeed, around the resonance acts as a natural cut-off for hard radiation. On the other hand, away from the -peak, the fast decrease of the cross section favours the radiation of hard photons that boost the effective two-fermion centre-of-mass energy back to the mass: this is the so-called return. Therefore, if for the inclusive two-fermion cross section one looks at the invariant mass of the fermion pairs, , one sees that a large sample is clustered around . The effect is quite dramatic for where, at , about 70% of the events are "LEP1-type pairs", as one can see from Fig. 3.

Still, considering the canonical integrated LEP2 luminosity ( ), one expects to measure the various two-fermion observables with a good precision (even after discarding the Z-return
events) especially if one combines the 4 experiments. For instance, at \( \sqrt{s} = 175 \), the expected experimental precision on the muon cross section is about 1.3\%, while for the hadronic one expects 0.7\% [2]. The corresponding error for the forward-backward asymmetry of muons is \( \Delta \mu_{FB} \sim 0.01 \) [2]. Note that this asymmetry is much larger than at LEP1. Another interesting observable is \( h \), with an expected overall accuracy of about 2.5\% [3]. These numbers should serve as benchmarks for the required accuracy on the theoretical calculations, \( \text{\textit{th}} \). One should aim, at least, at a theoretical precision below half the values quoted above. For instance, for \( h \) one needs \( \text{\textit{th}} h \sim 0.3\% \).

### 2.2 Radiative corrections and status of tuned comparisons

Although there have been many exhaustive studies of two-fermion final states and many programs have been successfully tested, the comparisons among these programs have been performed and optimized for energies around the peak. For a very recent state-of-the-art investigation see [4], where the main emphasis was put on the expected theoretical accuracy, assessed by comparing different codes with different implementations of the radiative corrections. However, as for the case of ISR pointed at earlier, a few characteristics of the two-fermion cross-sections are modified and new aspects appear when going to higher energies.

In order to address the issue of the status and the perspectives of tuned comparisons for \( + - \rightarrow + - \) processes at LEP2 energies, it is worth reviewing the various building blocks for computing observables related to the process \( + - \rightarrow + - \). We have:

---

*Here the WW events add as “backgrounds” that necessitate extra cuts.*
Figure 3: The invariant mass distribution of the hadrons at a centre-of-mass energy of 180 GeV before (solid) and after (dashed) cuts. The cuts reject an event if an isolated high energy photon is seen in the detector; if not the acollinearity of the two jets has to be less than 20° and the observed invariant mass larger than 0.4 √s. The inlet is a blow-up (logarithmic scale) showing what remains of the return events after cuts.

a) Pure electroweak corrections for the (kernel) deconvoluted distributions, including weak boxes (and internal lines). The latter were neglected at LEP1 energies since their relative contribution was of order 10^{-4}.

b) Final state (FS) QED and QCD corrections.

c) Initial state (IS) QED radiation.

d) IS lepton- and quark-pair production (PP).

e) Initial-final (IF) QED interference.

The result of the implementation of each block is to be compared between different codes before a global comparison, which incorporates all the parts, is made. This not only avoids eventual accidental cancellations, but also brings out the relative contribution of the various “ingredients” entering in the totally convoluted “realistic observables”. Within the study group, issue a) (deconvoluted observables) has been investigated by comparing the results of three codes based on different approaches for the implementation of the kernel: TOPAZ0 [5], WOH [7].

1Note that TOPAZ0 has been particularly designed to run around the Z resonance and it is not optimized for much higher energies. For the LEP2 study, TOPAZ0 has been modified by upgrading the radiator function according to ref. [6] and including the contribution from weak boxes.
and ZFITTER \cite{8}. TOPAZ0 results have been computed in the 't Hooft-Feynman gauge, $= 1$, and within the scheme, WOH has also $= 1$ but on shell (OS) renormalization scheme (RS) and, finally, ZFITTER works in the unitary gauge and in the OS RS. All three codes have adopted the input parameters:

$$
\begin{aligned}
\mu &= 91.1884 \text{ GeV} & \mu &= 175 \text{ GeV} \\
\mu &= 300 \text{ GeV} & \left(\frac{\mu}{\mu}\right)_0 &= 0.123 & \left(\frac{\mu}{\mu}\right)^{-1}_0 &= 128.896
\end{aligned}
$$

(1)

A complete comparison incorporating all a)-e) (realistic observables) has been restricted to TOPAZ0 (T) vs ZFITTER (Z) and covered a sample of six energies: the mass (in order to establish a link with the LEP1 calculations) and six LEP 2 energies, \textit{i.e.} 140 150 161 175 190 205 GeV.

Because of the critical issue of the hard radiation at LEP2 energies and since both (T) and (Z) now apply the same QED radiator function for the total cross section, issue c) has been independently investigated by the Pavia group \cite{9}.

\begin{itemize}
\item **Pure Electroweak Corrections: effective couplings and the box problem**
\end{itemize}

The genuine electroweak corrections are by far the most interesting aspect of the two-fermion observables. Indirect virtual effects of new physics can also mimic these corrections. Hence, one needs to verify whether the strategies and approximations applied at the peak are still at work. For example, a question related to the actual implementation of higher-order corrections is connected with the attempt of parametrizing physical observables in terms of ‘running’ effective couplings. This language of effective couplings, which has been so successful at LEP1, is deeply related to some factorization scheme that must be rediscussed at higher energies (for instance, weak boxes were neglected at LEP1). This language reduces the computational complexity, and does not introduce any -dependence in the amplitudes, leading to a most useful and successful parametrization in terms of effective (-dependent) vector and axial-vector couplings. Unfortunately, at LEP2 energies one expects the boxes to start resonating due to the (and, to a lesser extent, ZZ) thresholds. Moreover, as can be inferred from a cut across these boxes which reveals the $+ \rightarrow +$ -channel exchange, the box contribution is not gauge invariant; in the same way that the -channel for $\rightarrow$ is not unitary.

To quantify the effect of boxes one should first address some theoretical considerations about gauge invariance and give a procedure for isolating the effect of the weak boxes. It is well known that only a proper arrangement of the radiative corrections to $\rightarrow$, including all contributions up to a given order, is gauge invariant. Every procedure designed for subtracting some part from the whole answer, for instance deconvolution of QED radiation, must respect gauge invariance. Formally, one writes the amplitude in terms of full 1PI vector-boson self-energies, initial(final) vertex corrections and multiparticle exchange diagrams. Next, the complex pole is derived in terms of the bare Lagrangian and, after a Laurent expansion, we end up with the pole, the residue at the pole and the \textit{non-resonating background} (that encapsulates the -dependence of the two-fermion amplitude), each of which is separately gauge invariant. It turns out that at LEP2 energies the \textit{non-resonating background} (to which the boxes contribute) is not negligible. This is an unambiguous manifestation of the importance of the boxes.

Instead of using the complex pole formalism, which is difficult to implement in the codes, the effect of the boxes in the comparison has been handled by agreeing on a procedure for “extracting” the exchange box diagram. Schematically, this diagram is denoted by $ww( )$.
as computed in a general $\xi$ gauge. It may be split, non-uniquely though, according to

$$w_w( ) = w_w(1) + 2 - 1 \Delta( ) = w_w(0) + \tilde{\Delta}(0) \quad (2)$$

When working in the $\xi$ gauge, we can incorporate $\tilde{\Delta}$ into the rest of the amplitude, which is $\xi$-dependent, and compute explicitly the box diagram in any $0$ gauge. This approach is gauge invariant but not unique, especially when different procedures are adopted like keeping the weak boxes outside the QED convolution or performing re-summations. At this point we can adopt two different strategies. On the one hand, one can use the ZFITTER prescription of including the weak boxes into the form-factors. These then become explicit functions of the scattering angle. On the other hand, for comparison purposes, a proposal for “de-boxization” has been made. As one presents results for QED deconvoluted quantities, we could also subtract weak boxes from the data with few simple rules:

1. It was agreed to subtract $w_w( = 1)$. At LEP1 this contribution can be neglected, its relative effect being of order $10^{-4}$.

2. those who work in the $1 = 1$ gauge stop here,

3. those who work in any $0$ gauge compensate the rest of the amplitude with $\tilde{\Delta}(0,1)$.

The effect of weak boxes (as defined above) is studied on the deconvoluted observables $dec$ ($i.e.$ before the inclusion of any IS and FS radiation) through the quantity $B$:

$$B = \frac{dec}{0} - 1 \quad \text{where} \quad dec = 0 + \text{box} \quad (3)$$

and $0$ is the corrected cross section but without the inclusion of boxes.

First, for $+ - \rightarrow - -$ TOPAZ0 and ZFITTER are found to agree extremely well from $\sqrt{s} = z$ up to LEP2 energies, including the region around the threshold. The relative discrepancy is well below the per-mil level for both and at worst $1.3$ per-mil for $d$. There is some minor (in view of the expected experimental accuracy) disagreement with WOH($\cdots$): $| ( - )_\mu | 0.3\% \quad | ( - )_\mu | 0.5\% \quad | ( - )_d | 0.7\%$.

An important result, already pointed at, is that the effect of weak boxes is not negligible (a few per-cent in terms of $B$) especially around the threshold and at the highest LEP2 energies. For instance, at $205\text{GeV}$, $B$ for the channels is $-1.1\%(-1.2\%), -2.2\%(-2.6\%$, $-3.4\%(-4.1\%)$ for TOPAZ0/ZFITTER(WOH). The results for the other energies are displayed in Table 1. This table also shows the effect of the boxes for $b\bar{b}$ and $t\bar{t}$ channels. For these two observables the comparison only involves TOPAZ0 and ZFITTER.

Comparisons of the results for $b\bar{b}$ reveals a discrepancy between TOPAZ0 and ZFITTER which attains $\sim 2\%$ at $\sqrt{s} = 161$ and $205\text{GeV}$ while the agreement for $t\bar{t}$ is excellent. Looking in more detail, one sees that ZFITTER gives almost exactly the same $B$ for both and $t\bar{t}$. It should be remarked that, in ZFITTER (but not TOPAZ0) the top mass is neglected in the boxes. The TOPAZ0 results suggest that the inclusion of the mass decreases the relative effect of the box, as one would naively expect. This disagreement is in fact another indication
Table 1: The effect (in per-mil) of Weak Boxes, \( B \), on \( \mu, u, d, b \) and \( h \) before convolution.

<table>
<thead>
<tr>
<th>CM (GeV)</th>
<th>ZFITTER</th>
<th>TOPAZ0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \mu )</td>
<td>( u )</td>
</tr>
<tr>
<td>( Z )</td>
<td>+0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>100</td>
<td>-0.59</td>
<td>-3.56</td>
</tr>
<tr>
<td>140</td>
<td>+0.09</td>
<td>-11.09</td>
</tr>
<tr>
<td>150</td>
<td>+1.79</td>
<td>-10.00</td>
</tr>
<tr>
<td>161</td>
<td>+11.75</td>
<td>+3.52</td>
</tr>
<tr>
<td>175</td>
<td>+2.02</td>
<td>-12.64</td>
</tr>
<tr>
<td>205</td>
<td>-10.53</td>
<td>-34.89</td>
</tr>
</tbody>
</table>

of the special role played by the observables, a result reminiscent of the \( Z \rightarrow h \) at LEP1 and the top connection. Note, however, that the box result does not have the same conceptual importance as the \( Z \) vertex in the sense of the non-decoupling of the heavy top (or equivalently the contribution of the Goldstone Bosons). Anyhow, this disagreement largely gets diluted and disappears when considering the total hadronic cross section. For the hadrons, the largest difference in \( B \) (about 0.4%) shows up around the \( \sqrt{\frac{s}{M_{Z}}} \) threshold where the effect of the boxes is about 1%. The contribution of the boxes to \( h \) is larger at 190GeV and 205GeV, reaching about 2%.

Concluding on the effect of boxes and the comparison between the genuine weak corrections, we mention that the effect of boxes on the forward-backward asymmetry for muons is well below 0.01 and that TOPAZ0 and ZFITTER have been checked to agree perfectly for this observable.

Once one has subtracted the effect of the boxes, the remaining building block of the genuine electroweak corrections are essentially those one has dealt with at length at LEP1 (apart from the fact that these are now evaluated at \( \sqrt{s} \neq \frac{3}{2} M_{Z} \)). It is then worth inquiring about what one can learn from these “properly defined” observables that one has not from LEP1. In fact, one could further subtract the \( \sqrt{s} = \frac{3}{2} M_{Z} \) part and express the LEP2 observables in terms of the corresponding LEP1 quantities as suggested in [10]. The LEP1 observables were powerful enough in the sense that heavy particles, like the top, did not decouple, therefore allowing to put stringent limits on (or even ruling out) models beyond the SM. Unfortunately, after isolating the LEP1 observables, the remaining \( \sqrt{s} \) functions do not show much sensitivity to heavy particles, unless one is not far from their threshold. A most interesting topic concerning the \( \sqrt{s} \) dependence is the extraction of the running of \( \alpha_{em} \). If this could be done unambiguously, in a gauge-invariant way, one might hope to measure the non-Abelian contribution to the running that exhibits anti-screening and which, at high-energy, slows down the growing of \( \alpha_{em} \). It has been suggested to exploit the pinch technique [11], but more work related to this important issue is still needed.

- **ISR: Pure QED radiation**
  We have already stressed the qualitative difference between initial state radiation at LEP1
vs LEP2 energies. Because of this important difference and the overwhelming effect of this “correction” (see Fig. 1 and Fig. 2), it is crucial to reassess the implementations of the ISR and then see how the convoluted “realistic” observables compare in different codes. This is most conveniently done by convoluting the weakly-corrected cross section \( \sigma_{\text{dec}} \) (see Eq. 3) with a radiator function, \( ( = ' ) \), that encapsulates the results of the QED corrections (virtual corrections and real radiation)

\[
( ) = \frac{1}{x_{\text{cut}}} \sigma_{\text{dec}} (') ( )
\]

For a fully extrapolated set-up, \( x_{\text{cut}} = 4 \). To cut the “Z-return” at LEP2, one may take \( x_{\text{cut}} = 0.5 \).

Clearly the \( \mathcal{O}( ) \) result in \( ( ) \) is not sufficient. The \( \mathcal{O}(2) \) has been computed exactly[6] while one can resum (at least) the soft photons to all orders. This resummation is important especially for LEP1, and introduces an “exponentiation scheme ambiguity”. A typical scheme, or parametrization for \( ( ) \), after soft-photon resummation, is

\[
( ) = (1 - )^{2-1} s + V + H( ) = 2 - ( - 1) = \log \frac{s}{t}
\]

where \( s + V \) can be associated to the virtual and soft (bremstrahlung) corrections while the additive \( H \) is due to the hard-photon radiation (added linearly here). The large corrections are due to the “collinear logs” and formally one may write

\[
s + V = \sum_{n=0}^{\infty} \prod_{i=0}^{n} H_i ( ) = \sum_{n=1}^{\infty} \prod_{i=0}^{n} H_i ( )
\]

All schemes reproduce the leading logs, LL (i.e. \( mn \)) up to some order . For LEP1, \( = 2 \) is sufficient. However, not all schemes reproduce even the exact \( \mathcal{O}(2) \) result. This difference is reflected essentially in the hard part and explains why schemes and codes (reproducing only the leading logs) that agree perfectly at LEP1 energies, no longer do so away from the resonance. Thus, TOPAZ0 has partly upgraded its radiator to reproduce the exact \( \mathcal{O}(2) \) result. Using the definitions set in ref. [12], both TOPAZ0 and ZFITTER use now a radiator function of type \( A \) (i.e. of the same kind as in Eq. 5). \( A \) includes the complete \( \mathcal{O}(2) \) corrections computed in [6].

One also expects that different parametrizations of the hard part affect mainly the inclusive cross section, while if a large \( x_{\text{cut}} \) value is imposed, that cuts away the hard radiation, the difference between the two schemes gets substantially smaller. To quantify the impact of the hard radiation terms and make a comparison with the situation at LEP1, the effect of the order \( \mathcal{O}(3) \) LL versus the \( \mathcal{O}(2) \) (LL also) has been investigated in the case of \( \mu \). The implementation of the LL is most easily and conveniently done within the framework of the QED structure-function approach. The cross section obtained with a standard (LEP1) additive structure function with up to second-order hard contributions \( ( A_{\text{LL}}) \) is compared against a non-singlet additive structure function inclusive of up to third-order hard-photon effects [13, 14] \( ( A_{\text{LL}}) \). Applied to LEP1 the relative contribution of the latter is below \( 10^{-4} \). Figure 4 clearly
illustrates the points raised above [9]. While a stringent \( \text{cut} \) 0.5 makes the higher-order effects completely negligible (below \( 10^{-4} \)), for loose cuts \( \text{cut} \) 0.3 (inclusive set-up) the effect lies in the range (0.1-0.4)%.

While ZFITTER and TOPAZ0 use the same radiator function for the total cross section, only ZFITTER implements the leading-log correction to the forward-backward asymmetry as given in [15]. The latter involves a “non-symmetric” \( (i.e. \text{it would vanish upon angular integration}) \mathcal{O}( \frac{1}{\pi}) \) contribution. This might be responsible for a tiny difference in \( F_B \). The comparison between TOPAZ0 and ZFITTER for the purely photonic convolution has been made with two values of \( \text{cut} \): the first corresponding to an inclusive set-up with \( \text{cut} = 0.015 \) and the second to a loose cut on \( \text{cut} = 0.5 \). In this first comparison about the effect of ISR, the contribution of the boxes discussed above is switched off. The results are displayed for the muon and hadronic cross sections, as well as the forward-backward asymmetry, in Table 2. For \( \mu \mu \) in the inclusive set-up, there is excellent agreement for all the energies considered with an almost constant relative deviation of about 3 per-mil. With a more stringent cut the agreement is further improved and almost reaches the accuracy achieved at LEP1. As for \( F_B \), the cut has little effect on the absolute deviation which never exceeds 0.006 and is thus very satisfactory. For the hadronic cross section, the worst deviation occurs at the \( \pi \pi \) threshold, attaining 7 per-mil in the inclusive set-up, but is reduced by an order of magnitude when the \( \text{cut} \) is applied. This table also shows the large reduction in the event sample when the stricter cuts are applied, hence getting rid of the Z-return, as we discussed above.

- **IS Pair Production (PP)**
A consistent treatment of initial state radiation at \( \mathcal{O}( \frac{1}{\pi}) \) should include the radiation of ad-
<table>
<thead>
<tr>
<th>$c_m$ (GeV)</th>
<th>[0.015]</th>
<th>[0.05]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu()$</td>
<td>$\eta()$</td>
</tr>
<tr>
<td>91.1884</td>
<td>1477.8</td>
<td>3044.2</td>
</tr>
<tr>
<td></td>
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<td>3044.4</td>
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<tr>
<td></td>
<td>0.20</td>
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<tr>
<td>140</td>
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<td>243.49</td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
<td>3.10</td>
<td>7.07</td>
</tr>
</tbody>
</table>

Table 2: Comparing the results of the ISR convolution with boxes switched off. Two configurations are considered: \[0.015\] and \[0.05\]. The first row is ZFITTER and the second one is TOPAZ0. The third row is the relative deviation (in per-mil) for the cross sections and \[10^6 \times \] the absolute deviation for the forward-backward asymmetry.

Additional fermion pairs which also appear as a virtual correction at the two-loop level. Both TOPAZ0 and ZFITTER have used the available results at $O(\frac{1}{s^2})$ of the KKKS formulation [16]. This takes into account soft pair radiation with all events radiated up to some energy $\Delta \ll \sqrt{s}$ and hard-pair radiation

$$
\begin{align*}
S + V & = \frac{S + V_{\text{pair}}}{H_{\text{pair}}} + H_{\text{pair}} \\
S + V_{\text{pair}} & = \frac{\Delta^2}{4m^2} \left( \frac{\sqrt{s} - \sqrt{q^2}}{2} \right)^2 \left( \frac{2 f}{2} \right) \times \frac{4f}{2} \\
H & = \frac{s_{\text{min}} \sqrt{1 - \sqrt{s}/\Delta}^2}{4m^2} \left( \frac{2 f}{2} \right) \times \frac{4f}{2} \\
H_{\text{pair}} & = \frac{s_{\text{min}} \sqrt{1 - \sqrt{s}/\Delta}^2}{4m^2} \left( \frac{2 f}{2} \right) \times \frac{4f}{2} \\
\end{align*}
$$

The formula in Eq.7 involves two parameters $\Delta$ and $\text{min}$. The unnatural appearance of the infrared separator $\Delta$ makes questionable the exponentiation of soft pairs. In [17], an expo-
nentiated result is given which is valid for leptons. No analogous treatment is available for hadrons, where the $O(\alpha)$ result must be corrected for numerically when considering in addition IS photon radiation. No effort at all has been made so far in order to ‘adapt’ TOPAZ0 and ZFITTER for the treatment of radiated high-energy pairs. Both TOPAZ0 and ZFITTER do not exponentiate the “soft pairs” and $\text{pair}$ is added linearly to the cross section. There is also the so-called $z_{\text{min}}$ problem [16]. IS PP has been successfully compared around the resonance for various values of this parameter, and finally the default has been set to $z_{\text{min}} = 0.25$. This corresponds to an experimental selection of decays where the invariant mass of the products is at least 50% of the total and the soft–hard separator $\Delta$ has been fixed in the region where we see a plateau of stability. However, above the threshold the four fermion channel becomes competitive and one must establish a clear separation between real four-fermion events (see the section on four-fermion production below) and IS pair-production corrections to two fermion events. It looks plausible to include into the corrections for two-fermion events only very soft leptonic and hadronic pairs, i.e. something like $z_{\text{min}} = 0.05$ corresponding to 70% or 77.5% of $\sqrt{s}$ at $\sqrt{s} = 200$ GeV. For the following comparisons, $z_{\text{min}} = 0.5$ has been chosen.

The effect of pure photonic and pair-production initial state radiation on $\mu\mu$ is displayed in Table 3 in terms of the relative contribution of the “pairs”, $p$. Although IS PP is very small

<table>
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<th>$E_{\text{cm}}$ (GeV)</th>
<th>ZFITTER</th>
<th>TOPAZ0</th>
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</thead>
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<td>91.1884</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>-1.03</td>
</tr>
<tr>
<td>205</td>
<td>-4.27</td>
<td>-1.07</td>
</tr>
</tbody>
</table>

Table 3: The effect (in per-mil) of IS pair production, $p$, on $\mu\mu$ with a cut $\sqrt{s} > 0.015$ and $z_{\text{min}} = 0.5$. The first column is ZFITTER, the second is TOPAZ0.

(a few per-mil), we observe a much less satisfactory agreement between the two codes. With a $z_{\text{min}} = 0.5$ cut the agreement ZFITTER/TOPAZ0 is remarkable up to the maximum positive contribution, which happens to be around 100 GeV, after which there is a consistent difference of about 0.2 – 0.3%. Incidentally, for the inclusive $\mu\mu$, this is of the same order as the discrepancy between the two codes when the IS PP is switched off (see Table 2), with the result that the two effects largely cancel. Inclusion of pair production at high energies requires more theoretical work.

- **IF QED interference**
  Although the formulations in TOPAZ0 and ZFITTER are totally independent, the IF QED interference has been tested successfully over the whole range of energies.

- **Global comparisons and realistic observables.**
  For the global comparisons, all the ingredients listed above are included simultaneously. It
<table>
<thead>
<tr>
<th>( s_0 ) (GeV)</th>
<th>( \nu(,,,,,) )</th>
<th>( h(,,,,,) )</th>
<th>( \mu_{\text{pp}} )</th>
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<td>23.950</td>
<td>0.58565</td>
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</tr>
<tr>
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<tr>
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<tr>
<td></td>
<td>0.91</td>
<td>-8.44</td>
<td>-6.09</td>
</tr>
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</table>

Table 4: Overall comparison with a cut \( s > 0.5 \) and \( m_{\text{min}} = 0.50 \). The first entry is ZFITTER, the second one is TOPAZ0. The third entry is the relative deviation (in per-mil) for the cross sections and \( (10^3 \times \) ) the absolute deviation for the forward-backward asymmetry.

should be noted that the weak boxes are added linearly to the cross section and are not convoluted with QED radiation. The outcome of this final overall confrontation are collected in Table 4 for the case of \( s > 0.5 \) (and \( m_{\text{min}} = 0.5 \)). For this value of \( s \), the radiative return at LEP2 would be effectively discarded, and the observables would be more sensitive to the high energy component of the kernel with the genuine electroweak corrections. For the muon cross section, there is a remarkable agreement between the two codes almost equalling the level of accuracy reached at LEP1. It is always below 0.2% at all energies. In fact, even when relaxing the \( s \) cut to switch to the inclusive cross section, the agreement is excellent and much better than the relative deviation observed in the case of the inclusion of pair production. As pointed out above, the “more-than-needed” accuracy is partly due to some cancellation. For \( \frac{\mu}{FB} \), the relative deviation does not compete with the one observed at LEP1 energies, nonetheless it stays below the 0.01 mark. As mentioned earlier, part of the discrepancy may be attributed to the different inclusion of the pure QED ISR in TOPAZ0 and ZFITTER (the asymmetric \( O(\mu) \) is not implemented in TOPAZ0). For the hadronic cross section, the agreement is quite
satisfactory up to the threshold. Beyond this energy, it somehow degrades and reaches even 0.8%. However, at these energies the box contributions (before convolution) were found to show some discrepancy (see Table 1) that goes in the same direction as the discrepancy revealed in the "realistic observable". Some of the deviation here should be attributed to the different treatment of the boxes for $b\bar{b}$, and therefore one expects an improved agreement if the boxes are calculated with the same input parameters.

Let us also mention that KORALZ has also been upgraded for LEP2 energies. KORALZ[18] is a Monte-Carlo program for $e^+e^- \rightarrow 2$ which includes YFS exclusive exponentiation of initial and final state bremsstrahlung. Weak boxes are implemented. Full details may be found in [19].

As a conclusion, fermion pair production is under control. The study has also revealed which particular points require further investigation, i.e. especially the treatment of IS pairs. An important fact is the confirmation of the importance of the box contribution for all the two-fermion channels, mainly at the threshold and at the highest LEP2 energies. This should be kept in mind or compensated for when attempting to parametrize the two-fermion observables in terms of running effective couplings or $s$-dependent form factors. Another important aspect that needs a more detailed study is how different codes compare when realistic cuts (such as accolinearity cuts, cuts on the energy and scattering angle of the fermions) are applied on the fully dressed observables. A very preliminary investigation, restricted to the muon case, shows that the agreement between TOAPZ0 and ZFITTER somehow degrades when implementing an accolinearity cut. At the same time the integration error in TOPAZ0 is larger than what is at the $Z$ peak. All this shows that more optimisation for LEP2 energies is needed, especially when introducing specific cuts.

3 Single-photon production

When studying fermion pair production the special case of neutrinos was not addressed, since this contributes an invisible cross section. At LEP1 [20], the latter can be inferred from the measurement of the lineshape, once it is assumed that all the visible modes are counted in $\Gamma_{\tau,\mu,\pi}$ and $\Gamma_{\nu}$. Another, less competitive, method at LEP1, is the measurement of the single photon yield from $e^+e^- \rightarrow \gamma$. At LEP2 this technique is the only available way to reveal the production of stable neutrals. One may think of the supersymmetric neutralinos and sneutrinos or a fourth generation neutrino, to cite a few. For these heavy "beyond the SM" neutrals, one needs to retain sufficient energy to produce them. Consequently, the associated radiation will tend to be softer than the typical photons coming from the $S,M$ radiative neutrino background. Actually, the latter are mostly very energetic and are easy to trigger on, since they are predominantly photons that recoil against real $\rightarrow 0$ decays to the 3 light neutrino pairs. Once again, we are dealing with the radiative return, that produces very energetic photons. Thus the situation is much more promising than at LEP1.
3.1 Experimental requirements

Single-photon counting experiments at LEP1 have been rather delicate due to the essentially soft nature of the single photons. This necessitated low-trigger thresholds and high control of backgrounds in order to achieve sensitivity to the $^0$ invisible width [21]. In general, LEP1 experiments have required $\gamma \rightarrow 1.5$ GeV and have restricted the energy region $|\cos(\theta)| < 0.7$. At LEP2, the photon energy spectrum from $^+ - \rightarrow ^- (\gamma)$ gives highly energetic photons which are easy to trigger on and can be measured well. The detectors are expected to function with similar performance as at LEP1. One relevant difference is that the minimum polar angle at which one can detect electromagnetic particles (veto angle), is likely to increase from about 25 mrad at LEP1 to about 33 mrad at LEP2 due to the installation of additional background shields to protect the tracking chambers from backgrounds produced at the higher beam energies. This will lead to more stringent cuts on the transverse momentum scaled to the beam energy, $T = T_b$, designed in order to kinematically eliminate backgrounds from, *e.g.*, radiative Bhabha scattering (where the two electrons are below the veto angle). Moreover, LEP2 physics studies involving ISR can use the forward acceptance more readily than at LEP1, due to the much higher energies involved. Details depend on backgrounds and requirements: counting or measuring. Based on current analyses (studying for example $^+ - \rightarrow ^-$) acceptance in polar angle in the region $|\cos(\theta)| < 0.95$ should be achieved for photons with sufficient $T$. Using canonical cuts of $T = T_b 0.05$ and $|\cos(\theta)| < 0.95$, leads to a cross-section of about 5 pb for $\sqrt{s} = 180$ GeV. So, there is potential for a 2% measurement of the inclusive cross-section per experiment for an integrated luminosity of 500 pb$^{-1}$, indicating that a theoretical precision below 1% (4 experiments) is desirable. Given the striking nature of such events and the favourable energy spectrum, it is likely that measurements will be statistics limited and not limited by experimental systematics. On the other hand, to achieve this accuracy a precise knowledge of the SM cross section is needed, also taking into account that some approximations used at LEP1 are no longer valid. Moreover, of particular interest to experimentalists is the inclusion in Monte Carlo codes of additional hard photons to the single photon, as such photons affect the acceptance when emitted at detectable polar angles.

3.2 Calculations for $e^+ e^- \rightarrow \nu \bar{\nu} \gamma$: lowest-order and radiative corrections

Neutrino-pair production in association with a photon is not entirely due to decays. For $\gamma$, there are additional -exchange diagrams where the photon can be an ISR or from “internal radiation” (involving the non-Abelian vertex). These -exchange diagrams are not negligible at all at LEP2 energies, contrary to LEP1 [20]. For instance, comparing the purely -channel $\mu^- \mu^+$ with $e^- e^+$, there is about a factor 2 enhancement of the latter at $\sqrt{s} = 175$ GeV, brought about by the -channel. This applies for a visible photon with $\gamma 10$ and $|\cos(\gamma)| < 0.9$. Therefore, some of the approximations that worked so well for LEP1 and allowed for an easy implementation of the higher-order corrections are no longer valid.

An excellent approximation at LEP1, the so-called PIA [22], is obtained by taking the contribution complemented by the limit $w \rightarrow 0$ (and switching off the ). This reproduces the exact result within 1%. Another equally good approximation convolutes the neutrino-pair cross section (with $w \rightarrow 0$) with a radiator function[23]. These same approximations overestimate
the result of the full calculation by some 30%, already at \( \sqrt{s} = 150 \) \(^1\). Because of the failure of these approximations as the energy increases, the implementation of the higher-order corrections for this three-body reaction requires a special attention. Full one-loop QED corrections have been computed [25, 26], while complete \( \mathcal{O}(\alpha) \) weak corrections are presently known only for the "sub-process" \( + - \rightarrow \) [27]. Higher-order QED corrections, necessary to match the experimental precision reached at LEP, are taken into account in the Monte Carlo [24, 28] and semi-analytical codes [23] used by LEP collaborations, through the QED structure-function approach (SF) or the YFS algorithm to implement multiphoton effects. For instance, within the SF method [29], the QED-corrected cross section can be written as [30]

\[
(\gamma_1 \gamma_2) = \gamma_1 \gamma_2 (1) (2) \frac{D(x_1; s)}{D(x_2; s)}
\]

where \( \gamma_1 \gamma_2 \) is the exact spectrum of Ref. [25], the photon variables refer to the centre-of-mass frame after initial-state radiation, and \( (\gamma_1 \gamma_2) \) is the electron (positron) structure function. The explicit expression of \( (\gamma_1 \gamma_2) \), including soft multiphoton emission and hard collinear bremsstrahlung up to \( \mathcal{O}(\alpha) \), can be found in [30].

\[\text{Figure 5: The QED-corrected and the Born cross section for } + - \rightarrow \nu \nu \gamma \text{ as a function of the centre-of-mass energy at LEP2. The approximations are detailed in the text. The cuts on the photon are } \gamma_{\min} = 1 \text{ GeV and } \gamma_{\min} = 20^\circ.\]

Fig. 5 shows the QED-corrected cross section of \( + - \rightarrow \nu \nu \gamma \) as a function of the centre-of-mass energy at LEP2, assuming the cuts \( \gamma_{\min} = 1 \text{ GeV and } \gamma_{\min} = 20^\circ. \) The dash-dotted line

\(^1\)This is obtained with \( E_{\gamma} > 1 \text{ GeV}, |\cos \theta_{\gamma}| < 0.966, [24].\]

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represents the lowest-order total cross section obtained after integrating the exact photon spectrum \( \gamma \cos \gamma \) [25], the solid line is the result obtained according to the structure-function formulation of Ref. [30] (i.e. in the case of convolution of the full spectrum [25], including and diagrams), and the dotted line, reported for the sake of comparison, shows the results obtained by simulating the approach of Ref. [24], namely correcting the contribution only, and adding to this result the -exchange diagrams at tree level\(^1\). Two considerations are in order. First, the QED-corrected cross section at LEP2 is higher than the Born one as a consequence of the radiative return: the effect is to enhance, in this experimental set-up, the Born cross section by a factor of about 1.3. Secondly, the convolution of the full spectrum is in good agreement (within 1%) with the approach based on Ref. [24] because the QED-corrected cross section is largely dominated by the radiative return, and the tree-level contribution of diagrams and interference is almost flat over the full energy range spanning from LEP1 to LEP2. The agreement between the two calculations is within the expected experimental accuracy.

\[ \begin{align*}
\text{number of events} & \\
0 & 1000 \\
10 & 2000 \\
20 & 3000 \\
30 & 4000 \\
40 & 5000 \\
50 & 6000 \\
60 & 7000 \\
70 & 8000 \\
80 & 9000 \\
90 & 10000 \\
\end{align*} \]

\[ \begin{align*}
E_\gamma \text{[GeV]} & \\
0 & 10 \\
10 & 20 \\
20 & 30 \\
30 & 40 \\
40 & 50 \\
50 & 60 \\
60 & 70 \\
70 & 80 \\
80 & 90 \\
90 & 100 \\
\end{align*} \]

![Graph](image)

Figure 6: The energy distribution of the seen photon, without and with initial-state QED corrections, for a LEP1 ( \( \sqrt{s} = 48 \) GeV) and a LEP2 ( \( \sqrt{s} = 87.5 \) GeV) energy. The cuts on the seen photon are the same as with the previous figure. The numbers of events integrated in the case with ISR and without are proportional to the corresponding integrated cross sections.

The single-photon energy distribution is shown in Fig. 6 after including the higher order ISR. The results confirm the qualitative arguments given above concerning the LEP2 vs LEP1 comparison. Two peaks are clearly visible in the photon energy distribution, both at LEP1 and

\(^1\)Strictly speaking, in Ref. [24] the SF approach is applied to the complete \( O(\alpha) \) QED corrections to the \( Z \) exchange contribution of \( e^+e^- \rightarrow \nu\bar{\nu}\gamma \). In the comparison reported here, the SF is applied to the tree level \( e^+e^- \rightarrow \nu\bar{\nu}\gamma \) cross section. Weak corrections are implemented through an improved Born approximation in both cases. It has been checked that both versions agree within 1% with the approach of convoluting the full spectrum.
LEP2 energies: the higher one is located at the energy value of about \( (1 - \frac{2}{Z}) \sqrt{2} \), the lower one is due to \( 1 - \gamma \) (soft photon) peaking behaviour. As can be seen, the main modifications introduced by initial-state radiation are to reduce the higher peak and to enhance the lower one. The most important conclusion is that at LEP2 even after taking into account additional radiation, there still is a prominent peak around the recoil hard photon, hence allowing for a better discrimination of the heavy neutrals and improving the LEP1 limit on the number of neutrinos via the radiative method; even if this will not match the super-precision of the line-shape method. Further simulations of single-photon distributions at LEP2 versus LEP1 energies obtained analyzing the events generated by the Monte Carlo of Ref. [31] are given and commented in Ref. [30]. All the above results have been produced by means of a new Monte Carlo event generator [31] developed for radiative neutrino counting measurements at LEP1/LEP2 and based on eq. (8).

### 3.3 Towards a single-photon library

During this Workshop, the problem of finding a general approach to the computation of the single-photon spectrum associated to any process of the kind \( \gamma^+ \gamma^- \rightarrow (\text{invisible}) \) has been addressed. In particular, possible approximations have been studied that, starting from the \( \gamma^+ \gamma^- \rightarrow (\text{invisible}) \) cross section, could allow to get the corresponding single-photon spectrum in a straightforward way. The Standard Model process \( \gamma^+ \gamma^- \rightarrow \gamma^- \) can act as a benchmark for this purpose. Instead of the exact formula for the neutrino single-photon spectrum, one can use as a kernel in the convolution formula (8) an approximate factorized photonic spectrum given by

\[
\approx = \sigma((1 - \gamma) \gamma) \gamma \gamma; (9)
\]

where \( \sigma \) is the total Standard Model cross section of \( \gamma^+ \gamma^- \rightarrow (\gamma^-) \) and \( (\gamma \gamma; \gamma) \gamma \gamma; \) is the angular radiator proposed in Ref. [30] and derived from \( \mathcal{O}(s) \) \( t \)-dependent structure functions [32]. It describes the probability of radiating a photon with a given energy fraction \( \gamma \) at the angle \( \gamma \gamma \) in the charged legs, including the “universal”, factorized form of the photonic radiation. The above recipe has been checked against the exact Standard Model single-photon spectrum and found to be accurate at the level of a few per cent [30]. The same method has very recently been applied to the single-photon signature of the SUSY process \( \gamma^+ \gamma^- \rightarrow \) (for the most general gaugino/higgsino composition of neutralinos in the MSSM) [33]. Its cross section has been obtained by convoluting the cross section for the channel \( \gamma^+ \gamma^- \rightarrow \) with the radiator function and found to be very hard to disentangle from the neutrino background (see the Neutralino Section in the New Particles Report for some results on this channel).
4 Photon-pair production

Photon-pair production is essentially a pure QED process, that is not very sensitive to the genuine weak radiative corrections. Therefore, contrary to the single-photon production, there is no new phenomenon to take into account with respect to LEP1. One way to exploit this clean channel is to probe the indirect effects of alternative models such as the exchange of a heavy excited electron or a contact interaction. However, to conduct these tests it is essential to take into account the order \( O(3) \) QED corrections that could mimic new-physics effects. The corrected differential cross section may be written as:

\[
\frac{1}{\Omega} \alpha^3 = \frac{1 + \frac{2}{1 - \frac{2}{2}}}{1 + QED} \quad (10)
\]

where \( \Omega \) is the photon scattering angle with respect to the beam. \( QED \) includes the virtual, soft and hard bremsstrahlung corrections \( [34] \). This higher-order factor has been verified to be needed in order to reproduce the LEP1 data \( [35] \) as shown in Figure 7.

This correction will have to be included also at LEP2. However, one expects the sensitivity to the anomalous effects to be enhanced at LEP2, since the latter increase with energy, while the QED cross sections falls. For instance, the effect of an excited heavy electron that may be parameterized by a scale \( \Lambda \) (depending on the chirality of the coupling) \( [36] \) or a general dimension-6 contact interaction with a scale \( \Lambda \) \( [12, 37] \) modify the differential cross section.
Figure 8: Comparison of the measured differential cross section with the QED predictions including the deviations for the parameter values shown in the figure, as a function of $|\cos \theta|$. The cross sections are normalized to the radiatively corrected QED cross section. The functional effect of $\Lambda_+$ and $\Lambda$ is the same.

According to

\[
\frac{d\sigma}{d\Omega}(\Omega) = \frac{d\sigma}{d\Omega}_{\text{QED}} (1 + \text{new})
\]

where $\text{new} \approx \pm 2 \ 1 \ A^4_\pm (1 - \cos^2 \theta)$ for the excited electron assumption and with an analogous expression for the contact interaction. A comparison of the measured and QED predicted differential cross sections, including the deviation, are reproduced from the L3 experiment in Figure 8. At LEP2, with an integrated luminosity of about 66 pb$^{-1}$, the lower limit on the scale of the contact term $\Lambda$ is expected to increase from 600 to 800 GeV, while that describing the excited electron, $\Lambda_+$ and $\Lambda_-$, will go up to 200 GeV. These limits scale as the 1/4 power of the integrated luminosity.

5 Four-Fermion Processes

5.1 Classes of Feynman diagrams

At LEP2 centre-of-mass energies, four-fermion final states are produced with large cross sections. These are not only due to real and pair production with subsequent decays $\rightarrow$ and $\rightarrow$, but arise from several production mechanisms, each giving sizeable contributions to the four-fermion cross section in specific configurations of the final-particle phase space. In Fig. 9, all the possible classes of four-fermion production diagrams are shown. The largest total cross sections arise from the multiperipheral diagrams. Here, two quasi-real photons
are exchanged in the -channel, giving rise to forward (and undetected) electrons/positrons plus a \( - \) pair with a non-resonant structure (the so-called “two-photon” processes). For instance,

**Abelian Classes**

\[
\begin{align*}
\text{Conversion} & : e^+ \rightarrow B_1 f_1, B_2 f_2 \\
\text{Bremsstrahlung} & : e^- \rightarrow e^-, \nu_e \\
\text{Annihilation} & : e^+ \rightarrow f_1, \bar{v}_e
\end{align*}
\]

\[ (B=Z^0, \gamma; B_1, B_2, B_3=Z^0, \gamma, W^\pm; + Higgs Graphs. ) \]

**Figure 9:** Four-fermion production classes of diagrams.

one has \(( + - \rightarrow + - + - ) \sim 10^2 \text{ pb for } r \sim 10 \text{GeV}\). On the other hand, although interesting for QCD studies (see the Physics report) and as a main background for missing energy/momentum events (see the New Particles Physics report), these classes of processes do not sizeably contribute to final states that are of interest for the studies of and Higgs boson production. In the latter case, the main contributions come from double-resonant diagrams (conversion and nonabelian-annihilation diagrams in Fig. 9). Also single-resonant processes (proceeding through abelian-annihilation, bremsstrahlung, fusion and single-resonant conversion graphs) can give an important contribution to vector-boson physics, when the invariant mass constraint on one of the final fermion pairs is relaxed. A particular example is given by the single production, \(+ - \rightarrow \rightarrow \) and \(+ - \rightarrow \rightarrow \).
most of the cross section is due to single-resonant bremsstrahlung and fusion diagrams, where an almost real photon is exchanged in the -channel and one final electron escapes detection. In a sense, one could rename these channels as “three-(visible)fermion” processes. Some aspects of four-fermion processes are studied elsewhere in this report. Here we concentrate essentially on providing analytical (or semi-analytical) approaches. A particular attention is given to total cross sections especially in the case of forward electrons. We will also list the cross sections for the entire list of the four-fermion processes when some canonical cuts are imposed, as given by some available codes on the market, thus complementing the studies of the Events Generators for WW Physics group.

5.2 Single-W production

The cross section for single (on-shell) production is shown in Fig. 1 and is dominated by the -channel photon exchange. However, this is only one of the sub-processes that contributes to \( \gamma^* \rightarrow \ell^+ \ell^- \). Complete tree-level cross sections for the process \( \gamma^* \rightarrow \ell^+ \ell^- \) have been computed using the GRACE system [38] with the complete set of tree-level diagrams and taking into account all fermion-mass effects. This allows to integrate with no cuts over the forward-electron angle and exactly assesses the relative importance of double-resonant diagrams versus single-resonant and non-resonant diagrams [39]. In Fig. 10, after applying some realistic experimental cuts on the quark (\( u,d \), 1GeV and angular separation from the beam \( u,d \), 8°), the comparison of the total cross sections for all the diagrams (that is,
20 graphs) with the double-resonant subset (given by conversion plus nonabelian annihilation graphs, total of 3) and the \(-\) channel subset (given by bremsstrahlung, fusion and multiperipheral graphs, total of 10) is shown for a) no cut on the final electron, and b) a cut on the electron angle with respect to both beams \( \theta \). The dominant contribution to the \(-\) channel subset is given by the 4 diagrams where a photon is exchanged in the \(-\) channel, with the \( - \) scattered in the forward direction \(^{4}\). One can see that, below the \( W W \) threshold, the single-resonant and non-resonant diagrams give a substantial contribution to the total cross section. At \( \sqrt{s} = 190 \) GeV, their contribution is 4.4\% of the total, while it increases at larger \( \sqrt{s} \). On the other hand, imposing a cut on the forward electrons strongly depletes the \(-\) channel contribution.

![Graph](image)

Figure 11: Invariant mass distribution for \( + - \rightarrow -- - \) at \( \sqrt{s} = 180 \) GeV. The solid and dashed curves are, respectively, as for cases a) and b), of the previous figure.

It is also interesting to compare the effect of the single-resonant and non-resonant diagrams on the quark-pair invariant mass distribution. Figure 11 shows how \(-\) channel production can alter the \( \mu\bar{\nu} \) distribution and eventually play a role in the \( W \) mass determination.

### 5.3 Exact cross sections versus effective approximations

When including all the tree-level diagrams for a four-fermion process in a computer program, one can lose some insight on which subsets of diagrams are really dominant and which are “sub-leading”. On the other hand, in order to treat correctly the phase-space integrations and to get a reliable result, one should distinguish the main/secondary groups of diagrams. At

\(^{4}\)There is a very subtle problem with the implementation of the \( W \) width. A naive “running” width leads to disastrous predictions, see [39]. A general discussion about the implementation of the \( W \) width and gauge invariance is discussed in the \( WW \) Physics Report. See also [40].
the same time it is also useful to check the reliability of effective approximations that allow to evaluate given subsets of diagrams in a much simpler way. The natural way of forming subsets of diagrams is by isolating subgraphs that (with the in- and out- intermediate particles taken on mass shell) correspond to some gauge-invariant process of lowest order [41] (other ways of decomposition have not been successful, especially at high energies [42]). In this section, such a procedure is illustrated in the particular process \( \gamma^+ \to \gamma^- \). This channel is important as a background for Higgs bosons searches. Figure 12 shows the 48 diagrams that make up the complete set (excluding the two that involve Higgs bosons): 8 multiperipheral, 16 bremsstrahlung (single or non resonant, with a \( = \) in the -channel), 8 conversion (single- or double-resonant) and 16 annihilation (single- or non-resonant) graphs. The first three classes of diagrams involve the subprocesses \( \to \), \( \to (\, = \,) \) and \( \gamma^+ \to \gamma^- \), respectively. The contribution of each subset to the total cross section has been computed exactly at tree level by CompHEP[43], and then compared with the corresponding results obtained through appropriate effective approximations that are described in the following.

Note that, in general, interferences between different subsets are found to be negligible at LEP2 energies, with the exception of the interferences of the bremsstrahlung diagrams with the \( \to \) decay, and the conversion diagrams with the \( \to \) and \( \to \) decays (that gives \(-24\) fb at \( \sqrt{s} = 200\)GeV). Then, apart from the interference between the bremsstrahlung diagram with \( \to \) and the one with \( \to \), which gives \(-3.2\) fb at \( \sqrt{s} = 200\)GeV, all other interferences are found to be less than 1 fb at the same energy [41].

- **Effective approximation for multiperipheral diagrams.**

Using the equivalent photon spectrum in the Weizsäcker-Williams (WW) approximation [44], we can write the approximate formula for the total \( \| \) cross section corresponding to multiperipheral diagrams (first row in Fig. 12)

\[
(\gamma^+ \to \gamma^-) = \frac{1}{4m_B^2/s} \cdot \frac{1}{4m_s^2/s} \cdot 2^\ast (\gamma(1) \gamma(2))
\]

where \( \gamma(\ ) \) is given by [45]

\[
\gamma(\ ) = \frac{1}{2} \frac{1 + (1 - \ )^2}{\log \frac{1 - \ }{2} - \frac{1}{2} - \frac{1}{2}} + 2
\]

and \( \ast = 2\epsilon \cdot 4 \cdot \frac{2}{b} \). The subprocess cross section is given by (see, for instance, [46])

\[
\gamma^+ (\gamma^+ \to \gamma^-) = \frac{2}{27} \ast (3 - 4) \log \frac{1 + \ast}{1 - \ast} - 2 (2 - \ast)
\]

where \( \ast = 1 - 4 \cdot \frac{2}{b} \ast \). The results obtained through eq. (12) after a numerical integration are shown in Fig. 13 (dashed curve), and compared with the exact computation (solid curve) that includes also the multiperipheral \( \gamma^+ \) and \( \gamma^- \) exchange diagrams (the last two are found to be suppressed by a factor \( 10^{-5} \) and \( 10^{-6} \), respectively, relative to the dominant contribution). The agreement is excellent (indeed, the two curves overlap completely).

\( \| \) i.e. no cut on the invariant \( b\bar{b} \) mass, \( m_{b\bar{b}} \). The case including a cut on the invariant fermion mass and applications to the \( m_{ff} \) distribution are given below.
Figure 12: Complete set of diagrams for the process $^+ \rightarrow ^- \rightarrow ^+$. 
• Effective approximation for -channel photon exchange (bremsstrahlung) diagrams.

Diagrams including the subprocess \( ^* \rightarrow \) (second row in Fig. 12) are well approximated by

\[
\begin{align*}
( ^* \rightarrow ) &= \frac{x_{\text{max}}}{x_{\text{min}}} Q_{\text{max}}^2 Q_{\text{min}}^2 \frac{2}{2} \gamma \left( \frac{2}{2} \right) \gamma \left( ^* \rightarrow \bigg| ^* \bigg) \ ( ^* \rightarrow \bigg) \ ( \rightarrow \bigg)
\end{align*}
\]

where

\[
\frac{2}{2} = \frac{1}{2} \left( \frac{1}{2} \right) = 2 e^2 \left( \frac{1}{4} \right)
\]

with the integration limits

\[
\begin{align*}
\frac{2}{2} &= \frac{2}{e^2} \frac{2}{1 - 2 z} \\
\frac{2}{2} &= \frac{2}{e} \frac{2}{1 - 2 z} \\
\min &= \left( \frac{e + 2}{b} \right)^2 \\
\max &= \left( \sqrt{\frac{2}{2} - e} \right)^2
\end{align*}
\]

On the other hand, for the diagrams including the subprocess \( ^* \rightarrow \), one has

\[
\begin{align*}
( ^* \rightarrow ) &= \frac{x_{\text{max}}}{x_{\text{min}}} Q_{\text{max}}^2 Q_{\text{min}}^2 \frac{2}{2} \gamma \left( \frac{2}{2} \right) \gamma \left( ^* \rightarrow ^* \bigg| ^* \bigg) \Gamma ( ^* \rightarrow ^*)
\end{align*}
\]

with the integration limits

\[
\begin{align*}
\frac{2}{2} &= \frac{2}{e^2} \frac{2}{1 - 2 b} \\
\frac{2}{2} &= \frac{2}{e} \frac{2}{1 - 2 b} \\
\min &= \left( \frac{e + 2}{b} \right)^2 \\
\max &= \left( \sqrt{\frac{2}{2} - e} \right)^2
\end{align*}
\]

The cross section for the subprocess \( ^* \rightarrow \), where \( \gamma \) denotes or \( ^* \), can be written in the form

\[
\gamma \left( ^* \rightarrow \bigg| ^* \bigg) \right) = \frac{2}{e} \nu \left( 2 \left( \frac{2}{2} \nu - 2 \nu + 1 \right) \log \frac{1}{1} + \right.
\]

\[
+ \frac{\epsilon (7 \nu + 1) + \nu (3 \frac{2}{2} \nu - 2 \nu + 1)}{\epsilon + \nu (\nu - 1)} \bigg( \nu \bigg) \bigg( \nu \bigg) + \mathcal{O} (\delta \nu \delta \gamma)
\]

where

\[
\frac{2}{2} \nu, \quad \frac{2}{2} \nu, \quad \nu = \frac{2}{2}
\]

\[
\nu = 1 - \nu + \frac{e \nu}{2} - \nu (1 - \nu - \nu)
\]

\[
= (1 + (e - \nu)^2 - 2 \nu - 2 \nu)(1 + (\nu - \nu)^2 - 2 \nu - 2 \nu)^{1/2}
\]

\[
\nu = 2 \frac{e}{e}, \quad \nu = - \frac{2}{e} \gamma, \quad \gamma = \frac{2}{2}
\]

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• Effective approximation for conversion, single and double-resonant diagrams.
We start from the conversion subprocess $\gamma^- \rightarrow \gamma^+$ (diagrams in the third row in Fig. 12). In this case

$$(\gamma^- \rightarrow \gamma^+)\frac{1}{2} = \frac{(\sqrt{\gamma^2 - 2m_{\gamma^2}})^2}{4m_{\gamma^2}} \frac{2}{\gamma_1^2} \frac{2}{\gamma_2^2}$$

where the off-shell photon decay width is given by

$$\Gamma(\gamma^- \rightarrow \gamma^+) = \frac{2}{3} \frac{1}{\gamma_1^2} (1+2\gamma_1^2) \Gamma(\gamma^- \rightarrow \gamma^+)$$

where $\gamma_1^2 = 1/9$ for the b-quark and 1 for the electron, $\gamma_1^2 = 2/9$. The color factor $\gamma_1^2$ is equal to 3 for b-quark and 1 for the electron. The subprocess cross section is given by [47]

$$(\gamma^- \rightarrow \gamma^+) = \frac{2}{D} \log \frac{D+1}{D-3}$$

where

$$D = \frac{4}{1-\gamma_1^2}, \quad D = 1 + (1+\gamma_1^2)^2$$

For the single-resonant process $\gamma^- \rightarrow \gamma^+$ one has

$$(\gamma^- \rightarrow \gamma^+)\frac{1}{2} = \frac{(\sqrt{\gamma^2 - 2m_{\gamma^2}})^2}{4m_{\gamma^2}} \frac{2}{\gamma_1^2} \frac{2}{\gamma_2^2}$$

where the subprocess cross section is given by the formula eq. (23) with the parameters

$$D = \frac{4}{1-\gamma^* - z}, \quad D = 1 + (\gamma^* + z)^2$$

The cross section for the double resonant process $\gamma^- \rightarrow \gamma^+$, with the subsequent decays of $\gamma^+$ in the narrow width approximation, is given by the subprocess cross section eq. 23 with parameters

$$D = \frac{38}{16} \frac{\gamma^* - z}{\gamma^* + z} \frac{1}{1-2}, \quad D = 1 + 4 \frac{z}{2}$$

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Figure 13: Effective approximations (dashed lines) and exact calculations (solid lines) corresponding to various subsets of diagrams for the process $e^+e^- \rightarrow e^+e^- b\bar{b}$.

multiplied by $(\rightarrow 1\rightarrow 1) (\rightarrow 2\rightarrow 2)$.

In figure 13, one can see that the exact computation (solid) is always reasonably recovered by the above approximations (dashes). Indeed, adding the approximate formulae for multiperipheral, single and double conversion incoherently (with no interferences) the total cross section is reproduced within 5%.

It is also possible to improve on the approximation for the conversion diagrams that involves the , by including the finite-width effects and even the ISR, as we will discuss below.

5.4 Radiative corrections within the multiperipheral diagrams.

In this section, we discuss the accuracy of different versions of the Weizsäcker-Williams (WW) approximation [44] in describing both the integrated cross section (with a cut on the invariant mass of the fermions) as well as their $\tau$ distribution in two-photon processes. The effect of the QED corrections to the subprocess is also discussed within the approximation. In order to
isolate the effect of the WW approximation error from other uncertainties (like QCD effects in two-photon hadron production), we study the $\gamma \rightarrow \pi^+\pi^-$ production as a reference process for more general cases.

Within the approximation the tree-level cross section is given by eq. (12), implemented with a cut on the invariant mass of the $\pi^+\pi^-$ pair. Several functions for the photon flux can be found in the literature, with the aim of giving more accurate descriptions of the exact rates. Indeed, it can happen that one formula can reproduce the total cross section quite precisely, but is less successful as far as some distributions are concerned, or vice versa. In general, the accuracy of a given approximation is both process and experimental-cut dependent.

Here, we compare how two different flux functions fare with the exact tree-level and one-loop QED corrected result. This correction is only applied to the sub-process $\rightarrow$ [48]. The phase-space integration of the final state (7-dimensional for the 4-bodies and 10-dimensional for the 5-bodies) was performed by using the Monte Carlo integration package BASES [49].

The following two Weizsäcker-Williams spectra were examined

\[
\gamma^{(1)}(\ ) = \frac{2}{2} \ln \frac{2(1-\ )}{2(\ln -1) - \frac{2(2-\ )}{2}} - \frac{1}{2}
\]

(25)

\[
\gamma^{(2)}(\ ) = \frac{2}{2} \ln \frac{2(1-\ )}{2(\ln -1) - (1-\ )} - (1-\ )
\]

(26)

with $\gamma^{(1)}(\ )$ [WWA(1)] and $\gamma^{(2)}(\ )$ [WWA(2)] replacing $\gamma(\ )$ in Eq.13 (note that the integration limits depends on the cut on $\tau\tau$ now).

<table>
<thead>
<tr>
<th>(pb)</th>
<th>Born</th>
<th>soft + loop</th>
<th>hard</th>
<th>( ) corr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>exact</td>
<td>6.017(6)</td>
<td>-2 361(2)</td>
<td>2.403(2)</td>
<td>0.70(5)</td>
</tr>
<tr>
<td>WWA(1)</td>
<td>6.171(4)</td>
<td>-2 392(1)</td>
<td>2.463(3)</td>
<td>1.16(5)</td>
</tr>
<tr>
<td>WWA(2)</td>
<td>8.370(6)</td>
<td>-3 224(2)</td>
<td>3.316(4)</td>
<td>1.10(5)</td>
</tr>
</tbody>
</table>

Table 5: Total cross section for $\pi^+\pi^-$ pair production at $\sqrt{s} = 180 GeV$ with the invariant-mass cut $\tau\tau > 30 GeV$. The photon contribution is separated into soft and hard at $\gamma = 1$. The last column shows the $\mathcal{O}(\ )$ correction in %.

Table 5 summarizes the various components of the QED corrected total cross section calculated at $\sqrt{s} = 180$ GeV. The only kinematical cut applied is $\tau\tau > 30 GeV$. The first spectrum, with $\gamma^{(1)}(\ )$, reproduces the exact integrated cross section within 2% while the second choice overestimates the integrated cross section by almost 30%. Note that the $\mathcal{O}(\ )$ correction is small, about 1% and is reproduced in all three cases. The impact of the choice of the photon spectrum on the $\tau$ distribution for the $\pi^+\pi^-$ was also studied. The results are shown in Fig. 14. We observe that the first approximation reproduces nicely the exact distribution for
Figure 14: a) distribution of $\tau$ at $\sqrt{s} = 180\,\text{GeV}$ for $M_{\tau\tau} > 30\,\text{GeV}$ based on the exact calculation. b) shows the ratio of the WWA approximations over the exact result [see text for the definitions of WWA(1) and WWA(2)].

small $\tau$ ($\tau \sim 20\,\text{GeV}$) while the second one is more suited in the medium $\tau$ range, though both fall down too fast in the large $\tau$ region (where, however, the statistics is very poor). From this example, one can conclude that the best choice of the non-leading term in the WW approximations depends on which quantity one wants to reproduce. For instance, the WWA(2) has been preferred in the analysis of $\tau$ distributions of two-photon process with high $\tau$ at TRISTAN [50].

5.5 Improved semi-analytical calculations for conversion-type four-fermion final states

We have already discussed how the conversion type diagrams can be approximated. The above approximations can be further improved by including finite-width effects and inserting ISR. In this sub-section, we report on four-fermion cross sections and invariant mass distributions as obtained by the semi-analytical method. All angular degrees of freedom in the phase space (five at tree level, seven if the ISR is included) are integrated analytically. After these analytical integrations, elegant and short expressions are obtained for invariant mass distributions. Fast, numerically stable, and highly precise numerical algorithms are then used to integrate the remaining phase-space degrees of freedom, namely the two or three squared invariant masses. Semi-analytical results are, however, not suitable for experimental simulations. In this sense,
the semi-analytical and the Monte Carlo approach are complementary, and semi-analytical results may serve as benchmarks for numerical approaches, which usually rely on the Monte Carlo technique. A short review of semi-analytical calculations may be found in [51].

- **Convolution formulae at tree level**

In the framework of the semi-analytical technique, total four-fermion production tree-level cross sections are given by

\[
\text{Born}(s) = d_1 d_2 \sqrt{s_1 s_2} \cdot \frac{d^2 k_{12}}{d_1 d_2} \tag{27}
\]

Squared invariant masses for final-state fermion pair are represented by \(s_1, s_2, s_{12}\) with \(s_{12} = s_1 + s_2 - 2m^2\). The subscript index \(i\) labels cross section contributions from squared amplitudes or interferences with distinct Feynman topologies and coupling structure. Partial double-differential cross sections have the form

\[
\frac{d^2 k_{12}}{d_1 d_2} = C_k(s_{12}) \cdot \mathcal{G}_k(s_{12}) \tag{28}
\]

Coupling constants and off-shell boson propagators are collected in \(C_k\), while \(\mathcal{G}_k\) is a kinematical function obtained after fivefold analytical integration over the angular phase-space variables. Both \(C_k\) and \(\mathcal{G}_k\) are given by very compact expressions. For different charged current (CC) and neutral current (NC) processes, \(C_k\) and \(\mathcal{G}_k\) may be found in references [51, 52, 53, 54, 55].

- **Complete \(O(\beta)\) ISR with soft photon exponentiation**

A total four-fermion cross section with complete \(O(\beta)\) ISR corrections including soft photon exponentiation is given by

\[
\text{ISR}(s) = d_1 d_2 \sqrt{s_{12}} \cdot \frac{d'}{d_1 d_2 d'} \frac{d^3 \Sigma_k(\gamma; s_{12})}{d_1 d_2 d'} \tag{29}
\]

with the reduced squared center of mass energy \(\gamma\) and

\[
\frac{d^3 \Sigma_k(\gamma; s_{12})}{d_1 d_2 d'} = C_k(\gamma; s_{12}) \cdot \epsilon^{\beta-1} S_k + H_k \tag{30}
\]

where \(\epsilon = 2 \tan^2 \left( \frac{\gamma}{2} \right) - 1\) and \(\gamma = (1 - \gamma)\). Both the soft-plus virtual and hard contributions, \(S_k\) and \(H_k\), split into a universal, factorizing, process-independent and a non-universal, non-factorizing, process-dependent part. Using the twofold differential Born cross sections \(k_{0}(\gamma; s_{12}) = \frac{\sqrt{s_{12}}}{s_{12}} \cdot \mathcal{G}_k(\gamma; s_{12})\), one obtains

\[
S_k(\gamma; s_{12}) = 1 + \tilde{S}_k(s_{12}) + \tilde{\mathcal{G}}_k(\gamma; s_{12}) \tag{31}
\]

\[
H_k(\gamma; s_{12}) = \tilde{S}_k(s_{12}) + \tilde{\mathcal{G}}_k(\gamma; s_{12}) \tag{31}
\]

with the \(O(\beta)\) soft-plus virtual and hard radiators \(\tilde{S}_k\) and \(\tilde{\mathcal{G}}_k\) in the universal part given by

\[
\tilde{S}_k(\gamma) = -\frac{2}{3} - \frac{1}{2} + \frac{3}{4} \epsilon \quad \tilde{\mathcal{G}}_k(\gamma) = -\frac{1}{2} + \gamma - \epsilon \tag{32}
\]

238
The NC8 cross section. The solid line represents the Born cross section, the dash-dotted line includes universal, and the dotted line includes all ISR corrections. In the inset, the universally ISR corrected NC8 cross section is compared to the contributions from $\gamma\gamma$ and photon pair production.

If the index is associated with s-channel $^+ -$ annihilation diagrams only, non-universal ISR contributions are not present. Non-universal ISR contributions originate from the angular dependence of initial state $t$- and $u$-channel propagators. Since the non-universal cross section contributions $\hat{S}_k$ and $\hat{H}_k$ do not contain the large logarithm $\epsilon$, they only yield small cross section corrections up to a few percent. However, the analytical structure of $\hat{S}_k$ and $\hat{H}_k$ is very complex. An important feature of the non-universal corrections is the so-called screening property, i.e. an overall damping factor $1 - 2^{-2}$ in the non-universal corrections [52, 55, 56]. It is important to note that screening is a likely property with respect to the proper high energy unitarity behavior of the completely ISR corrected cross section. Semi-analytical treatments of complete ISR are presented in references [52, 55, 56]. Details of the non-universal contributions may be found in [56, 57].

As an example for numerical results, figures 15 and 16 present total cross sections for the NC8 process

$$^+ - \rightarrow (0 \ 0 \ 0) \rightarrow ^+ -$$

without and with invariant fermion-pair mass cuts [56]. In figure 15, the cross section correction due to universal the ISR varies between 12% at $\sqrt{s}=130$ GeV and 21% at 600 GeV. The
Figure 16: The effect of cuts of $2 \cdot \Gamma_Z$ and $5 \cdot \Gamma_Z$ around the $^0$ mass $Z$ on the NC8 ('All Graphs') and $^0$ pair ('ZZ Graphs') cross sections. The cuts were applied to both the $^+^-$ and the $^-^-$ pair invariant masses $s_1$ and $s_2$. All cross sections are universally ISR corrected.

Additional relative correction from the non-universal ISR increases from $9_{-8}^{+9}$ at 130 GeV to 4.2% at 600 GeV. From figure 16 one can see how the NC8 cross section approaches the cross section for the NC2 reaction $^+^- \rightarrow (^0^0) \rightarrow ^+^-^-$ when invariant fermion-pair mass cuts are tightened. For the NC2 reaction, the effect of universal ISR varies between $-28\%$ at the $^0$ pair threshold and approximately $+10\%$ at 600 GeV. Non-universal corrections to the NC2 reaction amount to less than half a percent below and around the threshold and rise to $1.5\%$ at 600 GeV. Results for the NC24 process, that is with complete set of diagrams contributing to $^+^- \rightarrow _1^1_2^2_2^2$, with $1 \neq 2 \neq e$, are found in reference [54] (see also below). Details of semi-analytical results for Higgs production and CC processes are reported by the working groups Higgs, WW cross sections and distributions, and Event Generators for WW Physics in this Report.

5.6 Cross sections for all four-fermion final states with inclusion of all diagrams

In this section, we report on the results of a study of the tree-level cross sections for all possible four-fermion final states, as listed in Tables 6-8. The complete set of diagrams is taken into account in each case (the corresponding total number of diagrams $d$ is shown in the same tables). Higgs-boson contributions are not included. This comparative study involves seven
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Table 6: Cross sections (in fb) for all the leptonic four-fermion final states. The superscript $^{[1]}$ marks all the results where complete fermion-mass effects are taken into account. The asterisks in the HIGGSPV column distinguish cross sections computed by the WWGENPV version of the program.
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</table>

Table 7: Cross sections (in fb) for all the semileptonic four-fermion final states. The notation is the same as in Table 6.
codes: ALPHA [58], CompHEP [43], EXCALIBUR [59], grc4f (a package for computing four-fermion processes based on GRACE [38]), WWGENPV/HIGGSPV [60], WPHACT [61] and WTO [62]. For a detailed description of the codes see the Event Generators for WW Physics Report. In this comparison ISR and gluon-exchange diagrams for the hadronic four-fermion final states (when implemented) are switched off. The effect of non-zero fermion masses for some of the processes has also been investigated by ALPHA and grc4f (see Tables 6–8). Total cross sections have been computed at the centre-of-mass energy $\sqrt{s} = 190\,\text{GeV}$, with the following cuts: $\epsilon^{\pm} 1\,\text{GeV}$, $q^{--} 3\,\text{GeV}$, $(\pm^{--} \pm^{-+}) 10^9$, $(\pm^{--} \pm^{--}) 5^9$, $(\pm^{--} \pm^{-+}) 5^9$, $q_{\mu}^f (t)$ 50 GeV (cuts on the fermion energy variables are loosened in the case of massive fermions). Furthermore, in order to better check the agreement among the different codes, a canonical set of input parameter has been agreed upon in all the computations, that is $z = 91.188\,\text{GeV}$, $\Gamma_Z = 2.4974\,\text{GeV}$, $\Gamma_W = 203.37\,\text{GeV}$, $-1(2\,\Gamma_W) = 128.0\,\text{MeV}$, $F = 1.16639\,10^{-5}\,\text{GeV}^{-2}$, $\sin^2 W$ from $\alpha(2\,\Gamma_W) = \frac{G_F m_W^2}{\sqrt{2}}$. In Table 6, the cross sections for all the four-lepton final states are shown, in Table 7 the ones for the semileptonic states and in Table 8 the ones for the hadronic four-fermion states. The error in the last one or two digits, corresponding to the Monte Carlo event generator, is also shown in parenthesis. One can see that the agreement among the different central values is in general at the level of a few per-mil, and even better in some cases. Note that, with the cuts above, the effect of the fermion masses can be not negligible, as can be seen by comparing the rates for muons to those for $\gamma$'s for instance, (cf. Tables 6–7).

<table>
<thead>
<tr>
<th>NN</th>
<th>$e^+e^- \to$</th>
<th>$N_d$</th>
<th>ALPHA</th>
<th>CompHEP</th>
<th>EXCALIBUR</th>
<th>grc4f</th>
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<td>25.36(17)</td>
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<td>47.11(3)</td>
</tr>
</tbody>
</table>

Table 8: Cross sections (in fb) for all the hadronic four-fermion final states. The notation is the same as in Table 6.
6 Three Vector-Boson Production

LEP2 can in principle be sensitive to quartic self-interactions of the electroweak vector bosons, through the production of two bosons plus one large-angle hard photon in the channels $+ - \rightarrow$, and $\gamma\gamma Z$. While the inclusion of quartic couplings is essential to maintain gauge invariance, these couplings cannot be simply isolated as subtle cancellations among many diagrams, including also trilinear couplings, take place. Nevertheless, triple vector-boson production can be used as a test for the presence of anomalous couplings, in particular $\gamma\gamma W W$ and $\gamma\gamma Z$. [63].

The cross-sections for the production of three vector bosons are shown in Figure 17, where (generous) angular cuts $\cos \theta_V 15^\circ (=\pm)$ and $\cos \theta V V 10^\circ$, as well as a cut on $\sqrt{s} 10 GeV$, have been imposed to avoid backgrounds. The cross section increases very sharply near 170GeV (just above threshold) but LEP2 has barely enough energy to produce these final states with healthy statistics. One must therefore strive for the highest possible energy in order to increase the statistics. Furthermore, the sensitivity to anomalous couplings
also rises with energy. Estimates using the above cuts have shown that even with a centre-of-
mass energy of 230 GeV, one would need a two-orders-of-magnitude increase in precision to
reach the level needed to test New Physics.

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[62] G. Passarino, program WTO.

QCD

Conveners: P. Nason and B.R. Webber


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1 Introduction

LEP1 has performed a gigantic task in testing QCD predictions. In this it has benefited from the very large statistics available, the substantial lack of background, and the fact that initial state radiation plays only a minor rôle on the resonance. At LEP2, QCD tests are more challenging. Initial state radiation is very important, there is a $W W$ production background, and statistics are somewhat limited. In fig. 1 we show the annihilation cross section as a function of the centre of mass energy. The figure reports the Born cross section for the production of hadronic final states through the $Z/\gamma$ annihilation process, and the same cross section with the inclusion of the initial state radiation. This increases the cross section considerably above the $Z$ resonance due to the $e^+ e^- \rightarrow Z \gamma$ process, in which the hadronic system has an invariant mass equal to the mass of the $Z$ boson. In the figure we also show the hadronic cross section at a fixed $E_{cm}$, as a function of a lower cut $E_{cut}$ on the invariant mass of the hadronic system (dot–dashed line). With $E_{cut} = 0$ this cross section coincides with the value of the dotted line at 175 GeV. As the cut is increased above the $Z$ mass, the cross section drops suddenly, and it approaches the partonic cross section at 175 GeV. As the cut approaches 175 GeV the cross section vanishes, but it is quite clear that if we allow for few GeV of initial state radiation, its value is very close to the Born cross section. Assuming therefore a 20 pb cross section, with an integrated luminosity of 500 pb$^{-1}$ we expect 10000 hadronic events. From the figure we see that the $W$ background is not a negligible one, and further cuts should be imposed to get rid of it. From statistics alone, the error on a measurement of the total hadronic cross section is 1%. Since $\sigma_{tot} = \sigma_{tot}^{[0]}(1 + \alpha_s/\pi + \ldots)$, we would expect a 25–30% error on a determination of $\alpha_s$ from the
hadronic cross section, not including systematics. Therefore, a useful measurement of $\alpha_s$ from the total cross section will not be possible at LEP2. Instead, it will be possible to determine $\alpha_s$ from jets. The rule of thumb in these cases is that we expect most events to be two–jet events, a fraction $\alpha_s$ of three–jet events, and a fraction $\alpha_s^2$ of 4–jet events. With 10000 hadronic events, we would have 1000 three–jet events, which will allow us to determine $\alpha_s$ with a statistical precision of 3%. Assuming that $\alpha_s(M_Z) = 0.123$, we expect $\alpha_s(175 \text{ GeV}) = 0.112$, a 10% variation. It seems therefore possible to see the running of $\alpha_s$ between LEP1 and LEP2.

A large fraction of this report will be dedicated to the problem of measuring $\alpha_s$ from jets at LEP2. In Section 2 the relevant experimental aspects of event selection and background corrections will be dealt with. In the Sections 3 and 4 the present status of theoretical calculations for jet shape variables will also be given.

Using the large number of hadronic events, studies of particle spectra will certainly be possible. Section 5 is dedicated to fragmentation function studies at LEP2. The study of fragmentation functions is a relatively recent topic at LEP1. Measurements of the various components of the quark and gluon fragmentation functions have been performed at LEP1, and they allow us to make an absolute prediction for the fragmentation function at LEP2 energies, and also for the fragmentation function in W decays. We will see that it is very difficult to see scaling violation effects from LEP1 to LEP2. It is nevertheless important to measure the fragmentation function to check for the consistency of the whole approach, since important assumptions are often made when performing the fit (for example, flavour SU(3) symmetry). A study of scaling violation towards the small $x$ region has not yet been performed even at LEP1, mostly because of the lack of a complete theoretical calculation. We will present the relevant theoretical ideas in Section 5.3.

Section 6 will be dedicated to the measurement of particle multiplicities at LEP2. QCD makes a prediction for the energy dependence of the multiplicity, and for the shape of the multiplicity distribution, based upon the assumption known as local parton–hadron duality. The measurement of the multiplicity in heavy–flavoured events has recently received some attention, and will also be considered here. Based again upon the idea of local parton–hadron duality, QCD predicts many features of the small–$x$ particle spectrum and correlations. Section 7 will deal with these topics.

2 Event Selection and Event Shapes – Experimental

2.1 Introduction

A number of interesting studies of QCD may be performed at LEP2 using $Z^0/\gamma \rightarrow q\bar{q}$ events. Although the number of events will be much smaller than at LEP1, it may be sufficient to

\footnote{The present Section is mostly work of D. Ward, including contributions from S. Bethke, G. Cowan, D. Lanske, and C. Padilla.}
explore aspects of the energy evolution of QCD. In this study we focus on the determination of \( \alpha_s \). The value of \( \alpha_s(M^2) \) has been determined using a number of techniques involving jet rates and event shape observables at LEP1\([1, 2, 3, 4, 5, 6, 7, 8]\) and at SLD \([9]\). For example, using a combination of resummed next-to-leading log (NLLA) and \( \mathcal{O}(\alpha_s^2) \) QCD calculations, an average measurement of

\[
\alpha_s(M_Z^2) = 0.123 \pm 0.006
\]

was obtained \([10]\). Taking the typical centre-of-mass energy at LEP2 to be 175 GeV, we may expect the value of \( \alpha_s \) to be reduced to 0.112. Although the change in \( \alpha_s \) is not great compared to the uncertainty on the LEP1 measurement, it should be noted that the error at LEP1 is predominantly theoretical in origin, and thus may be largely correlated between LEP1 and LEP2. We may therefore hope to make a useful measurement of the difference in \( \alpha_s \) between the two energies.

The experimental difficulties at LEP2 are somewhat different from those at LEP1. At LEP1 hadronic \( Z^0 \) decays could be readily identified with efficiencies in excess of 98%, and with negligible background. At LEP2 there are extremely large radiative corrections, and \( W^+W^- \) events may contribute a significant and troublesome background. Therefore, in Sect. 2.2 we investigate the problems of selecting a sample of non-radiative \( Z^0/\gamma \rightarrow q\bar{q} \) events, and discuss the extent to which these selection procedures may bias the events selected.

It will turn out that the events which may be selected most cleanly are those nearer to the two-jet region. Multi-jet events are much more susceptible to contamination from \( W^+W^- \) events. Since statistics are also meagre, and most of the events lie in the two jet region, this suggests that techniques based on the resummed NLLA QCD calculations will be most effective in determining \( \alpha_s \), since these calculations are expected to describe the two-jet region best. We have therefore focused on those event shape variables for which complete resummed NLLA calculations are available, namely Thrust \( (T) \), heavy jet mass \( (M_H) \), total jet broadening \( (B_T) \) and wide jet broadening \( (B_W) \) \([11]\). We also examine jet rates in the Durham jet-finding scheme, for which NLLA calculations are available – specifically the observable \( y_{23} \) which is the value of \( y_{cut} \) at which the event changes from two- to three-jet. All these variables are discussed in, for example, Ref. \([7]\). The next-to-leading order calculation \([12]\) for \( y_{23} \) as used so far by the LEP experiments was known to be incomplete. Recently, however, a more complete calculation has been presented \([13]\). We have not yet studied this new calculation, for compatibility with the existing LEP1 results. Resummed calculations are also now available for the \( C \)-parameter \([14]\), though they are not yet published, and are therefore not discussed here.

2.2 Selection of \( Z^0/\gamma \rightarrow q\bar{q} \) events

The discussion here will be based on events generated with PYTHIA \([15]\) version 5.715, with hadronization parameters tuned to LEP1 data \([16]\). The examples given below will relate to events processed through the OPAL detector simulation, but it is to be expected that similar results would hold for the other experiments. The cross-sections predicted for \( Z^0/\gamma \rightarrow q\bar{q} \) events

253
and for the principal source of background $W^+W^-\rightarrow q\bar{q}q\bar{q}$ are as follows:

<table>
<thead>
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<th>Reaction</th>
<th>Cross-section / pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$</td>
<td>1.69</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow Z^0/\gamma \rightarrow \bar{q}q$</td>
<td>149.6</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow Z^0/\gamma \rightarrow q\bar{q} ; E_{isr} &lt; 30$ GeV</td>
<td>39.0</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow Z^0/\gamma \rightarrow q\bar{q} ; E_{isr} &lt; 1$ GeV</td>
<td>26.1</td>
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The $Z^0/\gamma \rightarrow \bar{q}q$ cross-section is also given for two cuts on the amount of energy lost in initial state radiation. The useful cross-section for QCD studies is the non-radiative cross-section. The cut at 30 GeV corresponds roughly to the minimum in the hadronic mass spectrum $d\sigma/dM_h$ between the non-radiative process and radiation down to the $Z^0$ pole. Unless otherwise stated, the results shown relate to 175 GeV.

It is helpful to consider the selection of $Z^0/\gamma \rightarrow \bar{q}q$ events in two stages. In stage I we remove the leptonic and highly radiative events, and the $W^+W^-\rightarrow q\bar{q}\ell\nu\ell$ events, mainly using cuts on multiplicity and energy/momentum balance. These cuts introduce rather little bias into the $Z^0/\gamma \rightarrow q\bar{q}$ event sample. The stage II cuts are to remove $W^+W^-\rightarrow q\bar{q}q\bar{q}$ events, and are more problematic, since they turn out to bias the selected $Z^0/\gamma \rightarrow q\bar{q}$ sample significantly.

Typical stage I cuts would be as follows:

- Require $|\cos \theta_T| < 0.9$ to ensure reasonable containment of the event, where $\theta_T$ is the polar angle of the thrust axis.

- Require the number of charged tracks to be $N_{ch} > 6$ to remove purely leptonic events. This cut causes a negligible loss of $Z^0/\gamma \rightarrow q\bar{q}$ events.

- In Fig. 2 we plot $R_{vis}$ against $R_{miss}$ for various classes of events, where $R_{vis}$ is the visible energy scaled by the centre of mass energy $E_{c.m.}$, and $R_{miss}$ is the missing momentum scaled by $E_{c.m.}$. It is desirable to have the best resolution on the visible energy and missing momentum, which involves using an algorithm to combine the information from the charged tracks, electromagnetic and hadronic calorimeters so as to reduce double counting. We note that the non-radiative $Z^0/\gamma \rightarrow q\bar{q}$ events and the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ events are peaked around $R_{vis} = 1$ and $R_{miss} = 0$. The $W^+W^-\rightarrow q\bar{q}\ell\nu\ell$ events, and most of the radiative $Z^0/\gamma \rightarrow q\bar{q}$ events lie away from this point. Typical cuts are shown by the lines in Fig. 2.

- Fig. 2(b) reveals a group of radiative $Z^0/\gamma \rightarrow q\bar{q}$ events having $R_{vis} \sim 1$ and $R_{miss} \sim 0$. In these events, the radiative photons are detected in the electromagnetic calorimeter. Such photons may be identified using standard criteria on lateral shower shapes. The cluster should also be required to be be isolated, for example by demanding that within a cone of half angle 0.2 rad centred about the cluster less than 1 GeV is observed. If
Figure 2: Plots of $R_{vis}$ against $R_{miss}$ for (a) $Z^0/\gamma \rightarrow q\bar{q}$ events having less than 30 GeV initial state radiation (b) $Z^0/\gamma \rightarrow q\bar{q}$ events having more than 30 GeV initial state radiation (c) $W^+W^-\rightarrow q\bar{q}q\bar{q}$ events (d) $W^+W^-\rightarrow q\bar{q}l\bar{\nu}_l$ events. The lines show typical cuts. These plots are at $E_{cm} = 175$ GeV, though they are only weakly energy dependent.
the energy of the most energetic cluster satisfying the above criteria exceeds \( 0.6 \times p_\gamma \), the event is rejected. Here, \( p_\gamma \) is the expected photon momentum in an \( e^+e^- \to Z^0\gamma \) event, i.e. \( p_\gamma = (E_{c.m.}^2 - M_Z^2)/2E_{c.m.} \).

The cross-sections for the various channels of interest before and after these stage I selection cuts are listed in table 1. Hence, at \( E_{c.m.} = 175 \text{ GeV} \), in the region \(|\cos \theta_T| < 0.9\), the stage I cuts accept 92% of the non-radiative \( Z^0/\gamma \to q\bar{q} \) events, whilst accepting only around 1.5% of the radiative \( Z^0/\gamma \to q\bar{q} \) events and \( W^+W^-\to q\bar{q}\ell\bar{\nu}_\ell \) events. The \( W^+W^-\to q\bar{q}q\bar{q} \) events are accepted with high efficiency. The corresponding figures at 192 GeV and 161 GeV are essentially the same. Backgrounds from two-photon events, \( Z\gamma \to e^+e^- \) and \( W\nu \) final states have been examined, and appear to be negligible. \( ZZ\to q\bar{q}q\bar{q} \) does contribute, but at a much lower rate than \( W^+W^-\to q\bar{q}q\bar{q} \), with similar characteristics.

The main feature which distinguishes the \( W^+W^-\to q\bar{q}q\bar{q} \) events (and also the much smaller contribution from \( ZZ\to q\bar{q}q\bar{q} \) from the \( Z^0/\gamma \to q\bar{q} \) events is that the former contain four quarks, and thus generally have four or more jets, and are therefore less collimated. Furthermore, the invariant masses of appropriate pairs of jets should equal the mass of the \( W \) boson. We have examined the use of the following variables in separating these event classes:

1. The “narrow jet broadening”, \( B_N \). The event is divided into two hemispheres, \( S_\pm \), by the plane orthogonal to the thrust axis, \( \hat{n}_T \). In each hemisphere, the quantity \( B_\pm = \sum_{i \in S_\pm} |\vec{p}_i \times \hat{n}_T|/2 \sum_i |\vec{p}_i| \) is computed, where the sum in the denominator runs over all particles, whilst that in the numerator runs over one hemisphere. \( B_N \) is defined by \( B_N = \min(B_+, B_-) \).

2. The scaled “light hemisphere mass”, \( M_L/E_{vis} \). The event is divided into two hemispheres, \( S_\pm \), by the plane orthogonal to the thrust axis, and the invariant mass of each is computed, \( M_\pm \). Then, \( M_L \) is defined by \( M_L = \min(M_+, M_-) \).

3. The value of \( y_{cut} \) at which the event changes from 3-jet to 4-jet in the Durham jet finding scheme, \( y_{cut}^{(D)} \).

\[
\begin{array}{|c|c|c|}
\hline
\text{Channel} & \text{Cross-section /pb} & \text{Cross-section /pb} \\
& |\cos \theta_T| < 0.9 & \text{after stage I cuts} \\
\hline
Z^0/\gamma \to q\bar{q} (E_{vis} < 1 \text{ GeV}) & 17.90 & 16.51 \\
Z^0/\gamma \to q\bar{q} (E_{vis} < 30 \text{ GeV}) & 26.68 & 23.99 \\
Z^0/\gamma \to q\bar{q} (E_{vis} > 30 \text{ GeV}) & 73.37 & 1.07 \\
W^+W^-\to q\bar{q}\ell\bar{\nu}_\ell & 5.88 & 0.06 \\
W^+W^-\to q\bar{q}q\bar{q} & 6.08 & 5.74 \\
\hline
\end{array}
\]

Table 1: Cross-sections at \( E_{c.m.} = 175 \text{ GeV} \), based on PYTHIA.
Using the Durham jet finder, the event may be forcibly reconstructed as having four jets. The invariant masses of pairs of jets may be formed, from which we define the variable \( D^2 = \min [(M_{ij} - M_W)^2 + (M_{kl} - M_W)^2] \) where the minimum is taken over the permutations \((ij; kl) = (12; 34), (13; 24), (14; 23)\). Various ways of scaling the jet energies in order to improve the W mass resolution have been considered in connection with the W mass determination, but have not been used here.

In Fig. 3(a) we show the distributions of \( B_N \) for non-radiative \( Z^0/\gamma \rightarrow q\bar{q} \) events \((E_{isr} < 1 \text{ GeV})\) and for \( W^+W^- \rightarrow q\bar{q}q\bar{q} \) events, after the stage I cuts. In order to judge the correlation

![Figure 3](image)

Figure 3: (a) Distributions (at 175 GeV) of \( B_N \) for \( Z^0/\gamma \rightarrow q\bar{q} \) events having less than 1 GeV initial state radiation (open histogram) and for \( W^+W^- \rightarrow q\bar{q}q\bar{q} \) events (shaded) (b) average values of \((1 - T)\), \( B_W \) and \( y_{23}^{(D)} \) (scaled by their overall mean values) as a function of \( B_N \)

between \( B_N \) and the observables which we would wish to use for the determination of \( \alpha_s \) we show in Fig. 3(b) the average values of \((1 - T)\), \( B_W \) and \( y_{23}^{(D)} \) (normalized to their overall mean values) for non-radiative \( Z^0/\gamma \rightarrow q\bar{q} \) events as a function of \( B_N \). It is evident that the \( W^+W^- \rightarrow q\bar{q}q\bar{q} \) contribution can be reduced to almost any level desired by cutting on \( B_N \), but at an increasing cost in bias, and a corresponding loss in statistics. Generally, \( M_L \) and \( y_{34}^{(D)} \)
show similar behaviour to $B_N$. The $D^2$ variable offers a less clean separation between the $Z^0/\gamma \rightarrow q\bar{q}$ and $W^+W^-\rightarrow q\bar{q}q\bar{q}$ events, but it appears that it may introduce somewhat less, or different, bias, and may thus be complementary.

These observations may be quantified in table 2 below, where we show the effect of various possible stage II cuts on the $Z^0/\gamma \rightarrow q\bar{q}$ non-radiative signal and the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ background. We give the average values of $1 - T$, $B_W$ and $y_{23}^{(D)}$ as an indication of the bias caused by the cuts. We note that the stage I cuts cause only a small bias. We show several possible cuts on the $B_N$ variable. The background from $W^+W^-\rightarrow q\bar{q}q\bar{q}$ may be reduced, for example, to a level of 4% with an efficiency for selection $Z^0/\gamma \rightarrow q\bar{q}$ events of 82%. However, the sample of $Z^0/\gamma \rightarrow q\bar{q}$ events accepted is strongly biased. The bias, as measured by the mean value of the observable, tends to be greatest for $y_{23}^{(D)}$ and smallest for $B_W$. We show similar results for cuts on $M_L$ and $y_{34}^{(D)}$, where we have chosen cuts which yield roughly the same $Z^0/\gamma \rightarrow q\bar{q}$ efficiency as the $B_N < 0.06$ cut. Cutting on $M_L$ is less effective than $B_N$ at removing $W^+W^-\rightarrow q\bar{q}q\bar{q}$ background, while a cut on $y_{34}^{(D)}$ gives essentially the same performance as $B_N$. The cut on $D^2 > 600$ GeV$^2$ yields the same $W^+W^-\rightarrow q\bar{q}q\bar{q}$ contamination (7%) as the $B_N < 0.06$ cut, but for a significantly lower $Z^0/\gamma \rightarrow q\bar{q}$ efficiency (69% compared to 85%). Using $D^2$ yields a somewhat smaller bias on $1 - T$, but the bias on $y_{23}^{(D)}$ is a little greater. The two observables $B_N$ and $D^2$ are not strongly correlated (whereas, for example, $B_N$ and $M_L$ are highly correlated), suggesting that a joint cut on the two variables could give better separation. Examples are given in Table 2. The $W^+W^-\rightarrow q\bar{q}q\bar{q}$ background may, for example, be reduced to around the

<table>
<thead>
<tr>
<th>Cut(s)</th>
<th>$Z^0/\gamma \rightarrow q\bar{q} E_{isr} &lt; 1$ GeV</th>
<th>$W^+W^-\rightarrow q\bar{q}q\bar{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\cos \theta_T</td>
<td>&lt; 0.9$</td>
</tr>
<tr>
<td>Stage I only</td>
<td>16.51 0.0587 0.0701 0.0195</td>
<td>5.74 0.0195</td>
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<tr>
<td>$B_N &lt; 0.07$</td>
<td>15.73 0.0528 0.0668 0.0169</td>
<td>1.34 0.0169</td>
</tr>
<tr>
<td>$B_N &lt; 0.06$</td>
<td>15.27 0.0504 0.0653 0.0160</td>
<td>0.87 0.0160</td>
</tr>
<tr>
<td>$B_N &lt; 0.05$</td>
<td>14.53 0.0474 0.0632 0.0149</td>
<td>0.50 0.0149</td>
</tr>
<tr>
<td>$B_N &lt; 0.04$</td>
<td>13.26 0.0433 0.0601 0.0133</td>
<td>0.24 0.0133</td>
</tr>
<tr>
<td>$M_L &lt; 0.175$</td>
<td>15.26 0.0504 0.0656 0.0161</td>
<td>1.37 0.0161</td>
</tr>
<tr>
<td>$y_{34}^{(D)} &lt; 0.0065$</td>
<td>15.34 0.0501 0.0644 0.0156</td>
<td>0.88 0.0156</td>
</tr>
<tr>
<td>$D^2 &gt; 300$ GeV$^2$</td>
<td>14.17 0.0543 0.0667 0.0164</td>
<td>1.90 0.0164</td>
</tr>
<tr>
<td>$D^2 &gt; 600$ GeV$^2$</td>
<td>12.25 0.0504 0.0638 0.0140</td>
<td>0.89 0.0140</td>
</tr>
<tr>
<td>$D^2 &gt; 600$ GeV$^2 \text{ and } B_N &lt; 0.06$</td>
<td>11.71 0.0460 0.0613 0.0124</td>
<td>0.24 0.0124</td>
</tr>
<tr>
<td>$D^2 &gt; 300$ GeV$^2 \text{ and } B_N &lt; 0.05$</td>
<td>12.79 0.0457 0.0615 0.0132</td>
<td>0.24 0.0132</td>
</tr>
<tr>
<td>$B_N - \sqrt{D^2}/2000 &lt; 0.03$</td>
<td>13.84 0.0451 0.0612 0.0137</td>
<td>0.25 0.0137</td>
</tr>
</tbody>
</table>

Table 2: Cross-sections at 175 GeV accepted after the Stage I cuts, and after various possible Stage II cuts. The average values of various relevant observables are also shown, to indicate the level of bias introduced.

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2% level for a $Z^0/\gamma \to q\bar{q}$ efficiency of almost 80%, with somewhat less bias than a cut on $B_N$ alone. The precise cuts chosen for the separation of $Z^0/\gamma \to q\bar{q}$ and $W^+W^-\to q\bar{q}q\bar{q}$ events may therefore need to depend on the analysis being performed – whether a high purity is demanded, or whether a comparatively unbiased sample is required.

In Fig. 4(a) we show the distributions of a typical observable which may be used for the determination of $\alpha_s$, $(1 - T)$, after the stage I cuts. We compare the $Z^0/\gamma \to q\bar{q}$ non-radiative ($E_{isr} < 1$ GeV) signal with the $W^+W^-\to q\bar{q}q\bar{q}$ background. In Fig. 4(b) we show the same distributions after the stage II cuts, taking $B_N < 0.05$ as a typical stage II cut. As expected,

![Graph](figure.png)

Figure 4: (a) Distributions (at 175 GeV) of $(1 - T)$ after the stage I cuts. $Z^0/\gamma \to q\bar{q}$ non-radiative ($E_{isr} < 1$ GeV) events are shown as points with errors, and $W^+W^-\to q\bar{q}q\bar{q}$ events by the shaded histogram. (b) as (a), after applying the stage I cuts and the stage II cut $B_N < 0.05$. (c) Biases to the distribution of $(1 - T)$. The stage I cuts and the stage II cut $B_N < 0.05$ are applied. The closed points show the fraction of $Z^0/\gamma \to q\bar{q}$ non-radiative ($E_{isr} < 1$ GeV) events accepted after cuts. The open points show the ratio of all accepted $Z^0/\gamma \to q\bar{q}$ events after cuts to non-radiative ($E_{isr} < 1$ GeV) $Z^0/\gamma \to q\bar{q}$ events before cuts.

the background tends to be concentrated toward large values of $(1 - T)$, i.e. the region of hard gluon emission in the $Z^0/\gamma \to q\bar{q}$ reaction. The two-jet region of the $Z^0/\gamma \to q\bar{q}$ process is relatively free of background. Other stage II cuts give similar results. In Fig. 4(c) we show
the efficiency of the stage I+II cuts, taking $B_N < 0.05$ as a typical stage II cut, as a function of the $(1 - T)$, for $Z^0/\gamma \rightarrow q\bar{q}$ non-radiative ($E_{vis} < 1$ GeV) events (solid points). As expected, the cuts bias against large values of $(1 - T)$. In Fig. 4(c) we also show as open points the ratio of the distributions of all accepted $Z^0/\gamma \rightarrow q\bar{q}$ events (including radiative events) to those of the non-radiative events before selection cuts. In general the effect of initial state radiation is to bias the distribution towards higher values, but this is counteracted by the tendency of the cuts to reject events with high values of the observables. The net effect is that the distribution of the accepted radiative $Z^0/\gamma \rightarrow q\bar{q}$ events is quite similar to the distribution of non-radiative events before cuts, and so the ratios in Fig. 4(c) are increased roughly uniformly. Other stage II cuts give similar results, though the efficiencies may be systematically higher or lower.

2.3 Determination of $\alpha_s$

Before comparing with QCD calculations, the observed data must be corrected for the effects of detector resolution, the acceptance of selection cuts and the effects of background (Fig. 4). The influence of hadronization must then be accounted for, and one standard way of doing this is to multiply the corrected hadron level data by the ratio of the parton level to hadron level distributions from a Monte Carlo model. In Fig. 5 we show these ratios for $(1 - T)$, based on JETSET7.4, at 175 GeV (LEP2) and 91.2 GeV (LEP1). We note that the correction factors at LEP2 are significantly closer to unity, especially at small values of $(1 - T)$, corresponding to the two-jet region. Similar comments apply to the other observables. A requirement for a credible analysis is that the correction factors be not too far from unity.

In this study, we investigate three types of QCD calculations, which may be used as the basis of a measurement of $\alpha_s$ from event shape variables. These are:

$O(\alpha_s^2)$ The QCD matrix elements, expanded as a power series in $\alpha_s$ are fully known to $O(\alpha_s^2)$ [17]. From previous studies at LEP1 we know that these calculations are applicable in the “3-jet” region, i.e. the region dominated by hard gluon radiation. A significant uncertainty in applying the $O(\alpha_s^2)$ calculations is the choice of renormalization scale, $\mu$, represented by $x_\mu = \mu/E_{c.m.}$. The region over which the data can successfully be fitted can be extended further into the 2-jet region by choosing a small value of $x_\mu \sim 0.1$ (typically).

Figure 5: Hadronization corrections for the distributions of $(1 - T)$.

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NLLA. In the 2-jet region, the expansion in powers of \( \alpha_s \) is bound to fail, because large logarithms arise associated with collinear and soft gluon emission. In this region, "NLLA" calculations are available which resum the leading and next to leading logarithms to all orders in \( \alpha_s \). It has been shown in ref. [8] that such calculations may be used to derive \( \alpha_s \) at LEP1, but that it is necessary also to include a sub-leading term of the form \( G_2 \alpha_s^2 L \) in order to achieve a good description of the data.

**Combined NLLA+\( O(\alpha_s^2) \)** The most complete embodiment of our present knowledge of QCD comes from combining the \( O(\alpha_s^2) \) and NLLA calculations. It is necessary to match the calculations in such a way as to eliminate double counting of terms, and there are several ways of doing this. These have been studied at LEP1, based on which we choose the "\( \ln R \)" matching scheme for the present work.

To assess the range of validity of these calculations at LEP2 we proceed in the following empirical manner. We have generated distributions of the five observables, \((1-T), M_H, B_T, B_W \) and \( y_{23}^{(D)} \), at the parton level, using the \textsc{Jetset7.4} parton shower model without initial state radiation. We can assume that the data, after correction for detector acceptance, the effect of selection cuts and background, and hadronization, would closely resemble these distributions. For each observable, we then determine the largest range for which the theoretical calculations reproduce those from \textsc{Jetset} with an acceptable \( \chi^2/\text{DOF} \). The results are summarised in Table 3. We note that the \( O(\alpha_s^2) \) calculations may (in most cases) be extended to lower values of the observables by fitting \( x_\mu \). The NLLA or combined calculations allow a description down to still lower values, but, particularly in the case of the pure NLLA calculations, the higher values of the observables are less well modelled. The NLLA and combined calculations for \( B_W \) tend to give a rather poor description of the \textsc{Jetset} "data" (as seen at LEP1). The pure NLLA calculations are not applied to \( y_{23}^{(D)} \), since they are known to be incomplete, and in fact yield a poor fit to the \textsc{Jetset} distributions.

<table>
<thead>
<tr>
<th>Observable</th>
<th>( O(\alpha_s^2) ) (( x_\mu = 1 ))</th>
<th>( O(\alpha_s^2) ) (( x_\mu ) fitted)</th>
<th>pure NLLA</th>
<th>Combined ( O(\alpha_s^2) + \text{NLLA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1-T))</td>
<td>0.09–0.3</td>
<td>0.05–0.3</td>
<td>0.02–0.17</td>
<td>0.02–0.3</td>
</tr>
<tr>
<td>(M_H)</td>
<td>0.20–0.55</td>
<td>0.14–0.55</td>
<td>0.10–0.35</td>
<td>0.14–0.55</td>
</tr>
<tr>
<td>(B_T)</td>
<td>0.11–0.3</td>
<td>0.10–0.3</td>
<td>0.05–0.18</td>
<td>0.05–0.22</td>
</tr>
<tr>
<td>(B_W)</td>
<td>0.06–0.26</td>
<td>0.06–0.26</td>
<td>0.02–0.12</td>
<td>0.05–0.17</td>
</tr>
<tr>
<td>(y_{23}^{(D)})</td>
<td>0.015–0.2</td>
<td>0.005–0.2</td>
<td>–</td>
<td>0.005–0.2</td>
</tr>
</tbody>
</table>

**Table 3**: Approximate ranges of applicability of various types of QCD calculation.

If, for example, we require that the hadronization corrections lie between 0.8 and 1.2, that the \( Z^0/\gamma \to q\bar{q} \) acceptance be greater than 50% and that the \( W^+W^- \to q\bar{q}q\bar{q} \) contamination be less than 50%, the regions where the data can be used reliably would be roughly 0.03–0.2 for \((1-T)\), 0.15–0.4 for \(M_H\), 0.06–0.2 for \(B_T\), 0.03–0.18 for \(B_W\) and 0.005–0.09 for \( y_{23}^{(D)} \).
comparison with Table 3 it is evident that the regions in which reliable data may be obtained are best matched by the regions in which the combined NLLA+$\mathcal{O}(\alpha_s^2)$ calculations are valid. Since these are also the most complete calculations, this would appear to be the most promising approach.

We next assess the precision on $\alpha_s$ which could be achieved using 500 pb$^{-1}$ of data at LEP2. In order to do this, we take the JETSET7.4 parton level distribution, with statistical errors corresponding to this integrated luminosity (approximately 6500 $Z^0/\gamma \rightarrow q\bar{q}$ events). We then fit the QCD theory to infer $\alpha_s$, fitting in the range of the observable given by the overlap of the ranges in Tables 3 and the regions where reliable data may be obtained. A typical fit (of the $\mathcal{O}(\alpha_s^2)+$NLLA calculations to $(1 - T)$) is shown in Fig. 6. We find that

![Graph showing $\alpha_s$ fit](image)

Figure 6: Typical fit of the $\mathcal{O}(\alpha_s^2)+$NLLA QCD calculations to $(1 - T)$ in order to determine $\alpha_s$. The dotted lines delimit the fit region.

the $\mathcal{O}(\alpha_s^2)$ calculations yield typical statistical errors of $\pm 0.0024$, which are larger than the NLLA and combined $\mathcal{O}(\alpha_s^2)+$NLLA calculations (typically $\pm 0.0015$) because the former are only applicable towards the 3-jet region, where the few events are found. It also appears that the statistics are generally insufficient to permit a precise determination of the scale factor $x_\mu$ for the $\mathcal{O}(\alpha_s^2)$ fits. The pure NLLA and combined $\mathcal{O}(\alpha_s^2)+$NLLA calculations both appear to be competitive, and offer the possibility of measuring $\alpha_s$ with a statistical precision of around $\pm 0.0015$. For the event shapes $(1 - T)$, $M_H$, $B_T$, and $B_W$, the NLLA tend to yield smaller values of $\alpha_s$, and the $\mathcal{O}(\alpha_s^2)$ calculations larger values; the same trend was noted at LEP1 [8].

As at LEP1, the combined $\mathcal{O}(\alpha_s^2)+$NLLA method will probably be the preferred technique, because it represents the most complete theoretical calculations, and allows the largest fraction
of the data to be included in the analysis. For the discussion of possible systematic uncertainties, we therefore focus on these calculations. In ref. [7], for example, a wide range of systematic effects were investigated. The largest contribution was found to arise from variation of the renormalization scale factor $x_\mu$. Other significant effects arose from varying the hadronization model, particularly from the use of the HERWIG model, and from the influence of b-quark mass effects. We have estimated the systematic errors resulting from these effects at 175 GeV, and compared with the LEP1 experimental results.

- The renormalization scale factor is varied in the range $0.5 < x_\mu < 2.0$. The changes in $\alpha_s$ are highly correlated with those at LEP1, though about 20% smaller on average. If we assume that it makes sense to choose the same scale factor at LEP2 as at LEP1, then the effective systematic uncertainty on the change in $\alpha_s$ between LEP1 and LEP2 would be about $\pm 0.0015$ for $(1 - T)$, $\pm 0.0025$ for $M_H$, $\pm 0.0015$ for $B_T$, $\pm 0.0005$ for $B_W$ and $\pm 0.0003$ for $y_{23}^{(D)}$.

- The influence of b-quark mass effects may be crudely accounted for by basing the parton level distributions in the correction procedure only on udsc quark events. At LEP1 this correction was found to increase $\alpha_s$ by about 0.002 for most observables. Not surprisingly, the effect is much smaller at LEP2. However, the relevant point is the difference between the LEP1 and LEP2 uncertainties, which is of the order of 0.002 (somewhat larger for $B_T$ and smaller for $M_H$).

- The HERWIG model offers a quite different hadronization scheme from JETSET. Since the hadronization corrections are smaller at LEP2 than at LEP1, we would expect the uncertainty associated with the use of different models to be reduced. This is generally the case, but the correlation between the HERWIG uncertainties at LEP1 and LEP2 is unclear. This is partly because different fit regions have been used, and also different versions of the models. Clearly, in order to establish a reliable systematic uncertainty on the difference in $\alpha_s$ between LEP1 and LEP2 it would be necessary to make a more careful analysis using consistent versions of the models at the two energies. For some observables at least (e.g. $B_W$ and $y_{23}^{(D)}$) it seems plausible that the hadronization uncertainty could be quite small.

In summary, it appears that systematic errors would not preclude making a useful measurement of the difference in $\alpha_s$ between LEP1 and LEP2. The renormalization scale uncertainty seems to be comparable with or smaller than the statistical error. The uncertainty associated with b-quark mass effects could perhaps be reduced by further analysis and theoretical work. The uncertainties associated with the choice of hadronization models are less clear; it may be necessary to reanalyse the LEP1 data using the same models and parameter sets as employed in the LEP2 analysis, and the same fit regions, in order to minimise the uncertainties. Nonetheless, it seems that the systematic errors could be quite small, for some observables at least (especially $B_W$ and $y_{23}^{(D)}$, according to our study). It may be noted that recent studies of non-perturbative (power) corrections to the mean values of event shape observables [18] suggest
that certain observables or combinations of observables might be expected theoretically to have especially small hadronization uncertainties (e.g. $y_{23}^{(D)}$ or $T - 2C/3\pi$ [19]).

3 Event Shapes – Theoretical

Since the completion of the Yellow Report for LEP1 [20] much progress has been achieved in the theoretical calculations of shape variable distributions. A technique of resummation of contributions enhanced near the two–jet region has been studied and fully implemented in refs. [22, 11, 12, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Furthermore, new calculations of shape variable distributions (implemented as computer code) have become available.

Calculation of shape variables are all based upon the original work of ref. [17]. This calculation was also performed in ref. [21]. Although the analytic results did agree, several problems were found in the comparison of numerical results (see ref. [20] for a small review). While at the time of ref. [20] it was hard to find precision calculations of jet shape distributions that agreed with each other, today we have at least three general purpose programs that do agree. One, the program EVENT, was developed for ref. [20]. Results of shape variables distributions performed with this program are reported there, and have served as a benchmark for comparison with other computations. In ref. [36] a new computation was performed, which agrees with good accuracy with ref. [20]. Furthermore, very recently, yet another calculation was completed [37]. In ref. [37] also oriented events are implemented, and apparently they will also be implemented in ref. [36]. This means that it will be possible to compute distribution of shape variables that do depend upon the orientation of the incoming beams axis, unlike all shape variables that where used up to now (see the next section).

The most disturbing disagreement on shape variables was found to be on the Energy–Energy correlation (EEC). The computation performed in ref. [34] was found in important disagreement with other calculations, and in particular with ref. [20]. Recently, in ref. [35] the calculation of ref. [34] was repeated. The result of the new calculation was found in disagreement both with the result of ref. [20] and with ref. [34]. No clear statement is made in ref. [35] upon the origin of the discrepancy. It is however claimed that the disagreement comes from the region in which besides the quark–antiquark pair, two soft gluons have been radiated. The EEC is in fact peculiar, in the sense that even configurations with thrust near 1 can contribute to the EEC at angles far away from 0 and $\pi$. Because of the lack of a more complete theoretical paper form the authors of ref. [35], we thought that the most useful thing to be done for the present report is to perform a high-precision comparison of the different computations of the EEC, that can serve as benchmark for future calculations. In order to achieve high precision, instead of computing the EEC itself as a function of the angle, we computed its moments. The

2Written by P. Nason, including contributions of M.H. Seymour, N. Glover and K. Clay.
energy-energy correlation is defined as

\[
\text{EEC}(\chi) = \frac{1}{\sigma_{ij}} \int d^3 \vec{p}_i \, d^3 \vec{p}_j \frac{d\sigma}{d^3 \vec{p}_i \, d^3 \vec{p}_j} \frac{E_i E_j}{E^2} \delta(\vec{p}_i \cdot \vec{p}_j - \cos \chi).
\]  

(1)

We define

\[
\text{EEC}(\chi) \sin^{2+m} \chi \cos^n \chi \, d\cos \chi = \frac{\alpha_s}{2\pi} A^{(m,n)}_{\text{EEC}} + \frac{\alpha_s}{2\pi} B^{(m,n)}_{\text{EEC}} + \mathcal{O}(\alpha_s^3)
\]  

(2)

where \(\alpha_s = \alpha_s(E_{\text{cm}})\). The coefficients \(B^{(m,n)}_{\text{EEC}}\) have the following colour structure

\[
B^{(m,n)}_{\text{EEC}} = C_F \, C_A \, B^{(m,n)}_{C_A} + C_F \, B^{(m,n)}_{C_F} + T_f n_f B^{(m,n)}_{T_f}.
\]  

(3)

We then asked K. Clay (C), N. Glover (G), M. Seymour (S), and the author (N), to compute \(B^{(m,n)}_{C_A}, B^{(m,n)}_{C_F}, B^{(m,n)}_{T_F}\) with the programs of ref. [35], [36], [37] and [20] for \(m = 0, \ldots, 5\) and \(n = 0, 1\). All four computations agreed within errors for the \(B^{(m,n)}_{T_F}\) term. In the other two cases we found disagreements. The results are reported in tables 4 and 5.

<table>
<thead>
<tr>
<th>(m)</th>
<th>(n)</th>
<th>N</th>
<th>G</th>
<th>S</th>
<th>C</th>
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<tbody>
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<td>1</td>
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<td>-0.344 ± 0.006</td>
<td>-0.331 ± 0.003</td>
<td>-0.28 ± 0.01</td>
</tr>
</tbody>
</table>

Table 4: Comparison of different computations of the \(B^{(m,n)}_{C_A}\) coefficients.

It is clear that the results N, G and S agree with each other with high accuracy, while C is seriously different. Observe that, although for all practical purposes N, G and S agree with each other, there are among them discrepancies of several standard deviations. We attributed these differences as an underestimate of the errors, rather than to a real difference in the calculation. More details on the different characteristics of the three computer codes are given in the generator’s section [38].
The definition of thrust has a forward-backward ambiguity, so we are at liberty to define $\cos \theta > 0$. The leading order term is known analytically\cite{40},

$$A(T, \cos \theta) = C_F \frac{2(3T^2 - 3T + 2)}{T} \log \frac{2T - 1}{1 - T} - 3(3T - 2)(2 - T) \frac{3}{4}(1 + \cos^2 \theta)$$

\footnote{Author: M.H. Seymour}
Figure 7: The coefficients of the thrust distribution for five bins in $\cos \theta$, where $\theta$ is the angle between the thrust axis and the beam. The errors shown are purely statistical and are similar for each histogram, so we only show them for one.

\[ \frac{1}{2} \frac{d\sigma}{d\cos \theta} = 3 \left( 1 + \cos^2 \theta \right) \left( 1 + \frac{\alpha_s}{\pi} + \frac{\alpha_s}{\pi} \right) 8 \log \frac{3}{2} - 3 \left( 1 - 3 \cos^2 \theta \right) \approx \frac{3}{4} (1 + \cos^2 \theta) + \frac{\alpha_s}{\pi}, \]

where the approximation is good to better than 1%. The result is shown in Fig. 8b, where we see that the majority of the $\cos \theta$ dependence in Fig. 8a was from this dependence of the total event rate and the residual dependence is rather small. Nevertheless, it should be measurable with the full statistics of LEP1.

5 Fragmentation functions

The measurement of fragmentation functions at different energies and the comparison with the theoretical predictions, either implemented in the Monte Carlo programs or deduced from other measured data, can be used to perform different QCD tests and to tune the parameters describing the fragmentation processes inside the Monte Carlo programs.

\footnote{Author: C. Padilla.}
Figure 8: Predictions for the thrust distribution for five bins in $\cos \theta$ normalized to (a) the total number of events and (b) the number of events in each bin.

At the energies available at LEP II the scaled energy ($x = \frac{2E}{\sqrt{s}}$) distributions for charged particles can be measured for $q\bar{q}$ events in which the mass of the hadronic system is close to the centre-of-mass energy of the collision. Furthermore, the fragmentation function of the W boson can also be measured and compared to the expectation that comes from the measurement of the fragmentation functions for different enriched flavour samples at LEP I, after correcting for the small scaling produced for the different masses of the Z and the W boson and for the different flavour composition.

This Section describes how the measurement of the scaled energy distributions can be made and what can be expected in the measurement of $\alpha_s$ from scaling violations.

5.1 Measurement of scaled energy distributions

The measurement of the charged scaled energy distributions will follow the same procedure used at lower centre-of-mass energies. At centre-of-mass energies of the Z mass, hadronic events can be selected with very high purity and small backgrounds (coming mainly from $\tau$ events). At LEP II centre-of-mass energies, most of the $q\bar{q}$ events (more than 75%) will radiate an initial state hard photon such a way that the effective centre-of-mass energy of the collision will be reduced to below 120 GeV. These events have a high boost along the collision axis and have to be removed.

The selection of hadronic events will follow a procedure very similar to the one presented in subsection 2.2. After some minimal requirements on track quality, number of tracks and total measured energy of these tracks, additional selection variables have to be considered. Good containment of the events can be obtained with cuts in the sphericity or thrust axis.

Monte Carlo simulations performed in ALEPH, based upon DYMU3 and JETSET, including
full simulation of the detector response, show that requiring a visible mass of the event above 120 GeV and a normalized balanced momentum of the charged tracks along the beam axis below 0.3, a selection efficiency of $\sim 18\%$ can be achieved. The percentage of selected events such that the invariant mass of the propagator is below 120 GeV is reduced to approximately 7% with this selection procedure.

The backgrounds from dilepton events are small at this level. However, the background from WW events could still be substantial. A cut in missing momentum will remove most of the events in which one of the W has decayed leptonically. The remaining events in which both W decay hadronically can be removed by considering appropriate shape variables. Events resulting from the fragmentation of two W bosons will have a four-jet topology that makes them more spherical than the ones resulting from $Z/\gamma \rightarrow q\bar{q}$. In subsection 2.2 a discussion of the various possible approaches is given. For the case of the measurement of the scaled energy distributions, a cut on thrust $T > 0.925$ would be appropriate, since (unlike the case of shape variables) such a cut does not introduce strong biases in the shape of the fragmentation function.

The whole selection procedure should result in a cross section for $q\bar{q}$ events of $\sim 11$ pb with less than 1% of events with the effective centre-of-mass energy below 120 GeV and with a background of WW events below 5%. Assuming an integrated luminosity of 500 pb$^{-1}$ the expectation is to have $\sim 6000$ selected hadronic events.

It can be assumed that the background can be subtracted statistically using Monte Carlo techniques, and that the distribution is corrected using a hadronic event generator (with parameters adjusted to describe the data) for the effects of geometrical acceptance, detector efficiency and resolution, decays of long-lived particles (with $\tau > 1$ ns), secondary interactions and residual initial state photon radiation. The bin-to-bin correction factors are below 10% using the selection described above.

Figure 9 shows the Monte Carlo scaled energy distribution for the statistics of 6000 events. The energies of the particles before detector effects have been used to construct the distribution. Additional systematic uncertainties coming from possible discrepancies between the real detector performance and the simulated one and from the dependence on the hadron production model used to correct the data for detector effects will have to be considered.

The measurement of the W fragmentation function will require the selection of hadronic W events. The events in which one of the W decays leptonically can be selected using missing momentum or tagging a high-momentum lepton. The rest of the particles can be used to determine the momentum of the hadronically decaying W boson and to construct the scaled energy distribution, after boosting the particles into the rest frame of the parent W boson. In the case that both W bosons decay hadronically the techniques used in the measurement of the W mass can be used to unambiguously assign the jets to the corresponding W bosons.
5.2 Scaling violations: QCD tests

The analysis of scaling violations with the data available at LEP and data from lower centre-of-mass energy experiments (PEP, PETRA, TRISTAN) has focused on the measurement of \( \alpha_s \) [41, 42]. The prediction of scaling violations in fragmentation functions of quarks and gluons is similar to that predicted in structure functions in deep-inelastic lepton-nucleon scattering.

In an electron-positron collider, scaling violations are observed in the dependence of the distribution of the scaled energy of final-state particles in hadronic events on the centre-of-mass energy \( \sqrt{s} \). This comes about because with increasing \( \sqrt{s} \) more phase space for gluon radiation and thus for final-state particle production becomes available, leading to a softer \( x \)-distribution. As the probability for gluon radiation is proportional to the strong coupling constant, a measurement of the scaled-energy distributions at different centre-of-mass energies compared to the QCD prediction allows one to determine the only free parameter of QCD, \( \alpha_s \). A recent review of the relevant theoretical ideas has been given in ref. [43]. For another recent theoretical analysis see [44].

A reliable measurement of scaling violations has to disentangle the true QCD evolution from effects due to the dependence of the flavour composition upon the centre-of-mass energy. Since heavy flavours, after their decay into light particles, typically have softer fragmentation functions, when going from centre-of-mass energies below the Z mass towards the Z mass, the \( b \) content increases, and it decreases again when going towards higher energies. To analyse the data in a model independent way, final-state flavour identification and a measurement of
the gluon fragmentation function are needed. This procedure has been followed in the analysis
performed in ref. [41], where enriched \( uds, c, \) and \( b \)-quark scaled energy distributions, together
with the measurement of the gluon fragmentation functions and the longitudinal cross section,
have been used to constraint the fragmentation functions for the different flavours and the
gluon. It was assumed that the fragmentation functions of \( u, d, \) and \( s \) quarks are the same. In
the analysis presented there, a total of 15 parameters besides \( \alpha_s \) are fitted to all the available.
The parameters contain information on the fragmentation functions for the different quarks and
the gluon and also a parametrisation of the non-perturbative contributions to the evolution.
The value of the strong coupling constant obtained from this fit is

\[
\alpha_s(M_Z) = 0.126 \pm 0.007(\text{exp}) \pm 0.006(\text{theory}) = 0.126 \pm 0.009 \ .
\] (7)

The experimental error is the result of the combination in quadrature of the errors from the fit
(0.0053), the uncertainties in the flavour composition of the enriched scaled energy distributions
and the assumptions on the normalisation errors for those low-energy experiments where this
error is not specified. The theoretical error is estimated by varying the factorisation and
renormalisation scales.

A possible extension of this analysis has been investigated by including the predicted dis-
tribution measured at a centre-of-mass energy of 180 GeV (figure 9). Figure 10 shows the
result of the fit to the scaled energy distributions at three centre-of-mass energies (29 GeV,
91.2 GeV and 180 GeV). The fact that the variations with energy of the fragmentation func-
tions is logarithmic makes the difference between the distributions at 180 GeV and 91.2 GeV
smaller than that between 91.2 GeV and 29 GeV. This is accentuated by the fact that the
flavour composition changes between 91.2 GeV and 180 GeV, in particular the percentage of \( b \)
quarks diminishes when going to energies above the \( Z \) pole. Since the fragmentation function
for \( b \) quarks is softer, this hardens the inclusive distribution at LEP II energies.

The error in \( \alpha_s(M_Z) \) coming from the fit is not improved by including the distribution
measured at 180 GeV. It was found, however, that with four times the predicted available
statistics, a 10\% improvement in this error could be obtained. The conclusion is that the
analysis could serve as another consistency check of the predicted QCD scaling violations.
Improvement in the error on \( \alpha_s(M_Z) \) may come from several sources. A better understanding of
the flavour tagging algorithms used to measure the flavour-enriched distributions could improve
the experimental systematic error. Progress on the theoretical side, for example the extension
of the formalism to describe better the low-\( x \) region (see Section 5.3) could also be helpful.

Another consistency check can be performed by using the measured flavour-enriched dis-
tributions at the \( Z \) peak and the scaling violation formalism to predict the fragmentation function
in \( W \) decays. The fragmentation functions obtained from the fit to all data for the different
quark flavours can be evolved to the mass of the \( W \). Then the \( W \) scaled energy distribution
can be predicted using the \( W \) decay branching ratios for each flavour. Figure 11 shows, in the
continuous line, the prediction that results from this procedure. The points are the \( W \) scaled
energy distribution as predicted by the PYTHIA Monte Carlo.
Figure 10: Result of the scaling violation fit to the distributions at centre–of–mass energies at 29 GeV, 91.2 GeV and 180 GeV.

Figure 11: W fragmentation function predicted by the PYTHIA Monte Carlo (points), compared with the QCD prediction resulting from the analysis of scaling violations.
5.3 Small-\(x\) fragmentation\(^5\)

In the region of small values of the momentum fraction \(x\), the behaviour of the fragmentation functions may be significantly affected by phenomena related to the coherence of soft gluon radiation (for a review of this subject see, for instance, Ref. [46]). These effects are expected to result in a suppression of hadron production in the small-\(x\) (or soft) region, and to modify both the \(x\)-shape and the \(Q^2\)-dependence of the inclusive single-particle spectrum. In particular, as a consequence of coherence, when the momentum fraction becomes small the gluon fragmentation function is expected to peak at a value dependent on the hard scale of the process, and be damped in the soft region.

From the standpoint of perturbation theory, coherence effects show up as logarithmic corrections \(\alpha_S^k \log^m(1/x)\) (\(m \leq 2k - 2\)) to the splitting and coefficient functions which control the perturbative evaluation of the fragmentation functions. For example, the gluon splitting function \(P_{gg}(\alpha_S, x)\) has the small-\(x\) behaviour (\(\bar{\alpha}_S \equiv \alpha_S \, N_c / \pi\))

\[
P_{gg}(\alpha_S, x) \simeq \frac{\bar{\alpha}_S}{x} - \frac{\bar{\alpha}_S^2}{x} \log^2 x + \frac{\bar{\alpha}_S^3}{3x} \log^4 x + \ldots , \quad x \ll 1 . \tag{8}
\]

Small-\(x\) logarithms are present to all orders in \(\alpha_S\), and a systematic way to take coherence effects into account is to resum these logarithms to the leading accuracy, next-to-leading accuracy, and so on.

The leading-log results were determined in Refs. [22, 47], and can be best given in the moment space defined via the Mellin-Fourier transform

\[
\gamma_{gg}(\alpha_S, \omega) \equiv \frac{1}{\omega} \, dx \, x^\omega \, P_{gg}(\alpha_S, x) , \tag{9}
\]

and the analogous transform for any other function of \(x\). In the moment space logarithmic terms appear as multiple poles at \(\omega \to 0\), and the summation of the leading contributions \(\mathcal{O} \, \alpha_S^k / \omega^{2k-1}\) is encompassed by the formula [48]

\[
\gamma_{gg}(\alpha_S, \omega) = \frac{1}{4} \, \frac{\omega}{\omega^2 + 8 \bar{\alpha}_S - \omega} . \tag{10}
\]

The perturbative behaviour of this formula can be obtained by expanding it in the coupling \(\alpha_S\). The first terms of the expansion read as follows

\[
\gamma_{gg}(\alpha_S, \omega) \simeq \frac{\bar{\alpha}_S}{\omega} - 2 \frac{\bar{\alpha}_S^2}{\omega^3} + 8 \frac{\bar{\alpha}_S^3}{\omega^5} + \ldots , \tag{11}
\]

where in the \(\mathcal{O}(\alpha_S)\) and \(\mathcal{O}(\alpha_S^2)\) terms one may recognize the dominant part at small \(x\) of the standard one-loop and two-loop evolution kernels for the fragmentation functions (see [43] and references therein), whilst higher-order terms represent corrections due to coherent emission of

\(^5\)Author: F. Hautmann
soft gluons. An important feature which can be observed in Eq. (11) is the alternating sign of the expansion. As a matter of fact, this feature extends to the whole series, and the net effect of resumming all the leading logarithms turns out to be a damping of the fragmentation function in the soft region with respect to the lowest-order prediction.

The asymptotic properties of the resummed expression (10) are conversely determined by its behaviour near \( \omega = 0 \). This is given by

\[
\gamma_{gg}(\alpha_s, \omega) \sim \frac{\bar{\alpha}_s}{2}, \quad \omega \to 0.
\]

Note that the all-order summation of the perturbative poles \( \alpha_S^k/\omega^{2k-1} \) gives rise to a finite result at \( \omega = 0 \), and introduces on the other hand the non-analytic behaviour in \( \alpha_S \) of the square-root type.

The summation of the next-to-leading contributions \( \mathcal{O}(\alpha_s^2/\omega^{2k-2}) \) has also been performed [49]. The explicit expression of the next-to-leading correction to Eq. (10) reads

\[
\gamma^{\text{NL}}_{gg}(\alpha_s, \omega) = \gamma^L_{gg} + \bar{\alpha}_s - \frac{11}{12} - \frac{N_f}{6} + \frac{11}{4} + \frac{N_f}{3} - \frac{2}{3} \frac{N_fC_F}{C_A} \frac{\gamma^L_{gg}}{4\gamma^L_{gg} + \omega} \\
- \frac{11}{12} \frac{\omega \gamma^L_{gg}}{(4\gamma^L_{gg} + \omega)^2} - \frac{2}{3} \frac{N_f}{C_A} \frac{\gamma^L_{gg}}{4\gamma^L_{gg} + \omega^2},
\]

where \( \gamma^L_{gg} \) denotes the leading term (10), and \( N_f \) is the number of flavours. Next-to-leading contributions do not alter the qualitative behaviour determined by the leading-order analysis, but provide a \( \mathcal{O}(\sqrt{\bar{\alpha}_s}) \) correction to the position of the peak in the gluon fragmentation function.

Phenomenological studies of the soft region of the single-particle spectrum have been carried out in Ref. [50], on the basis of modified evolution equations which hold in the small-\( x \) regime. The central region of the spectrum, on the other hand, is known to be well described by second-order perturbation theory. It is therefore important to develop a procedure in which resummed contributions are consistently matched on to second-order perturbation theory, in order to get a uniform description of fragmentation over the whole phase space.

6 Charged Particle multiplicities\(^6\)

The study of hadron multiplicity distributions in high energy collisions is an important topic in multiparticle dynamics and is generally undertaken as soon as a new energy domain becomes accessible. It has been always considered a valuable tool to test our understanding of phenomenological approaches to multiparticle production and, in the framework of perturbative QCD (MLLA) with assumption of Local Parton Hadron Duality (LPHD) [52], the

\(^6\)Contributors: F. Fabbri and B. Poli (exp.), Yu.L. Dokshitzer and V.A. Khoze (th.)
average charged multiplicity $<n_{ch}>$ and the second binomial moment of the distribution, $R_2 = \frac{\langle n(n-1) \rangle}{\langle n \rangle}$, are predicted to evolve with energy [20]. The measurement of the average charged multiplicity in heavy flavoured events is also of interest to perturbative QCD and a theoretical discussion on this particular topic is presented in this Section.

6.1 Accompanying Multiplicity in Light and Heavy Quark Initiated Events

Perturbative QCD approach predicts a suppression of soft gluon radiation off an energetic massive quark $Q$ inside the forward cone of aperture $\Theta_0 = M_Q/E_Q$ (Dead Cone)[51]. This phenomenon is responsible for the “leading heavy particle effect” and, at the same time, induces essential differences in the structure of the accompanying radiation in light and heavy quark initiated jets. According to the LPHD concept[46], this should lead to corresponding differences in “companion” multiplicity and energy spectra of light hadrons.

In particular, a solid QCD prediction is that the difference of companion mean multiplicities of hadrons, $\Delta N_{Q\ell}$, from equal energy (hardness) heavy and light quark jets should be $W$-independent[53, 54] ($W$ is the energy available for soft particle production), up to power correction terms $\propto M_Q^2/W_Q^2$. This constant is different for $c$ and $b$ quarks and depends on the type of light hadron under study (e.g., all charged, $\pi^0$, etc). This is in a marked contrast with the prediction of the so called Naive Model based on the idea of reduction of the energy scale[55], $N_{Q\ell}(W) = N_{q\ell}(1-\langle x_Q \rangle)W)$, so that the difference of $q$- and $Q$-induced multiplicities grows with $W$ proportional to $N(W)$.

The data[56] for charged multiplicities in $b$- and $c$-quark events are in agreement with the energy independence of $\Delta N_{Q\ell}$. As far as the the value of multiplicity differences is concerned, an expression for $\Delta N_{Q\ell}$ has been derived within the MLLA accuracy[54] assuming $M_Q \gg \Lambda$:

$$\Delta N_{Q\ell} = N_{Q\ell}(W) - N_{q\ell}(W) = -N_{q\ell}(\sqrt{\varepsilon}M_Q) \ 1 + O(\alpha_s(M^2))$$

One usually consider the directly measurable quantity

$$\delta_{Q\ell} = \Delta N_{Q\ell} + N_D$$

where $N_D$ is the average multiplicity due to the heavy quark decay. Quantitative QCD expectation for the difference of measured charged multiplicities $\delta_{Q\ell}$ that includes decay products of heavy hadrons, based on (14) was obtained in [57]. For $b$ quarks, which are only relevant for LEP2, the MLLA estimate $\delta_{b\ell} = 5.5 \pm 0.8$ exceeds the experimental value 2.90 $\pm$ 0.30.

Recently an attempt has been made[58] to improve eq.(14), the result of which modification agreed with the data “significantly better than the original MLLA prediction” (W.Metzger, [56]). However, the very picture of accompanying multiplicity as induced by a single cascading gluon, implemented in [58], is not applicable at the level of subleading $O(\alpha_s)$ effects (see, e.g. [46]). Therefore a reliable theoretical improvement of the QCD prediction for the absolute value of $N_{Q\ell}$ remains to be achieved.
DELPHI has recently measured the number of $\pi^0$ in $b\bar{b}$ events to be close to that in all $Z^0$ events. The same difference should be there at LEP2.

6.2 Experimental

The main limitations at LEP2 in this kind of study will come from the limited statistics and the relatively high contamination of events from other physical processes, absent or totally negligible at LEP I. Hadronic decays of $W^+W^-$ pairs and highly radiative $Z^0/\gamma \rightarrow q\bar{q}$ events are expected to constitute the dominant background. It was shown in subsection 2.2 that this background can be reduced to a tolerable level, but the selection cuts needed, due to the particular nature of background events, will inevitably introduce a bias at both low and high multiplicities.

The present study is based on the analysis of events generated with PYTHIA version 5.715 at three different energies ($\sqrt{s} = 161, 175, 192$ GeV), with hadronization parameters tuned to LEP1 data [16]. The events were fully processed through the OPAL detector simulation and reconstruction program chain, but the conclusions drawn here are believed to be practically the same for the other experiments. The statistics used was large compared to the most optimistic assumption on the integrated luminosity achievable at LEP2. Following the usual convention, the charged multiplicity is defined as the total number of all promptly produced stable charged particles and those produced in the decays of particles with lifetimes shorter than $3 \cdot 10^{-10}$ sec. Non-radiative $Z^0/\gamma \rightarrow q\bar{q}$ events were selected following the criteria suggested in subsection 2.2, in particular we used a combined "stage I" and $B_X < 0.06$ cut. A further background reduction was obtained by rejecting events with a Thrust value $T < 0.8$. The selection efficiency achieved for non-radiative events, defined as those with an $E_{isr} < 1$ GeV (see subsection 2.2), was higher than 82% for all the considered energies. Due to detector acceptance and quality cuts, about 9% of the charged particles, on average, were lost in events surviving cuts while the predicted unbiased average charged multiplicity ($< n_{ch} > = 27.3$ at $\sqrt{s} = 175$ GeV) is about 14% higher than the observed one. Both those fractions were found to be practically energy independent. Approximately 33% of the events surviving cuts are radiative, namely with an $E_{isr} > 1$ GeV, but most of them have $E_{isr} < 20$ GeV. Background from $W^+W^-$ events never exceeds the 3% level. Residual background from other sources, like ZZ pairs, single W and single Z production, tau pairs and two-photon events was found to be negligible.

The observed multiplicity distribution must be corrected for detector effects (acceptance and efficiency in track reconstruction, spurious tracks from photon conversions and particle interactions in the material, selection cuts) and for effects induced by the residual background. In figure 12 we show the bias produced on the charged multiplicity distribution by the presence of residual $W^+W^-$ and radiative events as well as the bias produced by selection cuts. In figure 12-a we compare two normalized multiplicity distributions as they would appear in an ideal detector, namely without particle loss and interactions in the material, after event selection. In terms of real data, they would correspond to detector level corrected distributions including
residual initial state radiation (i.s.r.). The dotted distribution is relative to the pure q̅q sample.

Figure 12: Bias from: a) residual W⁺W⁻; b) radiative events; c) selection cuts.

which survived cuts, the other distribution (histogram) contains also the residual contamination from W⁺W⁻ events. As it can be seen from the bin-by-bin ratio shown in the bottom part, the bias due to this kind of background is relatively small. The estimated effect is 1.5% on <n_ch> and 0.6% on R₂. In figure 12-b a similar comparison is done to estimate the bias due to the presence of radiative events. Although this kind of contamination is relevant, the difference between the distribution containing residual radiative q̅q events (histogram) and the corresponding distribution for non-radiative events alone (dotted), is marginal. The effect is negligible on R₂, while on <n_ch> is similar in size and in the opposite direction with respect to the one produced by W⁺W⁻ events. The bias introduced by selection cuts is important at high
multiplicity. This can be seen in figure 12-c where the normalized distribution for non-radiative $q\bar{q}$ events surviving the selection criteria, (histogram), and the normalized distribution for an unbiased $q\bar{q}$ sample, (dots), are compared. In this case the effect on $<n_{ch}>$ and $R_2$ was estimated to be 3.7% and 1.3%, respectively.

Detector dependent corrections are usually carried out with unfolding matrix procedures and bin-by-bin coefficients[61]. In general matrices and coefficients are computed with the help of a very detailed detector Monte Carlo simulation in terms of material distribution, physical processes which particles undergo when interacting in the material and detector response to particles traversing the active media. After subtraction of the estimated residual $W^+W^-$ contamination, using for example a bin-by-bin correction, and provided one has a reliable simulation of the initial state radiation process, a global correction for particle loss due to detector effects is conceivable using a single unfolding matrix, computed from fully simulated events including i.s.r. The bias produced by event selection and residual i.s.r. can be corrected using bin-by-bin coefficients.

Considering our estimated selection efficiencies and assuming cross sections and multiplicity distributions as predicted by Pythia, we show in figure 13-a,b the expected relative statistical uncertainties on $<n_{ch}>$ and on $R_2$ as a function of the integrated luminosity, at three different energies. The integrated luminosities expected at LEP2 are such that statistical uncertainties on

![Figure 13: Expected relative statistical uncertainties on $<n_{ch}>$ and $R_2$.](image)

these parameters should comfortably stay below the 1% level. A sensible estimate of the magnitude of systematic uncertainties is difficult at this time. It will be most probably dominated by model dependent corrections needed to handle residual background and event selection biases. It is hard to believe it will be smaller than at LEP1 ($1-2\%$ on $<n_{ch}>$), but an uncertainty of a factor two higher may not be out of reach. We have fit the average charged multiplicity

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measured above the Upsilon threshold\cite{62} using the most popular parametrisations\cite{63}:

\begin{align*}
\langle n_{ch} \rangle &= a \cdot \alpha_s^3 \cdot \exp(\gamma / \sqrt{\alpha_s}) \\
\langle n_{ch} \rangle &= b \cdot s^a \\
\langle n_{ch} \rangle &= a + b \cdot \ln s + c \cdot \ln^2 s
\end{align*}

where a, b and c are free parameters. The predicted values extrapolated to LEP2 energies, however, only differ by a 3\% or less and it will be not obvious to disentangle among these models.

We also investigated the possibility to measure the average charged multiplicity in heavy-quark initiated events. Vertex-tagging methods have been shown to be very effective to select b-quark samples of high purity, and it is well known that secondary vertices with a relatively high associated multiplicity are likely to be produced in this kind of events. In the present study a method recently applied at LEP1\cite{64} was used to analyse events simulated at $\sqrt{s} = 175$ GeV. The method relies on the fact that independent samples of events with a different flavour composition can be selected by requiring events to have (at least) one secondary vertex with a certain decay length significance, defined as the decay length divided by its error. The fraction of a given flavour in the sample can be evaluated, as a function of this variable, from fully simulated and reconstructed events, figure 14-a. In general to a high value of the

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{figure14}
\caption{a) Flavour composition vs. decay length significance; b) $\langle n_{ch} \rangle$ for the unbiased event hemisphere vs. b-quark purity.}
\end{figure}
decay length significance corresponds a high probability to tag a b-quark event while at low significances the samples are predominantly populated by light-flavour events. To minimise the bias on multiplicity introduced by vertex tagging requirements, each event is divided in two hemispheres by a plane perpendicular to the thrust axis and the multiplicity is measured only in the hemisphere opposite to the one containing a secondary vertex, (unbiased hemisphere). The average charged multiplicity for pure samples of b-, c- and light-quark events is computed from a simultaneous fit to the corrected average multiplicity of samples selected with different decay length significance, i.e. with different flavour composition, figure 14-b. More details about the experimental procedure can be found in[64].

Due to extra selection requirements needed to insure the presence of secondary vertices, the original sample is reduced by a 30%. A b-tagging efficiency higher than 20% can be achieved in the highest purity bin. These values are similar to those obtained at LEP1. We studied the effects induced by the residual background and by the event selection cuts on the average charged multiplicity, measured in the unbiased hemisphere, as a function of the b-purity. Again we find that W+W− and radiative events produce only a marginal effect while the bias produced by selection cuts is important. The unfolding matrices to correct for detector acceptance, efficiency and spurious tracks must be calculated for each bin of decay length significance. A bin-by-bin correction method could be used to unfold residual i.s.r. and selection cuts effects.

In order to estimate the statistical precision attainable in this kind of measurement, we used the selection efficiencies found in this study and assumed the cross sections as well as the average multiplicity for different quark flavours, < nq >, predicted by Pythia. The fitting procedure mentioned above was applied to a high statistics sample of unbiased q̅q events to estimate the uncertainty on < nq >.

In figure 15-a we show the expected relative statistical uncertainties on < nq > (q = b, c,light) as a function of the integrated luminosity. The difference in charged multiplicity between b- and light-quark events, δb, and between c- and light-quark events, δc, is shown, taking into account

![Expected relative statistical uncertainties on < nq > and δq](image-url)
correlations, in figure 15-b. One can see that these measurements will be largely dominated by statistical uncertainties, at least using this method of analysis. A measurement of $\delta_{kl}$ could probably be attempted, while a determination of $\delta_{kl}$ seems to be precluded.

7 Hadron Momentum Spectra as a Test of LLA QCD\textsuperscript{7}

The shape of the momentum spectrum of hadrons produced in $e^+e^-$ collisions is successfully predicted in leading-log QCD (LLA). The LLA family of calculations together with the assumption of local parton hadron duality (LPHD) [52] predict that soft gluons should interfere destructively due to their coherent emission (or angular ordering), and this gives rise to a 'hump-backed' shape for the momentum distribution. At leading order, the distribution of $\ln(1/x)$ should have a Gaussian form, and this shape is modified to be a Gaussian with higher moments if next-to-leading order terms are calculated. The position of the peak of the distribution $\ln(1/x_0)$ is predicted in terms of the centre-of-mass energy, and therefore the evolution of the peak position with energy is well defined. This has been measured at centre-of-mass energies between 14 and 91 GeV, and the results are in agreement with a theoretical prediction that includes the effects of coherent, soft gluons. Models for hadron production based on phase space alone, or incoherent parton branchings predict a peak variation with energy that is twice as rapid and which is not supported by the data [62][65].

The increase of centre-of-mass energy afforded by the energy upgrade LEP2, allows the hadron $\ln(1/x)$ distribution to be measured in a new energy regime and provides the opportunity to further test the evolution of $\ln(1/x_0)$ from low energies. The energy increase is of the order of a factor two, so this represents a substantial 'lever arm' when compared to the existing data. In order to be able to challenge the predictions, the peak $\ln(1/x_0)$ should be measured to a precision of less than about 0.1 unit of $\ln(1/x_0)$.

The detailed shape of the $\ln(1/x)$ is predicted in terms of a small (typically three) number of parameters, and an energy evolution. These parameters have been fixed by fitting to the LEP1 data [66][65], and they can be used to predict the form of the data at higher energies. Clearly such a prediction of the shape of the $\ln(1/x)$ distribution constitutes an important potential measure of the success of the LLA approach to QCD calculations.

The LLA approach has been extended to predict the momentum distribution of pairs of gluons which has commonly been presented in terms of the two particle correlation [67]. This distribution has been measured at LEP1 [65][68] where it was found that the data were qualitatively, but not quantitatively described by next-to-leading order predictions. It was subsequently shown [69] that a satisfactory description of the data was possible if next-to-next-to-leading order terms with coefficients of order unity were added to the prediction. The study of the energy dependence of the two particle correlation is interesting as it is predicted solely in terms of a single free parameter and the energy scale.

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7.1 Monte Carlo Studies at 175 GeV

Monte Carlo events generated at a centre-of-mass energy of 175 GeV were used to study the likely precision and limitations of an analysis of hadron momentum spectra at LEP2. Events were generated using Pythia for the processes $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ and $e^+e^- \rightarrow Z^0/\gamma \rightarrow q\bar{q}$, and were subsequently passed through the Opal detector simulation program.

In contrast to analyses at LEP1 energies, the chief experimental problems are event statistics, and backgrounds in the event sample due to $W^+W^- \rightarrow q\bar{q}q\bar{q}$ events and events with a large amount of energy radiated by the initial fermions. The efficient selection of a clean sample of $Z^0/\gamma \rightarrow q\bar{q}$ events with propagator energies close to the centre-of-mass energy has been extensively studied in subsection 2.2. The present analysis uses the stage I cuts described there together with a stage II cut of $D^2 > 300\text{GeV}^2$ and $B_N < 0.05$. In total about 9000 $e^+e^- \rightarrow Z^0/\gamma$ events and about 130 $e^+e^- \rightarrow W^+W^-$ events would be selected assuming the nominal LEP2 luminosity of 500 pb$^{-1}$ and standard model cross sections. About 72% of the selected events have initial state radiation amounting to less than 2 GeV, and less than 2% of the events have radiation in excess of 60 GeV.

7.2 ln (1/x) Distributions at 175 GeV

The expected distribution of ln$(1/x)$ is shown in figure 16 (a) (and figure 16 (b) with a logarithmic vertical scale) for all events that pass the selection cuts. The statistical errors on the points correspond to a luminosity of 500 pb$^{-1}$. The contribution of the $W^+W^-$ background events is shown as the shaded region. The background is concentrated in the region around the peak of the ln$(1/x)$ distribution, varies smoothly, and is small compared to the level of the signal events—the signal to background ratio is almost 100:1. In practice, this background could be corrected for by a multiplicative correction factor, the background could be subtracted directly or a more complex matrix correction procedure could be applied.

Typical corrections for detector acceptance and resolution are shown in figure 16 (c). There is a correction of about 10% to the overall level of the distribution which is basically flat in the region around the peak of the ln$(1/x)$ distribution. Uncertainties in the determination of detector corrections are therefore unlikely to have a large effect on the position of the peak, and there is no evidence that a serious bias has been introduced into the ln$(1/x)$ distribution by the event selection. Figure 16 (d) shows the ratio of the ln$(1/x)$ distributions for events passing the selection cuts that did not radiate and those that radiated a photon of more than 2 GeV. This illustrates the component of the detector correction that accounts for initial state radiation. The bias introduced by the initial state radiation is most severe for low values of ln$(1/x)$ but is fairly uniform around the area of the peak. It is not expected that the event selection and detector corrections will seriously bias the measurement of the position of the peak.
7.3 Determination of Peak Position

The peak position may be determined by fitting a Gaussian to the \( W^+W^- \)-background subtracted \( \ln(1/x) \) distribution. The statistical error on the peak position is about 0.025 if data corresponding to 100 \( \text{pb}^{-1} \) are fitted, and this decreases to 0.011 when the expected 500 \( \text{pb}^{-1} \) data sample is analysed. A Gaussian function is only valid for the region close to the peak and is less successful at describing the shape of the distribution far away from it. This leads to a variation of the fitted peak position as data points far from the peak are included in the fit. Varying the fit range such that a reasonable \( \chi^2 \) is still obtained for the fit results in an uncertainty in the peak position of about 0.02.

A systematic error due to uncertainties in the level of the \( W^+W^- \) backgrounds has been estimated by varying the amount of the background subtracted by \( \pm 100\% \). The fitted peak position changes by less than 0.01 in all cases. It is not expected that there is a large uncertainty due to the details of the shape of the \( W^+W^- \) background. The process \( W^\pm \to q\bar{q} \) is very closely related to \( Z^0 \to q\bar{q} \) which has been very well understood thanks the the LEP1 data. In conclusion, it is expected that the position of the peak of the \( \ln(1/x) \) distribution may be measured with the data recorded at LEP2 to a sufficient precision in order to be able to test the LLA predictions.

7.4 Detailed Shape of \( \ln(1/x) \) Distribution

The expected statistical errors on the points of the \( \ln(1/x) \) distribution are small compared to the bin-to-bin variations of the distribution – there is no apparent scatter of the data points. This indicates that the data will most likely be of sufficient precision to allow a detailed comparison with the shape predicted by theoretical calculations with parameters fitted to LEP1 data. The comments regarding the influence of acceptance, initial state radiation and \( W^+W^- \) background corrections on the peak position also apply to the shape of the distribution – they are not expected to pose a major problem. If these systematic effects turn out to be troublesome, there is the still the prospect that a reasonable measurement of the ratio of \( \ln(1/x) \) distributions at LEP1 and LEP2 energies might be made in which many systematic effects may cancel.

It might be expected that the description of data by LLA predictions is more successful at higher energies as the LPHD assumption is more justified. This is supported in part by Monte Carlo studies that indicate that the differences between hadrons and partons are much reduced at LEP2 energies.
7.5 Two Particle Correlation

The two particle correlation at LEP2 energies has been studied in the same way as the single particle $\ln(1/x)$ distribution. If the correlation distribution is computed along lines in the $\ln(1/x_1) - \ln(1/x_2)$ plane as in reference [68] then the statistical error on each point would be of the order of 0.02 for the full 500 pb$^{-1}$ data ample. This should be compared to 0.005 achieved in reference [68] with about 21 pb$^{-1}$ of LEP1 data. Preliminary studies indicate that corrections for acceptance, resolution and initial state radiation will be small as anticipated for this distribution. As for the LEP1 analysis, it is also expected that other systematic effects such as the $W^+W^-$ background might also cancel when the normalized correlation distribution is calculated.

With the luminosity currently expected from LEP2 it is expected that any measurement of the two particle correlation would be statistics limited. There is however the hope that if the entire data sample is analysed, the possibility exists to test the energy evolution of the predicted correlation distribution in a meaningful way. In particular it can be tested whether the distribution at higher energies may be fitted by a prediction with the coefficients of the next-to-next-to-leading order terms fitted to LEP1 data. Such a prediction with coefficients fitted to LEP1 data is able to describe Pythia/Jetset events at both 91 and 175 GeV. Finally, the ratio of the two particle correlation at 91 and 175 GeV may be measured and compared to the theoretical prediction with the advantage that uncomputed higher order terms may cancel to some extent in the ratio.

7.6 Summary

In summary, measurements of hadron momentum spectra offer the possibility to make detailed tests of LLA QCD predictions, particularly in terms of their energy evolution. The peak position of the $\ln(1/x)$ distribution may be measured accurately with only a small amount of data allowing a powerful test of the extrapolation from lower energies. It should also be possible to determine the detailed shape of this distribution which will provide a stringent test of the energy evolution of predictions from LEP1 energies. Meaningful measurements of the two particle correlation will probably have to wait for the full 500 pb$^{-1}$ of luminosity to be delivered by LEP2.

References


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[62] See, for example, M. Schmelling, CERN/PPE 94-184, p. 30; subm. to Physica Scripta.

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Figure 16: Distributions of $\ln(1/x)$ for 500 pb$^{-1}$ of events passing selection cuts. Background from $W^+W^-$ events is shown as the shaded areas. Figures (c) and (d) show the the typical acceptance and initial state radiation corrections that might be expected.
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8 Exclusive channels
This chapter is devoted to QCD and, more generally, Strong Interaction studies in $\gamma\gamma$ collisions. For our purposes, LEP2 is a continuous energy $\gamma\gamma$ collider with a reach of up to 100 GeV center of mass energy for some observables. At low energy, the main studies concern resonance production and quasi two-body processes which probe the meson and baryon wavefunctions. At high energy, the partonic structure of the photon plays a dominant role and, as for hadronic processes, several tests of perturbative QCD, using many different observables, are possible. A specific feature of $\gamma\gamma$ collisions is the variability of the mass of the incoming photons which can be used to tune the non-perturbative component of the photon.

1 Introduction

While LEP1 was dedicated to the study of $Z^0$ production and decays the dominant process at LEP2 will be $e^+e^-\to e^+e^-X$ where the system $X$ is produced in the scattering of two quasi-real photons by $\gamma\gamma \to X$. It is well known that$^1$ this cross section grows like $(\ln s/m^2_{\text{electron}})^2$, where $s$ is the invariant energy squared of the incoming $e^+e^-$ pair, whereas the annihilation cross section decreases like $s^{-1}$. Thus LEP2 can be considered as the highest luminosity as well as the highest energy $\gamma\gamma$ collider presently available. When one of the outgoing electrons is tagged it is possible to probe the photon "target" at short distance in deep-inelastic experiments. In fact, concerning the study of the hadronic structure of the photon (i.e. its quark and gluon content) LEP2 is the analogue of both an ep collider and pp collider for the study of the proton structure. As in the purely hadronic case the main processes of interest in this respect will be, besides deep-inelastic scattering, large $p_T$ phenomena and heavy flavor production. The high luminosity (500 pb$^{-1}$) and the high energy (in the following we use $\sqrt{s} = 175$ GeV) available will make it possible to undertake precision phenomenology and obtain quantitative tests of perturbative QCD. Furthermore, combining LEP2 data with the lower energy TRISTAN and LEP1 data and with the high luminosity, high energy HERA results on photoproduction, a truly quantitative picture of the hadronic structure of the photon should emerge over a wide kinematical domain. Let us recall that a precise knowledge of the photon structure is required if reliable estimates are to be made of the background to new physics expected at LEP2.

Considering semi-inclusive or exclusive processes at high energy and relatively high momentum transfer it should be possible to probe diffractive phenomena and shed some light on the nature of the perturbative Pomeron (the so-called BFKL Pomeron) and the elusive Odderon (with vacuum quantum numbers but negative C-parity). These topics have undergone very interesting developments recently in connection with HERA results.

Finally, the traditional domain of $\gamma\gamma$ physics has been the formation of resonances and the study of two-body reactions of the type $\gamma\gamma \to$ meson-meson or $\gamma\gamma \to$ baryon-baryon. In the first case resonances in the C=+1 state, not directly accessible in $e^+e^-$ annihilation, are easily produced in a clean environment: heavy resonances like $\eta_c$ and $\chi_c$'s are produced more abundantly than in previous $e^+e^-$ colliders. In the second case, the present situation is often

$^1$taking into account the finite angular acceptance of any detector
unclear and the LEP2 results, at higher energy, will be helpful to distinguish between various models and hadron wave-functions.

One should point out interesting differences between LEP2, as a $\gamma\gamma$ collider, and a hadron-hadron collider. In particular, in the former case the initial energy is not fixed: this will turn out to be a major nuisance in the study of the deep-inelastic structure function of the photon but it could be an advantage in the study of the semi-inclusive channels (because it could help disentangle perturbative from non-perturbative effects). Furthermore, using the forward detectors of the LEP experiments one can vary the "mass" of the incoming virtual photons. This will be used to better constrain the non-perturbative component in the photon, which rapidly decreases with the photon virtuality, in the study of deep-inelastic, total cross section or large $p_T$ processes, for example. More generally, it will help understand the transition from a non-perturbative to a perturbative regime in QCD studies.

On the theoretical side, considerable progress has been recently achieved on the various topics mentioned above. Of particular interest for data analysis and the study of the event structure of $\gamma\gamma$ collisions is the existence of several general purpose Monte-Carlo codes (ARIDNE, JETSET, HERWIG, PHOJET, PYTHIA) which are described in the "$\gamma\gamma$ event generator" chapter. These generators are adapted from hadron-hadron and electron-positron studies and they have been (or are being) tuned to HERA data thus incorporating all the physics constraints necessary to reliably describe $\gamma\gamma$ reactions. The crucial test of confronting in detail the models with the LEP1 results on $\gamma\gamma$ physics is still in progress as both data and models are very recent and little discussion on this point will be given below. In any case, the situation is much improved compared to only a year ago, when essentially every experimental group had its own specific event generator, making the comparison between the various experimental results rather delicate. One interesting outcome of the recent studies is that the global features of $\gamma\gamma$ scattering are predicted to be rather similar to those of hadron-hadron scattering at the same energy.

For the anticipated quantitative studies in perturbative QCD one obviously needs theoretical predictions at (at least) the next-to-leading logarithmic order in perturbation theory. All relevant calculations for $\gamma\gamma$ processes have been performed or are being completed. Depending on the channel under study it will be seen that the sensitivity of the theoretical predictions under the various unphysical parameters (scales) is not perfect but, overall, the situation is not worse than in the purely hadronic channels.

The plan of the chapter is as follows. We first discuss, in some detail, the deep-inelastic scattering process on a photon target ($\gamma^*\gamma$ process) and its relevance for the determination of the parton distributions and the $\Lambda_{QCD}$ scale. We then turn to quasi-real $\gamma\gamma$ scattering and discuss the equivalent photon approximation, the (anti-)tagging conditions which define what we mean by $\gamma\gamma$ processes as well as the background to it. Global features of $\gamma\gamma$ events are described next. Large $p_T$ phenomena and heavy flavor production are then discussed in the context of next-to-leading QCD phenomenology. The chapter ends with the discussion of resonance production and exclusive processes.
Photon-photon physics has been the object of many review articles, see e.g. [1, 2]. A look at [3] shows to what extent the scope of γγ physics has extended since the previous LEP2 workshop. The topics discussed below are described from a different perspective, and with complementary details, in the “γγ Event Generators” chapter [4].

2 Structure functions

The measurements of the hadronic structure functions of the photon [5] at LEP1 and lower-energy \( e^+e^- \) colliders [6,7] can be extended in a number of important ways at LEP2. The higher beam energy will extend the kinematic reach both to lower Bjorken-x and to higher scales \( Q^2 \). Especially, the evolution of the real-photon structure function \( F_2^\gamma(x, Q^2) \) can be investigated experimentally via single-tag events up to \( Q^2 \leq 500 \text{ GeV}^2 \); and measurements can be done down to lower values of \( x \) than ever before, \( x \leq 10^{-3} \), entering the small-\( x \) region where the HERA experiments observe a strong rise of the proton structure function \( F_2^p \) [8]. The increased integrated luminosity will be equally important, in principle allowing for a so far unachieved statistical accuracy of \( F_2^\gamma \) data of a few per cent over a large part of the accessible region. See sec. 2.1 for a more detailed discussion of the kinematical coverage, and sec. 2.3 for a study of the sensitivity of \( F_2^\gamma \) at LEP2 to the QCD scale parameter \( \Lambda_{\text{QCD}} \).

Improved techniques for reducing systematic uncertainties will be needed to exploit fully these increasing statistics and kinematical coverage, and to approach the experimental precision achieved in lepton-hadron structure function studies, cf. sec. 2.2 and the report of the “γγ event generator” working group in these proceedings. In this context, some of the experiments are improving tracking and calorimetry in the forward region to obtain a more complete coverage for γγ events, but no major detector upgrades are planned. Masking the detectors against the increased synchrotron radiation expected at LEP2 will limit the coverage of structure function measurements at low \( Q^2 \) to values above about 3 GeV².

There is no prospect of measuring the longitudinal structure function \( F_L^\gamma \) at LEP2. However, in sec. 2.4 a new technique is presented which could allow for a measurement of related unintegrated structure functions via azimuthal correlations between the tagged electron and an outgoing inclusive hadron or jet. Moreover, a sufficient number of double-tag events is expected at LEP2 for a study of the transition from quasi-real \((P^2 \gg \Lambda_{\text{QCD}}^2)\) to highly-virtual \((P^2 \lesssim \Lambda_{\text{QCD}}^2)\) photon structure. Although these measurements will remain limited by statistics at LEP2, they can considerably improve upon the present experimental information obtained by PLUTO [9] as discussed in sec. 2.5.

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The kinematics of deep-inelastic lepton–photon scattering in $e^+e^-$ collisions is recalled in Fig. 1a. Shown is the ‘single-tag’ situation, where the electron or the positron is detected at $\theta_{\text{tag}} > \theta_0$, with a veto against a second tag anywhere in the detector. The bulk of the events useful for structure function studies are of this type; the generalization to ‘double-tag’ events is obvious. At LEP2, the limit of the main detector coverage will be about $\theta_0 \approx 30$ mrad, slightly higher than at LEP1 due to the synchrotron radiation shielding already mentioned in the introduction. For double-tag events, the very forward calorimeters which are used for online high-rate luminosity monitoring will be employed (see sec. 2.5).

Figure 1: (a) The kinematics of a single-tag inclusive $\gamma\gamma$ event. (b) The expected number of events for the determination of $F_2^\gamma$ (including the charm contribution) at LEP2 (see text for details). The standard antitag Weizsäcker–Williams photon spectrum [10] has been used with $\theta < 30$ mrad. The LO GRV parametrization of the photon structure [11] has been employed to estimate $F_2^\gamma$.

The cross section for (unpolarized) inclusive lepton–photon scattering reads to lowest order in the electromagnetic coupling $\alpha$:

$$\frac{d\sigma(e\gamma eX)}{dE_{\text{tag}} d\cos \theta_{\text{tag}}} = \frac{4\pi\alpha^2 E_{\text{tag}}}{Q^4 y} \left[ 1 + (1 - y)^2 F_2^\gamma(x, Q^2) + y^2 F_L^\gamma(x, Q^2) \right].$$

(1)

Here $F_{2,L}^\gamma(x, Q^2)$ denote the structure functions of the real photon. The virtuality of the probing photon and the invariant mass of the (hadronic) final state are given by

$$Q^2 = q^2 = 2E_{\text{beam}}E_{\text{tag}}(1 - \cos \theta_{\text{tag}}), \quad W_{\text{had}}^2 = (q + p)^2,$$

(2)

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and we have introduced the usual dimensionless variables

\[ x = \frac{Q^2}{Q^2 + W^2_{\text{had}}}, \quad y = 1 - \frac{E_{\text{tag}}}{E_{\text{beam}}} \cos^2 \frac{\theta_{\text{tag}}}{2}. \] (3)

Since usually \( y^2 \) is rather small due to background suppression cuts, typically at least \( E_{\text{tag}} > 0.5 E_{\text{beam}} \), only \( F_2 \) has been accessible experimentally so far. Under these circumstances \( Q^2 \) is limited to the region \( Q^2 > 3 \text{ GeV}^2 \) for \( \theta_{\text{tag}} = 30 \text{ mrad} \) at LEP2 energy, which in turn limits the reach towards small \( x \).

It was demonstrated already at the 1986 LEP2 workshop [3] that the longitudinal structure function \( F_L \) would be very difficult to measure also at LEP2. This statement remains valid. Even in the most favoured kinematic region with \( y > 0.5 \), the correction from \( F_L \) to the main part of the signal due to \( F_2 \) is only about 14\%. Achieving this marginal sensitivity in practice would require a costly (some 0.5 MChF per experiment) dedicated detector effort. The point is that events with \( y > 0.5 \) must have a low energy for the tagged electron. But experience at LEP1 has shown that there are significant numbers of off-momentum electrons that give spurious tags. To eliminate this background under present detector conditions, the so far published analyses for \( F_2 \) have required something like \( E_{\text{tag}} > 0.7 E_{\text{beam}} \) [6]. The off-momentum electrons come from beam–gas bremsstrahlung in the straight sections, and there is no reason to expect that their rate will be significantly reduced at LEP2.

The cross section (1) has to be convoluted with the Weizsäcker–Williams (WW) spectrum [10] for the photon, of virtuality \( P^2 \) (there is a high-\( P^2 \) tail which has to be corrected for in determinations of \( F_2^\gamma \)), emitted by the antitagged electron. For the explicit form of the WW spectrum see sec. 3. This fact leads to a key systematic problem in the determination of the photon structure functions: since \( p \) is unknown, \( W_{\text{had}} \) in eq. (2) and hence \( x \) in eq. (3) cannot be determined from the outgoing electron alone, in contrast to the situation in (electromagnetic) lepton–nucleon deep-inelastic scattering. This brings the hadronic final state into the game, of which only a part \( W_{\text{vis}} \) of the invariant mass is seen in the tracking regions of the detectors. The reconstruction \( W_{\text{vis}} \quad W_{\text{had}} \quad W_{\text{true}} \) requires a reliable modeling of the final state fragmentation. More on this issue can be found in sec. 2.2 and in the \( \gamma \gamma \) event generator report.

Estimated event numbers for the measurement of \( F_2^\gamma(x, Q^2) \), including the \( \gamma^*\gamma \quad c\bar{c} \) and \( \gamma^*g \quad c\bar{c} \) Bethe–Heitler charm contributions, are given in Fig. 1b in bins in \( x \) and \( Q^2 \). Only simple cuts have been applied here: on \( E_{\text{tag}}, \theta_{\text{tag}} \) and \( W_{\text{true}} \). The nominal LEP2 integrated luminosity has been used\(^2\). Some \((0.5 \ldots 1) \times 10^6 \) events, all in the deep–inelastic regime \( Q^2 > 3 \text{ GeV}^2 \), can be expected, a dramatic increase over the rates that have been available for \( F_2^\gamma \) determinations so far [6, 7].

If we put aside the \( W_{\text{vis}} \) problem for a moment, and assume that a systematic error between 5\% and 8\% (depending on the statistical accuracy of the bin under consideration) can be

\(^2\)The reduction in the event number due to further experimental final-state cuts (\( W_{\text{vis}} \) instead of \( W_{\text{true}} \), number of tracks etc.) will be approximately compensated by the presence of more than one experiment doing the measurements.
achieved, the potential of LEP2 on $F_2^\gamma$ is illustrated in Fig. 2. Note that in regions where a high-precision measurement is statistically possible at LEP2, $Q^2 \lesssim 100 \text{ GeV}^2$, such results will dominate our knowledge in the foreseeable future. A linear collider with 500 GeV center of mass energy is most likely to have no access to this region, since $\theta_{\text{tag}}$ would be too small to be accessible there.

![Figure 2: The kinematic coverage and maximal accuracy of the measurement of $F_2^\gamma$ at LEP2, using the event numbers of the previous figure for the statistical errors. The assumed systematic uncertainty of at least 5% has been added quadratically for the error bars shown. The central values have been estimated using the GRV(LO) parametrization [11].](image)

2.2 Determining $F_2^\gamma$ from the experimental information

Unfolding the photon structure function at small $x$

The rise in the proton structure function $F_2^p$ at small $x$ [8] is one of the most important results reported from HERA so far. It has been suggested that this rise is a signal for so-called BFKL [12] evolution. However, the observed rise has been obtained from “conventional” Altarelli-Parisi renormalization group $Q^2$-evolution, by starting from a sufficiently low scale $Q_0^2 < 1 \text{ GeV}^2$ [13], see also [14]. The small-$x$ coverage of $F_2^\gamma$ at LEP2 cannot compete with that of $F_2^p$ at HERA, hence the LEP2 measurement cannot be expected to shed any new light on the origin of this rise. But an important check of the universality of the rise is possible, which will also provide valuable constraints on the parton densities of the photon at small-$x$, where they are dominated by their hadronic (vector meson dominance, VMD) component.
So far all measurements of $F_2^\gamma$ have been at $x$-values above 0.01, which is outside the region where a rise would be expected. At LEP2, we expect to observe a significant number of events around and even below $x = 10^{-3}$ (see Fig. 1b), where a rise corresponding to that at HERA certainly should be visible. Measuring $F_2^\gamma$ at such small $x$ values is, however, far from trivial. While the value of $Q^2$ can be accurately determined to within a few per cent, since $\theta_{\text{tag}}$ and $E_{\text{tag}}$ are well measured, $x$ is not directly measurable because the energy of the target photon is unknown. The conventional procedure has been to measure the visible hadronic mass $W_{\text{vis}}$ in each event and calculate an $x_{\text{vis}}$ using eq. $1b/3$, where a rise corresponding to that at HERA certainly should be visible. Measuring $F_2^\gamma$ at such small $x$ values is, however, far from trivial.

While the value of $Q^2$ can be accurately determined to within a few per cent, since $\theta_{\text{tag}}$ and $E_{\text{tag}}$ are well measured, $x$ is not directly measurable because the energy of the target photon is unknown. The conventional procedure has been to measure the visible hadronic mass $W_{\text{vis}}$ in each event and calculate an $x_{\text{vis}}$ using eq. (3). Then, an unfolding is made to get the true $x$ distribution. This unfolding requires that the relationship between $W_{\text{vis}}$ and $W_{\text{true}}$ is well described by the event generator used.

During this workshop much work has been devoted to get a better understanding of how this unfolding behaves at small $x$. These results are described in more detail in the report from the “$\gamma\gamma$ event generator” working group in these proceedings. In that report methods to overcome these problems are also presented, using additional kinematic variables of the final state and limited information from the end-caps and the luminosity taggers, where much of the “missing” hadronic energy goes. The conclusion is that there is good hope that we will be able to measure at sufficiently small $x$ to detect a rise in $F_2^\gamma$.

### QED radiative corrections

In analogy to measuring $F_2^p$ at HERA, an accurate measurement of $F_2^\gamma$ must involve a careful treatment of radiative QED corrections to the basic one-photon exchange process described by eq. (1). Experiments have so far estimated the size of radiative corrections by comparing a Monte Carlo event generator for $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$ [15] (with appropriate changes of the muon mass and charge to conform with $q\bar{q}$ production) to one for $e^+e^-\rightarrow e^+e^-q\bar{q}$ via $e\gamma e\gamma$, with the photon energy distribution given by the Weizsäcker-Williams spectrum.

A more careful treatment would take into account the hadronic structure of the photon and effects of QCD evolution. Such a treatment is given in [16], in leading logarithmic approximation, which is known to be accurate to within a few percent for the proton case. It was found that the size of the corrections may vary from as much as 50% if one uses only so-called leptonic kinematic variables, to only a few per cent using only so-called hadronic variables. As the actual measurement will involve a mixture of such variables, a more extended study of the size of radiative corrections is needed.

#### 2.3 The $Q^2$ evolution of $F_2^\gamma$ and the QCD scale parameter $\Lambda_{QCD}$

At next-to-leading order (NLO) of the QCD improved parton model, the structure function $F_2^\gamma(x, Q^2)$ is related to the photon’s parton distributions [5] via

$$\frac{1}{x}F_2^\gamma = 2e_q^2 q^\gamma + \frac{\alpha_S}{2\pi} (C_q q^\gamma + C_G G^\gamma) + \frac{\alpha}{2\pi} e_q^2 c_q \gamma + \frac{1}{x}F_{2h} + 1/Q^2,$$  

(4)
where denotes the Mellin convolution. The summation extends over the light \( u, d \) and \( s \) quarks. The heavy flavour contribution \( F_{2,h}^\gamma \) has recently been calculated to second order in \( \alpha_S \) [17] and is discussed in sec. 7. \( C_{q,G} \) are the usual (scheme-dependent) hadronic NLO coefficient functions, and for the commonly used \( \overline{\text{MS}} \) factorization there is a 'direct' term \( C_q = 6 C_G \). Besides the leading-twist contribution written out in (4), at large \( x \) (close to and within the resonance region) power-law corrections \( \mu^2/Q^2(1-x) \) become important, with \( \mu \) being some hadronic scale. The \( Q^2 \)-evolution of the quark and gluon densities \( q^\gamma, G^\gamma(x, Q^2) \) is governed by generalized (inhomogeneous) Altarelli-Parisi evolution equations. For the singlet case the solution can be decomposed as

\[
q^\gamma = 2 \frac{q^\gamma}{G^\gamma} = q^\gamma_{\text{PL}} + q^\gamma_{\text{had}},
\]

where the well-known homogeneous ('hadronic') solution \( q^\gamma_{\text{had}} \) contains the perturbatively uncalculable boundary conditions \( q^\gamma(Q_0^2) \). The photon-specific inhomogeneous ('pointlike', PL) part is given by

\[
q^\gamma_{\text{PL}} = \frac{1}{\alpha_S} + \hat{U} \quad 1 \quad \frac{1}{1 + \hat{d}} \quad \frac{1}{\alpha_S/\alpha_S(Q_0^2)} + 1 \quad \frac{[\alpha_S/\alpha_S(Q_0^2)]_d}{\frac{1}{\hat{d}} \quad \hat{b} + \alpha_S}. \tag{6}
\]

Here \( \hat{a}, \hat{b}, \hat{d} \) and \( \hat{U} \) are combinations of the LO and NLO splitting-function matrices.

At asymptotically large \( Q^2 \) and large \( x \), eq. (6) reduces to the well-known asymptotic solution \( 1/\alpha_S \), suggesting a parameter-free extraction of \( \Lambda_{\text{QCD}} \) from the photon structure. At energies accessible at present and in the foreseeable future, however, the non-asymptotic contributions cannot be neglected even at large \( x \), and \( \Lambda_{\text{QCD}} \) determinations involve a model or a simultaneous free fit of the non-perturbative boundary conditions \( q^\gamma, G^\gamma(Q_0^2) \). In eq. (6), \( Q_0^2 \) is an arbitrary reference scale; hence \( q^\gamma_{\text{had}} \) in eq. (5) will in general contain not only the non-perturbative (coherent) hadronic part, but also contributions originating in the pointlike photon-quark coupling. However, final state information suggests that there is some low scale \( Q_0^2 \) close to the border of the perturbative regime, where (in NLO in some suitable factorization scheme, see [18–20]) the parton structure of the photon is purely hadronic and given by the fluctuations to virtual vector mesons (VMD) [7,21,22].

In order to estimate the possible sensitivity of \( F_2^\gamma \) to \( \Lambda_{\text{QCD}} \) at LEP2, deep-inelastic electron-photon collisions corresponding to an integrated luminosity of 500 pb\(^{-1} \) have been generated using the SaS1D distribution functions [23], passed through a (fast) detector simulation (DELPHI) and unfolded using the Blobel program [24]. Possible systematic errors due to the dependence on fragmentation parameters have been neglected. We have chosen six bins in \( Q^2 \) (logarithmically distributed) and let the unfolding program choose the number and sizes of \( x \) bins. In total 21 bins have been obtained at \( x > 0.1 \), shown in Fig. 3. Alternatively, we have used the theoretical error estimates and bins of sec. 2.1 as representative for the best possible measurement using the combined statistics of two experiments.

Next, fictitious \( F_2^\gamma(x, Q^2) \) data have been generated at these \( (x, Q^2) \) points. The input distributions of a simple toy-model have been evolved in NLO, which however yields very
similar numbers of events as the SaS [23] distributions. Thus the relative errors can be taken over from SaS. Specifically, the NLO parton distributions of the photon have been generated by a (coherent) sum of the three vector mesons $\rho^0$, $\omega$, and $\phi$ at $Q_S = 0.6$ GeV in the DIS$\gamma$ scheme [18].

![Graph showing $F_2^\gamma/\alpha$ as a function of $x$.]

**Figure 3:** The estimated accuracy of $F_2^\gamma$ from one experiment at LEP2 using a (linear) Blobel unfolding [24]. The (in-)sensitivity to $\Lambda_{QCD}$ is illustrated by the $1\sigma$ results described in the text. $\Lambda$ denotes $\Lambda_{MS}^{(4)}$ in MeV.

Finally, $\Lambda_{QCD}$ is fitted together with the shape parameters $N_v$, $a$, and $b$, of the vector-meson valence, sea and gluon input distributions

$$x_v(x) = \kappa N_v x^{\omega_v}(1 - x)^{b_v}, \quad x_S(x) = \kappa N_s x^{\omega_s}(1 - x)^{b_s}, \quad x_G(x) = \kappa N_g x^{\omega_g}(1 - x)^{b_g}$$

(7)

at $Q_{ref} = 2$ GeV ($N_v$ and $N_g$ are fixed by the charge and momentum sum rules of the vector mesons), the overall normalization $\kappa$, the scale $Q_S$, and the charm quark mass entering via $F_2^\gamma$ in eq. (4). The variation of the parameters is restricted to values reasonable for vector meson states, e.g., $0.8 < b_v < 1.3$ (the “data” were generated with the counting-rule value of 1.0).

Using the Blobel-unfolded “results” of Fig. 3, one finds an experimental $1\sigma$ accuracy of

$$\Lambda_{MS}^{(4)} = 200^{+190}_{-150} \text{ MeV} \quad \alpha_s(M_Z) = 0.109^{+0.010}_{-0.011}.$$  \hspace{1cm} (8)

The sensitivity is dominated by the large-$x$ region [25] as obvious from Fig. 3. If we use the “data” of sec. 2.1 instead, which need about the statistics of two experiments and some progress in the unfolding, especially at large $x$, we obtain

$$\Lambda_{MS}^{(4)} = 200^{+85}_{-65} \text{ MeV} \quad \alpha_s(M_Z) = 0.109 \quad 0.006.$$  \hspace{1cm} (9)
Hence, even under the most optimistic assumptions, the experimental error on $\Lambda_{QCD}$ as determined from $F_2^{\gamma}$ at LEP2 is a factor of two bigger than from fixed-target $ep/\mu p$ DIS. The theoretical scale-variation uncertainty of $F_2^{\gamma}$ has been studied in [17] for fixed parton distributions. The resulting theoretical error on $\Lambda_{QCD}$ is expected to be of similar size as in $e p$ DIS. It should be mentioned that without the VMD-based restrictions on the parameter ranges the determined from $F_2^{\gamma}$ is only structure function that contains additional information. If, instead of just measuring the total cross-section $F_T$, one triggers on a final-state ‘particle’ $a$ (either a hadron or a jet), more structure functions become accessible. The cross section as a function of the direction of $a$ can be written in terms of the unintegrated structure functions $F_P$, $P = T, L, A, \text{and } B$, as

$$\frac{d\sigma(e\gamma \to eX)}{dx \, dy \, d\Omega_a / 4\pi} = \frac{2\pi \alpha^2}{Q^2} \left[ 1 + (1 + \frac{y}{x})^2 \right] 2x F_T + \epsilon(y) F_L \rho(y) F_A \cos \phi_a + \frac{1}{2} \epsilon(y) F_B \cos 2\phi_a .$$

(10)

Here $\Omega_a$ represents the direction of $a$ in the $\gamma\gamma^*$ rest-frame, and $\phi_a$ is its azimuth around the $\gamma\gamma^*$ axis, relative to the electron plane. The functions $\epsilon(y)$ and $\rho(y)$ are both $1 + \frac{y^2}{2}$, and can be approximated by 1 throughout the accessible region of phase space. The standard structure functions $F_2$ and $F_L$ are related to the corresponding $F_P$ by integration over $\Omega_a$.

For leptonic final states $F_P$ are uniquely given by perturbation theory, while for hadronic final states these quantities involve a convolution over the parton densities of the photon:

$$F_P(x, z) = \int_{y, q, g} \frac{1}{x_p} F_{i/\gamma} \frac{x}{x_p} F_P^i(x_p, z) ,$$

(11)

where $f_{\gamma/\gamma}(x) = \delta(1 - x)$ and $z = (p_a - p_i)/(q - p_i) = \frac{1}{2}(1 + \beta \cos \theta)$, with $\beta$ and $\theta$ denoting the velocity and direction of $a$ in the $i\gamma^*$ rest-frame, respectively. One can find many incorrect formulae for $F_P^i$ in the literature. We have performed an independent calculation and confirm the leading-order results given in [26], obtaining

$$F_T^\gamma(x_p, z) = e q^4 4 \alpha \frac{\alpha}{2\pi} (x_p^2 + (1 - x_p)^2) \frac{z^2 + (1 - z)^2}{2z(1 - z)}$$

$$F_L^\gamma(x_p, z) = F_T^\gamma(x_p, z)$$

$$F_B^\gamma(x_p, z) = F_L^\gamma(x_p, z)$$

$$F_A^\gamma(x_p, z) = e q^4 4 \alpha \frac{\alpha}{2\pi} x_p (1 - x_p)(1 - 2z) \frac{x_p(1 - x_p)}{4z(1 - z)}$$
\[ F^q_B(x_p, z) = \frac{T_R^a}{e^2} F^\gamma_p, \]
\[ F^q_T(x_p, z) = e^2 C_F a_s \frac{z^2}{4\pi} \left( \frac{x_p^2 + z^2}{z} + 2(x_p z + 1) \right), \]
\[ F^q_A(x_p, z) = e^2 C_F a_s \frac{z^2}{\pi} \left( x_p (x_p z + 1) \right) \frac{4(1 - z)}{4(1 - x_p)(1 - z)}, \]

up to terms of order \( m^2_q x/(1 - z)Q^2 \). In the quark case, the azimuth is that of the outgoing quark – the equivalent expressions for the outgoing gluon are identical but with \( z \) replaced by 1 and \( F^q_A \) negated. The photon and gluon cases are identical for either outgoing parton.

Note that \( F^q_B = F^q_L \) for all parton types, so a measurement of \( \cos 2\phi_a \) gives the same information about the parton content of the photon as \( F^\gamma_L \), despite the fact that they arise from different spin states of the virtual photon (purely longitudinal for \( F_L \) and transverse-transverse interference for \( F_B \)). This is a consequence of the fact that the struck parton is a fermion. In the leptonic case, the two outgoing particles can be distinguished, and the above distributions directly measured. In the hadronic case, however, quark, antiquark and gluon jets cannot be distinguished, and one must sum over all assignments. Since each event consists of two jets with \( z \) and 1 \( z \), all three sub-processes give equal and opposite \( \cos \phi_a \)-dependence for the two jets, and \( \cos \phi_a \) defined naively is identically zero. But if we instead use only the more forward of the two jets (i.e., the one with larger \( z \), which is more often the quark in the \( q \) \( qg \) case), the constant and \( \cos 2\phi_a \) terms remain unchanged, but also the \( \cos \phi_a \) term becomes nontrivial, being always negative for quarks and taking either sign for the other two processes, depending on \( x_p \).

The measurement of azimuthal correlations involves reconstructing the hadronic final state to a much greater degree than does the measurement of the total cross-section. For this reason, a number of additional problems occur, including: How well do jet (or inclusive-particle) momenta mirror the underlying parton momenta? How well can the jets be reconstructed experimentally? How much are the azimuthal correlations smeared by the fact that the \( \gamma^* \) rest-frame is not exactly known, or by target photon mass effects? How much artificial (de)correlation is induced by the fact that any cuts made in the lab frame are azimuth-dependent in the \( \gamma^* \) rest-frame? A detailed detector-level study would be needed to answer these points, as discussed for CELLO in [27], but this has not yet been done for LEP2 energies. However, azimuthal correlations have been measured on a proton target at HERA [28], and in the leptonic final-state on a photon target at PETRA [29] and LEP1 [30], and all of the above problems have been addressed, and overcome, in one or other of these analyses. It thus seems hopeful that the measurement can be done at LEP2.

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4In fact this has only been proved at lowest order. Different definitions of \( F_B \) (e.g., single-particle vs. jet inclusive) will have different higher-order corrections, and it is currently unknown whether any obey the same relationship at higher orders.
Figure 4: Distribution of azimuthal angle of the more forward jet in two-jet events. Canonical cuts are made, plus a 2 GeV jet-$p_t$ cut. In (a) parton-level results are shown, (b) depicts generator-level results (data points with statistical errors corresponding to 500 pb$^{-1}$), in comparison with the total parton-level prediction of (a).

In the absence of a detailed experimental investigation, the group has performed a brief generator-level study using HERWIG, version 5.8d [31]. The jet reconstruction and cuts are loosely based on those used by the H1 collaboration [28], adapted for our assumed LEP2 detector coverage. The difficulties in measuring $x$ discussed in the context of $F_2$ are left aside—it is assumed that $x$ and $Q^2$ are perfectly known. Events passing the canonical cuts are selected ($E_{tag} > 0.5 E_{beam}$, $\theta_{tag} > 30$ mrad and $W_{vis} > 2$ GeV within $\cos \theta < 0.97$). All particles within the central region are boosted to the Breit frame of the virtual photon and target electron beam$^5$, and jets are reconstructed using the $k_t$ jet algorithm [32], with a cutoff of 2 GeV. No attempt was made to optimize this value. As discussed in [33], the maximum correlations are achieved for $p_t = Q/2$.

In Fig. 4a we show parton-level results in three $x$ and $Q^2$ bins, broken down into the contributions from the different parton types $\gamma$, $q$ and $g$. By ‘parton–level’ we mean at leading order, with a direct correspondence made between partons and jets, and with the photon assumed collinear with the incoming electron beam. The SaS1D parametrization of $f_{i/\gamma}$ [23] has been used. We see that the $\cos 2\phi_\gamma$-dependence arises predominantly from the gluon- and photon-induced sub-processes, and that a different initial-state parton dominates within each bin. In Fig. 4b generator-level results are presented, including parton showering, hadronization, jet reconstruction and target–mass smearing. The error bars shown correspond to the statistical errors one could expect from 500pb$^{-1}$ of data. While the correlations are somewhat smeared, a signal still persists. It is possible that one could improve the statistics by including lower

---

$^5$This frame has the same transverse boost as their rest-frame but a different longitudinal boost, which results in improved jet properties if a frame-dependent jet algorithm is used [32].
provided a hadronic plane could be cleanly defined. Hence it seems likely that the azimuth-dependent structure functions of the photon can be measured at LEP2, providing additional constraints on the parton content of the photon.

2.5 Virtual photon structure

Effects of a non-zero virtuality of the target photon, $P^2 = 0$, have attracted considerable interest recently [23, 34–36]. The non-perturbative hadronic (VMD) contribution to the photon structure is expected to go away with increasing $P^2$, allowing for a purely perturbative prediction for $F_2^\gamma(x, Q^2; P^2)$ at sufficiently high $P^2$ [37]. The fall-off of the non-perturbative part with increasing $P^2$ is theoretically uncertain and model dependent [23, 36], hence experimental clarification is required. An improved understanding of this transition is also relevant for the experimental extraction of the real-photon structure functions, since corrections of the order of 10% have to be made to take into account the finite range of $P^2$ in single-tag events. However, the present experimental knowledge is very poor: the older\footnote{Note that actually $F_{\gamma \text{eff}} = F_2 + 3F_L/2$ was measured by PLUTO.} measurement of $F_2^\gamma(x, Q^2; P^2)$ by the PLUTO collaboration [9] suffers from low statistics and a rather limited kinematical coverage: $Q^2 = 5$ GeV$^2$, $P^2 = 0.8$ GeV$^2$, and $x > 0.1$. Some new data are however being collected at HERA [38].

Especially because of its higher energy, but also due to its increased integrated luminosity, LEP2 can provide much improved information from double-tagged events. The rate of such events in the main forward luminometers ($\theta > 30$ mrad for the electron and the positron) is very small. The more important double-tagged results at LEP2 are expected to come from events with first tags in the main forward luminometers ($\theta_1 > 30$ mrad) and second tags in the very forward calorimeters which are used for online high-rate luminosity monitoring in each of the four LEP experiments. These small detectors (currently being upgraded in ALEPH and L3) are situated at 7 – 8 meters from the interaction point, beyond the minibeta quadrupole magnets. The defocusing effect of the quadrupoles distorts the acceptance of the detectors, but lepton tags can be reconstructed in the range $5 \lesssim \theta_2 \lesssim 15$ mrad, yielding $0.1 \lesssim P^2 \lesssim 1.0$ GeV$^2$ (the exact coverage varies between experiments).

For 500 pb$^{-1}$ of data collected at LEP2, it is expected that about 800 double-tagged events of this type will be seen within the range $3 \times 10^{-4} < x < 1$ and $3 < Q^2 < 1000$ GeV$^2$. The invariant mass of each event can be reconstructed from the two tagged leptons and hence the correlation between the measured value ($W_{\text{vis}}$) and the true value is good – there is no loss of correlation at high $W$ (unlike for single-tagged events, see sec. 2.2), and the structure function can be measured more easily down to low $x$.

Fig. 5 shows the virtual photon structure function $F_2^\gamma(x, Q^2; P^2)$ as predicted by the SaS1D, SaS2D [23] and GRS [36] models in two bins for $P^2$ and $Q^2$. The error bars indicate the statistical error expected on each point for a bin width leading to two points per decade in $x$, using the SaS1D parton distributions [23]. The SaS2D [23] and GRS [36] distributions, both
Figure 5: Expectations for the statistical accuracy of the virtual photon structure measurement at LEP2 in two different $P^2$ and $Q^2$ bins, using the SaS1D [23] distributions. The SaS1D prediction for the real photon and the $P^2 = 0$ results for the GRS [36] and SaS2D [23] distributions are shown as lines for comparison. The upper (lower) curves for GRS and SaS2D refer to $P^2 = 0.2\ (0.5)\ GeV^2$, respectively.

showing a rather different small-$x$ behaviour as compared to SaS1D, lead to similar results for the expected statistical errors. A measurement of $F^\gamma_2(x,Q^2;P^2)$, as distinct from the real ($P^2 = 0$) photon structure function (shown as solid curves for SaS1D), should be possible at LEP2 and could be compared to the results from PLUTO [9] in the region of overlap, as well as to different model predictions [23,36]. Additional information will be obtained in single tag events where high $p_T$ jets are produced (see sec. 6).

2.6 Summary

Due to its high beam energy and increased integrated luminosity, LEP2 will be a unique place for studying the hadronic structure functions of the photon. Some (0.5\ldots1) $10^9$ single-tag events for deep-inelastic ($Q^2 > 3\ GeV^2$) electron-photon scattering can be expected, for the first time allowing for a determination of $F^\gamma_2$ (and hence of the photon's quark content) with statistically high precision up to scales $Q^2$ of a few 100 GeV$^2$ and down to Bjorken-$x$ values as low as about $10^{-3}$. Depending on how well systematic uncertainties can be controlled, a determination of the QCD scale $\Lambda_{\text{QCD}}$ may become possible from $F^\gamma_2$. But even under optimistic assumptions, the experimental error on $\alpha_s(M_Z)$ will be about a factor of two bigger than that one obtained from electron-nucleon deep-inelastic scattering. It seems likely that supplementary information on
the parton densities can be obtained by measuring azimuthal correlations between the tagged electron and a final state hadron or jet. About 10^3 double-tag events are expected to be seen with 0.1 \lesssim P^2 \lesssim 1.0 \text{ GeV}^2 and Q^2 as above, allowing for a study of the virtual photon structure function over a much wider kinematical range than so far. It should be noted that the data taken at LEP2 on photon structure functions will dominate our knowledge in most of the accessible range discussed above for the foreseeable future, since a 500 GeV linear collider will most likely have no access to Q^2 < 100 \text{ GeV}^2.

### 3 The equivalent photon approximation

The cross section for a \( \gamma \gamma \) process is related to the cross section at the e^+e^- level, which is measured in the laboratory, by the formula

\[
\frac{d\sigma(e^+e^- \rightarrow e^+e^-X)}{dP_1^2 dP_2^2} = \sigma(\gamma \gamma) X \frac{d^2n_1}{dz_1 dP_1^2} \frac{d^2n_2}{dz_2 dP_2^2} \frac{dP_1^2 dP_2^2}{dP_1^2 dP_2^2}
\]

where \( z_i \) is the scaled photon energy in the laboratory frame and \( P_i^2 \) is the photon invariant mass. This is the equivalent photon approximation (EPA) [39] where the longitudinal polarization component as well as the mass of the incoming photons are neglected in \( \sigma(\gamma \gamma \rightarrow X) \). The \( P_i^2 \) integration can be carried out to give the photon “density” in the \( e^\pm \) (the photon flux) [10]

\[
f_{\gamma/e}(z, P_{\min}, P_{\max}) = \frac{P_{\max}^2}{P_{\min}^2} \frac{d^2n}{dz dP^2} = \frac{\alpha}{2\pi} \frac{1}{z} \frac{(1 + \frac{z}{z^2})^2 \ln \frac{P_{\max}^2}{P_{\min}^2}}{2m_e^2 z} \frac{1}{P_{\min}^2} \frac{1}{P_{\max}^2}.
\]

For untagged experiments \( P_{\min} \) is the kinematic limit

\[
P_{\min}^2 = \frac{m_e^2 z^2}{1 - z}
\]

and \( P_{\max} = E_{\text{beam}} \). The quality of the approximation is not guaranteed in this case as the EPA is derived under the hypothesis that \( P^2 \approx E^2_{\text{beam}} \) which is not always satisfied here. In most of the following we use antitagging conditions where the \( e^\pm \) are confined to small angles \( \theta < \theta_{\max} \) (typically \( \theta_{\max} = 30 \text{ mrad} \)) so that

\[
P_{\max}^2 = (1 + z) E_{\text{beam}}^2 \theta_{\max}^2.
\]

Using antitagging conditions rather than untagged conditions reduces somewhat the cross sections (about 30% in the case of heavy flavor production and a factor 2 for large \( p_T \) jet production) but improves the reliability of the theoretical calculations based on the EPA. Finally for tagged conditions both \( P_{\min}^2 \) and \( P_{\max}^2 \) are set by the detector configuration.

Even in the case of antitagging it is always worthwhile, whenever possible, to check the validity of the EPA for each of the considered process. For large \( p_T \) jets and heavy flavor production a possible check consists in comparing, with the appropriate choice of cuts and
kinematics, the lowest order matrix element calculation of $e^+e^- e^+e^-q\bar{q}$ to the approximate one. Good agreement is found provided the “constant” (non-logarithmic) term is kept in eq. (14). If the constant term is ignored the cross sections are over-estimated by roughly 10% when both $e^+$ and $e^-$ are antitagged. The same is true for minimum bias type physics. Special attention should be given to processes which involve the photon structure function, i.e., the so-called resolved processes. Under certain conditions it may be necessary to take into account the effect of the virtuality of the quasi-real photons initiating the process. Indeed as written in eq. (5) the structure function has two components

$$F_{i/\gamma}(x, Q^2; P^2) = F^{PL}_{i/\gamma}(x, Q^2; P^2) + F^{had}_{i/\gamma}(x, Q^2; P^2)$$

with a different dependence in $P^2$, the virtuality of the quasi-real photon. In particular $F^{had}_{i/\gamma}$ is roughly suppressed by the usual $VMD$ form factor $(m^2_{\rho}/(m^2_{\rho} + P^2))^2$. In the tagged case ($P^2 < 5 \text{ GeV}^2$) or in the case with relatively large antitagging angles this factor should be taken into account when carrying out the integration over the virtuality in eq. (14). The result is a relative reduction of the “had” component of the photon compared to the case when the photon is assumed to be real. This reduction obviously affects the rate of the observable cross section. If the aim is to obtain predictions at a 10% accuracy this effect should certainly be studied in further details.

Turning now to resonance production (see sec. 8), eq. (13) simplifies since one of the $z_i$ integration can be performed with the constraint $z_1 z_2 = \tau = M^2/s_{e^+e^-}$ where $M$ is the resonance mass. It is then customary to define luminosity functions (see e.g. [40])

$$\frac{d}{dM} = \frac{2\tau}{M} dz_1 dz_2 f_{ij/\gamma}(z_1) f_{ij/\gamma}(z_2) \delta(z_1 z_2 - \tau)$$

so that

$$d\sigma(e^+e^- e^+e^-X) = dM \frac{d}{dM} \sigma(\gamma\gamma X).$$

This luminosity curve makes it easy, in principle, to determine the counting rate for resonance production knowing the width of the resonance in the $\gamma\gamma$ channel. An important point however concerns the acceptance cuts of the detector which reduce the observed rates compared to the theoretical predictions. Such cuts are taken into account in sec. 8. Detailed studies of luminosity curves were done for the previous LEP2 and we do not repeat them here [3].

4 Tagging conditions, cuts and background to $\gamma\gamma$ processes

In this section we discuss what we call $\gamma\gamma$ events (as opposed to $\gamma^*\gamma$) i.e. events where the electron and positron of the diagram in Fig. 1 escape detection (i.e. essentially “go down the beam pipe”). This require a precise definition of the tagging conditions as well as the cuts necessary to reduce the background to $\gamma\gamma$ physics. Background sources come from all processes

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not initiated by $\gamma \gamma$ interactions but exhibiting similar features such that they can be taken, 
mistakingly, for genuine $\gamma \gamma$ events. Background events are, essentially, of two types (Fig. 6). 

![Type I processes](image1.png) ![Type II process](image2.png) 

**Figure 6: Two types of background processes for $\gamma \gamma$ physics**

Type I events have similar final states, although being produced by different processes: for example, the t-channel $\gamma Z$ exchange diagram or the initial state photon splitting in a $q\bar{q}$ pair. The final state is therefore two electrons and a $q\bar{q}$ pair as in the signal. Type II background processes can arise from detector acceptance and resolution when some particles are lost down the beam pipe or in detector cracks or are misidentified. In this category, one can mention s-channel initial state radiation: when the photon is lost in the beam pipe and the boosted $q\bar{q}$ pair detected, this event can be interpreted as a no tag or antitag $\gamma \gamma$ event. Four fermions channels where two of the fermions are lost in the beam pipe region are also part of this category of $\gamma \gamma$ no tag events. A series of dedicated cuts based on kinematic constraints must be set to reject most of the background (largest purity), although keeping most of the signal (largest efficiency).

The LEP2 total cross sections are orders of magnitude smaller than those found at the $Z^0$ peak. However, initial state radiations by emitting a high energy photon, can shift down the centre of mass energy to the $Z^0$ peak energy. In the following we concentrate on the study of this background as the Type I backgrounds can be argued away easily: the $\gamma Z$ exchange processes are suppressed due to the $Z$ propagator while the other process leads typically to a very small hadronic mass at large rapidity and leptons at large angle (which would be detected) or a large missing energy.

### 4.1 Tagging conditions and acceptances

Typical detector acceptances and thresholds have been selected in order to match an “average” LEP experiment. All momenta and angles are expressed in the laboratory frame. The events are selected by antitagging in the following way: both scattered electrons or photons have a polar angle $\theta \geq 30$ mrad or an energy $E \geq 1$ GeV. We define now the acceptance cuts on the final state particle system 

**cut 0:** no further constraint applied to the final state with the exception of the electron or photon tagging conditions.

**cut 1:** Only charged particles with $20^\circ \leq \theta \leq 160^\circ$, $p > 0.4$ GeV/c and neutral particles with $10^\circ \leq \theta \leq 170^\circ$, $E > 1$ GeV are accepted for the analysis. At least four charged particles have to survive the cuts mentioned above to accept the event.
The visible energy is calculated from the invariant mass of the four-momentum obtained by summing the four-momenta of all particles satisfying cut 1. For jet search, a cone algorithm with a cone radius $R = (\Delta \eta)^2 + (\Delta \phi)^2 = 1$ has been used. Only particles passing cut 1 enter the jet analysis.

### 4.2 Radiative return to the $Z^0$ cross-section

Background simulation have been performed at $\sqrt{s} = 175$ GeV using JETSET 7.4 [41]. We show in Fig. 7 the initial state radiated photon energy. The low energy peak reflects the $1/E_\gamma$ behaviour of the bremsstrahlung process, the high energy peak comes from the $\sigma_\gamma(\hat{s}) \sim 1/\hat{s}$ ($\hat{s} = (1 - x_\gamma) \hat{\gamma}$) singularity of the Born term as found in the hard radiative cross-section formula:

$$\frac{d\sigma}{dx_\gamma} = \frac{\alpha_{em}}{\pi} \left( \ln \frac{s}{m_e^2} \right)^1 1 + \left( \frac{x_\gamma}{x_\gamma^2} \right)^2 \sigma_0(\hat{s})$$

where $x_\gamma$ is the fraction of the beam energy carried by the real photon and $\hat{s}$ is the invariant mass squared of the virtual photon. The large peak, close to 64 GeV, is precisely due to the so-called return to the $Z^0$ for $E_\gamma = (s - M_Z^2)/2 \hat{s}$. The $\cos \theta$ distribution of the photon is shown in Fig. 8 where the forward and backward peaks reflect the cross section divergence for collinear photon production. If such a photon remains undetected, the boosted $q\bar{q}$ pair system may appear as an untagged $\gamma\gamma$ event. Even worse, the photon can be identified as an electron in the forward tagging detectors, this event would then be selected as a one tag $\gamma\gamma$ process.

The next three figures display clearly the differences between the signal and the radiative $Z^0$ production background. In these studies the charged hadrons angular acceptance has been
increased to $10^9$ with a realistic track reconstruction efficiency as expected in DELPHI. The two-photon events are generated with TWOGAM and are compared to PYTHIA Z\gamma and $W^+W^-$ all decays production contributions allowing initial state radiation. As expected the signal is characterised by a rapidly falling $W_{\text{vis}}$ distribution (due to the Weiszäcker-Williams convolution) while the background exhibits a clear peak slightly below the $Z^0$ mass and the $W^+W^-$ channel shows up at very large invariant mass. Similarly the $p_{\perp}^{\text{vis}}$, $p_{\parallel}^{\text{vis}}$ spectra from the signal are confined to low values while they have a long tail for the background (figs. 9, 10). The above

![Figure 9: Visible invariant mass of $\gamma\gamma$ (solid dots), $Z\gamma$ (open diamonds) and $W^+W^-$ (solid triangles).](image1)

![Figure 10: $p_{\perp}^{\text{vis}}$ distributions for the signal and the background. The symbols have the same meaning as in the previous figure.](image2)

features clearly dictate the following cuts to reject the background still retaining most of the genuine $\gamma\gamma$ events:

**cut 2**: the vector sum of the transverse momenta of all accepted particles satisfies $p_{\perp}^{\text{vis}} = 10 \text{ GeV}/c$.

**cut 3**: the vector sum of the longitudinal momenta of all accepted particles satisfies $p_{\parallel}^{\text{vis}} = 20 \text{ GeV}/c$.

**cut 4**: the invariant mass calculated from the four momenta of accepted particles satisfies $W_{\text{vis}} = 50 \text{ GeV}$.

Clearly these cuts will not affect the bulk of the $\gamma\gamma$ events (e.g. the total charm production cross section is hardly affected), however they will certainly reduce the rate of rare events in the signal characterized by a large invariant mass: this is the case of large $p_T$ jet production since one has in general $W = 2p_T$. This is illustrated in Fig. 12 where the effect of the cuts on jet searches is discussed. Histograms represent the contribution of the $Z^0$ return background. The upper line is for events satisfying cut 1 above, the next lower ones are for cut 1+2, cut 1+2+3 and cut 1+2+3+4 respectively. The solid lines are the results of an analytic calculation in the leading-logarithm approximation at the partonic level where the cuts are approximately
Figure 11: $\not{p}_{\text{vis}}$ distributions for the signal and the background.

Figure 12: Signal and background $p_T$ distributions for various cuts. The cross section is integrated over the jet rapidity range $\eta = 1$. See the text for explanations.

implemented: the top curve is for cut 1+2 (cut 2 is ineffective since $\not{p}_{\text{vis}} = 0$ by definition) and the lower two curves are as above. The $p_T$ threshold above which the background rate is larger than the signal rate goes from 13 GeV ($\not{p}_{\text{vis}}$ cut) up to 16 GeV ($\not{p}_T$ and $\not{p}_{\text{vis}}$). The final visible invariant mass cut has a strong effect on the background but reject also most of the signal over $p_T > 20\ 25$ GeV. A more detailed study of the effects of the cuts on the signal is discussed in the “Large-pt processes” section.

5 Soft and semihard physics, and event structure

Studies of minimum-bias physics and semihard interactions in two-photon events offer a good opportunity to investigate the high-energy behaviour of scattering amplitudes and the transition from perturbative to non-perturbative QCD.

5.1 Cross section predictions and general characteristics

The photon, in its high-energy interactions with hadrons, behaves very much like a hadron, however with cross sections reduced strongly relative to pure hadronic cross sections. Simi-

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larly to a hadron, the photon both undergoes soft hadronic interactions and has resolved hard interaction between its hadronic constituents and the hadronic constituents of the target. Additionally, the photon has a direct pointlike interaction with the hadronic constituents of the target.

Even at high energies, many features of hadronic interactions of photons are dominated by soft multiparticle production. Correspondingly, distributions measured in photoproduction are similar to those obtained in purely hadronic interactions (provided, of course, these are taken at the same center-of-mass energy). This is nicely illustrated in Fig. 13 for the central transverse energy density and the one-particle inclusive $p_T$ spectrum. It is only at high $p_T$ that photon-induced reactions differ because of the photon’s pointlike interactions and its correspondingly harder parton distribution functions (PDF). Based on these observations we can safely predict that minimum-bias physics of $\gamma\gamma$ interactions will follow that of $\gamma p$ or $pp$ interactions. At high $p_T$, the spectra should become harder when going from $pp$ to $\gamma p$ to $\gamma\gamma$ interactions.

In view of what we said above, any model that aims at an complete description of $\gamma\gamma$ interactions should better successfully describe the wealth of data taken in hadronic collisions, notably at $pp$ colliders. Reactions to be modelled include elastic scattering, diffractive dissociation and hard, perturbatively calculable interactions. On top of that, unitarity constraints have to be incorporated implying, in general, the existence of multiple (soft and hard) interactions (for a review and references see e.g. [44]). The SaS model (Schuler and Sjöstrand) [45] has
been implemented in \textsc{Pythia} \cite{PYTHIA} while the DPM (Dual-Parton-Model) \cite{DPM} has been extended (Engel and Ranft) to $\gamma p$ \cite{Engel:1995rm} and $\gamma\gamma$ reactions \cite{Bertone:2005cj} in \textsc{Picojet}. Minimum-bias physics in $pp$, $\gamma p$, and $\gamma\gamma$ collisions is currently being improved in \textsc{Herwig} by the inclusion of multiple hard scatterings \cite{Herwig}.

These event generators and the physics of the corresponding models are described in detail in the “Event generators” chapter. Here we discuss only some differences among the three models which have been used in Fig. 14.

To extend the description of $pp$ interactions to $\gamma p$ (and $\gamma\gamma$) ones it is convenient to represent the physical photon as the superposition \cite{Engel:1995ns}

$$\gamma = \frac{Z_3}{\beta} \gamma_B + \frac{P^{\gamma}_{\text{had}} \gamma_{\text{had}}}{\beta} + \frac{e}{f_{\text{low}}} \frac{q\bar{q}_{\text{low}}}{Q_0^2} + \frac{e}{f_{\text{high}}} \frac{q\bar{q}_{\text{high}}}{Q_0^2}, \quad (21)$$

where the (properly normalized) first term describes the pointlike interaction of the photon. The spectrum of hadronic fluctuations of the photon is split into a low- and a high-mass part, separated by some scale $Q_0$. Both contributions can, in general, undergo soft and hard interactions. The soft interactions are mediated by Pomeron/Reggeon exchange whose amplitudes can be inferred from the ones of $pp$ interactions assuming photon–hadron duality. Hard interactions are those that contain at least one hard scale and can be expressed in terms of the “minijet” cross section $\sigma_{\text{jet}}(s; p_{\text{T,min}})$, i.e. the cross section for perturbatively calculated partonic 2 → 2 scatterings above a $p_T$ cutoff $p_{\text{T,min}}$. Again, unitarization leads to multiple (soft and hard) scatterings.

In a most naïve scenario, the probability $P^{\gamma}_{\text{had}} = P(\gamma q\bar{q})$ is taken to be constant. At high energies, this cannot, however, be correct \cite{Engel:1995ns} since the contribution from high-mass hadronic fluctuations becomes important. These are perturbatively calculable and lead to a logarithmic increase of $P^{\gamma}_{\text{had}}$ with the hard scale $Q \sim p_T$, $(e/f_{\text{high}})^2 \ln(Q^2/Q_0^2)$. In the SàS approach \cite{SAS}, most parameters, in particular the coupling of the low-mass part of (21) are determined using VMD-type arguments. The only two additional parameters in the extension from $pp$ to $\gamma p$ collisions, namely $Q_0$ and the $p_T$ cutoff for the hard cross section originating from the high-mass part ($p_{\text{T, min}}^{\text{anom}}$) were fixed by low-energy $\gamma p$ data, prior to the HERA data. Elastic and diffractive cross sections as well as minimum-bias distributions were successfully predicted \cite{SAS}. The prediction for the $\gamma\gamma$ total cross section is shown in Fig. 14.

The DPM approach extended to $\gamma p$ collisions \cite{Engel:1995rm} (in the \textsc{Picojet} event generator) differs in several important aspects from the SàS approach. The unitarization requirements are obeyed by strictly sticking to the eikonal approach. This leads to multiple partonic scatterings also for high-mass photonic states. Furthermore the probabilities $e^2/f_{\text{low}}^2$ and $e^2/f_{\text{high}}^2$ as well as the Pomeron and Reggeon coupling constants and effective intercepts have been determined by fits to data on the total photoproduction cross section and the cross section for quasi-elastic $\rho^0$ production. Once these parameters are fixed, $\gamma\gamma$ collisions can be predicted without further new parameters \cite{Bertone:2005cj}. The predicted rise of $\sigma^{\gamma\gamma}_{\text{tot}}$ is shown in Fig. 14. It is governed by the small-$x$ behaviour of the PDF of the photon.

\footnote{This also leads to another difference compared to \textsc{Pythia}, which is already present for purely hadronic collisions, namely the generation of events containing multiple soft interactions in combination with any number (including zero) of hard interactions.}
The eikonalized mini-jet model is well described in the literature (see e.g. [1,44]). In addition to the above-mentioned parameters such as $P_{\text{had}}^\gamma$ and $p_{T\text{min}}$ the predictions of this model depend also on $\rho(b)$, the distributions of the photonic partons in the impact-parameter space. The new feature in the calculation in [52] is that for $\rho(b)$ they use the Fourier transform of the partonic transverse momentum distribution instead of the Fourier transform of the pionic form factor which is normally the case. The former has recently been measured [53] and has the form, in agreement with the expectations of perturbative QCD, $dN_\gamma/db^2 = 1/(k_0^2 + k_i^2)$ with $k_0 = 0.66 \pm 0.22$ GeV. Interestingly, the normal usage of pionic form factor corresponds to $k_0 = 0.735$. Predictions of the model are shown by the dotted lines in Fig. 14.

5.2 Production of hadrons and jets

In order to illustrate characteristic differences and similarities between $\gamma\gamma$, $\gamma p$, and $pp$ collisions we first show comparisons at fixed CM energy. Since elastic hadron-hadron collisions usually are excluded in studies of inclusive secondary distributions, we also exclude the analogous ones for the photon-induced reactions, i.e. the quasi-elastic diffractive channels $\gamma\gamma \to V + V'$, $\gamma p \to V + p$ ($V = \rho, \omega, \phi$) but we include all other diffractive processes.

First we show the transverse momentum distribution in Fig. 15. Both PYTHIA and PHOJET show a similar behaviour and agree very well with the behaviour of the data in Fig. 13b. In fact the distributions from both models for $\gamma\gamma$ interactions are very similar, differences between
Figure 15: Comparison at the collision energy $\sqrt{s} = 20$ GeV of the transverse momentum distribution in invariant form for all charged hadrons produced in $pp$, $\gamma p$ and $\gamma \gamma$ collisions. The calculation was done with PHOJET (left) and PYTHIA for inelastic collisions.

the two models appear mainly for $pp$ collisions. These differences are probably due to the use of different parton distribution functions and cutoffs for minijets. As expected, at low $p_T$, the

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Table 1: Comparison of average quantities characterizing hadron production in nondiffractive $pp$, $\gamma p$ and $\gamma \gamma$ collisions according to PHOJET at CM energies between 10 and 20 GeV.

distributions of $\gamma \gamma$, $\gamma p$, and $pp$ collisions are very similar, while the fraction of hard interactions in minimum bias interactions rises from $pp$ to $\gamma p$ to $\gamma \gamma$ collisions. The reason for this is the direct photon interaction and the fact that the photon structure function is considerably harder than the proton one. In $\gamma \gamma$ collisions it is easy to observe already with moderate statistics hadrons with transverse momenta approaching to the kinematic limit.

However, the differences in the hard scatterings hardly influence such average properties of the collision as average multiplicities or even average transverse momenta. This can be seen from Table 1, where we collect some average quantities characterizing nondiffractive $pp$, $\gamma p$ and $\gamma \gamma$ collisions in PHOJET at CM energies of 10 and 20 GeV. The total and charged multiplicities at all energies are rather similar to each other in all channels. The differences in the multiplicities of non-leading hadrons like $\pi^-$ and $\bar{p}$ are more significant and we find them
at all energies rising from $pp$ to $\gamma p$ to $\gamma \gamma$ collisions. Also the average transverse momenta rise as expected from $pp$ to $\gamma p$ to $\gamma \gamma$.

Next we consider an example of hadron and jet production in $e^+e^-$ collisions. In Fig. 16 the $e^+e^- \rightarrow e^+e^- X$ cross section is shown as a function of the visible photon-photon energy. Also shown in this Figure is the cross section for events with jets ($p_T^{\text{jet}} > 5 \text{ GeV/c}$). We predict that nearly all events have jets at large $W_{\text{vis}}$.

![Cross section plot](image)

Figure 16: Cross section at $\bar{s} = 175 \text{ GeV}$ as function of the visible $\gamma \gamma$ CM energy with cuts 1$+2+3$ on the final state system (full line) and for events with jets after the application of cut 1 (dotted line), (calculated with PHOJET; the cuts are defined in sec. 4).

Studies at LEP1 have started recently showing that measurements of minimum bias events are indeed possible. For example, Fig. 17 shows that small $p_T$ values are accessible. No detailed comparison between the multipurpose Monte-Carlo generators such as PHOJET and PYTHIA has yet been done with the existing LEP1 data. Instead, we show here the Aleph [56] results on the $W_{\text{vis}}$ distribution and on the charged track $p_T$ distribution for minimum bias untagged events compared to a specific in-house $\gamma \gamma$ event generator. The data has been modelled by a sum of four contributions (Fig. 17). The bulk of the data is described by a VMD model which includes only limited $p_T$ with respect to the $\gamma \gamma$ direction, and has been tuned to the data. At high $p_T$ and $W_{\text{vis}}$ the data require additional contributions from QCD. These are separated into the direct process (labelled QPM), and the sum of single- and double-resolved processes (labelled QCD-multijets).

5.3 Measurement of the $\gamma \gamma$ hadronic cross section

In untagged two-photon events the photons are nearly on-shell. The measurement of $\sigma_{\text{tot}}^{\gamma\gamma}$ is not so much limited by statistical accuracy as by a precise description of competing processes, such as beam-gas or beam-wall scattering, annihilation reactions, and a careful simulation of the process itself. The invariant mass from beam-gas scattering is effectively limited to $15 \text{ GeV}$ by the kinematics of electron scattering off slowly moving nucleons. It can be suppressed by requiring the event vertex to be at the interaction point. The contribution from the annihilation
The event characteristics are dominated by the Vector Dominance process, which accounts for about 70 to 80% of the events according to present models. Much information from two-photon events disappears along the holes in the detectors around the beams. As a hardware solution to this problem is not envisaged by the LEP experiments, one has to cope with partial event information; 30 to 50% of the hadronic energy is lost in the forward region, and the fraction increases with the $\gamma\gamma$ invariant mass and LEP beam energy. Some useful information can be collected from small-angle electromagnetic detectors. At low $\gamma\gamma$ invariant mass the measurements are limited by trigger requirements. Unfolding of the true $\gamma\gamma$ mass distribution from the data with the help of event simulation is imperative, and it gives rise to model dependence in the cross section. Results are expected up to about 80 GeV, somewhat below the $Z^0$ mass.

Interesting studies can be performed at LEP2 with double tag $\gamma\gamma$ events using Very Small Angle Tagger (VSAT) detectors due to the high enough cross section for the polar angle region (2 - 15 mrad) covered. The DELPHI collaboration had already obtained some new results studying the single tag events in their VSAT [54] (similar detectors are currently being used in OPAL or upgraded in ALEPH and L3). The double tag mode is attractive due to the possibility of a direct measurement of both the hadronic invariant mass produced and the absolute momentum transfers squared for both photons. Taking into account the experimental constraints and the efficiency of the hadronic system registration, for 500 pb$^{-1}$ of data, we expect about of 6000 VSAT double tagged events per experiment.

In Fig. 18 the $\gamma\gamma$ invariant mass distributions are illustrated for the visible $W_{\text{vis}}$ (black circles), the true $W_{\text{true}}$ (solid line) and $W_{\text{tag}}$ (triangles) reconstructed from the tag measurements. Also
Figure 18: Distribution in the number of events according to the various hadronic mass determinations.

displayed is the ratio $W_{\text{true}}/W_{\text{tag}}$ shown for two regions of $W_{\text{true}}$, above 40 GeV (triangles and solid line) and below (white circles). It appears that for $W$ above 40 GeV, there is a good agreement between the $W_{\text{tag}}$ and $W_{\text{true}}$. This means that one expects reliable results because there is little need to apply unfolding procedures in that region. The conclusion from this picture is that the extraction of the total $\gamma\gamma$ cross-section $\sigma_{\gamma\gamma}$ is possible in a wide region indeed: even above 80-90 GeV the statistics available is greater than 100 events.

Summarizing the above we can say that LEP2 will open a new opportunity to obtain reliable values of total $\sigma_{\gamma\gamma}$ for a $\gamma\gamma$ central energy up to 80 GeV (100 GeV) in the untagged (double-tag) case. This will increase five times the presently accessible range [55] (see Fig. 14). The extrapolation of the cross section for virtual photons in the double-tag case to $Q^2 = P^2 = 0$ should be safe since the average virtualities ($0.5\,\text{GeV}^2$) are much lower than where HERA sees a strong $x$ dependence in $F_2^p(x, Q^2)$. The uncertainty in the extrapolation will further be reduced by future HERA measurements at low $Q^2$. Finally, double-tag events provide a non-trivial check of hadronization models (see also sec. 3 in [4]).

5.4 Semihard quasidiffractive processes

We consider now the exclusive or semi-exclusive production of neutral mesons $M$

$$\gamma\gamma \to MM', \quad \gamma\gamma \to M + X$$

in the semihard region

$$W^2 = p_\perp^2 = t \mu^2, \quad W^2 = (p_{1\gamma} + p_{2\gamma})^2, \quad t = (p_{1\gamma} - p_M)^2, \quad \mu = 0.3\,\text{GeV}. \quad (23)$$
Here $M$ is a vector ($V = \rho^0, \omega, \phi, ..., \Psi$), or pseudoscalar ($P = \pi^0, \eta, \eta'$), or tensor ($T = a_2, f_2, f'$) neutral meson and $X$ is a hadron system with not too large invariant mass $M_X^2 < t$. Such processes are discussed in a number of papers (see, for example, [57–59]).

The condition $t = W^2$ determines these processes as quasi-Regge ones. In the production of the vector meson $V$ vacuum quantum numbers are transferred from the photon to the meson. The energy dependence of these processes is determined by the Pomeron singularity. In the production of pseudoscalar $P$ or tensor $T$ mesons the corresponding singularity is the odderon. In perturbative QCD (pQCD), the Pomeron (PP) and the odderon (PO) have the same status, however, the current data do not indicate unambiguously the odderon contribution to the total cross sections.

In the lowest order of pQCD the processes (22) are described by diagrams with two quark exchange. Their contribution to the cross section $d\sigma/dt$ decreases as $W^{-4}$ with increasing $W$ [57]. However, the contribution of the diagrams with the gluon exchange does not decrease with $W$ while $t$ is fixed. For the production of vector mesons $V$ the lowest nontrivial diagrams of pQCD corresponds to the two-gluon exchange and for the production of $P$ and $T$ mesons to the three-gluon exchange. The current lowest order calculations are performed in this approximation [57, 58]. In Tab. 2 the expected event rates are given.

It should be emphasized that the lowest order calculations (LO) provide a lower limit of the expected event rates, as, at high enough energies, higher-order terms in the perturbative series such as $\alpha_s(p_T^2)\ln(W^2/p_T^2)$ become large and lead to a considerable increase of the cross sections. For the PP case, this effect has been calculated in the leading logarithm approximation (LLA) (see Refs. [60] and references therein). Defining $z = 3\alpha_s/(2\pi)\ln W^2/W_0^2$ the expression for the differential cross section can be written [59], as a power series in $z$, i.e.,

$$
\frac{d\sigma}{dt} = \frac{d\sigma_{2\text{gluon}}}{dt}(1 + \sum_{n=1}^{\infty} c_n z^n) = \frac{d\sigma_{2\text{gluon}}}{dt} K(W^2, t)^2.
$$

The value of $z$ is very sensitive to the choice of parameters: for example, taking $\alpha_s = .2$ (small), $W_0^2 = 4(p_T^2 + m_V^2) = 16$ GeV$^2$ (large) and $W_{\text{min}} = 15$ GeV one obtains the conservative estimate $z = .25$. LEP2 can get statistics in the region $z = .25 \ldots .5$. If $z \geq .5$, the perturbative series needs to be summed to all orders and the striking power behaviour of the cross sections emerges, i.e.,

$$
\frac{d\sigma}{dt} f(W, t) \frac{W^2}{W_0^2}^{2\omega_0},
$$

where $1 + \omega_0$ is the “intercept” of the BFKL Pomeron and the $W$ dependence in $f(W, t)$ is weak. A corresponding strong enhancement over the two-gluon exchange results is thus anticipated. For large values of $z$ we have the LLA result $2\omega_0 = (12/\pi)\alpha_s(t)\ln 4$ leading eventually to a violation of the Froissart bound. At large enough $W^2$ this growth should be stopped by unitarity to satisfy approximately $K(W^2, t) < 25$ [57]. The PO estimations in [60] show that there is an increasing function of a parameter like $z$ also for the PO.

In the case of $J/\Psi$ production there exists a prediction [59] which takes into account the coupling of the reggeised gluons to the $c\bar{c}$-$J/\Psi$ system. For $p_T^2 = t < M_{\Psi}^2$ and large $z$ the
cross section is given by
\[
\frac{d\sigma(J/\Psi J/\Psi)}{dp_T^2} = 16\pi^2\alpha_s^2(\alpha_s C_F)\frac{4\pi^3}{4} \frac{\exp(16z\ln 2)}{(7\pi \zeta(3)\bar{z})^3} \frac{c_\psi f_\psi}{M_\psi^2} \frac{4}{\ln^4 \frac{M_\psi^2}{p_T^2}},
\]
where \(c_\psi = 3/4\) and \(f_\psi = 0.38\) GeV.

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</table>

Table 2: Number of expected events for LEP II with an integrated \(e^+e^-\) luminosity of 500 pb\(^{-1}\). The calculation was done for anti-tagged \(e^+e^-\) events with \(\Theta_e \geq 30\) mrad.

In Tab. 2 we present the number of expected events for LEP2 based on the two regimes (LO and LLA) described above. Since one does not know at which energy the asymptotic predictions become valid, we expect the number of experimentally observed events to be bracketed by the two sets of predictions. Further uncertainties are associated to the choice of \(\alpha_s\) and \(W_0\) and the fact that the enhanced rates are obtained using the large \(z\) approximation in solving the BFKL equation. More modest enhancements can be expected for the \(z\) values typical of LEP2 (see the numerical studies in [59]). The values for the \(\gamma\gamma\) cross sections in lowest order are the following: for the \(J/\Psi\) production we use the total cross sections obtained in [58], for \(\rho^0\) and \(\pi^0\) the cross section corresponds to the region \(t > t_{\text{min}}\). In the calculations, the photon-photon energy was restricted to \(W_{\gamma\gamma} < W_{\text{min}}\). Furthermore, results are given for the cross sections demanding the meson to emerge at angles larger than \(\Theta_{\text{min}}\). For the estimation of the effects of BFKL Pomeron we use the \(K\) factor calculated for the process \(\gamma\gamma \rightarrow \gamma\gamma\) via two \(c\bar{c}\) pairs [60]. To show the strong dependence of the results on the limitation of \(K\), two sets of numbers are given (last row), for \(K = 25\) and for \(K = 20\). In addition, the event rates according to Eq. (26) (neglecting unitarity corrections) are given. Note that the numbers obtained with the constraint on \(\Theta_{\text{min}}\) are calculated with a fixed value of \(\alpha_s = 0.32\) whereas for the other numbers running \(\alpha_s\) was used.

In conclusion, it appears that the study of diffractive phenomena in the semi-hard domain may yield interesting information on the nature of the Pomeron.
As it is the case in the study of the proton structure the production of jets or hadrons at large \( p_T \) in \( \gamma \gamma \) collisions is complementary to the deep-inelastic scattering of a real photon: indeed the latter reaction essentially probes the quark distribution while large \( p_T \) phenomena are also sensitive to the gluon distribution. In order to reliably understand the hadronic structure of the photon both types of processes (as well as heavy flavor production, see next section) should be studied and compared to the theory. Taken together, they will allow the determination of the parton distributions in the photon. For this purpose, we discuss below the jet inclusive cross section at the next-to-leading order (NLO) in QCD and related processes in the case of anti-tag or single tag experiments. The possibility to identify the gluon jet in three-jet events is also considered.

6.1 The structure of the hard process

Large \( p_T \) processes involving real photons are rather complex. This arises from the fact that the photon couples to the hard sub-process either directly or through its quark or gluon contents. The cross section for the production of a jet of a given \( p_T \) and pseudorapidity \( \eta \) can therefore be decomposed as (D: direct, SF: structure function, DF: double structure functions)

\[
\frac{d\sigma}{d\hat{p}_T d\eta} = \frac{d\sigma^D}{d\hat{p}_T d\eta} + \frac{d\sigma^{SF}}{d\hat{p}_T d\eta} + \frac{d\sigma^{DF}}{d\hat{p}_T d\eta}
\]

where each term is now being specified. In the NLO approximation [35] the “direct” cross section takes the form

\[
\frac{d\sigma^D}{d\hat{p}_T d\eta}(R) = \frac{d\sigma^{\gamma\gamma\rightarrow \text{jet}}}{d\hat{p}_T d\eta} + \frac{\alpha_s(\mu)}{2\pi} K^D(R; M).
\]

with the corresponding diagrammatic decomposition shown in Fig. 19a and b. The parameter \( R \) specifies the jet cone size, while \( \mu \) and \( M \) are the renormalization and factorization scales respectively. When one photon couples directly and the other one through its structure function, it leads to

\[
\frac{d\sigma^{SF}}{d\hat{p}_T d\eta}(R) = \sum_{i=q,G} dx_i F_{i/\gamma}(x, M) \frac{\alpha_s(\mu)}{2\pi} \frac{d\sigma^{\gamma\gamma\rightarrow \text{jet}}}{d\hat{p}_T d\eta} + \frac{\alpha_s(\mu)}{2\pi} K^{SF}(R; M, \mu) + 1 \quad 2
\]

where some diagrams representative of the \( (\alpha_s) \) and \( (\alpha_s^2) \) terms on the right hand side are shown in Fig. 19c and d, respectively. The underlined diagrams in Fig. 19b and c are in fact the same but they contribute to different regions of phase space. When the final state quark is not collinear to the initial photon (as in Fig. 19b) the exchanged propagator has a large virtuality (shown by the fat line) and the corresponding contribution is associated to the hard subprocess \( K^D \). When the final quark is almost collinear to the initial photon (as

Figure 19: Some diagrams contributing to jet production in $\gamma\gamma$ collisions in the NLO approximation.

in Fig. 19c)) the virtuality of the exchanged propagator is small and the interaction is soft (long range). Roughly speaking the factorization scale $M$ separates the hard region from the soft region and changing this arbitrary scale shifts contributions from $d\sigma^D$ to $d\sigma^{SF}$ but clearly should not affect the sum $d\sigma^D + d\sigma^{SF}$ [61]. A similar compensation occurs between $d\sigma^{SF}$ and $d\sigma^{DF}$ (see Fig. 19d and e). In conclusion, only the sum eq. (27) has a physical meaning and it is not legitimate to associate $d\sigma^{SF}$ and $d\sigma^{DF}$ to experimentally measured “once resolved” and “twice resolved” components. To illustrate quantitatively the variation of the theoretical predictions under changes of $M$ and $\mu$ we consider the production of jets at $p_T = 10$ GeV/c and $\eta = 0$: Fig. 20a is obtained when setting arbitrarily $K^D = K^{SF} = K^{DF} = 0$ (the so-called leading order (LO) predictions) while Fig. 20b takes into account the full expressions: the gain in stability is remarkable despite the fact that no saddle-point or extremum is found [35]. In the following we always use for definiteness $M = \mu = p_T$.

6.2 Transverse momentum and rapidity distributions

In order to compare theory and experiment one must convolute the above cross sections with the Weizsäcker-Williams spectrum of quasi-real photons emitted by the electrons, taking into account the experimental conditions. The usual antitagging conditions defined in sec. 4 are used here ($\theta_e < 30$ mrad or $E_e < 1$ GeV). The appropriate Weizsäcker-Williams spectrum is given in eq. (14). We display in Fig. 21 the jet $p_T$ distribution, the jet pseudo-rapidity being integrated in the interval $\eta < 1$. The complete NLO expressions are used. The solid curve is the prediction when using the full AFG parton distributions (with a VMD input at
\(Q_0^2 = .5 \text{ GeV}^2\) [19], while the dashed curve is obtained when one artificially sets the VMD component equal to 0. More precisely, in the latter case, the quark and gluon densities are vanishing at \(Q_0^2\) and they are generated by the evolution equations at larger \(Q^2\). It is clearly seen that the VMD component is quite important at \(p_T = 5 \text{ GeV/c}\) (about 40\%) but rapidly decreases as \(p_T\) increases since it is less than 10\% at \(p_T = 20 \text{ GeV/c}\). One may note that the VMD component of the quark is also tested in photon structure function studies but large \(p_T\) processes are also very sensitive to the VMD component of the gluon which dominates for \(x < .5\). With the luminosity expected at LEP2 the error bars in the data will be sufficiently small to constrain the size of the VMD input.

As mentioned in the structure function section above it will be possible to probe the virtual photon structure in deep-inelastic type experiments. We show here that it can also be probed in jet studies. Using the tagging conditions \(0.2 < Q^2 (\text{GeV}^2) < .8\) and \(z_e < .5\), typical of LEP2, on one photon and the usual anti-tagging condition on the other, one still obtains an appreciable jet cross section as shown on Fig. 21: about 100 events with \(p_T = 10-15 \text{ GeV/c}\) are expected.

Let us remark that, thanks to the usual VMD form-factor, the VMD contribution is rapidly reduced when considering tagged electron (it is reduced by roughly 75\% when going from a real photon to a photon of virtuality \(Q^2 = .5 \text{ GeV}^2\)).

Despite the warnings given above, we display in Fig. 22 the break-up of the single inclusive jet cross section, integrated in the range \(\eta < 1\), into the DIR, SF and DF components. These curves can serve useful purposes when comparing these analytic calculations with those based on Monte-Carlo generators. Of course, the size of the various components depend strongly on the choice of the factorization scale (here we use \(M = \mu = p_T\)). In the figure, obtained using the leading logarithmic expressions for the sub-processes we see that DF component dominates the lower end of the distribution but is less than 20\% at \(p_T = 20 \text{ GeV/c}\). Nowhere does the SF terms dominate. In Fig. 23 we show the rapidity dependence for \(p_T = 5 \text{ GeV/c}\) and \(p_T = 15 \text{ GeV/c}\) of the various components both in the LO and the NLO approximation. One sees that the pattern of higher corrections is quite different for the different subprocesses:
the DF component is appreciably increased by the higher-order corrections (about 40%) while the DIR component is decreased and (roughly 15%) the SF component remains stable. This pattern of higher order corrections is independent of the transverse momentum or rapidity. It is interesting to remark that at $p_T = 15$ GeV/c the hierarchy of cross sections is reversed when including the higher order corrections with the DF term dominating over the DIR one despite the fact that the overall cross section is not affected very much. In conclusion, one may say that the higher-order corrections affect the structure of the cross section more than its overall size.

It will be very useful to study a more exclusive observable namely the di-jet cross section. Quite interesting phenomenology is coming out of HERA [62]: it allows to separate on an experimental basis events dominated by the direct component from those dominated by the photon structure function. For an application of this technique to $\gamma \gamma$ reactions we refer to the ”High-$p_T$” section of [4]. NLO calculations are in progress [63].

Recently data on jet distributions have been published by the TOPAZ [64] and AMY [65] collaborations. The above calculations are in good agreement [35] with the single jet distribution of TOPAZ when using the AFG parton distributions [19] while both the NLO predictions [63] and the PHOJET [66] results using the GRV parametrizations [11] fall below the data. In contrast, a rather good agreement, at least at large $p_T$ values, is found with the two-jet data of TOPAZ in [63] and [66].

A word of caution should be said concerning large $p_T$ phenomenology. Even at LEP2 we are dealing with rather low $p_T$ jets and the comparison between the theoretical predictions, at the partonic level, and the experimental distributions at the hadronic level may not be easy because of the non negligible contribution to the jet transverse momentum from the “underlying event” [67]. Good event generators are required to understand this point and also a lot should

Figure 21: Jet pt distributions. Left: The NLO cross section using the full structure functions (solid line) compared to the case when only the perturbative part is kept (dashed line). Right: the same as above (solid line) in the LO approximation and the cross section when one of the photon is tagged (dashed line).
be learnt from present HERA studies. To avoid this difficulty it will be extremely interesting to consider the single hadron inclusive distribution which probes the same dynamics as jet production. New parameters come into play through the fragmentation functions of partons into hadrons but these distributions should soon be rather well constrained as several groups are at present extracting NLO parametrizations of fragmentation functions using data from both $e^+e^-$ colliders and $pp$ colliders [68]. Concerning lower energy data, the situation concerning single hadron production is rather confusing and paradoxical as the experimental results [69] are much above the NLO theoretical predictions [70] at large $p_T$ (where the DIR term is predicted to dominate) while they agree with the theory at lower $p_T$. Data are eagerly awaited to clarify this point.

6.3 Three-jet events and the separation of gluon jets

The first direct evidence for gluon jets was seen by the PETRA experiments as three-jet events, $e^+e^- q\bar{q}g$, where all three partons have high energy and are well-separated in angle. Ever since, one of the important aims of QCD studies has been to measure the similarities and differences between quark and gluons jets, as well as collective quark-gluon phenomena such as the string effect [71]. One of the important questions is to what extent such effects can be described by perturbative QCD, rather than non-perturbative models. However, many of the studies are inconclusive on this issue, because at any given energy, most non-perturbative models can be tuned to mimic the perturbative effects, and it is only in the energy-dependence that definitive differences can be seen. But comparing experiments at different energies typically

![Figure 22: The various components of the jet $p_T$ spectrum. The full cross section: full line; the DIR component: dashed; the SF component: dotted; the DF component: dash-dotted.](image)
Figure 23: The jet rapidity distributions at fixed $p_T$. The meaning of the curve is as in the previous figure. The dotted lines are the LO results while the starred line are the NLO results. Left: $p_T = 5$ GeV/c; right: $p_T = 15$ GeV/c.

Involves large uncorrelated systematic errors. In $\gamma \gamma$ collisions on the other hand, one can study a wide range of energies in the same experiment, and can thus study energy-dependent effects much more reliably.

We can make a rough estimate of the three-jet rate by taking the leading logarithmic approximation to the total two-jet rate \[72\] and simply multiplying it by a factor of $\alpha_s$ as an estimate of the fraction of three-jet events. We require each jet to have a transverse momentum above $p_{t\text{min}}$ both with respect to the beam axis and each other, and hence obtain

$$\sigma(s, p_{t\text{min}}) = \frac{4}{3} e_q^4 \alpha_s N_c \frac{\log s}{\log p_{t\text{min}}^2} \frac{p_{t\text{min}}}{m_q^2} \log^2 \frac{s}{m_q^2} \alpha_s.$$  

It is worth noting that the smallness of $\alpha_s$ cannot be compensated by any large logarithms, because the appropriate logarithms are $\log W^2 / p_{t\text{min}}^2$ and the $W^2$ distribution is dominated by $W^2 / p_{t\text{min}}^2$. Nevertheless, we obtain around $10^4$, $10^5$ and $10^2$ events in 500 pb$^{-1}$ for $p_{t\text{min}} = 5, 15$ and 35 GeV respectively. For such studies it is important to know which jet is the gluon. The cleanest way to do this is by flavour tagging the quark jets, but this severely reduces the rate. In $e^+ e^-$ annihilation, the fact that the gluon’s energy spectrum is softer than the quark’s was used, by always calling the softest of the three jets the gluon. By explicit analytical integration of the full five-dimensional matrix element for $\gamma \gamma \rightarrow q\bar{q}g$ down to a distribution over the energies of the quarks \[73\], we find that the same is also true for $\gamma \gamma$ collisions (Fig. 24, see \[73\] for further details). Thus the gluon jet in three-jet events can be statistically tagged by calling it the softest jet.

Although a more complete study incorporating realistic detector cuts and the effects of hadronization is clearly needed, it seems hopeful that a sample of three-jet events could be isolated and a study of the energy-dependence of quark-gluon jet effects made at LEP2.
6.4 Role of the experimental cuts on the inclusive jet spectrum

We have seen in sec. 4 that the radiative production of $Z^0$ bosons provides an important background to $\gamma\gamma$ physics when one looks for rare events such as large $p_T$ jets. Various cuts have been devised which considerably reduce the background. The study of the effect of these cuts on the signal requires the use of a Monte-Carlo generator and we use here the PHOJET program [47,48]. We have checked that the unbiased jet $p_T$ spectrum is in good agreement with the analytical results described above. In Fig. 25 we see that, as expected, only the upper end of the spectrum is reduced by the cuts (see sec. 4 for the meanings of the cuts) while at fixed $p_T$ mainly the large $\eta_{jet}$ regions are reduced. The net result is that, after cuts, the rate for producing jets with $p_T > 20$ GeV is comparable in the signal and in background. Needless to say that jets produced in single tag events are not affected by the background.

Figure 24: Differential spectra $d\sigma/dz$ of parent quark and gluon in the reactions $e^+e^- \rightarrow q\bar{q}g$ and $\gamma\gamma \rightarrow q\bar{q}g$

Figure 25: Jet cross section for events satisfying cut 1 (full line) and cut 1+2+3+4 (dotted line). Left: distribution in $p_T$ with $\eta_{jet} < 1$; right: distribution in $\eta_{jet}$ for fixed $p_T = 7$ GeV/c. (Calculated with PHOJET).
A favourable aspect of heavy flavour production in $\gamma\gamma$ collisions compared to other $\gamma\gamma$ processes is that the heavy quark mass ensures that the separation into a direct and resolved processes is, to next-to-leading order (NLO) in QCD, unambiguous, i.e. does not depend on an arbitrary separation scale. Experimentally one may perform this separation by using single tag events (see below), by using non-diffractively produced $J/\psi$'s, or by using the photon remnant jet, present in resolved processes, as a separator. Therefore heavy flavour production at LEP2 provides a good opportunity for simultaneously testing QCD (direct process) and measuring the poorly known gluon content of the photon (resolved processes).

7.1 Theory

We first discuss the theoretical aspects of the reaction $\gamma\gamma \rightarrow Q\bar{Q}$ where $Q$ ($\bar{Q}$) is a heavy (anti-)quark (charm or bottom). In practice the cross section for bottom production is too small to be observed at LEP2 so, in what follows, only charm production will be considered. Figure 26 shows some of the diagrams contributing to heavy quark production in two-photon physics. Diagrams (a)-(c) are examples of the so-called direct process in which the photon couples directly to a quark. Diagram (a) is the Born term direct process which is equivalent to the Quark Parton Model (QPM), (b) and (c) represent virtual and real QCD corrections to the Born amplitude. At low beam energy the direct process is completely dominant, however at LEP2 the production of open charm in the collision of two effectively on-shell equivalent photons receives contributions in about equal amounts from the direct- and once-resolved channels diagrams (d)-(f). In Ref. [74] this process was calculated to NLO in QCD, and all theoretical uncertainties (due to scale choice, mass of the charm, and choice of photonic parton densities) were thoroughly investigated. The largest part of the resolved process is given by the photon-gluon fusion process; this property offers the possibility of measuring the gluon content of the photon which is currently poorly known. Doubly resolved processes have been calculated to give a negligible contribution for presently available beam energies [74]. In Fig. 28 the total cross section and its theoretical uncertainty is shown as a function of the center of mass energy, together with some recent measurements. LEP2 offers the possibility of a serious comparison between a fairly well understood theory and experiment with considerably more statistics than hitherto. Given this larger statistics it will be interesting to make this comparison not just for the total cross section, but also for various differential distributions. Single particle distributions in the transverse momentum and rapidity of the heavy quark are given in [74]. Correlations between the heavy quarks in the direct process, including NLO effects, have recently been studied in [75], but will be difficult to observe at LEP2.

Furthermore, the NLO prediction for the one particle inclusive transverse momentum distribution contains potentially large terms $\alpha_s \ln(p_t/m)$ which might spoil the convergence of the perturbation series at $p_t \sim m$. In Ref. [76] the NLO cross section for large $p_t$ production of heavy quarks in direct and resolved channels has been calculated in the framework [77] of

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11M. Cacciari, A. Finch, M. Krämer, E. Laenen, S. Riemersma, G.A. Schuler, S. Sökner-Rembold
perturbative fragmentation functions. This approach allows for a resummation of the large logarithmic terms and thereby reduces the scale dependence of the NLO prediction.

The process $\gamma^*\gamma \rightarrow c\bar{c}$ is considerably more suppressed than the previous one due to the photon being off-shell. The structure functions $F_2^\gamma(x, Q^2)$ (and $F_L^\gamma(x, Q^2)$) for open charm production in deep-inelastic single-tag events were calculated to NLO in QCD in [17]. It was found that the contributions from the direct (or “pointlike”) and resolved (or “hadronic”) separate in the variable $x$: above $x > 0.01$ $F_2^\gamma(x, Q^2)$ is almost exclusively due to pointlike photons, and hence calculable, below this value it is mainly due to resolved photons, and essentially proportional to the gluon density in the photon, see Fig. 27a. Experimental studies of the reaction $e^+e^- \rightarrow e^+e^- D^{\pm}\bar{X}$ with one outgoing lepton tagged (“single-tag”) have been done by JADE [78] and by TPC/Two-Gamma [79] at low average value $Q^2$ of the momentum transfer squared of the tagged lepton (below 1 (GeV/c)$^2$) and by TOPAZ [80] at somewhat larger $Q^2$. The total number of events obtained was however very small (about 30 for TOPAZ).

Theoretical uncertainties in these quantities were investigated and are well under control. Numbers of events expected per bin in $x, Q^2$ over the lifetime of LEP2 are also given in Fig. 27b and in [81]. One may conclude from these that with a not too pessimistic charm acceptance (1-2%) a measurement of $F_2^\gamma(x, Q^2)$ should be feasible. Some single particle differential distributions in the transverse momentum and rapidity of the heavy quark have been studied in [81].

Let us finally mention onia production. The radiative decay width of the charmonium states $\eta_c, \chi_{c0}$ and $\chi_{c2}$ can directly be measured in two-photon collisions at LEP2. These $\gamma\gamma$ partial widths provide an important test of the non-relativistic quarkonium model. We refer to the section on resonances and exclusive states for more details. We comment here only further on the case of $J/\psi$. 

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Two-photon production of $J/\psi$ bound states is an attractive tool to determine the gluon distribution in the photon. In contrast to the case of open heavy flavour production, $J/\psi$ mesons are generated only via resolved photons and can be tagged in the leptonic decay modes. Higher order QCD corrections to $J/\psi$ production in photon-gluon fusion have been calculated in [82]. Including NLO corrections, the cross section for inelastic $J/\psi$ production in $\gamma\gamma$ collisions at LEP2 is predicted to be about 5 pb, suggesting that a measurement of this reaction may be feasible.

### 7.2 Experiment

We review in this section the experimental status of charm production in $\gamma\gamma$ collisions, list and evaluate presently available charm identification methods, and extrapolate published results to total charm cross sections for the purpose of comparison. We think this is useful as no such review is presently available in the literature.

Experimental measurements of charm production in $\gamma\gamma$ physics require some form of tag
to identify the presence of the charm quark. There are a number of different techniques that have been used for this in the past. They cover a spectrum in which generally the higher the selection efficiency of the tag, the larger the problems due to backgrounds from non charm contributions. These different techniques are described in the following and are summarised in Table 5 and Fig. 28. In order to enable the different experimental results to be compared we have attempted in this report to extrapolate the published results to a total charm cross section. Some caution is in order however when comparing theory and the results of different experiments in this way. This arises from the strong dependence of the cross section at low \( p_t \) on the choice of charm mass and renormalization scale. As all experimental results are made above some explicit or implicit \( p_t \) cut, extrapolating back to a total cross section increases the error on the measurement. This problem has been treated in different ways by each experiment, e.g. TOPAZ [80,83] chose only to quote a cross section in a limited acceptance. Table 3 summarizes the approach of each experiment.

![Graph]

Figure 28: Comparison of theoretical predictions and experimental results for the total charm cross section. The experimental results have been extrapolated to a total cross section from published measurements. Insert shows the results from Tristan experiments at a beam energy of 29 GeV. Results are tabulated in Table 6. The bands represent a range of theoretical predictions. See table 4 for details.

Figure 29: Distribution of transverse momentum \( p_t(K_s) \) of \( K_s \) simulated with PYTHIA 5.7 at \( E_{CM} = 91 \) GeV.

We now discuss the various charm tagging techniques in more detail. The \( D^* \) tagging technique exploits the fact that the available kinetic energy in the decay \( D^{*+} \rightarrow D^0 \pi^+ \) is only 6 MeV. The signal is typically displayed by plotting \( \Delta M = M_{D^{*+}} - M_{D^0} \) for all reconstructed decay product candidates. A \( D^0 \) decay mode can be used in this analysis if it has a reasonable
Table 3: Treatment of charm mass (m) in extracting experimental results. µ is the renormalisation scale; unless otherwise stated it is equal to the charm mass.

branching ratio (at least of order 1%). Published results have included the decays \( K^-\pi^+\), \( K^-\pi^+\pi^0 , K^-\pi^+\pi^+\pi^-\). Having formed a candidate \( D^0\) meson, which is within the accepted mass range, tracks identified as pions are added in turn to form candidate \( D^{\ast+}\) mesons, \( \Delta M\) being determined in each case. For background tracks the spectrum rises from a kinematic lower limit of 139.6 MeV/c\(^2\) (\( M_{\pi^+}\)), whilst the signal produces a peak at 145.5 MeV/c\(^2\), i.e. \( M_{D^{\ast+}} - M_{D^0}\), which is a region where the background is small.

Early measurements were reported by JADE [78], TPC/Two-Gamma [79], and TASSO [84]. More recently results have also been produced by TOPAZ [83] and ALEPH [85]. These results are summarised in Table 6. The adjusted figure in column 4 takes account of various factors. For TPC/Two-Gamma [79], JADE [78], TASSO [84] and the earlier TOPAZ result [83], the adjustment accounts for the latest values for the \( D^{\ast+}\) and \( D^0\) branching ratios [86]. The

<table>
<thead>
<tr>
<th>Electron Energy</th>
<th>Direct (Born term)</th>
<th>Direct (NLO)</th>
<th>Resolved (NLO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>75 14</td>
<td>103 19</td>
<td>17 6</td>
</tr>
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<td>17</td>
<td>87 16</td>
<td>120 22</td>
<td>23 8</td>
</tr>
<tr>
<td>29</td>
<td>146 26</td>
<td>200 35</td>
<td>70 22</td>
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<td>45</td>
<td>206 34</td>
<td>258 36</td>
<td>195 86</td>
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<td>80</td>
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<td>367 120</td>
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<tr>
<td>90</td>
<td>324 53</td>
<td>444 77</td>
<td>440 152</td>
</tr>
</tbody>
</table>

Table 4: Cross Section (pb) for charm production in two photon physics based on Ref. [74]. The highest values come from using a charm quark mass of 1.3 GeV/c\(^2\) with the scale equal to the charm mass. The lowest values use a charm mass of 1.7 GeV/c\(^2\) with the scale equal to \( \frac{1}{2} \) times the charm mass. The GRV [11] set of parton densities was used, and a lower cut of 3.8 GeV was applied for the invariant mass of the \( \gamma\gamma\) system.
Table 5: Summary of experimental methods for tagging charm in $\gamma\gamma$ physics. In column 1 are listed the experiments which have published results in each category. Column 3 is an estimate of the number of events likely to be observed at LEPII assuming similar selection efficiencies to those at current experiments. Column 3 indicates advantages ( ) and disadvantages ( ) of the different techniques.

published TOPAZ cross section is with the additional condition $p_{t}^{D^{*+}} > 1.6$ GeV/$c$. A total cross section was obtained from the published figures by multiplying the total QPM cross section by the ratio of the observed cross section to the QPM cross section for the same acceptance.

The soft pion method is similar to the $D^{*}$ tagging method in that it takes advantage of the small kinetic energy available to the soft pion in the decay of a $D^{*}$ to a $D^{0}$, but avoids the reduction of statistics which results in looking for fully reconstructed $D^{*}$. It has been found that if one plots the transverse momentum of all charged tracks in events with respect to the jet direction that there is a small excess at very low values which is ascribed to these soft pions. This excess sits on top of a background which is normally estimated by a fit to the $p_{t}$ distribution in the nearby bins and by Monte Carlo studies. Measurements of this type have been made by TOPAZ [80], and AMY [87] and are summarized in Table 6. Note that TOPAZ published their result for the restricted acceptance $p_{t}^{D^{*+}} > 1.6$ GeV/$c$, $\cos(\theta) < 0.77$.

In the lepton tagging technique one uses the fact that there is roughly a 10% branching fraction for a charmed meson decay to include electrons or muons. However the problem is that there are plenty of other sources of leptons in $\gamma\gamma$ events, so a measurement requires good modelling of the background. Results have been published by TOPAZ [88], VENUS [89], and AMY [87] and are summarised in Table 6.

Furthermore, kaons may be used for charm tagging; due to the quark charge, and neglecting quark masses, direct strange quark production is suppressed by a factor 16 compared to direct charm quark production in $\gamma\gamma$ collisions. Therefore a large fraction of the $K_{s}$ observed come from decays of primary charm quarks. However, a substantial number of kaons is also produced.
Table 6: Measurements of charm production in two photon physics. The third column shows the published measurement, either number of events or cross section. As these numbers are not directly comparable we have extrapolated them to a total charm cross section in the fourth column. The upper 7 measurements are of D*+ production, and out of these the first 5 measurement listed employed the ‘D’ Trick’, while the measurements in rows 6 and 7 used the ‘Soft Pion’ method. The remaining 3 rows report measurements where lepton tagging was employed.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Beam energy (GeV)</th>
<th>Published Measurement</th>
<th>$\sigma(e^+e^- \rightarrow c\bar{c}X)$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPC/Two-Gamma</td>
<td>14.5</td>
<td>74 32 pb</td>
<td>120 52</td>
</tr>
<tr>
<td>JADE [78]</td>
<td>17</td>
<td>20.5 events</td>
<td>365 150</td>
</tr>
<tr>
<td>TASSO [84]</td>
<td>17</td>
<td>97 29 pb</td>
<td>142 42</td>
</tr>
<tr>
<td>TOPAZ [83]</td>
<td>29</td>
<td>77 25 pb</td>
<td>430 140</td>
</tr>
<tr>
<td>ALEPH [85]</td>
<td>45</td>
<td>155 39 pb</td>
<td>326 87</td>
</tr>
<tr>
<td>TOPAZ [80]</td>
<td>29</td>
<td>23.5 4.6 pb</td>
<td>304 60</td>
</tr>
<tr>
<td>AMY [87]</td>
<td>29</td>
<td>169 48 pb</td>
<td>355 106</td>
</tr>
<tr>
<td>Venus [89]</td>
<td>29</td>
<td>68.4 13 events</td>
<td>340 65</td>
</tr>
<tr>
<td>TOPAZ [88]</td>
<td>29</td>
<td>19.3 3.4 pb</td>
<td>451 80</td>
</tr>
<tr>
<td>AMY [87]</td>
<td>29</td>
<td>1.0 0.23 pb</td>
<td>374 86</td>
</tr>
</tbody>
</table>

from strange quarks in soft VMD process and from u, d quarks in the fragmentation. Modelling such production introduces systematic errors. The statistical errors, however, are expected to be very small due to the large number of reconstructed $K$'s. In Fig. 29 the transverse momentum $p_t$ with respect to the beam axis is plotted for $K_s$ from $\gamma\gamma$ collisions reconstructed in a typical LEP detector for $E_{CM} = 91$ GeV. The events were simulated with PYTHIA 5.7 [41] including the VMD, anomalous and direct photon components. At $p_t(K_s) > 1$ GeV/c the production from primary $s$ and $c$ quarks dominates over the production from $u, d$ quarks. This effect will be even more pronounced at higher energies. The still strong presence of primary $s$ quarks makes the measurement quite dependent on how this contribution is modeled. The only published measurement of $K_s$ production in $\gamma\gamma$ events to date is by TOPAZ [90].

In principle it might be possible to observe the finite decay length of a charm quark using techniques such as those used so successfully to tag bottom quarks at LEP1. These techniques take advantage of the vertex detectors installed on LEP detectors and include impact parameter, secondary vertex finding and neural nets. At present none of these techniques has been studied in detail.

### 7.3 Prospects for LEP2

The cross section for charm production in both the direct and single resolved mode grows with energy. The cross sections at Petra, Tristan, LEP1, and LEP2 energies are shown in Table 4. Using these cross sections with the 500 pb$^{-1}$ promised for LEP2 and assuming that selection...
efficiencies will be similar to those at present experiments produces the estimated events rates given in column 3 of Table 5. Note that the total number of $c\bar{c}$ events at LEP2 will be around $5 \times 10^6$. It is clear that at LEP2 there is the prospect for high statistics measurements of charm production. With these statistics it should be possible to produce a clean measurement of the resolved and direct processes separately. Recently the direct and resolved contributions to charm production were measured [90] by identifying the energy from the remnant jet, present in resolved processes, seen close to the beam pipe. There is good reason to believe that LEP experiments will be at least as capable of observing this energy and thus extend these studies to higher energies where the contribution of resolved processes is larger. As mentioned in the theory section there is also particular interest in measuring charm production in events, i.e. in events with a tagged electron. The rates for this will be roughly 5-10% of the untagged rates given in Table 5.

8 Exclusive channels

The formation of light resonances by two-photon interactions is a powerful tool in understanding the hadron spectrum and the dynamics controlling the interaction of their constituents. A rigorous testing ground for nonperturbative QCD is provided by analyses of heavy quarkonia for which relativistic corrections and dynamical effects of gluons can be included in a systematic way. Similarly, the meson-photon transition form-factor at large $Q^2$ and exclusive (meson and/or baryon) pair production at large $p_T$ can reliably be calculated in QCD. Last but not least, high-energy $\gamma^*(Q^2)\gamma \rightarrow V_1+V_2$ and/or $V_1+X_2$ reactions, where either $Q^2$, the resonance mass or the momentum transfer is large, allow us to enter a new domain of perturbative QCD. Prospects for LEP2 will be discussed in the following.

8.1 Resonance production by quasi-real photons

Two-photon couplings provide a useful probe of the internal structure of mesons. Two quasi-real photons couple in a selective way to $C = +1$ states, thus simplifying the analysis of mass spectra where many hadrons are superimposed. The accessible resonances with spin 2 have $J^{PC} = 0^{--}$ (1$S_0$), 2$^{-+}$ (1$D_2$), 0$^{++}$ (3$P_0$), and 2$^{++}$ (3$P_2$) where we have given in parentheses the $q\bar{q}$ quark-model assignments in the spectroscopic notation. In particular the “classical” resonances have been observed at $e^+e^-$ machines with $\gamma\gamma$ partial widths consistent with quark-model predictions [91]. These are the light pseudoscalar 0$^{--}$ and tensor 2$^{++}$ states $\pi^0$, $\eta$, $\eta'$, $f_2(1270)$, $a_2(1320)$, $f'_2(1525)$, and, although with poor statistics, the $c\bar{c}$ states $\eta_c$, $\chi_{c0}$, and $\chi_{c2}$.

Particularly interesting are, of course, those resonances whose $\gamma\gamma$ couplings are not consistent with quark-model estimates. On the one hand, conventional $\gamma\gamma$-width calculations might not be reliable enough. As an example let us mention the $\pi_2(1670)$, thought to be the $^1D_2$ ($u\bar{u} - d\bar{d}$)/$\sqrt{2}$ quarkonium state. Here the discrepancy between the measured $\gamma\gamma$ partial width

---

and the non-relativistic calculation [92] could be due to large relativistic corrections [93]. On the other hand, mesons thought to be non-$q\bar{q}$ states generally have $\gamma\gamma$ widths far from expectations for a $q\bar{q}$ state. These anomalous states include the $f_0(1500)$, $f_{0/2}(1720)$, $f_J(2230)$, the $\eta(1410)$, $\eta(1460)$, and the $f_1(1420)$ (for a recent review see e.g. [94]). Measurements of their $\gamma\gamma$ widths have the potential to resolve the enigma of these mesons.

Exploring the exclusive channels at LEP2 has pros and cons compared to lower-energy machines. There are two advantages. First, the signal cross section $\sigma(e^+e^- e^+e^- R)$ rises\textsuperscript{13} with the $e^+e^-$ centre of mass (cm) energy $\tilde{s}$, while the background from the $s$-channel annihilation reaction decreases as $1/\sqrt{s}$. Second, since the energy released in annihilation events rises with $\tilde{s}$, the two contributions become more separated at higher energy [96].

There are also two disadvantages compared to experiments at lower energies. First, the detector acceptance is reduced since the photon-photon system receives on average a larger Lorentz boost. Table 7 displays the acceptance as a function of $\tilde{s}$ for the case of $\eta_c$ production [97-102] in the decay channel $\eta_c \to 4\pi$. All LEP detectors have a good solid angle coverage. For our studies we [103] use the following geometrical acceptance: $20^\circ < \theta < 160^\circ$ for tracks with a $p_t < 0.1$ GeV and $15^\circ < \theta < 165^\circ$ for photons with $E < 0.1$ GeV. As can be seen from Table 7 the acceptance slowly decreases with the centre of mass energy. Due to the increase in the cross section, we find still a net gain on the number of observed events. For lighter resonances the acceptance decreases, from \(20\%\) at 3 GeV to \(4\%\) at 1.3 GeV.

The second disadvantage concerns the triggers. Their efficiencies are more difficult to evaluate. Since the interest of the LEP experiments is centered on the maximum available energy, the two-photon events are mainly seen by triggers based on tracks, thus excluding the observation of purely neutral decays of resonances. The combined effect of the trigger and analysis cuts reduces the efficiency by factors varying from about two at a mass of 3 GeV to about 15 at 1.3 GeV. As examples we show in Table 8 the expectations for the lightest $0^+$ and $2^+$ states. With current triggers, only the $\eta'$ can be studied in the pseudoscalar octet. The expected rates

\textsuperscript{13}In the Low approximation, the two-photon cross-section rises as $ln^2 s$ for a $4\pi$ detector and as $ln^2 s$ for a realistic, limited angular acceptance detector [98]; compare with Table 7.
for the tensor octet are rather good, an analysis of these data has already started at LEP1 [104].

Let us emphasize the generally very low efficiencies, for example, the $a_2$ in Table 8. Its trigger could easily be improved to reach the acceptance limit [103]. We conclude that, if specialized triggers are installed, good results on light resonance physics can be achieved except for purely neutral decays. Hence even searches for glueballs might be within reach. Since their two-photon widths are expected to be at least one order of magnitude below that of normal $q\bar{q}$ states [105,106], searches for associated glueball ($G$) production, $\gamma\gamma \rightarrow \pi^0 G$ are welcome: of the order of 10 to 100 events should be produced above $p_T = 1$ GeV at 500 pb$^{-1}$ [107].

8.2 Resonance production with one off-shell photon

Resonances can also be studied in two-photon events in which one photon is far off the mass shell. Usually, this is the domain of single-tag events where the virtuality $Q^2$ is determined from a measurement of the scattered electron. The $Q^2$ range can be extended by including no-tag events, in which case $Q^2$ is reconstructed by the measurement of the $p_T$ of the resonance. In particular one may cover $Q^2$ ranges where the $Q^2$ determination from the electron is not possible (i.e. from about 0.8 GeV$^2$ to 7 GeV$^2$ at LEP2). Note that resonance production in single-tag events is just the exclusive limit of the photon structure function (i.e. $e\gamma \rightarrow eM$). The interest here is twofold. First, the meson–photon transition form factor can be measured, and secondly, spin-1 states can be produced (the Landau–Yang theorem forbids their production from two on-shell photons). Measurements of the pseudoscalar–photon transition form factors are now becoming quite precise [108]. Also the $1^{++}$ ($^3P_1$ $q\bar{q}$) state $f_1(1285)$ has been observed [109] with a $\gamma\gamma$ coupling consistent with quark-model estimates [110] for $Q^2 = 0$ photons.

The LEP experiments can cover a large $Q^2$ range with various tagging systems, e.g. for L3:

<table>
<thead>
<tr>
<th>Resonance</th>
<th>decay</th>
<th>cross sections (pb)</th>
<th>A</th>
<th>$\epsilon_{untag}$</th>
<th>Events</th>
<th>$\epsilon_{tag}$</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$</td>
<td>$\pi^+\pi^-\pi^0$</td>
<td>no trigger</td>
<td>3381</td>
<td>23</td>
<td>0.133</td>
<td>0.0188</td>
<td>10$^{-4}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$\pi^+\pi^-\pi^0$</td>
<td>no acceptance</td>
<td>1436</td>
<td>5.5</td>
<td>0.0416</td>
<td>0.0033</td>
<td>1658</td>
</tr>
<tr>
<td>$f_2$</td>
<td>$K_s K_s$</td>
<td>82.04</td>
<td>0.37</td>
<td>0.0935</td>
<td>0.0564</td>
<td>190</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

Table 8: Examples of low-mass resonances at LEP2 ($e^+e^- \rightarrow e^+e^- R$ at $\sqrt{s} = 175$ GeV for $= 500$ pb$^{-1}$). $A$ is the geometrical acceptance as described in the text and $\epsilon$ the efficiency including trigger and analysis cuts: $\epsilon_{untag}$ for untagged and $\epsilon_{tag}$ for tagged events ($0.2 \leq Q^2 \leq 0.8$ GeV$^2$, and $Q^2 \leq 7$ GeV$^2$). The third column gives the full resonance production cross-section, while the number of events is calculated taking into account the branching ratio.
VSAT (very small angle tagger) ($Q^2 = 0.2 - 0.8\text{ GeV}^2$), LUMI (luminosity monitor) ($Q^2 = 7\text{ GeV}^2$), and ECAL (the endcap electromagnetic calorimeter) ($Q^2 = 40\text{ GeV}^2$). Given the high cm energy, high $Q^2$ values should be reachable. We estimate that form-factor measurements will, even with current triggers, be possible for at least the $\eta'$, the $f_2$, and the $\eta_c$, see Table 8 and Fig. 30. Concerning the spin-1 mesons, the $f_1$ and $\chi_{c1}$ will certainly be observed. Otherwise

![Figure 30: Expected number of $\eta_c$ events as a function of $Q^2$, for an integrated luminosity of 500 $pb^{-1}$. All observable decay channels are included.](image)

the same remark as above applies: with more dedicated triggers more resonances will become accessible.

Theoretical predictions for the $\eta_c$-photon transition form factor $F_{\eta_c\gamma}(Q^2)$ [111] are shown in Fig. 31, as well as a similar calculation [112] for $F_{\pi\gamma}(Q^2)$ compared to recent data [108, 113]. For low $Q^2$, models based on vector-meson-dominance (VMD), constituent quarks, QCD sum rules or chiral perturbation theory work, in general, successfully for pseudoscalar mesons (P), see e.g. ref. [114]. The $Q^2$ dependence can be parametrized by

$$F_{\pi\gamma}(Q^2) = \frac{A_P}{1 + Q^2/A_P^2}.$$  (30)

For example, in VMD $A_P$ is related to the $V\pi\gamma$ and $V\ell^+\ell^-$ coupling constants (here $V$ is the corresponding vector state(s), i.e. $\rho$ and $\omega$ for the $\pi^0$, and $J/\psi$ for the $\eta_c$), and $A_P$ is given by the vector-meson mass.

Form factors are particularly interesting at high $Q^2$ since a factorization formula holds [115],

338
which expresses large-$Q^2$ exclusive reactions as a product of process-independent meson distribution amplitudes and perturbatively calculable short-distance coefficients. In fact, asymptotically, i.e. for $\ln Q^2/\mu_0^2$ ($\mu_0$ a typical hadronic scale 0.5-1 GeV), the distribution amplitudes are known and one derives the parameter-free result [115,116]

$$F_{\gamma\gamma}(Q^2) = \frac{2f_P}{Q^2}$$

(31)

where $f_P$ is the meson's decay constant (i.e. 130.7 MeV for the pion). A simple all-$Q^2$ formula is arrived at [115] by assuming (30) and fixing the two parameters from (31) ($\Lambda_P = 2\pi f_P$) and the PCAC value of the form factor at $Q^2 = 0$ ($\Lambda_P = 1/(2 \sqrt{2}\pi^2 f_P)$).

For finite $Q^2$, a comparison of the full calculation and data allows the determination of the distribution amplitude. The calculations shown in Fig. 31 are based on a modified hard-scattering-approach (HSA) to exclusive reactions, in which transverse degrees of freedom, representing higher twist effects, and Sudakov suppression are taken into account. The pion data nicely agree with the calculation if one uses a wave function $\exp\left[ a^2 k_+^2/2(1-x) \right]$, which leads, after $k_+$-integration, to the asymptotic distribution amplitude $x(1-x)$ ($x$ is the momentum fraction carried by the quark inside the pion). In contrast, the use of a wave function implying the Chernyak-Zhitnitsky distribution amplitude [117], leads to results in severe conflict with the data, see Fig. 31. That observation may have far-reaching consequences for our understanding of other large momentum transfer exclusive reactions, as for instance $\gamma \gamma \pi\pi$.

The $\eta_c$-photon transition form factor has not yet been measured. The predictions shown in Fig. 31 are obtained using the Bauer-Stech-Wirbel wave function [118]

$$\Psi_0(x, k_+) = N x(1-x) \exp\left[ a^2 M_{\eta_c}^2(x) 1/2 \right] \exp\left[ a^2 k_+^2 \right].$$

(32)

Its two parameters, namely the transverse size parameter $a$ and the normalization constant $N$ cannot be fully fixed from the $\eta_c$ $\gamma\gamma$ decay width. To illustrate the parameter dependence we
have taken the valence quark Fock state probability to be 0.8 and required a value of either 383 MeV or 284 MeV for the $\eta_c$ decay constant. Referring to Fig. 30, a measurement of $F_{\eta_c\gamma}(Q^2)$ up to $Q^2 \leq 10\text{GeV}^2$ seems feasible.

### 8.3 Charmonium

The heavy quark systems can, to first approximation, be described by nonrelativistic quark-potential models and many of their properties are expected to reflect the underlying dynamics of QCD. For example (see e.g. [119]),

$$\Gamma_{\eta_c \gamma\gamma} = \frac{64\pi\alpha^2}{27M_c^2} \Psi_{\eta_c}(0)^2 \frac{34}{\pi} \alpha_s(M_c).$$

(33)

Here $\Psi_{\eta_c}(0)$, the non relativistic wave function at the origin, contains all non-perturbative QCD effects. Replacing the two photons by two gluons gives $\Gamma_{\eta_c \gamma\gamma} \gamma\gamma$ which, in the potential model, is the total width $\Gamma_{\eta_c}$ of the $\eta_c$. It turns out [120] that the value of $\alpha_s(M_c)$ determined from the relation,

$$\frac{\Gamma_{\eta_c}}{\Gamma_{\eta_c \gamma\gamma}} = \frac{9\alpha_s(M_c)^2}{8\alpha^2} 1 + \frac{8.2}{\pi} \alpha_s(M_c)$$

(34)

is about 3 standard deviations below the value expected from other QCD measurements: $\alpha_s(M_c) = 0.33 \pm 0.02$ [96]. This indicates that corrections to the non-relativistic quark-potential model are non-negligible. Recently, a rigorous factorization of heavy quark-continuum decays has been developed [121, 122]. Both relativistic corrections and effects of dynamical gluons (those associated with the binding) can be calculated in a systematic way. Phenomenological studies [120, 123, 124] show results that depend crucially on the input data.

These problems can be solved by high precision LEP measurements of the two-photon widths $\Gamma_{\gamma\gamma}$ for charmonium states. In Table 9 we estimate the expected number of events for the charmonium states decaying into $4\pi$. For tagged events, the $Q^2$ dependence is modelled by a $J/\psi$ form-factor. Since the charmonia have very small branching ratios into $4\pi$ [86], the number of events can be increased substantially (factors 5–6) by measuring also other decay channels [101, 103]. Hence good measurements of the charmonium states, including the $\chi_{c1}$ are expected.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>cross sections (pb)</th>
<th>A</th>
<th>$\epsilon_{\text{untag}}$</th>
<th>Events untagged</th>
<th>$\epsilon_{\text{tag}}$</th>
<th>Events tagged</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c$</td>
<td>146.8 1.5</td>
<td>0.21</td>
<td>0.11</td>
<td>97</td>
<td>0.01</td>
<td>9.</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>25.9 0.3</td>
<td>0.22</td>
<td>0.12</td>
<td>57</td>
<td>0.01</td>
<td>6.</td>
</tr>
<tr>
<td>$\chi_{c2}$</td>
<td>18.2 0.11</td>
<td>0.22</td>
<td>0.12</td>
<td>17.5</td>
<td>0.01</td>
<td>2.</td>
</tr>
</tbody>
</table>

Table 9: Example of charmonium production expected at LEP2; given is the channel $e^+e^-\rightarrow e^+e^-\gamma R$, $R = 4\pi$ for $\bar{s} = 500\text{pb}^{-1}$ at $\bar{s} = 175\text{GeV}$. $A$ and $\epsilon$ as in Table 8.
8.4 Exclusive pair production

Two-photon exclusive meson and baryon production provides particularly clean tests of QCD. At large angles, a factorization holds [115] which tells us that the exclusive scattering amplitude is given as the product of a hard scattering amplitude and soft distribution amplitudes $\Phi_i(x, p_T^2)$. Although the latter are nonperturbative quantities, they are subjected to several constraints (evolution equation in $p_T$, asymptotic $(p_T^\infty)$ form, normalization in the case of mesons). This leads to a parameter-independent prediction for the fall-off with $s_{\gamma\gamma}$ at fixed angle $(d\sigma/dt$\nabla_s_{\gamma\gamma}^4, s_{\gamma\gamma}^6$ for meson and baryons, respectively) and essentially parameter-independent predictions for the angular distribution at moderate $s_{\gamma\gamma}$ (see e.g. [3]). Current experiments reach $s_{\gamma\gamma}$ values up to about 3 GeV, where a transition from the VMD-like angular distribution to the one predicted by the HSA just becomes visible. Production of pseudoscalar- and vector-meson pairs at LEP2 has been estimated during the last workshop: sufficient counting rates can be expected to test the predictions up to $s_{\gamma\gamma} \approx 5$–6 GeV [3].

Calculations within the HSA have been extended in various ways. On the one hand, the potentially dangerous endpoint regions in the $x$ integration have been shown to be suppressed by Sudakov form factors [125]. On the other hand, predictions now exist for the pair production of baryons [126], heavy mesons ($D, D'$) [127], and (light) mesons with non-zero orbital angular momenta [128]. Among the $L > 0$ mesons, $a_2^\pm a_2^- \pm a_2^\mp$ might have a rate large enough to be observable at LEP2, namely about 500 (50) events for a $p_T$-cut of 1 GeV (2 GeV) (without taking into account experimental cuts).

The standard (or possibly improved) HSA gives the leading-twist cross section at fixed cm scattering angle or, equivalently, at fixed $t/s_{\gamma\gamma}$. At LEP2, a new domain of perturbative QCD may be entered. This is the region $\mu_0^2 \ll t \ll s_{\gamma\gamma}$ (so-called semi-hard region) which is discussed in sec. 5 where numerical estimates are presented.

8.5 Summary

In summary, many light resonances can be studied with good counting rates at LEP2, provided the triggers of the experiments will be extended to low-momentum charged particles. The trigger efficiency and acceptance generally increases with the resonance mass. Good results can be expected for the four lowest-lying $C = +1$ charmonium states. Bottomonium resonances, however, are beyond the reach of LEP2. Measurements of spin-1 mesons (notably the $f_1$ and the $\chi_c(1)$ and various meson–photon transition form factors will be possible both in single-tag events and in untagged events through $Q^2$ reconstruction from the hadronic final state. Measurements of conventional pair production of $S$-wave mesons should be possible up to 5–6 GeV CM energy, allowing for definite tests of the HSA. Effects of QCD in a new perturbative domain are expected to become accessible in semi-hard reactions.

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1 Synopsis

1. The understanding of the mechanism responsible for the breakdown of the electroweak symmetry is one of the central problems in particle physics. If the fundamental particles – leptons, quarks and gauge bosons – remain weakly interacting up to high energies, then the sector in which the electroweak symmetry is broken must contain one or more fundamental scalar Higgs bosons with masses of the order of the symmetry breaking scale \( v \sim 174 \text{ GeV} \). Alternatively, the symmetry breaking could be generated dynamically by novel strong forces at the scale \( \Lambda \sim 1 \text{ TeV} \). However, no compelling model of this kind has yet been formulated which provides a satisfactory description of the fermion sector and reproduces the high precision electroweak measurements.

2. The simplest mechanism for the breaking of the electroweak symmetry is realized in the Standard Model (SM) [1]. To accommodate all observed phenomena, a complex isodoublet scalar field is introduced which, through self-interactions, spontaneously breaks the electroweak symmetry \( \text{SU}(2)_L \times \text{U}(1)_Y \) down to the electromagnetic \( \text{U}(1)_{\text{EM}} \) symmetry, by acquiring a non-vanishing vacuum expectation value. After the electroweak symmetry breakdown, the interaction of the gauge bosons and fermions with the isodoublet scalar field generates the masses of these particles [2]. In this process, one scalar field remains in the spectrum, manifesting itself as the physical Higgs particle \( H \).

The mass of the SM Higgs boson is constrained in two ways. Since the quartic self-coupling of the Higgs field grows indefinitely with rising energy, an upper limit on the Higgs mass is obtained by demanding that the SM particles remain weakly interacting up to a scale \( \Lambda \) [3]. On the other hand, stringent lower bounds on the Higgs mass can be derived from the requirement of stability of the electroweak vacuum [3, 4]. Hence, if the Standard Model is valid up to scales near the Planck scale, then the SM Higgs mass is restricted to the range between \( \sim 130 \text{ GeV} \) and \( \sim 180 \text{ GeV} \), for a top-quark mass \( M_t \sim 176 \text{ GeV} \). Moreover, if the Higgs particle is discovered in the mass range up to the 100 GeV accessible at LEP2, this will imply that new physics beyond the Standard Model should exist at energies below a scale \( \Lambda \) of order 10 TeV. [These bounds become stronger (weaker) for larger (smaller) values of the top quark mass].

The high precision electroweak data give a slight preference to Higgs masses of less than 100 GeV, despite the fact that the electroweak observables depend only logarithmically on the Higgs mass through radiative corrections [5]. They do not, however, exclude values up to \( \sim 700 \text{ GeV} \) at the 2\( \sigma \) level [6], thus sweeping the entire Higgs mass range of the Standard Model. By searching directly for the SM Higgs particle, the LEP experiments [7] have set a lower bound, \( m_H > 65.2 \text{ GeV} \) [95\% CL], on the Higgs mass.

The dominant production mechanism for the SM Higgs boson within the energy range of LEP2 is the Higgs-strahlung process \( e^+e^- \rightarrow ZH \) in which the Higgs boson is emitted from a virtual \( Z \) boson [8]. The cross section monotonically falls from \( \sim 1 \text{ pb} \) at \( m_H \sim 65 \text{ GeV} \) down to very small values for Higgs masses near the kinematical threshold. The cross section for the
production of Higgs bosons via $WW$ fusion \cite{9,10} is nearly two orders of magnitude smaller at LEP2, except at the edge of the phase space for Higgs--strahlung where both are small. In the mass range between 60 and 120 GeV, the dominant decay mode of the SM Higgs particle is $b \bar{b}$ \cite{12}. Branching ratios for Higgs decays to $\tau^+\tau^-$, $c\bar{c}$ and $gg$ final states are suppressed by an order of magnitude or more.

The experimental search for the SM Higgs boson at LEP2 will be based primarily on the Higgs--strahlung process. The $Z$ boson can easily be reconstructed in all charged leptonic and hadronic decay channels while the Higgs decay mostly leads to $b \bar{b}$ and, less frequently, to $\tau^+\tau^-$ final states. Moreover, neutrino decays of the $Z$ boson, augmented by $W$ fusion events, can be exploited in the experimental analyses. Higgs events can be searched for with an average efficiency of about 25%. Exploiting micro--vertex detection for tagging $b$ quarks, the Higgs events can be well discriminated from the main background process of $ZZ$ production even for a Higgs mass near the $Z$ mass. When the results of all four LEP experiments are combined, after accumulating an integrated luminosity $L = 150 \text{ pb}^{-1}$ per experiment, the SM Higgs boson can be discovered in the mass range up to $m_H \simeq 95$ GeV at LEP2 for a total center of mass energy of $\sqrt{s} = 192$ GeV.

3. If the Standard Model is embedded in a Grand Unified Theory (GUT) at high energies, then the natural scale of electroweak symmetry breaking would be close to the unification scale $M_{\text{GUT}}$, due to the quadratic nature of the radiative corrections to the Higgs mass. Supersymmetry \cite{13} provides a solution to this hierarchy problem through the cancellation of these quadratic divergences via the contributions of fermionic and bosonic loops \cite{14}. Moreover, the Minimal Supersymmetric extension of the Standard Model (MSSM) can be derived as an effective theory from supersymmetric Grand Unified Theories \cite{15}, involving not only the strong and electroweak interactions but gravity as well. A strong indication for the realization of this physical picture in nature is the excellent agreement between the value of the weak mixing angle $\sin^2 \theta_W$ predicted by the unification of the gauge couplings, and the measured value \cite{15}-\cite{21}. In particular, if the gauge couplings are unified in the minimal supersymmetric theory at a scale $M_{\text{GUT}} = \mathcal{O}(10^{16}\text{ GeV})$ and if the mass spectrum of the supersymmetric particles is of order $m_Z$, then the electroweak mixing angle is predicted to be $\sin^2 \theta_W = 0.2336 \pm 0.0017$ in the MS scheme for $\alpha_s = 0.118 \pm 0.006$, to be compared with the experimental result $\sin^2 \theta_W^{\text{exp}} = 0.2314 \pm 0.0003$. Threshold effects at both the low scale of the supersymmetric particle spectrum and at the high unification scale may drive the prediction for $\sin^2 \theta_W$ even closer to its experimental value.

In the past two decades a detailed picture has been developed of the Minimal Supersymmetric Standard Model. In this extension of the Standard Model the Higgs sector is built up of two doublets, necessary to generate masses for up- and down-type fermions in a supersymmetric theory, and to render the theory anomaly--free \cite{22}. The Higgs particle spectrum consists of a quintet of states: two CP--even scalar ($h, H$), one CP-odd pseudoscalar neutral ($A$), and a pair of charged ($H^\pm$) Higgs bosons \cite{23}.

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Since the tree–level quartic Higgs self–couplings in this minimal theory are determined in terms of the gauge couplings, the mass of the lightest CP–even Higgs boson $h$ is constrained very stringently. At tree–level, the mass $m_h$ has been predicted to be less than the $Z$ mass [24, 25]. Radiative corrections to $m_h^2$ grow as the fourth power of the top mass and the logarithm of the stop masses. They shift the upper limit to about $\lesssim 150$ GeV [26, 27], depending on the MSSM parameters.

The upper limit on $m_h$ depends on $\tan \beta$, the ratio of the vacuum expectation values associated with the two neutral scalar Higgs fields. This parameter can be constrained by additional symmetry concepts. If the theory is embedded into a grand unified theory, the $b$ and $\tau$ Yukawa couplings can be expected to unify at $M_{\text{GUT}}$. The condition of $b-\tau$ Yukawa coupling unification determines the value of the top–quark Yukawa coupling at low energies [28], thus explaining qualitatively the large value of the top quark mass [29–32]. In the small $\tan \beta$ regime, the top–quark mass is strongly attracted to its infrared fixed point [34], implying a strong correlation between the top–quark mass and $\tan \beta$. The large $\tan \beta$ regime is more complex because of possible large radiative corrections to the $b$ quark mass associated with supersymmetric particle loops [35, 36]. For small $\tan \beta$ and $M_t < 176$ GeV, the upper bound on the mass of the lightest neutral Higgs particle is reduced to $\sim 100$ GeV. This mass bound is just at the edge of the kinematical range accessible at a center of mass energy of 192 GeV [37] – raising the prospects of discovering this Higgs boson at LEP2.

The structure of the Higgs sector in the MSSM at tree level is determined by one Higgs mass parameter, which we choose to be $m_A$, and $\tan \beta$. The mass of the pseudoscalar Higgs boson $m_A$ may vary between the present experimental lower bound of 45 GeV [7] and $\sim 1$ TeV, the heavy neutral scalar mass $m_H$ is in general larger than $\sim 120$ GeV, and the mass of the charged Higgs bosons exceeds $\sim 90$ GeV. Due to the kinematics the primary focus at LEP2 will be on the light scalar particle $h$ and on the pseudoscalar particle $A$. In the decoupling limit of large $A$ mass [yielding large $H, H^\pm$ masses], the Higgs sector becomes SM like and the properties of the lightest neutral Higgs boson $h$ coincide with the properties of the Higgs boson $H$ in the Standard Model [38].

The processes for producing the Higgs particles $h$ and $A$ at LEP2 are Higgs–strahlung $e^+e^- \rightarrow Zh$, and associated pair production $e^+e^- \rightarrow Ah$ [39]. These two processes are complementary. For small values of $\tan \beta$ the $h$ Higgs boson is produced primarily through Higgs–strahlung; if kinematically allowed, associated $Ah$ production becomes increasingly important with rising $\tan \beta$. The typical size of the cross sections is of order 1 pb or slightly below. The dominant decay modes of the $h, A$ Higgs bosons are decays into $b$ and $\tau$ pairs, if we consider SM particles in the final state [12]. Only near the maximal $h$ mass for a given value of $\tan \beta$ do $c\bar{c}$ and $gg$ decays occur at a level of several percent, in accordance with the decoupling theorem. However, there are areas in the SUSY $[\mu, M_2]$ parameter space where Higgs particles can decay into invisible $\chi^0_1\chi^0_1$ LSP final states or possibly other neutralino and chargino final states [40, 41]. If the LSP channel is open, the $h$ and $A$ invisible decay branching ratios can be
close to 100% for small to moderate values of $\tan \beta$. However, the Higgs boson $h$ can still be found in the Higgs–strahlung process. The pseudoscalar $A$, produced only in association with $h$, would be hard to detect in this case since both particles decay into invisible channels for small $\tan \beta$.

The experimental search for $h$ in the Higgs–strahlung process follows the lines of the Standard Model, while for associated $Ah$ production $b\bar{b}b\bar{b}$ and $b\bar{b}\tau^+\tau^-$ final states can be exploited. Signal events of the $Ah$ type can be searched for with an efficiency of about 30%; the background rejection is somewhat more complicated than for Higgs–strahlung, due to two unknown particles in the signal final state. For small to moderate $\tan \beta$, $h$ particles with masses up to $\sim 100$ GeV can be discovered in the Higgs–strahlung process. For large $\tan \beta$ the experimentally accessible limits are typically reduced by about 10 GeV. The pseudoscalar Higgs boson $A$ is accessible for masses up to about 80 GeV. [These limits are based on the LEP2 energy of 192 GeV and an integrated luminosity of $L = 150$ pb$^{-1}$ per experiment, with all four experiments pooled.]

The supersymmetric theory may be distinguished from the Standard Model if one of the following conditions occurs: (i) at least two different Higgs bosons are found; (ii) precision measurements of production cross sections and decay branching ratios of $h$ can be performed at a level of a few per cent; and (iii) genuine SUSY decay modes are observed. Near the maximum $h$ mass, the decoupling of the heavy Higgs bosons reduces the MSSM to the SM Higgs boson except for the SUSY decay modes.

4. In summary. If a neutral scalar Higgs boson is found at LEP2, new physics beyond the Standard Model should exist at scales of order 10 TeV. In the framework of the Minimal Supersymmetric extension of the Standard Model, there are good prospects of discovering the lightest of the neutral scalar Higgs bosons at LEP2. Even though this discovery cannot be ensured, observation or non-observation will have far reaching consequences on the possible structure of low-energy supersymmetric theories.

In section 2 the theoretical analysis and experimental simulations for the search for the Higgs boson in the Standard Model are presented. In section 3 the Higgs spectrum and the couplings in the MSSM as well as the relevant cross sections and branching ratios are studied. In addition, the results of the experimental simulations are thoroughly discussed. Section 4 investigates opportunities of detecting Higgs particles at LEP2 within non-minimal extensions of the SM and the MSSM. In particular, the next-to-minimal extension of the MSSM with an additional isoscalar Higgs field (NMSSM) is studied.
2 The Standard–Model Higgs Particle

2.1 Mass Bounds

(i) Strong interaction limit and vacuum stability. Within the Standard Model the value of the Higgs mass $m_H$ cannot be predicted. The mass $m_H = \sqrt{2\lambda v}$ is given as a function of the vacuum expectation value of the Higgs field, $v = 174$ GeV, and the quartic coupling $\lambda$ which is a free parameter. However, since the quartic coupling grows with rising energy indefinitely, an upper bound on $m_H$ follows from the requirement that the theory be valid up to the scale $M_{\text{Planck}}$ or up to a given cut-off scale $\Lambda$ below $M_{\text{Planck}}$ [3]. The scale $\Lambda$ could be identified with the scale at which a Landau pole develops. However, in the following the upper bound on $m_H$ shall be defined by the requirement $\lambda(\Lambda)/4\pi \leq 1$ so that $\Lambda$ characterizes the energy where the system becomes strongly interacting. [This scale is very close to the scale associated with the Landau pole in practice.] The upper bound on $m_H$ depends mildly on the top-quark mass through the impact of the top-quark Yukawa coupling on the running of the quartic coupling $\lambda$,

$$\frac{d\lambda}{dt} = \frac{6}{16\pi^2} \lambda^2 + \lambda h_t^2 - h_t^4 + \text{elw. corrections}$$

(1)

with $t = \ln(Q^2/\Lambda^2)$. The first two terms inside the parentheses are crucial in driving the quartic coupling to its perturbative limit. On the other hand, the requirement of vacuum stability in the SM imposes a lower bound on the Higgs boson mass, which depends crucially on the top-quark mass as well as on the cut-off $\Lambda$ [3, 4]. Again, the dependence of this lower bound on $M_t$ is due to the effect of the top-quark Yukawa coupling on the quartic coupling of the Higgs potential [third term inside the parentheses of eq.(1)], which drives $\lambda$ to negative values at large scales, thus destabilizing the standard electroweak vacuum.

Fig.1 shows the perturbativity and stability bounds on the Higgs boson mass of the SM for different values of the cut-off $\Lambda$ at which new physics is expected. From the point of view of LEP physics, the upper bound on the SM Higgs boson mass does not pose any relevant restriction. The lower bound on $m_H$, instead, needs to be carefully considered. To define the conditions for vacuum stability in the SM and to derive the lower bounds on $m_H$ as a function of $M_t$, it is necessary to study the Higgs potential for large values of the Higgs field $\phi$ and to determine under which conditions it develops an additional minimum deeper than the electroweak minimum. The renormalization group improved effective potential of the SM is given by

$$V_{\text{eff}} = V_0 + V_1 \simeq -m^2(t)\phi^2(t) + \frac{\lambda(\phi)}{2} \phi^4(t)$$

(2)

where $V_0$ and $V_1$ are the tree–level potential and the one–loop correction, respectively. A rigorous analysis of the structure of the potential has been done in Ref.[4]. Quite generally it follows that the stability bound on $m_H$ is defined, for a given value of $M_t$, as the lower value of $m_H$ for which $\lambda(\phi) \geq 0$ for any value of $\phi$ below the scale $\Lambda$ at which new physics beyond the SM should appear. From eq.(1) it is clear that the stability condition of the effective potential demands new physics at lower scales for larger values of $M_t$ and smaller values of $m_H$. 358
From Fig. 1 it follows that for $M_t = 175$ GeV and $m_H < 100$ GeV [i.e. in the LEP2 regime] new physics should appear below the scale $\Lambda \sim$ a few to $100$ TeV. The dependence on the top-quark mass however is noticeable. A lower value, $M_t \approx 160$ GeV, would relax the previous requirement to $\Lambda \sim 10^3$ TeV, while a heavier value $M_t \approx 190$ GeV would demand new physics at an energy scale as low as $2$ TeV.

The previous bounds on the scale at which new physics should appear can be relaxed if the possibility of a metastable vacuum is taken into account [42]. In fact, if the effective potential of the SM has a non-standard stable minimum deeper than the standard minimum, the decay of the electroweak minimum by thermal fluctuations or quantum tunnelling to the stable minimum must be suppressed. In this case, the lower bounds on $m_H$ follow from requiring that no transition at any finite temperature occurs, so that all space remains in the metastable electroweak vacuum. In practice, if the metastability arguments are taken into account, the lower bounds on $m_H$ become gradually weaker. They seem to disappear if the cut-off of the theory is at the TeV scale; however, the calculations are technically not reliable in this energy regime. Moreover, the metastability bounds depend on several cosmological assumptions which may be relaxed in several ways.

(ii) Estimate of the Higgs mass from electroweak data. Indirect evidence for a light Higgs boson comes from the high-precision measurements at LEP [6] and elsewhere. Indeed, the fact that the SM is renormalizable only after including the top and Higgs particles in the loop corrections shows that the electroweak observables should be sensitive to these particle masses. Although
the sensitivity to the Higgs mass is only logarithmic, while the sensitivity to the top-quark mass is quadratic, the increasing precision of present experiments makes it possible to derive $\chi^2$ curves as a function of $m_H$. Several groups [6] have performed an analysis of $m_H$ by means of a global fit to the electroweak data, including low and high energy data. In the light of the recent direct determination of $M_t$, the results favor a light Higgs boson. With all LEP, SLD, $p\bar{p}$ and $\nu N$ data included, a central value for $m_H$ around 80 GeV and $M_t \sim 170$ GeV is obtained [6]. However, the recently reported LEP values of $R_b \equiv \Gamma_{Z\rightarrow b\bar{b}}/\Gamma_{Z\rightarrow hadrons}$ and $R_c \equiv \Gamma_{Z\rightarrow c\bar{c}}/\Gamma_{Z\rightarrow hadrons}$ which are more than 2 standard deviations away from the SM predictions, and the left-right asymmetries of SLD which still lead to a $2\sigma$ discrepancy in $\sin^2 \theta_W$ compared with LEP analyses, have drastic effects on the SM fits. Fig.2 shows $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ as a function of $m_H$; the curve is rather flat at the minimum due to the mild logarithmic dependence of the observables on $m_H$. It should be noticed in this context that the bounds on $m_H$ become very weak if $R_b$, $R_c$ and/or the left-right asymmetries are excluded from the data.
In summary. It is clear that the indirect bounds on $m_H$ cannot assure the existence of a light Higgs boson at the reach of LEP2. However, the fact that the best fit to the present high-precision data tends to prefer a light SM Higgs boson, indicates that this particle may be found either at LEP or LHC. On the other hand, the stability bounds imply that if the Higgs boson is light, new physics beyond the Standard Model should appear at relatively low energies in the TeV regime.

2.2 Production and Decay Processes

The main mechanism for the production of Higgs particles in $e^+e^-$ collisions at LEP2 energies is the radiation off the virtual $Z$-boson line [8],

$$\text{Higgs-strahlung : } e^+e^- \rightarrow ZH \quad (3)$$

The fusion process [9, 10, 11] in which the Higgs bosons are formed in $WW$ collisions, the virtual $W$'s radiated off the electrons and positrons,

$$\text{WW fusion : } e^+e^- \rightarrow \bar{\nu}_e\nu_e H \quad (4)$$

has a considerably smaller cross section at LEP energies. It is suppressed by an additional power of the electroweak coupling with respect to the Higgs-strahlung process, becoming competitive only at the edge of phase space in (3), where the $Z$ boson turns virtual. In this corner, however, both cross sections are small and the experimentally accessible mass parameter space will be extended only slightly by the fusion channel.

![Figure 3: Higgs-strahlung and WW fusion of the SM Higgs boson.](image)

2.2.1 Higgs-strahlung

The cross section for the Higgs-strahlung process can be written in the following compact form:

$$\sigma(e^+e^- \rightarrow ZH) = \frac{G_F^2 m_Z^4}{96\pi s} \, v_e^2 + a_e^2 \, \frac{\lambda + 12m_Z^2/s}{(1 - m_Z^2/s)^2} \quad (5)$$

where $\sqrt{s}$ denotes the center-of-mass energy, and $a_e = -1$, $v_e = -1 + 4s_\nu s$ are the $Z$ charges of the electron; $\lambda = (1 - m_H^2/s - m_Z^2/s)^2 - 4m_H^2m_Z^2/s^2$ is the usual two-particle phase space
The radiative corrections to the cross section are well under control. The genuine electroweak corrections [43] are small at the LEP energy, less than 1.5\% (for a recent review see Ref.[44]). By contrast, photon radiation [45] affects the cross section in a significant way. The bulk of the corrections, real and virtual contributions due to photons and $e^+e^-$ pairs, can be accounted for by convoluting the Born cross section in eq.(5) with the radiator function $G(x)$,

$$
\langle \sigma \rangle = \frac{1}{x_H} \int \frac{dx}{x} G(x) \sigma(x s)
$$

with $x_H = m_H^2/s$. The radiator function is known to order $\alpha^2$, including the exponentiation of the infrared sensitive part,

$$
G(x) = \beta (1 - x)^{\beta - 1} \delta_{VS} + \delta_H(x)
$$

where $\delta_{VS}$ and $\delta_H$ are polynomials in $\log s/m_e^2$ and $\beta = \frac{2\alpha}{s} [\log s/m_e^2 - 1]$. $\delta_{VS}$ accounts for virtual and soft photon effects, $\delta_H$ for hard photon radiation. The $\delta$'s are given in Ref.[45].

The cross-section for Higgs-strahlung is shown in Fig.4 for the three representative energy values $\sqrt{s} = 175$, 192 and 205 GeV as a function of the Higgs-mass [46]. The curves include all genuine electroweak and QED corrections introduced above. The $Z$ boson in the final state is allowed to be off-shell, so that the tails of the curves extend beyond the on-shell limit $m_H = \sqrt{s} - m_Z$. [The Higgs boson is so narrow, $\Gamma_H < 3$ MeV for $m_H < 100$ GeV, that the particle need not be taken off-shell.] From a value of order 0.3 to 1 pb at $m_H \sim 110$ GeV, the cross section falls steadily, reaching a level of less than 0.05 pb at the mass $m_H \sim 90$ GeV.

Since the Higgs particle decays predominantly to $b\bar{b}$ and $\tau^+\tau^-$ pairs, the observed final state consists of four fermions. Among the possible final states, the channel $\mu^+\mu^-b\bar{b}$, the $\mu$ pair being generated by the $Z$ decay, has a particularly simple structure. Background events of this type are generated by double vector-boson production $e^+e^- \rightarrow Z^*Z^*$, $Z^*\gamma^*$ and $\gamma^*\gamma^*$ with the virtual $Z^*$, $\gamma^*$ decaying to $\mu^+\mu^-$ and $b\bar{b}$; $Z$ final states generate by far the dominant contribution. Since these processes are suppressed by one and two additional powers of the electroweak coupling compared with the signal [except for $m_H \sim m_Z$], the background can be controlled fairly easily up to the kinematical limit of the Higgs signal. This is demonstrated in Tables 1/2 and Fig. 6 where signal and background cross sections for the process $e^+e^- \rightarrow \mu^+\mu^-b\bar{b}$ are compared for three Higgs masses at $\sqrt{s} = 192$ GeV. The invariant $\mu^+\mu^-$ mass is restricted to $m_Z \pm 25$ GeV and the invariant $b\bar{b}$ mass is cut at $m(b\bar{b}) > 50$ GeV. The following conclusions can be drawn from the tables and the figure: (i) The signal-to-background ratio decreases steadily with rising Higgs mass from a value of about three near $m_H = 65$ GeV; (ii) The initial state QED radiative corrections are large, varying between 10 and 20\%; (iii) The cross sections are lowered by taking non-zero $b$ quark masses into account, but only marginally at a level of less than 1\%. Since massless fermions are coupled to spin-vectors in $Z^*$ decays but to spin-scalars in Higgs decays, signal and background amplitudes do not interfere as long as $b$ quark masses are neglected.
Figure 4: The cross section for Higgs-strahlung as a function of the Higgs mass for three representative energy values [QED and electroweak radiative corrections included].

Figure 5: Higgs-strahlung (dashed) and WW fusion (long-dashed) processes for Higgs production in the cross-over region [without radiative corrections]. The solid line shows the total cross section for both processes including the (dotted line) interference term.
The angular distribution of the $Z/H$ bosons in the Higgs-strahlung process is sensitive to the spin-parity quantum numbers $J^P = 0^+$ of the Higgs particle. At high energies the $Z$ boson is produced in a state of longitudinal polarization according to the equivalence theorem so that the angular distribution approaches asymptotically the $\sin^2 \theta$ law, where $\theta$ is the polar angle between the $Z/H$ flight direction and the $e^+e^-$ beam axis. At non-asymptotic energies the distribution is shoaled \cite{47},

$$\frac{d\sigma}{d\cos \theta} \sim \lambda \sin^2 \theta + 8m_Z^2/s$$  \hspace{1cm} (8)

becoming independent of $\theta$ at the threshold. Were a pseudoscalar particle produced in association with the $Z$, the angular distribution would be given by $\sim (1 + \cos^2 \theta)$, independent of the energy; the $Z$ polarization would be transverse in this case. Thus, the angular distribution is sensitive to the assignment of spin-parity quantum numbers to the Higgs particle. The coefficients of the $\sin^2 \theta$ term and the constant term are independent and could be modified separately by additional effective $ZZH$, $\gammaZH$ couplings or $eeZH$ contact terms induced by interactions outside the Standard Model \cite{48}.

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2.2.2 The WW Fusion Process

The final state in which the Higgs particle is produced in association with neutrinos

\[ e^+e^- \rightarrow H + \nu\bar{\nu} \tag{9} \]

is built up by two different mechanisms, Higgs-strahlung with $Z$ decays to the three types of neutrinos and WW fusion [9, 10, 11, 49, 50]. For $\nu\bar{\nu}$ final states the two amplitudes interfere. At $e^+e^-$ energies above the $HZ$ threshold for on-shell $Z$, Higgs-strahlung is by far the dominant process, while below the $HZ$ threshold the fusion process becomes dominant. Correspondingly, the interference term is most important near the threshold where the cross-over between the two mechanisms occurs. The cross section for Higgs-strahlung above the $HZ$ threshold is of order $g^2_W$ while below the threshold it is suppressed by the additional electroweak vertex as well as by the off-shell $Z$ propagator. The fusion cross-section is of order $g^4_W$ and therefore small at LEP energies where no log $s/m_H^2$ enhancement factors are effective.\(^1\) The cross section for WW fusion can be expressed in a compact form [49]:

\[
\sigma(e^+e^- \rightarrow \nu\bar{\nu}H) = \frac{G_F^2 m_W^4}{4\sqrt{2}\pi^3} \int^1_{x_H} \int^1_x \frac{dy}{[1+(y-x)/xW]^2} \]

\[
F(x,y) = \frac{2x}{y^3} - \frac{1+3x}{y^2} + \frac{2+x}{y} - 1 - \frac{z}{1+z} \log(1+z) + \frac{xz^2(1-y)}{y^3} \tag{11} \]

with $x_H = m_H^2/s$, $x_W = m_W^2/s$ and $z = y(x-x_H)/(xxW)$. The more involved analytic form of the interference term between fusion and Higgs-strahlung [11] is given in the Appendix 5.1.

\(^1\)The cross-section for $ZZ$ fusion is reduced by another order of magnitude since the leptonic NC couplings are considerably smaller than the CC couplings.

Figure 6: Comparison of the Higgs signal with the background in the $\mu^+\mu^- bb$ final state for zero and non-zero quark mass.
The size of the various contributions to the cross section for the final state $e^+e^- \rightarrow H + \nu$ is shown in Fig. 5 at $\sqrt{s} = 192$ GeV. The Higgs-strahlung includes all three neutrinos in the final state. The nominal threshold value of the Higgs mass for on-shell $Z$ production in Higgs-strahlung is $m_H = 101$ GeV at $\sqrt{s} = 192$ GeV. A few GeV above this mass value the fusion mechanism becomes dominant while the Higgs-strahlung becomes rapidly more important for smaller Higgs masses. In the cross-over range, the cross-sections for fusion, Higgs-strahlung and the interference term are of the same size. With a cross section of the order of 0.01 pb only a few events can be generated in the cross-over region for the integrated luminosity at LEP. Dedicated efforts are therefore needed to explore this domain experimentally and to extract the signal from the event sample $e^+e^- \rightarrow b\bar{b} + \nu$ which includes several background channels. Nevertheless, $WW$ fusion can extend the Higgs mass range that can be explored at LEP2 by a few (perhaps very valuable) GeV.

2.2.3 Higgs Decays

The Higgs decay width is predicted in the Standard Model to be very narrow, being less than 3 MeV for $m_H$ less than 100 GeV. The width of the particle can therefore not be resolved experimentally. The main decay modes (Fig.7), relevant in the LEP2 Higgs mass range, are in the following channels [12, 46]:

\[
\begin{align*}
\text{quark decays} & : \ H \rightarrow b\bar{b} \text{ and } c\bar{c} \\
\text{lepton decay} & : \ H \rightarrow \tau^+\tau^- \\
\text{gluon decay} & : \ H \rightarrow gg \\
\text{W boson decay} & : \ H \rightarrow WW^*
\end{align*}
\] (12)

The $b\bar{b}$ decays are by far the leading decay mode, followed by $\tau$, charm, and gluon decays at a level of less than 10%. Only at the upper end of the mass range do decays of the Higgs particle to $W$ pairs start playing an increasingly important role.

![Diagram](Figure 7: The main decay modes of Higgs particles in the LEP2 mass range.)

The theoretical analysis of the Higgs decay branching ratios is not only important for the prediction of signatures to define the experimental search techniques. In addition, once the Higgs boson is discovered, the measurement of the branching ratios will be necessary to determine its couplings to other particles. This will allow us to explore the physical nature of the Higgs particle and to encircle the Higgs mechanism as the mechanism for generating the
masses of the fundamental particles. In fact, the strength of the Yukawa coupling of the Higgs boson to fermions, \( g_{\gamma H} = [\sqrt{2} G_F]^{1/2} m_f \), and the couplings to the electroweak \( V = W, Z \) gauge bosons, \( g_{V VH} = 2 [\sqrt{2} G_F]^{1/2} m_Z^2 \), both grow with the masses of the particles. While the latter can be measured through the production of Higgs particles in the Higgs-strahlung and \( WW \) fusion processes, fermionic couplings can be measured at LEP only through decay branching ratios.

**Higgs decay to fermions.** The partial width of the Higgs decay to \( \tau^+ \tau^- \) pairs is given by [51]

\[
\Gamma(H \to \tau^+ \tau^-) = \frac{G_F m_{\tau}^2}{4\sqrt{2\pi}} m_H \tag{13}
\]

For the decay into \( b\bar{b} \) and \( c\bar{c} \) quark pairs, QCD radiative corrections [52] must be included which are known up to order \( \alpha_s^2 \) [in the \( \delta' \) term up to order \( \alpha_s^3 \)],

\[
\Gamma(H \to q\bar{q}) = \frac{3G_F}{4\sqrt{2\pi}} m_q^2(m_H) m_H \left[ 1 + 5.67 \frac{\alpha_s}{\pi} + (35.94 - 1.36N_F + \delta + \delta') \frac{\alpha_s^2}{\pi} \right] \tag{14}
\]

\( \delta \) accounts for the top-quark triangle coupled to the \( q\bar{q} \) final state in second order by 2-gluon \( s \)-channel exchange [53], \( \delta = 1.57 - \frac{3}{2} \log(m_b^2/M_t^2) + \frac{1}{9} \log^2(m_H^2/m_b^2) \), while \( \delta' \) accounts for Higgs decays to two gluons with one gluon split into a \( q\bar{q} \) pair [12], discussed in detail below.

The strong coupling \( \alpha_s \) is to be evaluated at the scale \( m_H \), and \( N_F = 5 \) is the number of active flavors [all quantities defined in the \( \overline{\text{MS}} \) scheme]. The bulk of the QCD corrections can be absorbed into the running quark masses evaluated at the scale \( m_H \),

\[
m_q(m_H) = m_q(M_q) \frac{\alpha_s(m_H)}{\alpha_s(M_q)} \frac{(\pi^2 - 2N_F)}{1 + c_1 [\alpha_s(m_H)/\pi] + c_2 [\alpha_s(m_H)/\pi]^2} \tag{15}
\]

In the case of bottom (charm) quarks, the coefficients \( c_1 \) and \( c_2 \) are 1.17 (1.01) and 1.50 (1.39), respectively. Since the relation between the pole mass \( M_q \) of the charm quark and the \( \overline{\text{MS}} \) mass \( m_c \) evaluated at the pole mass is badly convergent, the running quark masses \( m_q(M_q) \) lend themselves as the basic mass parameters in practice. They have been extracted directly from QCD sum rules evaluated in a consistent \( O(\alpha_s) \) expansion [54]. Typical values of the running \( b, c \) masses at the scale \( \mu = 100 \ \text{GeV} \), which is of the order of the Higgs mass, are displayed in Table 3. The evolution has been performed for the QCD coupling \( \alpha_s(m_Z) = 0.118 \pm 0.006 \). The large uncertainty in the running charm mass is a consequence of the small scale at which the evolution starts and where the errors of the QCD coupling are very large. In any case the value of the c mass, relevant for the prediction of the c branching ratio of the Higgs particle, is reduced to about 600 MeV.

An additional mechanism for \( b, c \) quark decays of the Higgs particle [12] is provided by the gluon decay mechanism where virtual gluons split into \( b\bar{b}, c\bar{c} \) pairs, \( H \to g g^* \to g b\bar{b}, g c\bar{c} \). These contributions add to the QCD corrected partial widths (14) in which the \( b, c \) quarks are coupled to the Higgs boson directly. As long as quark masses are neglected in the final states, the two amplitudes do not interfere. In this approximation, the contributions of the
The splitting channels are obtained by taking the differences of the widths \( H \to gg, q\bar{q}g \) between \( N_F = 5 \) and 4 for \( b \), and \( N_F = 4 \) and 3 for \( c \) final states, given below in eq.\((16)\). The \( b/\bar{b} \) and the \( c/\bar{c} \) quarks are in general emitted into two different parts of the phase space for the two mechanisms; for the direct process the flight directions tend to be opposite, while by contrast for gluon splitting they are parallel.

The electroweak radiative corrections to fermionic Higgs decays are well under control \([55, 44]\). If the Born formulæ are parametrized in terms of the Fermi coupling \( G_F \), the corrections are free of large logarithms associated with light fermion loops. For \( b, c, \tau \) decays the electroweak corrections are of the order of one percent.

### Table 3: The running \( b, c \) quark masses in the \( \overline{\text{MS}} \) scheme at the scale \( \mu = 100 \) GeV. The initial values \( m_Q(M_Q) \) of the evolution are extracted from QCD sum rules; the pole masses \( M_Q^{\text{pt2}} \) are defined by the \( \mathcal{O}(\alpha_s) \) relation with the running masses \( m_Q(M_Q^{\text{pt2}}) = M_Q^{\text{pt2}}/[1 + 4\alpha_s/3\pi] \).

<table>
<thead>
<tr>
<th>( \alpha_s(m_Z) )</th>
<th>( m_Q(M_Q) )</th>
<th>( M_Q = M_Q^{\text{pt2}} )</th>
<th>( m_Q(\mu = 100 \text{ GeV}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b ) 0.112</td>
<td>(4.26 ± 0.02) GeV</td>
<td>(4.62 ± 0.02) GeV</td>
<td>(3.04 ± 0.02) GeV</td>
</tr>
<tr>
<td>0.118</td>
<td>(4.23 ± 0.02) GeV</td>
<td>(4.62 ± 0.02) GeV</td>
<td>(2.92 ± 0.02) GeV</td>
</tr>
<tr>
<td>0.124</td>
<td>(4.19 ± 0.02) GeV</td>
<td>(4.62 ± 0.02) GeV</td>
<td>(2.80 ± 0.02) GeV</td>
</tr>
<tr>
<td>( c ) 0.112</td>
<td>(1.25 ± 0.03) GeV</td>
<td>(1.42 ± 0.03) GeV</td>
<td>(0.69 ± 0.02) GeV</td>
</tr>
<tr>
<td>0.118</td>
<td>(1.23 ± 0.03) GeV</td>
<td>(1.42 ± 0.03) GeV</td>
<td>(0.62 ± 0.02) GeV</td>
</tr>
<tr>
<td>0.124</td>
<td>(1.19 ± 0.03) GeV</td>
<td>(1.42 ± 0.03) GeV</td>
<td>(0.53 ± 0.02) GeV</td>
</tr>
</tbody>
</table>

Higgs decays to gluons and light quarks. In the Standard Model, gluonic Higgs decays \( H \to gg \) are primarily mediated by top-quark loops \([56]\). Since in the LEP2 range Higgs masses are much below the top threshold, the gluonic width can be cast into the approximate form \([57]\)

\[
\Gamma(H \to gg, q\bar{q}g) = \frac{G_F \alpha_s^2(m_H)}{36\sqrt{2\pi^3}} \frac{m_H^3}{m_H} 1 + \frac{95}{4} - \frac{7}{6} N_F \frac{\alpha_s(m_H)}{\pi}
\]

The QCD corrections, which include the splitting of virtual gluons into \( gg \) and \( q\bar{q} \) final states, are very important; they nearly double the partial width.

It is physically meaningful to separate the gluon and light-quark decays of the Higgs boson \([12]\) from the \( b, c \) decays which add to the \( b, c \) decays through direct coupling to the Higgs boson. In this case, the partial width \( \Gamma(H \to \text{gluons + light quarks}) \) is obtained from \((16)\) by choosing \( N_F = 3 \) for the light \( u, d, s \) quarks and by evaluating the running QCD coupling at \( m_H \) for three flavors only [corresponding to \( \Lambda_3^{(3)}_{\overline{\text{MS}}} = 378^{+105}_{-92} \) MeV for \( \alpha_s^{(3)}(m_Z) = 0.118 \pm 0.006 \)].

Higgs decay to virtual W bosons. The channel \( H \to WW^* \to 4 \) fermions becomes relevant for Higgs masses \( m_H > m_W \) when one of the W bosons can be produced on-shell. The partial width for this final state is given by

\[
\Gamma(H \to WW^*) = \frac{3G_F^2 m_W^4}{16\pi^3} m_H R(x)
\]
$R(x) = \frac{1 - 8x + 20x^2}{(4x - 1)^{1/2}} \arccos \left( \frac{3x - 1}{2x^{3/2}} \right) - \frac{1 - x}{2x} (2 - 13x + 47x^2) - \frac{3}{2} (1 - 6x + 4x^2) \log x$

with $x = m_W^2 / m_H^2$. Due to the larger $Z$ mass and the reduced NC couplings compared with $W$ mass and the CC couplings, respectively, decays to $ZZ^*$ final states are suppressed by one order of magnitude.

Summary of the branching ratios. The numerical results for the branching ratios are displayed in Fig.8, taking into account all QCD and electroweak corrections available so far. Separately shown are the branching ratios for $\tau$'s, $c$, $b$ quarks, gluons plus light quarks, and electroweak gauge bosons. The analyses have been performed for the following set of parameters: $\alpha_s(m_Z) = 0.118 \pm 0.006$, $t$ pole mass $M_t = 176 \pm 11$ GeV, and the MS masses $m_b(M_b)$ and $m_c(M_c)$ as listed in Table 3. The dominant error in the predictions is due to the uncertainty in $\alpha_s$ which migrates to the running quark masses at the high energy scales.

Despite the uncertainties, the hierarchy of the Higgs decay modes is clearly preserved. The $\tau^+ \tau^-$ branching ratio is more than an order of magnitude smaller than the $b \bar{b}$ branching ratio, following from the ratio of the two masses squared and the color factor. Since the charm quark mass is small at the scale of the Higgs mass, the ratio of $BR_c$ to $BR_b$ is reduced to about 0.04, i.e. more than would have been expected naïvely.

Thus, the measurements of the production cross sections and of the decay branching ratios enable us to explore experimentally the physical nature of the Higgs boson and the origin of mass through the Higgs mechanism.

Figure 8: Branching ratios for the Higgs decays in the Standard Model. The bands include the uncertainties due to the errors in the quark masses and the QCD coupling.
Selection algorithms were developed by the four LEP experiments [58] towards the Higgs production via the Higgs-strahlung process, for the following event topologies:

(i) the four-jet channel, \((Z \to q\bar{q}) (H_{\text{SM}} \to b\bar{b})\);

(ii) the missing energy channel, \((Z \to \nu\bar{\nu}) (H_{\text{SM}} \to b\bar{b})\);

(iii) the leptonic channel, \((Z \to e^+e^-, \mu^+\mu^-) (H_{\text{SM}} \to \text{anything})\);

(iv) the \(\tau^+\tau^- q\bar{q}\) channel, \((Z \to \tau^+\tau^-) (H_{\text{SM}} \to \text{hadrons})\) and vice-versa;

altogether amounting to more than 90% of the possible final states in the LEP2 mass range.

All important background processes were included in the simulations. Whenever possible, the corresponding cross-sections were computed and events were generated using PYTHIA 5.7 [59]. The \(Z\nu\bar{\nu}\) process being not simulated in PYTHIA, the corresponding results were derived from a Monte Carlo generator based on Ref.[60]. The most relevant cross-sections are indicated in Table 4 for the three different center-of-mass energies at which the studies were carried out. Events from the Higgs-strahlung process were generated using either PYTHIA (DELPHI, L3, OPAL), the HZGEN generator [61] (DELPHI, for the \(HZ \to b\bar{b}\nu\bar{\nu}\) final state) or the HZHA generator [62] (ALEPH, for all signal final states), and the signal cross-section and Higgs boson decay branching ratios were determined from Ref.[46], or directly from the HZHA program in the case of ALEPH.

Table 4: The cross-sections for the most relevant background processes, in pb. Whenever a \(Z\) is indicated, the cross-section also includes the \(\gamma^*\) contribution. The \(\gamma\gamma \to f\bar{f}\) cross-section is given for a fermion pair mass in excess of 30 GeV/c\(^2\).

<table>
<thead>
<tr>
<th>(\gamma\gamma \to f\bar{f})</th>
<th>175 GeV</th>
<th>192 GeV</th>
<th>205 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^+e^- \to f\bar{f})</td>
<td>173.4</td>
<td>135.5</td>
<td>116.5</td>
</tr>
<tr>
<td>(e^+e^- \to WW)</td>
<td>14.63</td>
<td>17.74</td>
<td>18.07</td>
</tr>
<tr>
<td>(e^+e^- \to ZZ)</td>
<td>0.45</td>
<td>1.20</td>
<td>1.43</td>
</tr>
<tr>
<td>(e^+e^- \to Z e^+e^-)</td>
<td>2.75</td>
<td>2.93</td>
<td>3.05</td>
</tr>
<tr>
<td>(e^+e^- \to W e\nu)</td>
<td>0.68</td>
<td>0.90</td>
<td>1.10</td>
</tr>
<tr>
<td>(e^+e^- \to Z\nu\bar{\nu})</td>
<td>0.011</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>(\gamma\gamma \to f\bar{f})</td>
<td>22.3</td>
<td>24.9</td>
<td>26.3</td>
</tr>
</tbody>
</table>

The selection efficiencies and the background rejection capabilities were evaluated after a simulation of each of the four LEP detectors. Fully simulated events were produced by DELPHI [63, 64], L3 [65] and OPAL [66] for all the background processes and for the signal at several
Higgs boson masses, including all the detector upgrades foreseen for the LEP2 running. A fast, but reasonably detailed simulation was used in ALEPH [67] instead, with the current detector design (in particular, the gain expected from the installation of a new vertex detector was conservatively ignored), but it was checked in the four-jet topology and in the missing energy channel, at $\sqrt{s} = 175$ GeV and with $m_H = 70$ GeV, that this fast simulation faithfully reproduces the predictions of the full simulation chain both for the background rejection and for the signal selection, up to an accuracy at the percent level.

a) Search in the Four-jet Topology

The four-jet topology arises when the $Z$ decays into a pair of quarks, in 70% of the cases, and the Higgs boson decays into hadrons, in more than 90% of the cases. This topology represents therefore by far the most abundant final state (occurring in $\sim 65\%$ of the cases) produced by the Higgs-strahlung process. However, the search in this channel is affected by a large background consisting of multijet events from $e^+e^- \rightarrow q\bar{q}$, WW and ZZ production. For instance, at $\sqrt{s} = 192$ GeV, and for an integrated luminosity of 150 pb$^{-1}$, approximately 1500 $q\bar{q}$, 1000 WW and 80 ZZ events have at least four jets with all jet-jet invariant masses in excess of 10 GeV/$c^2$, while only 40 $H_{SM}Z$ events are expected if $m_H = 90$ GeV/$c^2$.

The selection procedures developed by the four collaborations to improve the signal-to-noise ratio are very close to each other. After a preselection aimed at selecting four-jets events, either from global events properties or directly from a jet algorithm such as the DURHAM or JADE algorithms, the four-jet energies and momenta are subjected to a kinematical fit with the four constraints resulting from the energy-momentum conservation, in order to improve the Higgs boson mass resolution beyond the detector resolution. Events consistent with the $e^+e^- \rightarrow WW$ hypothesis, i.e. events in which two pairs of jets have an invariant mass close to $m_W$, are rejected. Only events in which the mass of one pair of jets is consistent with $m_Z$ are kept, and they are fitted again with the Z mass constraint in addition. This last step improves again the Higgs boson mass resolution, which is found to be between 2.5 and 3.5 GeV/$c^2$ by the four LEP experiments.

However, these requirements do not suffice to reduce the background contamination to an adequate level. This is illustrated in Fig.9a where the distribution of the fitted Higgs boson mass (i.e. the mass of the pair of jets recoiling against the pair consistent with a Z) is shown, for the signal ($m_H = 90$ GeV/$c^2$) and for the backgrounds, at $\sqrt{s} = 192$ GeV and for a luminosity of 500 pb$^{-1}$ as obtained from the ALEPH simulation at this level of the analysis.
The high branching of the Higgs boson into $b\bar{b}$ must then be taken advantage of to further reduce the background, by requiring that the jets associated to the Higgs boson be identified as $b$-jets. This is done by means of a microvertex detector, either by counting the charged particle tracks with large impact parameters, or by evaluating the probability $P$ that these tracks come from the main interaction point [68], or by directly reconstructing secondary decay vertices [69]. Shown in Fig. 9b is the resulting Higgs boson mass distribution after such a $b$-tagging requirement is applied. The same distribution as seen by DELPHI is shown in Fig. 10, together with the efficiency of the DELPHI lifetime $b$-tagging requirement applied to four-jet events, in which four $b$-jets, two $b$-jets or no $b$-jets are present, as a function of the logarithm of the probability $P$. The OPAL result in this topology is shown in Fig. 11a. Due to the recent vertex detector installation, the L3 $b$-tagging algorithm is not yet fully optimized and its performance is thus expected to improve in the future.

![Figure 9: Distribution of the fitted Higgs boson mass as obtained from the ALEPH simulation, in the four-jet topology before (a) and after (b) a $b$-tagging requirement is applied, at 192 GeV, with 500 pb$^{-1}$ and for $m_H = 90$ GeV/c$^2$.](image)

![Table 5: Accepted cross-sections (in fb) for the signal and the backgrounds, as expected by ALEPH, DELPHI, L3, OPAL, for $m_H = 90$ GeV/c$^2$ at 192 GeV, in the four-jet topology.](table)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>ALEPH</th>
<th>DELPHI</th>
<th>L3</th>
<th>OPAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>58</td>
<td>43</td>
<td>43</td>
<td>46</td>
</tr>
<tr>
<td>Background</td>
<td>33</td>
<td>33</td>
<td>47</td>
<td>26</td>
</tr>
</tbody>
</table>

The numbers of background and signal events expected to be selected by ALEPH, DELPHI, L3, and OPAL in a window of $\pm 2\sigma$ around the reconstructed Higgs boson mass are shown in Table 5 for a Higgs boson mass of 90 GeV/c$^2$ and at a center-of-mass energy of 192 GeV.
Figure 10: (a) Distribution of the fitted Higgs boson mass as obtained from the DELPHI experiment, after a b-tagging requirement is applied, at 192 GeV, with 300 pb\(^{-1}\) and for \(m_H = 90 \text{ GeV/c}^2\), and (b) Evolution of the b-tagging efficiency as a function of the cut on \(P\) when applied to four jet events, with four, two or zero b-jets.

b) Search in the Missing Energy Channel

The topology of interest here, arising in 18% of the cases, is an acoplanar pair of b-quark jets with mass \(m_H\), accompanied by large missing energy and large missing mass, close to the Z mass. The background, with the exception of the \(ZZ \rightarrow b\bar{b}\nu\bar{\nu}\) or the \(Z\nu\bar{\nu}\) with \(Z \rightarrow b\bar{b}\) processes, either has no missing energy (\(e^+e^- \rightarrow q\bar{q}\) with no initial state radiation, \(WW, ZZ \rightarrow\) four-jets), or no missing mass and isolated particles (\(e^+e^- \rightarrow q\bar{q}(\gamma), WW \rightarrow \ell\nu +\) two jets, \(Z\nu\bar{\nu}\)), or no missing transverse momentum and small acoplanarity angle (\(e^+e^- \rightarrow q\bar{q}(\gamma\gamma), \gamma\gamma \rightarrow q\bar{q}\)), or light quark jets (\(e^+e^- \rightarrow (e)\nu W, WW \rightarrow \tau\nu +\) two jets, \(ZZ \rightarrow q\bar{q}\nu\bar{\nu}\)).

The four collaborations developed a selection procedure with a sequence of criteria, based on these differences between signal and background, including a b-tagging requirement. The mass of the Higgs boson can be either rescaled or fitted by constraining the missing mass to equal the Z mass, allowing mass resolutions from 3.5 to 5 GeV/c\(^2\) to be achieved. The mass distribution obtained by OPAL in this channel, for a Higgs boson mass of 90 GeV/c\(^2\) and at a center-of-mass energy of 192 GeV, is shown in Fig.11b.
Figure 11: Distribution of the fitted Higgs boson mass as obtained from the OPAL experiment, in the four-jet channel (a) and in the missing energy channel (b), at a centre-of-mass energy of 192 GeV, normalized to a luminosity of 1000 pb\(^{-1}\) and for \(m_H = 80\) and 90 GeV/c\(^2\), respectively. The signal (in white) is shown on top of the background (shaded histogram).

This selection procedure was supplemented in DELPHI by an alternative multi-variate probabilistic method, confirming (or slightly improving) the first analysis results. The contribution of the \(t\)-channel WW fusion to the \(H\nu\bar{\nu}\) final topology was also estimated by DELPHI with the recently released HZGEN event generator which includes both the Higgs-strahlung and the WW fusion diagrams together with their interference. As can be naively expected, the relative gain is only sizeable above the HZ kinematical threshold, and amounts to 28\% for a 100 GeV/c\(^2\) Higgs boson at 192 GeV, corresponding to 0.25 additional events expected for an integrated luminosity of 300 pb\(^{-1}\).

Table 6: Accepted cross-sections (in fb) for the signal and the backgrounds, as expected by ALEPH, DELPHI, L3, OPAL, for \(m_H = 90\) GeV/c\(^2\) at 192 GeV, in the missing energy channel.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>ALEPH</th>
<th>DELPHI</th>
<th>L3</th>
<th>OPAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>24</td>
<td>24</td>
<td>9</td>
<td>25</td>
</tr>
<tr>
<td>Background</td>
<td>13</td>
<td>17</td>
<td>11</td>
<td>20</td>
</tr>
</tbody>
</table>

The numbers of background and signal events expected to be selected by ALEPH, DELPHI, L3, and OPAL in a window of \(\pm 2\sigma\) around the reconstructed Higgs boson mass are shown in Table 6 for a Higgs boson mass of 90 GeV/c\(^2\) and at a center-of-mass energy of 192 GeV.
c) Search in the Leptonic Channel

Although occurring in only 6.7% of the cases, this topology can be selected in a simple way by requiring the presence of a high mass pair of energetic, isolated, and thus well identifiable leptons (e or $\mu$) in association with a high multiplicity hadronic system. The process $e^+e^- \rightarrow ZZ$ where one of the $Z$ bosons decays into a lepton pair and the other into $q\bar{q}$ and, to a much lesser extent, the $e^+e^- \rightarrow Z\mu^+\mu^-$ process, constitute the only irreducible background sources. A mild $b$-tagging requirement can also be applied, especially when $m_H \sim m_Z$, to improve the signal-to-noise ratio. Selection efficiencies varying from 50 to 80% were achieved by the four LEP experiments.

In addition to these high efficiencies, the mass of the Higgs boson can be determined with a very good resolution (typically better than 2 GeV/$c^2$) either as the mass recoiling to the lepton pair with the mass of the pair constrained to the $Z$ mass, or with a full fitting procedure using the energies and the directions of the leptons and of the Higgs decay products, the energy-momentum conservation and the $Z$ mass constraint. As shown in Fig.12 from L3, this drastically reduces the $ZZ$ background contamination, except if $m_H \sim m_Z$ when the two mass peaks merge together.

The numbers of background and signal events expected to be selected by ALEPH, DELPHI, L3, and OPAL in a window of $\pm 2\sigma$ around the reconstructed Higgs boson mass are shown in Table 7 for a Higgs boson mass of 90 GeV/$c^2$ and at a centre-of mass energy of 192 GeV.

d) Search in the $\tau^+\tau^- q\bar{q}$ Channel

At present, only ALEPH [67] and DELPHI [70] have investigated this topology, occurring in 9% of the cases when ($Z \rightarrow \tau^+\tau^-$) ($H_{SM} \rightarrow$ hadrons) (3%) or when ($H_{SM} \rightarrow \tau^+\tau^-$) ($Z \rightarrow$ hadrons) (6%). It is characterized by two energetic, isolated taus, defined as 1- or 3-prong slim jets, with masses compatible with $m_\tau$, not identified as an electron or a muon pair, and associated to a high multiplicity hadronic system. After a selection of this topology either by successive topological cuts (ALEPH) or by a single multi-dimensional cut (DELPHI), a fit to the four-body final state hypothesis with the energy-momentum conservation constraint is performed to reject most of the backgrounds.
Figure 12: Distribution of mass recoiling to the lepton pair as obtained from the L3 experiment, in the $He^+ e^-$ channel, at a center-of-mass energy of 192 GeV, normalized to a luminosity of 1000 pb$^{-1}$ and for $m_H = 60, 70, 80, 90$ GeV/$c^2$. The signal (in white) is shown on top of the ZZ background (in black).

Table 7: Accepted cross-sections (in fb) for the signal and the backgrounds, as expected by ALEPH, DELPHI, L3 and OPAL, for $m_H = 90$ GeV/$c^2$ at 192 GeV, in the leptonic channel.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>ALEPH</th>
<th>DELPHI</th>
<th>L3</th>
<th>OPAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>12</td>
<td>11</td>
<td>7</td>
<td>6.5</td>
</tr>
<tr>
<td>Background</td>
<td>12</td>
<td>24</td>
<td>10</td>
<td>9.4</td>
</tr>
</tbody>
</table>
The typical efficiency for such an analysis is 20 to 30%, corresponding to 6 to 8 signal events expected for a 90 GeV/c² Higgs boson with 1 fb⁻¹ at 192 GeV, and the $\tau^+\tau^-$ and the hadronic mass resolutions amount to approximately 3 GeV/c². These resolutions can be further improved by fitting the final state to the HZ hypothesis, with $m_H$ free and $m_Z$ constrained. As in the leptonic channel, the only really irreducible background source is the process $e^+e^- \rightarrow ZZ$ when one of the Zs decays into a $\tau$ pair and the other hadronically. The existence of the Higgs boson would then be observed as an accumulation around $(m_H, m_Z)$ in the folded two-dimensional distribution of these masses. A signal-to-noise ratio between 1 and 2 can be achieved when $m_H \sim m_Z$. It could be further improved by a factor of two with a b-tagging requirement, at the expense of a drastic efficiency loss, since two thirds of these events (when $H \rightarrow \tau^+\tau^-$) do not contain b-quarks.

**Summary: Numbers of Events Expected**

Tables 8, 9 and 10 summarize the results of the standard model Higgs boson search, with the total numbers of signal and background events expected by each experiment given for several Higgs boson masses, at $\sqrt{s} = 175, 192$ and 205 GeV, respectively. The uncertainties are due to the limited Monte Carlo statistics. No systematic uncertainties (due for instance to the simulation of the b-tagging efficiency) are included.

### Table 8: Accepted cross-sections (in fb) expected for the signal and the background, for various Higgs boson masses, at a center-of-mass energy of 175 GeV.

<table>
<thead>
<tr>
<th>$m_H$ (GeV/c²)</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ALEPH</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal</td>
<td>275 ± 5</td>
<td>234 ± 4</td>
<td>168 ± 4</td>
<td>115 ± 3</td>
<td>61 ± 2</td>
<td>7 ± 1</td>
</tr>
<tr>
<td>Background</td>
<td>51 ± 7</td>
<td>45 ± 7</td>
<td>38 ± 6</td>
<td>31 ± 6</td>
<td>24 ± 5</td>
<td>24 ± 5</td>
</tr>
<tr>
<td><strong>DELPHI</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal</td>
<td>210 ± 13</td>
<td>180 ± 11</td>
<td>147 ± 9</td>
<td>109 ± 7</td>
<td>64 ± 4</td>
<td>7 ± 1</td>
</tr>
<tr>
<td>Background</td>
<td>25 ± 4</td>
<td>25 ± 4</td>
<td>28 ± 5</td>
<td>28 ± 5</td>
<td>28 ± 5</td>
<td>15 ± 3</td>
</tr>
<tr>
<td><strong>L3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal</td>
<td>167 ± 10</td>
<td>142 ± 9</td>
<td>119 ± 7</td>
<td>88 ± 5</td>
<td>49 ± 3</td>
<td>7 ± 3</td>
</tr>
<tr>
<td>Background</td>
<td>79 ± 11</td>
<td>83 ± 10</td>
<td>87 ± 9</td>
<td>65 ± 7</td>
<td>44 ± 5</td>
<td>44 ± 5</td>
</tr>
<tr>
<td><strong>OPAL</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal</td>
<td>188 ± 9</td>
<td>160 ± 8</td>
<td>128 ± 6</td>
<td>98 ± 7</td>
<td>56 ± 4</td>
<td>6 ± 1</td>
</tr>
<tr>
<td>Background</td>
<td>27 ± 5</td>
<td>27 ± 5</td>
<td>26 ± 4</td>
<td>27 ± 5</td>
<td>17 ± 4</td>
<td>7 ± 3</td>
</tr>
<tr>
<td><strong>ALL</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal</td>
<td>840 ± 20</td>
<td>715 ± 17</td>
<td>561 ± 13</td>
<td>410 ± 12</td>
<td>229 ± 6</td>
<td>27 ± 3</td>
</tr>
<tr>
<td>Background</td>
<td>182 ± 14</td>
<td>180 ± 13</td>
<td>179 ± 13</td>
<td>151 ± 11</td>
<td>112 ± 9</td>
<td>89 ± 8</td>
</tr>
</tbody>
</table>
Table 9: Accepted cross-sections (in fb) expected for the signal and the background, for various Higgs boson masses, at a center-of-mass energy of 192 GeV.

<table>
<thead>
<tr>
<th>$m_H$ (GeV/$c^2$)</th>
<th>80</th>
<th>85</th>
<th>90</th>
<th>95</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH Signal</td>
<td>125 ± 3</td>
<td>115 ± 3</td>
<td>103 ± 3</td>
<td>64 ± 2</td>
<td>13 ± 2</td>
</tr>
<tr>
<td>Background</td>
<td>33 ± 6</td>
<td>48 ± 7</td>
<td>63 ± 8</td>
<td>57 ± 7</td>
<td>51 ± 7</td>
</tr>
<tr>
<td>DELPHI Signal</td>
<td>99 ± 4</td>
<td>108 ± 5</td>
<td>85 ± 4</td>
<td>60 ± 4</td>
<td>13 ± 2</td>
</tr>
<tr>
<td>Background</td>
<td>42 ± 5</td>
<td>68 ± 5</td>
<td>79 ± 5</td>
<td>50 ± 3</td>
<td>25 ± 2</td>
</tr>
<tr>
<td>L3 Signal</td>
<td>93 ± 5</td>
<td>77 ± 4</td>
<td>64 ± 3</td>
<td>45 ± 3</td>
<td>9 ± 1</td>
</tr>
<tr>
<td>Background</td>
<td>66 ± 6</td>
<td>67 ± 5</td>
<td>68 ± 5</td>
<td>44 ± 5</td>
<td>19 ± 3</td>
</tr>
<tr>
<td>OPAL Signal</td>
<td>98 ± 4</td>
<td>81 ± 3</td>
<td>72 ± 3</td>
<td>40 ± 2</td>
<td>13 ± 2</td>
</tr>
<tr>
<td>Background</td>
<td>28 ± 4</td>
<td>37 ± 4</td>
<td>46 ± 5</td>
<td>36 ± 5</td>
<td>26 ± 5</td>
</tr>
<tr>
<td>ALL Signal</td>
<td>414 ± 8</td>
<td>381 ± 8</td>
<td>323 ± 6</td>
<td>209 ± 6</td>
<td>47 ± 4</td>
</tr>
<tr>
<td>Background</td>
<td>169 ± 10</td>
<td>220 ± 1</td>
<td>255 ± 12</td>
<td>187 ± 11</td>
<td>121 ± 9</td>
</tr>
</tbody>
</table>

Table 10: Accepted cross-sections (in fb) expected for the signal and the background, for various Higgs boson masses, at a center-of-mass energy of 205 GeV.

<table>
<thead>
<tr>
<th>$m_H$ (GeV/$c^2$)</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>105</th>
<th>110</th>
<th>115</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH Signal</td>
<td>118 ± 3</td>
<td>90 ± 3</td>
<td>63 ± 2</td>
<td>46 ± 2</td>
<td>32 ± 3</td>
<td>5 ± 1</td>
</tr>
<tr>
<td>Background</td>
<td>48 ± 7</td>
<td>82 ± 9</td>
<td>28 ± 5</td>
<td>24 ± 5</td>
<td>20 ± 5</td>
<td>20 ± 5</td>
</tr>
<tr>
<td>DELPHI Signal</td>
<td>78 ± 4</td>
<td>84 ± 4</td>
<td>66 ± 4</td>
<td>48 ± 3</td>
<td>23 ± 2</td>
<td>3 ± 1</td>
</tr>
<tr>
<td>Background</td>
<td>56 ± 6</td>
<td>66 ± 6</td>
<td>52 ± 6</td>
<td>26 ± 4</td>
<td>13 ± 3</td>
<td>8 ± 2</td>
</tr>
<tr>
<td>L3 Signal</td>
<td>88 ± 6</td>
<td>68 ± 4</td>
<td>51 ± 3</td>
<td>38 ± 3</td>
<td>22 ± 3</td>
<td>4 ± 1</td>
</tr>
<tr>
<td>Background</td>
<td>70 ± 6</td>
<td>94 ± 7</td>
<td>59 ± 6</td>
<td>30 ± 6</td>
<td>20 ± 6</td>
<td>20 ± 6</td>
</tr>
<tr>
<td>OPAL Signal</td>
<td>55 ± 2</td>
<td>48 ± 2</td>
<td>39 ± 2</td>
<td>26 ± 2</td>
<td>13 ± 1</td>
<td>4 ± 0.3</td>
</tr>
<tr>
<td>Background</td>
<td>25 ± 4</td>
<td>45 ± 4</td>
<td>26 ± 4</td>
<td>20 ± 4</td>
<td>15 ± 4</td>
<td>15 ± 4</td>
</tr>
<tr>
<td>ALL Signal</td>
<td>339 ± 8</td>
<td>287 ± 7</td>
<td>220 ± 6</td>
<td>158 ± 5</td>
<td>89 ± 4</td>
<td>16 ± 1</td>
</tr>
<tr>
<td>Background</td>
<td>198 ± 12</td>
<td>288 ± 13</td>
<td>166 ± 11</td>
<td>101 ± 10</td>
<td>68 ± 9</td>
<td>62 ± 9</td>
</tr>
</tbody>
</table>
Based on the simulations described in Section 2.3, it is possible to derive the exclusion and discovery limits of the standard model Higgs boson as a function of the luminosity for the three center-of-mass energies specified earlier. The contours are defined at $5\sigma$ for the discovery in the case of the existence of the Higgs boson and at $95\%$ C.L. for the exclusion limits in the case of negative searches, with the specifications described in Appendix 5.3.

In Table 11, the minimum integrated luminosities needed to exclude or discover a given Higgs boson mass at the center-of-mass energies $\sqrt{s} = 175, 192$ and $205$ GeV are given for the combination of all channels for each of the four experiments separately, as well as for the combination of all channels for the four LEP experiments together. The results of the combination of the four experiments are graphically shown in Fig.13, and summarized in Table 12.

Combining the four LEP experiments, the required minimal integrated luminosity per experiment to discover or exclude a certain Higgs boson mass at a given center-of-mass energy is reduced to approximately a fourth of the average minimal integrated luminosity of each individual experiment. This implies that the maximal value of the Higgs boson mass will be reached at a given energy for luminosities which can be naturally expected at LEP2. The following conclusions can be drawn from detailed analyses of the figures and tables.

(i) At a center-of-mass energy of 175 GeV, the maximum integrated luminosity needed is of the order of 150 pb$^{-1}$ and this allows the discovery of a Higgs boson with a maximum mass of about 82 GeV/c$^2$. Indeed, combining the four experiments it follows that raising the luminosity leads only to a marginal increase of the exclusion and discovery limits, which are very close to each other.

(ii) At 192 GeV it is again sufficient to have an integrated luminosity of about 150 pb$^{-1}$, in this case to discover a Higgs boson with mass up to 95 GeV/c$^2$. Increasing the center-of-mass energy from 175 to 192 GeV leads to a significant extension in the discovery range of the Higgs boson mass. It is of great interest to observe that at $\sqrt{s} = 192$ GeV a 95 GeV/c$^2$ Higgs boson mass can be excluded at the 95% confidence level with an integrated luminosity as low as 33 pb$^{-1}$ while with 150 pb$^{-1}$ a Higgs boson mass close to 100 GeV/c$^2$ can be excluded.

(iii) This development continues up to 205 GeV, where a luminosity as low as 70 pb$^{-1}$ is sufficient to exclude Higgs boson masses up to about 110 GeV/c$^2$, and a 5$\sigma$ discovery of a Higgs boson with a mass of order 105 GeV/c$^2$ requires an integrated luminosity of $\sim 160$ pb$^{-1}$. More luminosity is needed in this case, since the cross section of the irreducible ZZ background increases. With an integrated luminosity of $\sim 300$ pb$^{-1}$ a Higgs boson mass close to 110 GeV can be discovered.

If each experiment is considered separately, the 5$\sigma$ discovery limit for an integrated luminosity of 500 pb$^{-1}$ is, on average, approximately given by $m_{H} = 82$ (95) (103) GeV/c$^2$ for $\sqrt{s}=$
Table 11: Minimum luminosity needed, in pb$^{-1}$, by ALEPH, DELPHI, L3, OPAL, and for a simple combination of the four experiments, at the three center-of-mass energies and for various Higgs boson masses. The first number holds for the 95% C.L. exclusion, the second one for the 5$\sigma$ discovery.

$$\sqrt{s} = 175 \text{ GeV}$$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$m_H =$ 60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>12:34</td>
<td>18:49</td>
<td>25:76</td>
<td>36:126</td>
<td>80:316</td>
</tr>
<tr>
<td>DELPHI</td>
<td>16:48</td>
<td>18:51</td>
<td>31:87</td>
<td>40:140</td>
<td>78:335</td>
</tr>
<tr>
<td>L3</td>
<td>29:127</td>
<td>39:180</td>
<td>56:244</td>
<td>75:334</td>
<td>152:727</td>
</tr>
<tr>
<td>OPAL</td>
<td>17:56</td>
<td>20:75</td>
<td>34:96</td>
<td>44:161</td>
<td>74:294</td>
</tr>
</tbody>
</table>

$$\sqrt{s} = 192 \text{ GeV}$$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$m_H =$ 80</th>
<th>85</th>
<th>90</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>33:117</td>
<td>42:166</td>
<td>59:238</td>
<td>103: 510</td>
</tr>
<tr>
<td>DELPHI</td>
<td>50:195</td>
<td>50:231</td>
<td>80:388</td>
<td>118: 529</td>
</tr>
<tr>
<td>L3</td>
<td>64:306</td>
<td>90:426</td>
<td>118:596</td>
<td>172: 832</td>
</tr>
<tr>
<td>OPAL</td>
<td>43:157</td>
<td>60:251</td>
<td>85:360</td>
<td>182: 825</td>
</tr>
<tr>
<td>All</td>
<td>12:44</td>
<td>15:60</td>
<td>20:87</td>
<td>33:149</td>
</tr>
</tbody>
</table>

$$\sqrt{s} = 205 \text{ GeV}$$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$m_H =$ 80</th>
<th>90</th>
<th>100</th>
<th>105</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>41:157</td>
<td>80:369</td>
<td>76:327</td>
<td>119: 504</td>
<td>186: 870</td>
</tr>
<tr>
<td>OPAL</td>
<td>87:372</td>
<td>149:735</td>
<td>151:719</td>
<td>267:1284</td>
<td>680:3500</td>
</tr>
</tbody>
</table>
Figure 13: Minimum luminosity needed per experiment, in pb$^{-1}$, for a combined 5$\sigma$ discovery (full line) or a 95% C.L. exclusion (dashed line) as a function of the Higgs boson mass, at the three center-of-mass energies.
Table 12: Maximal Higgs boson masses that can be excluded or discovered with a given integrated luminosity $L_{\text{min}}$ per experiment at the three representative energy values of 175, 192 and 205 GeV, when the four LEP experiments are combined.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>Exclusion: $m_H$ (GeV/$c^2$)</th>
<th>$L_{\text{min}}$ (pb$^{-1}$)</th>
<th>Discovery: $m_H$ (GeV/$c^2$)</th>
<th>$L_{\text{min}}$ (pb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>175</td>
<td>83</td>
<td>75</td>
<td>82</td>
<td>150</td>
</tr>
<tr>
<td>192</td>
<td>98</td>
<td>150</td>
<td>95</td>
<td>150</td>
</tr>
<tr>
<td>205</td>
<td>112</td>
<td>200</td>
<td>108</td>
<td>300</td>
</tr>
</tbody>
</table>

175 (192) (205) GeV. Similar results may be obtained by combining the four experiments for an integrated luminosity per experiment of about 150 pb$^{-1}$. For the combined exclusion limits, the maximum value of $m_H$ at $\sqrt{s} = 175, 192, 205$ GeV is reached for a luminosity per experiment of about 75, 150, 200 pb$^{-1}$. A further increase in luminosity is not very useful in case of negative searches. Clearly, energy rather than luminosity is the crucial parameter to improve the range of masses which can be reached at LEP2.

2.5 The LHC Connection

It has been shown in section 2.4 that LEP2 can cover the SM Higgs mass range up to 82 GeV at a total energy of $\sqrt{s} = 175$ GeV while the Higgs mass discovery limit increases to $\sim 95$ GeV for a total energy of 192 GeV. Since this mass range contains the lower limit at which the SM Higgs particle can be searched for at the LHC, the upper limit of the LEP2 energy is quite crucial for the overlap in the discovery regions of the two accelerators.

Low-mass Higgs particles are produced at the LHC predominantly in gluon–gluon collisions [71, 72] or in Higgs–strahlung processes [73, 74],

$$pp \to H \to \gamma\gamma$$

$$pp \to WH, ZH, t\bar{t}H \to \ell + \gamma\gamma \quad \text{and} \quad \ell + b\bar{b}$$

with the Higgs boson emitted from a virtual $W$ boson or from a top quark. In the gluon–fusion process the Higgs particle is searched for as a resonance in the $\gamma\gamma$ decay channel which comes with a branching ratio of order $10^{-3}$. Even though large samples of Higgs particles can be generated in this mass range, the signal–to–background ratio is only a few percent and the rejection of jet background events which are eight orders of magnitude more frequent, is a very difficult experimental task. Excellent energy resolution and particle identification is needed [75] to tackle this problem. It has been shown in detailed experimental simulations that the
significance $S/\sqrt{B}$ of the Higgs signal is expected to rise in this channel from a value $\sim 2.5$ at $m_H = 80$ GeV to a value $\sim 4.5$ at $m_H = 100$ GeV for $\mathcal{L} = 3 \times 10^4$ pb$^{-1}$ if ATLAS and CMS analyses are combined.

In the Higgs–strahlung process, the events can be tagged by leptonic decays of the $W/Z$ bosons or the $t$ quark to trigger the experiment and to reduce the jet background. In these subsamples the Higgs boson can be searched for in the $b\bar{b}$ decay mode with a branching ratio close to unity. This method is based on $b$ tagging by micro-vertex detection which is anticipated to be an excellent tool of the LHC detectors. After suitable cuts in the transverse momenta of the isolated lepton and the $b$ jets, a peak is looked for in the invariant $M(b\bar{b})$ mass. The experimental significance $S/\sqrt{B}$ of this method is biggest for small Higgs masses. For $\mathcal{L} = 3 \times 10^4$ pb$^{-1}$ and ATLAS/CMS combined, experimental simulations of the $[W]_{b\bar{b}}$ sample suggest that $S/\sqrt{B}$ falls from $\sim 8$ at $m_H = 80$ GeV down to $\sim 6$ at $m_H = 100$ GeV. It is not yet clear how the search can be extended to higher luminosities where the layers in the micro-vertex detectors closest to the beams may not survive, thus reducing significantly the $b$-tagging performance of the experiments.

Combining the prospective signals from the $\gamma\gamma$ and the $[W]_{b\bar{b}}$ analyses, an overall significance of 7 to 8 may be reached for Higgs masses below 100 GeV, based on a low integrated luminosity of $\mathcal{L} = 3 \times 10^4$ pb$^{-1}$ within three years. Raising the integrated luminosity to $\mathcal{L} = 10^5$ pb$^{-1}$ increases the discovery significance to almost 9 for $80 < m_H < 100$ GeV [76].
3 The Higgs Particles in the Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model leads to clear and distinct experimental signatures in the Higgs sector. Two Higgs doublets, $H_1$ and $H_2$, must be present, in order to give masses to the up and down quarks and leptons, and to cancel the gauge anomalies induced through the Higgs superpartners. In the supersymmetric limit, the Higgs potential is fully determined as a function of the gauge couplings and the supersymmetric mass parameter $\mu$. The breakdown of supersymmetry is associated with the introduction of soft supersymmetry breaking parameters, which are essential to yield a proper electroweak symmetry breaking. In the broken phase, the ratio of the Higgs vacuum expectation values, $\tan \beta = v_2/v_1$, appears as a new parameter, which can be related to the other parameters of the theory by minimizing the Higgs potential.

The physical Higgs spectrum of the MSSM contains two CP-even and one CP-odd neutral Higgs bosons, $h/H$ and $A$, respectively, and a charged Higgs boson pair $H^\pm$ [23]. The tree-level Higgs spectrum is determined by the weak gauge boson masses, the CP-odd Higgs mass, $m_A$, and $\tan \beta$. It is only through radiative corrections that the other parameters of the model affect the Higgs mass spectrum. The dominant radiative corrections to the Higgs masses grow as the fourth power of the top-quark mass and they are logarithmically dependent on the sparticle spectrum. The mass of the heavy Higgs doublet is controlled by the CP-odd Higgs mass and, for large values of $m_A$, the effective low energy theory contains only one Higgs doublet, which couples to fermions in the standard way. In a first approximation, the Higgs masses may be calculated by assuming that all sparticles acquire masses of order of the characteristic supersymmetry breaking scale $M_S$ which, based on naturalness arguments, should be below a few TeV. The low-energy effective theory below $M_S$ is a general two-Higgs doublet model, with couplings which can be calculated as a function of the other parameters of the theory. Under these conditions, a general upper bound on the lightest CP-even Higgs boson mass is derived for values of the CP-odd Higgs mass of order $M_S$. For smaller values of $m_A$, a more stringent upper bound is obtained. In the following, we shall discuss in detail the different methods to compute the Higgs spectrum in the MSSM and the bounds which can be derived in each case.

3.1 Higgs Mass Spectrum and Couplings

3.1.1 Tree-level Mass Bounds

The masses of the Higgs bosons at tree level are determined as a function of $m_A$, $\tan \beta$ and the gauge boson masses as follows,

$$ m_{h,H}^2 = \frac{1}{2} m_A^2 + m_Z^2 \mp \frac{(m_A^2 + m_Z^2)^2 - 4 m_Z^2 m_A^2 \cos^2 2\beta}{4 m_Z^2 m_A^2 \cos^2 2\beta} , $$

(18)
\[ m_{H^\pm}^2 = m_A^2 + m_W^2. \]  

The mass of the lightest MSSM neutral Higgs particle is bounded to be smaller than the Z mass \([24, 25]\),

\[ m_h^{\text{tree}} \leq m_Z \cos 2\beta, \]

and it approaches this upper bound for large values of \(m_A\). The bound is modified by radiative corrections, which raise the upper limit on the lightest CP-even Higgs mass to values close to 150 GeV.

### 3.1.2 Radiative Corrections to the Higgs Masses

The one- and partial two-loop radiative corrections to the Higgs mass spectrum in the MSSM have been calculated. Computations implying a variety of different approximations, which may be distinguished according to their level of refinement, exist. In general, the radiative corrections to the Higgs masses are large and positive, being dominated by the contributions of the third-generation quark superfields. Since the upper bound on \(m_h\) determines the limit for the detectability of the Higgs boson at LEP2, it is interesting to discuss the different methods in some detail.

#### a) Diagrammatic Approach.

Order by order, a precise method of computation of the radiative corrections to the Higgs masses is the full diagrammatic approach. At the one-loop level such calculations have been pursued by several authors \([24, 26, 80]\). Complete expressions, including all supersymmetric particle contributions are available \([81]\). The resulting Higgs masses are defined as the location of the pole in the Higgs propagator. In order to obtain a more accurate estimate of the Higgs spectrum in the diagrammatic approach, the two-loop effects must be evaluated. A first step in this direction was performed in Ref.\([82]\) for the case of large values of the CP-odd Higgs boson mass, large \(\tan \beta\), and degenerate squark masses. It was shown that these corrections may be quite significant, of order 10–15 GeV, underlining the need for a careful treatment of the two-loop effects on the Higgs mass spectrum.

#### b) Effective Potential Methods.

The leading corrections to the Higgs mass spectrum in the MSSM can be computed in a very simple way by means of effective potential methods \([27, 78]\). If all the contributions from the MSSM particles are included, the results within this scheme differ from those of the full diagrammatic approach in that the Higgs masses are evaluated at zero momentum. In order to simplify the calculations, it is possible to consider only the contributions of the third-generation quark superfields, neglecting all weak gauge coupling effects in the one-loop expressions \([27]\). This treatment of the effective potential has the virtue of displaying, in a compact way, the full dependence of the one-loop radiative corrections on the stop/stopbottom masses and mixing angles. For a given squark spectrum, the numerical results obtained in this case differ by only a few GeV from the results obtained within the full one-loop diagrammatic approach. This reflects the smallness of the one-loop contributions from superfields other than
top and bottom. Moreover, it shows that the one-loop vacuum polarization effects relating the Higgs pole masses to the running masses calculated through the effective potential approach are in general small. The effective potential computation can be improved by including the dependence of the stop and sbottom spectrum on the weak gauge couplings. In the limit $m_{t1}, m_{t2}, m_A \gg m_Z$, where $m_{t1,2}$ are the two stop mass eigenvalues, the expression of the lightest Higgs mass takes a simple form,

$$m_h^2 = m_Z^2 \cos^2 2\beta + (\Delta m_h^2)_{\text{LL}} + (\Delta m_h^2)_{\text{mix}}$$

(21)

where

$$\Delta m_h^2 = \frac{3m_W^4}{4\pi^2 v^2} \ln \frac{m_{t1} m_{t2}}{m_t^2} 1 + \mathcal{O} \left( \frac{m_W^2}{m_t^2} \right)$$

(22)

and

$$\Delta m_h^2 \text{mix} = \frac{3m_W^4 \tilde{A}_t^2}{8\pi^2 v^2} 2h(m_{t1}^2, m_{t2}^2) + \tilde{A}_t^2 f(m_{t1}^2, m_{t2}^2) 1 + \mathcal{O} \left( \frac{m_W^2}{m_t^2} \right)$$

(23)

In eq.(23), $\tilde{A}_t = A_t - \mu \cot \beta$ and the functions $h$ and $f$ are given by

$$h(a, b) = \frac{1}{a - b} \ln \frac{a}{b} \quad \text{and} \quad f(a, b) = \frac{1}{(a - b)^2} 2 - \frac{a + b}{a - b} \ln \frac{a}{b}$$

(24)

The above expression is particularly interesting since it provides the upper bound on $m_h$ for a given stop spectrum. Including two-loop effects remains, however, a necessary further step to obtain a correct quantitative estimate of the Higgs mass.

c) Renormalization Group Improvement of the Radiatively Corrected Higgs Sector.
The most important two-loop effects may be included by performing a renormalization group improvement of the effective potential, while taking into account, in a proper way, the effect of the decoupling of the heavy third-generation squarks. This program can be easily carried out in the case of a large CP-odd Higgs boson mass and degenerate squarks. Since only one Higgs doublet survives at low energies, the lightest CP-even Higgs mass may be calculated through the renormalization group evolution of the effective quartic coupling, assuming that the heavy sparticles decouple at a common scale $M_S$. The one-loop renormalization group evolution of the quartic couplings includes two-loop effects through the resummation of the one-loop result. The general result is, however, scale dependent but this dependence is reduced by taking into account the two-loop renormalization group improvement of the one-loop effective potential. The vacuum expectation value of the Higgs field and the renormalized Higgs mass scale (approximately) with the appropriate one-loop anomalous dimension factors within this approximation. The scale dependence of the Higgs mass is cancelled by adding the one-loop vacuum polarization effects, necessary to define the Higgs pole mass. For the case of small stop mixing and large values of $\tan \beta$, the Higgs spectrum evaluated through this method agrees with the diagrammatic computation at the two-loop level.

Analytical Expression for the Lightest CP-even Higgs Mass. The two-loop RG improvement of the one-loop effective potential includes two-loop effects in two different ways: through the
resummation of one-loop effects and through genuine two-loop effects. Numerically, the latter are small compared to the resummation effects [83]. Once an appropriate scale of order of the top-quark mass is adopted, the results of the one-loop RG improvement of the tree-level effective potential including the proper threshold effects of squark decoupling, are in excellent agreement with the pole Higgs masses computed by the two-loop RG improvement of the one-loop effective potential [85, 89]. This holds, for large values of the CP-odd Higgs mass, for any value of tanβ and the squark mixing angles. Based on this result, an analytical approximation may be obtained [89] which reproduces the dominant two-loop results [85] within an error of less than 2 GeV. Fig.14 shows the agreement of the one-loop and two-loop results for m_h evaluated at the appropriate scale M_t, and the accuracy of the analytical approximation. In the \( \overline{\text{MS}} \) scheme, the pole top-quark mass \( M_t \) must be related to the on-shell running mass \( m_t \equiv m_t(M_t) \) by taking into account the corresponding one-loop QCD correction factor

\[
m_t = \frac{M_t}{1 + \frac{4}{3\pi} \alpha_s(M_t)}
\]  \( \tag{25} \)

Top Yukawa effects have been neglected in eq.(25), since they are essentially cancelled by the two-loop QCD effects. Observe that eq.(25) gives the correct relation between the running and the pole top-quark masses only if the leading-log contributions to the running mass, associated with the decoupling of the heavy sparticles, are properly taken into account and the sparticles are sufficiently heavy so that the finite corrections become small [90]. The analytical approximation to the one-loop renormalization-group improved result, including two-loop leading-log effects, is given by [89]

\[
m_h^2 = m_Z^2 \cos^2 2\beta \left( 1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} \right. \\
\left. + \frac{9}{4\pi^2} \frac{m_t^4}{v^2} \frac{1}{2} \tilde{X}_t + t + \frac{1}{16\pi^2} \frac{3}{2} \frac{m_t^2}{v^2} - 32\pi \alpha_s \right) \tilde{X}_t t + t^2
\]  \( \tag{26} \)

where the angle \( \beta \) is defined at the scale \( m_A = M_S \) and \( t = \log(M_S^2/M_t^2) \). \( \tilde{X}_t \) is defined above and

\[
\tilde{X}_t = \frac{2\tilde{A}_t^2}{M_S^2} \left( 1 - \frac{\tilde{A}_t^2}{12M_S^2} \right)
\]  \( \tag{27} \)

Furthermore, \( \alpha_s(M_t) = \frac{\alpha_s(m_Z)}{1 + \frac{b_3}{4\pi} \alpha_s(m_Z) \log(M_t^2/m_Z^2)} \) with \( b_3 = 11 - 2N_F/3 \) being the one-loop QCD beta function and \( N_F \) the number of quark flavours [\( N_F = 5 \) at scales below \( M_t \)]. The supersymmetric scale \( M_S \) is defined as \( M_S = \frac{(m_{t1}^2 + m_{t2}^2)^{1/2}}{2} \). For simplicity, all supersymmetric particle masses are assumed to be of order \( M_S \). Notice that eq.(26) includes the leading \( D \)-term correction \( \mathcal{O}(m_Z^2m_t^2) \) [79].

A similar analytical result to eq.(26) has been obtained in Ref. [92]. In this approximation the two-loop leading-logarithmic contributions to \( m_h^2 \) are incorporated by replacing \( m_t \) in eq.(21) by the running top quark mass evaluated at appropriately chosen scales. For \( m_{t1} \approx m_{t2} \equiv M_S \) the result is:

\[
m_h^2 = m_Z^2 \cos^2 2\beta + (\Delta m_h^2)_{\text{LL}}(m_t(\mu_t)) + (\Delta m_h^2)_{\text{mix}}(m_t(M_S))
\]  \( \tag{28} \)

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Figure 14: The lightest Higgs mass as a function of the physical top-quark mass, for $M_S = 1$ TeV, evaluated in the limit of large $m_A$, as obtained from the two-loop RG improved effective potential (solid lines), the one-loop improved RG evolution (dashed lines) and the analytical approximation, eq.(26) (dotted-lines). The four sets of lines correspond to a) $\tan \beta = 15$ with maximal squark mixing, b) $\tan \beta = 15$ with zero-squark mixing, c) the minimal value of $\tan \beta$ allowed by perturbativity constraints for the given value of $M_t$ (IR fixed point) for maximal mixing and d) $\tan \beta$ the same as in c) for zero mixing.

Figure 15: The radiatively corrected light CP-even Higgs mass is plotted as a function of the MSSM parameters. The one-loop leading-log computation is compared with the RG-improved result which was obtained by numerical analysis and by using the simple analytic result given in eq.(28).
where $\mu_t = \sqrt{M_t M_S}$; the running top-quark mass is given by

$$m_t(\mu) = m_t \left( 1 - \frac{\alpha_t}{\pi} - \frac{3\alpha_t}{16\pi} \ln \frac{\mu^2}{M_t^2} \right)$$

(29)

with $\alpha_t = \alpha_i^2 / 4\pi$. All couplings on the right hand side of eq.(29) are evaluated at $M_t$. The requirement that the two stop mass eigenstates be close to each other allows an expansion of the functions $h$ and $f$, eq.(24), in powers of $m_t \tilde{A}_t / M_S^2$. The resulting expression for the Higgs mass is equivalent to the one obtained by performing an expansion of the effective potential in powers of the Higgs field $\phi$. Keeping only operators up to order four in the effective potential, eq.(28) reproduces the expression of eq.(26). This comparison holds up to small differences associated with the treatment of the effects due to the weak gauge couplings in the one-loop effective potential, and with the inclusion of the top Yukawa effects in the relation between the pole and running top-quark mass $[89, 92]$. A more detailed treatment of the dependence of the Higgs mass on the weak gauge couplings may be also found in Ref. [91].

Fig.15 shows the comparison between the results of the analytical approximation, eq.(29), and the one-loop RG improvement to the full one–loop leading–log diagrammatic calculation. In general, the prescription given in eq.(29) reproduces the full one-loop RG-improved Higgs masses to within 2 GeV for top-squark masses of 2 TeV or below. The dashed line in the figure shows the result that would be obtained by ignoring the RG-improvement; it reflects the relevance of the two-loop effects in the evaluation of the Higgs mass.

The Case $m_A \lesssim M_S$. A similar RG improvement of the effective potential method to the one already discussed can be applied to calculate all the masses and couplings in the more general case of a light CP-odd Higgs boson $m_A \lesssim M_S$. As above, the finite one-loop threshold corrections to the quartic couplings at the scale $M_S$ at which the heavy squarks decouple $[87]$ are also taken into account. The effective theory below the scale $M_S$ $[86, 87]$ is a two-Higgs doublet model where the tree–level quartic couplings can be written in terms of dimensionless parameters $\lambda_i$, $i = 1, \ldots, 7$, whose tree–level values are functions of the gauge couplings. The one-loop threshold corrections $\Delta \lambda_i$, $i = 1, \ldots, 7$, are expressed as functions of the supersymmetric Higgs mass $\mu$ and the soft supersymmetry breaking parameters $A_t$, $A_b$, and $M_S$ $[87]$. An analytical approximation, which reproduces the previous one–loop RG improved results for all values of $\tan \beta$ and $m_A$, can also be derived. For example, generalizations of Eq. (21) can be found in Refs.$[89, 92]$. The CP-even light and heavy Higgs masses and the charged Higgs mass are given as functions of $\tan \beta$, $M_S$, $A_t$, $A_b$, $\mu$, the CP-odd Higgs mass $m_A$ and the physical top-quark mass $M_t$ related to the on-shell running mass $m_t$ through eq.(25). The analytical expressions for the masses and mixing angle of the Higgs sector as a function of the parameters $\lambda_i$ are presented in Appendix 5.2. These expressions are the analogue of eq.(26) for the case in which two-Higgs doublets survive at low energies. Effects of the bottom Yukawa coupling, which may become large for values $\tan \beta \simeq m_t / m_b$ ($m_b$ being the running bottom mass at the scale $M_t$), are also included. A subroutine implementing this method is available $[93]$. 

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### Table 13: MSSM Higgs couplings relevant at LEP2.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>${h, H} W_\mu W_\nu$</td>
<td>$i2(G_F \sqrt{2})^{1/2} m_W^2 g_{\mu \nu} {\sin(\beta - \alpha), \cos(\alpha - \beta)}$</td>
</tr>
<tr>
<td>${h, H} Z_\mu Z_\nu$</td>
<td>$i2(G_F \sqrt{2})^{1/2} \frac{m_W^2}{\cos^2 \theta_W} g_{\mu \nu} {\sin(\beta - \alpha), \cos(\alpha - \beta)}$</td>
</tr>
<tr>
<td>${h, H, A} u \bar{u}$</td>
<td>$-i(\sqrt{2} G_F)^{1/2} m_u \frac{\sin \beta}{\sin \beta} {\cos \alpha, \sin \alpha, -i \gamma_s \cos \beta}$</td>
</tr>
<tr>
<td>${h, H, A} d \bar{d}$</td>
<td>$-i(\sqrt{2} G_F)^{1/2} m_d \frac{\cos \beta}{\cos \beta} {- \sin \alpha, \cos \alpha, -i \gamma_s \sin \beta}$</td>
</tr>
<tr>
<td>${h, H} A Z_\mu$</td>
<td>$-(\sqrt{2} G_F)^{1/2} m_Z (p + k)_\mu {\cos(\beta - \alpha), -\sin(\beta - \alpha)}$</td>
</tr>
</tbody>
</table>

Couplings. Notice that the radiatively corrected quartic couplings $\lambda_i$, $i = 1, \ldots, 7$, and hence the corresponding value of the Higgs mixing angle $\alpha$ as given in Appendix 5.2, permit us to evaluate all radiatively corrected Higgs couplings. For instance, the Yukawa and gauge Higgs couplings relevant for LEP2 energies are listed in Table 13 [$p_\mu$ ($k_\mu$) is the incoming (outgoing) CP-odd (CP-even) Higgs momentum]. The size of the couplings of the two scalar Higgs bosons to fermions and a gauge boson are shown in Fig.16 [95]. For fermions the charged Higgs particles couple to mixtures of scalar and pseudoscalar currents, with components proportional to $m_u \cot \beta$ and $m_d \tan \beta$ for the two $\pm$ chiralities. The couplings to left(right)-handed ingoing $u$ quarks are given by $g_{H^+ A_{\mu(R)}} = [\sqrt{2} G_F]^{1/2} m_u \cot \beta (m_d \tan \beta)$. For large $\tan \beta$ the down–type mass defines the size of the coupling; for small to moderate $\tan \beta$ it is defined by the up–type mass. Furthermore, the trilinear Higgs couplings can be explicitly written as functions of $\lambda_i$, $\alpha$ and $\beta$ [87, 88].

d) Renormalization Group Improvement of the Effective Potential: General Third–Generation Squark Mass Parameters. The above one–loop RG improved treatment of the effective potential relies on the definition of an effective supersymmetric threshold scale, $M_3^2 = (m_{t_1}^2 + m_{t_2}^2)/2$. Its validity is therefore restricted to the case of small differences between the squark mass eigenvalues. Quantitatively, the method is valid if $(m_{t_1}^2 - m_{t_2}^2)/(m_{t_1}^2 + m_{t_2}^2) \leq 0.5$. Furthermore, all the RG Higgs analyses performed in the literature, besides Ref.[91], rely on the expansion of the effective potential up to operators of dimension four. However, to safely neglect higher dimensional operators, the conditions $2|M_i A_i| \leq M_3^2$ and $2|M_i H_i| \leq M_3^2$ must be fulfilled.
Figure 16: MSSM Higgs couplings normalized to the SM couplings $g_{H_{ff}}^{SM} = [\sqrt{2}G_F]^{1/2}m_f$ and $g_{HVV}^{SM} = 2[\sqrt{2}G_F]^{1/2}m_V^2$. 
The case of large splitting in the stop sector is particularly interesting in the light of recent measurements of \( R_b = \frac{\Gamma(Z \rightarrow bb)}{\Gamma(Z \rightarrow \text{hadrons})} \), whose discrepancy of more than 3 standard deviations with the SM prediction can be ameliorated in the presence of a light higgsino together with a light right-handed stop (see the discussion in the chapter on New Particles) [101]-[104]. The left-handed stop must instead remain reasonably heavy to avoid undesirable contributions to the \( W \) mass and the \( Z \) leptonic width. It is hence important to generalize the results previously obtained by using the renormalization group improved one-loop effective potential, to the case of general values of the left- and right-handed squark masses and mixing parameters, \( m_Q, m_U, m_D, A_t \) and \( A_b \), respectively. In this case the contribution of higher dimensional operators to the effective potential must be properly taken into account; hence, the naive treatment in terms of quartic couplings is no longer appropriate.

In Ref.[91], a method has been developed for the neutral Higgs sector of the theory, in which each stop and sbottom particle is decoupled at its corresponding mass scale. Threshold effects, associated with the decoupling of the heavy sparticles, are frozen at the decoupling scales; they evolve, in the squared mass matrix, with the anomalous dimensions of the Higgs fields. The threshold effects achieve a complete matching of the effective potential for scales above and below the decoupling scales, and include all higher order (non-renormalizable) terms arising from the whole MSSM effective potential. The dominant leading-log contributions in the expressions of the renormalized Higgs quartic couplings involve the scale dependent contributions to the effective potential and are treated in the same way as in the RG improved approach described above. The way to proceed in evaluating the CP-even Higgs mass values and mixing angle \( \alpha \) is explained in detail in Ref.[91]. A subroutine implementing the method is available [94]. This approach makes contact with the computation of the Higgs masses by means of the effective potential performed in Ref.[27]. Moreover, it reproduces the results of Ref.[89] for small mass splitting of the squark masses. This comparison holds up to a tiny difference coming from the inclusion of the small dependence of the one-loop radiative corrections on the weak couplings and the vacuum polarization effects. Indeed, in Ref.[91] the definition of pole Higgs masses is introduced by including the most relevant vacuum polarization effects. The gaugino corrections, which are relatively small, have been also included by incorporating (only) the one-loop leading logarithmic contributions.

### 3.1.3 Results

The lightest CP-even Higgs mass is a monotonically increasing function of \( m_A \), which in the low \( \tan \beta \) regime converges to its maximal value for \( m_A \geq 300 \text{ GeV} \). In Fig.17 the upper limits on the lightest CP-even Higgs mass \( m_h \) [realized in the large \( m_A \) limit] are shown as a function of \( \tan \beta \). Since the radiative corrections to the Higgs mass depend on the fourth power of the top mass, the maximal top-quark mass compatible with perturbation theory up to the GUT scale has been adopted for each value of \( \tan \beta \). Apart from the natural choice of the mixing mass-parameters and the scale \( M_S \), this result is the most general upper limit on \( m_h \) for a given value of \( \tan \beta \) in the MSSM. The variation of the upper bound on \( m_A \) as a function of \( M_t \) is
shown by the solid line (a) of Fig.14. In Fig.18 the mass $m_h$ is plotted for different values of the mixing parameters $A = A_t$ and $\mu$. In fact, $A \approx |\mu| \ll M_S$ yields the case of negligible squark mixing, while $A = \sqrt{6} M_S$, $|\mu| \ll M_S$ characterizes the case of large mixing [i.e. the impact of stop mixing in the radiative corrections is maximized]; $A = -\mu = M_S$ yields moderate mixing for large $\tan \beta$ while the mixing effect is close to maximal for low $\tan \beta$. In Fig.19 we show the masses of the two CP–even Higgs bosons and of the charged Higgs boson as a function of $m_A$ for the case $A = -\mu = M_S = 1 \text{ TeV}$, $M_t = 175 \text{ GeV}$ and different values of $\tan \beta$. The peculiar behavior of $m_h$ and $m_H$ for large $\tan \beta$ will be explained in the following.

In general, for very large values of $\tan \beta$ and values of $\mu$, $A_t$ and $A_b$ of order or smaller than $M_S$, the mixing in the Higgs sector is negligible and the CP–even Higgs mass eigenstates are approximately given by $H_1$ and $H_2$. As a result, the properties of $h$ and $H$ mainly depend on the value of $m_A$. For $m_A^2 > 2 v^2 \lambda_2 \equiv m_b^2 + \text{rad.corr.}$, one approaches the decoupling limit and the relations $\sin \alpha \simeq -\cos \beta$ and $\cos \alpha \simeq \sin \beta$ hold. Hence, the CP–even Higgs mass eigenstates are given by $h \simeq \sin \beta H_2 + \cos \beta H_1 \simeq H_2$ and $H \simeq -\sin \beta H_2 + \cos \beta H_1 \simeq -H_2$. In this case the lightest CP–even Higgs couples to up (down) fermions as

$$hu\bar{u} \rightarrow h_u \sin \beta \quad \text{and} \quad Hd\bar{d} \rightarrow h_d \cos \beta$$

(30)

where $h_u \sin \beta$ ($h_d \cos \beta$) is the SM coupling $h_u^{SM}$ ($h_d^{SM}$) [Observe that $h_f^{SM} = g_H^{SM} \sqrt{2}$, with $f = u, d$]. The heaviest CP-even Higgs boson, instead, couples in highly non-standard way to fermions,

$$H u\bar{u} \rightarrow h_u \cos \beta = h_u^{SM} \cot \beta \quad \text{and} \quad H d\bar{d} \rightarrow h_d \sin \beta = h_d^{SM} \tan \beta$$

(31)

so that the coupling to up (down) quarks is highly suppressed (enhanced) with respect to the coupling in the Standard Model. For $m_A^2 < 2 v^2 \lambda_2$ instead, $\sin \alpha \simeq -\sin \beta$ and $\cos \alpha \simeq \cos \beta$. Hence, the CP–even Higgs mass eigenstates are given by $h \simeq \cos \beta H_2 + \sin \beta H_1 \simeq H_1$ and $H \simeq -\sin \beta H_2 + \cos \beta H_1 \simeq -H_2$. In this case the situation is interchanged; $h$ has the non-standard type of couplings to fermions, eq.(31), and $H$ has the SM couplings, eq.(30).

The values of the CP–even Higgs masses depend on the size of the $H_2$ or $H_1$ component. When the Higgs is predominantly $H_2$, its mass is given by eq.(26) for $|\cos 2\beta| = 1$, neglecting the small bottom–quark Yukawa effects. When the Higgs is predominantly $H_1$, instead, its mass is given by $m_A$. Hence, the mass of the lightest Higgs boson is given by $m_h \simeq m_A$ (and non-standard couplings to fermions) if $m_A^2 \lesssim 2 v^2 \lambda_2$, and it is given by eq.(26) for larger $m_A$, for which the couplings to fermions are SM-like. The complementary situation occurs for $H$ and this can clearly be observed in Fig.19.

The effects of the bottom quark are only relevant in the limit of large $\mu$ parameters. For values of $\mu$ larger than $M_S$ relevant corrections, which are dependent on the bottom mass, enter the Higgs mass formulae. This can be easily understood in the case $m_Q = m_U = m_D = M_S$, by studying the dependence of $\lambda_3$ on the supersymmetric Yukawa coupling $h_b$ [see appendix 5.2]. For values of $h_b$ of order of $h_t$, or equivalently for $\tan \beta \simeq m_t/m_b$, $\lambda_2$ depends significantly on the fourth power of the $\mu$ parameter. These radiative corrections are negative, lowering the mass of the CP–even Higgs associated with the $H_2$ doublet. Fig.20 shows the case of large $m_A$. 

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Figure 17: Upper limit on the mass of the lightest neutral Higgs boson mass $m_h$ as a function of $\tan \beta$ for zero mixing (dashed line) and for the maximal impact of mixing in the stop sector (solid line); $M_S = 1$ TeV.

Figure 18: Lightest neutral Higgs boson $h$ in the MSSM as a function of $m_A$ for zero mixing (dashed line), for intermediate mixing (dotted line) and for the maximal impact of mixing in the stop sector (solid line); for two values of $\tan \beta = 1.6$ (lower set), 15 (upper set); $M_S = 1$ TeV and $M_t = 175$ GeV.
Figure 19: *Lightest CP-even Higgs boson mass (dashed line), heaviest CP-even Higgs mass (solid) and charged Higgs mass (dotted line) in the MSSM as a function of $m_A$ for $A = \mu = M_S = 1 \text{ TeV}, M_t = 175 \text{ GeV}$ and different values of $\tan \beta$.*

Figure 20: *Plot of the pole Higgs mass $M_h$ as a function of $m_D$, for $M_t = 175 \text{ GeV}, \tan \beta = 60, A_h = 0, m_U = m_Q = 1 \text{ TeV}, A_0 = 0. 1.5, 2.4 \text{ TeV (from bottom to top)}$ and $\mu = 1 \text{ TeV (solid curves), } \mu = 2 \text{ TeV (dashed curves).})*
For large values of $\mu$ and small values of $m_A$, the charged Higgs mass also receives large negative radiative corrections, which grow as the fourth power of the $\mu$ parameter. Hence, large negative corrections to the charged Higgs mass may be obtained. Such large values of $\mu$, however, may be in conflict with the stability of the ordinary vacuum state.

### 3.1.4 Additional Constraints: $b$-$\tau$ Unification and Infrared Fixed Point Structure

The MSSM can be derived as an effective theory in the framework of supersymmetric grand unified theories. In addition to the unification of gauge couplings, the unification of the $b$ and $\tau$ Yukawa couplings, $h_b(M_{\text{GUT}}) = h_\tau(M_{\text{GUT}})$, appears naturally in most grand unified scenarios. Given this additional constraint, the experimental values of the $b$ and $\tau$ masses at low energies determine the value of $M_t$ as a function of $\tan\beta$ [18, 29, 30]. In fact, for the present experimental range of the top-quark mass $M_t = 180 \pm 12$ GeV [33], the condition of $b$-$\tau$ unification implies either low values of $\tan\beta$, $1 \leq \tan\beta \leq 3$, or very large values of $\tan\beta = O(m_t/m_b) \approx 50$ [18, 29]-[32]. To accommodate $b$-$\tau$ unification, large values of the top Yukawa coupling are necessary in order to compensate for the effects of the renormalization by strong interactions in the running of the bottom Yukawa coupling. Large values of $h^2_b(M_{\text{GUT}})/4\pi \simeq 0.1-1$ ensure the attraction towards the infrared (IR) fixed point solution for the top quark mass [34]. The strength of the strong gauge coupling as well as the experimentally allowed range of the bottom mass play a decisive role in this behavior [30]-[32]. In the low $\tan\beta$ case, for the presently allowed values of the electroweak parameters and of the bottom mass and for values of $\alpha_s(m_Z) \gtrsim 0.115$, $b$-$\tau$ unification implies that the top-quark mass must be within ten percent of its infrared fixed point values. A mild relaxation of exact unification $[0.85-0.9 \leq h_b/h_\tau|_{M_{\text{GUT}}} \leq 1.15]$ still preserves this feature, especially for values of $M_t \leq 4.95$ GeV. In the large $\tan\beta$ region, $h_b$ is $O(h_t)$ and the infrared fixed point attraction, within the context of $b$-$\tau$ Yukawa coupling unification, is much weaker.

The top-quark mass is also predicted to be close to its infrared-fixed point value in string scenarios, in which the top-quark Yukawa coupling is determined by minimizing the effective potential with respect to moduli fields [99]. Quite generally, the fixed point solution, $h_t = h_t^{IR}$, is obtained for large values of the top Yukawa coupling at high energy scales, which however remain in the perturbative regime. Within the framework of grand unification, one obtains $(h_t^{IR})^2/4\pi \simeq (8/9)\alpha_s(m_Z)$ for $M_{\text{GUT}} \sim 10^{16}$ GeV, and the running top-quark mass tends to its infrared fixed point value $m_t^{IR} = h_t^{IR} v \sin\beta$. Hence, relating the running top-quark mass $m_t$ with the pole top-quark mass $M_t$ by taking into account the appropriate QCD corrections we arrive in the low $\tan\beta$ regime at [100],

$$M_t^{IR} \simeq \sin\beta [1 + 2(\alpha_s(m_Z) - 0.12)] + 4\alpha_s(m_Z)/3\pi + O(\alpha_s^2) \times 196 \text{ GeV} \quad (32)$$

The strong $M_t$-$\tan\beta$ correlation associates with each value of $M_t$ at the infrared fixed point the lowest value of $\tan\beta$ consistent with the validity of perturbation theory up to scales of order $M_{\text{GUT}}$. If the physical top-quark mass is in the range 160–190 GeV, the values of $\tan\beta$
are restricted to the interval between 1 and 3. This is in agreement with the results from $b$-$\tau$ Yukawa unification.

The infrared fixed point solution can also be analysed in the large $\tan \beta$ case, where the effects of the bottom Yukawa coupling need to be taken into account in the RG evolution as well. For instance, if the values of the supersymmetric Yukawa couplings of the bottom and top quarks are very close to each other, $m_t(M_t) \simeq m_b(M_b) \tan \beta$, the infrared fixed point prediction for the top-quark mass is reduced by a factor $6/7$ with respect to eq.(32) [98, 105]. Still, the values of $M_t$ predicted in this regime are about 190 GeV.

After the above general discussions we shall describe their consequences for the Higgs sector:

(i) The infrared fixed point structure in the low $\tan \beta$ region have far-reaching consequences for the lightest CP-even Higgs mass in the MSSM [96]-[98]. Indeed, for $\tan \beta$ larger than one, the lowest tree-level Higgs mass is obtained at the lowest value of $\tan \beta$. Hence, in any theory consistent with perturbative unification, the fixed point solution is associated with the lowest value of the tree-level mass consistent with the theory. Even after including radiative corrections, the upper bound on the Higgs mass is considerably reduced at the fixed point solution: for a top mass of 175 GeV, the upper limit of the Higgs mass is less than 100 GeV, while for $M_t = 160$ GeV, it is even less than 80 GeV (see Fig.14). Hence, if the infrared fixed point solution for the top-quark with $M_t \leq 175$ GeV is realized in nature, the lightest CP-even Higgs mass must be accessible at LEP2 for $\sqrt{s} = 192$ GeV [37, 89]. Fig.14 shows also that for $M_t = 175$ GeV the upper bound on the lightest Higgs mass in the case of $b$-$\tau$ Yukawa coupling unification is nearly 25 GeV smaller than the unrestricted MSSM limit.

The present data indicate that the value of $R_h = \Gamma_b/\Gamma_h$ is more than $3\sigma$ above the SM prediction for this quantity. Large positive radiative corrections to $R_h$ are always associated with large values of the Yukawa couplings; they are therefore maximized at the infrared fixed point solution [102, 100]. Moreover, precision measurements also provide information about the structure of the soft supersymmetry breaking terms: Low values of the right-handed SUSY breaking stop mass $m_U$ and of the SUSY mass parameter $\mu$ are preferred, while the left-handed stop mass parameter $m_Q$ must be larger than $m_U$. For a fixed large value of $m_Q$, the upper bound on the Higgs mass is significantly lower in the case $m_U \ll m_Q$ than in the case $m_U \simeq m_Q$. Fig21 shows the Higgs mass as a function of $m_Q$ for $m_U = 100$ GeV, $m_A = 300$ GeV and $\tan \beta$ consistent with the fixed point solution for $M_t = 175$ GeV, for different values of the mixing mass parameter $A_t$. Even for the largest value of $A_t$ physically acceptable [i.e. $m_f$ above the experimental lower bound], the Higgs mass remains below 85 GeV. Hence, for the values of the supersymmetry breaking mass terms preferred by the precision electroweak data, associated with a light right-handed stop, lower values of $m_h$ than naively expected are obtained.

Furthermore, the most general upper bounds on $m_h$ at the infrared fixed point are valid for very large values of the mixing parameters in the squark sector [$A_t$, $A_b$ and $\mu$] which in general are hard to realize. Requiring radiative breaking of the electroweak symmetry [yet no colour breaking], and imposing the boundary conditions from experimental SUSY mass limits, the
range of upper values of $m_h$ is reduced further [100]. In the general framework of supergravity models, various analyses have been performed in the literature to study the spectrum of a constrained MSSM at different levels of refinement [106].

(ii) The condition of $b$-$\tau$ Yukawa coupling unification is also consistent with the values of the top-quark mass measured at the Tevatron for very large values of $\tan \beta \simeq m_t/m_b$. There are, however, large uncertainties in this sector associated with one-loop supersymmetric corrections to the bottom mass. These radiative corrections are strongly dependent on the structure of the supersymmetric spectrum and induce strong variations in the predictions for the top-quark mass and $\tan \beta$, once the unification of the $b$ and $\tau$ Yukawa couplings is implemented. Nevertheless, the large $\tan \beta$ regime with unification of the $b$-$\tau$ Yukawa couplings, although more model dependent, provides an interesting framework for Higgs particle searches at LEP2.

Large positive radiative corrections to $R_b$ can also be obtained for large values of $\tan \beta$, since the supersymmetric bottom–quark Yukawa coupling is enhanced in this regime. Indeed, the value of $R_b$ can be significantly increased if the CP-odd Higgs mass is below 70 GeV [101]-[103]. This is a result of the large positive one-loop corrections associated with the neutral CP-odd Higgs scalar sector of the theory. Low values of the CP-odd Higgs mass, $m_A \simeq m_Z$, imply that both the lightest CP-even and the CP-odd Higgs masses would be at the reach of LEP2. The charged Higgs mass is approximately determined through the CP-odd Higgs mass value, $m_{H^\pm} \simeq m_A^2 + m_W^2$, and hence, strong constraints on $m_A$ are obtained from the charged Higgs contributions to $BR(b \to s\gamma)$. Even conservatively taking into account the QCD uncertainties
associated with the branching ratio \( BR(b \to s\gamma) \) [i.e. assuming e.g. 40% QCD uncertainties],
the \( b \to s\gamma \) decay rate becomes larger than the presently allowed experimental values [107] for
\( m_{H^\pm} \lesssim 130 \text{ GeV} \), unless the supersymmetric particle contributions suppress the charged Higgs
enhancement of the decay rate. The most important supersymmetric contributions to this rare bottom decay come from the chargino-stop one-loop diagram [108]. The chargino contribution to the \( b \to s\gamma \) decay amplitude depends on the soft supersymmetry breaking mass parameter
\( A_t \) and on the supersymmetric mass parameter \( \mu \). For very large values of \( \tan \beta \), it is given by

\[
A_{\chi^+} \sim \frac{m_t^2 A_t \mu}{m_t^2 m_t^2} \tan \beta G \frac{m_t^2}{\mu^2}
\]  

(33)

where \( G(x) \) is a function with values of order unity when the characteristic stop mass \( m_t \) is
of order \( \mu \), and it grows as \( \mu \) decreases. For positive (negative) values of \( A_t \times \mu \) the chargino
contributions are of the same (opposite) sign as the charged-Higgs contributions. Hence, to
partially cancel the light charged-Higgs contributions and render the \( b \to s\gamma \) decay rate acceptable,
negative values for \( A_t \times \mu \) are required. This requirement has direct implications on
the corrections to the bottom mass mentioned above and puts strong constraints on models
with unification of the Yukawa couplings [36, 109].

3.1.5 MSSM Parameters

In the experimental simulations, we have chosen as the two basic parameters of the Higgs sector
the mass \( m_A \) of the pseudoscalar Higgs boson within the limits \( 40 \text{ GeV} \leq m_A \leq 400 \text{ GeV} \), and
the angle \( \beta \) within the bounds \( 1 \leq \tan \beta \leq m_t(M_t)/m_h(M_t) \approx 60 \). The upper limit on \( m_A \) is
introduced merely for convenience, since the variation of \( m_h \) with \( m_A \) becomes negligible for
values of \( m_A \geq 200-250 \text{ GeV} \). The upper value of \( \tan \beta \) is chosen such that the bottom Yukawa
coupling remains in the perturbative regime for scales below the grand unification scale. A given
value of \( \tan \beta \) implies an upper limit on the top mass for which the theory can be extended
perturbatively up to the GUT scale. For \( \tan \beta = 1 \) this upper limit is already close to 150 GeV
so that lower values of \( \tan \beta \) would be inconsistent with values of the top quark mass in the
experimental range. In the examples we shall discuss, we have chosen:

(i) Top mass, \( M_t = 175 \pm 25 \text{ GeV} \);

(ii) SUSY scale, \( M_S = 10^8 \text{ GeV} \);

(iii) SUSY Higgs mass parameter \( \mu \) and soft SUSY breaking parameter \( A_t = A_b = A \):

\( A = 0 \) and \( |\mu| \ll M_S \) [no mixing];

\( A = \sqrt{6}M_S \) and \( |\mu| \ll M_S \) [maximal mixing];

\( A = M_S = -\mu \) ["typical" mixing].

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We have taken $M_S$ of order 1 TeV to include the effects of possibly large radiative corrections to the lightest CP-even Higgs mass. In the same way the choice of the soft SUSY breaking parameter $A$ and of the SUSY mass parameter $\mu$ is motivated. The central top mass value is close to the central value measured at the Tevatron [33]. The upper and lower bounds are extreme, roughly corresponding to the $\pm 2\sigma$ limits of the CDF measurement. Although the central value for $M_t$ extracted from the LEP precision measurements in the MSSM for large masses of the SUSY particles would be somewhat lower than the central value observed in the Tevatron events, the lower values are still consistent at the $2\sigma$ level.

3.2 Production and Decay Modes of MSSM Higgs Particles

3.2.1 Higgs Production

The main production mechanisms of the neutral Higgs bosons $h$ and $A$ in the MSSM at LEP2 energies are through the following processes [110]:

Higgs-strahlung: $e^+e^- \rightarrow Z h$

Associated pair production: $e^+e^- \rightarrow A h$

The fusion processes, similar to the Standard Model, play only a marginal role at the kinematical limit of the Higgs-strahlung process for the production of the CP-even Higgs boson $h$. The CP-odd Higgs boson $A$ cannot be produced in Higgs-strahlung and in fusion processes to leading order.

The production of the heavy CP-even Higgs particle $H$ is very difficult at LEP energies. In the tiny corner of parameter space, for moderate to large $\tan\beta$, where associated $AH$ production would be allowed kinematically, the production cross section is suppressed by the small coefficient $\sin^2(\beta - \alpha)$, due to the $ZAH$ coupling discussed earlier, and the threshold P-wave factor. For $\tan\beta = 3 (50)$, $m_A = 60 \text{ GeV}$, $m_H = 123 (117) \text{ GeV}$, it is 4 (0.001) fb.

The cross sections (34) may be expressed in terms of the cross section $\sigma_{\text{SM}}$ for Higgs-strahlung in the Standard Model in the following way [110, 41]:

$$\sigma(e^+e^- \rightarrow Z h) = \sin^2(\beta - \alpha) \sigma_{\text{SM}}$$

$$\sigma(e^+e^- \rightarrow A h) = \cos^2(\beta - \alpha) \lambda \sigma_{\text{SM}}$$

The factor $\lambda = \lambda_{zh}^2 / \{ 12 m_Z^2 / s + \lambda_{zh} \}$ accounts for the correct suppression of the P-wave cross section near the thresholds. $[\lambda_{ij} = (1 - (m_i + m_j)^2 / s)(1 - (m_i - m_j)^2 / s)]$ is the usual momentum factor of the two particle phase space. The cross section for $WW$ fusion of $h$ is reduced by the same factor $\sin^2(\beta - \alpha)$ as is the cross section for Higgs-strahlung.

The cross sections for Higgs-strahlung $Zh$ and associated pair production $Ah$ are complementary to each other, coming either with the coefficient $\sin^2(\beta - \alpha)$ or $\cos^2(\beta - \alpha)$. The cross
sections are shown for two representative values of tan\(\beta = 1.6\) and 50 in Fig.22. The top-quark mass is varied, as usual, between 175 \(\pm\) 25 GeV. Since the upper limit on \(m_t\) depends strongly on \(M\), for small values of tan\(\beta\) where the tree-level mass is small, the endpoints of the curves are shifted upwards significantly with increasing top mass. For large values of tan\(\beta\), on the other hand, the dependence of the upper bound of \(m_t\) on the radiative corrections is weaker for rising top mass due to the large value of \(m_t\) at the tree level. The supersymmetric coefficient \(\cos^2(\beta - \alpha)\) is nearly independent of the top mass and it is very close to unity, so that the spread between the curves is negligible; the coefficient \(\sin^2(\beta - \alpha)\) is correspondingly small. In this large tan\(\beta\) case, the curves terminate at the kinematical limit before \(m_h^{\text{max}}\) can be reached, in contrast to the small tan\(\beta\) case. For large tan\(\beta\) the non-zero widths of the particles are taken into account.

For small tan\(\beta\), Higgs-strahlung \(Zh\) provides the largest production cross section while the cross section for \(Ah\) associated pair production is much smaller. With \(\sigma_{Zh} \sim \mathcal{O}(0.5 \text{ pb})\) at \(\sqrt{s} = 192\) GeV, this mechanism gives rise to a large sample of Higgs particles. For large tan\(\beta\), associated \(Ah\) production is the dominant mechanism with rates similar to the previous case.

The predictions for the cross sections \(e^+e^- \rightarrow Zh\) and \(Ah\) presented above have been based on the improved effective potential approximation which takes into account heavy \((s)\)quark effects on Higgs masses, mixings and couplings. It turns out \textit{a posteriori} that this scheme is quite accurate. Indeed, the box contributions to the cross sections are fairly small [111]. This is demonstrated in Fig.23 where the box contributions are compared with the Born term, defined for the effective value \(\tan 2\alpha = -(m_Z^2 + m_h^2) \tan \beta/(m_Z^2 + m_A^2 \tan^2 \beta - m_h^2) \cos^2 \beta\). The leading part of the box contributions is generated by the two-Higgs doublet diagrams while the contributions of the genuine SUSY particles are very small.

The angular distributions are of the standard form [47] for Higgs-strahlung and spin-zero pair production,

\[
\frac{d\sigma}{d\cos \theta} \sim \begin{cases} \\ \lambda \sin^2 \theta + 8m_Z^2/s & \text{for } e^+e^- \rightarrow Zh \\ \sin^2 \theta & \text{for } e^+e^- \rightarrow Ah \end{cases}
\] (37)

Since the main decay mode of scalar and pseudoscalar Higgs particles are \(b\bar{b}\) decays in the MSSM, it is interesting to study the 4-fermion process \(e^+e^- \rightarrow b\bar{b}b\bar{b}\) in greater detail. The final state includes the signal \(Zh \rightarrow (b\bar{b})_Z(b\bar{b})_h\) in the Higgs-strahlung process, and the signal \(Ah \rightarrow (b\bar{b})_A(b\bar{b})_h\) for associated pair production. The main component of the background is \(e^+e^- \rightarrow Z^+Z^-\) production followed by \(Z^+ \rightarrow b\bar{b}\) decays. These cross sections have been evaluated for a cut on the invariant \(b\bar{b}\) mass of \(m(b\bar{b}) > 20\) GeV. The results are shown for a variety of combinations \((m_A, \tan \beta)\) in Table 14 for \(\sqrt{s} = 192\) GeV.

The cross section for the production of charged Higgs bosons

\[
e^+e^- \rightarrow H^+ H^- \quad (38)
\]
is built up by s-channel \(\gamma\) and \(Z\) exchanges [41, 112]. It depends only on the charged Higgs
Figure 22: The cross sections for Higgs-strahlung $Zh$ and associated pair production $Ah$ in the MSSM for two values of $\tan\beta = 1.6$ and 50 and the top mass $M_t = 175 \pm 25$ GeV.
Figure 23: The box contributions to the cross sections for Higgs-strahlung Zh and associated pair production Ah for $\sqrt{s} = 192$ GeV; $m_A = 90$ GeV and a slepton mass $m_\tilde{t} = 100$ GeV.

mass and no extra parameter,

$$\sigma(e^+e^- \rightarrow H^+H^-) = \frac{2G_F^2 m_W^4 s_W^4}{3\pi s} 1 + \frac{2\hat{a}_e \hat{v}_H}{1 - m_Z^2/s} + \frac{(\hat{a}_e^2 + \hat{v}_H^2) \hat{v}_H^2}{(1 - m_Z^2/s)^2} \beta_H^2 \quad (39)$$

where the rescaled Z charges are defined by $\hat{a}_e = -1/4c_W s_W$ and $\hat{v}_e = (1 + 4s_W^2)/4c_W s_W$ and $\hat{v}_H = (-1 + 2s_W^2)/2c_W s_W$ [note that $s_W^2 = \sin^2 \theta_W$; $\beta_H = (1 - 4m_{H^\pm}^2/s)^{1/2}$ is the velocity of the Higgs particles. The cross section is shown in Fig. 24 as a function of the charged Higgs mass for the three representative LEP2 energy values $\sqrt{s} = 175, 192$ and 205 GeV. Within the MSSM the present lower limit of the charged Higgs boson mass is about 85 GeV so that only a small window is left for LEP2. Even though the cross section is not particularly small for

<table>
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<th>(m_A [GeV], tan $\beta$)</th>
<th>(75,30)</th>
<th>(400,30)</th>
<th>(75,1.75)</th>
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<th>$\infty$</th>
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</tr>
<tr>
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<td>23.286(58)</td>
<td>163.36(75)</td>
<td>74.04(31)</td>
<td>22.816(50)</td>
</tr>
<tr>
<td>with EXCALIBUR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23.045(23)</td>
</tr>
<tr>
<td>ISR HZHA/PYTHIA</td>
<td>118.60(58)</td>
<td>18.761(87)</td>
<td>151.75(75)</td>
<td>57.74(28)</td>
<td>18.384(80)</td>
</tr>
</tbody>
</table>

Table 14: The process $e^+e^- \rightarrow b\bar{b}b\bar{b}$ at $\sqrt{s} = 192$ GeV. Cross sections in fb.
$m_{H^\pm} \sim 80 \text{ to } 90 \text{ GeV}$, the signal is very hard to extract from the overwhelming background of $WW$ pair production in this mass range. The analysis of cascade decays $H^\pm \to W^* h, W^* A$ [95] can ameliorate the prospects of detecting the charged Higgs boson in this mass range for small $\tan \beta$.

### 3.2.2 Decay Modes of the MSSM Higgs Particles

Decays to SM particles. For $\tan \beta > 1$ the lightest CP-even neutral Higgs boson $h$ decays almost exclusively to fermion pairs if the mass $m_h$ is less than 100 GeV. Near the upper limit of $m_h$ for a given $\tan \beta$, i.e., in the decoupling region, the decay pattern becomes SM-like. Fermion pairs are also the dominant decay mode of the pseudoscalar Higgs boson $A$. The partial decay widths of all the neutral Higgs bosons $\Phi$ into fermions are given by

$$
\Gamma(\Phi \to f \bar{f}) = N_c \frac{G_F m_f^2}{4\sqrt{2}\pi} g_{\Phi ff}^2 m_\Phi \left( 1 + \frac{17 \alpha_s}{3 \pi} \right)
$$

in the limit $m_\Phi^2 \gg m_f^2$. The couplings $g_{\Phi ff}$ have been defined in Table 13. The small additional $O(\alpha_s^2)$ contributions have been summarized in Ref.[12]. As anticipated from chirality arguments, the widths, including the QCD radiative corrections, do not depend on the parity of the state apart from the overall coupling $g_{\Phi ff}$ in the limit of large Higgs masses. For quark decays, $m_f$ has to be chosen as the running quark mass evaluated at the scale $m_\Phi$. The electroweak corrections are incorporated at a sufficient level of accuracy by adopting the effective potential approximation for the couplings [113].
The partial width for charged Higgs decays to quark pairs is obtained from
\[
\Gamma(H^\pm \rightarrow U \bar{D}) = \frac{3G_F m_H^\pm}{4\sqrt{2}\pi} |V_{UD}|^2 \cot^2 \beta m_U^2 + \tan^2 \beta m_D^2 \quad 1 + \frac{17\alpha_s}{3\pi} \tag{41}
\]
This formula is valid if either the first or the second term is dominant. The up and down quark masses \(m_{U,D}\) are defined again at the mass scale of the charged Higgs boson.

Since the \(b\) quark couplings to the Higgs bosons are in general strongly enhanced and the \(t\) quark couplings suppressed in the MSSM [cf. Fig.16], \(b\) loops may contribute significantly to the \(gg\) coupling so that the approximation \(m_Q^2 \gg m_\phi^2\) cannot be applied any more in general. Nevertheless, it turns out \(a posteriori\) that this remains an excellent approximation for the QCD corrections. The CP-even and CP-odd Higgs decays to gluons and light quarks [12] are given by the expressions
\[
\Gamma(\Phi \rightarrow gg(g), q\bar{q}g) = \frac{G_F \alpha^2 m_\phi}{16\sqrt{2}\pi^3} A_Q^{\pm} 1 + \frac{95}{4} \frac{97}{4} - \frac{7}{6} N_F \frac{\alpha_s}{\pi} \tag{42}
\]
where the parentheses refer to the pseudoscalar particle. The form factors are defined by
\[
A_Q^{h,A} = g_Q^{h,A} \times \tau \left[ 1 + (1 - \tau) f(\tau) \right] \quad \text{and} \quad A_Q^{A} = g_Q^{A} \tau f(\tau) \tag{43}
\]
with \(f(\tau) = \arcsin^2(1/\sqrt{\tau})\) for \(\tau \geq 1\) and \(-1 [\log(1 + \sqrt{1 - \tau})/(1 - \sqrt{1 - \tau}) - i\pi]^2\) for \(\tau < 1\). The parameter \(\tau = 4m_Q^2/m_\phi^2\) is defined by the pole mass of the heavy loop quark \(Q\). In the same way as \(\alpha_s(m_\phi)\), the coefficient of the QCD corrections must be evaluated for \(N_F = 3\) if gluons and only light quarks are considered in the final state [cf. the SM section for details].

At the edge of the mass range accessible at LEP2, the CP-even Higgs boson \(h\) can decay into virtual gauge boson pairs \(W^+W^-/Z^+Z^-\). The widths are the same as in the Standard Model, yet suppressed by the MSSM coefficient \(\sin^2(\beta - \alpha)\).

(ii) Cascade decays. A variety of cascade decays could in principle play a role in some ranges of the MSSM parameter space accessible at LEP2, if sufficiently large samples of heavy Higgs bosons were generated. However, for the typical set of parameters discussed in this report, these decay modes are not very important in general and details may be traced back from [95, 114]. The only exception are the cascade decays of the charged Higgs bosons [95] for small to moderate \(\tan \beta\),
\[
\Gamma(H^\pm \rightarrow hW^{\mp} \rightarrow hf F^\mp) = \frac{9G_F^2 m_W^4}{8\pi^3} \cos^2(\beta - \alpha) m_{H\pm} G_{hW} \tag{44}
\]
\[
\Gamma(H^\pm \rightarrow AW^{\pm} \rightarrow Af F^\mp) = \frac{9G_F^2 m_W^4}{8\pi^3} - m_{H\pm} G_{AW} \tag{45}
\]
The coefficients \(G\) depend on the mass ratios of the particles involved,
\[
G_{ij} = \frac{1}{4} 2(-1 + \kappa_j - \kappa_i) \frac{\pi}{\lambda_{ij}} \frac{\pi}{2 + \arctan} \frac{\kappa_j(1 - \kappa_j + \kappa_i) - \lambda_{ij}}{(1 - \kappa_i) \lambda_{ij}} \tag{46}
\]
\[
+ (\lambda_{ij} - 2\kappa_i) \log(\kappa_i) + \frac{1}{3}(1 - \kappa_i) 5(1 + \kappa_i) - 4\kappa_j - 2 \frac{\kappa_{ij}}{\kappa_j}
\]

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with \( \kappa_i = m_i^2/m_H^2 \) and \( \lambda_{ij} = -1 + 2\kappa_i + 2\kappa_j - (\kappa_i - \kappa_j)^2 \). These decay modes are important for sufficiently light \( h/A \) Higgs bosons. If they are allowed, in particular \( H^\pm \to AW^* \), they reduce the \( \tau\nu_\tau \) branching ratio considerably, and they overrule \( cs \) decays as the second most important decay channel of the charged Higgs bosons.

Summary of the branching ratios. The branching ratios for the standard quark/lepton/gauge boson and the cascade decay modes discussed above, are shown for “typical mixing” and two representative values \( \tan\beta = 1.6 \) and 50 in Fig.25. Unless otherwise specified the top mass in Fig.25 has been fixed to \( M_t = 176 \) GeV. Increasing the top mass shifts the upper end of the \( b \) and \( \tau \) curves upwards while the \( cc \) and \( gg \) curves are transferred nearly parallel, a consequence of the larger Higgs mass values. The effect of varying \( \alpha_s = 0.118 \pm 0.006 \) is indicated by the hatched bands. The curves labeled \( bb \) and \( cc \) correspond to all mechanisms generating inclusive \( b,c \) quarks in the final states, while the curve labeled \( gg \) includes gluons and light quarks.

At \( \tan\beta = 1.6 \), interesting cascade decays are predicted for moderately small charged Higgs masses, \( H^\pm \to AW^* \) and \( hW^* \); they affect the experimental search techniques also in the \( \tau\nu_\tau \) channel by reducing this important decay branching ratio. For large \( \tan\beta \), \( bb \) and \( \tau^+\tau^- \) decays are overwhelming except in the decoupling regime near the upper limit of the \( h \) mass.

Predictions for decays of the heavy CP–even Higgs boson \( H \) are discussed in great detail in Ref.[12].

Neutralino decays. For \( h,A \) Higgs masses up to \( \sim 120 \) GeV, there are still windows open for decays into pairs of light neutralinos [41, 115]. These windows have been left by LEP1 and they cannot be closed by LEP1.5 either. The decay channels of interest are

\[
h, A \to \chi^0_1\chi^0_1 \tag{47}
\]

for small to moderate \( \tan\beta \).

Masses and couplings of the states \( \chi_i \) depend on \( \tan\beta \), the \( SU_2 \) gaugino mass parameter \( M_2 \), and the Higgs mass parameter \( \mu \). [We assume the relation \( M_1 = \frac{\mu}{3}\tan^2\theta_W M_2 \simeq \frac{1}{2} M_2 \).] For large \( M_2 \) and small (positive) \( \mu \) values, the lightest neutralino \( \chi^0_1 \) is predominantly built up by the higgsino component while for large values of \( \mu \) the light \( \chi^0_1 \) state is predominantly gaugino-like. The couplings of \( \chi^0_1 \) to \( h \) and \( A \) are given in terms of the neutralino mixing matrix \( Z \) by

\[
\kappa_h = (Z_{12} - \tan \theta_W Z_{11}) (\sin \alpha Z_{13} + \cos \alpha Z_{14})
\]

\[
\kappa_A = (Z_{12} - \tan \theta_W Z_{11}) (-\sin \beta Z_{13} + \cos \beta Z_{14})
\]

[see e.g. [116]]. Since \( Z_{11}/Z_{12} \) correspond to the gaugino components of \( \chi^0_1 \) while \( Z_{13}/Z_{14} \) correspond to the higgsino components, the couplings \( h\chi^0_1\chi^0_1 \) and \( A\chi^0_1\chi^0_1 \) can only be non-negligible if the state \( \chi^0_1 \) incorporates both components at a significant level. Moderate values of \( M_2, \mu \) are therefore the favorable domain for LSP decays. The widths for the \( h, A \) decays into \( \chi^0_1\chi^0_1 \) pairs can be written as
Figure 25: Decay branching ratios of the MSSM Higgs bosons $h, A, H^\pm$ into SM particles and cascade decays. The bands characterize the uncertainties in the predictions, except those due to the top mass, which are indicated by bars.
Figure 25: (cont’d).

Figure 26: Branching ratios for $h$, $A$ decays into pairs of the lightest neutralino for a set of $SU_2$ gaugino and higgsino mass parameters not excluded by LEP1/1.5. If $\chi^0_1$ is the LSP and if R parity is unbroken, these decays lead to invisible final states.
These decays of the Higgs particles are invisible if $\chi_1^0$ is stable. The Higgs particle $h$ can still be observed in the Higgs-strahlung process through the recoiling $Z$ at small to moderate $\tan\beta$ where this production channel is dominant. However, the pseudoscalar Higgs boson $A$ cannot be detected if, produced in associated $Ah$ production, both particles decay into invisible $\chi_1^0\chi_1^0$ channels for small to moderate values of $\tan\beta$.

Since for large $\tan\beta$ the $b\bar{b}$ decays of the $h$ and $A$ Higgs bosons are overwhelming, $\tan\beta$ needs to be small to moderate for $\chi_1^0$ decays to be relevant. Typical examples of large branching ratios for $h, A$ Higgs decays to LSP pairs are shown in Fig.26 for a set of $SU_2$ gaugino and higgsino mass parameters $M_2$ and $\mu$. The LSP masses can be read off the threshold values. The branching ratios are large whenever the LSP decay channels are open for $\mu > 0$. For $\mu < 0$ the LSP decays play a less prominent role; only in a small window close to $\mu \sim -M_2/2$ are the couplings large enough to allow for invisible $h$ and $A$ decays [115].

\[
\Gamma(h \to \chi_1^0\chi_1^0) = \frac{G_F m_W^2}{2\sqrt{2}\pi} \kappa_h^2 m_h \beta_X^3
\]

\[
\Gamma(A \to \chi_1^0\chi_1^0) = \frac{G_F m_W^2}{2\sqrt{2}\pi} \kappa_A^2 m_A \beta_X
\]
3.3 The Experimental Search for the Neutral Higgs Bosons

3.3.1 Searches in the Higgs-strahlung Process

For the $e^+e^- \rightarrow hZ$ process, as well as for $e^+e^- \rightarrow HZ$ when kinematically allowed, all the analyses [58] developed for the standard model Higgs boson and presented in Section 2.3 can be used with no modifications and with a similar efficiency, provided that the Higgs boson decays into supersymmetric particles, such as charginos and neutralinos, are not open. As soon as the decay into a pair of LSPs (Lightest Supersymmetric Particle) $h \rightarrow \chi\chi$ is allowed, it may even become dominant therefore rendering the existing analyses ineffective.

Two new selection algorithms were developed by ALEPH to take care of this particular situation where the Higgs boson would decay invisibly, for the following events topologies:

(i) the acoplanar lepton pair topology, $(Z \rightarrow e^+e^-, \mu^+\mu^-) (h \rightarrow \chi\chi)$;

(ii) the acoplanar jet topology, $(Z \rightarrow q\bar{q}) (h \rightarrow \chi\chi)$.

Only minor modifications to the selection procedure would be needed to extend the validity of these analyses to “almost invisible” Higgs boson decays, such as $h \rightarrow \chi'\chi$ or $\chi^+\chi^-$ when the mass difference between the LSP and the next-to-LSP is small.

a) Search in the Acoplanar Lepton Pair Topology

The acoplanar lepton pair topology arises when the $Z$ decays into a pair of leptons and the Higgs boson $h$ decays invisibly into a pair of neutralinos. Events can be selected by requiring a high mass $e^+e^-$ or $\mu^+\mu^-$ pair, compatible with the $Z$ hypothesis and with large missing energy and missing mass. Events from $e^+e^- \rightarrow \ell^+\ell^- (\gamma)$, $Ze^+e^-$ or $\gamma\gamma \rightarrow \ell^+\ell^-$ are characterized by a large missing momentum along the beam direction and a small acoplanarity angle, and can therefore easily be rejected. The only irreducible background sources are $e^+e^- \rightarrow WW \rightarrow e\nu\nu, \mu\nu\nu, e^+e^- \rightarrow ZZ \rightarrow e^+e^-\nu\bar{\nu}, \mu^+\mu^-\nu\bar{\nu}$, and to a lesser extent $e^+e^- \rightarrow Z\nu\bar{\nu}$.

An efficiency of 45 to 50% was achieved independently of $m_h$. The lepton momenta were subsequently fitted to the $Z$ mass hypothesis, and the missing mass calculated from the energy-momentum conservation as recoiling against the lepton pair, with a typical resolution of 2 GeV/$c^2$. Shown in Fig.27 are the mass distributions obtained with 500 pb$^{-1}$ at 175 and 192 GeV, for several Higgs boson masses. At 192 GeV, and for $m_h = 90$ GeV/$c^2$, the numbers of signal and background events expected in a window of ±2σ around the reconstructed Higgs
boson mass are 6.2 and 6.1 respectively, assuming a 100% branching fraction into invisible final states.

b) Search in the Acoplanar Jet Topology

In order to add to the numbers of signal events expected from the acoplanar lepton pair topology, the hadronic decays of the Z were also investigated. A very similar selection procedure as in Section 3.3.1a) was developed, in which the two hadronic jets played the role of the leptons. A similar selection efficiency was achieved, but with a much higher background from $e^+e^- \rightarrow WW$ and $(e)\nu W$ in particular, due to the much worse jet-jet invariant mass resolution. A b-tagging requirement may or may not be applied to reduce the background (at the expense of a 80% loss of efficiency), with almost no consequences on the minimum luminosity needed for the discovery or the exclusion.

Shown in Fig.28 are the mass distributions obtained in the same configurations as in Fig.27, when a tight b-tagging criterion is applied. At 192 GeV, and for $m_h = 90$ GeV/$c^2$, the numbers of signal and background events expected are 7.7 and 4.9 respectively, assuming a 100% branching fraction into invisible final states.

3.3.2 Search in Associated Pair-production $e^+e^- \rightarrow Ah$

The $e^+e^- \rightarrow hA$ associated production leads to two main final states, $b\bar{b}b\bar{b}$ in 83% of the cases and $\tau^+\tau^-q\bar{q}$ in 16% of the cases, if supersymmetric decays are absent.

a) The $b\bar{b}b\bar{b}$ Topology

This four-jet final state is similar to the four-jet topology arising from the Higgs-strahlung process, and a similar selection procedure can therefore be applied. Here, the Z mass constraint cannot be used, and the requirement of incompatibility with a WW final state must be removed to retain a sizeable efficiency for the case $m_h = m_A \sim 80$ GeV/$c^2$. However, since the b-quark content is much higher in this four-b-jet topology than in the Higgs-strahlung process, a much tighter b-tagging criterion can be applied. In terms of background rejection, this may even over-compensate the removal of the two previous requirements while keeping a high efficiency, varying between 10 and 35%.

The four-jet energies and directions can then be fitted to satisfy the total energy-momentum conservation constraint in order to achieve a good mass resolution. Shown in Fig.29a are the distributions of the sum of the fitted $m_h$ and $m_A$ values as obtained in the ALEPH detector with an integrated luminosity of 500 pb$^{-1}$ at 175 GeV, for $\tan \beta = 10$ and $m_A = 65$ and 75 GeV/$c^2$. & 411
Figure 27: Distribution of the missing mass recoiling against the $e^+e^-$ or $\mu^+\mu^-$ pair, in the acoplanar lepton pair topology, as obtained from the ALEPH simulation at 175 GeV (left) and 192 GeV (right), with an integrated luminosity of 500 pb$^{-1}$. The signal (in white) is shown on top of the background (shaded histogram), with Higgs boson masses of 60 (dashed), 70 (dotted) and 80 (dash-dotted) GeV/c$^2$ at 175 GeV, and 70 (dashed), 80 (dotted) and 90 (dash-dotted) GeV/c$^2$ at 192 GeV.

Figure 28: Same as in Fig.27, for the acoplanar jet topology, after a tight b-tagging (optional) requirement is applied.
The same distributions as seen by OPAL are shown in Fig. 29b with 500 pb⁻¹ taken at 192 GeV, for \( m_h = m_A = 70 \text{ GeV}/c^2 \). Since for large \( \tan \beta \) values the \( h \) and \( A \) masses are expected to be close to each other, the mass resolution is expected to improve by imposing this mass equality in the fit procedure. This was done by DELPHI for \( m_h = m_A = 79 \text{ GeV}/c^2 \) at 192 GeV, and the result is shown in Fig. 30a for an integrated luminosity of 300 pb⁻¹. Finally, the distribution of \( m_h \) vs \( m_A \) that would be obtained in L3 in the mass configuration (60 GeV/c², 80 GeV/c²) if the signal cross-section amounted to 0.5 pb is shown in Fig. 30b at 190 GeV, for an integrated luminosity of 1 fb⁻¹.

![Diagram](https://example.com/diagram.png)

Figure 29: Mass distributions obtained in the \( hA \rightarrow b\bar{b}b\bar{b} \) topology. (a) \( m_h + m_A \) from ALEPH (175 GeV, 500 pb⁻¹); and (b) \( m_h + m_A \) from OPAL (192 GeV, 500 pb⁻¹).

b) The \( \tau^+\tau^-b\bar{b} \) Topology

For this topology, the same analysis as for the Higgs-strahlung process was used by ALEPH and DELPHI (with the exception that the very last fit intended to improve the \( \tau^+\tau^- \) and hadronic mass resolution with the \( m_Z \) constraint does not apply). The background level is already very low, except when \( m_h \) and \( m_A \) are close to \( m_Z \) in which case the ZZ background can be reduced by tagging b-quarks. In this configuration, however, the signal is expected to have a very low cross-section except at the highest possible center-of-mass energy, \( \sqrt{s} = 205 \text{ GeV} \). Altogether, when added to the preceding one, this analysis increases the selection efficiency of the \( hA \) channel by about 20%.

c) The Case \( h \) or \( A \rightarrow \chi\chi \)

If either \( h \) or \( A \) decays predominantly into \( \chi\chi \), the relevant topology becomes that of an acoplanar jet pair, as already described in Section 3.3.1b). However, the pair of jets is actually
a pair of b-quarks in that case, therefore improving the selection efficiency of a b-tagging criterion with respect to the $e^+e^-$ configuration.

In the unfortunate situation where both $h$ and $A$ predominantly decay into a pair of LSPs, the resulting final state becomes totally invisible and cannot be found at LEP2. However, in that case, there is a fair chance to discover the lightest supersymmetric particle via a direct neutralino search.

### 3.4 Discovery and Exclusion Limits

Using the definitions of Appendix 5.3, a minimum signal cross-section was inferred for the $e^+e^-\rightarrow hA$ process from the expected number of background events, both for the discovery and the exclusion. Since the background mass distribution is mostly uniform over the $(m_h,m_A)$ plane, it turns out that this minimum signal cross-section does not depend on $m_h$ and $m_A$. For instance, at 192 GeV and with an integrated luminosity of 150 pb$^{-1}$, a cross-section in excess of 65 (30) fb can be discovered (excluded) in the $e^+e^-\rightarrow hA$ channel when supersymmetric decays are closed.

For the Higgs-strahlung process, the total cross-section is reduced with respect to the standard model expectation by a factor denoted $\sin^2(\beta - \alpha)$ in the MSSM. The number of events expected is also directly affected by the branching ratio of the $h$ decay into $bb$. For each $m_h$, 

![Figure 30: Mass distribution obtained in the $hA \rightarrow b\bar{b}b\bar{b}$ topology, (a) $m_h = m_A$ from DELPHI (192 GeV, 300 pb$^{-1}$); and (b) $m_h$ vs $m_A$ from L3, assuming a signal-cross-section of 0.5 pb (190 GeV, 1 fb$^{-1}$).](image-url)
a minimum value for \( R^2 = \sin^2(\beta - \alpha) \times BR(h \to b\bar{b}) \) was inferred for the discovery and the exclusion. The result, which is model-independent, is shown in Fig.31 for the three center-of-mass energies 175, 192 and 205 GeV, with integrated luminosities of 150, 150 and 300 pb\(^{-1}\), respectively [58]. The interpretation of the negative searches at LEP1 (from Ref.[117]) is also shown in this plot.

Since this limit becomes irrelevant when supersymmetric decays are dominant, it is supplemented by the 95\% C.L. upper limit on \( R^2 = \sin^2(\beta - \alpha) \times BR(h \to \text{invisible final states}) \), as shown in Fig.32. At LEP2, this limit is worse by about a factor of two than in the case where \( h \) predominantly decays into \( b\bar{b} \), while it was better at LEP1. In order to make easier the use of these curves to test other models, such as non-minimal supersymmetric extensions of the standard model, the exclusion and discovery limits on \( R^2 = \sin^2(\beta - \alpha) \times BR(h \to b\bar{b}) \) are presented in Table 15, for the three center-of-mass energies, and when combining the results at 175 and 192 GeV, on the one hand, and with the 205 GeV results in addition, on the other.

To interpret these results in the MSSM framework, both the \( e^+e^- \to hA \) cross-section and the \( \sin^2(\beta - \alpha) \) value were computed and compared to the above minimum values in a systematic scan of the \((m_A, \tan\beta)\) plane, for \( M_t = 175 \pm 25\text{ GeV}/c^2 \) and for the three different stop mixing configurations, (i) No mixing: \( A_t = 0 \) and \( |\mu| \ll M_S \); (ii) Typical mixing: \( A_t = M_S \) and \( \mu = -M_S \) (these values of \( A_t \) and \( \mu \) give a moderate impact of the stop mixing for large \( \tan\beta \) but a mixing effect close to maximal if \( \tan\beta \) is small); and (iii) Maximal mixing: \( A_t = \sqrt{6}M_S \) and \( |\mu| \ll M_S \), with \( M_S = 1 \text{ TeV} \).

The results of the above analysis are summarized in a series of figures (Figs.33 and 34), which display the areas in the \((m_A, \tan\beta)\) and \((m_h, \tan\beta)\) planes that can be covered for a given energy of LEP2, \( \sqrt{s} = 175, 192 \) and 205 GeV at the integrated luminosities 150, 150 and 300 pb\(^{-1}\), respectively. The top-quark mass and the stop mixing parameters are varied as specified above. The figures are obtained by combining the four LEP experiments. At \( \sqrt{s} = 175 \) GeV, an increase in luminosity beyond 150 pb\(^{-1}\) does not improve the potential of the machine for the discovery of the light Higgs boson in any relevant way. The analogous conclusion was reached in the standard model case. At \( \sqrt{s} = 192 \) GeV, still 150 pb\(^{-1}\) of integrated luminosity per experiment are sufficient to make proper use of the discovery potential of the machine, and a larger luminosity of \( \simeq 300 \text{ pb}^{-1} \) gives only a slight improvement in the upper bound on the Higgs boson mass which can be discovered or excluded. Although for any center-of-mass energy the variation from 150–200 pb\(^{-1}\) to 300–400 pb\(^{-1}\) of integrated luminosity yields a gain of at most 2 to 3 GeV/c\(^2\) in the maximal Higgs boson mass that can be reached, at \( \sqrt{s} = 205 \text{ GeV} \) the results are presented for 300 pb\(^{-1}\) since for this energy value the increase in luminosity translates into a quite impressive coverage of \( \tan\beta \).

Comparing the experimental limits for the center-of-mass energies 175 GeV and 192 GeV, Fig.33 a/b and c/d, it is clear that the higher energy value allows a remarkably larger part of the parameter space to be covered. For \( M_t = 175 \text{ GeV}/c^2 \) and in particular for \( \sqrt{s} = 175 \text{ GeV}, \)

\(^2\)Since this analysis was done in ALEPH only, it was assumed that the other experiments would contribute in the same proportions as for the visible decays to perform the combination.
Figure 31: 95% C.L. upper limits on $R^2$, as a function of $m_h$ for a visible Higgs boson, for the three center-of-mass energies: $R^2 = \sin^2(\beta - \alpha) \times \text{BR}(h \to \text{any visible final state})$ for the LEP1 part of the curve and $R^2 = \sin^2(\beta - \alpha) \times \text{BR}(h \to b \bar{b})$ for the LEP2 part of the curve.

Figure 32: 95% C.L. upper limits on $R^2$, as a function of $m_h$ for an invisible Higgs boson, for the three center-of-mass energies: $R^2 = \sin^2(\beta - \alpha) \times \text{BR}(h \to \text{any invisible final state})$. 

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Table 15: Minimum value for $R^2 = \sin^2(\beta - \alpha) \times BR(h \rightarrow b\bar{b})$ at the three center-of-mass energies, with integrated luminosities of 150, 150 and 300 pb$^{-1}$, respectively, and for various Higgs boson masses. Also shown is the combination of the 175 GeV and 192 GeV results, with an integrated luminosity of 150 pb$^{-1}$ taken at each energy, and the overall combination of the 175, 192 and 205 GeV results, assuming 300 pb$^{-1}$ at 205 GeV. The combination of several center-of-mass energies have not been used in Fig.33 and 34.

\[ \sqrt{s} = 175 \text{ GeV} \]

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\[ \sqrt{s} = 192 \text{ GeV} \]

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Combination of 175 and 192 GeV

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\[ \sqrt{s} = 205 \text{ GeV} \]

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Combination of 175, 192 and 205 GeV

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Figure 33: *Exclusion and discovery limits in the \([m_A, \tan \beta] \) plane for each of the center-of-mass energies, varying the values of the stop mixing parameters as specified in the text (a, c and e) and varying the values of \(M_t = 150, 175, 200 \text{ GeV}/c^2 \) for \(A = -\mu = M_S = 1 \text{ TeV} \) (b, d and f).*
Figure 33: (cont’d)
Figure 33: (cont’d)
Figure 34: Exclusion and discovery limits in the $[m_h, \tan \beta]$ plane, for $M_t = 175$ GeV and $A = -\mu = M_S = 1$ TeV for each of the center-of-mass energies. The dark shaded areas are excluded theoretically.
the potential of LEP2 is rather limited at large $m_A$, while for $192$ GeV it is possible to cover the moderate and large $m_A$ for small $\tan\beta$, $\tan\beta \leq 2 - 3$ (which is a difficult region for LHC). This is especially obvious if non-negligible mixing in the stop sector is considered. For large values of $m_A$, the limit of the standard model Higgs boson is approached (light gaugino channels are not considered to be open) and the lightest CP-even Higgs boson acquires its maximal value. Since the maximal value of $m_h$ increases with $\tan\beta$, the range in $\tan\beta$ covered by LEP2 directly reflects the range of standard model Higgs boson masses accessible to the experiments.

It is interesting to compare the upper limits on $m_h$ which can be reached experimentally, Table 16 and Fig.34, with the maximal $h$ masses expected in the MSSM. In particular, if the experimental limits are compared to the $h$ mass range preferred by gauge and $b$-$\tau$ Yukawa coupling unification, for $M_t \simeq 175$ GeV/$c^2$ it can be seen that a large part of the SUSY Higgs mass range is covered in the $192$ GeV version of LEP2, in contrast to the lower energy of $175$ GeV. In fact, the infrared fixed point solution can be excluded at the $95\%$ C. L. at $\sqrt{s} = 192$ GeV if $M_t \leq 175$ GeV/$c^2$. (For $M_t \simeq 185$ GeV/$c^2$ the infrared fixed point solution can be excluded only at $\sqrt{s} = 205$ GeV.)

3.5 MSSM vs. SM

No precise experimental analyses have yet been developed to cope with the situation in which a Higgs boson would be discovered and thus would need to be studied in detail. This can be
Table 16: Maximal $m_h$ (hZ and hA combined) and $m_A$ (hA only), in GeV/c^2, that can be directly discovered and excluded in the MSSM for $M_t = 175$ GeV/c^2 and typical mixing at various LEP2 energies for two representative values of tan $\beta$ (* These values of $m_h$ are already excluded theoretically in the MSSM for typical mixing, $M_S = 1$ TeV and for the values of $M_t$ and tan $\beta$ considered here).

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<td>300</td>
<td>108$^*$</td>
<td>80</td>
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partly explained by the relatively low luminosity currently expected at LEP2 which will not allow any accurate measurements to be performed in this field. It could however be imagined that if a new particle were discovered, a substantial extension of the LEP2 project could be decided upon with the purpose of identifying this particle.

The angular distribution of the Higgs boson produced in the Higgs-strahlung process is expected to be quite uniform at the LEP2 center-of-mass energies, and so is the angular distribution of the decay products in the Higgs boson rest frame. Even in the most difficult case $m_h \sim m_Z$, the numbers of events collected by the four experiments with 1 fb$^{-1}$ taken at 192 GeV (323 events from the signal and 255 from the backgrounds, see Table 9) suffice to exhibit the flatness of these distributions, and to characterize unambiguously a $J^P = 0^+$ particle.

Such an excess of events after a b-tagging requirement is also sufficient to claim that this object often decays into $b\bar{b}$, which also characterizes a Higgs particle. However, a precise measurement of the $b\bar{b}$ branching fraction can only be done with a channel in which the evidence for a signal can be claimed without b-tagging, i.e. the $H\ell^+\ell^-$ topology. This situation is particularly favourable if the Higgs boson mass is not degenerate with the Z mass, in which case the background is much reduced thanks to the excellent recoil mass resolution.

For instance, the cross-section for a 80 GeV/c^2 Higgs boson at $\sqrt{s} = 192$ GeV in the $H\ell^+\ell^-$ topology is 47 fb (after selection cuts, but with no b-tagging requirement), to be compared to 14 fb for the background. This already allows a measurement of the $b\bar{b}$ branching fraction (if close to 100%) with a ~ 20% statistical accuracy, if a luminosity of 1 fb$^{-1}$ is given to each experiment. Similarly, the quantity $\sigma(e^+e^- \to hZ) \times BR(h \to b\bar{b})$ can be determined with the same luminosity from the events collected in all the topologies with a statistical accuracy of ~ 5% (resp. 10%) for a 80 GeV/c^2 (resp. 90 GeV/c^2) Higgs boson.

Unfortunately, even if the systematics uncertainties were negligible (e.g. the errors related
to the b-tagging efficiency determination) this is not sufficient to distinguish the standard model and the MSSM in the region where only the Higgs-strahlung process plays a role. In this region, a statistical accuracy better than 1 or 2% is indeed required [118] to achieve this goal. The extension of the standard model would be manifested in this parameter range only if non-standard Higgs boson decays were observed. It is otherwise only in the region where the $e^+ e^- \rightarrow hA$ process or, less likely, charged Higgs bosons (see Section 3.6) can be discovered, that this distinction is possible at LEP2.

### 3.6 Search for Charged Higgs Bosons

In the MSSM the mass of the charged Higgs bosons $H^\pm$ is expected to be larger than $m_W$. In general, for non-extreme stop mixing configuration, it is of the order of $m_{H^\pm}^2 = m_A^2 + m_W^2$, i.e. larger than $\sim 90$ GeV/$c^2$ within a few GeV/$c^2$, rendering rather difficult the discovery of such heavy objects at LEP2. However, this does not hold necessarily in other, e.g. non-minimal supersymmetric extensions of the Standard Model where light charged Higgs bosons – although heavier than 44 GeV/$c^2$ as shown by LEP1 data – cannot be ruled out. Their discovery would unambiguously signal the existence of an extended Higgs sector.

#### 3.6.1 Production and Decays

Charged Higgs bosons are produced in pairs in the process $e^+ e^- \rightarrow H^+ H^-$ with a rate depending only on $m_{H^\pm}$ in the general two-doublet model. About 100 such events are expected to be produced with an integrated luminosity of 500 pb$^{-1}$ for $m_{H^\pm} \sim 70$ GeV/$c^2$, irrespective of the center-of-mass energy from 175 to 205 GeV, and this rate decreases rapidly with increasing mass due to the $\beta^3$ kinematic suppression factor.

Furthermore, if it is assumed that the cascade decay modes [95] like $H^+ \rightarrow W^+ h$ are kinematically suppressed, the charged Higgs bosons are expected to decay predominantly into the heaviest kinematically accessible fermion pair provided it is not suppressed by a small CKM matrix element, i.e. $H^+ \rightarrow \tau^+ \nu_\tau$ or $c\bar{s}$. Therefore the expected final states are $\tau^+ \nu_\tau \bar{\nu}_\tau$, $c\bar{s}$, and $c\bar{c}s^c$, thus leading to an important irreducible background from $e^+ e^- \rightarrow W^+ W^-$ in addition to the low expected signal rate. This renders almost hopeless the discovery of charged Higgs bosons with mass around and above the W mass.

Searches for these final states have been developed by L3 [119] and DELPHI [64], using full detector simulation of signal and background processes for an integrated luminosity of 500 pb$^{-1}$. A search for the four-jet final state $c\bar{s}s^c$ has so far been developed by L3.

**a) The $e^+ e^- \rightarrow H^+ H^- \rightarrow c\bar{s}s^c$ Channel**

The process $e^+ e^- \rightarrow H^+ H^- \rightarrow c\bar{s}s^c$ leads to four-jet hadronic events. In order to distinguish a Higgs signal from the main background of $e^+ e^- \rightarrow q\bar{q}$ and $W^+ W^-$, use is made of the different
topological prop erties. The simulated hadronic energy deposits are clustered into four jets. For example at \( \sqrt{s} = 175 \) GeV, for 60 and 70 GeV/c\(^2\) Higgs signals, 75\% and 69\% selection efficiencies are expected respectively. The numbers of expected background events are: 3140 q\(\bar{q}\), 4664 WW, and 90 ZZ for 500 pb\(^{-1}\).

The four jets can be combined into two jet-pairs in three possible ways, and their energies and directions are fitted to the energy-momentum conservation and to the \( m_{H^\pm} = m_H^-\) constraint. The combination with the smallest \( \chi^2 \) is chosen, provided that the measured mass difference between the two jet-pairs is smaller than 5 GeV/c\(^2\), and a mass resolution of about 1 GeV/c\(^2\) is achieved. Unlike the \( hA \to b\bar{b}b\bar{b} \) channel, no b-tagging requirement can be applied to reject the WW \( \to \) four-jet background, but the signal-to-noise ratio is improved by removing events where one jet pair combination is consistent with a W pair, at the expense of a suppression of the signal efficiency when \( m_{H^\pm} \simeq m_W \).

For \( m_{H^\pm} \sim 60 \) GeV/c\(^2\), the expected signal efficiency is about 7\% and the number of background events is about 2 q\(\bar{q}\) and 3 WW events, for an integrated luminosity of 500 pb\(^{-1}\) at \( \sqrt{s} = 175 \) GeV.

b) The \( e^+e^- \to H^+H^- \to c\bar{s}\tau^-\bar{\nu}_\tau \) Channel

The signature of an \( e^+e^- \to H^+H^- \to c\bar{s}\tau^-\bar{\nu}_\tau \) signal is one isolated slim jet with missing energy coming from the \( \tau \) decay recoiling against a hadronic system. In the DELPHI analysis, a preselection of hadronic final states is performed, and the events are clustered into three jets. The lowest multiplicity jet, i.e. the \( \tau \) candidate, is required to have at most three charged particle tracks. Further cuts on the mass and on the angle between the two most energetic jets are applied to reject events without a clear 3-jet topology. A kinematic fit of the neutrino direction and the \( \tau \) momentum (four unknowns) is performed by constraining the \( \tau \nu \) and cs systems to have the same invariant mass and the total energy-momentum to be conserved (five equations). A \( \chi^2 \) cut is applied to improve the signal-to-noise ratio. This retains from 16\% to 29\% of the signal events, depending on the \( H^\pm \) mass and approximately 38 background events remain for 500 pb\(^{-1}\) at 175 GeV.

In the L3 analysis, the kinematic fit is replaced by the following approximate method:

(i) the missing momentum, \( \vec{p}_{\text{miss}} \), the missing energy, \( E_{\text{miss}} \), and the invariant mass of the two most energetic jets, \( M_{cs} \), are calculated. In order to improve the mass resolution, this mass is rescaled by the factor \( E_{\text{beam}}/(E_{\text{jet1}} + E_{\text{jet2}}) \);

(ii) for the other hemisphere, the invariant mass \( M_{\nu\nu}^2 = (E_{\text{miss}} + E_{\tau})^2 - (\vec{p}_{\text{miss}} + \vec{p}_{\tau})^2 \) is calculated, where \( \vec{p}_{\tau} \) is the visible \( \tau \) momentum.

The reconstructed masses of \( M_{cs} \) and \( M_{\nu\nu} \) are shown in Fig.35. Cuts as given in the figure are applied. The expected signal efficiencies are about 5.6\% and about 2 WW background events are expected for 500 pb\(^{-1}\) at 175 GeV.
c) The $e^+e^- \rightarrow H^+H^- \rightarrow \tau^+\nu_{\tau}\tau^-\overline{\nu}_{\tau}$ Channel

The $e^+e^- \rightarrow H^+H^- \rightarrow \tau^+\nu_{\tau}\tau^-\overline{\nu}_{\tau}$ events are characterized by a low particle multiplicity and large missing energy. The background processes $e^+e^- \rightarrow \tau^+\tau^- (\gamma)$, $q\overline{q}$, WW and $\gamma\gamma \rightarrow \overline{f}f$ are relevant in this channel.

After applying selection criteria based on the acoplanarity, the total energy and the event thrust axis angle, the main remaining background comes from leptonic WW decays. Unlike the other two channels, a reconstruction of the Higgs boson masses is not possible because of too many unknowns due to the numerous missing energy sources. However, the energies of the decay products of the two taus can be measured, and part of the W decays into $e\nu_e$ and $\mu\nu_\mu$ can be removed by a cut on these two energies, which are expected to be larger in that case than for $\tau\nu_\tau$ final states due to the additional neutrinos from the $\tau$ decay.

The expected signal efficiencies are about 12% and about 1 WW background event is expected in the L3 analysis for 500 pb$^{-1}$ at 175 GeV. For DELPHI, a 23% signal efficiency is achieved, for a total of 85 background events expected.

### 3.6.2 Results

In the framework of a general two Higgs doublet model, the results of this analysis can be expressed as a function of $\text{Br}(H^+ \rightarrow \tau^+\nu_\tau)$ (if it is assumed that off-shell decay modes are
Figure 36: $3\sigma$ sensitivity regions as a function of the charged Higgs boson mass and its leptonic branching ratio at 175 GeV and for a total luminosity of 100, 200, and 500 pb$^{-1}$. The upper solid lines indicate the sensitivity limits from the $\tau\nu\bar{\nu}$ channel, the dashed lines come from the $c\bar{s}\tau\bar{\nu}_\tau$ channel, and the lower solid lines come from the $c\bar{s}s$ channel. (For simplicity, combined contour lines in overlapping sensitivity regions are not shown.)

suppressed) and of $m_{H^\pm}$. Combining the studies in the $c\bar{s}\bar{s}$, $c\bar{s}\tau\bar{\nu}_\tau$, and $\tau^+\nu_\tau\tau^-\bar{\nu}_\tau$ channels described in the previous sections, the regions which can be explored with $\sqrt{s} = 175$ GeV are shown in Fig.36 for luminosities of 100, 200, and 500 pb$^{-1}$ taken by one experiment. (These numbers would be roughly divided by four if the four LEP experiments were combined.)

Except for branching ratios into $\tau\nu_\tau$ near 0 or 1, all three channels contribute simultaneously which extends the sensitivity range by few GeV/c$^2$. Charged Higgs bosons with masses up to about 70 GeV/c$^2$ should be detectable independently of their decay branching ratios assuming a total luminosity of 500 pb$^{-1}$ at 175 GeV. The region where a 99.73\% CL (3$\sigma$) effect due to $H^+H^-$ production can be detected is strongly dependent on the assumed luminosity. The boundary lines also depend strongly on the detection sensitivities due to the small change of the charged Higgs boson production cross-section with different $m_{H^\pm}$. The expected variations of the number of background events for different Higgs masses is taken into account in the figure. The effect of increasing the center-of-mass energy to about 200 GeV is small since only a small change of the production cross section is expected.

The background at LEP2 from WW production with identical decay modes as for the

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charged Higgs bosons can be controlled. Irreducible background only occurs if \( m_{H^\pm} \sim m_W \). For leptonically decaying Higgs bosons, masses can however be explored up to about \( m_W \) due to the small branching fraction of the W boson into \( \tau \nu_\tau \).

To summarize, LEP2 has a good potential for a charged Higgs boson discovery already in its first phase at 175 GeV [well beyond the current mass limit of LEP1 of 44 GeV/\( c^2 \)] as soon as a sufficient luminosity of about 200 pb\(^{-1}\) is collected. Even with an increase of the machine energy to around 200 GeV, it is extremely difficult to explore the kinematic region around and above \( m_W \) because of the large irreducible background from WW production and the low production cross section.

### 3.7 Complementarity between LEP2 and LHC

As with the SM Higgs boson, it is an important task to compare the Higgs discovery potential of LEP2 with the potential of LHC for the minimal supersymmetric standard model MSSM [120]. The comparison will be based again on the LEP energy \( \sqrt{s} = 192 \) GeV with an integrated luminosity of \( \mathcal{L} = 150 \) pb\(^{-1}\) per experiment [yet all four experiments combined] and the presumably ultimate integrated luminosity \( \mathcal{L} = 3 \times 10^6 \) pb\(^{-1}\) of LHC, with the results from ATLAS and CMS combined.

The lightest of the neutral scalar Higgs bosons \( h \) can be produced at the LHC in gluon–gluon fusion [56, 72], and through Higgs–strahlung off W bosons [74] and top quarks [121]. This Higgs particle will be searched for in the \( \gamma\gamma \) and \( b\bar{b} \) decay channels. Tagging leptonic \( W/Z \) and \( t \) decays provides an experimental trigger for the \( b\bar{b} \) search, but also reduces considerably the huge backgrounds from non–\( b \) jets. The area of the \( [m_A, \tan\beta] \) parameter plane in which the light scalar Higgs boson \( h \) can be discovered in this way at the LHC is shown in Fig.37 by the shaded regions [122]. The boundary of the LEP2 discovery range is indicated by the full line. No method has yet been found which allows the discovery of \( h \) in the parameter range of large \( \tan\beta > 5 \) and \( m_A \) between \( \sim 90 \) and \( \sim 170 \) GeV at either of the two machines.

While the area in which the pseudoscalar Higgs boson \( A \) can be discovered at LEP is rather modest, a large domain is accessible at LHC, Fig.38. A clean signal of \( A \) comes from \( \tau^+\tau^- \) decays; in addition, cascade decays \( A \to Zh \) with subsequent leptonic \( Z \) decays provide promising search channels. The search for \( A \) in \( t\bar{t} \) decays requires the theoretical control of the top background production at a level between 10% and 2% which is an extremely difficult problem. A similar picture applies to the search for the heavy scalar Higgs particle \( H \) at the LHC, cf. Fig.38. In particular, the classical four–lepton decay of \( H \) via \( ZZ \) intermediate states can be exploited. At LEP2 the heavy Higgs \( H \) might be produced only in a small region of the MSSM parameter range. The search for charged MSSM Higgs particles is frustrated in either machine. While the LEP2 energy is not sufficient to produce these particles pairwise outside a tiny domain of the MSSM parameter space, the search technique at the LHC is restricted so far to \( t \to bH^+ \) decays with a rather limited range in the charged Higgs mass.
Figure 37: The regions in the $[m_A, \tan \beta]$ parameter space in which the lightest CP-even Higgs boson $h$ can be covered at LEP2 (Higgs-strahlung and associated production for $150 \text{ pb}^{-1}$ at $\sqrt{s} = 192 \text{ GeV}$) and at LHC (Higgs-strahlung and associated production for $3 \times 10^5 \text{ pb}^{-1}$ and $3 \times 10^6 \text{ pb}^{-1}$, $W^+W^-$ decay channel in associated $Wh$, $t\bar{t}h$ production for $3 \times 10^4 \text{ pb}^{-1}$, $3 \times 10^5 \text{ pb}^{-1}$, and $H \rightarrow hh \rightarrow b\bar{b}b\bar{b}$ decay channel for $3 \times 10^5 \text{ pb}^{-1}$). Parameters: $M_t = 175 \text{ GeV}$, $A = 0$, $|\mu| \ll M_S = 1 \text{ TeV}$, but the masses of all supersymmetric particles are set to $1 \text{ TeV}$. (Courtesy of D. Froidevaux and E. Richter-Was)

The predictions for the LHC have to be considered with some caution. The computation of the Higgs spectrum and couplings have been treated in analogy to the LEP2 simulations; however, the masses of all SUSY particles have been assumed heavy. Since the couplings in the $gg$ fusion cross sections as well as in the $\gamma\gamma$ branching ratios for $h$ decays are generated by loops, this channel could be affected strongly by light charginos and stop particles. Depending on the point considered in the SUSY parameter space, the variation of $\sigma\times\text{BR}$ through SUSY-loop effects can go either way, enhancing or spoiling the Higgs signal. The problem of SUSY loop corrections is much less severe in search channels which are based on reactions realized already at the Born level, as $Wh$ at the LHC, and all the search channels in $e^+e^-$ collisions. A problem of $pp$ collisions are the QCD corrections. They are known for the signal in $gg$ fusion [72], but not for all background processes; the assumption that significances are estimated in a conservative way by setting $K$ factors to unity is expected to be fulfilled in large parts of the SUSY parameter space, but this is not guaranteed yet for large $\tan \beta$. Moreover,
Figure 38: The regions in the \( [m_A, \tan \beta] \) parameter space which can be covered at LHC in all possible channels associated with the heavy scalar, the pseudoscalar and the charged Higgs bosons (parameters as in previous figure). (Courtesy of D. Froidevaux and E. Richter-Was)

if the Higgs bosons do not only decay to SM particles but instead to invisible LSP and other neutralino/chargino states with potentially large branching ratios \([41, 115]\), the analysis must be modified and the conclusions would eventually be altered rather dramatically.

Combining the discovery potentials of LEP2 at \( \sqrt{s} = 192 \text{ GeV} \) and of LHC by summing up all Higgs production channels, the entire \( [m_A, \tan \beta] \) parameter plane of the MSSM is predicted to be covered within the standard framework of non-SUSY Higgs decays \( \) [based on the parameter set \( M_t = 175 \text{ GeV}, M_S = 1 \text{ TeV} \) and \( A, \mu \ll M_S \)]. The discovery potential of LEP (LHC) in the search for \( h \) increases for smaller (larger) values of \( M_t, A, \mu \) and \( M_S \) which are associated with smaller (larger) values of \( m_h \). In the region of \( m_A \) values less than about 150 GeV, the search for \( h \) can be performed by LEP2 while the other heavy Higgs particles, \( H, A \) and \( H^\pm \), can be searched for at the LHC. As discussed before, the observation of at least two different Higgs states, at LEP2 or LHC, is a crucial step in disentangling the supersymmetric theory from the Standard Model. Moreover, the channels exploited in the search for \( h \) are different at LEP2 and LHC. This implies that the couplings involved will be different and hence the physics tested in both cases will be complementary.
4 Non–Minimal Extensions

4.1 The Next–to–Minimal Supersymmetric Standard Model

In this section we shall augment the MSSM by introducing a single gauge singlet superfield \( N \) leading to a model which is referred to as the NMSSM[124].

The classic motivation for singlets is that they can solve the so-called \( \mu \)-problem of the MSSM [125] by eliminating the \( \mu \)-term and replacing its effect by the vacuum expectation value (vev) \(< N > = z\), which may be naturally related to the usual Higgs vev's \(< H_i > = v_i\). However such models in which the superpotential contains only trilinear terms, possess a \( \mathbb{Z}_3 \) symmetry which is spontaneously broken at the electroweak breaking scale. This results in cosmologically stable domain walls [126] which make the energy density of the universe too large. This cosmological catastrophe can be avoided by allowing explicit and non-renormalizable \( \mathbb{Z}_3 \) breaking terms suppressed by powers of the Planck mass which will ultimately dominate the wall evolution [127] without affecting the phenomenology of the model. However such terms induce a destabilisation of the gauge hierarchy [128] due to tadpole contributions to the \( N \) mass in supergravity models with supersymmetry breaking in the hidden sector.

Where does all this leave the NMSSM? This depends on one's point of view. If there might be some [yet unknown] solution to the domain wall problem, then one can consider models with \( \mathbb{Z}_3 \) symmetry which is broken spontaneously. Another approach is to avoid the domain wall problem by considering more general NMSSM models without a \( \mathbb{Z}_3 \) symmetry. Note that \( \mathbb{Z}_3 \) violating terms, such as a \( \mu \) term, large enough to avoid the domain wall problem, can still be sufficiently small as to have no impact on collider phenomenology. These more general models allow arbitrary renormalizable mass terms in the superpotential including the \( \mu \) parameter, and linear and quadratic terms in \( N \). The question of Planck scale tadpole contributions arises in this case. However, such contributions depend on supersymmetry breaking in a hidden sector of a supergravity theory, and are hence model dependent.

According to this discussion we shall consider two quite different versions of the NMSSM:

(i) The "General NMSSM" is defined by the following superpotential:

\[
W = -\mu H_1 H_2 + \lambda N H_1 H_2 - \frac{k}{3} N^3 + \frac{1}{2} \mu' N^2 + \mu'' N + \ldots
\]  

This version of the model is essentially a generalization of the MSSM, and provides a more general realization of low-energy SUSY which is equally consistent with gauge coupling unification and high precision measurements. It reduces to the MSSM in the limit in which the \( N \) field is removed, and since it does not have a \( \mathbb{Z}_3 \) symmetry there is no domain wall problem.

(ii) The "Constrained NMSSM" is defined by the trilinear terms in eq.(50), i.e. \( \mu = \mu' = \mu'' = 0 \), plus the constraints of gauge coupling unification and universal soft SUSY-breaking parameters imposed at the unification scale \( M_{GUT} \approx 10^{16} \text{ GeV} \) [129, 130].
4.1.1 The General NMSSM

a) Masses and Couplings

The superfield \( N \) contains a singlet Majorana fermion, plus a singlet complex scalar. The real part of the complex scalar will be assumed to develop a vacuum expectation value. The singlet yields one additional CP-even state and one additional CP-odd state which are gauge singlets but can mix with the corresponding neutral Higgs states of the MSSM, leading to three CP-even Higgs bosons \( h_1, h_2, h_3 \) and two CP-odd Higgs bosons \( A_1, A_2 \). Although there are more neutral Higgs bosons than in the MSSM, they will have diluted couplings due to their singlet components, making their production cross-sections smaller.

The tree-level CP-odd mass matrix, after rotating away the Goldstone mode as usual, reduces to the \( 2 \times 2 \) matrix in the basis \((A, N)\) where \( A \) is the MSSM CP-odd field,

\[
M_A^2 = m_A^2 .
\]

The entries represented by dots are complicated singlet terms. Unlike the MSSM, the parameter \( m_A^2 \) here is not a mass eigenvalue due to singlet mixing. The CP-odd matrix is diagonalized by rotating through an angle \( \gamma \), leading to two CP-odd eigenstates \( A_1, A_2 \) of mass \( m_{A_1} \leq m_{A_2} \).

The tree-level CP-even mass squared matrix in the basis \((H_1, H_2, N)\) is

\[
M^2 = 
\begin{pmatrix}
  m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta & -(m_Z^2 + m_A^2 - 2\lambda^2 v^2) \sin \beta \cos \beta \\
  -(m_Z^2 + m_A^2 - 2\lambda^2 v^2) \sin \beta \cos \beta & m_Z^2 \sin^2 \beta + m_A^2 \cos^2 \beta
\end{pmatrix}
\]

where, as usual, \( v = 174 \ \text{GeV} \), \( \tan \beta = v_2/v_1 \); again the dots correspond to singlet terms. Apart from the terms involving \( \lambda \), the upper \( 2 \times 2 \) block of this matrix is identical to the MSSM CP-even matrix. However, whereas the MSSM matrix is diagonalized by rotation through a single angle \( \alpha \), the matrix in eq.52 is diagonalized by a \( 3 \times 3 \) unitary matrix \( V \), leading to three mass eigenstates \( h_1, h_2, h_3 \) with masses ordered as \( m_{h_1} \leq m_{h_2} \leq m_{h_3} \).

The singlets obviously cannot mix with charged scalars, and at tree-level the mass of the charged Higgs is

\[
m_{H^\pm}^2 = m_A^2 + m_W^2 - \lambda^2 v^2
\]

Clearly a non-zero \( \lambda \) tends to reduce the charged scalar masses which can now be arbitrarily small, and – in contrast to the MSSM – below the \( W \) mass.

We shall define the relative couplings \( R_i \equiv R_{ZZh_i} \) as the \( ZZh_i \) coupling in units of the standard model \( ZZH \) coupling, and similarly we shall define a \( Zh_iA_j \) coupling factor \( R_{Zh_iA_j} \). For example \( R_{ZZh_i} \) is a generalization of \( \sin(\beta - \alpha) \) and the \( R_{Zh_iA_j} \) are generalizations of \( \cos(\beta - \alpha) \) in the MSSM. The \( Zh_iA_j \) coupling factorises into a CP-even factor \( S_i \) and a CP-odd factor which depends only on the angle \( \gamma \) which controls singlet mixing in the CP-odd sector.
It can be shown that the CP-even Higgs bosons in this model respect the following relations [131, 132, 133]:

\[
\begin{align*}
m_{h_1}^2 &\leq \Lambda^2 = m_A^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta \\
m_{h_2}^2 &\leq \frac{\Lambda^2 - R_1 m_{h_1}^2}{1 - R_1^2} \\
m_{h_3}^2 &= \frac{\Lambda^2 - R_1^2 m_{h_1}^2 - R_2^2 m_{h_2}^2}{1 - R_1^2 - R_2^2}
\end{align*}
\] (54)

In the case \( \lambda = 0 \), \( \Lambda \) is equal to the lightest CP-even upper mass bound in the MSSM. The above results show that if \( R_1 \approx 0 \) we may simply ignore \( h_1 \) and concentrate on \( h_2 \) which then becomes the lightest physically coupled CP-even state and must satisfy \( m_{h_2}^2 \leq \Lambda^2 \). Similarly if both \( R_1 \) and \( R_2 \) are nearly zero, then \( m_{h_3}^2 = \Lambda^2 \).

It can also be shown that

\[
\begin{align*}
m_{A_1}^2 &\leq m_A^2 \\
m_{A_2}^2 &= \frac{m_A^2 - m_{A_1}^2 \cos^2 \gamma}{1 - \cos^2 \gamma}
\end{align*}
\] (55)

If the lighter CP-odd state is weakly coupled (\( \cos \gamma \approx 0 \)) then it is mainly singlet, and the heavier CP-odd state is then identified with the MSSM state of mass \( m_A \).

b) Theoretical Upper Limit on \( \Lambda \)

According to eq.(54), \( \Lambda \) is clearly a function of \( \tan \beta \) and \( \lambda \), and to find the absolute upper bound on the mass of the lightest CP-even Higgs boson we must maximize this function (\( \Lambda_{\text{max}} \)). Radiative corrections, which drastically affect the bound [134], are included using recently proposed methods [89].

For a fixed \( \tan \beta \), the maximum value of \( \Lambda \) is given by the maximum value of \( \lambda \) as derived from the triviality requirement that none of the Yukawa couplings becomes non-perturbative before the GUT scale of around \( 10^{16} \) GeV. Using the recently calculated two-loop RGEs [130], we find an upper limit on \( \lambda \) as a function of \( \tan \beta \). The maximum value of \( \lambda \) is typically in the range 0.6-0.7 for a wide range of \( \tan \beta \), depending on \( M_t \) and \( \alpha_3(m_Z) \), and falls off to zero for \( \tan \beta \rightarrow \approx 1.5 \) or 60 because \( h_t \) or \( h_s \), respectively, is very close to triviality. Having derived the maximum value of \( \lambda \) as a function of \( \tan \beta \), we can use this information to obtain an \( M_t \)-dependent maximum value of \( \Lambda \) shown as the upper solid curve in Fig.39. The MSSM bound is also shown (lower solid curve) for comparison. The dashed line is the corresponding upper mass bound in the constrained NMSSM (see later).

As well as being the upper bound on the mass of the lightest CP-even Higgs boson, the parameter \( \Lambda \) plays an important role in constraining all the CP-even Higgs boson masses and couplings. Thus the value of \( \Lambda_{\text{max}} \), corresponding to the upper solid line in Fig.39, also constrains \( h_2 \) and \( h_3 \) according to eq.(54).
c) Experimental Lower Limits on $\Lambda$

$\Lambda$ has a theoretical upper limit given by $\Lambda_{\text{max}} \approx 146$ GeV. Now we shall discuss how experiment may place a lower limit on $\Lambda$ which we shall refer to as $\Lambda_{\text{min}}$. The meaning of $\Lambda_{\text{min}}$ is as follows. For each value of $\Lambda$ there are many possible sets of parameters $(R_i, m_{h_i})$ subject to the bounds in eq.(54). Each of the three (CP-even) Higgs bosons in each set may or may not be discovered at LEP, depending on how light it is and how strongly coupled to the $Z$ it is. We can consider the present $R^2 - m_h$ 95% exclusion plots at LEP [119] and classify each of the three Higgs bosons in each set (for a fixed $\Lambda$) as excluded or not excluded. We may find, for some value of $\Lambda$, that for all the allowed sets at least one out of the three Higgs bosons is always excluded. In this case we classify this value of $\Lambda$ as being excluded by experiment. We now define $\Lambda_{\text{min}}$ as the largest value of $\Lambda$ which may be excluded by the LEP data. There will be a different $\Lambda_{\text{min}}$ for each of the expected LEP2 $R^2 - m_h$ 95% exclusion plots (see Fig. 31 and Table 15). If $\Lambda_{\text{min}}$ exceeds $\Lambda_{\text{max}}$ then the model is excluded.

$\Lambda_{\text{min}}$ is approximately determined by the “worst case” of all three CP-even Higgs bosons having equal masses $m_{h_i} \approx \Lambda$, and equal couplings $R_i^2 \approx 1/3$. Using this simple approximation, together with current $R^2 - m_h$ exclusion limits [119], LEP1 already places a limit on $\Lambda$ of $\Lambda > \Lambda_{\text{min}} = 59$ GeV, which is just equal to the mass limit for a CP-even Higgs boson with its mass.

\footnote{This approximate result is exact in the limit that 95% exclusion is equated to 50 produced events.  

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ZZh coupling suppressed by \( R^2 = 1/3 \).

The values of \( \Lambda_{\text{min}} \) which may be excluded by a future \( e^+e^- \) collider of a given energy and integrated luminosity [note that this is total luminosity of all four experiments pooled] are shown in Fig.40 where exclusion is approximately equated to 50 produced events.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.pdf}
\caption{Excluded values of \( \Lambda \) at \( e^+e^- \) colliders with energy \( \sqrt{s} \).}
\end{figure}

Focusing specifically on LEP2, we consider energies and integrated luminosities per experiment of \( \sqrt{s} = 175, 192, 205 \text{ GeV} \) and \( \mathcal{L} = 150, 150, 300 \text{ pb}^{-1} \), respectively. Using the \( R^2 - m_h \) 95\% exclusion plots [135], we find that LEP2 will yield the exclusion limits \( \Lambda_{\text{min}} = 81, 93, 105 \text{ GeV} \) respectively, for the three sets of LEP2 machine parameters above. These excluded values of \( \Lambda \) (corresponding to \( R^2 = 1/3 \)) are not far from the values of SM Higgs boson masses which may be excluded, due to the steep rise of the exclusion curves in the \( R^2 - m_h \) plane.

d) Exclusion Limits in the \( m_A - \tan \beta \) Plane

It is possible to obtain exclusion limits in this model in the \( m_A - \tan \beta \) plane, rather similar to the familiar MSSM plots. The excluded regions are obtained from the following three searches:

(i) For the processes \( Z \to Zh_i \), we exploit the fact that the upper 2×2 block of the CP-even mass squared matrix is completely specified (for fixed \( \lambda \)) in the \( m_A - \tan \beta \) plane. However, unlike the MSSM, the CP-even spectrum is not completely specified since it depends on three remaining unknown real parameters associated with singlet mixing (i.e. the dots in eq.(52)). Nevertheless, since each choice of these parameters completely specifies the parameters \( m_{h_i} \) and \( R_i \), we can scan over the unknown parameters; if the resulting spectrum is always excluded,
then we conclude that this point in the $m_A$-tan$\beta$ plane (for fixed $\lambda$) is excluded. For these excluded regions we use the available LEP1 and LEP2 $R^2 - m_h$ 95% exclusion lines.

(ii) An excluded lower limit on $m_A$, as a function of tan$\beta$ and $\lambda$ in this model comes from the non-observation of $Z \rightarrow h_i A_j$. The excluded lower limit on $m_A$ in this model is determined by the “worst-case” values of $m_{h_i}$, $m_{A_j}$, $S_i$, $\gamma$ consistent with this value of $m_A$. It turns out that the worst case experimentally is when the three CP-even Higgs bosons all have equal masses as heavy as possible

$$m_{h_i}^2 = m_A^2 + (m_Z^2 - \lambda^2 v^2) \sin^2 2\beta$$

and equal coupling factors, $S_i^2 \approx 1/3$, and the two CP-odd Higgs bosons both have masses equal to $m_A$ and $\gamma = \pi/4$, leading to $R^2_{Z h_i A_j} = 1/6$. For these excluded regions we use the simple approximation that 50 events corresponds to 95% exclusion.

(iii) Finally we shall present excluded regions for charged Higgs production, assuming detection up to the kinematic limit. We note that the charged Higgs signal is the same as in the MSSM as considered in Section 3.

In Fig.41 we show the excluded regions of this model corresponding to the choice of pa-
rameter $\lambda = 0$ \(^4\) for the three sets of LEP2 machine parameters $\sqrt{s} = 175, 192, 205$ GeV and $\mathcal{L} = 150, 150, 300$ pb\(^{-1}\), respectively, and using the LEP1 and LEP2 exclusion data, Fig. 31 and Table 15. In each case the solid lines correspond to exclusion limits from $Z \to Zh_i$ as obtained using procedure (i) above in the NMSSM, the dashed lines correspond to exclusion limits from $Z \to h_iA_j$ as obtained using procedure (ii) above in the NMSSM, and the dotted lines correspond to the exclusion limits from charged Higgs production using procedure (iii) above. These exclusion plots should be compared to similar exclusion plots in the MSSM for $M_t = 175$ GeV and degenerate squarks at 1 TeV, which are the parameters assumed in Fig.41.

In Fig.41 we also show a similar plot but with $\lambda = 0.5$. In this case the solid lines have disappeared beneath the $\tan \beta = 1$ horizon, because for larger $\lambda$ the bound $\Lambda$ may be larger, which allows a heavier CP-even spectrum which is consequently more difficult to exclude. The charged Higgs and $h_iA_j$ channels now give improved coverage, however, since larger $\lambda$ decreases the charged Higgs mass, and also decreases the $h_i$ masses for a fixed $S_i$ coupling.

### 4.1.2 The Constrained NMSSM

As noted in the introduction, the constrained NMSSM is defined by eq.(50) with $\mu, \mu', \mu'' \to 0$ and the condition of universal SUSY-breaking gaugino masses $M_{1/2}$, scalar masses $m_0$ and trilinear couplings $A_0$ at $M_{GUT}$. In addition, the effective potential has to have the correct properties, i.e. the $SU(2) \times U(1)$ symmetry has to be broken by Higgs vev's, but the vev's of charged and/or colored fields as sleptons, squarks and charged Higgs scalars have to vanish. Finally, present lower limits on sparticle masses due to direct searches have to be satisfied, and for the top-quark mass $M_t$ we require values between 150 GeV and 200 GeV.

A priori the constrained NMSSM has six parameters at $M_{GUT}$, three dimensionless couplings $\lambda, k$ and $h_t$ (the top-quark Yukawa coupling) and three dimensionful ones $M_{1/2}, m_0$ and $A_0$. The scale set by the known mass of the $Z$ boson reduces the number of free parameters of the model to five. A scan of the parameter space of the model, which is consistent with all the above constraints, has been performed in [129] and [130]. Below we present results, relevant for the Higgs search at LEP2, which are based on the data obtained in [129]. We will discuss the allowed Higgs masses and couplings within the constrained NMSSM.

First note that neither a CP-odd Higgs boson $A_j$ with sufficiently large coupling $R_{Zh_iA_j}$ nor a charged Higgs boson can be sufficiently light within the constrained NMSSM in order to be visible at LEP2. Thus we will concentrate on the neutral CP-even Higgs bosons in the following. Concerning their decays, it follows that neutralinos are too heavy to play a role, thus their branching ratios are close to the standard model ones (essentially $b\bar{b}$) and the same search criteria apply.

In general, the upper limit on the lightest Higgs mass as a function of the top-quark mass

\(^4\)Strictly speaking if $\lambda = 0$ then there is no singlet mixing. However these curves apply to the case where $\lambda$ is small (say less than 0.1) and singlet mixing is possible. Such small values of $\lambda$ are always found in the constrained NMSSM.
depends on the magnitude of the SUSY-breaking parameters due to radiative corrections to the Higgs potential. For a gluino mass beyond 1 TeV more and more fine tuning is required between these parameters; thus Fig. 39 shows the upper limit on the lightest Higgs boson within the constrained NMSSM as a dashed line, for gluinos lighter than 1 TeV.

As noted before, the lightest Higgs boson within the NMSSM can essentially be a gauge singlet state and thus couple very weakly to the Z boson. Fig. 42 (left) shows the logarithm of the coupling $R_1$ as a function of the mass $m_1$ of the lightest Higgs boson. Two different regions exist within the constrained NMSSM: A densely populated region with $R_1 \sim 1$ and $m_1 > 50 \text{ GeV}$, and a tail with $R_1 < (\text{or } \ll) 1$ and $m_1$ as small as $\sim 10 \text{ GeV}$. Within the tail, the lightest Higgs boson is essentially a gauge singlet state, which explains the small values of $R_1$.

The solid line in Fig. 42 (left) indicates (for $m_1 > 60 \text{ GeV}$) the boundary of the region which can be tested at LEP2 with a maximal c.m. energy of 192 GeV and a luminosity of 150 pb$^{-1}$; the dotted line corresponds to a maximal c.m. energy of 205 GeV and a luminosity of 300 pb$^{-1}$ [both after combining all experiments [135]]. A large part of the region with $R_1 \sim 1$, but only a small part of the tail can be tested.

![Figure 42: The logarithm of the $ZZh_{1(2)}$ coupling $R_{1(2)}$ squared vs. the mass of the lightest CP-even state $h_{1(2)}$ in the constrained NMSSM.](image)

Fortunately, as noted above, the second lightest Higgs boson cannot be too heavy if the lightest Higgs boson is essentially a gauge singlet state [131], [132], [133]. In the region of the tail of Fig. 42 (left), within the constrained NMSSM, the mass of the second Higgs boson $h_2$ varies between 80 GeV and the upper limit indicated in Fig. 39 as a dashed line. Its coupling
to the $Z$ boson $R_2$ is very close to 1 if $R_1 \ll 1$. Fig.42 (right) shows the logarithm of the coupling $R_2$ as a function of the mass $m_2$ of the second lightest Higgs. The tail of Fig.42 (left) corresponds to the region with $R_2 \sim 1$ in Fig.42 (right) and vice versa. Thus, a small part of the parameter space corresponding to the tail of Fig.42 (left) actually becomes visible at LEP2 through the observation of $h_2$, which behaves like the lightest Higgs boson of the MSSM in this case.

If a Higgs boson is observed, it will generally be very difficult, however, to distinguish the NMSSM from the MSSM [130]. This seems only possible if a Higgs boson happens to contain a sizeable amount of the singlet state (and hence a measurably reduced coupling to the $Z$ boson), but couples still strongly enough in order to be visible. Finally, the constrained NMSSM could actually be ruled out at LEP2 if a neutral CP-odd Higgs particle, a charged Higgs particle, or an invisibly decaying Higgs particle would be observed.

4.2 Non-linear Supersymmetry

Most of the supersymmetric models investigated so far are models based on linearly realized supersymmetry. However, supersymmetry may as well be realized nonlinearly. Whereas the linear supersymmetric models require supersymmetric partners for all conventional particles in the standard model, the nonlinear models do not lead to SUSY partners. Most global nonlinear supersymmetric models require only one new particle: the Akulov-Volkov field [136], which is a Goldstone fermion. This Goldstino can be removed by going over to curved space, i.e. to supergravity, where it can be gauged away. In the flat space limit, the supergravity multiplet decouples from ordinary matter so that supersymmetry can manifest itself only in the Higgs sector.

The formalism for extending the standard model to a supersymmetric theory in a nonlinear way was developed in ref.[137]. Recently, the general form of the nonlinear supersymmetric standard model has been constructed and the Higgs potential in the flat space limit [138] has been derived.

The Higgs sector of the nonlinear SUSY models is evidently larger than that of the Standard Model. It contains at least two dynamical Higgs doublets and an auxiliary Higgs singlet. In the case that both the dynamical and the auxiliary singlet are included in the theory, the spectra of Higgs bosons in the nonlinear models resemble those of the linear model with two Higgs doublets and one singlet (NMSSM). Both models have three scalar, two pseudoscalar Higgs bosons and one charged Higgs boson pair. However, the structure of the Higgs potential is quite different between nonlinear models and the NMSSM. For the general nonlinear model, the complete potential in the flat space limit is given by [138]

$$V = \frac{1}{8}(g_1^2 + g_2^2)(|H^1|^2 - |H^2|^2)^2 + \frac{1}{2}g_2^2|H^+_1 H_2|^2 + |\mu_1 + \lambda_1 N|^2 |H^1|^2$$
$$+ |\mu_2 + \lambda_2 N|^2 |H^2|^2 + |\lambda_0 H^1 e H^2 + \mu_0 N + k N^2|^2$$

(57)
involving novel types of interactions between the Higgs fields. The two Higgs doublets $H^1, H^2$ and the singlet $N$ develop the vev's $v_1, v_2$, and $x$ respectively. The three scalar Higgs bosons are the eigenstates of the scalar-Higgs mass matrix. In a way similar to the NMSSM, an upper bound for the mass $m_1$ of the lightest scalar Higgs boson $S_1$ can be derived at the lowest order,\(^5\)

\[
m_1^2 \leq m_{1, \text{max}}^2 = m_Z^2 (\cos^2 2\beta + 2\lambda_g^2 \sin^2 2\beta)
\]

where $\lambda_g = \lambda_0^2 / (g_1^2 + g_2^2)$. Hence, $m_1 \leq m_Z$ for $\lambda_0 \leq \frac{g_1^2 + g_3^2}{2} \approx 0.52$ and $m_1 \leq 1.92 \lambda_0 m_Z$ for $\lambda_0 > 0.52$. In the latter case, the upper bound of $\lambda_0$ determines the limit of $m_1$. For $M_t = 175$ GeV and with the GUT scale as cut-off scale, one obtains $m_1 \leq 130$ GeV. Even though at the c.m. energies of 175, 192, and 205 GeV for LEP2 the production of $S_1$ may be kinematically possible, the production rate is in general not large enough for $S_1$ to be detected. The main contributions to the production cross sections come from (i) the Higgs-strahlung process; (ii) the process where $S_1$ is radiated off leptons or quarks, and (iii) associated pair production $P_j S_1$, where $P_j$ ($j = 1, 2$) is a pseudoscalar Higgs boson.

![Contour lines](image)

Figure 43: Contour lines of the lightest scalar Higgs mass $m_1$ (dashed) and of the production cross section $\sigma$ (full) at $\sqrt{s} = 175$ GeV, as functions of $\lambda_1, \lambda_2$ for $\tan \beta = 6, \lambda_0 = 0.3, k = -0.02, m_C = 400$ GeV. The shaded area marks the parameter region excluded by LEP1, defined as the region where the production cross section at the $Z$ peak is greater than 1 pb.

We have first searched for parameter regions where the experimental lower limit on $m_1$ given by the LEP1 data is minimal; it turned out that there are regions where even $m_1 = 0$ is still

\(^5\)In the present exploratory analysis we have neglected the radiative corrections.
allowed. Fig.43 shows (dashed) the contour lines of $m_1$ and (full) the contour lines of the cross section $\sigma$ for the production of $S_1$ in $e^+e^-$ collisions at $\sqrt{s} = 175$ GeV, in the $\tan \beta$, $m_H$ plane for a representative set of the parameters $\tan \beta$, $\lambda_2$, $k$ and $m_C$ ($m_C$ being the charged-Higgs mass). In the shaded region, the cross section $\sigma$ at $\sqrt{s} = m_Z$ is greater than 1 pb, which corresponds to the discovery limit of LEP1 for $m_1 \approx 65$ GeV [135]. This region is excluded by LEP1 since the discovery limit decreases with decreasing $m_1$. The discovery limit at $\sqrt{s} = 175$ GeV with a luminosity of 500 pb$^{-1}$ is about 50 fb for $m_1 = 80$ GeV (about 30 fb for $m_1 = 40$ GeV) [135]. Thus the region accessible at LEP2 includes the area in Fig.43 where $m_1 < 80$ GeV and $\sigma > 50$ fb. As with LEP1, a massless $S_1$ could be undetectable.

For some parameter sets, the nonlinear SUSY model may be tested even if the lightest scalar is undetectable. If the production cross section of $S_1$ is smaller than the discovery limit, one can examine whether the production of the other Higgs bosons is kinematically possible and whether their production rates are large enough for discovery.

For the higher energies 192 and 205 GeV, the effects are similar to the 175 GeV case. Though the accessible region increases, an undetectable massless $S_1$ Higgs boson is still possible. Energies of 240 GeV and more are needed to test this nonlinear supersymmetric model conclusively.

### 4.3 Majoron Decays of Higgs Particles

There are a variety of well motivated extensions of the Standard Model (SM) with a spontaneously broken global symmetry. This symmetry could either be lepton number or a combination of family lepton numbers [139, 140]. These models are characterized by a more complex symmetry breaking sector which contains additional Higgs bosons. It is specially interesting for our purposes to consider models where such symmetry is broken at the electroweak scale [141, 142]. In general, these models contain a massless Goldstone boson, called majoron ($J$), which interacts very weakly with normal matter. In such models, the normal doublet Higgs boson is expected to have sizeable invisible decay modes to the majoron, due to the strong Higgs-majoron coupling. This can have a significant effect on the Higgs phenomenology at LEP2. In particular, the invisible decay could contribute to the signal of two acoplanar jets and missing momentum. This feature of majoron models allows one to strongly constrain the Higgs mass in spite of the occurrence of extra parameters compared to the SM. In particular, the LEP1 limit on the predominantly doublet Higgs mass is close to the SM limit irrespective of the decay mode of the Higgs boson [143, 144].

We consider a model containing two Higgs doublets ($\phi_{1,2}$) and a singlet ($\sigma$) under the $SU(2)_L \times U(1)_Y$ group. The singlet Higgs field carries a non-vanishing $U(1)_L$ charge, which could be lepton number. Here we only need to specify the scalar potential of the model:

$$
V = \mu_1^2 \phi_1^+ \phi_1 + \mu_2^2 \sigma^+ \sigma + \lambda_1 (\phi_1^+ \phi_1)^2 + \lambda_3 (\sigma^+ \sigma)^2 + \lambda_4 (\phi_1^+ \phi_1)(\phi_2^+ \phi_2) + \lambda_{13} (\phi_1^+ \phi_1)(\sigma^+ \sigma) + \lambda_{23} (\phi_2^+ \phi_2)(\sigma^+ \sigma)
$$
\[ + \delta(\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_1) + \frac{1}{2} \kappa \left[ (\phi_1^\dagger \phi_2)^2 + \text{h.c.} \right] \] (59)

where the sum over repeated indices \( i=1,2 \) is assumed.

Minimization of the above potential leads to the spontaneous \( SU(2)_L \times U(1)_Y \times U(1)_L \) symmetry breaking and allows us to identify a total of three massive CP even scalars \( H_i \) (\( i=1,2,3 \)), plus a massive pseudoscalar \( A \) and the massless majoron \( J \). We assume that at the LEP2 energies only three Higgs particles can be produced: the lightest CP-even scalar \( h \), the CP-odd massive scalar \( A \), and the massless majoron \( J \). Notwithstanding, our analysis is also valid for the situation where the Higgs boson \( A \) is absent [145], which can be obtained by setting the couplings of this field to zero.

At LEP2, the main production mechanisms of invisible Higgs bosons are the Higgs-strahlung process \( (e^+e^- \rightarrow hZ) \) and the associated production of Higgs bosons pairs \( (e^+e^- \rightarrow Ah) \), which rely upon the couplings \( hZZ \) and \( hAZ \) respectively. The important feature of the above model is that, because of its singlet nature, the majoron is not sizeably coupled to the gauge bosons and cannot be produced directly, thereby evading strong LEP1 constraints. The \( hZZ \) and \( hAZ \) couplings depend on the model parameters via the appropriate mixing angles, but they can be effectively expressed in terms of the two parameters \( \epsilon_A, \epsilon_B \):

\[
\mathcal{L}_{hZZ} = \epsilon_B \sqrt{2} G_F \frac{1}{2} \frac{m_H^2 Z_\mu Z^\mu h}{m_Z^2 Z_\mu Z^\mu h} \\
\mathcal{L}_{hAZ} = -\epsilon_A \frac{g}{\cos \theta_W} Z^\mu h \partial_\mu A
\]

The couplings \( \epsilon_{A(B)} \) are model dependent. For instance, the SM Higgs sector has \( \epsilon_A = 0 \) and \( \epsilon_B = 1 \), while a majoron model with one doublet and one singlet leads to \( \epsilon_A = 0 \) and \( \epsilon_B^2 \leq 1 \).

The signatures of the Higgs-strahlung process and the associated production depend upon the allowed decay modes of the Higgs bosons \( h \) and \( A \). For Higgs boson masses \( m_h \) accessible at LEP2 energies the main decay modes for the CP-even state \( h \) are \( b\bar{b} \) and \( JJ \). We treat the branching fraction \( B = BR(h \rightarrow JJ) \) as a free parameter. In most models \( BR \) is basically unconstrained and can vary from 0 to 1. Moreover we also assume that, as it happens in the simplest models, the branching fraction for \( A \rightarrow b\bar{b} \) is nearly one, and the invisible \( A \) decay modes \( A \rightarrow hJ, A \rightarrow JJJ \) do not exist (although CP-allowed). Therefore our analysis depends upon five parameters: \( m_h, m_A, \epsilon_A, \epsilon_B, \) and \( B \). This parameterization is quite general and very useful from the experimental point of view: limits on \( m_h, m_A, \epsilon_A, \epsilon_B, \) and \( B \) can be later translated into bounds on the parameter space of many specific models.

The parameters defining our general parameterization can be constrained by the LEP1 data. In fact Refs. [143, 146] analyze some signals for invisible decaying Higgs bosons, and conclude that LEP1 excludes \( m_h \) up to 60 GeV provided that \( \epsilon_B > 0.4 \). Similar results are obtained in fig. (32).

The \( b\bar{b} + p_T \) topology is our main subject of investigation and we carefully evaluate signals and backgrounds, choosing the cuts that enhance the signal over the backgrounds. Our goal is to evaluate the limits on \( m_h, m_A, \epsilon_A, \epsilon_B, \) and \( B \) that can be obtained at LEP2 from this final
state. There are three sources of signal events with the topology \( p_T + 2 \text{ b-jets} \): one due to the associated production and two due to the Higgs-strahlung.

\[
\begin{align*}
e^+e^- &\to (Z \to b\bar{b}) + (h \to JJ) \\
e^+e^- &\to (Z \to \nu\bar{\nu}) + (h \to b\bar{b}) \\
e^+e^- &\to (A \to b\bar{b}) + (h \to JJ)
\end{align*}
\]

(62)

(63)

(64)

The signature of this final state is the presence of two jets containing \( b \) quarks and missing momentum \( (p_T) \). It is interesting to notice that for light \( m_h \) and \( m_A \), the associated production dominates over the Higgs-strahlung mechanism [146].

There are several sources of background for this topology:

\[
\begin{align*}
e^+e^- &\to Z/\gamma Z/\gamma \to q\bar{q} \nu\bar{\nu} &\quad &e^+e^- \to (e^+e^-)\gamma\gamma \to [e^+e^-]q\bar{q} \\
e^+e^- &\to Z^*/\gamma^* \to q\bar{q}[m\gamma] &\quad &e^+e^- \to W^+W^- \to q\bar{q} \ell\nu \\
e^+e^- &\to W[e]\nu \to q\bar{q}' [e]\nu &\quad &e^+e^- \to Z\nu\bar{\nu} \to q\bar{q} \nu\bar{\nu}
\end{align*}
\]

where the particles in square brackets escape undetected and the jet originating from the quark \( q \) is identified (misidentified) as being a \( b \)-jet.

At this point the simplest and most efficient way to improve the signal-over-background ratio is to use that the \( A \) and \( h \) decays lead to jets containing \( b \)-quarks. So we require that the events contain two \( b \)-tagged jets. Moreover, the background can be further reduced requiring a large \( p_T \). We therefore have imposed a set of cuts which is based on the cuts used by the DELPHI collaboration for the SM Higgs boson search [63].

Depending on the \( h \) and \( A \) mass ranges, including or excluding an invariant mass cut \( m \pm 10 \text{ GeV} \) [where \( m \) is the mass of the particle decaying visibly] gives better or weaker limits on the \( ZhA \) and \( ZZh \) couplings. Therefore, for each mass combination four limits are calculated (with or without invariant mass cut, with thrust cut or the cut on the minimal two-jet energy) and the best limit is kept.

We denote the number of signal events for the three production processes (62 – 64), after imposing all cuts, \( N_{JJ} \), \( N_{SM} \), and \( N_A \) respectively, assuming that \( \epsilon_A = \epsilon_B = 1 \). Then the expected number of signal events when we take into account couplings and branching ratios is

\[
N_{\text{exp}} = \epsilon_B^2 [BN_{JJ} + (1 - B)N_{SM}] + \epsilon_A^2 BN_A
\]

(65)

In general, this topology is dominated by the associated production, provided it is not suppressed by small couplings \( \epsilon_A \) or phase space. The most important background after the cuts is \( Z/\gamma Z/\gamma \) production. The total numbers of background events summed over all relevant channels are 2.3, 2.8 and 5.9 for \( \sqrt{s} = 175 \), 190 and 205 GeV respectively.

In order to obtain the limits shown in Figs. 44-45, we assumed that only the background events are observed, and we evaluated the 95% CL region of the parameter space that can be excluded with this result. By taking the weakest bound, as we vary \( B \), we obtained the
Figure 44: Limits on $e_B^2$ as a function of $m_h$ for 500 pb$^{-1}$ and $\sqrt{s} = 175$ GeV and for 300 pb$^{-1}$ and $\sqrt{s} = 190$ GeV; for different values of $B = Br(h \to JJ)$

Figure 45: Limits on $e_A^2$ as a function of $m_h, m_A$ for $\sqrt{s} = 190$ GeV. The left plot shows the limits obtained for $B = Br(h \to JJ) = 1$, in the right plot $B$ is varied from 0 to 1.
absolute bound on $\epsilon_A$, $\epsilon_B$, and $m_h$ independent of the $h$ decay mode. The limits on $\epsilon_A$ obtained by searches for the $b\bar{b} + p_T$ final states are stronger than those obtained from the $b\bar{b}b\bar{b}$ topology. Moreover, the bounds in the limiting case $\epsilon_A = 0$ apply for the simplest model of invisibly decaying Higgs bosons, where just one singlet is added to the SM. A more complete presentation of these results will be given in Ref.[147].

4.4 Strongly Interacting Higgs Particle

The radiative corrections at LEP1 depend only logarithmically on the Higgs mass, and the measurements, although very precise, are not sufficient to determine the structure of the Higgs sector. It is therefore necessary to keep an open mind to the possibility that the Higgs sector is more complicated than in the Standard Model. Beyond the Standard Model various extensions have been suggested. One of the possibilities is supersymmetry which has been previously discussed. Another possibility is strong interactions in the form of technicolor, which at least in its simplest form is ruled out by the LEP1 data. Strong interactions in the Standard Model itself imply a heavy Higgs boson and can presumably be studied at the LHC.

However, the idea of strong interactions is more general. In particular it is possible that strong interactions are present in the singlet sector of the theory. In general the choice of representations in a gauge theory is arbitrary and presumably a clue to a deeper underlying theory. Singlets do not have quantum numbers under the gauge group of the Standard Model. They therefore do not feel the strong or electroweak forces, but they can couple to the Higgs particle. As a consequence, radiative corrections to weak processes are not sensitive to the presence of singlets in the theory, because no Feynman graphs containing singlets appear at the one-loop level. Because effects at the two-loop level are below the experimental precision, the presence of a singlet sector is not ruled out by any of the LEP1 precision data.

It is therefore not unreasonable to assume that there exists a hidden sector that affects Higgs physics only. Such an extension of the Standard Model involving singlet fields preserves the essential simplicity of the model, while at the same time acting as a realistic model for non-standard Higgs properties. Here we will study the coupling of a Higgs boson to an $O(N)$ symmetric set of scalars, which is one of the simplest possibilities introducing only a few extra parameters in the theory. The extra scalars may give rise to large invisible decay width of the Higgs particle. When the coupling is large enough, the Higgs resonance can become wide even for a light Higgs boson. This has led to the conclusion that this Higgs particle becomes undetectable at the LHC [148]. As one can measure missing energy more precisely at $e^+e^-$ colliders than at a hadron machine, LEP2 can give important constraints on the parameters of the model. However, it is clear that there will be a range of parameters where this Higgs boson can be seen neither at LEP nor at the LHC.
a) The Model

The Higgs sector of the model is described by the following Lagrangian,

\[ L = -\partial_{\mu} \phi^+ \partial^{\mu} \phi - \lambda (\phi^+ \phi - v^2/2)^2 \\
- 1/2 \partial_{\mu} \tilde{\phi}^\mu \tilde{\phi} - 1/2 m^2 \phi^2 - \kappa/(8N) (\phi^2)^2 - \omega/(2\sqrt{N}) \phi^2 \phi^+ \phi \]

where \( \phi \) is the normal Higgs doublet and the vector \( \tilde{\phi} \) is an N-component real vector of scalar fields, which we call phions. Couplings to fermions and vector bosons are the same as in the Standard Model. The ordinary Higgs field acquires the vacuum expectation value \( v/\sqrt{2} \). We assume that the \( \tilde{\phi} \)-field acquires no vacuum expectation value, which can be assured by taking \( \omega \) positive. After the spontaneous symmetry breaking one is left with the ordinary Higgs boson, coupled to the phions into which it decays. Also the phions receive an induced mass from the spontaneous symmetry breaking. The factor \( N \) is taken to be large, so that the model can be analyzed in the \( 1/N \) expansion. By taking this limit, the phion mass stays small, but because there are many phions, the decay width of the Higgs boson can become large. Therefore the main effect of the presence of the phions is to give a large invisible decay rate to the Higgs boson. The invisible decay width is given by

\[ \Gamma_H = \frac{\omega^2 v^2}{32\pi m_H} \]  

(66)

The Higgs width is compared with the width in the Standard Model for various choices of the coupling \( \omega \) in Fig.46. The model is different from Majoron models, since the width is not necessarily small. The model is similar to the technicolor-like model of Ref.[149].

Consistency of the model requires two conditions. One condition is the absence of a Landau pole below a certain scale \( \Lambda \). The other follows from the stability of the vacuum up to a certain scale. An example of such limits is given in Fig.47, where \( \kappa = 0 \) was taken at the scale \( 2m_Z \), which allows for the widest range. For the model to be valid beyond a scale \( \Lambda \) one should be below the indicated upper lines in the figure. One should be to the right of the indicated lower lines to have stability of the vacuum.

For the search for the Higgs boson there are basically two channels; one is the standard decay, which is reduced in branching ratio due to the decay into phions. The other is the invisible decay, which rapidly becomes dominant, eventually making the Higgs resonance very wide, Fig.46. In order to find the bounds we neglect the coupling \( \kappa \) as this is a small effect. We also neglect the phion mass. For non-zero values of the phion mass the bounds can be found by rescaling the decay widths with the appropriate phase space factor. The present bounds, coming from LEP1 invisible search, are included as a dashed curve in Fig.48 below.

b) LEP2 Bounds

In the case of LEP2 the limits on the Higgs mass and couplings in the present model come essentially from the invisible decay, as the branching ratio into \( \bar{b}b \) quarks drops rapidly with
increasing $\varphi$--Higgs coupling. To define the signal we look at events around the maximum of the Higgs resonance, with an invariant mass $m_H \pm \Delta$ for $\Delta = 5$ GeV, which corresponds to a typical mass resolution. Exclusion limits are determined by Poisson statistics as defined in Appendix 5.3. The results are given by the full lines in Fig.48. One notices the somewhat reduced sensitivity for a Higgs mass near the Z boson mass and a looser bound for small Higgs masses because there the effect of the widening of the resonance prevails. The small $\omega$ region is covered by visible search. There is a somewhat better limit on the Higgs mass for moderate $\omega$ in comparison with the $\omega = 0$ case; this is due to events from the extended tail of the Higgs boson which is due to the increased width.

We conclude from the analysis that LEP2 can put significant limits on the parameter space of such a model. However there is a range where the Higgs boson will not be discovered, even if it does exist in this mass range. This also holds true when one considers the search at the LHC. Assuming moderate to large values of $\omega$, i.e. in the already difficult intermediate mass range, it is unlikely that sufficient signal events are left at the LHC. In that case the only information can come directly from high-energy $e^+e^-$ colliders or indirectly from higher precision experiments at LEP1.
Figure 48: Exclusion limits at LEP2 (full lines), and LEP1 (dashed). The region where $\omega$ is small can be covered by the search for visible Higgs decays.
5 Appendices

5.1 Higgs-strahlung and WW Fusion

Compact forms can be derived for the cross section of the process \[ e^+ e^- \rightarrow H + \bar{\nu}\nu \] by choosing the energy \( E_H \) and the polar angle \( \theta \) of the Higgs particle as the basic variables in the \( e^+ e^- \) c.m. frame. The overall cross section that will be observed experimentally, receives contributions from Higgs-strahlung with \( Z \) decays into three types of neutrinos, \( \mathcal{G}_W \) from WW fusion, and \( \mathcal{G}_I \) from the interference term between fusion and Higgs-strahlung for \( \bar{\nu}_e \nu_e \) final states. We find:

\[
\frac{d\sigma(H\bar{\nu}\nu)}{dE_H \, d\cos \theta} = \frac{G_F^2 m_Z^8 \rho}{\sqrt{2} \pi^3 s} \left( 3\mathcal{G}_S + \mathcal{G}_I + \mathcal{G}_W \right)
\]

with

\[
\mathcal{G}_S = \frac{v_e^2 + a_e^2}{96} \frac{ss_{\nu} + s_1 s_2}{s - m_Z^2} \left( s_{\nu} - m_Z^2 \right)^2 + m_Z^2 \Gamma_Z^2
\]

\[
\mathcal{G}_I = \frac{(v_e + a_e)c_W}{8} \frac{s_{\nu} - m_Z^2}{s - m_Z^2} \left( s_{\nu} - m_Z^2 \right)^2 + m_Z^2 \Gamma_Z^2
\]

\[
\times \left[ 2 - (h_1 + 1) \log \frac{h_1 + 1}{h_1 - 1} - (h_2 + 1) \log \frac{h_2 + 1}{h_2 - 1} + (h_1 + 1)(h_2 + 1) \frac{\mathcal{L}}{\sqrt{r}} \right]
\]

\[
\mathcal{G}_W = \frac{c_W^8}{s_1 s_2 r} \left( h_1 + 1)(h_2 + 1) \right) \frac{2}{h_1^2 - 1} + \frac{2}{h_2^2 - 1} - \frac{6s_{\chi}^2}{r} + \frac{3t_1 t_2}{r} - c_{\chi} \frac{\mathcal{L}}{\sqrt{r}}
\]

where \( a_e = -1 \) and \( v_e = -1 + 4s_W^2 \). The cross section is written explicitly in terms of the Higgs momentum \( p = (E_H^2 - m_H^2)^{1/2} \), and the energy \( \epsilon_{\nu} = \sqrt{s} - E_H \) and invariant mass squared \( s_{\nu} = \epsilon_{\nu}^2 - p^2 \) of the neutrino pair. The expression for \( \mathcal{G}_W \) had first been obtained in Ref.[10]. The following abbreviations have been introduced:

\[
s_{1,2} = \sqrt{3}(\epsilon_{\nu} \pm p \cos \theta) \quad t_{1,2} = h_{1,2} + c_{\chi} h_{2,1}
\]

\[
h_{1,2} = 1 + 2m_W^2/s_{1,2} \quad r = h_1^2 + h_2^2 + 2c_{\chi} h_1 h_2 - s_{\chi}^2
\]

\[
c_{\chi} = 1 - 2s_{\nu}/(s_1 s_2) \quad \mathcal{L} = \log \frac{h_1 h_2 + c_{\chi} + \sqrt{r}}{h_1 h_2 + c_{\chi} - \sqrt{r}}
\]

\[
s_{\chi}^2 = 1 - c_{\chi}^2
\]

To derive the total cross section \( \sigma(e^+ e^- \rightarrow H\bar{\nu}\nu) \), the differential cross section must be integrated over the region \(-1 < \cos \theta < 1\) and \( m_H < E < \frac{1}{2} \sqrt{s} \) \( 1 + m_H^2/s \).

5.2 Higgs Mass Computation: analytical approximation in the limit of a common scale \( M_S \) and restricted mixing parameters

In this appendix we present the results of the analytical approximation which reproduces the two-loop RG improved effective potential results in the case of two light Higgs doublets below \( M_S \) \( (m_A \leq M_S) \)
The two CP-even and the charged Higgs masses read

\[ m_{h(H)}^2 = \frac{Tr M^2 \mp (Tr M^2)^2 - 4 \det M^2}{2} \]

\[ m_{H^\pm}^2 = m_A^2 + (\lambda_5 - \lambda_4)v^2, \]

where

\[ Tr M^2 = M_{12}^2 + M_{22}^2; \quad \det M^2 = M_{11}^2 M_{22}^2 - M_{12}^2, \]

with

\begin{align*}
M_{12}^2 &= 2v^2[\sin \beta \cos \beta(\lambda_3 + \lambda_4) + \lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta] - m_A^2 \sin \beta \cos \beta \\
M_{11}^2 &= 2v^2[\lambda_1 \cos^2 \beta + 2\lambda_6 \cos \beta \sin \beta + \lambda_5 \sin^2 \beta] + m_A^2 \sin^2 \beta \\
M_{22}^2 &= 2v^2[\lambda_5 \sin^2 \beta + 2\lambda_7 \cos \beta \sin \beta + \lambda_6 \cos^2 \beta] + m_A^2 \cos^2 \beta.
\end{align*}

The mixing angle \( \alpha \) is equally determined by

\[ \sin 2\alpha = \frac{2M_{12}^2}{(Tr M^2)^2 - 4 \det M^2}, \quad \cos 2\alpha = \frac{M_{11}^2 - M_{22}^2}{(Tr M^2)^2 - 4 \det M^2} \]

The above quartic couplings are given by

\[ \lambda_1 = \frac{g_1^2 + g_2^2}{4} - \frac{3}{8\pi^2} h_b^2 t + \frac{3}{8\pi^2} h_b^4 t + X_b \frac{1}{2} + \frac{1}{16\pi^2} \left( \frac{3}{2} h_b^2 + \frac{1}{2} h_t^2 - 8 g_3^2 \right) X_b t + t^2 - \frac{3}{96\pi^2} h_b^4 \frac{\mu^4}{M_s^4} \left( 1 + \frac{1}{16\pi^2} \right) 9 h_t^2 - 5h_b^2 - 16g_3^2 t \]

\[ \lambda_2 = \frac{g_1^2 + g_2^2}{4} - \frac{3}{8\pi^2} h_b^2 t + \frac{3}{8\pi^2} h_b^4 t + X_t \frac{1}{2} + \frac{1}{16\pi^2} \left( \frac{3}{2} h_t^2 + \frac{h_b^2}{2} - 8 g_3^2 \right) X_t t + t^2 - \frac{3}{96\pi^2} h_b^4 \frac{\mu^4}{M_s^4} \left( 1 + \frac{1}{16\pi^2} \right) 9 h_t^2 - 5h_b^2 - 16g_3^2 t \]

\[ \lambda_3 = \frac{g_2^2 - g_1^2}{4} - \frac{3}{16\pi^2} (h_t^2 + h_b^2) t + \frac{6}{16\pi^2} h_t^2 h_b^2 t + A_{tb} \frac{1}{2} + \frac{1}{16\pi^2} h_t^2 + h_b^2 - 8 g_3^2 A_{tb} t + t^2 + \frac{3}{96\pi^2} h_t^4 \frac{\mu^2 A_t^2}{M_s^4} \left( 1 + \frac{1}{16\pi^2} \right) 6 h_t^2 - 2h_b^2 - 16g_3^2 t + \frac{3}{96\pi^2} h_b^4 \frac{\mu^2 A_b^2}{M_s^4} \left( 1 + \frac{1}{16\pi^2} \right) 6 h_b^2 - 2h_t^2 - 16g_3^2 t \]
\[
\lambda_4 = -\frac{g_2^2}{2} \left( 1 - \frac{3}{16\pi^2}(h_t^2 + h_b^2) \right) t - \frac{6}{16\pi^2} h_t^2 h_b^2 t + \frac{A_{tb}}{2} + \frac{1}{16\pi^2} h_t^2 + h_b^2 - 8 g_3^2 \ A_{tb} t + t^2 \\
+ \frac{3}{96\pi^2} h_t^4 \frac{3\mu^2}{M_s^2} - \frac{\mu^2 A_t^2}{M_s^4} 1 + \frac{1}{16\pi^2} 6 h_t^2 - 2 h_b^2 - 16 g_3^2 \ t \\
+ \frac{3}{96\pi^2} h_b^4 \frac{3\mu^2}{M_s^2} - \frac{\mu^2 A_b^2}{M_s^4} 1 + \frac{1}{16\pi^2} 6 h_t^2 - 2 h_b^2 - 16 g_3^2 \ t 
\] (80)

\[
\lambda_5 = -\frac{3}{96\pi^2} h_t^4 \frac{\mu^2 A_t^2}{M_s^4} 1 - \frac{1}{16\pi^2} 2 h_b^2 - 6 h_t^2 + 16 g_3^2 \ t \\
- \frac{3}{96\pi^2} h_b^4 \frac{\mu^2 A_b^2}{M_s^4} 1 - \frac{1}{16\pi^2} 2 h_t^2 - 6 h_b^2 + 16 g_3^2 \ t 
\] (81)

\[
\lambda_6 = \frac{3}{96\pi^2} h_t^4 \frac{\mu^3 A_t}{M_s^4} 1 - \frac{1}{16\pi^2} \frac{7}{2} h_b^2 - \frac{15}{2} h_t^2 + 16 g_3^2 \ t \\
+ \frac{3}{96\pi^2} h_b^4 \frac{\mu}{M_s^4} \frac{A_b^3}{M_s^4} - \frac{6 A_b}{M_s^4} 1 - \frac{1}{16\pi^2} \frac{1}{2} h_t^2 - \frac{9}{2} h_b^2 + 16 g_3^2 \ t 
\] (82)

\[
\lambda_7 = \frac{3}{96\pi^2} h_b^4 \frac{\mu^3 A_b}{M_s^4} 1 - \frac{1}{16\pi^2} \frac{7}{2} h_t^2 - \frac{15}{2} h_b^2 + 16 g_3^2 \ t \\
+ \frac{3}{96\pi^2} h_t^4 \frac{\mu}{M_s^4} \frac{A_t^3}{M_s^4} - \frac{6 A_t}{M_s^4} 1 - \frac{1}{16\pi^2} \frac{1}{2} h_t^2 - \frac{9}{2} h_t^2 + 16 g_3^2 \ t 
\] (83)

They contain the same kind of corrections as eq.(26), including the leading D-term contributions, and we have defined,

\[
X_{t(b)} = \frac{2 A_{t(b)}^2}{M_s^2} 1 - \frac{A_{t(b)}^2}{12 M_s^2} ; \quad A_{tb} = \frac{1}{6} - \frac{6\mu^2}{M_s^2} - \frac{(\mu^2 - A_t A_t)^2}{M_s^4} + \frac{3(A_t + A_b)^2}{M_s^2} . \quad (84)
\]

All quantities in the approximate formulae are defined at the scale \( M_t \), and \( h_t = m_t(M_t)/(v \sin \beta) \) \( h_b = m_b(M_t)/(v \cos \beta) \) are the top and bottom Yukawa couplings in the two-Higgs doublet model.

For \( m_A \leq M_t \), \( \tan \beta \) is fixed at the scale \( m_A \), while for \( m_A \geq M_t \), \( \tan \beta \) is given by [79]

\[
\tan \beta(M_t) = \tan \beta(m_A) 1 + \frac{3}{32\pi^2}(h_t^2 - h_b^2) \log \frac{m_A^2}{M_t^2} . \quad (85)
\]

For the case in which the CP-odd Higgs mass \( m_A \) is lower than \( M_s \), but still larger than the top-quark mass scale, we decouple, in the numerical computations, the heavy Higgs doublet and define an
effective quartic coupling for the light Higgs, which is related to the running Higgs mass at the scale $m_A$ through $\lambda(m_A) = (m_h(m_A)/2v^2)$. The low energy value of the quartic coupling is then obtained by running the SM renormalization-group equations from the scale $m_A$ down to the scale $M_t$. In the analytical approximation, for simplicity the effect of decoupling of the heavy Higgs doublet at an intermediate scale is ignored but is partially compensated by relating the value of $\tan\beta$ at the scale $M_t$ with its corresponding value at the scale $m_A$ through its renormalization-group running, eq.(85). A subroutine implementing the above computations is available [93].

5.3 Deriving 5$\sigma$ Discovery and 95% C.L. Exclusion Contours

The minimum luminosity needed to assess the discovery or to exclude the existence of a Higgs boson with mass $m_H$ can directly be determined from the numbers of events expected from the signal and from the background processes at the three different center-of-mass energies. Given the rather small numbers of events involved in this process, it is preferable to use Poisson statistics to derive the result.

Several definitions for the “minimum luminosity needed” were proposed. For instance, the minimum luminosity needed for a 5$\sigma$ discovery can be defined either (i) as the luminosity needed by the typical experiment, i.e. by an experiment that would actually observe the number of events expected; or (ii) as the luminosity for which 50% of the experiments would make the discovery at the requested 5$\sigma$ level, where the a priori unknown numbers of events observed are properly generated according to a Poisson distribution around their expected values. Although both definitions lead to the same numerical result, a preference was given to the second one, which allows in addition the proportion of the experiments required to make the discovery to be varied.

In detail, let $b$ and $s$ be the numbers of background and signal events expected with a luminosity of 1 fb$^{-1}$, and $\alpha$ be the fraction needed for the discovery. The first definition corresponds to finding the smallest value of $\alpha$ that fulfills the condition

$$1 - \exp(-\alpha b) \sum_{i=0}^{N-1} \frac{(\alpha b)^i}{i!} \leq 5.7 \times 10^{-7},$$

(1)

where $N = \alpha(s + b)$, i.e. that renders the probability of a background fluctuation smaller than the probability of a 5$\sigma$ effect in the case of Gaussian distributions. The second requirement consists in finding the smallest value of $\alpha$ for which the number of events $N_1$ that would correspond to a 5$\sigma$ (high) fluctuation of the background alone is smaller than the numbers of events $N_2$ that would correspond to a 50% probability (low) fluctuation of the total number of events (signal included). This amounts to finding a value of $N$ which fulfills, in addition to (1), the following condition

$$\exp[-\alpha(s + b)] \sum_{i=0}^{N-1} \frac{[\alpha(s + b)]^i}{i!} \leq 0.5.$$

(2)

As to the exclusion of the existence of a signal at the 95% confidence level, the minimum luminosity needed has been similarly defined as the luminosity for which 50% of the experiments would actually exclude it in the case of the absence of signal. Again, this is equivalent to the luminosity needed by
the typical experiment, which is given by the value of $\alpha$ such that

$$
\exp[-\alpha(s + b)] \frac{N}{i!} \sum_{i=0}^{\alpha(s + b)} \left(\frac{(s + b)^i}{i!}\right) \leq 0.05,
$$

where $N = ab$.

In both instances, when deriving the result, $b$ and $s$ were conservatively increased (resp. reduced) by their systematic uncertainties, mainly coming from the yet limited Monte Carlo statistics. The numbers of events expected by each of the four experiments were then added together, and the individual uncertainties were added in quadrature.

However, one caveat should be mentioned. Even if it is legitimate to compute the minimum luminosity needed by each of the four individual experiments by requiring only 50% of the Gedanken-experiments to make the discovery/exclusion, this becomes unclear for the combined experiment: this minimum luminosity would not suffice in 50% of the cases, and this would not be “compensated” by having two (or four) such combined experiments. Since a choice for this fraction cannot be uniquely defined, the combined results have been presented with a fraction of 50% too.
References


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SEARCHES FOR NEW PHYSICS

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1 Supersymmetry

1.1 Introduction
1.2 Supersymmetry and $R_b$

\begin{align*}
R_b & : \pm & R_c & : \pm & R_b & : \pm & R_c & : \pm & \sigma \\
- & . \sigma & R_b & : \pm & R_c & : \pm & \text{SM} & \text{SM} & m_t \\
& & & & & & R_b & : \pm \\
\beta R_b & \quad \quad \beta \quad \quad CP- \\
M_A m_{\chi^\pm} & \quad \beta \quad \quad R_b \quad m_t, m_{\chi^\pm} \quad \beta \\
i) & \quad \chi^2 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{ii)} \quad BR b \to s\gamma \quad \chi^2 <
\end{align*}
Contours of constant $R_b$ for $m_t = 180$ GeV a) in the chargino–lighter stop mass plane for $\tan \beta = 1.6$; b) in the chargino–$CP$–odd Higgs boson mass plane for large $\tan \beta = 50$.

- $\times -4$

ii) $BR \ t \to bW >$

iv) $Z^0 \to \chi_1^0 \chi_1^0 <$

v)

$R_b$

$\beta \ M_t \ m_{\chi^+} \ R_b \ m_{\chi^+}$

$\beta \ M_A \ R_b \ \chi^2$

$R_b >$

$BR \ b \to c\tau \nu$

$m_{\chi^\pm} >$

$m_{\chi^\pm} - m_{\chi_1^0} >$

$m_{\chi^\pm} - m_{\chi_1^0} \ R_b$

$R_b$

$|\mu| < M_2$
Chargino production cross sections at LEP$^2$, $\sqrt{s} = 190$ GeV, as a function of $m_{\tilde{\chi}^\pm_1}$. We show the ranges obtained by varying $M_2$, $\mu$, $\tan \beta$ and $m_{\tilde{\nu}_e}$ throughout the parameter space, requiring $m_{\tilde{\nu}_e} > 45$ GeV. The solid and dotted lines correspond to the maximum and minimum production rates. The dashed (dash-dotted) line corresponds to the minimum cross section if $m_{\tilde{\nu}_e} = 2$ TeV ($m_{\tilde{\nu}_e} = 200$ GeV).

1.3 Chargino

\[ \gamma \rightarrow \chi_{1}^{\pm} \nu_e \]

\[ M_2 \quad \mu \]

\[ m_{\tilde{\chi}^\pm_1} \]

\[ \beta \quad m_{\tilde{\nu}_e} \]

\[ m_{\tilde{\nu}_e} > m_{\tilde{\chi}^\pm_1} \]

\[ \chi_{1}^{\pm} \rightarrow \chi_{1}^{0} f f' \]
with $f$ and $f'$ being fermions of the same weak isospin doublet and the lightest neutralino, which is assumed to be the lightest SUSY particle. The chargino decay can occur via virtual exchange of $W$, sfermions or charged Higgs boson. If $H^+$ and all sfermions are very massive $m_{H^+}; m_\tilde{f}$, the BR are the same as those of the $W$. If the slepton masses are significantly smaller than the squark masses and of the order of $m_W$, the leptonic BR are enhanced. Suppression of the hadronic modes due to phase space can also take place when the mass difference between the chargino and the lightest neutralino is small. Moreover, for some values of the SUSY parameters the second lightest neutralino $\tilde{f}_0$, can be lighter than the lightest chargino and therefore the decay $\tilde{f}_0 \rightarrow \tilde{f} f'$ be possible. This can reduce the BR of decay mode $\tilde{f}_0 \rightarrow \tilde{f} f'$ and give rise to cascade events with more leptons and more jets. The possible outcomes in terms of cross sections, decay modes and branching ratios as a function of the SUSY parameters have been extensively studied in the literature.

If $\tilde{f}_1$ or $\tilde{f}_2$ are lighter than the chargino, the decay modes $\tilde{f}_1 \rightarrow \ell \nu$, $\tilde{f}_2 \rightarrow \ell \nu$ are accessible. The decays to lepton/-sneutrino or slepton/-neutrino would yield signals similar to those from slepton pair production and could be looked for in similar ways. As mentioned earlier, the region in which the chargino cross section is suppressed corresponds to a light sneutrino and a gaugino/-like chargino. In this situation, the decay mode $\tilde{f}_1 \rightarrow \ell \nu$ becomes dominant, see $\tilde{f}_2$. The efficiency for chargino detection is therefore improved and can compensate for the lower production cross section. The experimental studies of the four LEP experiments that we present here have only considered the three body decays in $\ell \nu$, so three kinds of events can be observed depending on whether the charginos decay to leptons or quarks: a pair of acoplanar leptons that may have different lepton modes; a relatively isolated lepton with two hadronic jets and missing energy and hadronic events with missing energy. These modes may lead to different experimentally observed topologies. For example a $\ell j j \ell$ event where the lepton is a $\tilde{f}$ that decays hadronically may look rather like a four-jet event than two jets plus a lepton.

In the Monte Carlo studies carried out by the LEP experiments, the BR of the $\tilde{f}_1$ have been assumed to be those of the $W$, so that the probability of the above modes are taken to be $1/0\%$, $4/3\%$ and $4/6\%$, respectively. The relevant backgrounds to this process are given in table. The most dangerous background is $W^+W^-$ pair production due to the presence of missing energy and visible states similar to those of the signal. Even though $\tilde{f}_2$ events are very sensitive to the cuts used and should be a manageable background, care should be taken to have them well under control through a good knowledge of the apparatus. Indeed, the extremely high cross section of this kind of events makes it unfeasible to generate luminosities of simulated events large enough to match the number of expected real events. The experiments have used a preselection in order to discard at the generation level those events that would in any case be rejected after simulation. This procedure keeps only the tails of the distributions which may be a dangerous background of fake missing energy events. Cross-checks have been performed by the experi...
### Table 1:

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sqrt{s}$</th>
<th>$\sqrt{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+ e^- \rightarrow f f \gamma$</td>
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</tr>
<tr>
<td>$e^+ e^- \rightarrow W^+ W^- \gamma$</td>
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</tr>
<tr>
<td>$e^+ e^- \rightarrow Z^0 Z^0 \gamma$</td>
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<tr>
<td>$e^+ e^- \rightarrow W e \nu \gamma$</td>
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<tr>
<td>$e^+ e^- \rightarrow Z^0 e e \gamma$</td>
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</tbody>
</table>

Relevant backgrounds to chargino production and their cross sections at $\sqrt{s} = 175$ and 192 GeV. The cross section for the $\gamma \gamma$ process is not given since it is highly dependent on the initial cuts used.

### 1.3.1 Mode $jj\ell$

$jj\ell$

$jj\ell$

$\tau$

$\gamma\gamma$

$W^+ W^-$

$W^+ W^-$
Left: selection efficiency as a function of the chargino–neutralino mass difference in the jjl mode for an average experiment. Right: same efficiency for the 4j mode. The two curves correspond to chargino masses of 70 and 90 GeV.

\[ m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}^0} < \]

\[ m_{\tilde{\chi}_1^0} < \]

\[ p_T \]

\[ W^+W^- \]

\[ W \]

\[ \gamma \gamma \]

\[ \sqrt{s} \]

\[ qq\gamma \]

\[ qq\gamma \]

\[ W^+W^- \]

\[ jjl \]

i.e.
The $jj\ell$ mode has been studied by DELPHI, L3, and OPAL for several points in the SUSY parameter space with full simulation of the detectors.

In this mode, high multiplicity and the absence of an isolated lepton are the first requirements. Again, the background is rejected by means of cuts in minimum missing $p_T$ and minimum acoplanarity. It is also required that only a limited amount of the total energy be present in the far-forward/backward region and that the missing momentum vector does not point to this region. For the non-$jj\ell$ background, cuts have been applied on the minimum missing mass, on the maximum visible energy, and on the maximum hadronic mass. As before, these cuts rely on the fact that the missing mass is low for the background and high for the signal, while the hadronic mass is around the center of mass energy for the background and low for the signal. Even though the background is in this case slightly higher than in the $jj\ell$ mode, it is nevertheless still in the $1000\,fb$ range. The experiments agree on the three main sources of background: $W^+W^-$, $qq\gamma$, and $W\ell\nu$ events, but the percentage of each one changes from experiment to experiment, reflecting the different choices of cuts. It is worth noting that for this mode the $W\ell\nu$ process is a non-negligible source of background amounting to $30\%$ of the total.

The signal efficiency depends only very slightly on the chargino and neutralino masses unless we are close to the limit of a small neutralino mass or small chargino/neutralino mass difference. However, in this mode the search can be extended down to chargino/neutralino mass differences of the order of 5 GeV, see the other right. This lower mass difference is due to the higher multiplicity of the $jj\ell$ mode, to the absence of the requirement of an isolated lepton, and to the smaller missing $p_T$ needed to reject the $jj\ell$ background. Another feature of this mode is that it recovers events missed in the $jj\ell$ mode. Indeed, some of the $jj\ell$ events that are lost during the $jj\ell$ selection are kept in the $jj\ell$ selection. In case of discovery, this "migration" might be a serious problem to compute branching ratios with the present selection criteria, but during the search stage it is a way of increasing the overall efficiency. The efficiency to select $jj\ell$ events is, as in the preceding mode, in the $40\%$ range. Additionally, around $15\%$ to $20\%$ of the $jj\ell$ events are classified also as $jj\ell$. If we take into account also these events in our final sample, we obtain an efficiency of $60\%$ to $70\%$.
Minimum signal cross sections required for 5-σ chargino discovery in the $(m_{\tilde{\chi}^\pm}, m_{\tilde{\chi}_1^0})$ plane, with 150 pb$^{-1}$ integrated luminosity. The case of the jj$\ell$ mode (left) and of the 4j mode (right) are shown.
1.3.4 Results

For $p = 190$ GeV, an integrated luminosity of $150$ pb$^{-1}$, and assuming for the chargino the BR of the $W$, the LEP experiments are able to discover at the 5$\sigma$ confidence level a chargino signal in the $jj\ell$ mode provided that its production cross section is above 0.05 to 0.07 pb, depending on the experiment. An integrated luminosity of 5000 pb$^{-1}$ would allow the exploration of the chargino signal in this same mode down to cross sections of 0.02 to 0.04 pb. Although the search can be carried out almost to the kinematical limit for both luminosities, when the chargino and neutralino are close in mass or the mass of the neutralino is very light, the experimental efficiencies decrease very quickly. However, it is not excluded that more specialized selection criteria can be envisaged to recover, at least partially, these regions.

In the $jj\ell$ mode, the minimum reachable cross section at $p = 190$ GeV to discover the chargino at the 5$\sigma$ confidence level is in the 0.04 to 0.07 pb range, which can be reduced to 0.02 to 0.04 pb with an integrated luminosity of 5000 pb$^{-1}$. In this mode, chargino/neutralino mass differences down to 5 GeV can be explored.

In $\ell\ell$, we show the minimum cross-section at the 5$\sigma$ confidence level for an integrated luminosity of 150 pb$^{-1}$ for an average experiment in the chargino/neutralino mass plane, for the $jj\ell$ and $\ell\ell$ modes.

In the $\ell\ell$ channel, the corresponding minimum cross section at $p = 190$ GeV is in the range 0.04 to 0.07 pb with 150 pb$^{-1}$ and 0.02 to 0.04 pb with 5000 pb$^{-1}$. The minimum chargino/neutralino mass difference is 1.0 GeV.

Combining the three modes and assuming an integrated luminosity of 1500 pb$^{-1}$, the chargino search can go down to a minimum cross section of 0.03 to 0.05 pb depending on the experiment. This conclusion is reached under the assumption that the chargino BR are the same as those of the $W$. A variety of enhancements and suppressions of the leptonic and hadronic BR of the $\chi_{1}^{\pm}$ can take place depending on the relevant SUSY parameters, as well as pointed out above. For these cases the above results can be properly rescaled.

1.4 Scalar Lepton Searches

Each $e_{L,R}$, $\mu_{L,R}$, $\tau$, $Z$, $\gamma$, $s$ can pair produce at LEP/2 via $Z$ and $W$-channel exchange. Their production cross section, corrected for ISR, is shown in $\chi_{1}^{\pm}$, in the limit of vanishing left-right mixing. In the case of the selectron, neutralino-exchange in the $t$-channel can also contribute to the production. Now the cross section is not uniquely...
Cross sections for the production of various slepton pairs at 190 GeV, as a function of the slepton mass. In the case of $\tilde{\ell}_L\tilde{\ell}_R$ production we assume $m_{\tilde{\ell}_L} = m_{\tilde{\ell}_R}$. ISR corrections are included throughout. For the selectron processes, the solid lines represent minimum and maximum rates obtained by varying $M_2$, $\mu$ and $\tan\beta$ in the range allowed by the LEP1 constraints. The shaded areas have the additional requirement $m_{\chi^\pm} > 95$ GeV. Notice that the minimum cross section for $\tilde{\ell}_L\tilde{\ell}_R$ is off scale when the $m_{\chi^\pm} > 95$ GeV requirement is not applied.
Dominant slepton decay modes in the $(M_2, \mu)$ plane, for $m_\tilde{e} = 80$ GeV and $\tan \beta = 1.3$. The regions labeled I are excluded by LEP1 data. The regions II do not satisfy $m_\tilde{e} > m_{\tilde{\chi}^0_1}$. The large unmarked regions correspond to $\tilde{\ell} \rightarrow \tilde{\chi}^{\pm}_1 \tilde{\chi}^0_1$ being the dominant decay mode. In the dotted regions the dominant decay is $\tilde{\ell} \rightarrow \nu_\ell \tilde{\chi}^0_1$, while in the hatched area the dominant decay is $\tilde{\ell} \rightarrow \nu_\ell \tilde{\chi}^0_2$.

\[
\ell^\pm \rightarrow \ell^\pm \chi^{0}_i, \quad i = 1, 2, 3, 4,
\]

\[
\ell^\pm \rightarrow \nu_\ell \chi^{\pm}_i, \quad i = 1, 2, 3, 4,
\]

\[m_\nu < m_\ell < m_{\tilde{\chi}^0} \quad \ell \]

\[\ell^\pm_R \rightarrow \ell^\pm \chi^{0}_1 \]

\[a \quad \beta \]

\[\ell^\pm_L \rightarrow \nu_\ell \chi^{\pm}_1 \quad b \quad \beta \]

\[\ell_L \quad \chi^{\pm} \quad \chi^{0}_i, \quad i > \]
Efficiency of the selection cuts I and II on selectron pairs of various masses, as a function of $m_{\tilde{e}}$. 

$e^+ e^- \rightarrow ll, W^\pm W^-, ZZ,$

$\gamma\gamma \rightarrow ll \quad e\gamma \rightarrow \nu W^\pm, e^\pm Z.$

$m_{\tilde{e}} \quad m_{\tilde{\chi}_1^0}$
Limits of detectability of sleptons with $5\sigma$, at 190 GeV, $\tan\beta = 4$ and $\mu = -200$ GeV, for integrated luminosities of $100\,pb^{-1}$ and $500\,pb^{-1}$, in the plane $m_{\tilde{e}_R} - m_{\chi_0^0}$, (a) for $\tilde{e}_R^+ \tilde{e}_R^-$ in the L3 detector at LEP II and (b) for right selectrons and smuons, in an ideal LEP detector.

\[ \begin{align*}
L = 500\,pb^{-1} \\
L = 100\,pb^{-1}
\end{align*} \]

\[ \begin{align*}
5\sigma \text{ discovery regions for L3}
\end{align*} \]

\[ \begin{align*}
\sqrt{s} = 190 \text{ GeV} \\
tan\beta = 4 \\
\mu = -200 \\
m_{\tilde{e}} - m_{\chi_0^0} < m_{\chi_1^0}
\end{align*} \]

\[ \begin{align*}
e^+ e^- &\rightarrow ff, \nu\nuZ \\
gg &\rightarrow ff.
\end{align*} \]
In selection /2/, the missing energy cut is moved from /50 GeV to /30 GeV/. In selection /1/ the acoplanarity angle has to be smaller than /60\degree_0e/, the total transverse momentum larger than /5 GeV and the difference of longitudinal momentum between the negative and the positive lepton/, has to be smaller than /40 GeV/. As the /1c/ can decay either leptonically or hadronically /, the /0c/al state searched for is either /2 jets or a jet and a lepton/. Two lepton /0c/al states/, representing /3% of the total/, would hardly be distinguished from smuon or selectron production/. The total number of charged particles is limited to four/, the acoplanarity angle must lie between /5_0e/ and /150_0e/, the total transverse momentum between /15 and /29 GeV and the missing energy must be larger than /50 GeV/.

After applying the above selections/, the cross section corresponding to the total remaining background/, is reduced to the values shown in table /2/. The difference in background remaining for selectron production/, compared with the L/3 study /, is due to the more realistic detector efficiencies used there/. The /5_0b/ detectability ranges obtained after applying the above cuts on simulated signal events are shown in /c/. /8b/, assuming tan /c/ = /4 and /0c/ = /16_00 GeV/. The results for selectrons and smuons are shown in the m/~/m plane for \( L = 100 \) pb \(-1\) and \( L = 500 \) pb \(-1\). The agreement between selectron limits in both analyses brings some confidence in the crude analysis of /c/. /8b/, which is the only one existing for smuons/. With the above selection status are not observed with /5_0b/ at these luminosities/, for this set of MSSM parameters/. It is only possible in the most favourable case/: \( /3c/ /50 \) or \( /3e/ /30_00 GeV and \( /0c/ '1_0 /5_00 GeV \). The agreement between selectron limits in both analyses brings some confidence in the crude analysis of /c/. /8b/, which is the only one existing for smuons/. With the above selection status are not observed with /5_0b/ at these luminosities/, for this set of MSSM parameters/. It is only possible in the most favourable case/: \( /3c/ /50 \) or \( /3e/ /30_00 GeV and \( /0c/ '1_0 /5_00 GeV \). The agreement between selectron limits in both analyses brings some confidence in the crude analysis of /c/. /8b/, which is the only one existing for smuons/. With the above selection status are not observed with /5_0b/ at these luminosities/, for this set of MSSM parameters/. It is only possible in the most favourable case/: \( /3c/ /50 \) or \( /3e/ /30_00 GeV and \( /0c/ '1_0 /5_00 GeV \). The agreement between selectron limits in both analyses brings some confidence in the crude analysis of /c/. /8b/, which is the only one existing for smuons/. With the above selection status are not observed with /5_0b/ at these luminosities/, for this set of MSSM parameters/. It is only possible in the most favourable case/: \( /3c/ /50 \) or \( /3e/ /30_00 GeV and \( /0c/ '1_0 /5_00 GeV \). The agreement between selectron limits in both analyses brings some confidence in the crude analysis of /c/. /8b/, which is the only one existing for smuons/. With the above selection status are not observed with /5_0b/ at these luminosities/, for this set of MSSM parameters/. It is only possible in the most favourable case/: \( /3c/ /50 \) or \( /3e/ /30_00 GeV and \( /0c/ '1_0 /5_00 GeV \). The agreement between selectron limits in both analyses brings some confidence in the crude analysis of /c/. /8b/, which is the only one existing for smuons/.
OPAL's $5\sigma$ limits of detectability for stau pairs in the $m_\tau - m_{\tilde{\chi}_1^0}$ plane. $\sqrt{s} = 190$ GeV, for an integrated luminosity of 300 pb$^{-1}$. 
1.5 Stop and Sbottom

1.5.1 Phenomenological Aspects

\[ t_L \quad t_R \]
\[ b_1 \]
\[ t_1 \]
\[ \beta \quad \beta > \]
\[ t_L, \, t_R \]
\[ M^2_i \quad M^2_Q \quad m^2_t \quad m^2_Z \]
\[ m_t \quad A_t - \mu \quad \beta \]
\[ T_{3q} - Q_q \quad 2 \theta_W \]
\[ m_t \quad A_t - \mu \quad \beta \]
\[ M^2_U \quad m^2_t \quad m^2_Z \]
\[ \beta Q_t \quad 2 \theta_W \]

\[ T_{3u} \quad Q_t \quad b \]
\[ \theta_i \quad t_R \quad t_2 \quad - \quad \theta_i \quad t_L \]
\[ \theta_i \quad t_R \quad t_2 \quad - \quad \theta_i \quad t_L \]
\[ \theta_i \quad t_R \quad m_b \quad A_b - \mu \quad \beta \]
\[ \theta_i \quad t_R \quad m_b \quad A_b - \mu \quad \beta \]
\[ \theta_i \quad t_R \quad \theta_i \quad t_L \]
\[ M_Q \quad M_U \quad M_D \quad A_t \quad A_b \]
\[ e^+ e^- \rightarrow t_1 \; t_1 \]
\[ e^+ e^- \rightarrow t_1 \; t_1 \]
\[ Z^0 \]
\[ T_{3q} \quad 2 \theta_q - Q_q \quad 2 \theta_W \]
\[ Z^0 \]
\[ \sqrt{s} \]
\[ \sqrt{s} \]
\[ \theta_t \quad \theta_t \]
\[ \theta_b \quad \theta_b \]
\[ \sqrt{s} \]
\[ \sqrt{s} \]

\[ \theta_{t,b} \approx \]

\[ \sigma [\text{pb}] \]

\[ \sigma [\text{pb}] \]

Figure 10: Total cross section in pb at $\sqrt{s} = 175$ GeV (dashed lines) and $\sqrt{s} = 192.5$ GeV (solid lines) as a function of the mixing angle for squark masses of 50, 60, 70, 80, and 90 GeV, for (a) $e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1$ and (b) $e^+ e^- \rightarrow \tilde{b}_1 \tilde{b}_1$. 
already been observed and its main properties are \( e^+ e^- \to t_1 t_1 \), \( c\chi_1^0 \), \( m_{t_1} > m_{\chi_1^\pm} m_b \), and \( \beta \). 

\[
\begin{align*}
M_2, \mu & \quad m_{t_1} \\
\beta & \quad c\chi_2^0 \\
b\chi_1^+ & \quad E \\
t_1 \to b\chi_1^+ & \quad E \\
t_1 & \quad \chi_1^+
\end{align*}
\]

\[
m_{t_1} - m_{\chi_1^\pm} \gtrsim
\]

\[
\sigma (\tilde{t}_1 \tilde{t}_1) [pb]
\]

\[
\sqrt{s} = 192.5 \text{ GeV} \\
\cos \theta_t = 0.4
\]

Figure 11: Born approximation (- - -), QCD corrected (- -), and QCD+ISR corrected (---) cross section for \( e^+ e^- \to \tilde{t}_1 \tilde{t}_1 \) as a function of \( m_{t_1} \) for \( \sqrt{s} = 192.5 \) GeV and \( \cos \theta_t = 0.4 \).

\[
\begin{align*}
t_1 & \quad \tilde{t}_1 b \\
t_1 \to b\chi_1^+ & \quad \theta_t > .
\end{align*}
\]

Figure 12: Parameter domains in the \( (M_2, \mu) \) plane for the various \( \tilde{t}_1 \) decay modes, for \( m_{t_1} = 80 \) GeV and \( \tan \beta = 2 \). The grey area is excluded by LEP1.

Figure 13: Contour lines for the branching m-
tio (in %) of \( \tilde{b}_1 \to b\chi_1^0 \), for \( m_{\tilde{b}_1} = 80 \) GeV, \( \tan \beta = 30 \), and \( M_2 = 60 \) GeV. The grey area is excluded by LEP1.
5 σ discovery reach (left) and 95% CL limits (right) for the $\tilde{t}$ search at 190 GeV. The solid (dashed) lines correspond to maximal (minimal) coupling to the Z. The shaded area corresponds to the current 95% CL Tevatron limits [44].

\[ b_1 b_1 \rightarrow b\chi^0 \quad b_1 \rightarrow b\chi^0_{2} \quad b_1 \rightarrow b\chi^0_{i} \quad M_2 < m_{b_1} - m_b \quad |\mu| < m_{b_1} - m_b \]
\[ |\theta_b| > \beta \]  

1.5.2 Search Strategy for Stop

\[ t_1 \quad t_1 \]
\[ c\chi^0_1 \]
\[ t_1 t_1 \rightarrow c\chi^0_1 c\chi^0_1 \quad \sqrt{s} \quad t_1 t_1 \]
\[ t\chi^0_1 \quad m \]
\[ \chi^0_1 \]
\[ E_{\text{vis}} > \sqrt{s} \]
\[ p_T \]
\[ t_1 t_1 \quad m \]
\[ \approx \theta_{\text{min}} \sqrt{s} \quad \theta_{\text{min}} \]
The maximum $t_1$ mass for $5\sigma$ discovery and 95% C.L. exclusion at $\sqrt{s} = 190$ GeV as a function of the integrated luminosity. The numbers are for one typical LEP experiment for the two cases of the full and zero coupling of $t_1$ to the Z boson.

$$
\begin{array}{c|c|c|c}
\hline
m & \sigma & \sigma \\
\hline
-1 & & \\
\hline
-1 & & \\
\hline
-1 & & \\
\hline
\end{array}
$$

$E_{\text{vis}}$, $p_T$

$m \gtrsim W^+W^- \rightarrow \ell\nu q q'$

$W \rightarrow \ell \nu$

$t_1 t_1$

$ZZ$

$E_{\text{vis}}$, $d_{\text{join}}$

$m_{t_1}$

$W^+W^-$

$q q \gamma$

$Z \gamma \rightarrow q q \gamma$

$\phi_{\text{acop}} > \sigma E_{\text{vis}} \cdot \sqrt{s}$

$\gamma$

$\phi_{\text{acop}} > \sigma E_{\text{vis}} \cdot \sqrt{s}$

$E_{\text{vis}} > \cdot \sqrt{s}$

$\phi_{\text{acop}}$
Contour lines for the cross section (fb) of $e^+e^- \rightarrow \chi_1^0 \tilde{\chi}_2^0$ visible, in the $(\mu, M_2)$ plane, for $\tan \beta = 1.5$, $m_0 = M_Z$, $M_A = 3M_Z$. The central empty region is excluded by LEP1 data. The bold lines represent the kinematical limits for $e^+e^- \rightarrow \chi_1^0 \tilde{\chi}_2^0$ (‘$N$’) and $e^+e^- \rightarrow \chi_1^0 \tilde{\chi}_1^-$ (‘$C$’) at LEP2 ($\sqrt{s} = 190$ GeV).

$$t_1 \rightarrow c\chi_1^0$$

1.6 Neutralinos

$$e^+e^- \rightarrow \chi_1^0 \chi_2^0$$

$$e^+e^- \rightarrow \chi_1^0 \chi_2^0$$

$$m_0 \quad \chi_2^0 \quad \chi_1^0 \quad \ell^+\ell^- \quad \nu\nu \quad qq \quad \ell^+\ell^-\nu\nu \quad \ell^\pm\nu q \quad q_1\overline{q}_1 q_2\overline{q}_2$$

$$\chi_2^0$$

$$\chi_2^0$$

$$\chi_1^0$$

$$\chi_2^0$$

i.e. \(|\mu| \lesssim M_Z \quad M_2 \gtrsim M_Z\)
B.R.'s of all $\tilde{\chi}_2^0$ decay channels for $\mu = m_Z$, $M_2 = 1.5m_Z$, $m_{A0} = 3m_Z$ as functions of $m_0$ (with $\tan \beta = 1.5$) and of $\tan \beta$ (with $m_0 = m_Z$).

$\chi_1^0$ $\chi_2^0$ $i.e.$ $|\mu| \gtrsim M_Z$

$e^+e^- \rightarrow \chi_1^0\chi_2^0$ $\mu$, $M_2$

$|\mu| \gtrsim M_Z$

$\gamma$ $i.e.$ $m_0 \simeq M_Z$

$e$ $i.e.$ $m_0 \simeq M_Z$

$\chi_2^0$

$e^+e^- \rightarrow \chi_1^0\chi_2^0$

$\chi_2^0$ $m_0$

$\beta$

$\chi_2^0$

$i.e.$ $\mu = -m_Z$ $M_2$ $m_Z$
Distributions of the most discriminating variables for the signal from $e^+ e^- \rightarrow \chi_1^0 \chi_2^0 \rightarrow \text{jets} + \not{E_T}$, $\sqrt{s} = 190$ GeV. The distributions for signal (empty histogram) and the sum of the backgrounds from standard physics processes (hatched histogram) are normalized to the same integral.
\[ M_2 \quad \mu \quad m_W \]

\[ m_{\chi_2^0} - m_{\chi_1^0} \quad m_{\tilde{\chi}^0} - m_{\chi_1^0} \]

\[ e^+ e^- \rightarrow \chi_1^0 \chi_2^0 \quad M_2 \gg \mu, m_W \]

\[ R_b \]

\[ e^+ e^- \rightarrow \chi_1^0 \chi_1^0 \gamma \quad \chi_1^0 \]

i.e. \( p_\gamma / E_{\text{beam}} > \cdot |\theta_\gamma| < . \quad E_\gamma / E_{\text{beam}} < . \)

\[ e^+ e^- \rightarrow \gamma \nu \bar{\nu} \]

1.6.1 Search Strategy for Neutralinos

\[ \sqrt{s} \]

\[ m_{\chi_1^0}, m_{\chi_2^0} \]

\[ \chi_1^0 \chi_2^0 \quad \chi_2^0 \rightarrow \chi_1^0 \quad q \bar{q} \text{ or } \ell^+ \ell^- \quad \chi_1^0 \]

\[ \chi_2^0 \]

\[ e^+ e^- \rightarrow q \bar{q} \gamma \quad WW \quad W_{\text{ev}} \quad ZZ \quad Z_{\text{ee}} \quad \gamma \gamma \]

\[ f f \quad e \rightarrow e \gamma \rightarrow f f \]

\[ e \rightarrow e \gamma \rightarrow f f \]
1.6.2 Neutralinos in the NMSSM
Accessible parameter space at LEP2 ($\sqrt{s} = 190$ GeV). The different shadings denote regions where the $\tilde{\chi}_0^0$ production cross section is larger than 500, 200, and 100 fb (from dark to light). The contour line for $m_{\tilde{\chi}^\pm} = 95$ GeV is also shown.

$\begin{array}{ccc}
\lambda & k & x \\
M_1 & M_2 & \beta
\end{array}$

*e.g.* $x \approx$
Production cross section for $(\nu\tilde{\chi}_1^0)$ (a) and $(\ell^\pm\tilde{\chi}_1^\mp)$ (b) final states produced via R-parity violating s-channel $\tilde{\nu}_e$ exchange.

$$\mu \quad \lambda x$$

$$x \lesssim$$

$$\lambda$$ / $b$
1.7 R-Parity Violation

\[ \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \lambda''_{ijk} U_i D_j D_k. \]

\[ Q_i U_i D_i L_i E_i i, \]

\[ R = \frac{2S+3B+L}{s} S B L \]

\[ e.g. \]

\[ \lambda > \times -6 \sqrt{\gamma L} m_j / m_j^2 \]

\[ \gamma L \]

\[ M_{\rm LSP}^{5/2}, \]

\[ L_1 L_{2,3} E_1 \]

\[ e^+ e^- \rightarrow \nu_{2,3} \rightarrow \nu_1^0, \]

\[ \chi_1^\pm \]

MSSM-Production followed by LSP Decay.

\[ \text{all} \]

Resonant Sneutrino Production.

\[ \nu_e \]

\[ \nu_{2,3} \rightarrow e^+ e^- \]

\[ \nu_{2,3} \rightarrow \nu_{2,3} \chi_1^0 \]

\[ \ell_{2,3}^+ \chi_1^0 \]

\[ |M|^2 e^+ e^- \rightarrow \chi_1^0 \ell_{2,3}^+ \]

\[ g^2 \lambda^2 |V_{11}|^2 \frac{s M_{\chi_{1/3}}^2 - s}{|R s|^2} \frac{t M_{\chi_{1/3}} - t}{|D t|^2} = \frac{R I s, t, u}{R s D t} \]
The cross sections are plotted in $/n_0c/$ as functions of $m_t$ due to the ~
$/n_1f/$. The branching fraction depends strongly on the size of $R_b$ where $R_b$/1./8.

Multi-mode Search for Minimal Supergravity at LEP2

The regions denoted by TH are excluded by the theoretical constraints built in to the
$/n_16/n_3e$

scale $M_{/n_16}$ radiative EW symmetry breaking, along with universal soft SUSY-breaking terms at a GUT
$/n_16$ conserving Higgsino mixing term $A_0$. The model parameters are thus

$\mu$, the weak-scale sparticle masses and mixings, which in turn allow all sparticle and Higgs boson

of parameter space accessible to searches at LEP2 and to distinguish the different SUSY signals

$p p$

$\tau \tau$ $W W$ $Z Z$

$\mu >$ $\beta$ $\mu >$ $\epsilon$ $\beta$ $\mu <$ $d$ $\beta$
Regions of the minimal supergravity parameter space explorable at LEP2 with $\sqrt{s} = 190$ GeV.

Cumulative reach of various LEP2 options (and Tevatron MI) for SUSY particles (excluding Higgs bosons).
2 New Fermions

2.1 Introduction
2.2 New Elementary Fermions

- \( i.e. \)
- \( i.e. \)

\[
\begin{align*}
Z^0 & \\
\text{\( m_f \geq \sim \)} & \\
S & \quad \epsilon_3
\end{align*}
\]
Left panel: mass reconstruction for $N\bar{N}$ for the process of eq. 2.2. Right panel: the luminosity needed for heavy neutrino 5σ discovery, as a function of $N$-mass, for $\sqrt{s} = 192$ GeV. The limits for $\sqrt{s} = 175$ GeV are similar, but the mass reach is less.

2.2.1 Pair Production

\[ s \quad \gamma/Z \]

\[ \zeta \quad \zeta \gg -6 \]

\footnote{While this is not strictly true for the exotics, constraints [69] on FCNC imply that pair production dominates over single production for much of the range $m_\ell < \sqrt{s}$.}

\footnote{The only exception is the process $e^+e^- \rightarrow N\nu$ for a heavy neutrino $N$ which may proceed through a $t$-channel $W$-exchange diagram. We shall discuss this case later.}
The effective cross section (at various c.m. energies) for the processes of eqs.2.4 and 2.1, after imposing the cuts of eqs.2.5 and 2.6 respectively.

\[ bb \rightarrow e^-\nu Q q_j \rightarrow e^- e^- \nu \nu \]

\[ e^+e^- \rightarrow N\bar{N} \rightarrow l_i^+l_i^- W^{(*)} W^{(*)} \rightarrow l_i^+l_i^- \]

\[ e^+e^- \rightarrow N\bar{N} \rightarrow l_i^+l_i^- W^{(*)} W^{(*)} \rightarrow l_i^+l_i^- l_a \]

\[ \phi_T \]

\[ p_T \]

\[ l_i \quad e/\mu \]

\[ \tau \]

\[ l \quad j \]

\[ E_l \sim E_{beam} - E_{had} \]

\[ \phi \]

\[ ZZ \]

\[ Z^{-1} \]

\[ N \]

\[ \sigma \]

\[ EE \]
Total cross section (a) and angular distribution (b) for the single production of exotic leptons in association with their ordinary light partners at LEP2 with $\sqrt{s} = 190$ GeV. The solid (dashed) lines are for first generation neutral leptons with a left-handed (right-handed) mixing, the long-dashed-dotted line is for second/third generation leptons with a left-handed or right-handed mixing, and the dotted (dot-dashed) lines are for first generation charged leptons with a left-handed (right-handed) mixing. For the angular distribution the symmetric solid curve is for a Majorana neutrino.

\[ e^+ e^- \rightarrow E \bar{E} \rightarrow \nu_i \bar{\nu}_i W^{(*)} W^{(*)} \rightarrow \mu_T, \]

\[ p_T j > , \ E j > , \ p_T > , \ \theta_j > , \ \theta_{jj} > , \ \theta_{j\mu} > , \]

\[ m_E \approx \sqrt{s}/ -1 \]

\[ p_T j, l > , \ E j > , \ \theta_j, \theta_l < , \ \theta_{jj} > , \ \theta_{ll}, \theta_{lj} > , \]

\[ NN \]
2.2.2 Single Production

\[
\begin{align*}
Z^0 & \quad s \\
t & \quad N\nu \\
Z^0 & \quad eE \\
\end{align*}
\]

\[
\begin{align*}
\zeta_{L,R} & \\
Qq & \\
E_l & \\
E & \quad l\nu \\
l^+l^- & \quad l\nu W \quad l^+l^- Z \\
l\nu & \quad N \quad \nu
\end{align*}
\]

W^+W^-

\[\not{p}\]

2.3 Excited Leptons

\[F^*\]
The singly produced first generation heavy neutrino: reconstructed invariant masses (left) with $W^+W^-$ background at 190 GeV, and (right) limits for 176 GeV.

$e^+e^-\to \nu\bar{\nu}N$, $\sqrt{s}=190$ GeV

$\xi_{\nu N}=0.01$

The production cross sections are dominated by $s$-channel diagrams involving $Z$ exchange. While $e^\pm$ or $\mu^\pm$ production can receive contributions from the magnetic piece of the Lagrangian, for the allowed range of $f_i/a$, these terms are negligible only for $p_{T}\ll f_i/a$. The production rates are thus similar to those for the heavy fermions. The presence of the radiative decay modes thus provides us with tell-tale signatures, especially in the context of LEP2. The SM background may be further reduced by imposing isolation cuts on the photons.

We examine below the specific case of $\nuN$ production.

Events with two photons above 10 GeV and either two or four tracks are required, all isolated from each other by a cone of at least $0.5$, with the exception of the track triplet, which must be a candidate. Signal events were simulated by smearing Monte Carlo vectors in accordance with the ALEPH detector resolution for tracks and photons, after appropriate "losses," such as photon conversion. Background events were looked for, using standard EW.

2.3.1 Pair Production

$e^+e^-\to \nu\bar{\nu}N$, $\sqrt{s}=190$ GeV

$\xi_{\nu N}=0.01$

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$e^+e^-\to \nu\bar{\nu}N$, $\sqrt{s}=190$ GeV

$\xi_{\nu N}=0.01$

The production cross sections are dominated by $s$-channel diagrams involving $Z$ exchange. While $e^\pm$ or $\mu^\pm$ production can receive contributions from the magnetic piece of the Lagrangian, for the allowed range of $f_i/a$, these terms are negligible only for $p_{T}\ll f_i/a$. The production rates are thus similar to those for the heavy fermions. The presence of the radiative decay modes thus provides us with tell-tale signatures, especially in the context of LEP2. The SM background may be further reduced by imposing isolation cuts on the photons.

We examine below the specific case of $\nuN$ production.
Limits for excited lepton pairs, decaying radiatively at LEP2 (190 GeV): 205 GeV is very similar, with more mass reach.

\[ \gamma/Z \]

\[ E_0 \]

\[ \sigma \]

\[ \ell^* \]

\[ \tau^* \bar{\tau}^* \]

2.3.2 Single Production

\[ f_i \]

\[ F^* \]
Total cross section (a) and angular distribution (b) for the single production of vector-like excited leptons in association with their ordinary light partners at LEP with $\sqrt{s} = 190$ GeV and $\Lambda = 1$ TeV. The solid (dashed) lines are for first generation charged (neutral) excited leptons and the dotted (dash-dotted) lines are for the second/third generation charged (neutral) excited leptons.

For excited neutrinos, the situation is similar to that of exotic neutrinos (although the angular distributions are different, as shown in fig. 2T) and one has to look for $e\nu jj$ events. A detailed analysis has not been performed here. Note that the single photon signal has a large associated background.
Exclusion and discovery limits (at 190 and 205 GeV) in the mass-coupling plane for singly produced chiral $e^\ast$. The two sets derive from 2-track ("s-channel") and 1-track ("t-channel") signals respectively. $\mu^\ast\mu$ limits are almost identical to $e^\ast e$ (2-track), and $\pi^\ast\pi$ only a little worse.

$$e^+ e^- \gamma$$
2.3.3 Virtual Effects

\[ m_{e^*} > \]

\[ g \]

\[ * \]

\[ * \]

\[ \theta^* \]

\[ e^* \]

\[ \lambda \]

\[ \gamma \]

\[ \sqrt{s} \]

3 Leptoquarks

\[ SU_e \times SU_L \times U \]

\[ Z \]

\[ \frac{\lambda}{m_{LQ}} \]

\[ \lambda \]

\[ \lambda \]

\[ \alpha \]

\[ \frac{e^2}{4\pi} \]

\[ \lambda \]

\[ m_{LQ} \]

\[ \lambda \]

\[ e \]
LEP and TeV. The LEP experiments set a mass limit of $456$ GeV for any leptoquark species. Searches with the D0 detector exclude $0_{n^{0}}^{n_{d}}_{c}r_{s}t_{g}$ generation scalar leptoquarks below $m_{LQ}=133$ GeV, while the CDF experiment puts a bound on second generation scalar leptoquarks provided these leptoquarks decay to electron plus jet with $100\%$ branching ratio:

$$\frac{m_{LQ}}{n_{28}}=\phi_{29}l_{+}$$

Since there is at least one state in each isospin multiplet with $Br_{LQ}!l_{+}je_{t}$, the absolute lower mass limit for $0_{n^{0}}^{n_{d}}_{c}second/n_{29}generation$ scalar leptoquarks is $120$ GeV, at least if the members of a multiplet are nearly mass degenerate. Otherwise, states decaying exclusively into $17+jet$ final states are only required to respect the less severe LEP mass limit. For sufficiently small $1_{15}$, the LEP bound is also the only one applying to third generation leptoquarks.

At LEP, leptoquarks can be pair produced via $s$ and $Z$ exchange and, in the case of first generation leptoquarks carrying the electron number, also via $t$ channel exchange of a $u$ or $d$ quark. The latter process involves the unknown Yukawa coupling. The production cross section depends very much on the leptoquark quantum numbers. At $p_{s}=190$ GeV and for $m_{LQ}=80$ GeV it varies from $0.04$ pb for the scalar isosinglet $S_{1}$ with charge $1/3$ to $12$ pb for the vector isotriplet $U_{3}$ with charge $+5/3$. Possible $t$ channel contributions are disregarded.

Here, we follow the notation introduced in $\phi_{28}$.$1/n_{5d}$. Furthermore, leptoquarks in the LEP mass range are very narrow. The partial width for a massless decay channel is expected to be of the order of $100$ MeV for $1_{15}=e$.

The signatures for leptoquark pair production and decay are:

i) two electrons/muons plus two hadronic jets;

ii) one electron/muon, two hadronic jets, and missing energy;

iii) two hadronic jets and missing energy;

iv) two tau leptons plus two hadronic jets. Given the Tevatron bounds pointed out above, the focus is on the last two signals. To estimate the acceptance for leptoquark final states we have written a Monte Carlo program and generated events for $m_{LQ}=500$ GeV, $0_{n^{0}}^{n_{3c}}_{c}L_{;R}=e_{n_{3c}}$ and $p_{s}=150$, $175$ and $190$ GeV. The generator is based on the analytical formulae for cross sections and angular distributions given in ref. $\phi_{28}$.$2/n_{5d}$. Initial state radiation is included. Fragmentation is implemented according to JETSET $7.3/n_{5d}$.

The following background processes have been considered and generated for $p_{s}=150$, $175$ and $190$ GeV using Pythia $5.7/n_{28}$,$8$:

$e^{+}e^{-} \rightarrow W^{+}W^{-}$, $W_{e}W_{\nu}$, $ZZ$, $Zee$.

The numbers of generated events correspond to an integrated luminosity of $500$ pb$^{-1}$, except for the two-photon processes, where they correspond to only $20$ pb$^{-1}$. The events were passed through the standard L3 simulation and reconstruction program $\phi_{28}$.$9/n_{5d}$.

Two electrons (muons) and two jets

$$\times \sqrt{s} \quad ZZ \quad Z$$

$Z$
One electron (muon) and two jets

\[ \times \sqrt{s} \quad \times \sqrt{s} \quad WW \quad W_{\ell \nu} \]

Two neutrinos and two jets

\[ \times \sqrt{s} \]

\[ \times \sqrt{s} \quad ZZ \quad Z \quad WW \quad W_{\ell \nu} \quad W \]

Two tau leptons and two jets

\[ \times \sqrt{s} \quad \times \sqrt{s} \quad \times \sqrt{s} \]

\[ \times \sqrt{s} \quad ZZ \quad WW \quad Z \quad W_{\ell \nu} \]

\[ M_{jj} \frac{\sqrt{s}}{E_{\text{vis}}} < \min \sqrt{s} - m_Z, m_W \]

Table 4 shows the estimated acceptance and number of surviving background events for different leptoquark decay channels. Using these numbers and the leptoquark production cross sections, we estimated the 95\% confidence level discovery limits as a function of the integrated luminosity. Some illustrative results are shown in (29). Note that the large variation of the discovery limit for the first generation leptoquark \( S_{\ell 1} \) is mainly due to the variation of the Yukawa coupling \( \lambda \).
5σ discovery limits for a) $S_1^\tau$ at 175 GeV, b) $S_1^\tau$ at 190 GeV taking $\text{Br}(S_1^\tau \rightarrow \nu_\tau b) = 1$, c) $S_3^\tau (4/3)$ at 150 GeV, and d) $U_1^{\nu}$ at 150 GeV. The shaded areas show the effect of varying the Yukawa couplings $\lambda_{R,L} / e$ from zero to unity, and the acceptance within the estimated range given in table 4.
<table>
<thead>
<tr>
<th>(&lt; M_{LQ} &lt;)</th>
<th>(ZZ)</th>
<th>(WW)</th>
<th>(\gamma ZZ)</th>
<th>(WW ZZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^+e^-qq)</td>
<td>(e\nu qq)</td>
<td>(\mu^+\mu^-qq)</td>
<td>(\mu\nu qq)</td>
<td>(\nu\nu qq)</td>
</tr>
</tbody>
</table>

Acceptance and number of background events for different leptoquark signals assuming an integrated luminosity of 500 pb\(^{-1}\).

4 The BESS Model for Dynamical EW Symmetry Breaking
95\% C.L. upper bounds on $g/g''$ vs. $M$ from LEP1 data (continuous line) and CDF data (dotted line) compared with the expected bounds from LEP2 (dashed line).

\[ W^\pm \quad Z \quad \gamma \quad SU \otimes U \]

\[ g'' \quad L^\pm \quad L_3 \quad R^\pm \quad R_3 \quad SU \otimes SU \]

\[ g'' \rightarrow \infty \quad M \rightarrow \infty \]

\[ M \quad W^\pm \quad Z \quad W_L Z_L \quad W_L W_L \]

\[ q^2/M^2 \quad M \]
### Virtual Effects

\[ e^+ e^- \rightarrow W^+ W^- \]
Relative deviations in the differential cross section, due to gauginos contribution (LL channel) for $M_2 = 150, 300, 600, 1000$ GeV
\(\lambda^{\pm}\)

\[
M = -\frac{\sqrt{-}}{2\theta} \delta_{\Delta\sigma,-1} - \frac{2\theta}{\theta} r_w - e_6 \frac{\beta^2 - \beta}{\Delta\sigma,-1}
\]

\(|\lambda| \leq \)

\[
M^\gamma = -\beta \delta_{\Delta\sigma,1} \quad \alpha s \quad A_{\lambda\lambda}^\gamma \quad \delta A_{\lambda\lambda}^\gamma s
\]

\[
M^Z = \beta \rho s \quad \frac{\theta}{2\theta} k s \quad \delta_{\Delta\sigma,1} - \frac{\delta_{\Delta\sigma,-1}}{2\theta} k s \quad \frac{s}{s - m_Z^2} A_{\lambda\lambda}^Z \quad \delta A_{\lambda\lambda}^Z s
\]

\[
M^\nu = \frac{\theta}{2\theta} \beta \delta_{\Delta\sigma,-1} - \frac{2\theta}{\theta} r_w - e_6 \quad B_{\lambda\lambda} - \frac{\beta^2 - \beta}{\Delta\sigma,-1} C_{\lambda\lambda}
\]

\[
\beta = m_W^2 / s^{1/2} \quad \sigma \quad \sigma \quad \lambda \quad \lambda
\]

\[
B_{\lambda\lambda} \quad C_{\lambda\lambda} \quad \sqrt{s}
\]

\[
\sqrt{s}
\]

\[
\delta A_{\lambda\lambda}^\gamma \quad \delta A_{\lambda\lambda}^Z
\]

\[
\delta f_{i}^{\gamma,Z} i \quad \ldots,
\]

\[
\delta A_{++}^V \quad \delta A_{--}^V \quad \delta f_{1}^{V}
\]

\[
\delta A_{\pm 0}^V \quad \delta A_{\pm 0}^V \quad \delta f_{3}^{V} - i \delta f_{4}^{V}
\]

\[
\delta A_{-}^V \quad \delta A_{+}^V \quad \delta f_{2}^{V} \quad i \delta f_{4}^{V}
\]

\[
\delta A_{00}^V \quad \gamma^2 - \beta^2 \delta f_{1}^{V} \quad \gamma^2 \beta^3 \delta f_{2}^{V} \quad \delta f_{3}^{V}
\]

\[
\gamma \quad \sqrt{s} / m_W \quad \delta f_{i}^{V} i \quad \ldots \quad \nu \quad \gamma, Z
\]

\[
W
\]

\[
V \quad \gamma, Z
\]

\[
W
\]

\[
\delta f_{i}^{V}
\]

\[
k s \quad \rho s \quad r_w \quad e_6 \quad \delta A_{\lambda\lambda}^\gamma \quad \delta A_{\lambda\lambda}^Z \quad \alpha s
\]

\[
LL \quad \gamma \quad W \quad \gamma^2 \quad \gamma
\]

\[
TL \quad \gamma^2 \quad \gamma \quad \gamma \quad \gamma^2 \quad \gamma
\]

\[
\delta A_{\lambda\lambda}^\gamma \quad \delta A_{\lambda\lambda}^Z
\]
On the other hand, one of the possibilities to have appreciable deviations in the cross section is to delay the behaviour required by unitarity. This may happen if in the energy window $\sqrt{s}/M_{2}/\mu \gg M_{2} \gg m_{W}$, denoting the mass of the new particles, the above cancellation is less efficient and terms proportional to positive powers of $\mu$ survive in the total amplitude. Only if were sufficiently large, beyond the LEP2 value, could sizeable deviations from the SM be expected.

Figure 3 shows the relative deviation from SM results, $R = d\sigma_{ii}/d\sigma_{SM} - d\sigma_{LL}/d\sigma_{SM}$, as function of $\cos \theta$ at $p_{s} = 200$ GeV for several values of the gaugino mass $M_{2}$ and negligible higgsino contribution. The deviations, even in the most favourable case $M_{2} = 150$ GeV, are unobservable, being smaller than $3\times10^{-3}$. Similar magnitudes have been found for the channels TL and TT and when the higgsino contribution is singled out. Only when one considers SUSY particles very close to the production threshold, $M_{2} = 105$ GeV at $p_{s} = 200$ GeV, deviations of the order $1\%$ are obtained. When model ii is considered, the deviations at LEP2 are at the percent level in the LL and TL channels, even smaller in the TT one and in any case well below the observability level.

Finally, we would like to mention that at higher energies $p_{s} = 500$ GeV or even more, the deviations $R$ in model ii remain at the percent level, making questionable the possibility of observing such effects even in next generation $e^{+}e^{-}$ colliders. While more interesting is the case of model i, in which a delay of unitarity in the LL, LT channels, due to enhancement factor, gives deviations from SM of the order $1\% - 5\%$, for a wide range of new particles masses.

6 CP-odd Correlations at LEP2

$e^{+}e^{-}$

i.e. $bb$

$W^{+}W^{-}$

$\gamma$
and

e^+ e^-

\begin{align*}
e^+ p_+ & \quad e^- p_- \rightarrow \tau^+ k_+ \quad \tau^- k_- \rightarrow A q_- \quad B q_+ \quad X, \\
\pi & \quad \pi \\
O q_+, q_- & \quad -O -q_-, -q_+ \\
\sqrt{s} & \quad pb^{-1}
\end{align*}

\begin{align*}
d^2 \frac{\sqrt{s}}{m_Z} & \quad \tau \gamma \\
\delta d^2 \frac{\sqrt{s}}{m_Z} & \quad \simeq \cdots \times -17e \quad \sigma, \\
\delta d^\gamma \frac{\sqrt{s}}{m_Z} & \quad \simeq \cdots \times -16e \quad \sigma.
\end{align*}

\begin{align*}
\tau^+ & \quad k \\
\tau^\pm & \quad \sigma_{1,\pm}
\end{align*}

\begin{align*}
O_1 & \quad p \times k \cdot \sigma_+ - \sigma_- \\
O_2 & \quad k \cdot \sigma_+ p \times k \cdot \sigma_- - k \cdot \sigma_- p \times k \cdot \sigma_+, \\
O_2 & \quad d^\gamma \\
O_1 & \quad d^2 \frac{\sqrt{s}}{m_Z}
\end{align*}
The distributions are given, for example, in ref. More over one can use optimized observables $O_{i} \rightarrow O_{i}^{opt}$ with maximal signal-to-noise ratio. At $p_{s}=190 \text{ GeV}$ and with an integrated luminosity of $5000 \text{ pb}^{-1}$ we find that with these observables, using the channels $A; B = \pi, \rho, a_{1}$, the following sensitivities can be reached:

- $\text{Re} \delta^{Z} \sqrt{s} \geq 2 \times 10^{-7} \text{ e}^{-1} \sigma$,
- $\text{Re} \delta_{r}^{Z} \sqrt{s} \geq 2 \times 10^{-7} \text{ e}^{-1} \sigma$.

We mention that one can find spin-momentum observables whose expectation values are proportional to the imaginary parts of the form factors.

Also of interest are searches of non-standard CP violation in the reactions $e^{+}e^{-} \rightarrow bb$ gluon $s \rightarrow$ jets

- $\gamma bbg$ $Z bbg$
- $\sqrt{s}$ $\delta^{Z}$ $\sqrt{s}$ $d_{r}^{Z}$

In conclusion, measurements of CP-odd correlations in tau pair production at LEP2 would test the weak dipole form factor $d^{Z}$ with a sensitivity slightly below the accuracy reached at LEP1, but at higher energy. In addition, one can probe the EDM form factor $d$ directly, and with better sensitivity than the direct test at LEP1. Essentially the same conclusions are reached at $p_{s}=175 \text{ GeV}$ assuming the same integrated luminosity.

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TRIPLE GAUGE BOSON COUPLINGS

Conveners: G. Gounaris, J.-L. Kneur and D. Zeppenfeld


1. Introduction
2. Parametrization, models and present bounds on TGC
3. The W pair production process
4. Statistical techniques for TGC determination
5. Precision of TGC determination at LEP2: generator level studies
6. Analysis of the $jj\nu\ell$ and $jj\mu\nu$ final states
7. Analysis of the $jj\tau\nu$ final state
8. Analysis of the $jjjj$ final state
9. Analysis of the $\ell\nu\ell\nu$ final state
10. Other anomalous couplings and other channels
11. Conclusions
1 Introduction

Present measurements of the vector boson-fermion couplings at LEP and SLC accurately confirm the Standard Model (SM) predictions at the 0.1 – 1% level [1], which may readily be considered to be evidence for the gauge boson nature of the W and the Z. Nevertheless the most crucial consequence of the $SU(2) \times U(1)$ gauge theory, namely the specific form of the non-Abelian self-couplings of the W, Z and photon, remains poorly measured to date. A direct and more accurate measurement of the trilinear self-couplings is possible via pair production of electroweak bosons in present and future collider experiments ($W^+W^-$ at LEP2, $W\gamma$, $WZ$ and $W^+W^-$ at hadron colliders).

The major goal of such experiments at LEP2 will be to corroborate the SM predictions. If sufficient accuracy is reached, such measurements can be used to probe New Physics (NP) in the bosonic sector. This possibility raises a number of other questions. What are the expected sizes of such effects in definite models of NP? What type of specifically bosonic NP contributions could have escaped detection in other experiments, e.g. at LEP1? Are there significant constraints from low-energy measurements? Although we shall address these questions, the aim of this report is mostly to elaborate on a detailed phenomenological strategy for the direct measurement of the self-couplings at LEP2, which should allow their determination from data with the greatest possible accuracy.

2 Parametrization, Models and Present Bounds on TGC

We shall restrict ourselves to Triple Gauge boson Couplings (TGC) in most of the report (possibilities to test quartic couplings at LEP2 are extremely limited). Analogous to the introduction of arbitrary vector and axial-vector couplings $g_V$ and $g_A$ of the gauge bosons to fermions, the measurements of the TGC can be made quantitative by introducing a more general WWV vertex. We thus start with a parametrization in terms of a purely phenomenological effective Lagrangian $^1$ [2, 3] $[V = \gamma$ or $Z]$

$$i\mathcal{L}^{WWV}_{eff} = g_{WWV} g^V_{\mu} W^-_{\mu} W^+_{\nu} - W^+_{\mu} W^-_{\nu} + \kappa_V W^+_{\mu} W^-_{\nu} V^{\mu\nu} +$$

$$+ \frac{\lambda_V}{m_W^2} W^\nu_{\nu} W^+_{\mu} W^-_{\mu} + i g_{d}^V \varepsilon_{\mu\nu\rho\sigma} \left( \partial^{\rho} W^{-\mu} \right) W^+_{\nu} - W^{-\mu} \left( \partial^{\rho} W^{+\nu} \right) V^{\sigma}$$

$$+ i g_{d}^V W^-_{\mu} W^+_{\nu} \left( \partial^{\mu} V^{\nu} + \partial^{\nu} V^{\mu} \right) - \frac{\kappa_V}{2} W^+_{\mu} W^-_{\nu} \varepsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} - \frac{\lambda_V}{2 m_W^2} W^-_{\mu} W^+_{\nu,\varepsilon^{\mu\nu\rho\sigma}} V_{\rho\sigma},$$

which gives the most general Lorentz invariant WWV vertex observable in processes where the vector bosons couple to effectively massless fermions. Here the overall couplings are defined as $g_{WW\gamma} = e$ and $g_{WWZ} = e \cot \theta_W$, $W^-_{\mu} = \partial_{\mu} W^- - \partial_{\nu} W_{\mu}$, and $V_{\mu\nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}$. For on-shell

$^1$We use $\epsilon^{0123} = 1$. 
photons, \( g_\gamma^2(q^2 = 0) = 1 \) and \( g_\delta^2(q^2 = 0) = 0 \) are fixed by electromagnetic gauge invariance \(^2\) Within the SM, at tree level, the couplings are given by \( g_\gamma^2 = g_\gamma = \kappa_Z = \kappa_\gamma = 1 \), with all other couplings in (1) vanishing. Terms with higher derivatives in (1) are equivalent to a dependence of the couplings on the vector boson momenta and thus merely lead to a form-factor behaviour of them. We also note that \( g_\gamma^V \), \( \kappa_\gamma \) and \( \lambda_\gamma \) conserve \( C \) and \( P \) separately, while \( g_\delta^V \) violates \( C \) and \( P \) but conserves \( C P \). Finally \( g_\gamma^V \), \( \kappa_\gamma \) and \( \lambda_\gamma \) parameterize a possible CP violation in the bosonic sector, which will not be much studied in this report, as it may be considered a more remote possibility for LEP2 studies \(^3\). However, there exist definite and simple means to test for such CP violation, see section 3. The \( C \) and \( P \) conserving terms in \( \mathcal{L}_{\text{eff}}^{WW\gamma} \) correspond to the lowest order terms in a multipole expansion of the \( W \)–photon interactions: the charge \( Q_W \), the magnetic dipole moment \( \mu_W \) and the electric quadrupole moment \( q_W \) of the \( W^\pm \) [5]:

\[
Q_W = e g_\gamma^Z, \quad \mu_W = \frac{e}{2m_W} (g_\gamma^\gamma + \kappa_\gamma + \lambda_\gamma), \quad q_W = -\frac{e}{m_W^2} (\kappa_\gamma - \lambda_\gamma).
\]

For practical purposes it is convenient to introduce deviations from the (tree-level) SM as

\[
\Delta g_\gamma^Z \equiv (g_\gamma^Z - 1) \equiv \tan \theta_W \delta_Z, \quad \Delta \kappa_\gamma \equiv (\kappa_\gamma - 1) \equiv x_\gamma, \quad \Delta \kappa_Z \equiv (\kappa_Z - 1) \equiv \tan \theta_W (x_Z + \delta_Z), \quad \lambda_\gamma \equiv y_\gamma, \quad \lambda_Z \equiv \tan \theta_W y_Z.
\]

For completeness (and easy comparison) the correspondence of the most studied \( C \) and \( P \) conserving parameters has also been given for another equivalent set \((\delta_Z, x_V, y_V)\), which was used in some recent analyses [6, 7].

### 2.1 Gauge-invariant Parametrization of TGC

Any of the interaction terms in (1) can be rendered \( SU(2) \times U(1) \) gauge invariant by adding to it interactions involving additional gauge bosons [8], and/or additional Would Be Goldstone Bosons (WBGBs) and the physical Higgs (if it exists) [9, 10, 11]. However, one needs to consider \( SU(2) \times U(1) \) gauge invariant operators of high dimension in order to reproduce all couplings in (1). For example, if the Higgs particle exists one needs to consider operators of dimension up to \( d = 12 \). Depending on the NP dynamics, such operators could be generated at the NP mass scale \( \Lambda_{NP} \), with a strength which is generally suppressed by factors like \( (m_W/\Lambda_{NP})^{d-4} \) or \( (\sqrt{s}/\Lambda_{NP})^{d-4} \) [12, 13]. Accordingly, the gauge invariance requirement alone does not provide any constraint on the form of possible interactions. Rather it is a low energy approximation, the neglect of operators of dimension greater than 4 or 6, which leads to relations among the various TGCs.

Such relations among TGCs are highly desirable, given the somewhat limited statistics accessible at LEP2. They were first derived in [14, 8] by imposing approximate global \( SU(2)

\(^2\)For \( q^2 \neq 0 \) deviations due to form factor effects are always possible, see section 2.4 below in this connection. 

\(^3\)Data on the neutron electric dipole moment allow observable effects of e.g. \( \kappa_\gamma \) at LEP2 only if fine tuning at the \( 10^{-3} \) level is accepted [4].
symmetry conditions on the phenomenological Lagrangian (1). In the next subsection we present them following an approach based on $SU(2) \times U(1)$ gauge invariance and dimensional considerations. The connection to the approach based on "global $SU(2)$" symmetry will be discussed at the end.

In order to write down all allowed operators of a given dimensionality one must first identify the low energy degrees of freedom participating in NP. We assume that these include only the $SU(2) \times U(1)$ gauge fields and the remnants of the spontaneous breaking of the gauge symmetry, the WBGBs that exist already in the standard model. If a relatively light Higgs boson is assumed to exist, then NP is described in terms of a direct extension of the ordinary SM formalism; i.e. using a linear realization of the symmetry. On the other hand, if the Higgs is absent from the spectrum (or, equivalently for our purpose, if it is sufficiently heavy), then the effective Lagrangian should be expressed using a nonlinear realization of the symmetry.

2.1.1 Linear Realization

In addition to a Higgs doublet field $\Phi$, the building blocks of the gauge-invariant operators are the covariant derivatives of the Higgs field, $D_\mu \Phi$, and the non-Abelian field strength tensors $\tilde{W}_{\mu\nu} = W_{\mu\nu} - g W_\mu \times W_\nu$ and $B_{\mu\nu}$ of the $SU(2)_L$ and $U(1)_Y$ gauge fields respectively.

Considering CP-conserving interactions of dimension $d = 6, 11$ independent operators can be constructed [15, 9, 10]. Four of these operators affect the gauge boson propagators at tree level [16] and as a result their coefficients are severely constrained by present low energy data [9, 10]. Another subset of these operators generates anomalous Higgs couplings and will be discussed in section 10.4 below. Here we consider the three remaining operators which do not affect the gauge boson propagators at tree-level, but give rise to deviations in the C and P-conserving TGC. Denoting the corresponding couplings as $\alpha_{W,\phi}$, $\alpha_{B,\phi}$, and $\alpha_W$, the TGC inducing effective Lagrangian is written as

$$\mathcal{L}_{d=6}^{TGC} = ig \frac{\alpha_{B,\phi}}{m_W^2} (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi) + ig \frac{\alpha_{W,\phi}}{m_W^2} (D_\mu \Phi)^\dagger \gamma \cdot \tilde{W}^{\mu\nu} (D_\nu \Phi) + g \frac{\alpha_W}{6m_W^2} \tilde{W}^{\mu\nu} \cdot (\tilde{W}_\rho \times \tilde{W}_\mu),$$

with $g, g'$ the $SU(2)_L$ and $U(1)_Y$ couplings respectively. Replacing the Higgs doublet field by its vacuum expectation value, $\Phi^T \to (0, v/\sqrt{2})$, yields nonvanishing anomalous TGCs in (1),

$$\Delta g_1^Z = \frac{\alpha_{W,\phi}}{c_W^2}, \quad \Delta \kappa = -\frac{c_W^2}{s_W^2} (\Delta \kappa_Z - \Delta g_1^Z) = \alpha_{W,\phi} + \alpha_{B,\phi}, \quad \lambda_{W,\phi} = \lambda_Z = \alpha_W,$$

where $s_W \equiv \sin \theta_W$, $c_W \equiv \cos \theta_W$. The normalization of the dimension 6 operators in (4) has been chosen such that the coefficients $\alpha_i$ correspond directly to $\Delta \kappa_{\gamma}$ and $\lambda_{\gamma}$. It should be noted that, as the NP scale $\Lambda_{NP}$ is increased, the $\alpha_i$ are expected to decrease as $(m_W/\Lambda_{NP})^2$.

This scaling behaviour can be quantified to some extent by invoking (tree-level) unitarity constraints [17, 18, 19, 13]. A constant anomalous TGC leads to a rapid growth of vector boson pair production cross-sections with energy, saturating the unitarity limit at $\sqrt{s} = \Lambda_Y$. A larger
value of $\Lambda_U$ implies a smaller TGC $\alpha_i$. For each of them the unitarity relation may be written as [17, 18]

$$ |\alpha_W| \simeq 19 \frac{m_W}{\Lambda_U}^2, \quad |\alpha_{W\phi}| \simeq 15.5 \frac{m_W}{\Lambda_U}^2, \quad |\alpha_{B\phi}| \simeq 49 \frac{m_W}{\Lambda_U}^2. \quad (6) $$

For any given value of $\alpha_i$ the corresponding scale $\Lambda_U$ provides an upper bound on the NP scale $\Lambda_{NP}$. Conversely, a sensitivity to small values of an anomalous coupling constant is equivalent to a sensitivity to potentially high values of the corresponding NP scale. Applying (6) for $\Lambda_U = 1$ TeV, we get $|\alpha_W| \simeq 0.12, \quad |\alpha_{W\phi}| \simeq 0.1, \quad |\alpha_{B\phi}| \simeq 0.3$. Since these values are larger than the expected LEP2 sensitivity by less than a factor 3, it is clear that LEP2 is sensitive to $\Lambda_{NP} \lesssim 1$ TeV. Thus a caveat is in order: for these low values of $\Lambda_{NP}$ the neglect of dimension 8 operators may no longer be justified, leading to deviations from the relations (5)[20].

2.1.2 Nonlinear Realization

In the absence of a light Higgs a non-linear approach should be used to render $L^{W,W'}_{\text{eff}}$ gauge invariant. The SM Lagrangian, deprived of the Higgs field, violates unitarity at a scale of roughly $4\pi v \sim 3$ TeV, so that the new physics should appear at a scale $\Lambda_{NP} \lesssim 4\pi v$. Technically the construction of gauge-invariant operators follows closely the linear case above, except that in place of the scalar doublet $\Phi$ a (unitary, dimensionless) matrix $U \equiv \exp(i\omega \cdot \vec{r}/v)$, where the $\omega_i$ are the WBGBs, and the appropriate matrix form of the $SU(2)_L \times U(1)_Y$ covariant derivative are used. The so-called “naive dimensional analysis” (NDA) [22] dictates that the expected order of magnitude of a specific operator involving $b$ WBGB fields, $d$ derivatives and $w$ gauge fields is $\sim v^2\Lambda_{NP}^2 (1/v)^b (1/\Lambda_{NP})^d (g/\Lambda_{NP})^w$. Applying NDA to the terms in Eqs. (1), we see that $\Delta g_Y^i$ and $\Delta \kappa_Y$ are of $O(m_W^4/\Lambda_{NP}^4)$. In other words, just as in the linear realization, these terms are effectively of dimension 6 (in the sense that there is an explicit factor of $1/\Lambda_{NP}^2$). On the other hand, we see that the $W^i_{\mu\nu} W^{\mu\nu} V_{\gamma\rho}$ term is effectively of dimension 8, i.e. the coefficient $\lambda_Y$ is expected to be of order $m_W^4/\Lambda_{NP}^4$. Thus, within the nonlinear realization scenario, the $\lambda_Y$ terms are expected to be negligible compared to those proportional to $\Delta g_Y^i$ and $\Delta \kappa_Y$. Accordingly there remain three parameters at lowest dimensionality, which can be taken as $g_1^2$, $\kappa_Z$ and $\kappa_\gamma$.

2.1.3 Operators of Higher Dimension and Global Symmetry Arguments

As mentioned in section 2.1, one may argue that relations like in (5) would not even be approximately correct if $\Lambda_{NP}$ is substantially smaller than 1 TeV, since higher dimensional operators are no longer suppressed, and may even be more important than the $\text{dim} = 6$ operators [20]. In fact, as far as the 5 C and P conserving TGC in (1) are concerned, the most general choice can be realized by invoking two $\text{dim} = 8$ operators in addition to the 3 terms in (4) [10, 11, 23].

---

4One should caution that this estimate of $\Lambda_{NP}$ follows directly from analogy with low energy QCD and Chiral perturbation theory [21], where $v \equiv f_\pi$ and $\Lambda \simeq M_P$ are known, while in the present context $\Lambda_{NP}$ is essentially unknown. It should be taken as a rough order of magnitude estimate only.
Requiring restoration of an SU(2) global ("custodial") symmetry for \( g' \to 0 \) (i.e. in the limit of decoupling B field) implies [23] the coefficient of one of these two operators to vanish, because it violates SU(2) global\(^5\) independently of the B field. In that way one recovers the constraints between \( \Delta \kappa_\gamma \) and \( \Delta \kappa_Z \) in (5), in both the nonlinear realization and in the linear realization at the \( \text{dim} = 8 \) level. Nevertheless a second \( \text{dim} = 8 \) operator spoils the relation, \( \lambda_\gamma = \lambda_Z \) in (5). One may neglect this term (which vanishes in the limit \( g' \to 0 \)) by imposing exact SU(2) at the scale \( \Lambda_{\text{NP}} \), which in our context is similar to neglecting the \( \pi^\pm - \pi^0 \) mass difference in strong interaction physics.

Largely these are simplifying assumptions only, intended to reduce the number of free parameters. Motivated by the previous discussion we recommend two sets of three parameters each for full correlation studies between anomalous couplings at LEP2:

- set1 = \( (\Delta g_1^Z, \Delta \kappa_\gamma, \Delta \kappa_Z) \) with \( \lambda_\gamma = \lambda_Z = 0 \). These correspond to the operators of lowest dimensionality in the nonlinear realization. A reduction to 2 parameters (using \( \Delta \kappa_\gamma = -\frac{\kappa_\gamma}{Z}(\Delta \kappa_Z - \Delta g_1^Z) \)) is achieved by assuming [6, 23] custodial SU(2) for \( g' \to 0 \).

- set2 = \( (\Delta g_1^Z, \Delta \kappa_\gamma, \lambda_\gamma) \) with \( \lambda_Z \) and \( \Delta \kappa_Z \) given by (5). It is this set which has been used in this report for the determination of precisions achievable from WW production at LEP2, presented in sections 5–9 as limits on the parameters \( \alpha_{B\phi}, \alpha_{W\phi} \) and \( \alpha_W \) defined by (4).

Expressing results in terms of \( \Delta g_1^Z, \Delta \kappa_\gamma, \) etc. will be useful for ease of comparison with published hadron collider data [25, 26].

In addition, it would clearly be of interest to present fits to each of the parameters in \( \mathcal{L}_{\text{eff}}^{WW} \) in order to reduce the dependence of the analysis on specific models. However, this can only be achieved bearing in mind the limited data which will be available from LEP2, and the correlations inherent in the extraction of many parameters from the data. We return to this point in sections 3.1, 4.2 and 5.1 below.

2.2 Present constraints on TGC

The errors of present direct measurements, via pair production of electroweak bosons at the Tevatron, are still fairly large. The latest, best published 95% CL bounds by CDF and D0 are obtained from studies of \( W\gamma \) events [25, 26]

\[
-1.6 < \Delta \kappa_\gamma < 1.8 , \quad -0.6 < \lambda_\gamma < 0.6
\]

but constraints from the study of WW, \( WZ \to \ell\nu jj, \ell\ell jj \) events are becoming competitive and should lead to 95% CL bounds of roughly \(-0.65 < \Delta \kappa_\gamma < 0.75, \quad |\lambda_\gamma| = |\lambda_Z| < 0.4\), once the\(^5\)Note that there is no contradiction with the SU(2)\(_L\) x \(U(1)_Y\) local invariance of all these operators, since SU(2) custodial is a different symmetry from the SU(2)\(_L\) global [24].
already collected run 1b data are fully analyzed. Increasing the integrated luminosity to 1 fb⁻¹ with the Fermilab main injector is expected to improve these limits by another factor 2 [27]. Note that these latter bounds assume the relations between anomalous couplings as given by (5) with \( \alpha_{W^\pm} = \alpha_{B^\pm} \). In addition, the Tevatron measures these parameters at considerably larger momentum transfers than LEP2 and, hence, form factor effects could result in different measured values at the two machines.

Alternatively, constraints may be derived also from evaluating virtual contributions of TGC to precisely measured quantities such as \((g - 2)_\mu\) [28], the \(b \to s\gamma\) decay rate [29, 30], \(B \to K^{(*)}\mu^+\mu^-\) [31], the \(Z \to b\bar{b}\) [32] rate and oblique corrections [9, 10] (i.e. corrections to the W, Z, \(\gamma\) propagators). Oblique corrections combine information from the recent LEP/SLD data, neutrino scattering experiments, atomic parity violation, \(\mu\)-decay, and the \(W\)-mass measurement at hadron colliders.

When trying to derive TGC bounds from their virtual contributions one must make assumptions about other NP contributions to the observable in question. In the linear realization, for example, Higgs contributions to the oblique parameters tend to cancel the TGC contributions and as a result the TGC bounds are relatively weak for a light Higgs boson [10]. In general, there are other higher dimensional operators which contribute directly to the observable, in addition to the virtual TGC effects. Bounds on the TGC then require to either specify the underlying model of NP completely or to assume that no significant cancellation occurs. The bounds on the TGC parameters in (1) due to virtual effects thus depend on the underlying hypotheses and are of \(\mathcal{O}(0.1)\) to \(\mathcal{O}(1)\) [9, 10, 33].

More stringent bounds are obtained [9] by comparing the higher dimensional operators which induce TGC with those operators which directly induce oblique effects (see Section 10.1). In simple models the coefficients of these two sets of operators are of similar size and hence the stringent LEP1 bounds on the latter [34] indicate that one should not expect anomalous TGC above \(\mathcal{O}(0.01)\). One should stress, however, that no rigorous relation between oblique effects and TGC can be derived except by going to specific models of NP. Therefore, these stringent bounds must be verified, by a direct measurement of the TGC at LEP2.

### 2.3 Virtual Contributions to TGC in the MSSM

Definite TGC contributions are certainly present at the loop level in any renormalizable model, although such loop effects contribute to TGC with a factor of \((g^2/16\pi^2) \approx 2.7 \times 10^{-3}\), being therefore too small a priori to be observed at LEP2. For instance, SM one-loop TGC predictions are known [35, 36, 37] and give, at \(\sqrt{s} = 190\) GeV, \(\Delta\kappa_\gamma (\Delta\kappa_Z) \approx 4.1-5.7 \times 10^{-3}\) (3.3-3.1 \(10^{-3}\)), for \(m_{Higgs} = 0.065-1\) TeV and \(m_{top} = 175\) GeV [38]. (Contributions to \(\lambda_\gamma\) are about a factor of 3 smaller). One may, however, expect that the “natural scale” \((g^2/16\pi^2)\) could be substantially enhanced if, for example, some particles in the loop have strong coupling and/or are close to

---

6A complementary study of virtual MSSM contributions to the \(e^+e^- \rightarrow W^+W^-\) cross-section is done in the New Particle chapter of these proceedings.
their production threshold. To obtain a “reference point” it is thus important to explore more quantitatively how far one is from the LEP2 accuracy limit, within some well-defined model of NP. We here use the contributions of the (MSSM) [39] as an example. These contributions were calculated independently by two groups in the framework of the Workshop. We summarize the main results, referring for more details to refs. [37, 38].

<table>
<thead>
<tr>
<th>SUGRA-GUT MSSM</th>
<th>Unconstrained MSSM (maximal effects)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0, m_0, M_{1/2} = 300, 300, 80$ (GeV), $\tan \beta = 2$ ($\mu &lt; 0$)</td>
<td>$\tan \beta = 1.5$; $M_{1, 2}^{\chi} \simeq 95, 130$; $M_1^{\chi} \simeq 20-132$ (GeV); $m_{H^+} \simeq 95$; $m_{\tilde{\nu}<em>i} \simeq 45$; $m</em>{\tilde{g}} \simeq 92-110$; $m_{\tilde{q}} \simeq 45-800$ GeV</td>
</tr>
<tr>
<td>$</td>
<td>\Delta \kappa_\gamma</td>
</tr>
</tbody>
</table>

Table 1: $\Delta \kappa_{\gamma, Z}$ (as defined in eq. 1) in MSSM at $\sqrt{s} = 190$ GeV. (Contributions to $\lambda_Y$ are about a factor of 2-3 smaller).

Naively, TGC are obtained by summing all MSSM contributions to the appropriate parts in eq. (1) from vertex loops with entering $\gamma$ (or $Z$) and outgoing $W^+$, $W^-$. But as is well-known, the vertex graphs with virtual gauge bosons need to be combined with parts of box graphs for the full process, $e^+e^- \rightarrow W^+W^-$, to form a gauge-invariant contribution. The resulting combinations define purely $s$-dependent TGC [36]. In table 1 we illustrate our results for ($s$-dependent) contributions in two different cases. First, for a representative choice of the free parameters in the more constrained MSSM spectrum obtained [37] from the SUGRA-GUT scenario [40]: the only parameters are the universal soft terms $m_0, M_{1/2}, A_0$ at the GUT scale, $\tan \beta$ (and the sign of $\mu$). Second, we give one illustrative contribution, obtained [38] from a rather systematic search of maximal effects in the unconstrained MSSM parameter space. The largest contributions are mostly due to gauginos and/or some of the sleptons and squarks being practically at threshold. One may note, however, that some individual contributions, potentially larger, were quite substantially reduced when the present constraints on the MSSM parameters are taken into account [38]. Even these maximal contributions hardly reach the level of the most optimistic accuracy limit expected on TGC (compare section 5 below). One should also note that radiatively generated TGC generally have a complicated $\sqrt{s}$ form factor dependence as well as contributions from boxes, which are well approximated by an expansion in $1/\Lambda_{NP}$ only when one probes well below threshold.

2.4 TGC from extra $Z'$

A light and weakly coupled $Z'$ provides an illustrative example of relatively large deviations of the TGC from their SM values and of strong form-factor effects [41]. Consider an extra

---

7By definition, $t$ and $u$-dependent box contributions are left over in this procedure. We have evaluated [38] a definite (gauge-invariant) sample of this remnant part, the slepton box contributions, and found them negligible, $\simeq 0.1 (g^2/16\pi^2) \simeq 3 \, 10^{-4}$ at most, at LEP2 energies.

8A complementary study can be found in the $Z'$ working group chapter of these proceedings.
gauged $U(1)'$ symmetry with associated coupling $g'_i$, whose vector boson $Z'$ is relatively light, say $M_{Z'} \approx 200\text{GeV}$. For such a boson to remain undetected at LEP1 and CDF, it must have rather small couplings to fermions: $\lambda \equiv \sin(\theta_W)g'_i/g_1 < 0.2$ or less [41]. However, this new $Z'$ might be only part of the new physics beyond the SM, and we parametrize this by gauge invariant higher dimensional operators. For illustration, let us focus on the $\text{dim} = 6$ operator

$$\mathcal{L}_{B'W} = \frac{e}{v^2}O_{B'W} = \frac{e}{v^2}\phi^\dagger B'^{\mu\nu} \tilde{W}_{\mu\nu} \cdot \tau\phi$$

where $B'_\mu$ is the new $U'(1)$ field strength. This operator has a part linear in $W_\mu$ inducing unusual mixing through the kinetic terms, from which LEP1 data put upper bounds on $\lambda$ and $\epsilon$. The other piece is quadratic in $W_\mu$ and brings anomalous contributions to $W$-pair production at LEP2, which may be enhanced at will by approaching the $Z'$ pole. Within a gauge-invariant framework, enlarging the symmetry group has given us enough freedom to escape the more stringent LEP1 constraints on the coefficient of the similar operator $C_{B'W}$ [9, 10] of Eq. (29). Having such an (admittedly contrived) counter-example to [9] (depending on the 3 parameters $M_{Z'}, \lambda < 0.2$ and $|\epsilon| < 0.2$), it is instructive to see how it fits into our TGC parametrization.

The normal way of extracting the predictions of this model for $W$-pair production would be to add all the amplitudes for $e^+e^- \rightarrow W^+W^-$, namely the $t$-channel $\nu$ pole, and $s$-channel $\gamma$, $Z$ and $Z'$ poles, including the contributions of $O_{B'W}$ in the latter. Alternatively, the correct angular dependence in $e^+e^- \rightarrow W^+W^-$ from such a $Z'$ is recovered through the introduction of “process-dependent” TGC form factors: the $Z'$ exchange only contributes to the $J = 1$ partial wave and the TGC of Eq. (1) allow to parameterize the most general $J = 1$ amplitude. For the case at hand one can always find TGC $(g'^Z_1, \kappa^Z, g'_{1\gamma}, \kappa^\gamma)$ matching the $Z'$ parameter dependence in this particular $e\bar{e}$-$WW$ channel, but these TGC will depend on the incoming electron’s coupling to the $Z$ and the photon.

In general, a non-zero $\Delta g'^Z_1$ is needed to match the precise $t$-dependence, but in such a process-dependent approach, this does not imply any violation of charge conservation. Finally one should note that the $Z'$ described above would also appear in $e^-e^+ \rightarrow q\bar{q}$, $\ell^-\ell^+$ at LEP2 and thus all channels need to be searched for NP effects.

\[\Delta g'^Z_1 vs. \Delta g'_{1\gamma} for \sqrt{s} = 205\text{GeV}\]

and $M_{Z'} = 210\text{GeV}$. For each $\lambda$ ranging from 0 (plain curve) to 0.2 (smallest dashes), $\epsilon$ is limited to satisfy today’s $W$ mass accuracy, $|\delta M_W| < 160\text{MeV}$ (LEP2’s $|\delta M_W| < 45\text{MeV}$ for the thick curves).

In general, a non-zero $\Delta g'^Z_1$ is needed to match the precise $t$-dependence, but in such a process-dependent approach, this does not imply any violation of charge conservation. Finally one should note that the $Z'$ described above would also appear in $e^-e^+ \rightarrow q\bar{q}$, $\ell^-\ell^+$ at LEP2 and thus all channels need to be searched for NP effects.
3 The $W$ Pair Production Process

3.1 Phenomenology of On-shell $WW$ Production

Deviations of the TGC's from their SM, tree level form are most directly observed in vector boson pair production. At LEP2 this is the process $e^-e^+ \rightarrow W^-W^+$, which, to lowest order, proceeds via the Feynman graphs of Fig. 1. We start by describing the core process, including the $W$ decay into fermion anti-fermion pairs in the zero-width approximation, since most of the effects of anomalous couplings can already be understood at this level. A full simulation of the signal will, of course, need refinements such as finite width effects, the ensuing contributions from final state radiation graphs and the inclusion of $t$-channel vector boson exchange graphs for specific final states such as $e^- \nu \bar{d}$. The simulation of this full $e^+e^- \rightarrow 4$ fermions process will be discussed later. It is instructive to consider first the individual contributions of $s$-channel photon and $Z$ exchange and of $t$-channel neutrino exchange to the various helicity amplitudes for the process $e^-e^+ \rightarrow W^-W^+$ [3],

$$\mathcal{M}(\sigma, \lambda, \bar{\lambda}) = \mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_\nu.$$  \hspace{1cm} (8)

Here the $e^-$ and $e^+$ helicities are given by $\sigma/2$ and $-\sigma/2$, and $\lambda$ and $\bar{\lambda}$ denote the $W^-$ and $W^+$ helicities. Let us define reduced amplitudes $\mathcal{M}$ by splitting off the leading angular dependence in terms of the $d$-functions \[d^6 \] where $J_0 = 1, 2$ denotes the lowest angular momentum contributing to a given helicity combination. In the c.m. frame, with the $e^-$ momentum along the $z$-axis and the $W^-$ transverse momentum pointing along the $x$-axis, the helicity amplitudes are given by\footnote{As compared to Ref. [3] a phase factor $(-1)\bar{\lambda}$ is absorbed into the definition of the $W^+$ polarization vector.}

$$\mathcal{M}(\sigma, \lambda, \bar{\lambda}; \theta) = \sqrt{2} \sigma e^2 \mathcal{M}_{\sigma,\lambda,\bar{\lambda}}(\theta) d^6_{\sigma,\lambda,\bar{\lambda}}(\theta).$$ \hspace{1cm} (9)

For $(\lambda, \bar{\lambda}) = (\pm, \mp)$, i.e. $|\lambda - \bar{\lambda}| = 2$, only $t$-channel neutrino exchange contributes and the incoming electron must be lefthanded. The corresponding amplitudes are given by

$$\mathcal{M}(-1, \lambda, \bar{\lambda} = -\lambda; \theta) = -\sqrt{2}e^2 \frac{\sqrt{2}}{\sin^2\theta_W} \frac{1}{1 + \beta^2 - 2\beta \cos \theta} \lambda \sin \theta (1 - \lambda \cos \theta)/2.$$ \hspace{1cm} (10)

![Diagram](image-url) (Figure 1: Feynman graphs for the process $e^+e^- \rightarrow W^+W^-$.)
The $s$-channel photon and $Z$ exchange is possible only for $|\lambda - \bar{\lambda}| = 0, 1$. The corresponding reduced amplitudes can be written as

$$\tilde{M}_\gamma = -\beta A^\gamma_{\lambda\bar{\lambda}},$$

$$\tilde{M}_Z = +\beta A^Z_{\lambda\bar{\lambda}} \left( 1 - \delta_{\sigma,-1} \right) \frac{1}{2 \sin^2 \theta_W} \frac{s}{s - m_Z^2},$$

$$\tilde{M}_\nu = +\delta_{\sigma,-1} \frac{1}{2 \beta \sin^2 \theta_W} B_{\lambda\bar{\lambda}} - \frac{1}{1 + \beta^2 - 2 \beta \cos \theta} C_{\lambda\bar{\lambda}}. \quad (11)$$

Here $s$ denotes the $e^+e^-$ center of mass energy and $\beta = 1 - 4m_W^2/s$ is the $W^\pm$ velocity. The subamplitudes $A^\gamma, B$ and $C$ are given in Table 2.

Table 2: Subamplitudes for $J_0 = 1$ helicity combinations of the process $e^-e^+ \rightarrow W^-W^+$, as defined in Eq. (11). $\beta$ denotes the $W$ velocity and $\gamma = \sqrt{s}/2m_W$. The abbreviation $f_3^V = g_3^V + \kappa_V + \lambda_V$ is used.

<table>
<thead>
<tr>
<th>$\lambda\bar{\lambda}$</th>
<th>$A^V_{\lambda\bar{\lambda}}$</th>
<th>$B_{\lambda\bar{\lambda}}$</th>
<th>$C_{\lambda\bar{\lambda}}$</th>
<th>$\delta_{\sigma,-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>++ $g_1^V + 2\gamma^2\lambda_V + \frac{i}{\beta}(\kappa_V + \lambda_V - 2\gamma^2\lambda_V)$</td>
<td>1</td>
<td>$1/\gamma^2$</td>
<td>$-\sigma \sin \theta / \sqrt{2}$</td>
<td></td>
</tr>
<tr>
<td>-- $g_1^V + 2\gamma^2\lambda_V - \frac{i}{\beta}(\kappa_V + \lambda_V - 2\gamma^2\lambda_V)$</td>
<td>1</td>
<td>$1/\gamma^2$</td>
<td>$-\sigma \sin \theta / \sqrt{2}$</td>
<td></td>
</tr>
<tr>
<td>+0 $\gamma(f_3^V - ig_4^V + \beta g_5^V + \frac{i}{\beta}(\kappa_V - \lambda_V))$</td>
<td>2$\gamma$</td>
<td>$2(1 + \beta)/\gamma$</td>
<td>$(1 + \sigma \cos \theta) / 2$</td>
<td></td>
</tr>
<tr>
<td>0$-$ $\gamma(f_3^V + ig_4^V + \beta g_5^V - \frac{i}{\beta}(\kappa_V - \lambda_V))$</td>
<td>2$\gamma$</td>
<td>$2(1 + \beta)/\gamma$</td>
<td>$(1 + \sigma \cos \theta) / 2$</td>
<td></td>
</tr>
<tr>
<td>0$+$ $\gamma(f_3^V + ig_4^V - \beta g_5^V + \frac{i}{\beta}(\kappa_V - \lambda_V))$</td>
<td>2$\gamma$</td>
<td>$2(1 - \beta)/\gamma$</td>
<td>$(1 - \sigma \cos \theta) / 2$</td>
<td></td>
</tr>
<tr>
<td>0$-$ $\gamma(f_3^V - ig_4^V - \beta g_5^V - \frac{i}{\beta}(\kappa_V - \lambda_V))$</td>
<td>2$\gamma$</td>
<td>$2(1 - \beta)/\gamma$</td>
<td>$(1 - \sigma \cos \theta) / 2$</td>
<td></td>
</tr>
<tr>
<td>00 $g_1^V + 2\gamma^2\kappa_V$</td>
<td>$2\gamma^2$</td>
<td>$2/\gamma^2$</td>
<td>$-\sigma \sin \theta / \sqrt{2}$</td>
<td></td>
</tr>
</tbody>
</table>

One of the most striking features of the SM are the gauge theory cancellations between $\gamma$, $Z$ and neutrino exchange graphs at high energies. Within the SM the only non-vanishing couplings in the table are $g_1 = \kappa = 1$ and $f_3 = 2$ for both the photon and the $Z$-exchange graphs. As a result $A^V_{\lambda\bar{\lambda}} = A^Z_{\lambda\bar{\lambda}}$ and the $\beta A^V$ terms in Eq. (11) cancel, except for the difference between photon and $Z$ propagators. Similarly, the $B_{\lambda\bar{\lambda}}$ term in $\tilde{M}_\gamma$ and the $\delta_{\sigma,-1}$ term in $\tilde{M}_Z$ cancel in the high energy limit for all helicity combinations. While the contributions from individual Feynman graphs grow with energy for longitudinally polarized $W$'s, this unacceptable high energy behavior is avoided in the full amplitude due to the cancellations which can be traced to the gauge theory relations between fermion–gauge boson vertices and the TGC’s.

LEP2 will operate close to $W$ pair production threshold and these cancellations are not yet fully operative. For example, at $\sqrt{s} = 190$ GeV one has $\beta = 0.54$, $\beta s/(s - m_Z^2) = 0.70$, and $1/\beta = 1.87$ instead of unity. As a result, the linear combinations of couplings which enter in $\tilde{M}_\gamma$ and $\tilde{M}_Z$ are quite different from their asymptotic forms. In particular the $\gamma^2$ enhancement factors are still small, the $(\pm, \pm)$ and $(0, 0)$ amplitudes are not yet dominated by individual couplings, and interference effects between different TGC are very important.
Table 2 shows that only seven $W^- W^+$ helicity combinations contribute to the $J_0 = 1$ channel and the various $WWV$ couplings enter in as many different combinations. This explains why exactly seven form factors or coupling constants are needed to parameterize the most general $WWV$ vertex. Since we have both $WWZ$ and $WW\gamma$ couplings at our disposal, the most general $J = 1$ amplitudes $M_L = \mathcal{M}(\sigma = -1, \lambda, \bar{\lambda})$ and $M_R = \mathcal{M}(\sigma = +1, \lambda, \bar{\lambda})$ for both left- and right-handed incoming electrons can be parameterized. Turning the argument around one concludes that all 14 helicity amplitudes need to be measured independently for a complete determination of the most general $WW\gamma$ and $WWZ$ vertex.

A first step in this direction is the measurement of the angular distribution of produced $W$'s, $d\sigma/d \cos \theta$. In terms of the reduced amplitudes $\tilde{\mathcal{M}}_{\sigma,\lambda,\bar{\lambda}}$ of (9) this distribution is given by

\[
\frac{d\sigma}{d \cos \theta} = \frac{\pi a^2 \beta}{4s} \sigma = \pm 1 \frac{\sin^2 \theta}{2} \left[ |\tilde{\mathcal{M}}_{\sigma,+1}|^2 + |\tilde{\mathcal{M}}_{\sigma,-1}|^2 + |\tilde{\mathcal{M}}_{\sigma,0}|^2 \right] + \frac{(1 + \sigma \cos \theta)^2}{4} |\tilde{\mathcal{M}}_{\sigma,+0}|^2 + |\tilde{\mathcal{M}}_{\sigma,-0}|^2 \right] + \frac{1}{2} \frac{1}{2(1 + \cos^2 \theta) \sin^2 \theta} \frac{2}{\sin^4 \theta_W} \frac{1}{(1 + \beta^2 - 2\beta \cos \theta)^2}.
\] (12)

Due to the different $d$-function factors amplitudes with different values of $\lambda - \bar{\lambda}$ can be separated in principle. In practice, the additional $\theta$-dependence of the neutrino exchange graphs (the $C_{\lambda\bar{\lambda}}$ terms in Eq. (11)) distorts these angular distributions and leads to contributions from the individual $W^- W^+$ helicity combinations as shown in Fig. 2. In fact, the interference with the $\nu$-exchange graphs can be used to further separate the various $s$-channel helicity amplitudes.

Due to the $V - A$ structure of the $W$-fermion vertices the decay angular distributions of the $W$'s are excellent polarization analyzers and a further separation of the various $W^+ W^-$ helicities can be obtained [3, 6]. These decay distributions are most easily given in the rest frame of the parent $W$. Choose the $e^- e^+ \rightarrow W^- W^+$ scattering plane as the $x - z$ plane with the $z$-axis along the $W^-$ direction and obtain the $W^\pm$ rest frames by boosting along the $z$-axis. In the $W^-$ frame we define the momentum of the decay fermion for $W^- \rightarrow f_1 \bar{f}_2$ as

\[
p'_i = \frac{m_W}{2} \left( 1, \sin \theta_1 \cos \phi_1, \sin \theta_1 \sin \phi_1, \cos \theta_1 \right),
\] (13)

and, similarly, for $W^+ \rightarrow f_3 \bar{f}_4$, the anti-fermion momentum in the $W^+$ frame is given by

\[
p'_4 = \frac{m_W}{2} \left( 1, \sin \theta_2 \cos \phi_2, -\sin \theta_2 \sin \phi_2, -\cos \theta_2 \right),
\] (14)

Thus, $\theta_i = 0$ corresponds to the charged lepton or the down-type (anti)quark being emitted in the direction of the parent $W^\pm$.

Neglecting any fermion masses, the $W^- \rightarrow \ell^- \bar{\nu}$ decay amplitude is given by [3]

\[
\mathcal{M}_D(\lambda) = \frac{e m_W}{\sqrt{2} \sin \theta_W} \ell_3(\theta_1, \phi_1),
\] (15)
Figure 2: Angular distributions $d\sigma/d\cos \theta$ for $e^-e^+ \rightarrow W^-W^+$: SM contributions from fixed $W^-W^+$ helicities $(\lambda\bar{\lambda})$ at $\sqrt{s} = 190$ GeV.

where the angular dependence is contained in the functions

$$ (\ell_-, \ell_0, \ell_+)(\theta_1, \phi_1) = \frac{1}{\sqrt{2}}(1 + \cos \theta_1) e^{-i\phi_1}, -\sin \theta_1, \frac{1}{\sqrt{2}}(1 - \cos \theta_1) e^{i\phi_1} .$$

An analogous expression is obtained for the $W^+$ decay amplitude.

The production and decay amplitudes can easily be combined to obtain the five-fold differential angular distribution for the process $e^-e^+ \rightarrow W^-W^+ \rightarrow f_1 f_2 f_3 f_4$, in the narrow $W$-width approximation [3, 6],

$$ \frac{d^5 \sigma (e^-e^+ \rightarrow W^-W^+ \rightarrow f_1 f_2 f_3 f_4)}{d\cos \theta d\cos \theta_1 d\phi d\cos \theta_2 d\phi_2} = \frac{\beta}{128\pi s} \frac{3}{8\pi} B(W \rightarrow f_1 f_2) B(W \rightarrow f_3 f_4) $$

$$ \times M(\sigma, \lambda, \bar{\lambda}) M^*(\sigma', \lambda', \bar{\lambda}') $$

$$ \times D_{\lambda, \sigma}(\theta, \phi) D_{\bar{\lambda}, \sigma'}(\theta_2, \phi_2 + \pi) .$$

Here the production amplitudes $M(\sigma, \lambda, \bar{\lambda})$ are given in Eq. (9) and the $D_{\lambda, \sigma}$ are given by

$$ D_{\lambda, \sigma}(\theta, \phi) = \ell_{\lambda}(\theta, \phi) \ell_{\sigma}(\theta, \phi) .$$

The information contained in the five-fold differential distribution (17) can be used to isolate different linear combinations of $WWV$ couplings and hence reduce the possibility of cancellations between them. For example, by isolating $W^+W^-$ pairs which are both transversely
polarized (and hence give $1 + \cos^2 \theta$; decay distributions) the combinations $g_1^V + 2\gamma^2 \lambda_V$ are determined which appear in the production amplitudes $\mathcal{M}_{++}$ and $\mathcal{M}_{--}$ (see Table 2). Similarly, longitudinal $W$'s produce a characteristic $\sin^2 \theta$; decay distribution. The isolation of LT+TL and of LL polarizations of the two $W$'s allows independent measurements of the combinations $f_3^V = g_1^V + \kappa \nu + \lambda_V$ and $g_1^V + 2\gamma^2 \kappa_V$, respectively, and thus the three $C$- and $P$-conserving anomalous couplings\textsuperscript{10} may be isolated.

Additional information is obtained from the azimuthal angle distributions of the decay products. A nontrivial azimuthal angle dependence arises from the interference between helicity amplitudes for different $W^+$ or different $W^-$ polarizations. The large $\mathcal{M}_{++}$ and $\mathcal{M}_{--}$ amplitudes, which arise solely from neutrino exchange, can thus be put to use: interference with these large amplitudes can amplify the effects of anomalous couplings.

The observation of azimuthal angular dependence and correlations is particularly important for the study of $CP$-violating effects in $W^-W^+$ production [3, 43]. The methods suggested in section 4 below for TGC determination from data can all be used for this purpose, and the reader is referred to the literature for details of procedures using density matrix [43] and optimal observable [44] analyses. Similarly, the study of rescattering effects between the produced $W$ pairs, i.e. the presence of nontrivial phases in the production amplitudes, relies on the interference with the phase factors introduced by the azimuthal angle dependence of the decay amplitudes. We do not explicitly discuss these techniques here but rather refer to the literature [3, 45].

<table>
<thead>
<tr>
<th>WW decay channel</th>
<th>Decay fraction</th>
<th>Available angular information</th>
</tr>
</thead>
<tbody>
<tr>
<td>$jjl\nu$</td>
<td>$l = e$: 14%</td>
<td>$l = \mu$: 14%</td>
</tr>
<tr>
<td></td>
<td>$l = \tau$: 14%</td>
<td>$\cos \theta$ $(\cos \theta, \phi_1)$ $(\cos \theta, \phi_2)_{\text{folded}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$l = j$: 49%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\cos \theta$ $(\cos \theta, \phi_1)<em>{\text{folded}}$ $(\cos \theta, \phi_2)</em>{\text{folded}}$</td>
</tr>
<tr>
<td>$jjjj$</td>
<td>9%</td>
<td>$\cos \theta$ $(\cos \theta_1, \phi_1)$ $(\cos \theta_2, \phi_2)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 solutions</td>
</tr>
</tbody>
</table>

Table 3: Availability of angular information in different WW final states. The production angle is denoted by $\theta$ and $(\theta_{i,j}, \phi_{i,j})$ denote decay angles for $W \rightarrow$ (leptons, jets) respectively. $(\cos \theta_j, \phi_j)_{\text{folded}}$ implies the ambiguity $\cos \theta_j \leftrightarrow -\cos \theta_j$, $\phi_j \leftrightarrow \phi_j + \pi$ incurred by the inability to distinguish quark from antiquark jets.

The application of (17) to experimental data must take account of some restrictions in the ability to determine the angles involved: in the case of hadronic $W$ decays, and in the absence of any quark charge or flavour tagging procedure, the fermion and anti-fermion cannot be distinguished; also, in the case where both $W$'s decay leptonically, a quadratic ambiguity is

\textsuperscript{10}Note however that if relations among TGC such as those in eq. (4) are relaxed, it will not be easy to distinguish $\kappa_1$ from $\kappa_2$ (or $\lambda_1$ from $\lambda_2$) with unpolarized beams, since these both feed the same helicity amplitudes in table 2.
encountered. The ambiguities in each of the three $WW$ final states $jj\ell\nu$, $jjjj$ and $\ell\ell\ell\nu$, where $j$ represents the jet fragmentation of a quark or antiquark and $(l\nu)$ the products of $W$ decay into lepton-antilepton, are summarized in table 3.

### 3.2 Four-fermion production and non-standard TGC

Most studies of TGC so far have been made with zero width simulated data and with an analysis program based on the same assumptions. This procedure might neglect some important effects, however, and the corresponding physics issues will be discussed in this subsection. These are the influence of a finite $W$-width, of background diagrams, i.e. graphs other than the three $W$-pair diagrams of Fig. 1, and the influence of radiative corrections (RC) in particular the dominant QED initial state radiation (ISR).

At the moment there are many Monte Carlo (MC) programs for four fermion production, but only two of them can at present study the above issues, namely ERATO\cite{46} and EXCALIBUR\cite{47,48}. For a detailed description we refer to the WW event generator report, but we make a few comments here. Although the programs can study non-standard TGC effects\cite{46,49} for all the channels of Table 3, we will only consider the $jj\ell\nu$ case in the following. More specifically we will study $e^-\bar{\nu}_e u\bar{d}$ or $\mu^-\bar{\nu}_\mu u\bar{d}$ final states. The amplitude for these final states consists of 20 and 10 diagrams, respectively, of which 3 are the $W$-pair diagrams of Fig. 1. Since the four fermions are assumed to be massless in the calculations, cuts have to be applied to avoid singularities in the phase space. Experimental cuts usually have this effect as well. In the case of only three diagrams such cuts are not required. ISR is incorporated following the prescription of Ref. \cite{50}. In table 4, we list a number of differential cross-sections which have been calculated,

<table>
<thead>
<tr>
<th>Standard Model</th>
<th>non-standard TGC</th>
<th>physical assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{SM,on}$</td>
<td>$\sigma_{AN,on}$</td>
<td>$\Gamma_W = 0$</td>
</tr>
<tr>
<td>$\sigma_{SM,off}$</td>
<td>$\sigma_{AN,off}$</td>
<td>3 diagrams</td>
</tr>
<tr>
<td>$\sigma_{SM,off,cuts}$</td>
<td>$\sigma_{AN,off,cuts}$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{SM,all}$</td>
<td>$\sigma_{AN,all}$</td>
<td>20 diagrams, cuts</td>
</tr>
<tr>
<td>$\sigma_{SM,ISR}$</td>
<td>$\sigma_{AN,ISR}$</td>
<td>3 diagrams, ISR</td>
</tr>
<tr>
<td>$\sigma_{SM,all,ISR}$</td>
<td>$\sigma_{AN,all,ISR}$</td>
<td>20 diagrams, cuts, ISR</td>
</tr>
</tbody>
</table>

Table 4: Cross sections and the corresponding physical assumptions under which they have been calculated. The subscripts $SM$, $AN$, $on$, $off$ refer to Standard Model, non-standard TGC, on-shell and off-shell, respectively.

and correspond to different physical assumptions. The first column refers to the SM and the second one to a non-standard TGC case (usually with only one of the CP-conserving couplings being different from its SM value). For the cross-sections labeled $\sigma_{cuts}$, cuts are applied mainly
to lepton and quark energies and angles in the laboratory frame:

\[ E_{e-ud} > 20 \text{ GeV} , \quad |\cos \theta_{e-ud}| < 0.9 , \quad \cos \theta_{u-d} < 0.9 , \quad m_{u\bar{d}} > 10 \text{ GeV} . \]  

The calculations were performed with input parameters as prescribed in the WW cross-section Working Group chapter. Results from the two programs agree within the MC errors. The particular case of \( d\sigma_{AN,\text{off}}/d\cos \theta \) (for the full phase space) has also been calculated by M. Bilenky in a semi-analytical method and full agreement with EXCALIBUR has been obtained for all CP conserving TGC.

![Figure 3: Ratios of differential cross-sections at various levels of the simulation of the 4-fermion processes, (a) \( R_1 = \sigma_{SM,\text{off}}/\sigma_{SM,\text{on}} \), (b) \( R_2 = \sigma_{SM,\text{ISR}}/\sigma_{SM,\text{off}} \) and (c) \( R_3 = \sigma_{SM,\text{all}}/\sigma_{SM,\text{off,cut}} \).](image)

Different physical mechanisms could influence the angular distribution of the produced \( W \)s and thus simulate the effect of non-standard TGC. Typical examples are shown in Fig. 3, namely the effect of a finite \( W \) width, of ISR and of background graphs on \( d\sigma/d\cos \theta \). ISR, for instance, lowers the available \( \sqrt{s} \) of the event and thus reduces the forward peak of the \( W^-W^+ \) production cross-section. In addition, the recoil of the \( W^-W^+ \) system against the emitted photon further smears out the \( W \) angular distribution [51]. A similar effect, relative depletion of forward as compared to backward produced \( W^- \)s can also arise from negative TGC parameters. This is evident from Fig. 4, where ratios of a non-standard \( d\sigma/d\cos \theta \) and SM cross-sections are presented, both having been calculated under the same physical assumptions. Fig. 4(b) demonstrates the quantitative importance of this phenomenon. For final state electrons the background graphs, if not included in the analysis, could mimic a \( \delta_Z \) of the order of \(-0.2\). While the shape of the angular distribution \( d\sigma/d\cos \theta \) for negative TGC parameters shows a trend similar to that induced by ISR, finite width or background graph effects, the normalization of the cross-section might provide some discriminating power, as do the decay angular distributions. Another very important message coming from Fig. 4 is that the sensitivity to the TGC remains the same at the different levels of the simulation (from on-shell \( W \)s up to four-fermion production). Conversely, the influence of the various physics effects on production
and decay angular distributions is largely independent of whether or not non-standard TGC are present.

We conclude that it is clearly important to account for and to correct the effects considered above in experimental analyses. We return to the effects of ISR and finite W width in Section 5.2 where their neglect in TGC determination at LEP2 is quantified. In Section 6.2 we indicate how they contribute to the overall bias in a typical simulated TGC determination.

Figure 4: Ratio of anomalous to SM differential cross-section. (a) $\sigma_{AN,off}/\sigma_{SM,off}$ (solid line), $\sigma_{AN,ISR}/\sigma_{SM,ISR}$ (dotted line), $\sigma_{AN,all}/\sigma_{SM,all}$ (dashed line), and $\sigma_{AN,all,ISR}/\sigma_{SM,all,ISR}$ (dashed-dotted line) for $y_\gamma = +0.1$. (b) $\sigma_{AN,off}/\sigma_{SM,off}$ (solid line), $\sigma_{AN,all}/\sigma_{SM,all}$ for muons (dashed-dotted line) and electrons (dashed line) for $\alpha_W = 0.2$, $\delta_Z = 0.2$, $\delta_Z = -0.2$, $\alpha_W = -0.2$ (bottom-top) and $\sigma_{SM,all}/\sigma_{SM,off}$ for muons (squares) and electrons (circles).

4 Statistical techniques for TGC determination$^{11}$

Three different methods have thus far been proposed for the determination of TGCs at LEP2, — the density matrix method, the maximum likelihood method and the method of optimal observables. These methods are outlined in the following subsections and their application to common simulated datasets is compared. In devising these methods, two considerations have been borne in mind: first, — as will be elaborated in the next section — that it is advantageous to use as much of the available angular data for each $WW$ event as possible; second, that the

$^{11}$The experimental sections, 4–9, have been coordinated by R. L. Sekulin
expected LEP2 data (a total of \( \approx 8000 \) events for an integrated luminosity of \( 500 \text{pb}^{-1} \) at 190 GeV) will not be sufficient, for instance, to bin the data into the five angular variables appearing in the \( WW \) production and decay distribution (17) and subsequently to perform a \( \chi^2 \) fit. The studies reported in this section have been performed assuming that the final state momenta of the four partons from \( W^- \) and \( W^+ \) decay have been successfully reconstructed from the data; the practical difficulties of doing this are discussed in section 5.

### 4.1 Density matrix method

In this method, TGC parameters are extracted from the data in a two-stage analysis. First, experimental density matrix elements and their statistical errors are determined from the angular distribution (17) in bins of \( \cos \theta \); then the predictions of different theoretical models are fitted to the resulting distributions using a \( \chi^2 \) minimization method. The joint \( WW \) helicity density matrix elements \( \rho_{\lambda \lambda'} \) are defined from (17) as the sums \( \sigma \mathcal{M}(\sigma, \lambda, \lambda'), \mathcal{M}^*(\sigma, \lambda', \lambda') \) of bilinear products of production amplitudes and the dependence of the cross-section on the TGC parameters is fully contained in the complete density matrix thus evaluated. Similarly, by integrating over the observables of one \( W \), single \( W \) density matrix elements \( \rho_{\lambda \lambda'} \) and \( \rho_{\lambda' \lambda} \) can be defined.

The density matrix elements can be calculated in two ways:

- Using the orthogonality properties of the \( W \) decay functions \( D_{\lambda \lambda'} \) and \( D_{\lambda' \lambda} \) in (17), density matrix elements can be extracted by integrating over the \( W \) decay angles with suitable projection operators. Thus, unnormalized density matrix elements of the leptonically decaying \( W \) in \( jj\ell\nu \) events can be found from the lepton spectrum as

\[
\rho_{\lambda \lambda'} \frac{d\sigma(e^+e^- \to W^+W^-)}{d\cos\theta} = \frac{1}{B_{Wj\ell\nu}} \frac{d\sigma(e^+e^- \to W^+W^- \to jj\ell\nu)}{d\cos\theta d\cos\theta_l d\phi_l} \Lambda_{\lambda \lambda'}(\theta_l, \phi_l) d\cos\theta_l d\phi_l
\]

(20)

where \( B_{Wj\ell\nu} \) is the branching ratio for the \( jj\ell\nu \) channel, the angular variables are as defined in (13), (14), with the decay angles and helicity indices now referring to the leptonically decaying \( W \). Expressions for the normalized operators \( \Lambda_{\lambda \lambda'} \) are given in [52]; for example, \( \Lambda_{00} = 2 - 5 \cos^2\theta \) projects out the longitudinal cross-section \( \rho_{00} \frac{d\sigma}{d\cos\theta} \) of the leptonically decaying \( W \).

- In the second method [6], the production and decay angular distribution is expressed in terms of the density matrix elements and, in each bin of \( \cos \theta \), they are determined using a maximum likelihood fit to the distribution of the decay angles.

Fig 5 shows some of the density matrix elements calculated from a sample of simulated events by the two methods as a function of \( \cos \theta \) and fitted to the prediction of the Standard Model. It can be seen that there is good agreement between the density matrix elements as calculated by the two methods, and with the fit to the Standard Model.
Figure 5: $\cos \theta$ dependence of density matrix elements $\rho_{11}$ and $\rho_{-10}$ for a sample of 2930 simulated $e^+e^- \to W^+W^-$ events at 190 GeV, calculated using the projection method (full circles) and the maximum likelihood method (triangles) and compared with the prediction of the Standard Model (fitted curve).

4.2 Maximum likelihood method

In this method, the distribution of some or all of the observed angular data is used directly in an unbinned maximum likelihood fit [7], in which parameters $P$, denoting one or more of the Lagrangian contributions (4), are varied to maximize the quantity

$$ \ln \mathcal{L}_{ML} = \sum_i \ln p(\Omega_i, \mathbf{P}) - N_{obs} \ln \int p(\Omega, \mathbf{P}) d\Omega, $$

(21)

where the sum is over events in the sample, $\Omega_i$ represents, for the i’th event, the angular information being used, $p(\Omega, \mathbf{P})$ is derived from the cross-section (17), $N_{obs}$ is the observed number of events, and the integral is over the whole of phase space. Many of the results shown here have been obtained using the method of extended maximum likelihood, in which the absolute prediction for the magnitude of the cross-section is also tested [53]:

$$ \ln \mathcal{L}_{EML} = \sum_i \ln p(\Omega_i, \mathbf{P}) - N(\mathbf{P}), $$

(22)

where, for integrated luminosity $L$, the predicted number of events $N(\mathbf{P})$ in the sample is
\[ I = \frac{d\sigma}{d\Omega}(\Omega, P) d\Omega. \]

It may be noted that, while in the evaluation of \( N(P) \) in (22) the absolute normalization of the cross-section must be used (as given in (17)), constant factors such as the flux factor may be omitted from the unnormalized expression \( p(\Omega, P) d\Omega \) in (21). Furthermore, since for any event the probability \( p \) is proportional to the product of a phase space factor, which is independent of \( P \), and a matrix element squared, \(| \mathcal{M} |^2\), which contains the dependence on the TGC parameters, the sums over events in (21) and (22) may be replaced by \( \ln | \mathcal{M} |^2(\Omega_\ast, P) \), and the maximum of the likelihood function will be unchanged. While this replacement is trivial for the 2-body cross-section given by (12), it is essential in the evaluation of the log-likelihood sum when the reaction is analyzed in terms of the 4-fermion processes, in which the phase space factor is different for every event.

While the maximum likelihood method is able to use all the available angular information for each event, it has the disadvantage compared with a \( \chi^2 \) fit of being unable to provide a goodness of fit criterion. Nonetheless, the goodness of fit of a hypothesis represented by the likelihood function \( \mathcal{L}_1(p) \) can be compared with that of \( \mathcal{L}_2(P) \) if the parameters \( p \) of \( \mathcal{L}_1 \) satisfy the condition \( p \in P \). Then the quantity \(-2 \ln (\mathcal{L}_1^{\text{max}}/\mathcal{L}_2^{\text{max}})\), derived from the ratio of their likelihood functions, has a \( \chi^2 \) distribution [54]. This property has been applied to event samples generated with non-SM values of one TGC, \( P_1 \), and used to distinguish this hypothesis from a wrong one, when a different TGC, \( P_2 \), is fitted to the data. — In general, a fit of \( P_2 \) produces a result differing significantly from the SM value. Fig 6 shows the results of applying this test to the correct and wrong models in two alternative ways. In both cases, \( \mathcal{L}_1 \) is taken as the likelihood function when \( P_1 \) varies; in the “same family” case (a), \( \mathcal{L}_2 \) is the likelihood function when both \( P_1 \) and \( P_2 \) vary, while, in (b), \( \mathcal{L}_2 \) describes a “composite” hypothesis,

\[
\mathcal{L}_2(P_1, P_2; \beta) = \prod_{i=1}^{N} [\beta p(P_1) + (1 - \beta) p(P_2)],
\]

where \( \beta \) is the probability that model 1, represented by the probability density function \( p(P_1) \), is correct, and \( P_1, P_2 \) and \( \beta \) vary in the fit. It can be seen that a simple comparison between the values of these probabilities indicates the correct model for the majority of the cases. In addition, the absolute probability value indicates the goodness of the fit.

### 4.3 Optimal observables method

Optimal observables are quantities with maximal sensitivity \(^{12}\) to the unknown coupling parameters [44, 56]. To construct them, a particular set of couplings \( P_i \) is chosen which are zero at Born level in the Standard Model (for instance, the TGCs defined by (4)). Then, recalling

\(^{12}\)This method has been used to search for CP violation in \( \tau^+\tau^- \) production at LEP1, with a clear increase of sensitivity [55].
Figure 6: Hypothesis testing using a) the “same family” and b) the “composite hypothesis” methods, for data sets of about 2500 $jjl\nu$ events generated with TGC values deviating from the SM values by one to five times the expected LEP2 precisions.

that the amplitudes for the four-fermion process are linear in the couplings, the differential cross-section may be written

$$\frac{d\sigma}{d\Omega} = S_0(\Omega) + \sum_i S_{1,i}(\Omega) P_i + \sum_{i,j} S_{2,ij}(\Omega) P_i P_j,$$

where $\Omega$ represents the kinematic variables as before. Kinematic ambiguities, such as those described in table 3, can readily be incorporated into (24). The distributions of the functions

$$O_i(\Omega) = \frac{S_{1,i}(\Omega)}{S_0(\Omega)}$$

are measured, and their mean values $\langle O_i \rangle$ evaluated\(^{13}\). An example is shown in fig 7. To first order in the $P_i$, the mean values $\langle O_i \rangle$ are given by

$$\langle O_i \rangle = \langle O_i \rangle_0 + \sum_j c_{ij} P_j,$$

from which the couplings $P_j$ can be extracted because $\langle O_i \rangle_0$ and $c_{ij}$ are calculable given (24) and (25). From the distributions of the $O_i$, the statistical errors on their mean values can be

\(^{13}\)The functions $O_i(\Omega)$ for the TGC parameters used in [3] are available as a FORTRAN routine [44].

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evaluated, the observables having been constructed to minimize the induced errors on the $P_j$. If the linear expansion in the couplings is good, the method has the same statistical sensitivity as a maximum likelihood fit. It can also be extended to incorporate total cross-section information in a manner analogous to the use of the extended maximum likelihood method discussed in the previous section.

**Optimal Observables - 190 GeV**

![Graph showing distribution of $O_{\alpha W\phi}$](image)

**Figure 7:** Distribution of $O_{\alpha W\phi}(\cos \theta, (\cos \theta_1, \phi_1), (\cos \theta_2, \phi_2))$ for a large sample (50000) of simulated $e^+e^- \rightarrow W^+W^-$ events at 190 GeV. The experimentally determined mean value is to be compared with the expectation value of this observable in the SM, $\alpha_{W\phi} = 0$, used to generate the events.

### 4.4 Comparison of methods

In this section a comparison is presented of fits of the TGCs $\alpha_{W\phi}$, $\alpha_{B\phi}$, and $\alpha_W$, defined in (4), to common datasets generated with the PYTHIA[57] Monte Carlo simulation program.

We precede this by mentioning the results of a comparison of the use of the maximum and extended maximum likelihood (ML and EML) methods, in which both of these methods were used in fits of the three TGCs to a large sample (50000) of events using first only the $W$ production angle, and then the complete angular information (production and decay angles). The extra information contained in the EML method gave a substantial improvement (10%) in precision only in one case — the fit of $\alpha_{B\phi}$, generally the least well determined parameter, to the production angular distribution. In the other fits the improvement was only $\sim 1\%$. Similar conclusions have been obtained when applying the optimal observables method with and without total cross-section information.

In the comparison of the density matrix (DM), EML and optimal observables (OO) methods, the three analyses were applied to datasets at 175 and 190 GeV simulating both the expected
LEP2 statistics ($\approx 2000$ events) and much larger statistics ($50000$ events). Sample results are given in fig 8, in which precisions obtained using the three methods in 1- and 2-parameter fits to the large dataset at $190$ GeV are plotted. In all cases, the precisions obtained using the three methods are very similar when the same angular data is used in the fit. This can be seen in the figure, where the precisions from the EML and OO methods, both of which used angular data $\cos \theta_i$, $(\cos \theta_{i1}, \phi_i)$ and $(\cos \theta_{i2}, \phi_i)$ folded, are almost identical. The DM results shown used the differential cross-section, $\frac{d\sigma}{d\cos \theta}$, density matrix elements $\rho_{00}, \rho_{1-1}, \rho_{10}$ and $\rho_{-10}$ of the leptonically decaying $W$, and the part symmetric in both polar decay angles of the transverse element $\rho_{TT} \equiv \rho_{11,11} + \rho_{-1-1,-1-1} + \rho_{11,-1-1} + \rho_{-1-1,11}$ of the joint $WW$ density matrix, representing somewhat less than the full 35 (CP-conserving) elements of the full joint density matrix. (Other density matrix elements can in principle be included in the analysis).

**Comparison of methods**

![Comparison of methods graph]

Figure 8: Comparison of TGC fits to a large sample of simulated events at 190 GeV using the density matrix (DM), maximum likelihood (EML) and optimal observables (OO) methods.

a): 1 s.d. precisions in 1-parameter fits to $\alpha_w$, $\alpha_{W\phi}$ and $\alpha_{B\phi}$. b): 95% confidence contours in 2-parameter fits to $(\alpha_{W\phi}, \alpha_{B\phi})$.

A difference between the EML or DM analyses and the OO analysis can be seen in the 2-parameter fit shown, where a second allowed region, remote from the SM region ($\alpha_{W\phi} = 0, \alpha_{B\phi} = 0$) where the events were generated, is seen by the EML and DM methods. This effect is discussed in detail in ref. [7], where it is shown to arise naturally from the amplitude structure of $WW$ production, and in particular from the fact that the helicity amplitudes are linear in the TGCs. It is not seen in the OO results, because here the cross-section (24) has
been linearized with respect to the TGCs about their SM values\textsuperscript{14}.

In considering possible extensions to the analyses, two comments may be made. First, the EML and OO methods could readily be used in a 4-fermion treatment by replacement of the matrix elements. The DM method does not lend itself to this adaptation, as the form (17) used in the projection of the density matrix elements assumes $J = 1$ for the two final state $f \bar{f}$ pairs. Second, all three methods can in principle be adapted to the analysis of events with the experimental and other effects discussed later in this chapter; however, we have not made an assessment of the relative ease with which this can be done for the different methods.

With the above points borne in mind, we can recommend all three methods for consideration in the analysis of LEP2 data. The studies reported in the following sections have, except where otherwise indicated, used ML or EML fits to obtain the results shown.

\section{Precision of TGC determination at LEP2: generator level studies}

In this section, the precisions to be expected in TGC determination from the anticipated LEP2 integrated luminosity are summarized and an estimate of the biases and systematic errors accessible at generator level is given.

\subsection{TGC precisions in fits to simulated events}

Precisions in TGCs obtained from 1-parameter fits to simulated $e^+e^- \rightarrow W^+W^-$ events at 176 and 190 GeV are shown in table 5, and confidence limits in the planes of two of the three possible combinations of two of the parameters in eq. (4) are shown in fig 9. Results are shown using various combinations of the angular data appropriate to each of the three final states $jj\ell\nu$, $jjjj$ and $\ell\ell\nu$, as indicated in table 3, as well as to the “ideal” case without angular ambiguities. For the first two channels (and for the “ideal” analysis), 1960 (2600) events were fitted at 176 (190) GeV; for the $\ell\ell\nu$ channel, 280 (370) events were used. These figures emulate the statistics anticipated from an integrated luminosity of 500pb\textsuperscript{−1} after experimental efficiency cuts of $\sim$ 95\%, 60\% and 95\% for the three channels respectively, and excluding leptonic decays into $\tau\nu$. The extended maximum likelihood method was used in the fits, and the events were generated and analyzed in the narrow $W$ width approximation and without initial state radiation (ISR). In the analysis, the generated values of parton momenta were used, so that no account has been taken of the subsequent quark fragmentation nor of possible experimental effects. No kinematic cuts have been made on the data. The analysis reported here is therefore to be considered as an idealized one; the implications of the additional effects mentioned above are considered in detail in subsequent sections.

\textsuperscript{14} An extension of the OO method to incorporate second order terms in the parameters is under development.
Several conclusions may be drawn from inspection of the table and figure. As anticipated by the discussion of section 3, substantial gains in precision are achievable by running at higher energy. Also, use of as much as possible of the available angular data serves to increase the precision and, in 2-parameter fits, to reduce the (quite pronounced) correlations between the fitted TGCs. The use of the $jjjj$ channel, even with the angular ambiguities incurred by the inability to distinguish quark from antiquark jets, can be seen to provide a modest but worthwhile improvement in the overall precision attainable. Finally, the occurrence of a second region in the $(\alpha_{W\phi}, \alpha_{B\phi})$ plane, remote from the Standard Model region $(0, 0)$ at which the events were generated but acceptable at the chosen significance level, has already been noted in the previous section.

<table>
<thead>
<tr>
<th>Model</th>
<th>Channel</th>
<th>Angular data used</th>
<th>176 GeV</th>
<th>190 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{B\phi}$</td>
<td>$jjlv$</td>
<td>$\cos \theta$ $\cos \theta, (\cos \theta_1, \phi_1)$</td>
<td>0.222</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\cos \theta, (\cos \theta_1, \phi_1)$</td>
<td>0.182</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\cos \theta, (\cos \theta_1, \phi_1), (\cos \theta_1, \phi_1)_{\text{folded}}$</td>
<td>0.159</td>
<td>0.080</td>
</tr>
<tr>
<td>$jjjj$</td>
<td>$\cos \theta, (\cos \theta_1, \phi_1)$</td>
<td>0.376</td>
<td>0.149</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\cos \theta, (\cos \theta_1, \phi_1), (\cos \theta_1, \phi_1)_{\text{folded}}$</td>
<td>0.123</td>
<td>0.328</td>
</tr>
<tr>
<td>$lvlv$</td>
<td>$\cos \theta, (\cos \theta_1, \phi_1), (\cos \theta_2, \phi_2)$, 2 solutions</td>
<td>0.099</td>
<td>0.061</td>
<td></td>
</tr>
<tr>
<td>Ideal</td>
<td></td>
<td>$\cos \theta, (\cos \theta_1, \phi_1), (\cos \theta_2, \phi_2)$</td>
<td>0.118</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Table 5: 1 s.d. errors in fits of $\alpha_{B\phi}$, $\alpha_{W\phi}$ and $\alpha_W$ to various combinations of the angular data at 176 and 190 GeV. The simulated data corresponds to integrated luminosity of 500pb$^{-1}$. Details of the data samples are given in the text.

In a first step towards a more realistic simulation of the data, some of the fits described above have been repeated using calculations corresponding to 4-fermion rather than $WW$ production both in event generation and analysis. In so doing, contributions are included from the complete set of relevant diagrams and the finite $W$ width effects ignored in the previous analysis are taken...
Figure 9: 95% confidence limits in the planes of 2-parameter TGC fits at 176 and 190 GeV, using various combinations of angular data. a), b), c), d): Fits to \((\alpha_{WF}, \alpha_{B})\); e), f), g), h): Fits to \((\alpha_{WF}, \alpha_{W})\). In the legend, the notation \(\Theta_{l,j}\) implies a pair of decay angles \((\theta_{l,j}, \phi_{l,j})\) for \(W \rightarrow (\text{leptons, jets})\) respectively, and \(|\Theta_{j}|\) implies the ambiguity \(\cos \theta_{j} \leftrightarrow -\cos \theta_{j}, \phi_{j} \leftrightarrow \phi_{j} + \pi\) incurred by the inability to distinguish quark from antiquark jets. In plots a), b), e), f), the angular data simulates channel \(jj\ell \nu\) (and the “ideal” case, with no ambiguities); in c), d), g), h), it simulates channel \(jjjj\).
into account. Using events generated with the ERATO [46] program corresponding to the expected statistics at 175 and 190 GeV, similar precisions to those shown above are obtained in fits of $\alpha_W$ and $\alpha_W$ to angular data $\cos \theta$, $(\cos \theta_1, \phi_1)$ and $(\cos \theta_2, \phi_2)$ folded\(^{15}\). In addition, in fits to a sample of $j j \ell \nu$ events generated at 161 GeV corresponding to an integrated luminosity of 100pb\(^{-1}\) (as suggested for the determination of the $W$ mass from its threshold excitation [58]), 1 s.d. precisions of 0.18 and 0.43 were obtained in fits of $\alpha_W$ and $\alpha_W$ respectively. It is interesting to note that these values compare well with current experimental limits [25, 26], implying that TGC measurements from this exposure may also be of interest. This conclusion, however, remains to be tested when backgrounds and other experimental effects are included.

5.2 Biases and systematic errors in TGC determination calculable at generator level

It was pointed out in the previous section that the analyses presented there are idealized, in the sense that effects due to finite $W$ width (unless a 4-fermion calculation is used), ISR, QCD and experimental reconstruction have been ignored. In this section, we consider the biases introduced in TGC determinations, first, if events generated with a realistic $W$ mass distribution are nonetheless analyzed in the narrow width approximation, and, second, if ISR effects are also present, but ignored in the analysis. The discussion of the overall bias to be expected in TGC determination is pursued in the next section, where biases arising due to event selection and reconstruction are added to those discussed here. The systematic errors incurred both in the assessment of these biases and from other sources calculable at generator level are also estimated in this section.

Figs. 10a) and b) show the effects of ignoring finite $W$ width and ISR in the analysis of events generated with these effects included. Results are shown for several different generators, all operating in $e^+e^- \rightarrow W^+W^-$ (CC03) mode. It can be seen, first, that the bias incurred by neglect of ISR is greater than that from neglect of $W$ width effects, second, that the biases are smaller when a fit involving more angular data is used, and, third (from b), that the biases are different for different values of a typical TGC parameter. Finally, we note that the overall bias is $\lesssim$ the statistical error expected from LEP2 data.

The systematic errors arising from these and other sources calculable at generator level are summarized, using a particular TGC fit as an example, in table 6\(^{16}\). The first three entries come from the effects discussed above, the next two represent two different ways of expressing the uncertainty in the other electroweak parameters which are important in the evaluation of

\(^{15}\)A computational point may be made here: in the evaluation of the differential and total cross-sections needed in the likelihood expression (22), time may be saved by noting that, since the amplitudes for the process $e^+e^- \rightarrow f_1f_2f_3f_4$ (or $e^+e^- \rightarrow W^+W^-$) are linear in the TGCs, an exact parametrization of the cross-section dependence on any one TGC may be found from a quadratic fit to its values for any three values of the TGC parameter. This procedure can be extended in an obvious way to fits of two or more parameters.

\(^{16}\)The magnitude of some of these errors, in particular those arising from finite $W$ width and ISR effects, depend on the angular data used in the fit, (c.f. fig 10).
Figure 10: Effect of ignoring finite $W$ width and ISR in TGC fits. a): Results of fits of $\alpha_{W,\phi}$ to events generated with SM parameters at three energies using various generators. Left-hand plots: fit to $\cos \theta$ only; Right-hand plots: fit to $\cos \theta$, $(\cos \theta_1, \phi_1)$, $(\cos \theta_2, \phi_2)$. b): as a), for EXCALIBUR events at 190 GeV, using $\cos \theta$, $(\cos \theta_1, \phi_1)$, $(\cos \theta_2, \phi_2)$, as a function of $\alpha_{W,\phi}$. The legend for both plots is shown on b).

the matrix element, and the final pair represent two independent uncertainties coming from machine and detector considerations. In any analysis which does not compare total cross-section predictions with the observed data, the second and last entries will not contribute to the overall uncertainty. It can be seen that, even when all the relevant entries are added in quadrature, the total is small compared with the statistical precision expected from LEP2 data, and we expect the larger component of the systematic error to come from uncertainties in the experimental effects considered in the next sections.

In addition to the effects considered above, it is legitimate to ask whether colour recombination effects among the two $W$s could affect TGC measurements in the $jjjj$ channel. It has recently been advocated that such effects may produce a shift of up to 400 MeV in $M_W$ [59]. Therefore, by analogy with the effects of ISR, it may produce a bias in TGC measurements which would need to be accounted for, and, if not understood, would have an associated systematic error. However, a preliminary study [60] has indicated that the $W$ production angular distribution, reconstructed from the hadronization products of generated $jjjj$ events, is little affected by application of the colour recombination models of ref [59], and hence that it is unlikely that the shift in TGC values determined from the data in this channel will be significant.
Table 6: Systematic errors from various sources incurred in fits of \(\alpha_{W\phi}\) to angular data \(\cos \theta, (\cos \theta_l, \phi_l), (\cos \theta_j, \phi_j)\) folded at 190 GeV. The 1 s.d. statistical precision estimate for this fit from LEP2 data (c.f. table 5) is ±0.022.

compared to the expected statistical error.

6 Analysis of the \(jj e\nu\) and \(jj \mu\nu\) final states

In the following we address some of the experimental aspects of the analysis of the \(e^+e^- \rightarrow W^+W^- \rightarrow jj e\nu\) channel. In this section, we concentrate on the muon and electron channels, these being the cleanest and very similar in many respects. The tau channel is considered separately in the following section. For simplicity, the data are analyzed in terms of the five angles describing \(WW\) production and decay, by analogy with the generator-level analysis reported in section 5.1. In its extension to a four-fermion treatment, also described in that section, the effect of the experimental selection and reconstruction procedures are expected to be the same.

In section 6.1 we describe the efficiencies and purities obtained after the application of typical selection criteria and of kinematic constraints to the events. In the process of reconstructing and analyzing \(jj e\nu\) events, there are many experimental effects which can potentially bias the angular distributions, and hence the fitted values of TGC parameters. The scale of such effects is estimated in section 6.2, and in section 6.3 we discuss briefly some methods proposed to allow for them in the analysis. The numbers presented result from a comparison of the work of several different groups and should be regarded as broadly typical of the four LEP experiments.

6.1 Event selection, kinematic reconstruction and residual background

The \(jj e\nu\) event selections used typically demand the following:
- that the event contains a minimum number, typically five or six, of charged track clusters;
- that there is an identified electron or muon, or alternatively a high energy isolated track;
- that the lepton has a momentum greater than its kinematic minimum, $\sim 20$ GeV;
- that the lepton be isolated, by requiring low activity in a cone around the track (typically that the energy deposited in a cone of 100-200 mrad be less than 1-2 GeV).

The effect of these cuts corresponds approximately to a fiducial cut in the centre-of-mass polar angle of the lepton of $|\cos \theta_{\text{lepton}}| < 0.95$. The acceptance for jets, which have some angular size, extends further but with falling efficiency. These numbers vary for specific detectors.

The non-lepton system is then split into two (or more) jets using a conventional jet-finding algorithm. The following kinematic constraints [61] can then be applied to impose energy and momentum conservation, and to improve the measurements using the fact that the system is overconstrained:

1C fit:  \[ E = E_{\text{cm}}, \quad \vec{p} = 0, \quad m_\nu = 0; \]

3C fit:  In addition to 1C, $M_{\text{reconstructed}} = M_W$ for both $W$ candidates;

3C' fit: In addition to 1C, $M_{\text{reconstructed}}$ for both $W$ candidates is constrained to a central value of $M_W$ but is allowed to vary approximately within the $W$ width\textsuperscript{17}.

In the above, $m_\nu$ is the neutrino mass and $M_W$ the $W$ mass. A $\chi^2$ probability cut, typically of 0.1-1%, is applied to the constrained fit result. Typical efficiencies after these stages are shown in table 7 for centre-of-mass energies $\sqrt{s} = 175$ and 192 GeV. The main loss is due to geometrical acceptance and lepton identification in the basic selection. The kinematic fits themselves are of the order of 90% efficient for such a probability cut.

The background estimation was made using event samples, simulated with PYTHIA, of the final states $WW$ (with neither of the bosons decaying to an electron or a muon), $Z\gamma$, $ZZ$ and $Zee$. Also, contamination from $\gamma\gamma$ events, generated with TWOGAM [62], were studied. Backgrounds from the last two channels were found to be negligible; those from the other final states are summarized in table 7. Contributions from the non-resonant graphs leading to the $jj\ell\nu$ final state and containing TGCs have also been studied. It is found that, taken in isolation and ignoring interferences, they are rejected by the selection procedure. The main contribution to the $WW$ background comes from events where one of the $W$s decays into a tau and then into an electron or muon. Although this channel is sensitive to the TGCs, it will be seen in section 6.2 that the inclusion of such events into the analysis does not significantly bias the result.

\textsuperscript{17}This is achieved by including either Gaussian approximations or true Breit-Wigner constraints in the fit procedure.
Table 7: Efficiencies and purities of the $jj\ell\nu$ sample at progressive stages of selection and kinematic fitting.

<table>
<thead>
<tr>
<th>Selection</th>
<th>Efficiency %</th>
<th>Background %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{cm}$ = 175 GeV</td>
<td>$E_{cm}$ = 192 GeV</td>
</tr>
<tr>
<td>Basic Selection</td>
<td>77</td>
<td>75</td>
</tr>
<tr>
<td>$1C$ fit</td>
<td>75</td>
<td>73</td>
</tr>
<tr>
<td>$3C$ fit</td>
<td>70</td>
<td>66</td>
</tr>
<tr>
<td>$3C'$ fit</td>
<td>72</td>
<td>71</td>
</tr>
<tr>
<td>$Z\gamma$</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>$WW$ (non-$jj\ell\nu$)</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>$ZZ$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 8: Resolutions on $WW$ production and decay angles using simulated events at 192 GeV. The ranges indicate the spread of values obtained from different experimental simulations.

Other approaches can be used instead of the selection procedure described above. In particular, if one wishes to avoid the use of the constrained fit, a cut requiring the missing momentum direction to be away from the beam pipe, typically $\cos\theta < 0.95$, can be used to reduce the background from the $ZZ$ and $Z\gamma$ channels. In this case, an algorithm has to be applied to impose energy and momentum conservation. Nonetheless, in the rest of this section we adopt the $3C$ fits as representative of the efficiency and purity which can be achieved.

The resolutions obtained for the $WW$ production and decay angles before and after kinematic fitting are shown in table 8. The values shown are averages over the whole fiducial region; however, in general, the resolutions depend upon the values of the kinematic variables themselves and, following kinematic fitting, they are correlated. It can be seen that a modest improvement in resolution is obtained, the main qualitative effect being due to the recovery of mis-measured events.
Table 9: Biases in the measurement of $\alpha_{W\phi}$ estimated from studies of large samples of fully simulated events. In the last part of the table the additional biases due to residual backgrounds are shown.

### 6.2 Systematic biases and statistical precision

We now consider potential systematic biases, and the degradation of statistical precision due to experimental effects in the $jj\ell\nu$ channel. In this we include a) the neglect of ISR and $\Gamma_W$, b) experimental acceptance, c) reconstruction and detector resolution, and d) residual background contamination. The first item has been discussed in detail in section 5.2; the result is included here for completeness. We use as example fits to $\alpha_{W\phi}$ only.

The overall bias due to a)-d) has been determined using a total of approximately 20,000 simulated $jj\ell\nu$ events at 175 GeV and 30,000 events at 192 GeV. A maximum likelihood or extended maximum likelihood fit was used, assuming in the analysis that the events originate from $WW$ production with narrow $W$ width and without initial state radiation. We emphasize that, since the purpose of this study is to show explicitly the scale of the biases, no corrections for the effects listed above have been applied in the analysis.

The results are shown in table 9. The column labelled 1D refers to fits using only the production angle $\cos \theta$. The column labelled 5D refers to fits using the production and decay angles (with the angles of the hadronically decaying $W$ folded to take account of the ambiguity described in table 3). The bias due to ISR and $\Gamma_W$ is derived as described earlier. The
bias due to event selection and acceptance was determined by comparing fits to the generated angles before and after event selection, and the bias due to reconstruction and resolution was determined by comparing fits to generated angles with fits to fully reconstructed angles. In the last part of the table the additional biases due to background are shown. However the reader should be aware that these were measured by adding small numbers of events to the sample, and in the absence of a systematic study should be considered to be very approximate.

We conclude that the size of the biases from ISR and $\Gamma_W$, acceptance and reconstruction are up to a few times the expected statistical error in the case of 1D fits, and somewhat smaller when all the angular information is used. In order that these effects do not present a serious source of systematic error compared to the statistical error, they will eventually have to be understood and corrected for, incurring an error of less than $\sim 10\%$ of their values.

Finally, we investigate the extent to which the statistical precision in TGC determination is degraded due to the effects mentioned above. The large simulated sample was divided into subsamples corresponding to the expected LEP2 statistics. The TGC parameter fit was performed on each sample, and the standard deviation of the spread of the results calculated. The precisions given for fits to generator level data for the $jj\ell\nu$ channel in table 5 assume an efficiency of 95%; thus the ideal precision in this channel is better by a factor $\sqrt{0.95} = 0.97$. Taking this and the estimated experimental efficiency of 70% shown in table 7 into account, we expect a statistical degradation of $\sim \pm 20\%$ with respect to this ideal case. This is indeed observed, together with an additional degradation of $\pm 10\%$ to $\pm 20\%$ after application of the analysis procedure described above, showing the effect of the extra randomization from ISR, $\Gamma_W$ and experimental effects.

6.3 Strategies for allowing for systematic biases

In the previous section the scale of the potential systematic bias due to detector and other effects was quantified. The simplest method of correction for such a bias is to determine its value for many simulated samples, subtract the mean bias from the experimentally measured TGC value and assign a systematic error on the basis of the width of the bias distribution and the experimental number of events. If the spread on the bias is large compared with the statistical error, this procedure will clearly be far from optimal. A second method is to use a Monte Carlo simulation to produce a correction function to map between “true” and “measured” values. This can easily be applied when fitting to a small number of variables, for instance to the $\cos \theta$ distribution alone, but is more difficult to apply in 5 dimensions simply because of the number of events required to characterize a 5D function in several bins per variable (unless corrections for each variable can be assumed to factorize). It has previously been shown at generator level that the precision is maximized by using all variables; it may however be that when systematic errors are taken into account the best overall precision is obtained by using a different strategy.

It is nonetheless possible to formulate methods which take resolution effects into account.
in fits using all the kinematic variables. For instance, if the resolution/acceptance function for the variables $\Omega$ is known, then the probability function $p(\Omega, P)$ used for each event in the maximum likelihood expressions (21) and (22) given in section 4.2 can be replaced by

$$p_{\text{eff}}(\Omega_{\text{meas}}, P) = p(\Omega_{\text{true}}, P) \times \rho(\Omega_{\text{true}} \rightarrow \Omega_{\text{meas}})d\Omega_{\text{true}}$$

(27)

(where $P$ represents the TGC parameters of the fit). The resolution/acceptance function $\rho$ gives the probability that the true value $\Omega_{\text{true}}$ would be reconstructed as $\Omega_{\text{meas}}$.

There are several potential problems with the application of (27): (i) a 5-D integration is required; (ii) the resolution and acceptance functions will almost certainly not be simple, nor will they factorize; (iii) the correlations between angles must be known and included (in particular if kinematically fitted quantities are used). One suggested method [63] uses fully simulated Monte Carlo events which are passed through the same events selection as data, in order to calculate the effective likelihood function. The variation of the TGC parameters is performed by reweighting the Monte Carlo events at their generated coordinates, while the comparison with data is performed at the reconstructed coordinates. This method can be applied for any fit dimension and can in principle take into account the effect of acceptance cuts, experimental resolution, any kinematic fitting procedure and background contamination in the data.

7 Analysis of the $jj\tau\nu$ final state

This channel requires special attention for two reasons. First, it comprises a sizeable part of the semileptonic $WW$ decays and therefore could provide a useful addition to the available statistics and, second, it is a background mainly for the hadronic channel and therefore methods are required to reject it.

In this study we consider only the hadronic decays of the $\tau$ and describe criteria to select this final state. The resulting efficiency and purity expected for the sample and the resolution expected in the angular variables are presented. We find that an increase in the overall number of events selected for analysis in the $jj\ell\nu$ channel of between $10 - 20\%$ can be expected.

7.1 Selection and reconstruction of $jj\tau\nu$ events

To select $jj\tau\nu$ events, we make use of the characteristics of the $\tau$ jet, namely small jet opening angle and low jet-charge multiplicity and of the global characteristics of the event, mainly missing energy and event acoplanarity.

The signal for the $jj\tau\nu$ final state has a 3-jet topology, while the main sources of background ($WW \rightarrow jjjj$ and $WW \rightarrow Z\gamma(s) \rightarrow q\bar{q}\gamma$) fall into the 4-jet and 2-jet topologies respectively.
Thus the choice of the resolution parameter in a jet-clustering algorithm is quite significant. Requiring at least 3 jets in the event, we find a $\tau$-reconstruction efficiency of $70 - 80\%$ while only $30 - 40\%$ of $Z\gamma$ events survive. The clustering algorithm itself ensures isolation for the $\tau$ jet.

Jets from $\tau$ decays can be distinguished from quark and gluon jets by the distribution of quantities such as the track multiplicity (total or charged), the maximum angle of any charged track in the jet to the jet axis, and the fractional energy of the jet contained within a cone of a specified angle (say, 0.1 rad) about the jet axis. A likelihood function based on such parameters has been constructed, giving a typical efficiency of about 70% with a rejection factor for quark and gluon jets close to 50. The charge of the $\tau$ lepton can be estimated rather reliably from the total charge of the tracks in the jet (excluding those with momenta $< 1$ GeV/c from the sum in order to reduce the contribution from soft tracks from neighbouring jets).

The $\tau$ signal can be further enhanced by requiring that the event contains less than five jets and that the sum of the missing energy and the energy of the reconstructed $\tau$ candidate should exceed $\sqrt{s}/2$. This results in a selection efficiency for $\tau$ events of about 90% with a rejection factor against the $WW \rightarrow jjjj$ channel and against $ZZ$ events of greater than 10. In addition, constraints on the polar angle of the missing momentum and the acoplanarity of the event can be imposed to reduce further the background from $Z\gamma$ events. A rejection factor of 10 is obtained while about 20% of the signal is lost. Finally, the very forward electromagnetic calorimetry can be used to detect ISR photon(s) in cases where they have not escaped in the beam pipe.

It may be noted from the above that missing energy and missing momentum are key variables for the rejection of all types of background, and therefore the hermiticity of the detector is an important factor.

The efficiencies and purities obtained for $jj\tau\nu$ events from a sample of simulated events at 192 GeV are shown in table 10. The background from the $jj\ell\nu$ channel stems mainly from inefficiencies in muon detection in the simulation used, and some improvement may be possible here. The application of a 2-constraint kinematic fit\textsuperscript{18} can also be seen to provide background rejection, with a small decrease in the $\tau$ selection efficiency.

An improvement to the kinematic fit may result by constraining the $\tau$ momentum, using the fact that the direction of the $\tau$ can be accurately estimated from the combined momentum of its visible decay products, so that the $\tau$ energy can then be computed from the $W$ decay kinematics [64].

\textsuperscript{18}The 2C fit imposes energy and momentum conservation and constrains the $jj$ and $\tau\nu\tau$ systems to have the $W$ mass, leaving the momentum of the neutrino from $W$ decay and the $\tau$ energy as free variables (with a lower limit on $E_\tau$ given by the visible energy of the $\tau$ decay products).
Table 10: Typical efficiencies and purities for the \( j\bar{j}\tau \nu \) channel with no kinematic fit and with a 2-constraint kinematic fit.

<table>
<thead>
<tr>
<th>Selection</th>
<th>Efficiency %</th>
<th>Background %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Z\gamma )</td>
<td>( WW \rightarrow j\bar{j}j )</td>
</tr>
<tr>
<td>No fit</td>
<td>35 - 45</td>
<td>4 - 6</td>
</tr>
<tr>
<td>2C fit</td>
<td>32 - 42</td>
<td>0 - 2</td>
</tr>
</tbody>
</table>

7.2 Resolution in reconstructed quantities

The resolution in the centre-of-mass polar and azimuthal angular variables of the \( \tau \), evaluated using 2-Gaussian fits to the differences between reconstructed and generated values, is of the order of 5 mrad in 75% of the events, and is not changed much by the kinematic fit. The energy of the original \( \tau \) can only be estimated at a level of \( \Delta E/E = 0.15 \) with no kinematic fit, but after the fit has a resolution \( \Delta E/E = 0.05 \) in 80% of events. The resolutions in the \( W \) production and decay angles, evaluated after the kinematic fit, are found to be \( \Delta \cos \theta = 0.11 \), \( \Delta \cos \theta_\tau = 0.13 \) and \( \Delta \phi_\tau = 250 \) mrad respectively.

7.3 TGC determination from \( j\bar{j}\tau \nu \) events

The precision with which TGCs can be determined from \( j\bar{j}\tau \nu \) events has been investigated using a sample of 937 fully simulated events, generated at 192 GeV with finite \( W \) width and ISR, corresponding to an integrated luminosity of 500pb\(^{-1}\). Of these events, 390 survived the selection and reconstruction procedures described above. The parameter \( \alpha_{W\phi} \) was fitted to the cross-section (17) (i.e. in the narrow width, no ISR approximation), using the extended maximum likelihood method described in section 4.2 and folding over the 2 ambiguous solutions. The 1 s.d. precision in \( \alpha_{W\phi} \) was found to be ±0.06 with estimated biases of −0.04 from the neglect of \( W \) width and ISR, −0.025 from the effects of reconstruction, and +0.03 from the presence of background events.

8 Analysis of the \( j\bar{j}j \bar{j} \) final state

The advantage of the high branching fraction of this channel is somewhat reduced by experimental difficulties associated with the purely hadronic nature of the final state. Background rejection in the four-jet channel is difficult, since no high-energy charged lepton is present to tag one \( W \) as in the semileptonic case. The largest background is expected from the high cross-section channel \( e^+e^- \rightarrow Z/\gamma^* \rightarrow q\bar{q}(\gamma) \) leading to multi-jet final states. Also, since the decay modes of the two \( W \)s are both hadronic, the problem arises of selecting the correct pairs...
of jets to form the two $W$s and of assigning their charges.

In the following we suggest an analysis of the $jjjj$ channel, including event selection, jet reconstruction and kinematic fitting, and indicate the expected efficiency and background levels. A section is devoted to jet- and $W$-charge tagging. We then discuss the determination of TGCs from the selected events, and draw conclusions on the sensitivity of the four-jet channel.

### 8.1 Selection of events and reconstruction of 4 jets in the final state

The general criteria for the selection of $jjjj$ events are based on the fact that the hadronization of four quarks gives rise to a high multiplicity of particles in the final state, and to a large visible energy. Other types of events with hadrons in the final state can have similar characteristics, mainly the $jj\ell\nu$ channel and the reactions $e^+e^- \rightarrow q\bar{q}\gamma$ with $M_{q\bar{q}} > 120$ GeV\(^{19}\) and $e^+e^- \rightarrow ZZ \rightarrow q\bar{q}q\bar{q}$. The first two reactions can mimic 4 jets when gluon radiation has occurred.

The following variables were typically used to select $jjjj$ events:

- A large multiplicity of particles in the detector ($N_{\text{charged}} > 25$, or $N_{\text{charged}} + N_{\text{neutr}} > 25 - 40$). This cut helps to reject $jj\ell\nu$ and QCD background, where the observed hadrons originate from a smaller number of initial quarks;

- Small thrust and/or large sphericity ($T < 0.9 - 0.97$ or $S > 0.1$). The QCD background generally consists of two back-to-back jets ($T \rightarrow 1$, $S \rightarrow 0$), while the $WW$ hadronic decays are more isotropic. However, the discriminating power of these variables becomes smaller as $\sqrt{s}$ increases;

- Large total visible energy (charged + neutral);

- Small missing energy ($E_{\text{miss}} < 40 - 50$ GeV). Large missing energy and momentum are associated with the neutrino in leptonic $W$ decays, and with an undetected high energy photon in $q\bar{q}\gamma$ events.

Events from the $jj\ell\nu$ channel can also be suppressed by requiring that no energetic isolated track or high energy identified lepton be present. The efficiency of the selection criteria at this stage is typically around 80% and the purity of the surviving sample is around 60%.

After the cuts described above, the final state particles are grouped into four jets. For this purpose, several clustering algorithms have been tried, which fall into two categories, namely transverse momentum-based clustering, such as LUCLUS [65], PUJET4 [66], DURHAM [67] or GENEVA [68], and scaled invariant mass squared clustering, such as JADE [69]. Comparative studies have shown that differences are contained to within about 3%, with the algorithms based\(^{19}\) on events with a lower invariant mass of the $q\bar{q}$ system correspond to radiative return to the $Z^0$ peak and can be easily rejected either because the photon radiated from the initial state is detected or because the missing momentum associated with it is very high.

\(^{19}\) Events with a lower invariant mass of the $q\bar{q}$ system correspond to radiative return to the $Z^0$ peak and can be easily rejected either because the photon radiated from the initial state is detected or because the missing momentum associated with it is very high.
on transverse momentum reproducing the initial parton directions somewhat better, leading to better jet definition and hence better resolution in invariant mass.

Further rejection of background can be achieved by application of the following additional cuts to the reconstructed jets, leading to a $jjjj$ purity of $\sim 80\%$:
- Minimum number of particles inside each jet (2 to 5);
- Minimum angular separation between jets ($20^\circ$);
- Minimum energy of the 2 least energetic jets (15-20 GeV);
- Minimum jet-jet invariant mass ($Y_{\text{cut}} = 0.002\sqrt{s}$).

8.2 Kinematic fitting

The kinematic fit is used as a tool to improve the resolution on measured quantities by imposing external constraints. For the $jjjj$ channel, the measured quantities are the energies and polar and azimuthal angles of the four reconstructed jets (and, for massive jets, their invariant masses). The external constraints which can be imposed are as follows:

1) energy-momentum conservation (4C),
2) as 1), plus equality of the two reconstructed invariant jet-jet masses (5C), or
3) as 1), plus equality of the two reconstructed invariant jet-jet masses with $M_W$ (6C).

The importance and the limits of kinematic fitting have been discussed in previous sections of this report, and technical details can be found in references [66, 70]. As in their application to TGC determination in the $jj\ell\nu$ channel (see section 6) the second and third constraints can be imposed without fear of introducing biases, as they would if applied to $W$ mass determination. Nonetheless, a comparison of results using different constraints has shown that there is negligible gain in going from the 4C fit to the 5C or 6C fits, and the results given below have used a 4C fit, followed by cuts on the invariant masses of the jets assigned to each $W$. In order to choose the best pairing of the four jets into two $W$s, kinematic fits are made to each of the three pairings, and that with the largest $\chi^2$ probability is taken as the correct combination.

8.3 Results in efficiency and resolution

After additional cuts on the fitted quantities to reduce background contamination, the efficiencies, purities and remaining background content of selected event samples generated with different detector simulations and at two centre-of-mass energies are as shown in table 11.

The resolutions in the radial and azimuthal jet angles $\theta_{\text{jet}}, \phi_{\text{jet}}$ and the resolution $\Delta E_{\text{jet}} / E_{\text{true}}^{\text{jet}}$ in the jet energy can be estimated by comparing each reconstructed jet with the closest generated quark direction. They show little dependence on the centre-of-mass energy and on the different detector simulations. Results for the resolutions in jet energy and in the reconstructed $W$ production angle $\cos \theta$ for simulations at 192 GeV are shown in table 12. It can be seen that the kinematic fit substantially improves the resolutions in the variables shown (by a factor

562
of between 30 and 50%). However, it has less effect on the jet angular resolutions, which are typically between 20 and 30 mrad for about 2/3 of the selected events.

<table>
<thead>
<tr>
<th></th>
<th>$\sqrt{s} = 175$ GeV</th>
<th>$\sqrt{s} = 192$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency (%)</td>
<td>54 - 59</td>
<td>52</td>
</tr>
<tr>
<td>Purity (%)</td>
<td>92</td>
<td>90</td>
</tr>
<tr>
<td>Background (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e^+e^- \rightarrow q\bar{q}\gamma$</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow ZZ \rightarrow q\bar{q}'\bar{q}'$</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow WW \rightarrow jj\nu$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 11: Efficiency and purity of samples of events selected with the cuts described in the text at two centre-of-mass energies.

<table>
<thead>
<tr>
<th></th>
<th>Before kinematic fit</th>
<th>After kinematic fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E_{\text{jet}} / E_{\text{true}}^{\text{jet}}$</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>$\Delta \cos \theta$ (mrad)</td>
<td>50.0</td>
<td>40.0</td>
</tr>
</tbody>
</table>

Table 12: Resolutions in jet energy and $W$ production angle before and after the kinematic fit at 192 GeV.

### 8.4 $W$ charge assignment

The ambiguities in angular data arising from the inability to distinguish quark from antiquark jets in $W$ decay have been listed in table 3, and the generator level studies simulating the $jjjj$ channel described in section 5.1 were made using distributions folded in both production and decay angles. In order to attempt to resolve the ambiguity on the production angle, a jet charge can be defined by weighting the charge $Q_i$ of each particle assigned to the jet with some function of its momentum, $P_i$,

$$Q_{\text{jet}} = \sum_{i:\in \text{jet}} Q_i \cdot F(P_i).$$

(28)

Different weight functions have been tried, based on transverse momentum, on rapidity, and on a power of the momentum [71, 72, 73, 74]. It appears very difficult to identify the charges of each individual jet. But, since the separation between the charges of the two $W$s is equal to 2, one can more easily distinguish the $W^-$ from the $W^+$ and therefore determine the production angle in the lab frame. The charges of the two jets assigned to one $W$ on the basis of the kinematic fit are therefore added together to evaluate the charge of the $W$. The fraction of selected events where the charge is correctly assigned is found to be 80%. No significant difference among the various weight functions was found. The $W$ charge identification implies a gain in sensitivity in TGC determinations.

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8.5 TGC determination from $jjjj$ events

The precision obtained in TGC determination after application of the procedures outlined above has been estimated using a fully simulated sample of 2292 events at $\sqrt{s} = 192$ GeV, corresponding to an integrated luminosity of 500 pb$^{-1}$. Two types of fit were performed to the observed angular distributions, namely, a $\chi^2$ fit to the production angle $\cos \theta$ only, and an unbinned maximum likelihood fit (as described in section 4.2) to the production angle and folded $W$ decay angles ($\cos \theta_{j_1}$, $\phi_{j_1}$)$\text{folded}$, ($\cos \theta_{j_2}$, $\phi_{j_2}$)$\text{folded}$. In both fits, the ambiguity in production angle was resolved using the jet charge assignment. Precisions obtained in fits to the TGC parameters $\alpha_{W\phi}$ and $\alpha_W$ are shown in table 13.

<table>
<thead>
<tr>
<th>Fitted parameter</th>
<th>Fitting method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi^2$ method</td>
</tr>
<tr>
<td>$\alpha_{W\phi}$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\alpha_W$</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 13: 1 s.d. errors in fitted values of parameters $\alpha_{W\phi}$ and $\alpha_W$ to a sample of 2292 fully simulated $jjjj$ events at 192 GeV. $\chi^2$ fits were made to the production angle only and maximum likelihood fits to production and folded decay angles.

The data used in the fits were generated according to the Standard Model using PYTHIA, with $\Gamma_W = 2.1$ GeV and with ISR. The theoretical expectations [52, 6] were calculated with $\Gamma_W = 0$ and without ISR. In these conditions, a biased result is expected, as indicated from the results shown in fig 10 (section 5.2). In addition, experimental biases due to the selection and reconstruction procedures are to be expected, as found in the analysis of the $jj\ell\nu$ channel and discussed in section 6. In the case of the $jjjj$ channel, the angular distributions are quite severely distorted by bad association of pairs of jets to the parent $W$ and by wrong $W$ charge assignment, and the resulting biases can easily simulate an anomalous TGC. The results shown for the fit to the production angle only include the effect of the application of a procedure to correct for the bias. Although based at present on the use of very limited Monte Carlo statistics, the fitted central values are found to remain within $\sim 1\sigma$ of the SM values after application of the correction. However, a full study of the biases in this channel and the development of methods to correct for them in fits using several angular variables have yet to be carried out.

9 Analysis of the $\ell\nu\ell\nu$ final state

The analysis of the channel in which both $W$s decay leptonically presents particular problems. It is the least statistically significant final state (with branching ratio $\sim 11\%$ for $l = e, \mu, \tau$), the missing neutrino momenta imply that the $W$ direction cannot be determined unambiguously, and, if one or both of the $W$s decay into $\tau\bar{\nu}_\tau$ or its charge conjugate, the presence of the extra neutrino from $\tau$ decay makes it impossible to reconstruct the event, reducing the useful branching ratio of such events to around $5\%$. On the other hand, the knowledge of the $W$
charge and the small reconstruction errors of leptons favour this channel in contrast to the 4 jet channel. In this section the usefulness of the purely leptonic decay channel for TGC determination is discussed in the light of these considerations.

### 9.1 Selection of \(\ell\ell\nu\ell\nu\) events

The \(\ell\ell\nu\ell\nu\) event signature is very simple: two leptons and large missing energy. This makes the channel easy to identify, but the background contributions, chiefly from \(Z\gamma\), are high. Also, \(\ell\ell\nu\ell\nu\) events with one or two leptonic \(\tau\) decays \((BR_{\tau\to e,\mu} \approx 35\%)\) constitute a possible background of about 1.8\% of the total number of \(WW\) events. The typical selection criteria used for \(\ell\ell\nu\ell\nu\) events aim at reducing these backgrounds by requiring large missing transverse momentum and, for equal weights, that the mass of the lepton-lepton system should not be close to \(M_Z\).

In addition it is also necessary that physical solutions to the reconstructed neutrino directions exist – this turns out to give the strongest background rejection.

For purely leptonic \(WW\) events the momenta of the 2 neutrinos are unknown. However, in the absence of ISR and for fixed \(M_W\), we have six constraints allowing the momenta of the neutrinos to be reconstructed \([3]\). The quadratic nature of these constraints results in a two-fold ambiguity, corresponding to flipping both neutrinos with respect to the lepton-antilepton plane, hence only affecting \(\cos \theta, \phi_1\), and \(\phi_2\), while leaving \(\cos \theta_1\) and \(\cos \theta_2\) unchanged.

The efficiencies and purities after each stage in event selection and reconstruction are shown in table 14 for fully simulated events generated with ISR and finite width. It can be seen that the required existence of solutions to the six constraints provides a very strong background suppression. However, it is important to note that the solution of these equations requires the use of all the kinematic information available in the event, leaving no possibility, for instance, of accounting for ISR or finite \(W\) width effects. Thus, with these effects included, no solution is found at generator level for about 20\% of the events.

<table>
<thead>
<tr>
<th>Cut</th>
<th>(N_{\text{leptons}} = 2) lepton (\in {e, \mu})</th>
<th>(P_T^{\text{miss}} &gt; 1.5) GeV</th>
<th>(l = \ell, M_{\ell\ell} &lt; m_Z - \Gamma_Z), (M_{\ell\ell} &gt; m_Z + \Gamma_Z)</th>
<th>Reconstruct</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{s}) (GeV)</td>
<td>175</td>
<td>190</td>
<td>175</td>
<td>190</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>82.7</td>
<td>80.1</td>
<td>82.6</td>
<td>79.9</td>
</tr>
<tr>
<td>Purity (%)</td>
<td>9.70</td>
<td>9.35</td>
<td>25.2</td>
<td>24.5</td>
</tr>
<tr>
<td>Background (%)</td>
<td>(Z\gamma)</td>
<td>86.0</td>
<td>87.0</td>
<td>63.6</td>
</tr>
<tr>
<td>(ZZ)</td>
<td>0.4</td>
<td>0.5</td>
<td>1.22</td>
<td>1.32</td>
</tr>
<tr>
<td>(WW \to \ell\nu\ell\nu)</td>
<td>3.83</td>
<td>3.16</td>
<td>9.99</td>
<td>8.28</td>
</tr>
</tbody>
</table>

Table 14: Efficiencies and purities in selection of \(\ell\ell\nu\ell\nu\) events at 175 and 190 GeV.
9.2 TGC measurements from $\ell\nu\ell\nu$ events

The precision with which TGCs can be determined from $\ell\nu\ell\nu$ events has been investigated using samples of fully simulated events, generated at 175 and 190 GeV with finite $W$ width and ISR, corresponding to an integrated luminosity of 500 pb$^{-1}$. The parameter $\alpha_{W\phi}$ was fitted to the cross-section (17) (i.e. in the narrow width, no ISR approximation), using the extended maximum likelihood method described in section 4.2 and folding over the 2 ambiguous solutions. The 1 s.d. precision in $\alpha_{W\phi}$ was found to be $\pm 0.15$ at 175 GeV, with estimated biases$^{20}$ of $-0.04$ from the neglect of $W$ width and ISR, $-0.05$ from the same sources plus the effects of reconstruction, and a combined bias of $-0.07$ when, in addition, background events are added. At 190 GeV the precision in $\alpha_{W\phi}$ was found to be $\pm 0.09$ and the same biases $-0.04$, $-0.13$ and $-0.21$, respectively.

Taking into account the small number of $\ell\nu\ell\nu$ events ($\approx 220$) in the sample, it is clear that the sensitivity to TGCs is highly preserved in this channel, despite the two-fold ambiguity. However, it is clear that, due to the very limited statistics, they will have to be used in combination with events from other decay channels.

10 Other Anomalous Couplings and Other Channels

10.1 Constraints on $WW\gamma$ Coupling from $e^+e^- \to \bar{\nu}\nu\gamma$

The $W^+W^-$ production process suffers from the drawback that both $W^+W^-\gamma$ and $W^+W^-Z$ couplings contribute and it is not easy to disentangle the various contributions. However, there does exist a process, namely $e^+e^- \to \bar{\nu}\nu\gamma$, which allows us to concentrate solely on the $W^+W^-\gamma$ vertex. The matrix-element for $\bar{\nu}\nu\gamma$ production in terms of the $WW\gamma$ TGCs $\kappa_\gamma$, $\lambda_\gamma$ in (1) has been calculated in Ref.[75]. In the numerical analysis we set acceptance cuts of a minimum photon angle of 20$^\circ$ and transverse momentum of 10 GeV. To increase the sensitivity to anomalous couplings the background from the $Z$ exchange graphs, $e^+e^- \to Z\gamma \to \bar{\nu}\nu\gamma$, is eliminated by requiring the energy of the photon to be at least $5\Gamma_Z$ away from the energy corresponding to the $Z\gamma$ final state which essentially amounts to an upper limit on photon energy of 53 GeV. With these cuts the cross-section$^{21}$ for the standard model is 1 pb at $\sqrt{s} = 175$ GeV, which still leads to an appreciable number of events at design luminosity of 500 pb$^{-1}$. Cross-sections for non-standard TGC, within these cuts, differ by less than 0.1 pb for $|\Delta\kappa|$ and/or $|\lambda| < 2$, so not much sensitivity is expected from the total cross-sections alone. Looking, however, at the deviations of the differential cross-sections from the standard model predictions one can set some limits on the parameters. We consider a $\chi^2$ fit to SM data, adding in

$^{20}$Due to limited statistics the statistical errors on the results from which the biases are estimated are of the same order as the biases themselves, but since the samples are correlated the statistical error of the biases are expected to be smaller.

$^{21}$We have not included effects of initial state radiation.
quadrature a relative systematic error of $\varepsilon = 0.02$. In Fig. 11 we show the contour plots for the $\chi^2$ distributions as functions of $\Delta \kappa_{\gamma}$ and $\lambda_{\gamma}$ as extracted from a) the energy, b) the transverse momentum distributions of the photon$^{22}$. We used equal size binning with 17 and 16 bins for the two cases respectively. This process is, in general, more sensitive to $\Delta \kappa_{\gamma}$ than to $\lambda_{\gamma}$. It

is thus somewhat complementary to Tevatron bounds which are derived from $W\gamma$ production. While quantitative improvements on the constraints may be made by considering two-variable distributions or by adopting maximum likelihood methods, these would still not be competitive with those deduced from $W^+W^-$ production. However, the $\bar{\nu}\nu\gamma$ channel isolates the $WW\gamma$ couplings and probes them in a different $q^2$ region. Therefore it complements the information obtained from $W$-pair production.

10.2 Anomalous $Z\gamma$ couplings$^{23}$

While the measurement of $WW\gamma$ and $WWZ$ couplings at LEP2 has deservedly received considerable attention, it is also important to search for couplings between the neutral gauge bosons$^{76, 77}$. For the trilinear $ZV\gamma$ vertex ($V=Z,\gamma$) the most general vertex function invariant under Lorentz and electromagnetic gauge transformations can be described in terms of four independent$^{24}$ dimensionless form factors$^{78}$, denoted by $h^V_i$, $i=1,2,3,4$. The parts of the vertex function proportional to $h^V_1$ and $h^V_2$ are CP-violating while those involving the other pair of form factors are CP-conserving. As is well known, all $Z\gamma$ form factors are zero at the tree level in the SM. At the one-loop level, $h^V_1$ and $h^V_2$ are zero while the CP-conserving

$^{22}$The angular distributions are less sensitive to the anomalous couplings.

$^{23}$We are grateful to Ulrich Baur for making his $Z\gamma$ event generator available to us.

$^{24}$As for the WWV TGCs of Eq. (1), constraints on the different $h^V_i$ can be obtained from restriction to the lowest terms of a gauge-invariant expansion in $1/\Lambda_{NP}$.
form factors are nonzero but too small to lead to observable effects at any present or planned experiment. Observation of $Z\gamma$ couplings would, therefore, signal physics beyond the SM.

We have carried out a generator-level study to estimate the sensitivity at LEP2 to anomalous $Z\gamma$ couplings. Following reference [78], the form factors were parameterized as $h_i^V = h_0^V/(1 + (P^2/\Lambda^2_V)^{n_i})$ where $P$ is the effective center-of-mass energy, and $h_0^V$, $\Lambda_V$, and $n_i$ are parameters. For comparison with present limits on $Z\gamma$ couplings, we chose $n_1 = n_3 \equiv n_6 = 3.0$ and $n_2 = n_4 \equiv n_8 = 4.0$. Two channels, $e^+e^- \rightarrow \mu^+\mu^-\gamma$ ($\mu\mu\gamma$) and $e^+e^- \rightarrow \nu\bar{\nu}(1\gamma)$, have been studied in detail. At LEP2 energies it turns out that the $1\gamma$ channel is much more sensitive to anomalous $Z\gamma$ couplings than the $\mu\mu\gamma$ channel. This is due mainly to anomalous couplings being dominated by the case where the detected photon recoils against a resonant $Z$ and that $\Gamma(Z \rightarrow \nu\bar{\nu}) \approx 6\Gamma(Z \rightarrow \mu^+\mu^-)$. Below we thus report on the sensitivity expected from the $1\gamma$ channel alone.

Experimentally, anomalous couplings in the $1\gamma$ channel would populate the same energy range as "radiative return" to the $Z$ pole through initial-state radiation (ISR), namely, the reflection of the $Z$ pole centered on $E_0 \equiv (s - m_Z^2)/(2\sqrt{s})$. Unlike photons from ISR, however, photons from anomalous couplings are distributed almost uniformly in solid angle. In our sensitivity analysis, which employed event counting rather than fits to distributions, we therefore required (a) the photon energy to be within 10 GeV of $E_0$ and (b) $|\cos\theta_\gamma| < 0.8$ in order to maintain good acceptance for anomalous couplings while suppressing the background from ISR. For $1\gamma$ events passing these cuts, a combined trigger and reconstruction efficiency of 90% was assumed.

Figure 12(a) shows the $ZZ\gamma$ couplings that would be excluded at the 95% C.L. for $\sqrt{s} = 175$ GeV and 500 pb$^{-1}$ assuming that the SM expectation is observed$^{25}$. The limits are shown for two different values of $\Lambda_Z$ to provide some indication of how much they depend on the particular choice of parameter values. Limits on these couplings have been published recently by L3[79], CDF[80], and D0[81]; the L3 and CDF limits are also plotted. It is evident that the expected sensitivity of LEP2 is comparable to the combined sensitivity of searches by LEP1 and Tevatron experiments. Figure 12(b) shows the corresponding estimated sensitivity to anomalous $Z\gamma\gamma$ couplings. As can be seen from comparison with the limits from CDF[80] (competitive limits have also been published by D0[81]), LEP2 is expected to extend considerably the sensitivity to $Z\gamma\gamma$ couplings.

The sensitivity to anomalous $Z\gamma$ couplings increases rapidly with center-of-mass energy, the effect being more pronounced for sensitivity to $h_2^V$ and $h_4^V$, which correspond to dimension-8 operators compared to dimension-6 operators in the case of $h_1^V$ and $h_5^V$. For example, sensitivity to $h_4^\gamma (h_5^\gamma)$ is improved by about 25% at 192 GeV, even with a smaller integrated luminosity of 300 pb$^{-1}$. Although backgrounds are expected to be more severe, analysis of the event sample consisting of hadrons and an isolated, energetic photon may provide another way of significantly increasing sensitivity to $Z\gamma$ couplings.

$^{25}$The effects of QED corrections on LEP2 sensitivities are not reflected in Fig. 12. These corrections reduce the sensitivity to anomalous $Z\gamma$ couplings but by less than 10%.
Figure 12: Estimated LEP2 sensitivity limits (95% C.L.) to anomalous $Z\gamma$ couplings and 95% C.L. limits from present experiments. The LEP2 estimate is for $\sqrt{s}=175$ GeV and 500 pb$^{-1}$. See text for explanation of the parameters.

10.3 Constraints on gauge boson interactions from $e^-e^+ \to q\bar{q}, l\bar{l}$

The description of NP for $e^-e^+ \to q\bar{q}, l\bar{l}$ in terms of dimension 6, purely bosonic, $SU(2)\times U(1)$ gauge invariant operators necessitates the consideration of the interactions

$$L_{NP} = \frac{f_{DW}g^2}{2\Lambda_{NP}^2}(D_\mu \bar{W}_{\nu \rho}) \cdot (D^\mu \bar{W}_{\nu \rho}) - \frac{f_{DB}g^2}{2\Lambda_{NP}^2}(\partial_\mu B_{\nu \rho})(\partial^\mu B^{\nu \rho})$$

$$- \frac{f_{BW}gg^2}{4\Lambda_{NP}^2} \Phi^\dagger B_{\mu \nu} \bar{\Phi} \cdot \Phi + \frac{f_{\phi_1}}{\Lambda_{NP}^2}(D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi),$$

(29)

in addition to the ones mentioned in Section 2.1.1. Such interactions affect the gauge boson propagator at the tree level and are thus rather strongly constrained by LEP1 measurements. Nevertheless LEP2 can significantly improve these constraints, particularly for the first two terms in (29) which give a $q^4$ contribution to the gauge boson propagator [34]. It has been remarked in [82], that if the physical quantities measurable in $e^-e^+ \to q\bar{q}, l\bar{l}$ at LEP2 are expressed in terms of $Z$-peak observables, then the aforementioned $q^4$ contribution allows the remaining anomalous dependence of the results to be described in terms of only the two couplings $f_{DW}$ and $f_{DB}$. Thus by looking at $\sigma_{\text{hadrons}}$, $\sigma_{\mu\tau}$, very strong constraints on these couplings should be possible; (see Fig. 13).
Figure 13: Sensitivity of LEP2 to $f_{DW}$ and $f_{DB}$ from $e^-e^+ \rightarrow q\bar{q}, l\bar{l}$ at $\sqrt{s} = 175$ GeV and 500 pb$^{-1}$ (one experiment). Constraints from $\sigma_{hadrons}$ (solid lines); $\sigma_{\mu^+\mu^+}$ (dashed lines); $A_{FB}^{\mu\tau}$ (dash-dotted lines); $\sigma_b$ (dotted lines); global fit (solid ellipse). $\Lambda_{NP} = 1$ TeV is assumed.

10.4 Higgs anomalous couplings

Anomalous couplings could also arise for the Higgs interactions with itself and the gauge bosons. In fact, dynamical considerations indicate that it is easier to generate anomalous couplings for the Higgs rather than for the gauge bosons [12, 13]. The dimension 6, $SU(2) \times U(1)$ invariant, CP conserving interaction is

$$L_{NP} = \frac{1}{v^2} (\Phi^\dagger \Phi - \frac{v^2}{2})(d \tilde{W}^{\mu\nu} \cdot \tilde{W}_{\mu\nu} + d_B B^{\mu\nu} B_{\mu\nu}) + \frac{4f_{\phi_2}}{v^2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi).$$

The first two of the above terms generate Higgs-gauge boson interactions while the last one induces anomalous Higgs interactions through a renormalization of the Higgs field.

As in section 2.1.1, unitarity can be used to associate to any given value of each of these anomalous couplings the largest allowed scale $\Lambda_U$ where New Physics generates it. For the first two operators these relations are

$$d \simeq \frac{104.5}{1 + 6.5 \frac{M_W}{\Lambda_U}} \frac{M_W}{\Lambda_U}^2 \quad \text{for } d > 0, \quad d \simeq - \frac{104.5}{1 - 4 \frac{M_W}{\Lambda_U}} \frac{M_W}{\Lambda_U}^2 \quad \text{for } d < 0,$$

$$d_B \simeq \frac{195.8}{1 + 200 \frac{M_W}{\Lambda_U}} \frac{M_W}{\Lambda_U}^2 \quad \text{for } d_B > 0, \quad d_B \simeq - \frac{195.8}{1 + 50 \frac{M_W}{\Lambda_U}} \frac{M_W}{\Lambda_U}^2 \quad \text{for } d_B < 0.$$
Thus, for $\Lambda_U = 1$ TeV, the largest allowed values are $d \simeq 0.4$ or $-1$ and $d_B \simeq 0.6$ or $-1$.

The above anomalous Higgs couplings may be studied at LEP2 through the processes $e^-e^+ \rightarrow ZH$, provided $m_H < \sqrt{s} - M_Z$, or via $e^-e^+ \rightarrow \gamma H$ if $m_H < \sqrt{s}$. Considering tree level anomalous contributions and restricting to cases where only one of the operators above is active [83, 84, 85, 86], we get the results given in the figures below. Thus, from Fig. 14a, presenting $e^-e^+ \rightarrow ZH$, we deduce observability limits $|f_{\phi \gamma}| \simeq 0.01$ and $|d| \simeq 0.015$ ($|d_B| \simeq 0.05$) corresponding to $\Lambda_U \simeq 6 - 7$ TeV ($\Lambda_U \simeq 5$ TeV) for $m_H \simeq 80$ GeV.

More striking is the process $e^-e^+ \rightarrow \gamma H$ which is unobservable at LEP2 in the SM [87, 88], but may become observable in the presence of NP interactions for the Higgs. A sensitivity to $|d| \simeq 0.05$ or $|d_B| \simeq 0.025$ should be possible from this process for $m_H \sim 80$ GeV, which means testing NP scales up to 3 and 7 TeV, respectively [86].

11 Conclusions

Experiments at LEP2 will allow a precise direct measurement of the most immediate consequence of the non-Abelian character of the electroweak bosons, the TGC of the $W$ to the photon and the $Z$. Various channels can provide information on non-standard interactions in the bosonic sector. The process $e^-e^+ \rightarrow f \bar{f}$ determines oblique parameters which are complementary to LEP1 results. $Z\gamma$, $HZ$ and $H\gamma$ production allow one to search for non-standard
boson couplings in the neutral sector. $e^- e^+ \rightarrow \nu \bar{\nu} \gamma$ is marginally sensitive to the $WW\gamma$ coupling in isolation. However, the most important process is clearly $e^- e^+ \rightarrow W^- W^+$ or its generalization, 4 fermion production.

Of the various decay channels, the semileptonic modes $W^- W^+ \rightarrow j j e \nu$, $jj \mu \nu$ will provide the most precise individual measurements of TGCs, since high statistics and almost complete information on the decay distributions are combined. Of particular importance is the identification of the $W$ charge which is needed to measure the full production angle distribution $d\sigma/d\cos\theta$. Also, the decay angular distributions and their correlations with each other and with the $W$ production angle are needed to resolve the correlations between different TGCs to a maximal extent.

A priori, the $jjjj$ final state provides incomplete information on the $W$ charges. However, correct charge assignments at the 80% level can be obtained by determining weighted jet charges, providing potentially valuable additional information in TGC determination. While more limited in statistics, the leptonic channel, $\ell \nu \ell \nu$, is particularly clean, and the $jj \tau \nu$ channel will also be of use in TGC analyses.

Using $jje \nu$ and $jj \mu \nu$ data alone, measurements of particular TGC parameters at $\sqrt{s} = 192$ GeV appear possible at generator level with a precision of $\approx \pm 0.02$ for an integrated luminosity of 500 pb$^{-1}$. The effects of ISR and finite $W$-width and the application of experimental selection, acceptance and reconstruction procedures lead to a degradation estimated at $\approx 30 - 40\%$ in the precision, and to a systematic shift which is a factor 3 larger than the statistical error, but our studies indicate that this bias can be corrected. For more general TGCs, considerable cancellation between different parameters is possible, resulting in weaker bounds. It is for this case that information from the full five-fold angular distribution of $W^- W^+$ production and decay angles or its generalization to 4-fermion final states becomes particularly important.

References


[40] For a review see e.g R. Arnowitt and P. Nath, in Brazil Summer School 1993 (hep-ph/9309277).


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1 Introduction

Direct production of new particles (with special emphasis on Higgs and supersymmetric partners) and possible indirect effects due to deviations from the predictions of the Standard Model (hereafter denoted as SM), in particular in the presence of anomalous triple gauge couplings, will be soon thoroughly searched for at LEP2 in the forthcoming few years. In both cases, typical and sometimes spectacular experimental signatures would exist, allowing to draw unambiguous conclusions if a certain type of signal were discovered.

At LEP2, one extra $Z$ (to be called $Z'$ from now on) would not be directly produced, owing to the already existing mass limits from Tevatron. Its indirect effects on several observables might be, though, sizeable, since it would enter the theoretical expressions at tree level. In this sense, $Z'$ effects at LEP2 would be of similar type to those coming from anomalous triple gauge couplings (hereafter denoted as AGC), although the responsible mechanism would be of totally different physical origin. This peculiar feature of a $Z'$ at LEP2 has substantially oriented the line of research of our working group. In fact, we have tried in this report to answer two complementary questions.

The first one was the question "which information on a $Z'$ can one derive if no indirect signal of any type is seen at LEP2?". To answer this point leads to the derivation, to a certain conventionally chosen confidence limit, of (negative) bounds on the $Z'$ mass $M_{Z'}$. This has been done for a number of "canonical" $Z'$ models, and the resulting bounds (that are typically in the TeV range) will be shown in section 2 together with those for more general $Z'$'s that might not be detected at an hadronic collider.

The second question is: "if a signal of indirect type were seen at LEP2, would it be possible to decide whether it may come from a $Z'$ or, typically, from a model with anomalous triple gauge couplings?". The answer to this question, which is essentially provided from measurements in the final leptonic channel, is given in section 3. In section 4, the role of the final WW channel, that might a priori not be negligible for a $Z'$ of most general type, has also been investigated and shown to be irrelevant at LEP2. Section 5 is devoted to a short final discussion, that will conclude our work.

2 Derivation of bounds

Theoretical motivations for the existence of a $Z'$ have already been given by several authors, and excellent reviews are available [1], where the most studied models are listed and summarized. In the following, we will limit ourselves in defining as "canonical" cases those of a $Z'$ from either $E_6$ [2] or Left-Right symmetric models [3] type. For sake of completeness we shall also consider the often quoted case of a "Sequential" Standard Model $Z'$ [4] (hereafter denoted as SSM), whose couplings to fermions are the same of those of the SM $Z^0$. For these models, derivations of bounds for $Z'$ parameters ($Z - Z'$ mixing angle and $M_{Z'}$) have been obtained from present
data[5],[6] and calculations of discovery limits for $M_{Z'}$ performed for future colliders[7],[8], including also a discussion of $Z'$ model identification. Therefore, the first question that we shall answer in our report will be that of how do the LEP2 indirect mass limits compare to the direct ones achievable at Tevatron now and in a not too far future (i.e. assuming an integrated luminosity of $1 fb^{-1}$). In fact a motivation of our work was also that of deriving limits on a $Z'$ whose couplings to fermions are completely free, including cases that would not be detectable by any present or future hadronic collider (for example for negligibly small $Z'$ quark couplings). For this purpose, the final leptonic channel at LEP2 provides all the necessary experimental information, and we shall consequently begin our analysis with the detailed examination of the role of this channel.

To fix our normalization and conventions, we shall write the expression of the invariant amplitude for the process $e^+e^- \to l^+l^-$ (where $l$ is a generic charged lepton) in the Born approximation and in the presence of a $Z'$. Denoting $q^2$ as the squared center of mass energy this amplitude reads in our notations:

$$A^{[0]}_{l' l'}(q^2) = A_{l' l'}^{[0]γZ}(q^2) + A_{l' l'}^{[0]Z'}(q^2)$$

where:

$$A_{l' l'}^{[0]γZ}(q^2) = \frac{i e_0^2}{q^2} \overline{u}_l γ_μ u_l γ_μ ν_{l'}$$

$$A_{l' l'}^{[0]Z'}(q^2) = \frac{i}{q^2 - M_{Z'}^2} \overline{u}_l γ_μ (g_{Vl}^{[0]} - γ^5 g_{Al}^{[0]}) u_l γ_μ (g_{Vl'}^{[0]} - γ^5 g_{Al'}^{[0]}) ν_{l'}$$

and (note the particular normalization):

$$A_{l' l'}^{[0]Z'}(q^2) = \frac{i}{q^2 - M_{Z'}^2} \overline{u}_l γ_μ (g_{Vl}^{[0]} - γ^5 g_{Al}^{[0]}) u_l γ_μ (g_{Vl'}^{[0]} - γ^5 g_{Al'}^{[0]}) ν_{l'}$$

In the previous equations $e_0 = g_0 s_0$, $e_0^2 = 1 - s_0^2$, $g_{Al}^{[0]} = I_{2L} = -\frac{1}{2}$ and $g_{Vl}^{[0]} = -\frac{1}{2} + 2s_0^2$. Following the usual approach we shall treat the SM sector at one loop and the $Z'$ contribution at tree level. The $Z'$ width will be assumed "sufficiently" small with respect to $M_{Z'}$ to be safely neglected in the $Z'$ propagator. Moreover the $Z - Z'$ mixing angle will be ignored since the limits for this quantity provided by LEP1 data from the final leptonic channel are enough constraining to rule out the possibility of any observable effect in the final leptonic channel at LEP2 (this has been shown in a previous paper [6] for the "canonical" cases and for a general $Z'$ case in a more recent preprint [8]). If we stick ourselves to the final charged leptonic states, we must therefore only deal with two "effective" parameters, that might be chosen as the adimensional quantities $g_{Vl}^{[0]}\sqrt{\frac{r^2}{M_{Z'}^2-q^2}}$ and $g_{Al}^{[0]}\sqrt{\frac{r^2}{M_{Z'}^2-q^2}}$ (this would be somehow reminiscent of notations that are common for models with AGC, with $M_{Z'}$ playing the role of the scale of new physics). In practice, for the specific purpose of the derivation of bounds, a convenient choice was that of
the following rescaled couplings [9]:

\[ v_N^l = gV_1 \sqrt{\frac{q^2}{M_{Z'}^2 - q^2}} \sqrt{\frac{\alpha}{16c_W^2 s_W^4}} \]  \hspace{1cm} (5)

and

\[ a_N^l = -g'_{A_{l}} \sqrt{\frac{q^2}{M_{Z'}^2 - q^2}} \sqrt{\frac{\alpha}{16c_W^2 s_W^4}} \]  \hspace{1cm} (6)

with \( \alpha = \frac{1}{137} \) and \( s_W^2 = 1 - c_W^2 = 0.231 \).

As previously stressed our first task has been that of the derivation of constraints for the two previous rescaled couplings from the non observation of effects in the leptonic channel. We considered as observables at LEP2 the leptonic cross section \( \sigma_l \) and the forward-backward asymmetry \( A_{FB} \), obtained from measurements of \( \mu \) and \( \tau \) final states, and also the final \( \tau \) polarization \( A_\tau \) (we have not yet included the electron channel because a full assessment of the corresponding experimental precision is more complicated at this stage). Three different energy-integrated luminosity configurations were considered, i.e. \( \sqrt{q^2} = 140 \) GeV and \( \int L dt = 5pb^{-1}, 175(500) \) and \( 192(300) \). Table 1 gives the SM predictions for the three leptonic quantities together with the expected experimental accuracies in the three cases. For each energy the first block of three lines contains convoluted quantities, whereas the second does not.

A short technical discussion about the way we have calculated the effects of QED emission is now appropriate. In fact two main approaches exist that use either complete Feynmann diagrams evaluation to compute photonic emission from external legs [10] or the so called QED structure function formalism [11], based on the analogy with QCD factorization and on the use of the Lipatov-Altarelli-Parisi evolution equation [12]. In the calculation of the limits on rescaled parameters performed in this section we have used the code ZEFIT [13] together with ZFITTER [14]. These programs utilize the first approach [10]. More precisely ZFITTER has all SM corrections and all possibilities to apply kinematical cuts. The code ZEFIT contains the additional \( Z' \) contributions including the full first order QED correction with soft photon exponentiation. The first version of this combination, applied to LEP1 data, which was restricted to definite \( Z' \) models, has been adapted for the model independent analysis that we are now performing at LEP2.

In practice the largest contribution is due to initial state radiation. The corresponding expressions for the cross section and forward-backward asymmetry read:

\[ \sigma_T = \int_0^{\Delta} dk \sigma_T^0(s') R_T^*(k) \]  \hspace{1cm} (7)

\[ A_{FB} = \frac{1}{\sigma_T} \int_0^{\Delta} dk \sigma_{FB}^0(s') R_{FB}^*(k) \]  \hspace{1cm} (8)

The reduced energy \( s' \) reads \( s' = q^2(1 - k) \) and \( \Delta = \frac{E_H}{E_{beam}} \). To first order in \( \alpha \), improved by soft photon exponentiation, the two functions \( R_T^*(k) \) and \( R_{FB}^*(k) \) are given by the following
expressions:

\[ R_{T,FB}(k) = (1 + S_e)\beta_e k^{\beta_e - 1} + H_{T,FB}(k) \]  

(9)

where

\[ \beta_e = 2 - \frac{\alpha e^2}{\pi} \ln \left( \frac{q^2}{m_e^2} - 1 \right) , \]  

(10)

the soft radiation part reads:

\[ S_e = \frac{\alpha e^2}{\pi} \left( \frac{\pi^2}{3} - \frac{1}{2} + \frac{3}{2} \ln \left( \frac{q^2}{m_e^2} - 1 \right) \right) \]  

(11)

and the hard radiation parts:

\[ H_T(k) = \frac{\alpha e^2}{\pi} \left\{ \frac{1 + (1 - k)^2}{k} \ln \left( \frac{q^2}{m_e^2} - 1 \right) \right\} - \frac{\beta_e}{k} \]  

(12)

\[ H_{FB}(k) = \frac{\alpha e^2}{\pi} \left( \frac{1 + (1 - k)^2}{k} \frac{1 - k}{(1 - \frac{k}{2})^2} \ln \left( \frac{q^2}{m_e^2} - 1 \right) - \ln \left( \frac{1 - k}{1 - \frac{k}{2}} \right) \right) - \frac{\beta_e}{k} \]  

(13)

The value of \( \Delta \) is chosen by requiring that the invariant mass of the final fermion pair \( M_{\ell\ell} = (1 - \Delta)q^2 \) is "sufficiently" greater than \( M_Z \), to exclude the radiative return to the \( Z \) peak. This has very important implications for searches of \( Z' \) effects, since it is well known that the radiative tail can enhance the SM cross section by a factor 2 \(- 3\), then completely diluting the small \( Z' \) effects, as fully discussed in a previous paper [15]. Results shown in table 1 are obtained for an invariant mass of the final fermion pair \( M_{\ell\ell} \) greater than 120 GeV.

One clearly sees from inspection of Table 1 that the most promising of the three energy-luminosity combinations for what concerns the relative size of the error is 175 GeV and 500pb\(^{-1}\).

From now on we shall therefore concentrate on this configuration and evaluate the bounds on \( Z' \) rescaled parameters that would follow from the non observation of any virtual effect. With this purpose we have made full use of the code ZEFIT and chosen \( \Delta = .7 \), although smaller values like for instance the one used in table 1 would lead to practically the same conclusions.

To obtain exclusion limits, we calculate the SM predictions of all observables \( O_i(SM) \) and compare them with the prediction \( O_i(SM, v_{i}^{N}, a_{i}^{N}) \) from a theory including a \( Z' \). In our fits we use the errors \( \Delta O_i \) calculated using the same assumptions as in Table 1 and define:

\[ \chi^2 = \sum_{O_i} \left( \frac{O_i(SM) - O_i(SM, Z')}{{\Delta O_i}} \right)^2 . \]  

(14)

\( \chi^2 < \chi^2_{\min} + 5.99 \) corresponds to 95% confidence level for one sided exclusion bounds for two parameters.
Table 1: SM predictions for leptonic observables including experimental accuracies. As a simple simulation of the detector acceptance, we impose that the angle between the outgoing leptons and the beam axis is larger than 20°, leading to an acceptance of about 0.9. The first line gives the muon cross section and the forward-backward asymmetry and errors. The second line gives the averaged \( \mu \) and \( \tau \) cross section and asymmetry (and errors) whereas the third line contains the \( \tau \) polarization (obtained by using only the \( \rho \) and \( \pi \) channels and assuming an average sensitivity of 0.5). Concerning systematics we assumed 0.5% relative error for \( \mu \) and \( \tau \) selections and also for luminosity. For all asymmetries we did not consider any systematic error. All quoted errors refer to a single LEP experiment. Taking into account the type of systematic errors and the relative size of systematics vs statistics, it is a good approximation to divide the error by 2 to estimate the combined error of the four experiments.

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Fig. 1 The areas in the \((\alpha_i^N, v_i^N)\) plane excluded with 95\% CL at LEP 2 by different observables i.e. \(\sigma_t\) (ellipse) and \(A_{FB}^t\) (crossed lines). The remaining contours, that do not improve the limits, would correspond to a measurement of \(A_t\) with an accuracy twice better than the realistic one quoted in Table 1.

Fig. 2 The area in the \((\alpha_i^N, v_i^N)\) plane excluded with 95\% confidence at LEP 2 by the combination of all the leptonic observables.

Figures 1 and 2 give our model independent constraints to the rescaled leptonic \(Z'\) couplings including all radiative corrections, i.e. the QED radiation and the electroweak corrections, that have a very small influence on the results. In figure 1 the constraint from each observable is shown separately. The combined exclusion region is depicted in the next figure 2 and a few comments on the previous figure are now appropriate. It represents in fact the most general type of constraints that can be derived on a \(Z'\) from the absence of signals in the leptonic channel at LEP2, under the assumption that the \(Z'\) couples to charged leptons in a universal way. In particular, from this figure one might derive bounds on the parameters of \(Z'\) that were only coupled to leptons and would therefore escape detection at any hadronic machine. For such models, the limit on \(M_{Z'}\) would be then derivable to a very good (and conservative) approximation, for given \(Z'\) couplings, by the simple expression (derived from eq. (5) and eq. (6)):

\[
\frac{M_{Z'}^2}{q^2} \geq \frac{1}{r^2} \left( \frac{g'^2 v_{l1} + g'^2 a_{l1}}{\frac{\alpha}{16\pi^2} r^4} \right)
\]

(15)

where \(r\) is the distance from the origin in the \((v_N^N, \alpha_i^N)\) plane of the intersection between the boundary region of figure 2 and the straight line:

\[
\frac{g_{l1}^N}{g_{l1}^N} = -\frac{v_i^N}{a_i^N} = k
\]

(16)
where $k$ is fixed by the considered model and $r$ will vary between .01 and .015. (Numerically $\frac{a}{16\pi^2 \alpha_s} \approx 81$.)

<table>
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<td>9.181</td>
<td>0.177</td>
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<tr>
<td>$R_h$</td>
<td>140.</td>
<td>5.</td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$R_b$</td>
<td>140.</td>
<td>5.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>140.</td>
<td>5.</td>
<td>0.25</td>
<td>10.62</td>
<td>2.92</td>
<td>.32</td>
<td>2.93</td>
<td>.21</td>
<td>27.6%</td>
<td>.509</td>
<td>.373</td>
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<tr>
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</tr>
<tr>
<td>$R_b$</td>
<td>140.</td>
<td>5.</td>
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<tr>
<td>$b$</td>
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<td>500.</td>
<td>0.25</td>
<td>5.15</td>
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<td>.43</td>
<td>.24</td>
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<tr>
<td>$R_b$</td>
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<td>500.</td>
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<tr>
<td>$b$</td>
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<td>500.</td>
<td>.25</td>
<td>4.69</td>
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<td>.14</td>
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<tr>
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<td>28.90</td>
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<td>.32</td>
<td>.40</td>
<td>.24</td>
<td>1.4%</td>
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</tr>
<tr>
<td>$R_h$</td>
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<td>500.</td>
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</tr>
<tr>
<td>$R_b$</td>
<td>175.</td>
<td>500.</td>
<td></td>
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<tr>
<td>$b$</td>
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<td>300.</td>
<td>0.25</td>
<td>4.08</td>
<td>0.23</td>
<td>.12</td>
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<td>1.00</td>
<td>25.22</td>
<td>0.29</td>
<td>.28</td>
<td>.40</td>
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<td>$R_h$</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$R_b$</td>
<td>192.</td>
<td>300.</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>192.</td>
<td>300.</td>
<td>.25</td>
<td>3.62</td>
<td>0.22</td>
<td>.11</td>
<td>.25</td>
<td>.25</td>
<td>6.8%</td>
<td>.577</td>
<td>.078</td>
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<td>1.00</td>
<td>22.79</td>
<td>0.28</td>
<td>.25</td>
<td>.38</td>
<td>.25</td>
<td>1.6%</td>
<td>6.942</td>
<td>0.159</td>
</tr>
<tr>
<td>$R_h$</td>
<td>192.</td>
<td>300.</td>
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<td></td>
</tr>
<tr>
<td>$R_b$</td>
<td>192.</td>
<td>300.</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Table 2: SM predictions for hadronic observables including experimental accuracies. The first line gives the $b$ quark cross section and forward-backward asymmetry and the corresponding experimental errors. The second line gives the total hadronic cross section (and errors) whereas the third line contains the ratio $R_h = \frac{\sigma_h}{\sigma_t}$ and the fourth line the ratio $R_b = \frac{\sigma_b}{\sigma_t}$. Concerning systematics we assumed 1% relative error for hadron selection and 3% relative error for $b$ quark selection. We did not consider any systematic error for all asymmetries. Concerning the $b$ quark cross section we assume a tagging efficiency of 25% (vertex tag) and 10% (lepton tag) for the asymmetry. The first block of numbers (upper four lines) refer to convoluted quantities where proper cuts have been applied, whereas the lower four lines contain deconvoluted quantities.
In a less special situation, the $Z'$ couplings to quarks will not be vanishing. In these cases, to derive meaningful bounds, the full information coming from the final hadronic channel should be also exploited. At LEP2, we assumed the availability of three different measurements, i.e. those of the total hadronic cross section $\sigma_h$ and those of the cross section and forward-backward asymmetry for $b\bar{b}$ production, $\sigma_b$ and $A_{FB}$. In table 2 we give the related expected experimental accuracies, for the three energy-luminosity configurations already investigated for the final leptonic channel in Table 1, and under the same general assumptions listed in the discussion preceding the presentation of this table.

From the combination of the leptonic and hadronic channels, a fully general investigation of the six rescaled $Z'$ couplings (there would be four extra rescaled couplings for "up" and "down" type quarks) might be, in principle, carried through if at least four hadronic independent observables were measured at LEP2. This could be obtained if one more hadronic asymmetry were measured. In practice, though, the utility of such an approach is somehow obscured by practical considerations (like the realistic achievable experimental accuracy). For these reasons, we have therefore decided to make full use of the hadronic observables to derive limits on $M_{Z'}$ only for a number of "canonical" models where the $Z'$ couplings to fermions are constrained. As relevant examples to be investigated, we shall discuss $E_6$ models [2] and Left-Right symmetric models [3], for which the $Z'$ current can be decomposed as:

$$J_{Z'}^\mu = J_X^\mu \cos \beta + J_6^\mu \sin \beta$$

or

$$J_{Z'}^\mu = J_{LR}^\mu \alpha_{LR} - J_{B-L}^\mu \frac{1}{2\alpha_{LR}}$$

Table 3a

<table>
<thead>
<tr>
<th>$f$</th>
<th>$\frac{g' v_f}{v_w}$</th>
<th>$\frac{g' A_f}{A_w}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>$\frac{2}{\sqrt{6}} \cos \beta$</td>
<td>$\frac{1}{\sqrt{6}} \cos \beta + \frac{\sqrt{10}}{6} \sin \beta$</td>
</tr>
<tr>
<td>u</td>
<td>0</td>
<td>$-\frac{1}{\sqrt{6}} \cos \beta + \frac{\sqrt{10}}{6} \sin \beta$</td>
</tr>
<tr>
<td>d</td>
<td>$-\frac{2}{\sqrt{6}} \cos \beta$</td>
<td>$\frac{1}{\sqrt{6}} \cos \beta + \frac{\sqrt{10}}{6} \sin \beta$</td>
</tr>
</tbody>
</table>

Table 3b

<table>
<thead>
<tr>
<th>$f$</th>
<th>$\frac{g' v_f}{v_w}$</th>
<th>$\frac{g' A_f}{A_w}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>$\frac{1}{\alpha_{LR}} - \frac{\alpha_{LR}}{2}$</td>
<td>$\frac{\alpha_{LR}}{2}$</td>
</tr>
<tr>
<td>u</td>
<td>$-\frac{1}{\alpha_{LR}} + \frac{\alpha_{LR}}{2}$</td>
<td>$-\frac{\alpha_{LR}}{2}$</td>
</tr>
<tr>
<td>d</td>
<td>$-\frac{1}{\alpha_{LR}} - \frac{\alpha_{LR}}{2}$</td>
<td>$\frac{\alpha_{LR}}{2}$</td>
</tr>
</tbody>
</table>

Table 3: Couplings of ordinary fermions ($f=l,u,d$) to $Z'$ boson a) from $E_6$ models as a function of the parameter $\cos \beta$ b) from Left-Right models as a function of the parameter $\alpha_{LR}$.  

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In table 3 we have given the $Z'$ couplings to $l$, $u$ and $d$ fermions for the two models. Some specific relevant cases in the $E_6$ sector are the so called $\chi$ model (corresponding to $\cos \beta = 1$), $\psi$ model ($\cos \beta = 0$) and $\eta$ model ($\arctan \beta = -\sqrt{\frac{5}{3}}$). Special cases for Left-Right symmetric models are obtained for $\alpha_{LR} = \sqrt{\frac{2}{3}}$ (this case reproduces the $\chi$ model) and $\alpha_{LR} = \sqrt{2}$ (the so called manifestly L-R symmetric model). Finally we also chose the $Z'$ of the Sequential Standard Model (which has the same fermionic couplings as those of the SM $Z$) as an additional benchmark.

Table 4 shows the CL bounds on $M_{Z'}$ obtainable from the non observation of any effect at LEP2 in the configuration: $175$ GeV, $500 \text{pb}^{-1}$. In fact one can easily show that for virtual $Z'$ searches this configuration is the best of the three that we have considered for LEP2, since the simple scaling law for the achievable limit $(M_{Z'})_{\text{max}} \sim (q^2 f L)^{\frac{1}{2}}$ applies. The different lines show the influence of the hadronic observables. As one sees, this is indeed relevant for the SSM $Z'$. In the other cases it improves the bounds derived from purely leptonic observables by an amount of less than (typically) a relative 10%.

<table>
<thead>
<tr>
<th>$\sigma_t, A_{FB}^t$</th>
<th>$\chi$</th>
<th>$\psi$</th>
<th>$\eta$</th>
<th>$\text{LR}$</th>
<th>$\text{SSM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_t, A_{FB}^t, \sigma_{\text{had}}$</td>
<td>900</td>
<td>642</td>
<td>550</td>
<td>854</td>
<td>1530</td>
</tr>
<tr>
<td>$\sigma_t, A_{FB}^t, R_b, A_{FB}^b, \sigma_{\text{had}}$</td>
<td>930</td>
<td>666</td>
<td>560</td>
<td>880</td>
<td>1580</td>
</tr>
</tbody>
</table>

Table 4: Maximal $Z'$ masses $M_{Z'}$ excluded by leptonic and hadronic observables. $\chi^2 < \chi_{\text{min}}^2 + 2.7$ (95% CL, one sided limits).

The more general analysis of the two models of extra gauge, that corresponds to values of $\cos \beta$ ranging from $-1$ to $+1$ (positive $\sin \beta$) and $\alpha_{LR}$ ranging from $\sqrt{\frac{2}{3}}$ to $\sqrt{2}$, has been summarized in figures 3 and 4 (full lines). One sees that the best $M_{Z'}$ limits correspond to models where $\cos \beta \sim 1$ and where $\alpha_{LR}$ is at the boundary of its allowed interval, for which the bounds derivable at LEP2 would be about 1 TeV.
Maximal $Z'$ masses excluded at LEP2 (full curve) and at Tevatron with a luminosity of $1 fb^{-1}$ (dashed curve) or $20 pb^{-1}$ (dotted curve) for $E_6$ models (Fig. 3) and for Left-Right models (Fig. 4).

The values that we have derived should be compared with those already available and with those reachable in a not too far future at Tevatron. To fix the scales for the comparison, we have considered the limits that would correspond to an energy of 1.8 TeV with an integrated luminosity of $1 fb^{-1}$ and drawn on the same figures 3 and 4 (dashed lines) the expected Tevatron limits, that would "compete" with the LEP2 results (the present Tevatron limits for $20 pb^{-1}$ correspond to the dotted curves). The limits correspond to 95% CL bounds on $M_{Z'}$ based on 10 events in the $e^+e^- + \mu^+\mu^-$ channel, assuming that $Z'$ can only decay in the three conventional fermion families. The values that we plot are in agreement with those quoted in a recent report [16]. One sees from figures 3 and 4 that for the $E_6$ models the LEP2 limits are in a sense complementary to those of Tevatron in the future configuration, providing better or worse indications depending on which range is chosen for $\cos \beta$. LEP2 is better if $\cos \beta$ lies in the vicinity of $-1$ and $\cos \beta \geq 0.4$. On the contrary, for Left-Right symmetric models LEP2 appears to do much better, except for $\alpha_{LR}$ ranging between 1 and 1.2 where Tevatron could provide limits a bit higher; concerning the SSM $Z'$, LEP2 does systematically better since it can reach 1.5 TeV whereas the future Tevatron limit is around 900 GeV. Note that, should other exotic or supersymmetric channel be open for $Z'$ decay, the Tevatron limits might decrease by as much as 30%, depending on the considered model [16]. We conclude therefore that, until
the Tevatron luminosity will reach values around $10 fb^{-1}$, the canonical LEP2 bounds will be, least to say, strongly competitive.

Our determination of bounds is at this point finished for what concerns the final fermionic channel. In the next section we shall try to derive some model-independent criterion to identify $Z'$ signals at LEP2.

3 Search for signals: the leptonic channel

In this section, we shall assume that some virtual signal has been seen at LEP2 in the leptonic channel. In this case, we shall show that it would be possible to conclude whether this signal is due to a $Z'$ or not.

This can be easily understood if one compares the $Z'$ effect to a description that includes the SM effects at one loop, and we shall briefly summarize the main points. For what concerns the treatment of the SM sector, a prescription has been very recently given [17], that corresponds to a ”Z-peak substracted” representation of four fermion processes, in which a modified Born approximation and ”substracted” one loop corrections are used. These corrections, that are ”generalized” self-energies, i.e. gauge-invariant combinations of self-energies, vertices and boxes, have been called in [17] (to which we refer for notations and conventions) $\tilde{\Delta}\alpha(q^2)$, $R(q^2)$ and $V(q^2)$ respectively. As shown in ref [17], they turn out to be particularly useful whenever effects of new physics must be calculated. In particular, the effect of a general $Z'$ would appear in this approach as a particular modification of purely ”box” type to the SM values of $\tilde{\Delta}\alpha(q^2)$, $R(q^2)$ and $V(q^2)$ given by the following prescriptions:

$$
\tilde{\Delta}\alpha^{(Z')}(q^2) = -\frac{q^2}{M_{Z'}^2 - q^2} \frac{1}{4c_1^2 s_1^2} g_{Vl}(\xi_{Vl} - \xi_{Al})^2
$$

$$
R^{(Z')}(q^2) = \left( \frac{q^2 - M_Z^2}{M_{Z'}^2 - q^2} \right) \xi_{Al}^2
$$

$$
V^{(Z')}(q^2) = -\left( \frac{q^2 - M_Z^2}{M_{Z'}^2 - q^2} \right) \frac{g_{Vl}}{2c_1 s_1} \xi_{Al}(\xi_{Vl} - \xi_{Al})
$$

where we have used the definitions:

$$\xi_{Vl} = \frac{g_{Vl}}{g_{Vl}}$$

$$\xi_{Al} = \frac{g_{Al}}{g_{Al}}$$

with $g_{Vl} = \frac{1}{2}(1 - 4s_1^2); g_{Al} = -\frac{1}{2}$ and $c_1 s_1 = \frac{s_1}{\sqrt{2}G_F M_Z^2}$. To understand the philosophy of our approach it is convenient to write the expressions at one-loop of the three independent leptonic

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observables that will be measured at LEP2, i.e. the leptonic cross section, the forward-backward asymmetry and the final $\tau$ polarization. Leaving aside specific QED corrections extensively discussed in the previous section, these expressions read:

$$\sigma_l(q^2) = \sigma_l^{\text{Born}}(q^2) \left\{ 1 + \frac{2}{\kappa^2(q^2 - M_Z^2)^2 + q^4}[\kappa^2(q^2 - M_Z^2)^2 \Delta \alpha(q^2)] - q^4(R(q^2) + \frac{1}{2}V(q^2)) \right\}$$  \hspace{1cm} (24)

$$A_{FB}^l(q^2) = A_{FB}^{l,\text{Born}}(q^2) \left\{ 1 + \frac{q^4 - \kappa^2(q^2 - M_Z^2)^2}{\kappa^2(q^2 - M_Z^2)^2 + q^4}[\Delta \alpha(q^2) + R(q^2)] + \frac{q^4}{\kappa^2(q^2 - M_Z^2)^2 + q^4}V(q^2) \right\}$$  \hspace{1cm} (25)

$$A_r(q^2) = A_r^{\text{Born}}(q^2) \left\{ 1 + \left[ \frac{\kappa(q^2 - M_Z^2)}{\kappa(q^2 - M_Z^2) + q^2} - \frac{2\kappa^2(q^2 - M_Z^2)^2}{\kappa^2(q^2 - M_Z^2)^2 + q^4}\right][\Delta \alpha(q^2)] + R(q^2) - \frac{4c_1\delta_1}{v_1}V(q^2) \right\}$$  \hspace{1cm} (26)

where $\kappa$ is a numerical constant ($\kappa^2 = \left(\frac{\alpha}{4\pi M_Z^2}\right)^2 \approx 7$) and we refer to [17] for a more detailed derivation of the previous formulae.

A comparison of eqs. (24-26) with eqs. (19-21) shows that, in the three leptonic observables, only two effective parameters, that could be taken for instance as $\xi_{VL} \frac{M_Z}{\sqrt{M_Z^2 - q^2}}$ and $(\xi_{VL} - \xi_{A_r}) \frac{M_Z}{\sqrt{M_Z^2 - q^2}}$ (to have dimensionless quantities, other similar definitions would do equally well), enter. This leads to the conclusion that it must be possible to find a relationship between the relative $Z'$ shifts $\delta \xi_{VL}$, $\delta A_{FB}^l \bar{A}_{FB}^l$ and $\delta A_r \bar{A}_r$ (defining, for each observable $O_i = O_i^{SM} + \delta O_i^{Z'}$) that is completely independent of the values of these effective parameters. This will correspond to a region in the 3-d space of the previous shifts that will be fully characteristic of a model with the most general type of $Z'$ that we have considered. We shall call this region "$Z'$ reservation".

To draw this reservation would be rather easy if one relied on a calculation in which the $Z'$ effects are treated in first approximation, i.e. only retaining the leading effects, and not taking into account the QED radiation. After a rather straightforward calculation one would then be led to the following approximate expressions that we only give for indicative purposes:

$$\left(\frac{\delta A_r}{A_r}\right)^2 \approx f_1 f_3 \frac{8c_1^2 s_1^2}{v_1^2} \frac{\delta \sigma_l}{\sigma_l} \left(\frac{\delta A_{FB}^l}{A_{FB}^l} + \frac{1}{2}f_2^2 \frac{\delta \sigma_l}{\sigma_l}\right)$$  \hspace{1cm} (27)
where the $f_i$'s are numerical constants, whose expressions can be found in [17].

Eq. (27) is an approximate one. A more realistic description can only be obtained if the potentially dangerous QED effects are fully accounted for. In order to accomplish this task, the QED structure function formalism [11] has been employed as a reliable tool for the treatment of large undetected initial-state photonic radiation. Using the structure function method amounts to writing, in analogy with QCD factorization, the QED corrected cross section as a convolution of the form:

$$
\sigma(q^2) = \int dx_1 dx_2 D(x_1, q^2) D(x_2, q^2) \sigma_0((1 - x_1 x_2)q^2)(1 + \delta_{fs}) \Theta(cuts)
$$

where $\sigma_0$ is the lowest order kernel cross section, taken at the energy scale reduced by photon emission, $D(x, q^2)$ is the electron (positron) structure function, $\delta_{fs}$ is the correction factor taking care of QED final-state radiation and $\Theta(cuts)$ represents the rejection algorithm to implement possible experimental cuts. Its expression, obtained by solving the Lipatov-Altarelli-Parisi evolution equation in the non-singlet approximation, can be found in [12] together with a complete discussion of the method. In order to proceed with the numerical simulation of the $Z'$ effects under realistic experimental conditions, the master formula eq. (28) has been implemented in a Monte Carlo event generator which has been first checked against currently used LEP1 software [18], found to be in very good agreement and then used to produce our numerical results. The $Z'$ contribution has been included in the kernel cross section $\sigma_0$ computing now the s-channel Feynman diagrams associated to the production of a leptonic pair in $e^+e^-$ annihilation mediated by the exchange of a photon, a SM $Z$ and an additional $Z'$ boson. In the calculation, which has been carried out within the helicity amplitude formalism for massless fermions and with the help of the program for algebraic manipulations SCHOONSCHIP [19], the $Z'$ propagator has been included in the zero-width approximation. Moreover, the bulk of non QED corrections has been included in the form of the Improved Born Approximation, choosing $\hat{\alpha}(s)$, $M_Z$, $G_F$ and $\Gamma_Z$ as input parameters. The values used for the numerical simulation are [20]: $M_Z = 91.187$ GeV, $\Gamma_Z = 2.4979$ GeV. The center of mass energy has been fixed to $\sqrt{q^2} = 175$ GeV and the cut $x_1 x_2 > 0.35$ (that would correspond to the choice $\Delta = 0.65$ in the notations of the previous section) has been imposed in order to remove the events due to $Z$ radiative return and hence disentangle the interesting virtual $Z'$ effects. These have been investigated allowing the previously defined ratios $\xi_{VI}$ and $\xi_{AI}$ to vary within the ranges $-2 \leq \xi_{AI} \leq 2$ and $-10 \leq \xi_{VI} \leq 10$. Higher values might be also taken into account; the reason why we chose the previous ranges was that, to our knowledge, they already include all the most known models.

The results of our calculation are shown in figure 5 [21]. One sees that the characteristic features of a general $Z'$ effect are the fact that the shifts in the leptonic cross section are essentially negative. This can be qualitatively predicted from the Born approximation formula eq. (24) because the dominant photon exchange contribution to $\sigma_1$ is clearly negative since $\Delta(Z')\alpha_s(q^2)$ is negative. Away from $\xi_{AI} \simeq 0$ the forward-backward asymmetry will be also negative, as easily inferred from eq. (25).
Fig. 5 $\frac{\delta A^\tau}{A^\tau}$ versus $\frac{\delta \sigma^\mu}{\sigma^\mu}$ and $\frac{\delta A^\mu_{PB}}{A^\mu_{PB}}$. The central "dead" area where a signal would not be distinguishable corresponds to an assumed (relative) experimental error of 1.5\% for $\sigma^\mu$ and to 1\% (absolute) errors on the two asymmetries. The region that remains outside the dead area represents the $Z'$ reservation at LEP2, to which the effect of the most general $Z'$ must belong.

Fig. 6 The same as Fig. 5, comparing the realistic results obtained via Monte Carlo simulation with the approximate ones according to Born approximation.

Fig. 7 The region corresponding to Anomalous Gauge Couplings according to a Born approximation.
One might be interested in knowing how different the realistic figure 5 is from the approximate Born one, corresponding to the simplest version given in eq. (27). This can be seen in figure 6 where we have drawn the allowed regions, the points corresponding to the realistic situation, already shown in figure 5. The region inside the parallelepiped, where a signal would not be detectable, corresponds to an assumed relative experimental error of 1.7% for \( \sigma_l \) and to an absolute error of 1% for \( A_{FB}^l \). For the \( \tau \) asymmetry an absolute error of 2% has been assumed, that is extremely optimistic. The domain that remain outside this area represents the \( Z' \) reservation at LEP2, to which the effect of the most general \( Z' \) must belong. One sees that the simplest Born calculation is, qualitatively, a reasonable approximation to a realistic estimate, which could be very useful if one first wanted to look for sizeable effects.

The next relevant question that should be now answered is whether the correspondence between \( Z' \) and reservation is of the one to one type, which would lead to a unique identification of the effect. We have tried to answer this question for one specific and relevant case, that of virtual effects produced by anomalous gauge couplings. In particular, we have considered the case of the most general dimension 6 \( SU(2) \times U(1) \) invariant effective lagrangian recently proposed [22]. This has been fully discussed in a separate paper [23], where the previously mentioned "\( Z \)-peak subtracted" approach has been used. The resulting AGC reservation in the \((\sigma_l, A_{FB}^l, A_\tau)\) has been calculated for simplicity in the Born approximation, as suggested by the previous remarks. This AGC area is plotted in figure 7. As one sees, the two domains do not overlap in the meaningful region. Although we cannot prove this property in general, we can at least conclude that, should a clear virtual effect show up at LEP2, it would be possible to decide unambiguously to which among two well known proposed models it does belong.

The results that we have shown so far have been obtained by exploiting the information provided by the final fermionic channel. We shall devote the next section 4 to a brief discussion of the WW channel.

### 4 Search for effects in the WW channel

The virtual effects of a \( Z' \) in W pair production from \( e^+e^- \) annihilation can be described, at tree level, by adding to the photon, SM Z and neutrino exchanges the diagram with an additional \( Z' \) boson exchange. The overall effect in the scattering amplitude reads:

\[
A_{IW}^{(q^2)} = A_{IW}^{(q^2, Z, \nu)} + A_{IW}^{(q^2, Z')} \tag{29}
\]

where we assume universal couplings. Separate expressions can be easily derived for eq. (29). We shall only give here the relevant \( Z' \) contribution:

\[
A_{IW}^{(q^2, Z')} = \frac{i}{q^2 - M_{Z'}^2} \frac{g_\nu g_\mu}{2c_0} \gamma_\mu (g^{(\nu)}_V - \gamma^5 g^{(\nu)}_A) \eta_0 g_{ZW} W P^{\alpha\beta\mu} \epsilon^*_\alpha(p_1) \epsilon^*_\beta(p_2) \tag{30}
\]

where:

\[
P_{\alpha\beta\mu} = g_{\mu\beta}(p_1 + 2p_2)_\alpha + g_{\beta\alpha}(p_1 - p_2)_\mu - g_{\mu\alpha}(2p_1 + p_2)_\beta \tag{31}
\]
and $p_{1,2}$ are the four momenta of the outgoing W bosons. In the expression eq. (30) we have assumed that the $Z^\prime WW$ vertex has the usual Yang-Mills form. We do not consider here the possibility of anomalous magnetic or quadrupole type of couplings. An analysis with anomalous $ZWW$ and $Z^\prime WW$ couplings is possible along the lines of [24] but is beyond the scope of this report. Our analysis will be nevertheless rather general as the trilinear $Z^\prime WW$ coupling will be treated as a free parameter, not necessarily proportional to the $Z - Z^\prime$ mixing angle as for example it would appear in a "conventional" $E_6$ picture.

For the purposes of this working group, it will be particularly convenient to describe the virtual $Z^\prime$ effect as an "effective" modification of $Z$ and $\gamma$ couplings to fermions and W pairs. As one can easily derive, this corresponds to the use of the following modified trilinear couplings that fully describe the effect in the final process $e^+e^- \rightarrow W^+W^-:$

$$g_{\gamma WW}^* = g_{\gamma WW} + g_{Z\gamma WW} \frac{q^2}{M_Z^2 - q^2} g_{Vl}(\xi_{Vl} - \xi_{Al})$$

$$g_{Z\gamma WW}^* = g_{Z\gamma WW} - g_{Z\gamma WW} \frac{q^2 - M_Z^2}{M_Z^2 - q^2} \xi_{Al}$$

In the previous equations, the same definitions as in eq. (22) and in eq. (23) have been used. In the following we shall use the results obtained on $\xi_{Vl}$ and $\xi_{Al}$ in the previous section. Our normalisation for trilinear couplings is such that: $g_{\gamma WW} = 1$ and $g_{Z\gamma WW} = \frac{\cos \theta}{\sin \theta}$.

Adopting the notations that are available in the recent literature [25], we find for the $Z^\prime$ effect:

$$\delta^{(Z^\prime)}_{\gamma} = g_{\gamma WW}^* - 1 = g_{Z\gamma WW} \frac{q^2}{M_Z^2 - q^2} g_{Vl}(\xi_{Vl} - \xi_{Al})$$

$$\delta^{(Z^\prime)}_{Z} = g_{Z\gamma WW}^* = -\cos \Theta_{W} = g_{Z\gamma WW} \frac{q^2 - M_Z^2}{M_Z^2 - q^2} \xi_{Al}$$

From eq. (34) and eq. (35) one can derive the following constraint:

$$\frac{\delta^{(Z^\prime)}_{\gamma}}{\delta^{(Z^\prime)}_{Z}} = \frac{-q^2}{q^2 - M_Z^2} \left( \frac{\xi_{Vl} - \xi_{Al}}{\xi_{Al}} \right) g_{Vl}$$

We then notice that the virtual effect of a general $Z^\prime$ in the WW channel is, at first sight, quite similar to that of a possible model with anomalous gauge couplings, that would also produce shifts $\delta_{\gamma, \delta_{Z}}$ both in the $\gamma WW$ and in the $ZWW$ couplings. But the $Z^\prime$ shifts satisfy in fact the constraint given in eq. (36), that corresponds to a certain line in the $(\delta_{\gamma, \delta_{Z}})$ plane whose angular coefficient is fixed by the model i.e. by the values of $\xi_{Al}$ and $(\xi_{Vl} - \xi_{Al})$.

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We shall now introduce the following ansatz concerning the theoretical expression of $g_{Z'WW}$, that we shall write as:

$$g_{Z'WW} = \left( c \frac{M_Z^2}{M_{Z'}^2} \right) \cot \Theta_W$$  \hspace{1cm} (37)

The constant $c$ would be of the order of one for the "conventional" models where the $Z'$ couples to $W$ only via the $Z - Z'$ mixing (essentially contained in the bracket of eq. (37)). But for a general model, $c$ could be larger, as one can see for some special cases of composite models[26] or when the $Z'$ originates from a strong coupling regime[27]. In fact, a stringent bound on $c$ comes from the request that the partial $Z'$ width into $WW$ has to be "small" compared to the $Z'$ mass. Imposing the reasonable limit:

$$\Gamma_{Z'WW} \leq \frac{1}{10} M_{Z'}$$  \hspace{1cm} (38)

leads to the condition:

$$c \leq 10$$  \hspace{1cm} (39)

that we consider a rather "extreme" choice.

We shall now discuss the observability limits on $\delta_\gamma$ and $\delta_Z$. According to [25], six equidistant bins in the cosine of the production angle are chosen for the generation of data, such that each bin contains a reasonable number of events ($\geq 4$). A binned maximum likelihood method has been used. The result for one parameter fit $\delta_Z$ is: $-0.2 \leq \delta_Z \leq 0.25$ for the configuration $\sqrt{s} = 175$ GeV and $\int L dt = 500 pb^{-1}$ and similarly for $\delta_\gamma$. We have then considered a number of possible illustrative situations, as extensively discussed in [28] and found that even in correspondence to the available present CDF limits and for the optimistic choice $c = 10$, one would get an effect of about 1%, i.e. well below the expected LEP2 observability limit.

In conclusion, a $Z'$ of even pathologically small mass, for extreme values of its assumed couplings, would be unable to produce observable effects in the $WW$ channel at LEP2. Therefore in the derivation of bounds or searches for visible effects, the final fermionic channel provides all the relevant information.

5 Concluding remarks

We have tried in this report to be as concise and essential as possible, partially owing to the lack of space. In this spirit, we feel that a proper conclusion to our work might be that of stressing that LEP2, under realistic experimental conditions and in a rather near future, will be able to perform a clean and competitive, in some respects quite unique, search for effects of
a $Z'$ whose mass is not above the TeV boundary. For $M_{Z'}$ values beyond this limit, only more energetic machines will be able to continue this task.

Acknowledgements

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