THE PROTOTYPE LAMINATED QUADRUPOLE FOR THE
BEAM TRANSFER SYSTEM

by

P. Brummer and N. Siegel

CONTENTS

1. Introduction
2. List of Parameters
3. Construction
   3.1. Magnetic Circuit
   3.2. Coils
4. Measurements
   4.1. Method
   4.2. Definitions
   4.3. Measurement error due to coil size
   4.4. Discussion of graphs
5. Comparison with computed field
6. Graphs
1. **Introduction**

A prototype quadrupole magnet for the ISR Beam Transfer System has been designed and constructed in order to obtain the information necessary for the design of the final quadrupoles.

The core of the quadrupole is divided into four quadrants, each quadrant consists of a stack of laminations glued together with a hot setting epoxy resin. The radius of the inscribed circle is 50 mm and the core has a length of 1.00 m. Each pole is excited by one coil made of hollow water-cooled copper conductor. The maximum gradient is 1800 gauss/cm.

Since the cross-sectional shape of the proton beam in the F quadrupoles is elliptical, a larger than usual ratio of pole width to inscribed circle radius (R) was chosen in order to extend the good field region along the median plane at low and medium fields up to about 1.2 R. This results, for a given width of the good field region, in a more compact design and in a reduction of the necessary number of ampere-turns.

The cross-section of the quadrupole is shown in Fig.1. The present report describes the construction of the quadrupole and the results of the magnetic measurements.

2. **List of Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of inscribed circle</td>
<td>100 mm</td>
</tr>
<tr>
<td>Length of the core</td>
<td>1.00 m</td>
</tr>
<tr>
<td>Maximum field gradient</td>
<td>1800 gauss/cm</td>
</tr>
<tr>
<td>Maximum current</td>
<td>380 Amp</td>
</tr>
<tr>
<td>Maximum power</td>
<td>19.7 kW</td>
</tr>
<tr>
<td>Total resistance at 20°C</td>
<td>131.3 mΩ</td>
</tr>
</tbody>
</table>
Inductance 143 mH
Time Constant 1.09 sec
Number of turns per coil 51

3. Construction of the quadrupole
3.1. Magnetic Circuit

Each quadrant has a girder bolted onto its back flat side. The four quadrants of the quadrupole are held together with 8 bolts and turnbuckles which are fixed onto these girders.

The steel sheet from which the laminations were punched is a low carbon, cold rolled sheet 1.5 mm thick with a low coercive force and high permeability. The specifications of the magnetic characteristics of this steel sheet are the same as the specifications of the steel sheet for the main magnet of the ISR. Magnetic measurements were performed on ring samples cut out from sheets taken from different parts of the steel band. The results of these measurements are shown in Table I.

<table>
<thead>
<tr>
<th>Steel band</th>
<th>Position of the sample in the steel band</th>
<th>Induction (Gauss) at 0.5 Oe</th>
<th>Induction (Gauss) at 15 Oe</th>
<th>Induction (Gauss) at 300 Oe</th>
<th>Coercive force Oe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Beginning</td>
<td>660</td>
<td>15170</td>
<td>20270</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>800</td>
<td>15280</td>
<td>20310</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>End</td>
<td>1100</td>
<td>15300</td>
<td>20330</td>
<td>0.77</td>
</tr>
<tr>
<td>2</td>
<td>Beginning</td>
<td>1140</td>
<td>15260</td>
<td>20230</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>1300</td>
<td>15400</td>
<td>20280</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>End</td>
<td>1470</td>
<td>15440</td>
<td>20320</td>
<td>0.705</td>
</tr>
</tbody>
</table>
The laminations were punched in two successive press operations. In the first operation the contour of the lamination was punched, leaving a small margin along the precision surfaces, namely the pole profile and the V shaped mating surfaces. In the second operation the precision surfaces were cut, by removing the small margin left along those surfaces.

The pole profile consists of a rectangular hyperbola terminated in two straight lines tangent to the hyperbola shown in Fig. 2. The position of the tangents have been determined with the aid of analog models to be such as to extend the quadrupole field along the median plane as far as possible. Finally the resulting field has been calculated with the version of the MARE programme for quadrupoles \(^1\). (See section 5).

The mechanical tolerance on the pole profile of the punched lamination and on the relative location of the V shaped mating surfaces with respect to the pole profile was specified to be \(\pm 0.02\) mm. Measurements performed on different laminations showed that the required tolerances had been maintained throughout the punching. A total of about 4000 laminations were punched.

The laminations were stacked in a precision frame. Alternating with the laminations a sheet of epoxy pre-impregnated polyester felt was inserted in the stack. This sheet carried the epoxy resin for bonding the laminations together and at the same time served as insulation of the laminations. The whole stack was then compressed and heated in an oven to polymerize the epoxy resin. Two 30 mm thick end plates were glued together with the laminations and terminate the stack at both ends.
No shimming was done during the stacking since the bonding resin compensates for the differences in thickness of the lamination. A packing factor of 96.8% was obtained on the glued quadrants. The interlaminar resistance was measured every centimeter along the quadrant and the measured values ranged from 20 Ω/cm up to one M Ω/cm. It was specified that the quadrants be straight within 0.1 mm. The deviations from straightness, measured along the centre of the pole face and along the mating surfaces were within 0.07 mm.

The strength of the bond in the longitudinal direction of the glued stack was tested on a fifth quadrant. The quadrant broke under a total force of 4 tons. In addition to gluing, each quadrant is provided with three tie bolts to ensure its mechanical strength.

During the assembly of the quadrupole a mylar sheet was inserted in between the mating surfaces of adjacent quadrants in order to insulate one quadrant from another.

The diameter of the inscribed circle and the distances between opposite pole corners were checked all along the quadrupole. The deviations from the nominal values at both ends and in the middle of the quadrupole are indicated in Table II. A precision of ± 0.1 mm was requested on the diameter of the inscribed circle. The maximum difference between the 4 distances measured between opposite pole corners at a given cross-section of the quadrupole was specified to be less than 0.1 mm, the absolute value had to be within ± 0.1 mm of the nominal one.
### TABLE II

Diameter of inscribed circle
(Deviations from nominal value)

<table>
<thead>
<tr>
<th>Measured between poles No</th>
<th>1 - 3</th>
<th>2 - 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connection side</td>
<td>- 0.05 mm</td>
<td>- 0.04 mm</td>
</tr>
<tr>
<td>Middle</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Non connection side</td>
<td>- 0.05 mm</td>
<td>- 0.09 mm</td>
</tr>
</tbody>
</table>

Distance between opposite pole corners.
(Deviations from nominal value).

<table>
<thead>
<tr>
<th>Measured between poles No</th>
<th>1 - 2</th>
<th>2 - 3</th>
<th>3 - 4</th>
<th>4 - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connection side</td>
<td>0.02 mm</td>
<td>- 0.05 mm</td>
<td>- 0.02 mm</td>
<td>- 0.08 mm</td>
</tr>
<tr>
<td>Middle</td>
<td>-0.05</td>
<td>- 0.03</td>
<td>- 0.04</td>
<td>- 0.03</td>
</tr>
<tr>
<td>Non connection side</td>
<td>-0.07</td>
<td>- 0.03</td>
<td>- 0.09</td>
<td>- 0.06</td>
</tr>
</tbody>
</table>

3.2. The coil

The coils were manufactured with hollow, water-cooled copper conductors. The inter turn insulation was machine applied onto the conductor before winding the coil. The insulation to ground was supplied after all the empty spaces within the coil had been filled with solid pieces made of fibreglass epoxy of suitable shape. The coil was then dried, evacuated and impregnated with resin under vacuum in a single operation. The coils were then placed into moulds in order to confine their external dimensions and placed in an oven to polymerize the impregnating resin.
All the coils withstood successfully the prescribed high voltage test, namely, 7 kV applied for 1 minute between the conductor and the ground after immersion in water for 24 hours, and the interturn insulation test, during which a voltage of 1,02 kV, 150 Hz, was induced between the terminals during 1 minute.

4. Measurements
   4.1. Method

   The gradient inside the quadrupole was measured with a pair of matched coils which were moved from the central position to the measurement position. (See Fig. 3).

   The integrated voltage difference of the two coil-outputs is a measure for the difference in gradient between the two positions.

   Some preference was given to this system to rotating coils. (Ref. 3). This can be explained as follows:

   The scalar potential of a two dimensional magnetic field which has the \( y = 0 \) plane as a symmetry plane can be written as (Ref. 2):

   \[
   V = - \sum_{n=0}^{\infty} a_n \frac{r^n}{n} \sin n \phi
   \]  \hspace{1cm} (1)

   We find:

   \[
   B_r = - \frac{\partial V}{\partial r}
   \]

   and

   \[
   B_\phi = - \frac{1}{r} \frac{\partial V}{\partial \phi} \hspace{1cm} (2)
   \]

PS/6986
In the case of a symmetrically mounted quadrupole which has four symmetry-lines the power series leads to the following equations.

\[ -V = a_2 \frac{r^2}{2} \sin 2 \phi + a_6 \frac{r^6}{6} \sin 6 \phi + a_{10} (...) \] (3)

\[ \frac{B_r}{r} = -\frac{3V}{2r} = a_2 \frac{r}{2} \sin 2 \phi + a_6 \frac{r^5}{5} \sin 6 \phi + a_{10} (...) \] (4)

\[ \frac{B_\phi}{r} = -\frac{1}{r} \frac{\partial V}{\partial \phi} = a_1 \cos \phi + a_6 \frac{r^5}{5} \cos 6 \phi + (...) \] (5)

And after transformation:

\[ \frac{B_x}{x} = -\frac{2V}{3x} = a_2 y + a_6 (5 x^4 y - 10 x^2 y^3 + y^5) + (...) \] (6)

\[ \frac{B_y}{y} = -\frac{2V}{3y} = a_2 x + a_6 (x^5 - 10 x^3 y^2 + 5xy^4) + (...) \] (7)

For detecting higher order field components it is an advantage to measure with high values of \( r \); the measured term being as large as possible. With a rotating coil the maximum value obtainable is the radius of the inscribed circle of the quadrupole. Small matched coils can be used used even between two poles.

As the field in the \( y = 0 \) plane will be used up to 1.2 times the radius of the inscribed circle, one likes to measure the field at that position, which can only be done with matched coils. A measuring machine that moves the coils in the \( x = 0 \) plane and \( y = 0 \) plane can also measure in other planes by a combined movement of \( x \) and \( y \).

A special check can now be done by measuring with two coils in the \( 45^\circ \) plane \( (x = y) \).
The gradient in the $45^{\circ}$ plane can be found from eq. (4):

$$
\left( \frac{\partial B}{\partial r} \right)_{45^{\circ}} = a_2 \sin 2\phi + 5 a_6 r^4 \sin 6\phi
$$

$$
\left( \frac{\partial B}{\partial r} \right)_{45^{\circ}} = a_2 - 5a_6 r^4 + \ldots
$$

The gradient along the $x$-axis ($y = 0$), is obtained from eq. (6)

$$
\frac{\partial B_y}{\partial x} = a_2 + 5a_6 x^4 + \ldots
$$

We see that the gradient in the horizontal plane as function of $x$ will vary contradictory to the gradient in the $45^{\circ}$ plane as a function of $r$.

The end effect was measured in the same manner, with a long pair of matched coils half-way in and half-way out of the quadrupole.

4.2. Definitions (See Fig. 4)

The graphs show the relative error in the gradient which is defined as:

$$
\frac{AG}{G_o} = \left( \frac{\partial B_y}{\partial x} \right) x = a = \left( \frac{\partial B_y}{\partial x} \right) x = b \quad \left( \frac{\partial B_y}{\partial x} \right) x = c
$$

RS/6886
The effective length \( L_{G}(a) \) at one end of the quadrupole is defined as:

\[
L_{G}(a) = \frac{\int_{0}^{a} \left( \frac{\partial B}{\partial x} \right) x = \infty \, dz}{\frac{1}{2} L_{I} \left( \frac{\partial B}{\partial x} \right) x = 0, \, z = 0}
\]

where

\( L_{I} \) = the length of the iron of the quadrupole.

The variation in effective length along the x-axis is now defined as:

\[
\Delta L_{G}(a) = L_{G}(a) - L_{G}(0)
\]

In the \( 45^\circ \) plane the radial gradient error is defined as:

\[
\frac{\Delta G_{R}}{G_{R}} = \frac{\left( \frac{\partial B}{\partial r} \right) r = a}{\left( \frac{\partial B}{\partial r} \right) r = 0}
\]

The definitions for effective length and variation of effective length in the \( 45^\circ \) plane are in accordance with those of the horizontal plane.
4.3. Measurement error due to the coil size

Since the coils are at a certain distance from each other, the gradient variations can in reality be higher than the measurement indicates. This effect was computed and taken into account in section 5.

4.4. Discussion of the graphs

The measurements were made, primarily, to check up on the field distribution resulting from the designed form of the pole profile. Secondly, measurements were made at the ends of the quadrupole, in order to find a shape of the ends of the poles that would give an effective length independent of the position in the x-y plane. This end-form should be easy to machine.

Graphs 1 and 2 indicate the effect of saturation of the steel on the gradient in the centre of the quadrupole. Saturation sets in at about 1600 Gauss/cm (8000 Gauss on the pole tips). At 1800 Gauss/cm the loss of ampere-turns in the steel is about 4%.

Graphs 3 and 4 show the field distribution inside the quadrupole at different excitation levels. The differences between the left and right hand side of the quadrupole (x negative and x positive respectively), were always small, the measurement points are therefore combined into a single graph.

A gradient increase at the sides (bump) of 1 part in $10^3$ at about $x = 50$ mm can be seen. The three low field measurements do not differ significantly. Above gradient values of 1300 G/cm a saturation effect that decreases the bump and the gradient at the sides, sets in, for instance at the point $x = 50$ mm the decrease in gradient is about 0.2%.

PS/6886
The effective length of the quadrupole without shaped ends is strongly dependent on the position (graph 5). Saturation also has an influence (graph 6). For low field values (1000 G/cm) and \( x = 50 \text{ mm} \) the effective length is 7 mm shorter than in the centre. Saturation makes this even worse (8 mm). An end-shape that corrects for this effect needs to have more iron at the sides of the poles, than in the middle. A simple way of arriving at such a form is cutting steel away from the top of the pole under an angle of 45 degrees. (See Fig. 5). The correction effect becomes stronger when more material is taken away. Having a quadrupole with straight ends, we started by screwing plates with this special chamfer (shims) on the pole ends. Later on the chamfer was machined on the quadrupole itself, in the end-plates.

The results obtained with the 10 mm shim (Fig. 5, shim 1) are given in graph 7. At low gradient, overcorrection (a bump) of 1 mm can be found at \( x = 50 \text{ mm} \). This was wanted for correction of saturation losses at higher gradients.

By varying the thickness of the shim and the depth of the cutting, many end effect corrections could be achieved. Other shims with round noses and different cutting angles were measured at the same time. No significant difference with the 45° cutting was noted.

As the 10 mm shim (Fig. 5 shim 1) gave for our case the best results, this form was machined in the end plates. The resulting end-effect is shown in graphs 8 and 9.
In graph 10 the equivalent gradient as seen by a particle passing a quadrupole of 1.20 meter length is given. Two ends of the form of graph 8 are supposed and the effective length variations are calculated as if it were variations in the gradient field. The equivalent gradient length of this quadrupole is now the same everywhere in the x-y plane, for a given field level.

5. Comparison between measured and computed field

The computed and measured variations of the gradient along the median plane at low field level is shown in graph 11. The field distribution was computed with the programme MARE A\(^1\), using the special version for symmetrical quadrupoles. The computation was done using a square mesh of 1.5 mm side and assuming an equipotential iron surface. The result is shown in curve No. 1. Curve No. 2 has been derived from the computations taking into account the actual measuring procedure, with two coils that are spaced 12 mm apart obtaining thus a variation of gradient as would be measured on a field distribution like the computed one. This corrected curve is compared with the results of a measurement at a low field level (gradient of 750 gauss/cm). The difference between both curves is smaller than 1.5% for values of x up to 60 mm.
References

1) R. Perin. and S. van der Meer.
   The program MARE for the computation of two-dimensional static magnetic fields, CERN 67-7.

2) B. de Raad

3) F. Ferger
   Magnetic Measurements on a quadrupole lens.
   CERN, AR/Int. SR 61-12.
Fig. 1  Cross-Section of Quadrupole
Fig. 3 Coordinate axis, magnet seen from non-connection side. Coils, outer dimensions. Integrator circuit.
Fig. 4  Definition of effective length
Fig. 5  SHIMS
Graph 1  Gradient in center, function of current

\[ G = \mu_0 \frac{2NI}{R^2} \]
Graph. 2  Saturation loss in gradient
Graph. 3  Gradient changes along X-axis
Graph. 4  Gradient changes along r-axis (45°)
Graph. 5  Variation of effective length on $X$-axis.
(N.B. One end only)  No shims
Graph. 6  Variation of effective length as function of Gradient at the center
Graph. 7  Variation of effective length with shim 1
Graph. 8  Variation of effective length (one end)
End plates machined off under 45° (flat nose)
Depth of cut 9.6 mm (like shim 1)
Graph. 9  Variation of effective length as function of Gradient at the center.
Graph. 10 Variation of gradient over the total field length, $(2L_G(0) + L_I)$, for a 1.20 meter quadrupole with end plates machined off as for graph 8.
Graph. 11  Gradient changes along X-axis

Curve 1. Computed field
Curve 2. Computed field as measured by 2 coils at distance 12 mm

\[ G(x) = \frac{B(x + 6) - B(x - 6)}{12} \]

Curve 3. Measured at 750 G/cm