ABSTRACT

Audition:

PLACE: 6.7.9, 401 November, 11:00 to 12:00 PM

TIME: Introduction to Detectors

TITLE: A H. WALENIA / University of Siegen, Germany

SPEAKER:

Lecture Series for Postgraduate Students

1999-1999 Academic Training Programme
Introduction to Detectors

A. H. Waldau, Univ. of Siegen, Germany

1) Introduction

Survey of detector applications and properties

2) Gas (filled) detectors

- Energy loss of charged particles
- X-ray absorption
- Energy resolution, gas amplification
- Multichannel detectors and position
- Resolution: MWPC, DC, IOC, TPC, TIGAC...
- Ion chambers and imaging: OBE, HIF...
- Single photon counting and imaging

3) Semiconductor detectors

- P-n junctions, detector properties
- Analogue electronics, noise, filters
- Spectroscopy

4) Scintillators

- Light detection (PM tubes)
- Scintillating mechanism, efficiency and time resolution
- Light guides, fibres

5) 2D detectors

- Diffraction measurements with X-rays
- Standard imaging devices: film, image plates, screen & image intensifier
- Counting pixel detectors,
o detection of radiation

- coincidence method
- particle aspect
- high sensitivity (single electron)
23 MeV
H: 25 \times 10^4 Gauss cm

63 MeV, pos.
H: 2.1 \times 10^4 Gauss cm

C: Imaging detector
C: Momentum measurement (B-field)
R = \frac{P}{0.3 m} \text{ (m, s/10, tesla)}

1.6 mg/cm² solid with longer range
(200) few percent with 63 MeV Hc.

\begin{align*}
\frac{\Delta P}{P} &= \frac{\sigma P}{0.3 B L^2} \sqrt{\frac{220}{N+H}} + \frac{0.05}{B L} \sqrt{\frac{1.43 L}{X_0}} \\
S &= 0.3 \frac{B L^2}{\sigma P} \text{ measure } S \text{ with precision } 0.5
\end{align*}

\text{Detector thickness causes scattering, radiation length } x/\chi_0
$\bar{\nu} + p \rightarrow n + \beta^+$

$\beta^+ + e^- \rightarrow 2\gamma$

\[ \rightarrow \text{capture, delayed } \gamma \text{-rays} \]

- massive detector, time measurement
- specific information for particle identification (pulse height, timing)

- calorimeter hits
- tracks

- 4\Pi detectors
- event reconstruction
two electromagnetic clusters

92 events

QED Background shape

$\text{mass (GeV/c^2)}$

(a)

UA2 final selections

153 events

11.3 events background

(b)

Schneider et al.

complex scattering with bound electrons

$\gamma^*$ momentum distribution

use position sensitive semiconductors
x-ray emitting Cygnus X-1
pulsating, candidate for black hole

Doppler shift of companion

x-ray intensity, correlated
Bore hole spectroscopy
determine quality of coal
before exploitation (ash contents)
Prompt γ-rays from thermal neutron capture
Nuclear medicine, scanner determines life process
Radio nuclides as tracers
chest x-ray

lung activity (Xe)
Energy loss spectra of protons

\[ \Delta E/L \sim U_p \]

(1 m, 90% Ar + 10% CH₄)
Energy loss of charged particles

electrons in atom $q_0 = 2e_0$

\[ q_T \]

charged particle

Momentum transfer:

\[ p^2 = \int F(t) \, dt = \int q_0 E(t) \, dt \quad E: \text{electric field} \]

\[ E' = \frac{p_0^2}{2m} \quad \text{for free electron} \]

\[ \phi(E', E) \, dE' \, dx : \text{probability for encounter in } E' \text{.. } E' + dE' \text{ in } dx \]

\[ \Rightarrow \]

1) Number of encounters

\[ \frac{dN}{dx} = \frac{\phi(E', E) \, dE'}{E_{\text{min}}} \]

2) Energy loss $E_{\text{max}}$

\[ \frac{dE}{dx} = \int E' \, \phi(E', E) \, dE' \quad E_{\text{min}} \]

Calculation of $\phi(E', E)$

\[ 4\pi p_0 \int E \, dF = \int E' 2\pi b v \, dt = 2\pi b v \int E \, dt \]

\[ q_0 \int E(t) \, dt = \frac{2q_0}{bv} \Rightarrow p^2 = \frac{22e^2}{b \beta c} \]

\[ E' / k = \frac{p_0^2}{2m} = \frac{42^2 e^2}{2mc^2 \beta^2} = \frac{1}{b^2} \]

\[ \frac{dE}{db} = -2 \alpha \frac{1}{b^3} \quad db = -\frac{b^3}{2\alpha} \, dE' \]

\[ \phi(E') \, db = \frac{2\pi b}{\alpha} \, db \]

\[ \phi(E') \, dE' = \frac{2\pi^2 22^2 e^2}{F dx \, m \alpha \beta^2} \frac{1}{E_{\text{ii}}^2} \, dE' \, dx \]

\[ \phi(E') \, dE' = \frac{N 2\pi 2^2 e^2}{m \alpha \beta^2} \frac{1}{E_{\text{ii}}^2} \, dE' \, dx \quad \text{for } N: \text{electron density} \]
\[ \phi(E)dE = \frac{\tilde{\lambda} g}{\beta^2} \frac{dE}{E^{1.5}} \, dx \]

Energy Loss:

\[ \langle \frac{dE}{dx} \rangle = \frac{E_{max}}{E_{min}} \]

\[ \frac{\lambda g}{\beta^2} \ln \frac{E_{max}}{E_{min}} \]

1) \( \beta_{max} \)

\( \beta = \frac{\beta_s}{\sqrt{1 - \beta^2}} \)

\[ \beta_{max} = \frac{\beta_s}{\sqrt{1 - \beta^2}} \]

\[ E_{min} = \frac{\sqrt{V_s^2(1 - \beta^2)}}{\beta c^2} \]

2) \( b_{min} (E_{max}) \)

\[ b_{min} = \frac{\hbar}{\beta} \cdot \frac{\hbar}{m_e \beta \tilde{c}} = \frac{\hbar (1 - \beta^2)^{1/2}}{m_e \beta \tilde{c}} \]

\[ E_{max} = \frac{mc^2 \beta^2}{\hbar^2 (1 - \beta^2)} \]

\[ \tilde{\lambda} = \hbar \langle \nu \rangle : \text{mean ionisation potential} \]

\[ \tilde{\nu} = \frac{E_{ph}}{E_{nu} \ln I_u} \text{ mixture } V \]

\[ \frac{\tilde{\lambda}}{\nu} = \frac{3}{2} \frac{2\tilde{\nu}}{\tilde{\nu} - \nu} \text{ e/a} \]

\( \tilde{\nu} : \text{electrons/atom} \)
Range

\[ R = \int_0^\infty \frac{dE}{dE/dx} \]

\[ \text{PLATEAU} \]

rest probable energy loss (Landau)
Longitudinal drift

$^{241}$Am $U_a = 700 V$ $U_b = 500 V$

TPC data, $E_{tot} = 29$ GeV
full lines: predicted values
Fig. 2

View of ISIS looking downstream during construction.

4m, 2m x 5.12 m
Electrode arrangement of the 1515 detector
A: anode, P: potential wire

4 anodes 640
4 channels 320
E 500 V/cm
Gas 80% Ar + 20% CO₂ (Oxidor)
Gain 10⁴
Gate 200 μs
Fig. 1. JADE experimental apparatus

Fig. 2. Cross section of a jet cluster cell.
1, 2, 3: particle tracks, 4, 5: clans, 6: wire space, 7: wire assembly.

Fig. 3. Cross section of a WITI space and associated electronics

One segment of the JADE central detector at the Heidelberg institute.
Particle Identification using $dE/dx$

Technical "details":
- broad distribution: Landau distribution
- results in considerable overlap

![Graph showing pions and electrons with $p = 0.5 \text{ GeV/c}$]
Oscilloscope signal of minimum ionizing particle. Longitudinal drift shows clusters.

Cluster counting with longitudinal drift:

- 52 -

![Diagram showing number of clusters vs number of events with measured distribution and best fit Poisson distribution.]
Geiger / Streamer

Drift - Chamber

Landau - Distribution

\[ \frac{dn}{de} \]

\[ E_{mp} \]

\[ \text{FWHM} \]

\[ E \]

\[ \phi(x) dx : \text{Probability for encounter with } x \]

\[ E = \sum_{i} E_i \text{ with } N \text{ Poisson distributed} \]

\[ \bar{E} = \int \phi(x) x dx \text{ with } \phi(x) dx = \frac{A \rho x}{\rho + x} \]

\[ \bar{E} \text{ depends on limits!} \]

\[ \left( \frac{\Delta E}{E} \right)_{\text{rms}} = \frac{1}{N} \int (E - \bar{E})^2 \phi(x) dx = \frac{1}{N} \frac{E_{\text{max}} - E_{\text{min}}}{E_{\text{min}} E_{\text{max}}} \]

worse!
MONTE CARLO II (without density effect)

Cluster size distribution:

\[
\frac{dN}{dE} = \frac{\text{Const}}{E^2} \left\{ \frac{1}{E} \rho(E) \ln \left( \frac{2mE^2}{\hbar^2} \right) + \frac{1}{E} \int_0^E \rho(E') dE' \right\}
\]

Resonant

Rutherford

\( f(E) \): oscillator strength

Ansatz: \( f(E) \sim \sigma(E) \) for photoabsorption

\( \sigma(E) = \frac{1}{E} \)

\( \rho(E) = \begin{cases} \frac{N}{E} & \text{for } E > E_i; \\ 0 & \text{for } E < E_i; \end{cases} \)

Integration of Rutherford part

\[
\int_0^E \rho(E) dE = \int_{E_i}^E \frac{N}{E'} dE' = \frac{N}{2} \left( \frac{1}{E_i} - \frac{1}{E} \right) \quad \text{for } E > E_i;
\]

\[
\frac{dN}{dE} = \frac{\text{Const}}{E^2} \left\{ \frac{1}{E} \rho(E) \ln \left( \frac{2mE^2}{\hbar^2} \right) - \rho(E) \left( \frac{1}{E_i} - \frac{1}{E} \right) \right\}
\]

\( V(E) \)

Normalized on number of clusters:

\[
\text{Net} = \int_0^\infty \frac{dN}{dE} dE = \text{splitting in shell contributions}
\]

\[
\text{Net} = \frac{\text{Const}}{E} \sum_i \left\{ \int_0^{\infty} \frac{\rho(E)}{E} \left[ V(E) + \frac{1}{E} \right] dE' \right\}
\]

\[
\text{Net} = \frac{\text{Const}}{E} \sum_i \left\{ \left[ \frac{E}{E_i} \left[ V(E) - 1 \right] + \frac{1}{E_i} \right] \right\}
\]

\[\begin{align*}
\text{Beam:} & \quad \Delta E = \int E \frac{dN}{dE} dE
\end{align*}\]
Application to Monte Carlo:

1) Determination of contributions from shells

\[ P_i = \frac{n_i/n; (V_i + 1)}{\sum n_i/n; (V_i + 1)} \]

2) Determination of resonant contribution

\[ P_{i, res} = \frac{V_i}{V_i + 1} \quad \left( P_{i, max} = \frac{1}{V_i + 1} \right) \]

3) Choice of \( N_f \)

4) Cut-off for low energies: 
   \( E \) is adjusted such that
   \[ W = \frac{E}{n_e} = \text{measured value} \]
Fig. 1. Observed\textsuperscript{11} and calculated\textsuperscript{12} photo-absorption cross sections for photons incident on argon.

The locations of equivalent configurations used in this paper (rows labelled S) are values of S through 6s.

\textit{Allison}
Effect of rel. rise

\[ \frac{dN}{dx} \]  

Emin to Emin

Number of collisions for Emin = 0 (ionization + gas minimum plateau excitation)

<table>
<thead>
<tr>
<th>Gas</th>
<th>( \frac{dN}{dx} )</th>
<th>5.3</th>
<th>18.8</th>
<th>29.3</th>
<th>35.4</th>
<th>41.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>5.3</td>
<td>8.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ne</td>
<td>18.8</td>
<td>19.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ar</td>
<td>29.3</td>
<td>40.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kr</td>
<td>35.4</td>
<td>50.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xe</td>
<td>41.1</td>
<td>66.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Energy Loss in 1cm Material for Minimum Ionising Particles ($\beta y = 3.5$)

<table>
<thead>
<tr>
<th>Material</th>
<th>$a_r$ (keV)</th>
<th>$\Delta E_{\text{mean}}$ (keV)</th>
<th>$\Delta E_{\text{most probable}}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2$</td>
<td>0.0147</td>
<td>1.54</td>
<td>0.862</td>
</tr>
<tr>
<td>$CH_4$</td>
<td>0.0716</td>
<td>4.30</td>
<td>2.16</td>
</tr>
<tr>
<td>$C_3H_8$</td>
<td>0.188</td>
<td>3.20</td>
<td>1.77</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>0.157</td>
<td>2.30</td>
<td>1.17</td>
</tr>
<tr>
<td>$P$-10</td>
<td>0.122</td>
<td>5.99</td>
<td>2.05</td>
</tr>
<tr>
<td>$X$-10</td>
<td>0.352</td>
<td>2.43$\cdot$10$^3$</td>
<td>1.8$\cdot$10$^3$</td>
</tr>
<tr>
<td>Plastic sc.</td>
<td>1.06$\cdot$10$^2$</td>
<td>2.43$\cdot$10$^3$</td>
<td>1.8$\cdot$10$^3$</td>
</tr>
</tbody>
</table>
Energy loss distribution

W: FWHM  E: most probable energy loss

\[ \int \text{measured} \]

\[ \text{Landau (calculated)} \]

\[ \frac{W}{E} \]

\[ \frac{dN/\text{d}E}{E(\text{MeV})} \]

Historical remark:

- Landau has no mean free path
- Block & Langmuir have too small MC
- Block & Langmuir is right but needs additional atomic information
**Application**

**Particle identification**

In region of rel. rise differences are small

\[ \text{K-N} = 10\% \]

as measurement of energy loss \( \Delta E \% < 5\% \)

Best resolution in prop. gas counter

Landau limit \( \Delta E / E < 0.2 \)

Therefore sampling and statistical treatment

\[ \begin{array}{c}
E \\
\text{track} \\
\text{total ionisation} \quad \Delta E / E \approx 0.2 \\
\end{array} \]

\[ n \text{ cells} \]

\[ E_i \]

\[ \frac{H_b^{(n)}}{E_b} = \frac{1}{\sigma_n} \frac{H_b^{(n)}}{E_b} \quad \text{for n independent measurements} \]

\[ \frac{1}{\sigma_n} = \frac{1}{\sqrt{n}} \quad \text{for Gaussian distributions} \]

Landau distributions are not Gaussian

therefore:

- trimmed mean
- max. likelihood improvement factor

\[ q(n) = n^{0.428} \]

**Cluster counting**

\[ \frac{\Delta H}{N} = \frac{1}{\sqrt{N}} \quad \text{independent of sampling} \]

\[ N = 20 \text{ cm}^{-1} \quad \Rightarrow \frac{\Delta H}{N} = 2.2\% \quad \text{for 1 m track length} \]

rel. rise limited

measurements not always in agreement with calculations.
Performance plot of delta detectors
n = number of samples
P = pressure
l = length of sample
w = width of sample (cavomicro)
R = room for constant resolution
pL (atm·m)

Fig. 5 - Eichmann et al.
### Experimental Results

<table>
<thead>
<tr>
<th>Device</th>
<th>Particles</th>
<th>p(%)</th>
<th>exp. %</th>
<th>th. %</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISIS^*</td>
<td>p/e</td>
<td>0.5</td>
<td>8.0</td>
<td>8.1</td>
<td>1.01</td>
</tr>
<tr>
<td>CRISIS^*</td>
<td>p/n</td>
<td>40</td>
<td>2.18</td>
<td>3.2</td>
<td>1.48</td>
</tr>
<tr>
<td>EPI</td>
<td>p/n</td>
<td>50</td>
<td>5.1</td>
<td>6.1</td>
<td>1.20</td>
</tr>
<tr>
<td>TPCT</td>
<td>p/e</td>
<td>0.8</td>
<td>11.8</td>
<td>15.0</td>
<td>1.36</td>
</tr>
<tr>
<td>JADE</td>
<td>p/e</td>
<td>0.45</td>
<td>4.2^*</td>
<td>7.8</td>
<td>1.86</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>5.0</td>
<td>7.0</td>
<td>1.46</td>
</tr>
<tr>
<td>HRS^*</td>
<td>p/e</td>
<td>4</td>
<td>3.2</td>
<td>4.1</td>
<td>1.32</td>
</tr>
<tr>
<td>Cleo</td>
<td>p/e</td>
<td>0.45</td>
<td>8.25</td>
<td>11.1</td>
<td>1.50</td>
</tr>
</tbody>
</table>

* analysis in jet

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### Response of detectors to x- and \( \gamma \)-rays

- Ion applications in industry
- Non destructive testing (NDT)
- Performance defined by
  - Interaction of photons in detector
  - Mechanism of gas amplification
Fig. 10. Microperforations in a precision cast turbine blade as revealed in an enlarged image by microfocal radiography (compared with a conventional x-ray radiograph). Partida and Cason (1977).

Magnification: $A = \frac{s_2}{s_1}$

Problem for film: “low rate” $\approx 10^8$ s$^{-1}$ cm$^2$
ideal for fast photon counter
Interaction of Photons with Matter

1) Thomson scattering

incoming EN-wave:

\[ E(t) = E_0 \sin \omega t \quad \text{polarisation} \]

acceleration of atomic electron:

\[ \mathbf{a} = \frac{e E}{m} = \frac{e E_0}{m} \sin \omega t \]

\[ \langle a^2 \rangle = \frac{e^2}{m^2} E_0^2 \]

Larmor's formula of radiation by accelerated charge:

\[ \frac{dP}{d\omega} = \frac{e^2}{4mc^2} (\mathbf{a})^2 \sin^2 \theta \]

\[ \frac{dP}{d\omega} = \frac{c}{8\pi mc^2} E_0^2 \sin^2 \theta \]

Energy flow/time/area: Pointing Vector \( S \)

\[ \langle S \rangle = 4 \pi \frac{E^2}{8\pi} \]

\[ \langle S \rangle = \frac{1}{4\pi} \mathbf{E} \cdot \mathbf{H} = \frac{c^2}{8\pi} E_0^2 \]

\[ \frac{d\mathbf{S}}{d\omega} = \frac{dP}{d\omega} \frac{1}{4\pi} = r_0^2 \sin^2 \theta \]
Mean over polarization in $x$-$y$-plane

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{tot}} = \frac{r_0^2}{2} \int_0^{2\pi} \frac{1}{1 + \cos^2 \theta} \, d\Omega$$

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{tot}} = \frac{r_0^2}{2} \frac{1}{1 + \cos^2 \theta}$$

Total cross section:

$$\sigma_{\text{tot}} = \int_0^{2\pi} \frac{d\sigma}{d\Omega} \, d\Omega$$

$$\sigma_{\text{tot}} = \frac{r_0^2}{2}$$ per electron

Dense material: $N$ electrons/volume

$$dn : \equiv \frac{dN}{dx} = -n \mu \, dx$$

$$n(x) = n(0) e^{-\mu x}$$

$\mu$: absorption coefficient

Thomson cross section, absorption

**Numerical exercise:**

$$\mu = N \sigma = 6.625 \times 10^{-25} \cdot 6.651 \times 10^{-22} \cdot \frac{\text{g} \cdot \text{cm}}{\text{cm}^3}$$

$$\mu = 0.40 \ \text{g/cm}^2$$

$g = 1 \ \text{g/cm}^3$ (water), $\frac{\mu}{N} = 0.5$

$\mu = 0.2 \ \text{cm}^{-1}$

$\lambda = \frac{1}{\mu} = 5 \ \text{cm}$ (soft scale, foetal doses)

Resonant absorption (photo-effect)

Absorption by bound electron:

- energy conservation: $h\nu \geq E_i$
- momentum conservation: falls off with $h\nu$

$$G = \frac{G_{\text{TH}}}{4} \gamma \left( \frac{\mu}{N} \right) \left( \frac{m c^2}{\hbar \nu} \right)^{3.5}$$
Absorption: Krypton, Xe, 30...50 bar, 5 cm

Radiation → drift cathode → ionisation → fluorescence photon

Anode plane → cathode strip → amplifier

Kα fluorescence: \( E_{\text{f}} = E_{K} - E_{L} \)

Auger electron: \( E_{K} = E_{K} - 2E_{L} \)...

Fluorescence yield: \( \eta = \frac{N_{\text{f}}}{N_{0} + N_{\text{Auger}}} \)
Influence of fluorescence radiation (Xe)
Position resolution:

![Incident beam diagram]

Photoelectron range $\propto E_{\text{max}}$

Electron range and resolution

![Graph showing electron range vs. log Ekin (keV)]

b) Katz-Penfold: $R(g/cm^2) = 0.442(E/keV)^n$

$n = 1.205 - 0.0954 \ln E/keV$

a) Grain: $R(g/cm^2) = 0.665(E/keV)^{1.25}$

c) Effective resolution including photo-absorption

$\Delta X(4\mu m) = 3.8\text{mg/cm}^2(E/keV)^{1.25}$
no cracks bad:
- additional information
- X-ray picture of a material composition
- welding seam
- inclusions
- no cracks
Dual Energy Method

measurement at two energies:

\[ S_1 = \ln \frac{I_1}{I_0} = (\mu_{1,1}x_1 + \mu_{1,2}x_2) \]

\[ S_2 = \ln \frac{I_2}{I_0} = (\mu_{2,1}x_1 + \mu_{2,2}x_2) \]

or

\[ \vec{S} = \begin{pmatrix} \mu_{1,1} & \mu_{1,2} \\ \mu_{2,1} & \mu_{2,2} \end{pmatrix} \circ \vec{x} \]

Solutions for rows and columns linear independent.

Method of measurement:
- two monoenergetic radiation sources
- detector with energy resolution
$\sigma_c$: total Compton cross section

$$\sigma_a = \frac{T_a}{hv} \sigma_c$$

$$\sigma^s = \frac{hv^2}{hv} \sigma_c$$

$$\sigma_a + \sigma^s = \sigma_c$$
LETI, Grenoble

make code simulation of composite materials

density

material
problem: simultaneously good position resolution $\Delta x \leq 0.25$ mm

implemented in BRITE/EURAM project:
CTS, ISO-Test, LETI, SIEGEN
+ industrial partners
Gas Amplification in Proportional Counter

\[ E(r) = \frac{V_0}{r} \ln \frac{r}{L} \quad \nu(r) = \frac{V_0}{L} \ln \frac{r}{P} \]

**avalanche:**

\[ dn = n(r) \alpha dr \quad \alpha : \text{Townsend coeff.} \]

\[ n = n_0 \exp \left( \int \alpha(r) dr \right) \]

From definition: \[ \alpha = \sigma N \]

\[ \sigma \text{: cross-section for ionisation by electron impact} \]

\[ N = N_0 \rho \text{ density of atoms} \]

\[ \alpha = \sigma N_0 \rho \]

\[ \% = N_0 \sigma \left( \frac{E}{\rho} \right) \]

Since \( \sigma \) depends on energy distribution of electrons in gas at reduced field \( E/\rho \):

"physics" scales with mean free path

**Parametrisation:**

\[ \frac{\alpha}{\rho} = A \cdot \exp \left\{ -B \cdot \left( \frac{E}{\rho} \right)^k \right\} \]

best: \[ k = 0.65 \]

Townsend: \[ k = 1 \]

\( \text{Ward: } k = \frac{1}{2} \)

**Problem:** in counters mixtures are used.

\[ n_{\text{air}}/n_{\text{CH}_4} (90/10) \]

\[ \text{best: } n_{\text{air}}/n_{\text{CH}_4} \]
Gas gain: measured and fitted to
\[ \frac{\Delta U}{V} = A \exp \left( - \frac{E}{\Delta E} \right) \]
Statistical Fluctuation and Energy Resolution

1) Primary ionisation:

\[ \bar{N}_0 = \frac{E_0}{W} \]

\[ (\Delta \bar{N}) = F \bar{N}_0 \quad F < 1 : \text{Fano factor} \]

\[ E \]

\[ E_0 + \Delta E \]

\[ N \text{ (steps)} \]

\[ n \text{ for all steps equal} \]

\[ \bar{N}_0 \text{ for steps with fluctuation} \]

\[ o \text{ for equal steps error is independent of } n \]

\[ \text{and given by } \Delta E / \sqrt{E} \]

\[ o \text{ for fluctuation in energy deposition it becomes dependent on number of steps:} \]

\[ (\Delta \bar{N})^2 = \bar{N}_0 \quad F = 0.2 \]
2) Gas amplification

avalanche \[ n = n_0 e^{a_n} \]

\( r = \frac{1}{n} \int \alpha \, dr \) \( n = 1 \) electron

\[ P(n) = \frac{1}{m} \left( 1 - \frac{1}{m} \right)^{m-1} \] (Furry)

\[ \langle \alpha n \rangle_{\text{max}} \cdot \bar{n}^2 (1 - \frac{1}{m}) = f \cdot \bar{n}^2 \]

\( \bar{n} \gg 1 \Rightarrow P(n) = \frac{1}{n} e^{-\frac{n}{m}}, \quad f = 1 \)

observe for large gain deviation (J.P.)

Physics background:

\[ \text{electron gains energy } E \sim eE \times \] cross section for ionisation.

\[ \text{mean free path becomes meaningless, electron ionises in well defined intervals.} \Rightarrow \text{less fluctuation} \]
Polya distribution:

\[ P(m, z) = \frac{1}{\Gamma(m)} \frac{m^m}{\Gamma(m)} z^{m-1} e^{-mz} \]

Detailed calculations by Alkhazov:

\[ m = 1 \Rightarrow f = 0.5 \]
Signal fluctuation:

\[ (\Delta S_{\text{tot}})^2 = (\bar{n})^2 F N_0 + (\bar{n})^2 N_0 \]

\[ \left( \frac{\Delta S_{\text{tot}}}{\bar{n} N_0} \right)^2 = \left( \frac{\Delta S}{S} \right)^2 = \frac{F + f}{N_0} = \left( \frac{F + f}{N_0} \right) \frac{W}{E} \]

Examples:

- \( W = 26 \text{eV} \), \( E = 1 \text{keV} \), \( F = 0.2 \), \( f = 0.6 \)
  - \( p = 5 \text{bar} \), \( \frac{\Delta S}{S} = 0.14 \) (5 bar)
  - \( p = 30 \text{bar} \), \( \frac{\Delta S}{S} = 0.08 \) (10 bar)

Measurements at 50 bar.
Energy resolution at 50 GeV

1) Pressure, recombination, attachment
   → clean gas, etc.

2) Limited streamer mode

   streamer (UV-mediated)

   Townsend av.
   anode

   → gas mixture, geometry
   (Ar/C4H4, 95:5 → 98:2)
Detectors for Position Resolution

Frank, Cambridge 1951
Principles of Position Resolution

1) Charge Division (Lautenjung, 1959)

\[ x \sim \frac{A}{A+B} \]

\[ \Delta \theta_e = \frac{\theta}{2} = 10^{-3} \]

2) Delay Line

\[ x \sim t_1 - t_2 \]

Signal Formation

\[ \text{electrons} \]

\[ \text{avalanche} \]

\[ \text{useful approximation:} \]

\[ s_0 \text{ at } t=0 \]

\[ \frac{q}{s_0} \text{ center of gravity of avalanche} \]

\[ r_0 \]

1) calculate

\[ V_{0t} = \frac{dS_{\text{sim}}}{dt} = \mu^+ E(s_{\text{sim}}), \quad \mu^+ \text{ mobility of ions} \]

\[ E(s_{\text{sim}}) = \frac{V_0}{s_{\text{sim}} \ln \gamma} \]

\[ \frac{dS_{\text{sim}}}{dt} = \mu^+ V_0 \frac{1}{s_{\text{sim}} \ln \gamma} \cdot \frac{d\ln s_{\text{sim}}}{dt} \]

\[ s_{\text{sim}} = \frac{2 \mu^+ V_0}{\ln \gamma} t + S_0^2 \]
Induction:

\[ Q^+ = \frac{|q^+|}{\ln \frac{r_s}{r_e}} \quad \text{en} \quad \ln \frac{r_s}{r_e} = q \]

\[ Q^- = \frac{|q^-|}{\ln \frac{r_s}{r_e}} \quad \text{en} \quad r_s \]

\[ Q_{tot} = Q^+ + Q^- = \frac{|q^+|}{\ln \frac{r_s}{r_e}} \left( \ln \frac{r_s}{r_e} - \ln \frac{r_s}{r_e} \right) \]

\[ Q_{tot} = \frac{|q^+|}{\ln \frac{r_s}{r_e}} \quad \text{en} \quad \frac{r_s}{r_e} \]

Electrons fast (< 1 m) on wire: \( s_e = r_s \)

\[ Q_{tot} = \frac{|q^+|}{\ln \frac{r_s}{r_e}} \quad \text{en} \quad \sqrt{\frac{2m_e}{e}} \left( \frac{s_e}{r_s} \right)^2 \]

\[ Q_{tot} = \frac{|q^+|}{\ln \frac{r_s}{r_e}} \quad \text{en} \left[ \frac{1}{2} + \left( \frac{s_e}{r_s} \right)^2 \right] \]

Example:

\[ r_i = 10^{-3} \text{cm} \quad r_s = 0.5 \text{cm} \quad \ln \frac{r_s}{r_e} = 6.2 \]

\[ \mu^+V_0 = 3 \text{ cm}^2 / \text{ms} \quad \text{(Ar/CH}_4\text{)} \]

\[ \frac{r_s}{r_e} = 1 \text{ ms} \]

\[ s_e = 2 \lambda_0 \quad \lambda_0 = \frac{1}{\rho \tau} \]

\[ n_0 = 7.6 \times 10^8 \text{ cm}^{-1} \text{atm}^{-1} \]

\[ s_e = 2 \frac{1.3}{\text{plam}} \text{ (mum)} = \frac{2.6 \text{ pm}}{\text{plam}} \]
Current/charge

\[ \frac{1}{q} \frac{dq}{dt} = \frac{1}{ln(3)} \left[ \frac{\gamma}{\gamma + 1} \right] = \frac{1}{2\ln(\gamma) + 1 + e^{-\gamma}} \]

\[ t_0 = \frac{S_0^2 \ln(\gamma)}{2\mu \gamma V_0} \quad S_0 = \frac{r}{\gamma} \text{ for } \gamma \geq 10 \mu m \]

Electron component

\[ Q_{tot} = Q(\gamma) + Q(r) = \frac{1}{\ln(\gamma)} \ln \frac{S_0}{S_0 + 1} \]

\[ R_e = \frac{Q_{tot} (S_0 = S_0 + 1) - Q_{tot} (S_0 = r)}{Q_{tot} (S_0 = r)} \]

with \( S_{av} = \frac{1}{2} \left( \frac{S_0^2}{\ln(\gamma)} + S_0^2 \right) = S_0 \sqrt{\frac{1}{\ln(\gamma) - 1}} \)

\[ R_e = \frac{\ln \frac{S_0}{r}}{\ln \frac{S_0}{r} + \frac{1}{2} \ln \left( \frac{S_0}{r} + 1 \right)} \]

Very small structure or low pressure
$t_0$ and electron component vs. range

$P_n(\mu m)$

$Q_{dl}(\mu m)$

$Q_{dl}(t+\infty)$

$Q_{lim}$

$Q_{vl}$

$Q_{vl}(t+\infty)$

$Q_{lim}$

$\frac{Q_{vl}}{Q_{lim}}$

$\frac{Q_{vl}}{Q_{lim}}$ vs. $P_n(\mu m)$

1967 Heidelberg, Boll, Keulie & Waentig
spark chamber

$^3H \rightarrow p + d$

large area
proportional
counter (TRIGGER!)

scintillator $\rightarrow$ te

$
\begin{array}{c}
\Delta E
\end{array}$

$\rightarrow$ to be used as position detector
Right-Left Problem

\[ a) \]

\[ b) \]

\[ \alpha \]

\[ y_0 \]

\[ x_0 \]

\[ P \]

\[ A_1, P_1, A_2 \]

\[ D \]

\[ C \]

\[ X_1 \]

\[ t_1, t_2 = \text{const} \]
a) CERN

<table>
<thead>
<tr>
<th>Potential wires</th>
<th>Drift space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transparent grid</td>
<td>Proportional chamber or counter</td>
</tr>
</tbody>
</table>

b) Saclay

- HT (2500 Volts)
- Gas
- Proportional chamber
- Drift space
- Single cell drift chamber

---

\[ t(n_s) \]

---

\[ x \]

---

\[ 5 \]

---

\[ 1 \]

---

\[ 10 \]

---

\[ 200 \]
Drift and Diffusion I

A.I.H. Walton

Free charges in electric field in gas

\[ \mathbf{F} = q \mathbf{E} \]

collisions with atoms
stationary

\[ \mathbf{E} = \frac{\mathbf{E}}{e^2} \]
random energy (\( \approx \) Maxwell distribution)
\( \mathbf{W} = \mu \mathbf{E} \) directed motion "drift"

Total current: superposition of directed motion + random motion

1) \( \mathbf{F} = n \mathbf{W} - \partial \mathbf{W} / \partial t \) continuity equation

2) \( \frac{\partial n}{\partial t} + \text{div} \mathbf{F} = 0 \)

interest in detector: \( n(x, y, z, t) \)

\[ \frac{\partial n}{\partial t} + \nabla \cdot \mathbf{F} = 0 \]

solution

\[ n(x, y, z, t) = \frac{1}{8\pi^4D^4} \int_0^t \int_{|y-y'|<b} n(x, y, z, \tau) \frac{b^2}{4Dt} \, dy \, d\tau \]

\[ n(x, y, z, t) = n(x, y, z, 0) e^{-t^2/4Dt} \quad \text{(Einstein formula)} \]

at detector \( n(x_0, t) \) distortion

\[ x_0 - x_0(t) = \frac{1}{2} \frac{D}{\mu^2} \]

for max. of \( n(x_0, t) \) signal on detector

\[ x_0 = 0 \]

for max. of \( n(x, t) \) centroid
1) Momentum conservation

\[ m \dot{\mathbf{W}} = \left\{ \int_{0}^{t_{s}} \dot{e} \, dt \right\} \]

\[ \mathbf{W} = V_{\text{brill}} \]

- \( t_{s} \): time between encounters
- \( \bar{t}_{s} \): mean collision time
- \( \tilde{q} \): statistical factor = 0.95

\[ m \dot{W} = \tilde{q} \int_{0}^{\bar{t}_{s}} e \, E \, dt \]

\[ \dot{W} = \tilde{q} \frac{e E}{\bar{t}_{s}} \]

\[ \dot{W} = \frac{e E}{\bar{t}_{s}} \]

\[ \dot{W} = \frac{e E}{0.95} \]

2) Energy conservation

\[ dE : \text{energy from } E/t \text{ time interval} \]

\[ dE = e E \frac{dx}{dt} \]

\[ = e E \dot{W} \]

\[ dE^3 : \text{energy absorbed/mean collision time} \]

\[ dE^3 = \tilde{q} e / \tilde{t} \]

\[ \tilde{q} : \text{mean energy loss/impact} \]

\[ e E \dot{W} = \tilde{q} e / \tilde{t} \]

\[ W^3 = \tilde{q} \tilde{q} \tilde{t} \]

\[ W = \tilde{q} \sqrt{\tilde{q} \tilde{t}} \]

\[ \tilde{q} = 0.95 \]

3) From transport equation

\[ \frac{dE}{dt} = \frac{3}{2} E \]
inelastic encounters (sub. & cond. excitations)

$W = 0.65 \sqrt{E}$

total cross section

$V_{th} \sim \frac{5}{2} \frac{m}{C} \quad \bar{E} : \text{thermal}$
$C : \text{charge}$
\[ W = 0.85 \frac{E/p}{\sigma C} \]

\[ V/cm(\mu s^{-1}) \]

\[ E/P \ (V/cm\cdot Torr^{-1}) \]

- CH$_4$: Cottell & Walker
- Ar/0.5$\%$ CH$_4$: Nagy \\& D'Ani
- Hakeem \\& Nathanson
- CO$_2$: Hake \\& Phelps
Diffusion

\[ \sigma^2 = 2Dt \quad \sigma = \mu \sqrt{t} \]

rel. resolution: \[ \frac{\sigma^2}{X} = \frac{1}{\sqrt{X}} = \frac{\sigma^2}{X} = \frac{1}{\sqrt{X}} \]

\[ \frac{\sigma^2}{X} = \frac{2D}{\mu} \text{ or } \frac{2D}{\nu} \]

\[ \nu = \frac{e^2}{E} \]

\( \nu \): characteristic energy

\( \nu = kT \) for \( E = 0 \) or no heating by electric field

Small diffusion:

\[ \frac{\sigma^2}{X} = \frac{2D}{V} = \frac{2D}{\nu \eta} \]

no explicit field or var.

dependence!

\[ \frac{\sigma^2}{X} = \eta \frac{2}{\nu \eta} \]

\( \eta \): statistical factors = 1

\( \eta \propto \) N large (pressure), \( V \) large, \( \eta \lbrack \text{large} \)
Diffusion with respect to drift direction

Experimentally discovered $D_L < D_0$  
(Wagner, Davis, Hurst)

Reason for $D_L \neq D_0$

\[ \frac{\partial f}{\partial t} + \text{div} \cdot \text{E} f = \text{div} \cdot \left( a(x) \phi \right) f = \frac{\partial f}{\partial t} \text{rec} \]

This term had been neglected!

Formal solution by Parker & Crowe

Simple calculation of $D_i/D_0$

$W$ change of energy distribution

If $W_t (E)$ decreases with incr. $E$:

\[ W = \frac{nE^2}{m} \quad \nu = n\phi \sigma \]

Approx. $\nu = \nu_0 + \frac{\partial \nu}{\partial E} \Delta E$

$\Delta E = E_0 + \Delta E$

Energy conservation:

\[ \gamma \frac{2m}{h} \Delta \nu = \frac{E}{n} \Gamma \quad \Gamma = \text{total current} \]
\[ \bar{\nabla}\cdot n = \bar{v} \cdot (\frac{\partial}{\partial x}) \]

\[ \Rightarrow \]

\[ \Gamma_{av} = \frac{\varepsilon E}{m} \bar{v} - \frac{2}{3} \frac{\varepsilon}{m} \bar{v}^2 \]

\[ = \frac{\varepsilon}{m} \bar{v} \]

solve for $\Delta \varepsilon$

\[ \Delta \varepsilon = -\frac{2}{3} \frac{\varepsilon}{m} \frac{1}{1+2\varepsilon} \bar{v} \]

influence on current:

\[ \bar{v} = \frac{1}{\nu} \left( \frac{n \varepsilon E}{m} - \frac{2}{3} \frac{\varepsilon}{m} \bar{v} \bar{v} \right) \]

\[ = \frac{1}{\nu} \left( 1 - \frac{\bar{v}}{v_0} \right) \frac{n \varepsilon E}{m} - \frac{2}{3} \frac{\varepsilon}{m} \bar{v} \bar{v} \]

\[ = \frac{n \varepsilon E}{m} \bar{v} + \frac{\bar{v}}{1+2\varepsilon} \frac{\varepsilon}{m} \bar{v} \bar{v} - \frac{2}{3} \frac{\varepsilon}{m} \bar{v} \bar{v} \]

\[ \bar{v} = n \bar{w}_0 - \frac{\bar{v}}{1+2\varepsilon} \frac{\varepsilon}{m} \bar{v} \bar{v} \]

\[ \frac{D \bar{v}}{D_0} = \frac{1+2\varepsilon}{1+2\varepsilon} \]

Application to gases:

\[ \sigma = C_0 \left( \frac{\varepsilon}{\bar{v}} \right) \]

\[ \Rightarrow \]

\[ v = v_0 \left( \frac{\varepsilon}{\bar{v}} \right) \frac{\varepsilon}{2} \]

\[ \Rightarrow \]

\[ D \bar{v}/D_0 = \frac{1}{2} \frac{3\varepsilon}{2+\varepsilon} \]

Better formula:

\[ D \bar{v}/D_0 = \frac{1}{2+\varepsilon} \]

**Examples**

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>Simple</th>
<th>Robson</th>
</tr>
</thead>
<tbody>
<tr>
<td>$=-1$</td>
<td>$D_\bar{v}/D_0 = 1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$=0$</td>
<td>$D_\bar{v}/D_0 = \frac{\varepsilon}{2}$</td>
<td>$\frac{\varepsilon}{2}$</td>
</tr>
<tr>
<td>$=1$</td>
<td>$D_\bar{v}/D_0 = \frac{\varepsilon}{3}$</td>
<td>$\frac{\varepsilon}{3}$</td>
</tr>
</tbody>
</table>

**Limit for $\varepsilon$ large**

| $D_\bar{v}/D_0 = \frac{\varepsilon}{2}$ | $0$ |
$D_v/D_0 = 0.24 \quad \text{observed (data)} \quad D_{v0}/D_0 = 0.285$

**Fig. 1.12:** Anisotropie der Diffusionskoeffizienten parallel ($D_v$) und senkrecht ($D_0$) zum elektrischen Feld $E$ als Funktion des reduzierten elektrischen Feldes $E/p$ (Einheit V cm$^{-1}$ Torr$^{-1}$) bzw. $E/N$ (Einheit $8.6 \times 10^{-15}$ cm$^2$ V$^{-1}$ Torr$^{-1}$). Kreispunkte nach [80 86], berechnete Kurven nach [90 68] und [80 72].
Effect of clusters

-27-
2. In TEC signal follows ionisation structure.

3. Measurement of center of gravity of He electrons:

\[ \sigma = \frac{1}{\sqrt{2} m} \sigma_0 \]

Center of Gravity

Leading Edge (Standard Drift Chamber)

Timing for drift distance determination is given by first electron arriving at the anode.

\[ \sigma_{1st} = \frac{\pi}{2 \sqrt{3} \alpha n m} \sigma_0 \]
$\sigma = F \cdot G$

$\frac{dW}{dx} = n g e^{n-1}$

$g$: Gaussian function
$e$: error

<table>
<thead>
<tr>
<th>n</th>
<th>f(x) = e^{-x^2}</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.79</td>
<td>0.66</td>
<td>0.40</td>
</tr>
</tbody>
</table>

$X$ (in units of $\sigma$)
Drift Chamber Adaption

\[ U \]

- \( T_s \)

\[ B = 200 \mu m \]

\[ \frac{T_s \cdot v_0}{3} = \frac{200 \mu m}{30 ns} \cdot 7 \mu m/\mu s \]

\[ \Rightarrow \]

1) slow \( v_0 \)

2) bandwidth of "gas-amplifier" = 500 MHz

Charge Distribution  Signal  Signal after Shaper

\[ g \]

\[ u \]
Diffusion for TEC

Einstein relation: $\sigma^2 = 2Dt$

using: $D = \frac{\xi}{\mu\epsilon} \Rightarrow$

$$\sigma^2 = 2 \frac{\xi}{\mu \epsilon}$$

Drift Velocity for TEC

\[
\begin{align*}
W &= \frac{e}{m} \frac{E}{N} \frac{1}{\sqrt{c}} \\
W \frac{\sigma^2}{X} &= \frac{e}{m} \frac{E}{N} \frac{1}{\sqrt{3}} \frac{1}{\sqrt{c}} \frac{1}{\sqrt{3e}} \frac{1}{\sqrt{E}} \frac{1}{\sqrt{2mc^2}} \\
&= \frac{2 \epsilon}{3 NC^2}
\end{align*}
\]

\(c\) small (thermal)
\(N\) large (density)
\(\sigma\) large (molecules)

\(\Rightarrow\) small $\frac{\sigma^2}{X}$
COMPARISON BETWEEN TWO ANALYSIS METHODS:
<< LEADING EDGE >>: • [0.8°] ○ [3.2°]
<< CENTER OF GRAVITY >>: ■ [0.8°] □ [3.2°]
(1) BEAM ANGLE
RUN: 28-32 ETH/Siegen

CERN DATA: A = C_{60} + methyl
MWPC: $d_s = 0.5 \text{ mm} \ldots 2 \text{ mm}$

NSG: $d_s = 0.25 \text{ mm}$

**Small Cells**
Support of wires to prevent electrostatic instabilities
b) Glass: D263

- 55Fe (5.9 keV)
- Anode potential: +650 V
- Cathode potential: -2000 V
- Gas: Ar/CH₄ 90/10

Pulse height vs. number of events graph:

Field lines on insulator: no space charge
Field lines on conductor: no charging of insulator, stable operation
Fig. 1) A cross section of the detector internal structure

Fig. 2) A photograph of one anode-cathode micro-gap
Fig. 8. Gain as a function of rate measured with MS1 and MS2 at an avalanche size of $3 \times 10^5$ electrons.

Conductive glass

Latest achievement:

Conductive coating with C, N, Si

Fig. 9. The 5.4 keV signal from the OR of several anode strips at a flux of $2 \times 10^6$ photons s$^{-1}$.

Fig. 10. The normalized gain as a function of rate.