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ντ Oscillation Experiments and Present Data

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Our goal in this paper is to examine the discovery potential of laboratory experiments searching for the oscillation $\nu_\mu \rightarrow \nu_\tau$, in the light of recent data on solar and atmospheric neutrino experiments, which we analyse together with the most restrictive results from laboratory experiments on neutrino oscillations in a four-neutrino framework.

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1 Introduction

If neutrinos have a mass, a neutrino produced with flavour $\alpha$, after travelling a distance $L$, can be detected in the charged-current (CC) reaction $\nu N \rightarrow l_\beta N$ with a probability

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha,\beta} - 4 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \text{Re}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin^2 \left( \frac{\Delta_{ij}}{2} \right),$$

where $U$ is the mixing matrix. The average includes the dependence on the neutrino energy spectrum, the cross section for the process in which the neutrino is detected, and detection efficiency for the experiment. The probability, therefore, oscillates with oscillation lengths $\frac{\Delta_{ij}}{L} = 1.27 \frac{[m_i^2 - m_j^2]}{E} \frac{L}{m/\text{MeV}}$ where $E$ is the neutrino energy. For oscillation lengths such that $\Delta_{ij} \gg 1/(L/E)$ the oscillating phase will have been over many cycles before the detection and therefore it will have averaged to $1/2$. On the other hand, for $\Delta_{ij} \ll 1/(L/E)$, the oscillation did not have time to give any effect.

Present data from solar and atmospheric neutrino experiments favour the hypothesis of neutrino oscillations. All solar neutrino experiments find less $\nu_e$ than predicted theoretically. As for atmospheric neutrino experiments, two

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of them measure a ratio $\nu_\mu/\nu_\tau$ smaller than expected from theoretical calculations. Nevertheless, this interpretation requires confirmation from further experiments, in particular from laboratory experiments, where the experimental conditions, in particular the shape, energy, and flux of the neutrino beam are under control.

At present all laboratory neutrino experiments report no evidence for neutrino oscillation with the possible exception of LSND, which looks for $\nu_\mu \rightarrow \nu_\tau$ oscillations. In addition a number of new experiments are in the process of starting taking data of being proposed both at CERN and Fermilab. Our goal is to examine the discovery potential of these experiments searching for the oscillation $\nu_\mu(\nu_\tau) \rightarrow \nu_\tau$, in the light of recent data on solar and atmospheric neutrino experiments, which we analyse together with the most restrictive results from laboratory experiments on neutrino oscillations in a four-neutrino framework.

2 Four-Flavour Models

Naive two-family counting shows that it is very difficult to fit all experimental information mentioned above with three neutrino flavours. In the spirit of Pauli, one is tempted to introduce a new neutrino as a "desperate solution" to understand all present data. The nature of such a particle is constrained by LEP results on the invisible $Z$ width as well as data on the primordial He abundance. Those rule out the existence of additional, light, active neutrinos. In consequence the fourth neutrino state must be sterile.

If one assumes a natural mass hierarchy with two light neutrinos with their main projection in the $\nu_\tau$ and $\nu_\mu$ directions and two heavy neutrinos with their largest component along the $\nu_\mu$ and $\nu_\tau$ flavours the mixing matrix can be parametrized in a general way as

$$U = \begin{pmatrix}
\nu_1 & c_{es} & s_{es}c_{e\mu}c_{e\tau} & s_{es}s_{e\mu} & s_{es}c_{e\mu}s_{e\tau} \\
\nu_2 & -s_{es} & c_{es}c_{e\mu}c_{e\tau} & c_{es}s_{e\mu} & c_{es}c_{e\mu}s_{e\tau} \\
\nu_3 & 0 & -c_{\mu\tau}s_{e\mu}c_{e\tau} - s_{\mu\tau}s_{e\tau} & c_{\mu\tau}c_{e\mu} & -c_{\mu\tau}s_{e\mu}s_{e\tau} + s_{\mu\tau}c_{e\tau} \\
\nu_4 & 0 & s_{\mu\tau}s_{e\mu}c_{e\tau} - c_{\mu\tau}s_{e\tau} & -s_{\mu\tau}c_{e\mu} & s_{\mu\tau}s_{e\mu}s_{e\tau} + c_{\mu\tau}c_{e\tau}
\end{pmatrix}$$

(2)

with $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$. For the sake of simplicity we have assumed no CP violation in the lepton sector. We also required that the sterile neutrino does not mix directly with the two heavy states to verify the constraints from big bang nucleosynthesis. Such a hierarchy appears naturally, for instance if one advocates an $L_\mu \pm L_\mu \mp L_\mu$ discrete symmetry for the mass matrix. In Ref. a similar mass pattern is also generated via a combination of see-saw
mechanism and loop mechanism. In this approximation \( m_1, m_2 \ll m_3, m_4 \) and 
\[ |m_3^2 - m_4^2| \ll m_3^2, m_4^2. \]

The value of \( m_3 \approx m_4 \) is inferred from the dark matter data. Currently, the best scenario to explain the data considers a mixture of 70% cold plus 30% hot dark matter. This implies \( m_3 \simeq m_4 = 2-3.5 \text{ eV} \). Such a mass pattern has been argued to yield satisfactory results in Cold + Hot Dark Matter scenarios.

We define
\[
\Delta^2_{\text{sol}} = |m_1^2 - m_2^2|, \quad \Delta^2_{\text{AT}} = |m_3^2 - m_4^2|
\]
\[
\Delta^2_{\text{DM}} = |m_1^2 - m_3^2| \approx |m_1^2 - m_4^2| \approx |m_2^2 - m_3^2| \approx (4 - 10) \text{ eV}^2.
\]

Transition probabilities between the different flavors will have therefore contributions from the three oscillation lengths due to the three different mass differences in the problem which we will denote \( \sin^2(\Delta_{\text{sol}}/2) \), \( \sin^2(\Delta_{\text{AT}}/2) \), and \( \sin^2(\Delta_{\text{DM}}/2) \), respectively.

3 Global Analysis

At present the most precise laboratory experiments searching for neutrino oscillations are the reactor experiment at Bugey, which looks for \( \nu_e \) disappearance, and the CDHSW experiment at CERN, which searches for \( \nu_\mu \) disappearance. The E776 experiment at BNL searches for the \( \nu_\mu \to \nu_\tau \) appearance channel and the E531 experiment at Fermilab for the \( \nu_\mu \to \nu_e \) channel. Neither of these experiments shows evidence for neutrino oscillation on those channels.

Recently the Liquid Scintillator Neutrino Detector (LSND) experiment has announced the observation of an anomaly that can be interpreted as neutrino oscillations in the channel \( \nu_\mu \to \nu_e \). Most of the oscillation parameters required as explanation are already excluded by the E776 and KARMEN experiments.

For the Bugey reactor experiment the relevant transition probability is the \( \nu_e \) survival probability. For any value of the atmospheric mass difference this probability will always verify
\[
0.93 \lesssim P_{ee}^{\text{Bugey}} \leq 1 - 2c^2_{\nu\mu}c^2_{\nu\tau}(1 - c^2_{\nu\mu}c^2_{\nu\tau}) \Rightarrow c^2_{\nu\mu}c^2_{\nu\tau} \geq 0.96. \quad (4)
\]

For CDHSW the relevant probability is the \( \nu_\mu \) survival probability
\[
0.95 \lesssim P_{\nu\mu}^{\text{CDHSW}} \leq 1 - \frac{1}{2} \sin^2(2\theta_{\nu\mu}) \Rightarrow \sin^2(2\theta_{\nu\mu}) \lesssim 0.1. \quad (5)
\]

For E776 the situation is somehow more involved, since the value of the oscillating phase \( \langle \sin^2(\Delta_{\text{DM}}/2) \rangle \) varies in the range \( \Delta_{\text{DM}} = 4-10 \text{ eV}^2 \) due to
the wiggles of the resolution function of the experiment. Also, the experiment is sensitive to the atmospheric mass difference. We find that the limit $1.5 \times 10^{-3} \geq P_{\mu}^{E531}$ is verified for any value of the atmospheric mass difference and the $\mu \tau$ mixing angle, if

$$\sin^2(2\theta_{e\mu})c_{e\tau}^2 \leq (2.5) \times 10^{-3} \quad (6)$$

The limit from E531 on the mixings $e\mu$ and $e\tau$ is always less restrictive than the previous ones for any value of $\Delta m^2_{AT}$ and $\Delta M^2_{DM}$.

Combining these constraints we obtain that $e\mu$ and $e\tau$ mixings are constrained to

$$\sin^2(2\theta_{e\mu}) \leq (2.5) \times 10^{-3} \quad \sin^2(2\theta_{e\tau}) \leq 0.16 \quad (7)$$

where the range of $\sin^2(2\theta_{e\mu})$ depends on the specific value of $\Delta M^2_{DM}$.

If we now turn to the effect due to the oscillation with $\Delta_{AT}$, we can rewrite the relevant probabilities for the different experiments expanding in the small angles $e\mu$ and $e\tau$:

$$\begin{align*}
P_{\mu \mu}^{CHSW} & \simeq 1 - \frac{1}{2} \sin^2(2\theta_{e\mu}) - \sin^2(2\theta_{\mu\tau}) \sin^2(\frac{\Delta m^2_{AT}}{2}) \\
P_{\mu \tau}^{E531} & \simeq \sin^2(2\theta_{\mu\tau})c_{e\tau}^2 \sin^2(\frac{\Delta M^2_{DM}}{2}) \\
P_{\mu \mu}^{E531} & \simeq \sin^2(2\theta_{e\mu})c_{\tau e}^2 \sin^2(\frac{\Delta m^2_{AT}}{2}) + \sin^2(2\theta_{\mu\tau})s_{e\tau}^2 \sin^2(\frac{\Delta M^2_{DM}}{2}) . \quad (8)
\end{align*}$$

With the constraints in Eq. (7), the Bugey experiment is not sensitive to oscillations with $\Delta_{AT}$. The relevant exclusion contours for each channel are shown in the Figures.

We now turn to the atmospheric neutrino data. Neutrinos are produced when cosmic rays hit the atmosphere and initiate atmospheric cascades. The mesons present in the cascade decay leading to a flux of $\nu_e$ and $\nu_\mu$ which reach the Earth and interact in the different neutrino detectors. Naively the expected ratio of $\nu_\mu$ to $\nu_e$ is in the proportion $2:1$, since the main reaction is $\pi \rightarrow \mu \nu_\mu$ followed by $\mu \rightarrow e\nu_\mu \nu_e$. However, the expected ratio of muon-like interactions to electron-like interactions in each experiment depends on the detector thresholds and efficiencies as well as on the expected neutrino fluxes. Currently four experiments have observed atmospheric neutrino interactions. Two experiments, Kamiokande $^{3,4}$ and IMB $^5$, have observed a ratio of $\nu_\mu$-induced events to $\nu_e$-induced events smaller than the expected one. In particular Kamiokande has performed two different analyses for both sub-GeV neutrinos $^3$ and multi-GeV neutrinos $^4$, which show the same deficit. On the other hand, the results from Fréjus and NUSEX $^5$ appear to be in agreement with the predictions.
The results of the three most precise experiments in terms of the double ratio $R_{\mu/e}/R_{\mu/e}^{MC}$ of experimental-to-expected ratio of muon-like to electron-like events are

\[
R_{\mu/e}/R_{\mu/e}^{MC} = 0.55 \pm 0.11 \quad \text{for IMB}
\]
\[
R_{\mu/e}/R_{\mu/e}^{MC} = 0.6 \pm 0.09 \quad \text{for Kamiokande sub-GeV}
\]
\[
R_{\mu/e}/R_{\mu/e}^{MC} = 0.59 \pm 0.1 \quad \text{for Kamiokande multi-GeV}
\]
\[
R_{\mu/e}/R_{\mu/e}^{MC} = 1.06 \pm 0.23 \quad \text{for Fréjus}
\]

The statistical and systematic errors have been added in quadrature. The systematic error contains a 5% contribution due to the neutrino flux uncertainties. For the Monte Carlo prediction we have used the expected fluxes from $^{16}$ depending on the neutrino energies. Use of other flux calculations would yield similar numbers.

In each experiment the number of $\mu$ events, $N_{\mu}$, and of $e$ events, $N_{e}$, in the presence of oscillations will be

\[
N_{\mu} = N_{\mu}^{\nu} \langle P_{\mu\mu} \rangle + N_{\mu}^{\nu} \langle P_{\mu\mu} \rangle, \quad N_{e} = N_{e}^{\nu} \langle P_{ee} \rangle + N_{e}^{\nu} \langle P_{ee} \rangle,\tag{10}
\]

where

\[
N_{\alpha\beta}^{\nu} = \int \frac{d^2 \Phi_\alpha}{dE_\nu d \cos \theta_\nu} \frac{d\sigma}{dE_\beta} \epsilon(E_\beta) dE_\nu dE_\beta d(\cos \theta_\nu)
\]

and

\[
\langle P_{\alpha\beta} \rangle = \frac{1}{N_{\alpha\beta}^{\nu}} \int \frac{d^2 \Phi_\alpha}{dE_\nu d \cos \theta_\nu} P_{\alpha\beta} \frac{d\sigma}{dE_\beta} \epsilon(E_\beta) dE_\nu dE_\beta d(\cos \theta_\nu).
\]

Here $E_\nu$ is the neutrino energy and $\Phi_\alpha$ is the flux of atmospheric neutrinos $\nu_\alpha$; $E_\beta$ is the final charged lepton energy and $\epsilon(E_\beta)$ is the detection efficiency for such charged lepton; $\sigma$ is the interaction cross section $\nu N \rightarrow N’ l$. The expected rate with no oscillation would be $R_{\mu/e}^{MC} = N_{\mu}^{\nu}/N_{e}^{\nu}$ . The double ratio $R_{\mu/e}/R_{\mu/e}^{MC}$ is then given by

\[
\frac{R_{\mu/e}}{R_{\mu/e}^{MC}} = \frac{\langle P_{\mu\mu} \rangle + N_{\mu}^{\nu} \langle P_{\mu\mu} \rangle}{\langle P_{ee} \rangle + N_{e}^{\nu} \langle P_{ee} \rangle + N_{e}^{\nu} \langle P_{mu} \rangle}.
\]

We perform a global fit to the data in Eq. (9). In Fig. 3 the results are shown for zero mixings $e\mu$ and $e\tau$ as in a two-family scenario. Figure 4 shows the effect of the inclusion of the mixings. As seen in the figure the inclusion of the $e\tau$ mixing leads to a more constrained area for the oscillation parameters. The effect of the $e\tau$ mixing is to increase the value of the double ratio since there is
Table 1: Summary of the performance parameters for CHORUS and NOMAD.

<table>
<thead>
<tr>
<th>Sensitivity:</th>
<th>NOMAD</th>
<th>CHORUS</th>
<th>E803/NAUSICAA (CERN)</th>
<th>MINOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{e\tau}$</td>
<td>$1.3 \times 10^{-2}$</td>
<td>$0.8 \times 10^{-2}$</td>
<td>$7 \times 10^{-4}$</td>
<td>$-0.012$</td>
</tr>
<tr>
<td>$P_{\mu\tau}$</td>
<td>$2 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2: Accessible regions (90\% CL) for the $\nu_\mu \rightarrow \nu_\tau$ optimum value of the mixing $c_\tau$.

The fine solid line corresponds to the CHORUS/NOMAD experiments and the dotted line corresponds to the Fermilab experiment E803 and to the improved CERN experiment. The dot-dashed line corresponds to NAUSICAA at Fermilab and the dashed line corresponds to MINOS. Also shown in the figure are the regions at present excluded by E776 data (shaded area) and the favoured range on $\Delta M_{DM}^2$ from dark matter considerations (solid horizontal lines). For comparison also the LSND data are shown.

The two upcoming $\nu_\mu(\nu_\tau) \rightarrow \nu_\tau$ experiments, CHORUS and NOMAD\(^8\), are $\nu_\tau$ appearance experiments, i.e., they search for the appearance of $\nu_\tau$'s in the CERN SPS beam consisting primarily of $\nu_\mu$'s, with about 1\% $\nu_\tau$'s. The mean energy of the $\nu_\mu$ beam is around 30 GeV and the detectors are located approximately 800 m away from the beam source. Their expected performance are summarized in Table 1. There are a number of future $\nu_\mu(\nu_\tau) \rightarrow \nu_\tau$ experiments being discussed at present. As a specific example of these experiments we have considered a suggestion to upgrade the NOMAD detector\(^7\). We will refer to this future detector with the generic name of Neutrino ApparatUS with Improved CApAbilities (NAUSICAA). The detector performance is summarized in Table 1.

At Fermilab, a neutrino beam will be available when the main injector becomes operational, around the year 2001. Compared with the CERN SPS beam, the main injector will deliver a beam 50 times more intense, but with an average energy around one third of that of the SPS neutrinos. There are
currently two experiments proposed to operate in this beam. One is a short-
baseline experiment, E803, and MINOS a long-baseline experiment, which
proposes two detectors, separated by 732 km. This experiment can perform sev-
eral tests to look for a possible oscillation $\nu_\mu \rightarrow \nu_\tau$ in the small mass difference
range. The expected performance of E803 and MINOS are summarized in
Table 1. Finally we will consider the possibility of installing the NAUSICAA
detector as an alternative or a successor to E803 in the Fermilab beam which
will improve by one order of magnitude its sensitivity.

After implementing the limits derived in Sec. 3 and considering the sen-
sitivity of the experiments, one sees that for all facilities the only observ-
able $\nu_e \rightarrow \nu_\tau$ transition oscillates with an oscillation length $\Delta_{DM}$ such that
$P_{\mu\tau} \simeq \sin^2(2\theta_{\mu\tau})\sin^2(\frac{\Delta_{DM}}{2})$. Figure 1 shows the regions accessible to the
experiments in the $(\sin^2(2\theta_{\mu\tau}), \Delta M^2_{DM})$ plane. For transitions $\nu_\mu \rightarrow \nu_\tau$ a
four-neutrino framework predicts (unlike the naive two-family framework) two
oscillations, dominated by the characteristic lengths $\Delta_{DM}$ and $\Delta_{AT}$. All ex-
periments are in principle sensitive to both oscillations depending on the values
of the mixing angles

$$P_{\mu\tau}^{DM} \simeq \sin^2(2\theta_{\mu\tau})\sin^2(\theta_{\mu\tau})\sin^2(\frac{\Delta_{DM}}{2}) \quad P_{\mu\tau}^{AT} \simeq \sin^2(2\theta_{\mu\tau})\cos^2(\theta_{\mu\tau})\sin^2(\frac{\Delta_{AT}}{2})$$

In Fig. 2 we show the regions accessible for the oscillation with oscillation
length $\Delta M^2_{DM}$ to the different experiments for an optimum value of the $\epsilon\tau$
mixing.

Figures 3 and 4 show the region accessible to the experiments for the
oscillation in $\Delta m^2_{AT}$ for different values of the other mixings.

References

1. Based on the work by J. J. Gomez-Cadenas, and M. C. Gonzalez-Garcia,
   Phys. C.
Figure 3: Accessible regions [90% CL] for the $\nu_\mu \rightarrow \nu_\tau$ oscillation in the $[\Delta m^2_{AT}, \sin^2(2\theta_{\mu\tau})]$ plane for zero value of the mixing $\sin^2(\theta_{\mu\tau})$. The fine solid line corresponds to the CHORUS/NOMAD experiments and the dotted line corresponds to the Fermilab experiment E803 and to the improved CERN experiment. The dot-dashed line corresponds to NAUSICAA at Fermilab and the dashed line to MINOS. Also shown in the figure are the regions at present excluded by CDHSW data (dark shaded area) and the atmospheric neutrino analysis (light shaded area).

Figure 4: Same as previous figure for maximum mixing $\sin^2(\theta_{\mu\tau})$. Also shown in the figure are the region at present excluded by CDHSW data (dark shaded area) and the atmospheric neutrino analysis (light shaded area). These are the only constraints for zero or values of the $e\mu$ mixing $\sin^2(2\theta_{e\mu}) < 0.003$. For maximum value $\sin^2(2\theta_{e\mu}) \approx 0.003$ the E776 limit (thick dot-dashed line) is relevant.