I. INTRODUCTION

Absolute beam intensity measurements have been the privilege of beam transformers. When it was considered convenient to turn to other instruments, transformers were still used for calibration purposes. The reason for this is the following. A simple, straightforward and reliable relation exists between the beam current (which can not be intercepted to be measured) and the response of the transformer, i.e. the turns ratio. The approach reported in this paper is that the output of an electromagnetic monitor can be known with the same confidence level as for a transformer. From then onwards it is sufficient to analyse and measure the various signal processing building blocks, including the acquisition system, that complete the measuring chain.

The bunch intensity measurement chain contains the following elements: an electro-magnetic coupler, summing hybrids, (long) signal transmission cables, signal filters, narrow band homodyne receivers, amplifiers and a peak signal detection system. The performance of the most important elements will be described below. The layout of the monitors is sketched in Figure 1.

II. TRANSFER IMPEDANCE OF COUPLER

It may be interesting to note that the monitor was designed and built to measure beam position. It consists of four $50\Omega$ striplines shorted on one end. In [1] a formula was derived to compute the relevant properties of a stripline. This formula was found to be reliable based on laboratory tests on several existing monitors and on different prototypes developed for the LHC. A typical monitor is sketched in Figure 2.
The maximum value of the transfer impedance of a stripline electrode is:

$$Z_t = \frac{\mu_0}{\varepsilon_0} \frac{2}{\pi} \ln \left( \frac{r}{r-h} \right),$$  \hfill (1)

where $r=0.0665\,\text{m}$ is the outer radius of the monitor body and $h$ the height of the stripline. The stripline impedance $Z$ can be found with:

$$Z = \frac{\sqrt{\mu_0/\varepsilon_0}}{2\pi} \frac{2}{\Phi} \ln \left( \frac{r}{r-h} \right),$$  \hfill (2)

where $\Phi$ is the total subtended angle which is larger than the mechanical subtended angle by an amount $\Delta\Phi$. The angles $\phi$ and $\Delta\phi$ are determined by:

$$\Phi = \Delta\phi + \frac{w}{r-h-t}$$

and

$$\Delta\phi = 2\pi \frac{h}{2r-h}$$ \hfill (3)

$w=0.027\,\text{m}$ is the width of the line and its thickness is $t=0.0015\,\text{m}$. The structural dimension $h$ is not always as well known as the other dimensions $r,w$ and $t$ since it is adjusted during mounting such that $Z=50\,\Omega$. However, an accurate value for $h$ can be derived from the set of equations (2),(3). This yields $h=0.0067\,\text{m}$ and $Z_t=6.37\,\Omega$.

The actual value of the transfer impedance depends upon the working frequency and the length of the stripline:

$$Z_t = Z_t \sin \left( \frac{\omega t}{c} \right).$$ \hfill (4)

The length $l$ of the stripline in a first type of monitor is 0.6m. An operating frequency of 50 MHz yields $Z_t=3.75\,\Omega$. The length of a second type is 0.34m. For an operating frequency of 50 MHz we find $Z_t=2.22\,\Omega$ and for 200 MHz (special application see below) $Z_t=6.3\,\Omega$.

### III. THE RECEIVERS

The bunch signal can in general be written as:

$$f(t) = A(\omega \pm \delta\omega) e^{\pm j(\omega \pm \delta\omega)t},$$ \hfill (5)

where $A(\omega \pm \delta\omega)$ is defined by the bunch spectrum and the characteristic of the filter. The homodyne receiver is a non linear system in which the input signal is shifted by its central frequency $\omega$. This yields a low frequency component at $\pm \delta\omega$ and a component at twice the input frequency. The single sideband low frequency demodulation product is then:

$$f_{dem}(t) = A(\omega \pm \delta\omega) e^{\pm j\delta\omega t}. \hfill (6)$$

### IV. THE FILTERS

The complete system contains two filters. The first one is a bandpass filter which selects the operating frequency (50 or 200MHz) of the receiver. The second one is a low pass filter which rejects the high frequency demodulation harmonics of the receiver. The bandwidth of the second filter is the largest one. The combined filter integration time $\Delta t = 1/\Delta\phi$ is a result of the convolution of both filters and can be derived rather accurately from the measured transfer functions (amplitude and phase). It should be noted that the acquisition is based on the peak detection of the signal impulse. The peak signal $v$ at the output of the filter of a single spectral sideband is:

$$v = Z_w N e \frac{\pi}{2} \int h(\omega) d\omega = Z_w \frac{Ne}{\Delta t} \hfill (7)$$

where $N$ is the number of particles in the bunch, $e$ is the electronic charge and $Z_w$ is the effective transfer impedance taking all gains and attenuations into account up to that point of the measuring chain.

The bunch spectrum is $h(\omega)$. For the 50MHz filters we found $\Delta t = 1/4.54\,\text{MHz} = 220\,\text{ns}$ and for 200MHz this was $\Delta t = 1/4.93\,\text{MHz} = 203\,\text{ns}$.

### V. INFLUENCE OF BUNCH LENGTH ON MONITOR RESPONSE

The spectrum of a bunch with a Gaussian profile is:

$$h(\omega) = (Ne)^{-\frac{1}{2}} e^{-\frac{1}{2}(\omega \sigma)^2} \hfill (8)$$

where $\sigma$ is the r.m.s. bunch length in time. It is operationally advantageous to work with a small value for $\omega \sigma$, so that $h(\omega) \approx Ne$. The expected
bunch length of the leptons injected in the SPS is \( \sigma_t \leq 0.9 \text{ ns} \). The operational frequency is 50 MHz so that \( \omega \sigma_t \leq 0.28 \). Hence, the measurement is low by 3.5\% when the simplified formula is used. This can be corrected as will be pointed out later. The error at the SPS extraction is entirely negligible since \( \sigma_t = 0.13 \text{ ns} \) and \( \omega \sigma_t \leq 0.04 \) for the same operational frequency.

The spectral dependence on bunch length can be turned into an interesting application. Indeed, the same bunch signal can be sampled at \( \omega_1 = 50 \text{ MHz} \) and \( \omega_2 = 200 \text{ MHz} \) which are the frequencies for which receivers are available. The signal reduction factor computed with (8) can be applied to (7) and the following signal ratio is found:

\[
\frac{v_1}{v_2} = e^{\frac{1}{2} \sigma_t^2 (\omega_2^2 - \omega_1^2)}.
\]  

(9)

The r.m.s. bunch length follows:

\[
\sigma_t = \sqrt{\frac{2}{\omega_2^2 - \omega_1^2} \ln \left( \frac{v_1}{v_2} \right)}.
\]  

(10)

Putting in the numbers gives:

\[
\sigma_t (\text{ns}) = \frac{2}{\pi} \sqrt{\frac{1}{0.3} \ln \left( \frac{v_{50 \text{ MHz}}}{v_{200 \text{ MHz}}} \right)}.
\]  

(11)

This measurement of \( \sigma_t \) relies exclusively on the relative signals measured at 50 and 200 MHz. Hence it is in principle possible to correct the 50 MHz intensity value for bunch length. With the available hardware it is not possible to make a useful bunch length measurement on the beams extracted from the SPS since the expected signal difference between 50 and 200 MHz is too small (1.26\%).

VI. INFLUENCE OF BEAM POSITION ON INTENSITY SIGNAL

The four electrodes cover somewhat less than 20\% of the full circumference of the monitor. The signal is enhanced when the beam approaches one of the electrodes and it is reduced when the beam is rather in between two electrodes. This effect can and has been computed. It is very non linear and increases quickly with increasing distance from the centre of the monitor. However, in practice it can be neglected. For example, the variation of the signal induced by a beam circulating at a distance from the centre equal to 1/3 of the aperture (20 mm) is less than 3\%. Beams tend to be at less than 10 mm from the centre. A dedicated monitor built especially for intensity would look like a coaxial line and cover the full circumference so that the signal becomes totally independent from beam position.

VII. OTHER ELEMENTS

The performance of all elements have to be accurately measured in order to guarantee an intensity measurement of sufficient quality. The other elements are cables, amplifiers, attenuators, hybrids. Their attenuation/gain has been determined with great precision such that the total error is estimated at less than 5\% (0.4 dB). A lot of care is required. As an example it may be worth mentioning that there is a slight discrepancy in gain in the receiver units between pulsed mode operation and continuous wave operation. Therefore it was necessary to do all the measurements in pulsed mode, which is somewhat more cumbersome. Figure 3 shows the schematic diagram of the measuring chain.

![Figure 3](image)

FIGURE 3

VIII. CONCLUSION

The system has been operational for two years. One monitor is installed in each lepton injection line (bunch length available for positrons only) of the SPS and two monitors (up- and downstream) in each SPS extraction line. The intensity evolution measured from injection, SPS circulating beam (DC monitor), extraction are quite acceptable from a physical point of view. Typically the loss at injection is rather large (up to 50\% due to high frequency transverse mode coupling instability), some 10\% loss in the SPS, 3 to 4\% during extraction and a few % in the transfer line.

REFERENCES