Muon Pair Production
in $e^+e^-$ Collisions at the Z Resonance

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Dehong Zhang
Muon Pair Production
in $e^+e^-$ Collisions at the Z Resonance

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To my mother
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Introduction

The questions of what the ultimate constituents of matter are and how they are bonded together to form such a rich and colourful world have been with us for some time. At present, it is understood that the constituents of matter are six leptons and six quarks and that the interactions between these particles can be classified into four types of markedly different strengths: gravitational, weak, electromagnetic and strong interactions. The Standard Glashow-Salam-Weinberg Model unifies weak and electromagnetic interactions [1, 2, 3].

Within the framework of the Standard Model, weak interactions are mediated via a doublet of charged bosons $W^\pm$ and a neutral boson $Z$. The introduction of the $W^\pm$ was initiated by experimental observations, while the existence of the $Z$, or the neutral current, was proposed out of consistency considerations. The existence of a neutral current was experimentally verified in the middle of the 1970's when weak interactions could be directly studied by neutrino-induced reactions at center-of-mass (C.M.) energies of $\sqrt{s} = 10 - 20$ GeV. These neutrino experiments established that the neutral current behaves in a similar way as the charged current. Analyzing the results from these experiments within the framework of the Standard Model, the masses of the $W^\pm$ and $Z$ bosons were predicted to be $M_W = 65$ GeV and $M_Z = 80$ GeV. These predictions led to the discovery of the $W^\pm$ and $Z$ at the CERN $p\bar{p}$ collider with masses of $M_W = 81$ GeV and $M_Z = 92$ GeV [4].

To test the validity of the Standard Model, including the Higgs mechanism which generates masses for all particles, motivated the construction of a Large Electron-Positron (LEP) collider with a beam energy ranging from 40 (phase I for Z physics) to 100 GeV (phase II for W pair production) [5]. In addition to the study of weak interactions, very high energy $e^+e^-$ collisions could produce high mass quark-antiquark pairs and new particles.

Starting from August 1989, LEP has been running in the vicinity of the $Z$ resonance. By the end of 1992, large statistics were accumulated and it became important to reduce the systematic uncertainties on the measurements in order to determine precisely the $Z$ parameters. In this thesis measurements of the muon pair production with small systematic errors are presented using the $L_3$ detector. The relevant theoretical aspects are outlined in chapter 1. Then, the LEP collider and the $L_3$ detector are briefly described in chapter 2. In
chapter 3, details are given for the Z chambers of the muon spectrometer, while chapter 4 is devoted to the muon identification with the calorimeters. Chapters 5 and 6 elaborate on measurements of the muon pair cross section and charge asymmetry. In chapter 7, detailed studies are presented of the photon radiation in muon pair production. The results are interpreted in chapter 8.

Unless otherwise specified, all quantities in this thesis are expressed in the so-called natural units corresponding to: $\hbar = c = 1$. 
Chapter 1

The Standard Model of Electroweak Theory

At present, it is believed that all elementary particles and the fundamental interactions between them can be described by a gauge field theory, where the symmetry transformations are space-time dependent. However, the imposition of local symmetry implies that all gauge bosons are massless, which contradicts the interpretation of low-energy weak interaction phenomena: weak interactions must involve massive intermediate vector bosons. To obtain masses for particles, the gauge symmetry must be broken. One may attempt to introduce explicit mass terms, but then the high-energy behaviour of the theory is altered such that the theory becomes nonrenormalizable. In the case of spontaneous symmetry breaking, the Lagrangian can still be fully invariant under the symmetry transformations but the vacuum, the ground state, is no longer a singlet of the symmetry group. The choice of one from all the possible degenerate ground states as the vacuum breaks the symmetry. According to the Goldstone theorem, this implies the existence of a set of massless scalar bosons [6].

Fortunately, the massless gauge bosons and the massless Goldstone bosons can be combined to form massive vector particles, without ruining the good high-energy behaviour of the symmetric theory [7]. Also, the resulting renormalizable theory [3] displays the desired unity of weak interactions with electromagnetism. All these findings have brought the present Standard Model of electroweak theory into being.

1.1 The Standard Model

Historically the $W^\pm$ were introduced to describe the charged weak currents which have a pure (V–A) Lorentz covariant structure [1]. Thus in a unified gauge theory of weak
and electromagnetic interactions, there are at least three vector gauge bosons \(W^\pm\) and the photon. The simplest group with three generators is SU(2). However, the three currents do not close under commutation. The neutral gauge boson, the Z which ought to be an unequal mixture of the V and A Lorentz covariants, was then introduced [2] to complete the group SU(2) × U(1). The amount of mixing between the SU(2) and U(1) groups is determined by the so-called weak mixing angle \(\theta_W\), which is related to the masses of the \(W^\pm\) and Z bosons:

\[
\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.23.
\]

(1.1)

Based on the SU(2) × U(1) symmetry group, the Standard Model of electroweak theory states that the fundamental constituents of matter are spin 1/2 fermions (the left handed ones form weak isospin doublets, the right handed ones are singlets), while the electroweak interactions between them are generated through the exchange of gauge bosons (see table 1.1). While the photon remains massless, the \(W^\pm\) and Z bosons, together with one new particle of unknown mass, the Higgs particle, acquire masses through the so-called Higgs mechanism. This mechanism also introduces mass terms for the fermions (except the \(\nu\)'s) through Yukawa couplings. The strengths of these couplings are free parameters, the masses of the fermions therefore remain unpredicted.

Within the framework of the Standard Model, fermions can be grouped in families, each containing 2 leptons and 2 quarks (see table 1.1). However, the number of families is not specified in the theory.

At tree level, the matrix elements, which reflect the dynamic characteristic of the theory, can be constructed in the same way as in electromagnetism: current × propagator × current,

<table>
<thead>
<tr>
<th>Fermions</th>
<th>Family</th>
<th>(Q)</th>
<th>(I_3)</th>
<th>(N_C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\nu_e) (\nu_\mu) (\nu_\tau)</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>e (\mu) (\tau)</td>
<td>-1</td>
<td>-1/2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>u (c) (t)</td>
<td>2/3</td>
<td>1/2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>d (s) (b)</td>
<td>-1/3</td>
<td>-1/2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Bosons</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(W^\pm)</td>
<td>±1</td>
<td>±1</td>
<td>±1</td>
<td></td>
</tr>
<tr>
<td>Higgs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| \(H^0\) | 0 | -1/2 |

Table 1.1 The properties of (left handed) fermions, bosons and the Higgs particle. \(Q\) stands for the electrical charge in units of \(e\), \(I_3\) is the third component of the weak isospin and \(N_C\) the "colour factor" indicating the corresponding particle being either a singlet or a triplet in QCD.
while the couplings of the $\gamma$, $W^\pm$ and $Z$ bosons to the fermions take the following forms:

$$\gamma_f = -ieQ_f\gamma_i \quad \quad (f \text{ can be any fermion and} \quad e = \sqrt{4\pi\alpha})$$

$$Z_j^f = -ie\gamma_i (g_V^f - g_A^f) \quad \quad (f \text{ can be any fermion})$$

$$W^+_{li} = -ie\gamma_i (1 - \gamma_5) \quad \quad (l = e, \mu \text{ or } \tau \text{ with } v_l \text{ the corresponding (anti-)neutrino})$$

$$W^z_{q_i} = -ie\gamma_i (1 - \gamma_5) \cdot U_{ij} \quad \quad (q_i = u, c \text{ or } t; q_j = d, s \text{ or } b; \quad U \text{ is the CKM matrix})$$

where

$$g_V^f = t_3^f - 2Q_f \sin^2 \theta_W$$

$$g_A^f = t_3^f$$

are the vector and axial vector coupling constants of the $Z$ boson to fermion $f$ with $t_3^f$ the third component of its weak isospin and $Q_f$ its electrical charge. The CKM matrix describes the transformation from quark mass eigenstates to weak interaction eigenstates. In the following chapters we will be dealing with the cases $f = e, \mu$, the coupling constants then become:

$$g_V^{e\mu} = -\frac{1}{2} + 2\sin^2 \theta_W \quad \quad (1.2)$$

$$g_A^{e\mu} = -\frac{1}{2} \quad \quad (1.3)$$

To fix the theory completely, a number of parameters must be used as input; all other parameters, some of them experimental observables, can then be calculated. In the on-shell scheme, which ensures that all input parameters have a clear physical meaning and can in principle be measured directly in suitable experiments, the basic input parameters are the electromagnetic coupling constant $\alpha$, the strong coupling constant $\alpha_s$, the muon decay constant $G_\mu$, the $Z$ mass $M_Z$, the fermion masses $m_f$ and the Higgs mass $m_H$. Among the fermion masses, the lepton masses are better known than the quark masses.

To allow direct comparison between theoretical calculations and experimental measurements, higher order corrections, which in the on-shell scheme can be separated into weak corrections and QED corrections, ought to be implemented. By expressing the theory in
terms of the redefined effective coupling constants, $\tilde{g}_V(M_Z^2)$ and $\tilde{g}_A(M_Z^2)$ which depend on all parameters of the model, in particular the $m_t$ and $m_W$ which do not enter the tree level results, most of the corrections can be incorporated while keeping the theory in a simple form [8]. Since $g_V$ and $g_A$ are common to any model where the vector boson has vector and axial vector couplings to fermions, it is of special interest to experimentally determine $\tilde{g}_V(M_Z^2)$ and $\tilde{g}_A(M_Z^2)$, so to check the validity of the Standard Model in a manner which does not depend on the Standard Model itself.

Studying the muon pair production in the vicinity of the $Z$ pole provides one way of fulfilling this primary goal by determining the $\tilde{g}_V(M_Z^2)$ and $\tilde{g}_A(M_Z^2)$ through the cross section and asymmetry measurements [9, 10]. Furthermore, the cross section measurements can be used, in combination with measurements from other $Z$ decay channels, to determine the number of fermion families assuming that all neutrinos are "light" ($m_\nu < M_Z/2$).

### 1.2 Muon Pair Production in $e^+e^-$ Collisions

#### 1.2.1 Born Approximation

In lowest order, i.e. in the so-called Born approximation, the muon pair production

$$e^+e^- \rightarrow \mu^+\mu^- (\gamma)$$

gets contributions from $Z$ exchange and from $\gamma$ exchange (Higgs exchange is neglected because of the small Yukawa coupling to the electron):

![Figure 1.1](image)

*Figure 1.1* The lowest-order contributions to $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$.

Neglecting terms of the order $m^2_e/s$, the total differential cross section can be written as:

$$\frac{d\sigma_0(s)}{d\cos \theta_+} = G_1(s) \cdot (1 + \cos^2 \theta_+) + G_2(s) \cdot \cos \theta_-$$  \hspace{1cm} (1.4)

where $\theta_+$ is the polar angle of the outgoing $\mu^-$ with respect to the incoming $e^-$ in the C.M. frame, $G_1(s)$ and $G_2(s)$ are functions of $s$, $\alpha$, $g_V^{\text{eff}}$, $g_A^{\text{eff}}$ and $M_Z$. $G_1(s)$ corresponds to the magnitudes of the $Z$ and $\gamma$ exchanges and thus determines the total cross section, $G_2(s)$ is due to the interference between these two exchanges and causes angular asymmetry.
The Cross Section of $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$

Integrating the total differential cross section over the full solid angle, the total cross section as a function of $s$ becomes:

$$\sigma_0(s) = \frac{8}{3} \cdot G_1(s)$$

$$= \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^0} \left\{ \frac{12\pi \Gamma_{ee}^0 \Gamma_{\mu\mu}^0}{M_Z^2} + \frac{I_0 \cdot (s - M_Z^2)}{s} \right\} + \frac{4\pi\alpha^2}{3s}$$ \hspace{1cm} (1.5)

where

$$\Gamma_Z^0 = \sum_f N_C^f G_f M_Z^2 \sqrt{1 - \frac{4m_f^2}{s}} \left( g_V^2 \cdot (1 + 2m_f^2/s) + g_A^2 \cdot (1 - 4m_f^2/s) \right)$$ \hspace{1cm} (1.6)

$$\Gamma_{\mu\mu}^0 = \frac{G_F M_Z^2}{6\sqrt{2}\pi} \left( g_V^2 + g_A^2 \right)$$ \hspace{1cm} (1.7)

$$I_0 = \frac{2\sqrt{2}}{3} \alpha G_F M_Z^2 g_V g_V^\mu$$

with $l = e, \mu$. Here, $\Gamma_{ee}^0$ and $\Gamma_{\mu\mu}^0$ are the partial widths of $Z$ decaying to $e^+e^-$, $\mu^+\mu^-$ respectively, while $\Gamma_Z^0$ being the tree-level total $Z$ width.

In equation 1.5, the first term is the Breit-Wigner form for the spin one resonance of the $Z$ exchange, the last term is from the $\gamma$ exchange while the second term is the $\gamma$-$Z$ interference. In the vicinity of the $Z$ pole, the production is dominated by the $Z$ exchange.

The Forward-Backward Charge Asymmetry in $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$

Besides the total cross section, the shape of the angular distribution in $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ also depends on $g_V^{e\mu}$, $g_A^{e\mu}$ and other parameters in the theory. This shape is characterized by the so-called forward-backward charge asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$ \hspace{1cm} (1.8)

where

$$\sigma_F = 2\pi \int_0^1 d\cos\theta \frac{d\sigma}{d\cos\theta}$$ \hspace{1cm} and \hspace{1cm} $$\sigma_B = 2\pi \int_{-1}^0 d\cos\theta \frac{d\sigma}{d\cos\theta}$$.

Using equation 1.4, the first order asymmetry becomes:

$$A_{FB}^0(s) = \frac{3G_2(s)}{8G_1(s)} - \frac{3\sqrt{2}\pi\alpha}{G_F M_Z^2 g_V^{e\mu} g_A^{e\mu}} \left( 1 - \frac{M_Z^2}{s} \right) + \frac{3g_V^{e\mu} g_V^\mu}{g_A^{e\mu} g_A^\mu}$$ \hspace{1cm} (1.9)

Here we use $g_V^2 \ll g_A^2$ (see equations 1.1, 1.2 and 1.3), and neglect terms of the order $(\Gamma_Z^0 / M_Z)^2$. 
1.2.2 Weak Corrections and the Improved Born Approximation

Weak corrections include propagator corrections, vertex corrections and corrections due to box diagrams with massive bosons (see figure 1.2 a-c). The propagator corrections involve all fundamental particles of the model, in particular the top quark and the Higgs boson, and thus depend on $m_t$ and $m_H$. In the case of muon pair production, the vertex corrections are practically independent of $m_t$. In the vicinity of the Z pole, the box diagrams are non-resonant. Their contribution to the differential cross section at $s = M_Z^2$ is smaller than 0.02% and can be neglected.

![Diagram (a)](image1)

![Diagram (b)](image2)

![Diagram (c)](image3)

![Diagram (d)](image4)

**Figure 1.2** Examples of weak and radiative corrections: a) virtual fermion propagator correction; b) virtual photon vertex correction; c) box diagram and d) real photon initial state bremsstrahlung.

The large terms from the propagator and vertex corrections can be incorporated in an approximation, which keeps the relations between the parameters and the observables unchanged, by [8]

- Replacing the electromagnetic coupling constant $\alpha$ by an energy dependent effective coupling constant:
  $$\alpha \rightarrow \alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$
  where $\Delta\alpha(M_Z^2) = 0.0602$.

- Fixing the value of the strong coupling constant $\alpha_s$ at $Q^2 = M_Z^2$:
  $$\alpha_s \rightarrow \alpha_s(M_Z^2) = 0.123.$$

- Replacing the vector and axial vector coupling constants $g_V$ and $g_A$ by energy dependent effective coupling constants:
  $$g_V \rightarrow g_V(M_Z^2) = \rho^{1/2} \cdot (I_3 - 2Q \sin^2 \theta_W)$$


\[ g_A \longrightarrow \tilde{g}_A(M_Z^2) = p^{1/2} \cdot I_3 \]

where

\[ p = \frac{1}{1 - \Delta p} \quad \text{and} \quad \Delta p = \frac{3G_p m_e^2}{8\sqrt{2}\pi^2} \cdot \frac{\sqrt{2} G_p M_Z^2}{10\pi^2} \cdot (\ln m_H - 4.8) \quad (m_H \gg M_W). \]

\( m_H \) is in GeV and \( \tilde{\theta}_W \) is the so-called effective weak mixing angle defined through:

\[ \sin^2 \tilde{\theta}_W = (1 + \Delta \kappa_{\text{f},\text{vertex}}) \cdot (\sin^2 \theta_W + \Delta \rho \cdot \cos^2 \theta_W) \]

with \( \Delta \kappa_{\text{f},\text{vertex}} (= 0.034 \text{ for } Z \rightarrow l\bar{l}) \) being a small non-universal correction from the vertex diagrams.

- Replacing the tree-level Z width by an energy dependent width:

\[ \Gamma_Z^0 \longrightarrow \frac{s}{M_Z^2} \cdot \Gamma_Z. \]

Here, we include in \( \Gamma_Z \) also the QED correction due to final state radiation by multiplying the non-neutrino contributions with a factor

\[ 1 + \delta_{\text{QED}} = 1 + \frac{3\alpha}{4\pi}, \]

and the QCD correction by further multiplying the quark contributions with a factor

\[ 1 + \delta_{\text{QCD}} = 1 + \frac{\alpha_s(M_Z^2)}{\pi} + 1.405 \cdot \left( \frac{\alpha_s(M_Z^2)}{\pi} \right)^2. \]

Figure 1.3 shows the effects of the top and Higgs masses on \( \tilde{g}_V(M_Z^2) \) and on \( \tilde{g}_A(M_Z^2) \), as calculated with ZFITTER [11] according to the Standard Model. Besides the top and Higgs masses, two other input parameters are used: \( M_Z = 91.195 \text{ GeV} \) and \( \alpha_s = 0.123 \).

Weak corrections are small compared to QED corrections. Due to their factorization, they do not affect the Born-type angular distribution. For this reason, the above incorporation scheme is usually referred to as the "improved Born approximation". In the following, a superscript "B" is used to indicate that the corresponding quantity is in the improved Born approximation.

### 1.2.3 QED Corrections

QED corrections are due to initial state radiation (see figure 1.2 d), final state radiation and the interference between initial and final state radiation.

The interference between initial and final state radiation introduces a non-Born shaped term to the differential cross section, which modifies the total cross section by a negligible
amount but induces an additional asymmetry. When the cut on the photon phase space is not too tight*, this additional asymmetry is also negligible (< 0.001).

Due to final state radiation, the symmetric part of the differential cross section gets a multiplicative factor \((1 + \delta_{\text{QED}})\), while the asymmetric part stays untouched. Therefore the total cross section and the partial widths are changed to:

\[
\begin{align*}
\sigma &= \sigma^B \cdot (1 + \delta_{\text{QED}}) \\
\Gamma_{ll} &= \Gamma_{ll}^B \cdot (1 + \delta_{\text{QED}})
\end{align*}
\]  

while the asymmetry becomes:

\[
A_{FB} = A_{FB}^B \cdot (1 - \delta_{\text{QED}}).
\]

Here the relative changes are small (~ 0.17%).

The actual C.M. energy is reduced from \(\sqrt{s}\) to \(\sqrt{s'} < \sqrt{s}\) due to initial state radiation. The observed cross section is therefore the sum of the "all other corrections included" cross section \(\sigma^B(sz) \cdot (1 + \delta_{\text{QED}})\), weighted by the probability \(G(z)\), the radiator function, to reduce \(s\) to \(s' = sz\) through photon emission:

\[
\sigma(s) = (1 + \delta_{\text{QED}}) \cdot \int_{4m_e^2/s}^{1} dz \ G(z) \ \sigma^B(sz) ,
\]

*\(E_\gamma / E_{\text{beam}} < \epsilon, \) with \(\epsilon > 0.1\). For the asymmetry analysis, we require the acollinearity angle between the muon pair to be less than 15° (see chapter 6), which translates to \(\epsilon \sim 0.23\).
while the asymmetry can be well approximated by [12]:

\[ A_{FB}(s) = \frac{1}{\sigma} \cdot \int_{z_0}^{1} dz \ G(z) \ \sigma^B(sz) \ A_{FB}^B(sz). \]

Here \( z_0 \approx 4m_{e}^2/s \) depends on the actual experimental cut-offs, such as the acollinearity cut.

Since the cross section is strongly peaked around the Z pole, the relative reduction from \( \sigma^B(M_Z^2) \cdot (1 + \delta_{\text{QED}}) \) to \( \sigma(M_Z^2) \) due to initial state radiation is about 30% (see figure 1.4 a). Similarly, since the asymmetry is a steeply increasing function of \( s \) around the Z, when no cut on the photon phase space is imposed, the asymmetry close to the Z peak is shifted by

\[ \delta A_{FB} = -0.025, \]

which is of the order of the on-peak asymmetry itself (see figure 1.4 b). More importantly, the Born-type angular distribution \( d\sigma^B / d\cos \theta \) gets smeared by the transformation from the reduced C.M. system, where the scattering angle \( \theta \) is defined, to the lab frame. This smearing effect makes it less clear how the asymmetry can be extracted from the data in a simple way.

It is found [12] that even when hard initial state radiation is allowed, the Born-type angular distribution can be well restored if we take instead of \( \theta \) defined in the lab frame another angle, the scattering angle in the reduced C.M. system, \( \theta_c \). This angle can be calculated according to

\[ \cos \theta_c = \frac{\sin(\theta_e - \theta_-)}{\sin \theta_e + \sin \theta_-}, \]

assuming the initial state photons have zero transverse momenta. In terms of this \( \theta_c \), the asymmetry can be obtained though a fit of

\[ \frac{d\sigma(s)}{d\cos \theta_c} = \sigma(s) \cdot \left\{ \frac{3}{8} \cdot (1 + \cos^2 \theta_c) + A_{FB}(s) \cdot \cos \theta_c \right\} \]

to the experimentally observed angular distribution. The asymmetry \( A_{FB}(s) \) extracted through this approach has a systematic uncertainty of less than 0.001 [12].

Due to the restricted solid angle coverage of the experiment and the necessity to suppress background contamination from \( e^+e^- \rightarrow \tau^+\tau^- (\gamma) \), a few cuts are applied on the data when selecting events for the cross section and asymmetry measurements (see chapters 5 and 6). In the case of the cross section measurement, the effects of these cuts are eliminated by simply extrapolating to the full solid angle. In the case of the asymmetry measurement, only the effect of the fiducial volume cut can be easily removed through extrapolation; the effect of the acollinearity angle cut is taken care of in a later stage — when extracting the Z parameters from our measurements with the fitting program ZFITTER.

Since LEP only operates in the vicinity of the Z pole, the normal measurements are all done in between \( \sqrt{s} = 88 — 94 \text{ GeV} \). In chapter 7, an attempt is presented to obtain
Figure 1.4  a) The $s$-dependence of the cross section and b) the forward-backward charge asymmetry of $e^+e^- \rightarrow \mu^+\mu^- (n\gamma)$ as calculated with ZFITTER in the framework of the Standard Model. The solid line in b) indicates the actual asymmetry for muon pairs with acollinearity angles smaller than 15°.
measurements of the cross section and asymmetry at $\sqrt{s} = 80$ GeV. These measurements are of significance in the sense that they are done in an energy region which was never tested before. In this region, the cross section is close to a minimum and the asymmetry attains its largest negative value.

1.3 The ZFITTER Package

ZFITTER calculates cross sections, forward-backward asymmetries for fermion pair production in $e^+e^-$ annihilation and Bhabha scattering with a semi-analytical approach. In the case of $\tau$ pair production, it also calculates the final-state polarization. It includes complete $O(\alpha)$ QED corrections and soft photon exponentiation. Higher order QED corrections have also been implemented for initial-state radiation contributions to the total cross section and asymmetry. Using correct radiator functions for the total and differential cross sections, the asymmetry and for different bremsstrahlung origins, ZFITTER can be applied at energies far away from the $Z$ peak.

For the calculation of hard scattering processes, ZFITTER consists of several branches including the Standard Model treatment and two model-independent approaches. In the Standard Model branch, ZFITTER includes complete $O(\alpha)$ weak loop corrections with a resummation of leading higher-order terms. Although the accuracy of the Standard Model branch has been optimized near the $Z$ pole, it can be used at PETRA and TRISTAN energies.

With the two model-independent approaches, it is assumed that the $Z$ has real constant vector and axial-vector couplings to fermions and that the scattering through the $Z$ can be considered as subsequent formation and decay of a resonance. These two approaches are in principle completely model independent, except the interference term which is adopted from the Standard Model.

1.4 Monte Carlo Generators

Monte Carlo events are needed to test and improve the reconstruction software, to calculate the theoretical expectations for certain quantities like the detector acceptance, and to check background contamination. For this purpose, physics generators have been developed according to existing theories such as the Standard Model. These generators "produce" events for a certain process, e.g. $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$, with kinematic distributions according to the corresponding theoretical probability densities. These probability densities are calculated including relevant Feynman diagrams up to a certain order.

The generated events are then fed to packages which simulate the evolution of particles in time and their interactions with the detector materials. When only a rough but quick estimation is wanted, the generated events can be directly used without detailed detector
simulation and reconstruction in order to save computing time.

The generators used in the analysis of this thesis are shortly described below:

- **KORALZ-3.8** generates events of the type $e^+e^- \rightarrow f\bar{f}$. It incorporates electroweak corrections, second order initial state radiation with exponentiation and first order final state radiation [13].

100,000 $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ events and 105,000 $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ events were generated in the full solid angle at $\sqrt{s} = 91.250$ GeV with a cross section of 1.475 nb. These events were then subjected to the detector simulation which includes effects from: multiple scattering, bremsstrahlung, nuclear interactions, decays, the spatial distribution of the interaction vertex, the finite resolutions of the detector components and their hardware status. The simulations were done three times according to different hardware conditions in the three running periods of 1991 pre-scan, 1991 scan and 1992. Finally, the events were reconstructed with the same package used for the data. These events are used to calculate the detector acceptance for $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ and to check background contamination from $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$.

In addition, seven sets of 100,000 $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ events were generated at energies between 88 and 94 GeV. These events are used, without detailed detector simulation, to check the variation of the detector acceptance with the C.M. energy.

- **KORALZ-4.0** incorporates the YFS3 package [14]. It generates $e^+e^- \rightarrow \mu^+\mu^-(\gamma\gamma)$ events according to the Yennie-Frautschi-Suura scheme [15] with multiple collinear and soft photon radiation in both the initial and final states. It also includes the additional leading-log terms for one or two hard photons. The production cross section for events with hard and isolated photons as calculated with this program is found to be in good agreement with the exact $O(\alpha^2)$ matrix element calculations [16].

In total, $10^6 e^+e^- \rightarrow \mu^+\mu^-(\gamma\gamma)$ events were generated at 91.25 GeV with a cross section of 1.491 nb. The events with two muons inside $|\cos \theta| \leq 0.9$ and with at least two isolated photons inside $|\cos \theta| \leq 0.99$ are selected, simulated and reconstructed in the same way as done with the KORALZ-3.8 events. The photons are required to have at least 0.35 GeV in energy and be at least 4° away from both muons. These events are used to check the multiple hard photon production in $e^+e^- \rightarrow \mu^+\mu^-(\gamma\gamma)$.

- **TWOGAMMA** generates events with four fermions in the final state. It includes 4 subprocesses: multiperipheral, bremsstrahlung, conversion and annihilation [17].

In total, 40,000 $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ events were generated at 91.25 GeV with a 4 GeV invariant mass cut on the muon pair. The production cross section is $1.052 \pm 0.005$ nb. These events are simulated according to the 1992 hardware conditions. They are reconstructed and used to check background contamination from $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ in the $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$ samples.
Chapter 2

LEP and $L_3$

This chapter presents the experimental apparatus, including a few aspects of the LEP machine. A description is given of the $L_3$ detector, with emphasis on the muon spectrometer.

2.1 The LEP Collider

The LEP complex [5] of CERN, the largest $e^+e^-$ collider in the world, is located at the French-Swiss border near Geneva. It consists of five successive parts (see figure 2.1): the LEP Injector Linac (LIL, which includes two linacs), the Electron-Positron Accumulation

![Diagram of the LEP complex](image.png)

**Figure 2.1** The LEP complex.
ring (EPA), the modified Proton Synchrotron (PS), the modified Super Proton Synchrotron (SPS), and the main LEP storage ring. The function of the LIL is two-fold: one high current linac shoots 200 MeV electrons onto a target where positrons are created via photon conversion; the second linac accelerates positrons as well as electrons to 600 MeV. The EPA serves as a buffer to allow sufficiently intense beams to be built up. The PS receives electrons and positrons from the EPA and accelerates them to 3.5 GeV and finally, the SPS completes the pre-acceleration chain, boosting the electrons and positrons to 20 GeV, ready for filling the LEP ring. The electron and positron bunches are then brought to the final energy of ~ 45 GeV (~ 100 GeV in phase II) and focused to collide in the centres of four detectors: ALEPH, DELPHI, L3 and OPAL.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Design</th>
<th>1991</th>
<th>1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy range, $E_{\text{beam}}$ (GeV)</td>
<td>40 — 100</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>Peak luminosity, $L$ ($10^{31}$ cm$^{-2}$s$^{-1}$)</td>
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<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Circumference (m)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Bending radius (km)</td>
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<td></td>
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<tr>
<td>RF frequency (MHz)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Betatron amplitude function, $\beta_y$ (cm)</td>
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<td>5.0</td>
<td>4.3</td>
</tr>
<tr>
<td>Vertical beam-beam strength, $\xi_y$</td>
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<td>0.022</td>
<td>0.020</td>
</tr>
<tr>
<td>Current per bunch, $I_b$ (mA)</td>
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<td>0.45</td>
<td>0.32</td>
</tr>
<tr>
<td>Number of bunches per beam, $k_b$</td>
<td>32</td>
<td>4</td>
<td>4.8</td>
</tr>
<tr>
<td>Beam lifetime (h)</td>
<td>10</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

| Table 2.1  | The main parameters of LEP. |

The main machine parameters (see table 2.1) have been chosen such that the luminosity $L$ exceeds $10^{31}$ cm$^{-2}$s$^{-1}$, thus allowing each experiment to accumulate within five to six years of running time several million Z events. For a given process with a cross section $\sigma$, the total number of produced events $N$ is directly proportional to the time integrated luminosity $\mathcal{L}$:

$$N = \sigma \cdot \mathcal{L} = \sigma \cdot \int L \, dt.$$  

For beams colliding head-on, the luminosity at a collision point is related to the number of bunches per beam ($k_b$), the current per bunch ($I_b$), the vertical beam-beam strength parameter ($\xi_y$), the betatron amplitude function ($\beta_y$) and beam energy ($E_{\text{beam}}$) in the following manner:

$$L \propto k_b \cdot I_b \cdot \xi_y \cdot E_{\text{beam}} / \beta_y^2,$$

assuming that all bunches in both beams have the same number of particles.

In an attempt to increase the luminosity, $k_b$, $I_b$ and $\xi_y$ were gradually changed from 4, 0.45 mA and 0.022 to 8, 0.32 mA and 0.020 respectively, while $\beta_y^2$ shrank from 5.0 cm to
Figure 2.2 The total integrated luminosity delivered to \(L_3\) (dotted line) and that being used in the data analysis described in later chapters (solid line) as functions of time.

4.3 cm during the 1991 and 1992 running periods. As a result, the peak luminosity rose from \(\sim 0.5 \times 10^{31} \text{ cm}^{-2}\text{s}^{-1}\) to \(\sim 0.8 \times 10^{31} \text{ cm}^{-2}\text{s}^{-1}\) (see table 2.1). The total integrated luminosity delivered to \(L_3\) and that being used in the data analysis described in later chapters as functions of time are shown in figure 2.2.

With large number of \(Z\) events at hand, properties of the \(Z\) can be determined from the energy dependence of the cross section for \(e^+e^- \rightarrow \text{hadrons}\) and \(e^+e^- \rightarrow \text{leptons}\), and from the energy dependence of the asymmetry for \(e^+e^- \rightarrow \text{leptons}\) around the \(Z\) resonance. The LEP energy calibration contributes systematic errors of 7 MeV on \(M_Z\) and 5 MeV on \(\Gamma_Z\). The former is due to errors on the absolute energy scales, which are 18, 53 and 18 MeV for the 3 periods of 1991 pre-scan, 1991 scan and 1992 respectively. The latter is dominated by the uncertainties in the relative energy scales [18].

2.2 The \(L_3\) Detector

The \(L_3\) detector [19, 20] enables high resolution energy and direction measurements of electrons, photons and muons. On its way outwards, a muon, created in the centre of the detector in an \(e^+e^-\) collision, may be detected by (see figures 2.3, 2.4 and 2.5):

- Central Track Detector (TEC) as one charged track (TTRK);
- Electromagnetic Calorimeter (ECAL) as one geometrical cluster (EGCL);
- Scintillation Counters (SCNT) as one hit;
- Hadron Calorimeter (HCAL) as one geometrical cluster (HGCL);
- Muon Filter (HMFL) as one cluster of hits;
- Muon Spectrometer (MUCH) as one track (MUTK).

These subdetectors are complemented by a luminosity monitor, triggering and data-taking electronics, a cluster of online computers and a mainframe computer for offline analysis.

All subdetectors except the muon spectrometer are installed inside a 32-m long, 4.45-m diameter steel cylinder, the support tube. The muon spectrometer is mounted on the outer surface of the support tube. The whole detector is supported at the two extremities of the support tube by adjustable jacks placed on concrete pillars, and housed inside a 7800-t octagonally shaped solenoid providing a 0.5-T magnetic field over a 11.9-m long and 11.4-m across effective volume. To optimize the muon momentum resolution, which improves linearly with the strength of the magnetic field but quadratically with the track length, a relatively low field in a large volume has been chosen.

The whole $L_3$ detector is aligned such that it is concentric with the LEP beam line and symmetric with respect to the interaction point. The coordinate system is defined with the origin at the centre of the detector, and with the positive $z$ along the outgoing $e^-$ beam direction, $\theta$ and $\phi$ being the polar and azimuthal angles (see figure 2.3). The main component of the magnetic field is in the positive $z$ direction.

![Diagram of the $L_3$ Detector](image)

**Figure 2.3** The $L_3$ Detector (the scintillation counters are too small to be seen).
Figure 2.4 Endview of the L3 Detector showing one muon being detected by: 1) the TEC as one charged track, 2) the ECAL as one geometrical cluster, 3) the SCNT as one hit, 4) the HCAL barrel as one geometrical cluster, 5) the HMFL as one cluster of hits and, 6) the MUCH as one track consisting of 3 segments.
Figure 2.5 Sideview of the L3 Detector showing one muon being detected by the subdetectors.
2.2 The $L_3$ Detector

It has to be noted that due to an oversight, the gaps between the muon chambers are aligned with those between the HCAL modules, those between the scintillators and those between the TEC sectors (see figure 2.4). The lack of hardware redundancy in these gap regions poses a great challenge to determine the muon pair cross sections with small systematic errors.

2.2.1 Central Track Detector

A charged particle traversing a drift chamber ionizes the gas, leaving along its track a number of well separated electron clusters. Under the combined influence of the electric field and interactions with the gas molecules, the electron clusters drift along certain drift paths toward the anode wires with nearly constant drift velocities. In the high field region near the anode wires, the electrons further ionize the gas, producing more electrons. In the end, up to $2 \times 10^6$ electrons arrive at one anode wire producing a sufficiently large pulse. This signal is then used to derive a space point along the track of the incident particle, thus allowing the reconstruction of the track.

With a normal drift chamber, one only records the arrival time of an anode signal, which is determined by the leading edge of the earliest-arriving electron cluster; the spatial resolution and the ability to resolve multiple tracks are therefore limited. By reducing the dimension of the gas amplification region and by lowering the drift velocity (to $\sim 6 \mu m/\text{ns}$) such that the electronics can follow the cluster structure of the ionization, allowing "centre of gravity" measurements of the ionization clusters, a higher single wire resolution ($\sigma = 60 \mu m$) and a better multi-track separation ($\sim 500 \mu m$) can be achieved [21]. These considerations have been taken into account by the design and construction of the $L_3$ central detector and led to the Time Expansion Chamber, TEC [19, 20, 22].

![TEC wire configuration](image)

**Figure 2.6** TEC wire configuration
The TEC is 900-mm long and has an inner radius of 85 mm, an outer radius of 457 mm. It consists of two concentric cylindric parts, the inner and the outer, surrounded by two cylindric proportional chambers with cathode strip readout, the Z-detector.

The inner and outer TEC are radially divided into 12 and 24 sectors respectively, with each inner TEC sector covering two outer TEC sectors (see figure 2.6). There are 8 and 54 anode wires in one inner and outer sector respectively; among which 2 and 9 wires are the so-called charge division wires which are read out at both ends and thus can be used to determine \( r \Phi \) coordinates and \( z \) coordinates at the same time. At either side of the sense plane, there is a plane of grid wires which is to separate the low field drift region from the high field amplification region. The cathode wires are located along the sides of the sector.

In the analyses described in later chapters, the TEC measured momenta, as well as the track multiplicity, are used to distinguish muon pair events from background such as inclusive muon pairs, tau pairs and two photon events. A TEC reconstructed track is accepted only if it satisfies the following criteria:

- There are more than 10 hits used in the track fitting;
- The transverse momentum, \( p_t \), of the track is larger than 200 MeV;
- The transverse distance of the closest approach to the actual beam axis, DCA, is smaller than 4 mm.

### 2.2.2 Electromagnetic Calorimeter

The electromagnetic calorimeter [23] uses \( \text{Bi}_4\text{Ge}_3\text{O}_{12} \) (BGO) crystals as both the showering and detecting medium. The BGO crystals have short radiation length for photons and electrons but large nuclear interaction length.

The calorimeter is split into a barrel part and two endcaps (see figure 2.7). The barrel part, which has a cylindric shape with a inside radius of 520 mm and a inside length of \(~1170\) mm giving a polar angle coverage of \( 42^\circ < \theta < 138^\circ \), is divided into two half-barrels. The endcaps cover the polar angle regions from \( 10^\circ \) to \( 35^\circ \) and from \( 145^\circ \) to \( 170^\circ \).

![Figure 2.7](image)

**Figure 2.7** Left: sideview of the ECAL showing the fragmentation in both the barrel and end-cap regions; right: endview of the barrel part.
2.2 The $L_3$ Detector

Each half-barrel consists of 160 modular 24-crystal "slices", while each endcap contains 1536 BGO crystals. All crystals are truncated pyramids 240 mm long (corresponding to 21.4 radiation lengths, or 1.1 nuclear interaction lengths), 20 × 20 mm$^2$ at the inner end and about 30 × 30 mm$^2$ at the outer end. They all point to the interaction region, with a small angular offset (10 mrad) in $\phi$ to suppress particle leakage.

The energy resolution is 5% at 100 MeV and about 2% for energies above 1.5 GeV [24]. The typical signature of a muon in the ECAL is a one-crystal cluster with an energy deposit of $\sim$ 0.26 GeV. This feature is being used to identify muons (see chapter 4).

2.2.3 Hadron Calorimeter

The hadron calorimeter measures the energy and direction of hadrons [25]. It has a structure similar to that of the ECAL: one barrel, two endcaps (see figures 2.3, 2.4 and 2.5). The barrel covers the central region $35^\circ < \theta < 145^\circ$, the endcaps cover $5.5^\circ < \theta < 35^\circ$ and $145^\circ < \theta < 174.5^\circ$. For hadronic events from Z decays, the hadron calorimeter as a whole covers 99.5% of the full solid angle. An energy resolution of about 10% has been achieved using the hadron calorimeter in combination with the ECAL [26].

The Barrel Hadron Calorimeter

The barrel hadron calorimeter is a fine sampling calorimeter made of depleted uranium absorber plates interspersed with proportional wire chambers. It has a modular structure consisting of 9 rings of 16 modules each, with a total length of 4725 mm, an outer radius of 1795 mm and an inner radius of 885 mm for the three inner (long) rings and 979 mm for the outer (short) rings. The innermost ring is centred at the interaction vertex.

There are 60 planes of proportional chambers in one long module and 53 planes in one short module. The sense wires in alternating planes are perpendicular to each other, running either parallel to the beam axis (referred to as $\Phi$ chambers) or normal to the beam (called $Z$ chambers).

The wires are grouped to form readout towers in order to reduce the amount of readout electronics. There are 9 towers in $\Phi$ and in $Z$ for both the long and short modules, whereas 10 and 8 layers in the radial direction for the long and short modules respectively. In the $\Phi$ projection the towers point to the beam axis with a constant angular interval, while in the $Z$ projection they have a constant width. The number of wires in each tower depends on the position of the tower and ranges from 3 to 28 (see figure 2.8).

The barrel hadron calorimeter acts as a filter as well as a calorimeter, allowing only nonshowering particles to reach the precision muon spectrometer. It is also used as a muon tracker (see chapter 4).
The Hadron Calorimeter Endcaps

The hadron calorimeter endcaps consist of three separate rings: one outer ring and two inner rings (see figures 2.3 and 2.5). Each ring is vertically split into two half-rings. This modularity permits fast withdrawal of the endcap parts to provide access to other central detector components.

The endcap half-rings are stainless steel containers filled with alternating layers of brass tube proportional chambers and depleted uranium absorber plates. Within a half-ring, a chamber layer consists of four chambers, each covering an interval of $\Delta\phi = 45^\circ$ with wires stretched azimuthally to measure the polar angle $\theta$ directly. The even numbered chamber layers are rotated by $\Delta\phi = 22.5^\circ$ with respect to the odd numbered ones. This stereo-angle arrangement allows measurement of the coordinate $\phi$ orthogonal to $\theta$ and the gaps between chambers do not coincide in successive layers.

The azimuthal segmentation of the absorbers is twice that of the chambers. Again the gaps between plates do not coincide in successive layers; nor do they coincide with the gaps between chambers. The gaps also do not point to the beam axis [20].

This arrangement of chambers and absorbers between successive layers permits full coverage over the azimuthal angle $\phi$. 
2.2.4 Scintillation Counters

In between the barrel parts of the electromagnetic and the hadron calorimeter, there are 30 10-mm thick Bicron BC-412 plastic scintillation counters, which stretch along the beam direction (see figures 2.4 and 2.5). The counters are bent to follow the shape of the hadron calorimeter: they are 875 mm away from the beam at the position of the inner rings of the barrel hadron calorimeter but 969 mm away from the beam at the position of the outer rings. In the azimuthal direction $\phi$, from $0^\circ$ to $180^\circ$, and from $202.5^\circ$ to $337.5^\circ$, 28 dumbbell shaped counters, with widths of 167 mm in the middle and 182 mm at the ends, cover 14 HCAL modules. The other two counters are wider, one covering $186^\circ < \phi < 202.5^\circ$, another $337.5^\circ < \phi < 354^\circ$ (see figure 2.4).

The projected length of the counters is 2.9 m, thus providing a polar angle coverage of $34^\circ < \theta < 146^\circ$. In the $\phi$ direction, the counters cover 96.7% of the solid angle.

Using the signals recorded at both ends of one counter, both the mean time ($t_{sc}$), the time the particle hits the counter, and the longitudinal position of the hit are reconstructed. The $t_{sc}$, after correction for the time of flight $t_{flight}$, scatters around zero with a $\sigma$ of $\sim 0.4$ ns for muons originating from genuine $e^+e^-$ collisions, as can be seen in figure 2.9 a. For cosmic rays in which single cosmic muons pass near the interaction point resembling pairs of muons produced in $e^+e^-$ collisions, $t_{sc}$ is uniformly distributed. Therefore $t_{sc}$ can be used to distinguish muons from cosmic ray background.

To further remove in-time cosmic rays from the muon pair sample, one can check the time lag between the $t_{sc}$ from the upper counter and that from the lower counter. For genuine events, this time lag should be around 0 ns; for cosmic ray background, it is about

![Figure 2.9](image)

**Figure 2.9** a) The scintillator mean time after correction for time of flight; b) the scintillator time difference between the upper and lower counters for muon pair candidates.
6 ns (see figure 2.9 b).

2.2.5 Muon Filter

The muon filter is mounted on the inside wall of the support tube and is divided into eight identical octants corresponding to the octagonal structure of the muon spectrometer (see figure 2.4). Each octant consists of six 10-mm thick brass (65% Cu + 35% Zn) absorber plates, interleaved with five layers of proportional chambers and followed by five 15-mm thick absorber plates matching the circular shape of the support tube. They are 4 m long, 1.4 m wide and 0.2 m thick in the radial direction.

In the angular region of $53^\circ \leq \theta \leq 127^\circ$, the muon filter adds about one absorption length to the barrel hadron calorimeter, thus reducing hadronic punch-through to the muon spectrometer. The muon filter hits are used to improve the muon identification efficiency (see chapter 4).

2.2.6 Muon Spectrometer

Muon tracks are first reconstructed in the muon spectrometer region; after this, they are traced back to the interaction point, through the support tube and the inner subdetectors, to correct for energy losses ($\sim 2.6$ GeV) and to optimize the $\theta$ and $\phi$ angles at the interaction vertex [27].

The muon spectrometer is designed [19, 20] to measure muon momenta using a configuration of three layers of precision drift chambers located in the region between the support tube and the magnet coil (see figure 2.3). Along the beam direction, the 12-m long muon spectrometer is split into two Ferris wheels ("master" in $z > 0$, and "slave" in $z < 0$), each having eight independent octants. In each octant, there are two chambers in the outer layer (MO at a distance of 5425 mm from the beam line, see figure 2.10), two chambers in the middle layer (MM at 4010 mm from the beam line), and one chamber in the inner layer (MI at 2530 mm from the beam line). In $\theta$, the regions from $44^\circ$ to $87^\circ$ and from $93^\circ$ to $136^\circ$ are covered by three layers of precision, or "P" chambers, while the regions from $35^\circ$ to $44^\circ$ and from $136^\circ$ to $145^\circ$ are covered by only two layers (MI and MM). In $\phi$, the muon spectrometer covers $\sim 95\%$ of $2\pi$ due to the gaps between P chambers and between octants. In total, about 64% of the full solid angle is covered by at least two layers of P chambers.

The P chambers, with the sense wires stretching along the beam direction, provide measurements of the track coordinates in the bending plane. When measurements of all three chamber layers are available (three track segments), the transverse momentum $p_t$ (in GeV) of a muon can be calculated according to (see figure 2.11):

$$p_t = \frac{3}{80} L^2 B / s,$$

(2.1)
where $L$ is the distance in meters between the positions at MI and MO, $B$ the magnetic field in Tesla while the quantity

$$s = d_2 - (d_1 + d_3)/2$$

is the so-called sagitta, i.e. the maximum deviation of the muon track from a straight line due to the magnetic field. The reconstructed muon track is referred to as a "triplet". When only track segments in two of the three layers of P chambers are available, the $p_t$ can be determined according to:

$$p_t \approx \frac{3}{40} LB \tan \frac{\alpha_i - \alpha_j}{4}$$

(2.2)

where $\alpha_{ij}$ ($i, j = 1, 2, 3$, but $i \neq j$) are the tangent angles measured by two layers of P chambers, $L$ the distance ($\sim 1.5$ m) between the positions of the two track segments (in the $i$-th and $j$-th layers). The reconstructed muon track is then referred to as a "doublet".

The top and bottom covers of the MI and MO P chambers are also drift chambers with sense wires running normal to the beam. These chambers measure the $Z$ coordinates of a muon along the beam, thus determining the $\theta$ angle (see chapter 3).

For a 45.6 GeV muon originating from the interaction point, the momentum is reduced
to \(-43\) GeV by the time it reaches the muon spectrometer. A typical triplet at \(\theta = 64^\circ, \phi = 11.25^\circ\) with \(p_t = 39\) GeV and a typical doublet at \(\theta = 40^\circ, \phi = 11.25^\circ\) with \(p_t \approx 28\) GeV are taken as examples for a discussion of the momentum resolution. These tracks have \(s \approx 4\) mm and \(|\alpha_1 - \alpha_2| = 8\) mrad respectively.

As will be shown in chapter 3, the \(\theta\)-angle of muons can be determined to \(\Delta \theta = 1\) mrad. So from the error propagation:

\[
\frac{\Delta p}{p} = \sqrt{\left(\frac{\Delta p_t}{p_t}\right)^2 + \left(\frac{\Delta \theta}{\tan \theta}\right)^2}
\]

one can see that the uncertainty in the momentum measurement is dominated by the uncertainty in the \(p_t\). From equation 2.1, one estimates that in order to achieve \(\Delta p_t / p_t < 2\%\) for triplets, one must measure \(s\) to \(\Delta s < 80\) \(\mu\)m. Similarly, one can deduce from equation 2.2 that for doublets, even \(\Delta p_t / p_t < 15\%\) already requires \(\Delta(\alpha_i - \alpha_j) < 1.2\) mrad.

Monte Carlo study shows that more than 75\% of the muons with energies larger than 5 GeV are confined within one octant. For these muons, the main contributions to the errors in the sagitta measurements are:

- intrinsic resolution of the drift chambers;
- multiple scattering;
- alignment of chambers belonging to different layers.

**Precision Chambers**

The P chambers are about 6 m long and are constructed of two machined aluminium end frames, and two extruded aluminium side panels. The MI and MO chambers are closed on the top and bottom by Z chambers, the MM chambers are closed by aluminium honeycomb panels to minimize the effect of multiple scattering on the momentum resolution.

![Figure 2.12 Details of a P chamber cell.](image)
Each MI, MM or MO chamber has 19, 15 or 21 cells separated from one another by (mesh) planes of cathode wires. The width of a cell, i.e. the distance between the mesh planes, is 101.5 mm. In the middle of a MI, MM or MO cell, 50.75 mm away from the mesh planes, there are 20, 28 or 20 sense wires, spaced 9 mm apart and interspersed with field wires (see figure 2.12). On the top and bottom of each cell, beyond the outmost sense wires, there are three guard wires which together with the two outmost sense wires equalize the drift time behaviour of the remaining 16, 24 or 16 sense (signal) wires within 0.2 ns. Multiple sampling of coordinates along the track of one particle improves the position measurement by a factor $\sqrt{n}$ over the single wire resolution, where $n$ is the number of samplings [28].

Inside one chamber, the wires are supported by three precision ladders (see figure 2.13), one at each end, one in the middle of the chamber. The one in the middle is needed to:

- reduce the gravitational sag of the wires by a factor four (to 96 $\mu$m),
- reduce the positional uncertainty due to electrostatic forces,
- double the natural frequency of the wires hence reducing the peak self-vibration amplitude.

The ladders consist of Pyrex glass pieces, glued at the top and bottom to two carbon fibre bars. Each edge of a glass piece defines a wire plane, one sense plane on one side and one mesh plane on the other side. The two end ladders are positioned with respect to external reference surfaces in such a way that they can only be moved in the direction $x$; the middle ladder is supported by precision actuators allowing adjustments in the $x$ and $y$ directions. Integrated into the structure of the ladders are three calibrated RASNIK straightness monitors which consist of LEDs, lenses and 4-quadrant photodiodes [29]. Light from the LED mounted on one end ladder is imaged by the lens in the middle ladder onto the 4-quadrant photodiode at the opposite end ladder. A displacement $\delta$ of the middle ladder moves the image on the photodiode by $2\delta$. During data taking, the displacement of the

![Figure 2.13](image.png)  
*Figure 2.13* Three ladders support the wires. Also indicated are the three straightness monitors and the directions in which the three actuators move.
middle ladder with respect to the end ladders are measured continuously by the RASNIK systems. This information is written to the database and later used by the reconstruction program to correct the wire positions with an accuracy of ~ 10 µm in the x direction and 40 µm in the y direction.

A gas mixture of 61.5% argon and 38.5% ethane is chosen for the chambers in order to maintain nearly constant drift velocity around the working high voltage point. To obtain a very uniform electric field throughout the active region inside one cell and to control the gas amplification, four different high voltages are applied to the sense, field, cathode and guard wires. At nominal voltage settings, with an electric field of 114 V/mm in the drift region, in a 0.5 T magnetic field and at 740 mm Hg pressure, the drift velocity is about 48 µm/ns, the drift angle due to the Lorentz force is about 19°.

Data taken in test beam runs with a small test chamber in a magnetic field of 0.5 T show that, averaged over all positions and slopes expected for high momentum tracks, the overall rms single-wire resolution is ~ 170 µm [20, 30]. For data taken with the real chambers in 1991 and 1992, this resolution is about 210 µm (see appendix A).

**Alignment between the Precision Chambers**

Inside one octant, the P chambers are supported by a structure consisting of a longeron and two end frames. The structure also contains about 300 special parts to provide precise spacing in the radial direction between chambers and to maintain long term chamber alignment to better than 30 µm [31]. In the tangential direction, the relative positions of the sense wires are adjusted (during the assembly phase) and monitored (during data taking).

![Figure 2.14](image)

**Figure 2.14** The laser beacon system defines a plane through the centre of an octant using a He-Ne laser beam which is reflected by 90° by a highly accurate rotating pentaprismatic mirror assembly. Together with components of the straightness monitors, the laser sensors (multichannel photodiode arrays) are mounted on gauge blocks.
with the help of a laser beacon system [32] (see figure 2.14), and four straightness monitors (see figure 2.10).

As external reference for the locations of the wire planes, one gauge block containing one laser sensor and two LEDs is attached to the middle of each end frame of the MI chamber. By moving the end ladder so that a certain field wire just makes electrical contact with one insulated brass pin that is referenced to the LEDs, the wire planes are positioned to a few μm.

At each end of the MM or MO chamber pair, one gauge block keeps the edge sense-wire planes of opposing chambers at a precise separation of 203 mm, using two touching pins — one touches one field wire in the left hand chamber, one touches one field wire in the right hand chamber. On each of these gauge blocks, there is also a laser sensor. In addition, on the one between the MMs, there is a lens; on the one between the MOs, there are two 4-quadrant photodiodes. Thus, in each end of one octant, there are two straightness monitors, which operate on the same principle as the RASNIK systems.

Cosmic ray and UV laser verifications with the 16 octants show that at each end of one octant, the chamber centres have been brought into a straight line within 30 μm [20]. These two octant-centre lines have been tuned to be parallel to each other and to pass the beam line within ±2 mm.

During data taking, the displacements of the MM chamber layer with respect to the MI-MO-centre lines at both ends of each octant are measured by the straightness monitors. This information is written to the database and fed to the reconstruction program later.

**Z Chambers**

The Z chamber layers consist of two planes of about 58 drift cells offset by one half cell with respect to each other to resolve left-right ambiguities (see figure 2.15). Each cell, 91.8 mm wide and 29.3 mm high, has two parallel aluminium I beams at −2.4 kV and one gold-plated molybdenum anode wire of 50 μm diameter at 2.15 kV in the centre. The cell is

![Figure 2.15](image-url)  
**Figure 2.15** Endview of a Z chamber layer showing the double-plane structure (the numbers are in mm).
closed by two aluminium sheets at ground potential and isolated from the I beam profiles by fibre glass strips. In order to keep the variation of the drift velocity (~ 30 μm/ns, averaged over the cell) small, a gas mixture of 91.5% argon and 8.5% methane has been chosen.

Within one octant, corresponding wires of opposite MO chambers are electrically connected. Signals from the total of 7456 anode wires are processed by time recording channels. The time-to-distance conversion function, i.e. the cell-map function, has been mapped in test beam runs with a prototype. The measured single wire resolution, both in test beam runs with the prototype and with cosmic rays in production chambers without magnetic field, is about 500 μm [33].

The Z chambers cover θ from 24° to 156°. In z, they are positioned with a tolerance of ± 400 μm. In the radial direction, the uncertainty in the Z chamber positioning is negligible.

More details concerning the Z chambers, such as the construction of the cell-map function, the pattern recognition and the simulation of its response, are given in chapter 3.

Momentum Resolution

During the 1991 and 1992 running periods, momentum resolutions of 2.5% for triplets [34] and 20% for doublets (see figure 5.2 b) at ~ 45.6 GeV were obtained. These results correspond to momentum resolutions of 2.47% for triplets and 20.9% for doublets with the muon spectrometer at \( p_t = 39 \text{ GeV} \) and 28 GeV respectively. The correction due to the energy loss of ~ 2.6 GeV in the inner subdetectors is estimated to have a relative accuracy of better than 15%. This corresponds to a contribution to the final momentum resolution of at most 0.8%.

As shown in appendix A, the single wire resolution of the P chambers is about 210 μm including the inaccuracy of the cell-map function and the fluctuations introduced by the electronics. Following the procedure described in [35], this single wire resolution leads to a sagitta measurement accuracy of

\[ \Delta s = 75 \text{ μm} , \]

including a 31 μm contribution due to multiple scattering and a 33 μm contribution due to internal alignments. This accuracy of the sagitta measurement implies \( \Delta p_t / p_t = 1.9% \) for triplets at \( p_t = 39 \text{ GeV} \), and hence the final triplet momentum resolution should be about 2% for muons at 45.6 GeV.

The discrepancy between the achieved resolution of 2.5% and the expectation can be attributed to an additional error of about 62 μm in the sagitta measurement. This additional error may be due to imperfections in the internal alignments within each octant, incomplete understanding of the performance of the electronics, etc.

The above study shows that, while improvement in the doublet resolution can be a direct result of improving the P chamber single wire resolution, noticeable improvement
in the triplet resolution can only be achieved by resolving the unknown error in the sagitta measurement.

2.2.7 Luminosity Monitor

The luminosity provided by LEP at the $L_3$ interaction point is determined by measuring the rate of Bhabha events ($e^+e^- \rightarrow e^+e^-$) at small scattering angle. This process has very little contribution from the $Z$ exchange, but is for more than 99% dominated by the elastic scattering between the electron and the positron. The cross section for the elastic scattering can be calculated to very high precision within the framework of QED.

The luminosity monitor, see figures 2.3, 2.5 and 2.16, consists of two identical parts, which are located at either side of the interaction point at a distance of 2.7 m and cover the polar angle range $1.4^\circ < \theta < 3.9^\circ$. Each part of the luminosity monitor is a highly segmented BGO crystal array. In order to protect the crystals from radiation damage due to beam loss, the array is split vertically into two half-cylindrical shells that can be moved away from their nominal positions close to the beam pipe before each fill of LEP.

During the 1991 and 1992 running periods, an accuracy of 0.6% on the luminosity determination was obtained [26].

![Figure 2.16 A Bhabha event in the luminosity monitor.](image)

2.2.8 Trigger System

After each bunch crossing, all the subdetectors are read out by front end electronics. The trigger system has to decide before the next bunch crossing, i.e. within 22 $\mu$s whether an interaction of physics interest took place. In order to go from the 45 kHz bunch crossing rate to a few Hz of tape writing rate, three levels of triggers act as a filter.

The level-1 and level-2 triggers make their decisions based on special trigger data with coarse granularity and lower resolution provided by the subdetectors, while the level-3 trigger is embedded in the main flow of the data acquisition.
The level-1 trigger

The level-1 trigger is a logical OR of trigger conditions from different sources: muon trigger, TEC trigger, calorimeter trigger and scintillator trigger.

The **muon trigger** searches for tracks originating at the interaction point according to a two-step (load and search) procedure. In the load step, information from all the "trigger cells" are loaded into memory. For the P chambers, the trigger cells coincide with the physical chamber cells (16 or 24 wires); for the Z chambers, they are the combinations of the two adjacent wires of each double-plane. For each P chamber trigger cell, the number of hits is counted and compared to a preset threshold number, and then a decision on the presence of a track segment is made. In the search step, the presence of tracks are determined by looking for track segments in a number of predefined "roads", each corresponding to a certain region of the muon production angle in $\phi$ (all the possible tracks with $p_t > 2$ GeV are defined) and being parametrized by its central cell number and its half width in each one of the 3 chamber layers. The following conditions give a trigger:

- **Di-muon trigger**: At least two octants should have tracks, each defined as the coincidence of identified track segments in any two of the three P chamber layers, and the two tracks should have an acoplanarity of less than 90°. This trigger can cover down to $\theta = 36^\circ$.

- **Single muon trigger**: At least one octant should have an identified track, defined in this case as the coincidence of track segments in all the three P chamber layers. This trigger covers the $\theta$-angle down to $44^\circ$.

- **Small-angle muon trigger**: At least two tracks, each defined as a track segment in MI, and a coincidence of both layers of the MI Z chambers, identified in two back-to-back octants. This trigger is meant to be the backup of the di-muon trigger because in the small angle region ($36^\circ < \theta < 44^\circ$), there are only two layers of P chambers.

The **TEC trigger** uses the information given by 14 pre-appointed sense wires in each outer TEC sector to search for tracks originating from the beam line. It fires when at least two tracks with $p_t > 0.15$ GeV and with an angular separation larger than 120° in $\phi$ are observed. In $\theta$, it can cover down to $35^\circ$.

The **calorimeter trigger** uses the information from the ECAL, the HCAL and the luminosity monitors to perform the following trigger calculations: total energy trigger, cluster trigger, single photon trigger, hit counting trigger, luminosity trigger and single tag trigger. The one relevant to the muon pair analysis is the total energy trigger, which fires when more than 10 GeV is found in the ECAL barrel, or 15 GeV jointly in the ECAL barrel and the HCAL barrel, or 20 GeV in all calorimeters, including the endcaps. It acts as a backup trigger for muon pair events with hard photons.

The **scintillator trigger** requires that at least 5 out of the 30 scintillation counters
2.2 The $L_3$ Detector

fired within 10 ns of the bunch crossing and that among those counters which fired, at least two should be separated by more than 45° in $\phi$.

After each bunch crossing, all the above trigger conditions are evaluated within 22 $\mu$s. If the event is rejected, the readout system is reset and ready for the next event. If the event is accepted, all the detector components start digitizing and buffering, which takes about 500 $\mu$s. During this time period all the detector components are protected from accepting new events, but after that the detector is active again. During the 1991 and 1992 running periods, the level-1 trigger rate was typically 8 Hz, corresponding to a dead time of 0.4%.

The level-2 trigger

Four XOP processors [36] are used in a "round robin" mode to reduce the level-1 triggered event rate by about 25%. The following checks are made:

- the clustered energies in the ECAL and two lateral layers of the HCAL have to be correlated in the $\theta$-$\phi$ plane;
- the clustered energies have to be balanced in both the longitudinal and transverse directions;
- a rough vertex along the beam axis is reconstructed based on information from the charge division wires of the TEC.

The level-3 trigger

The level-3 trigger has access to the complete digitized data with finer granularity and higher resolution. It selects events according to:

- the correlation of the energies deposited in the ECAL and HCAL;
- the reconstruction of the muon tracks in the Z chambers;
- the reconstruction of the vertex in the TEC.

During the 1991 and 1992 running periods, this level-3 trigger reduced the number of level-2 accepted events by a factor 2. The final tape writing rate was typically 3 Hz.

2.2.9 Offline Processing

Tapes from the data acquisition system contain groups of individual ADC and TDC* counts (the raw data). These numbers, together with information from the databases, are passed

*Analogue to Digital and Time to Digital Converters.
onto the offline reconstruction package. Based on information from individual subdetectors, items such as TEC tracks, ECAL clusters and bumps, scintillator hits, HCAL clusters, muon filter hits and muon chamber tracks are constructed using preliminary calibration constants. These items are then combined to form physical objects like muons, electrons, photons and jets. Quantities such as momenta, energies and spatial orientations are determined for these objects.

Due to the large volume of the raw data, a significant reduction is necessary for easier handling later on. For this purpose, loose criteria are applied to select various types of events according to various physics processes. For each stream of event types, the selected events are written to a separate tape, the Data REconstruction (DRE) tape. Among the streams is the $\mu\mu$ stream containing muon pair candidates. A combined tape, the Master Data REconstruction (MDRE) tape, is also obtained containing all events which are selected as candidates of any one of the event types.

For the muon pair analysis described in this thesis, the $\mu\mu$ DRE tapes are further reduced to a set of Lepton-Lepton Data SUmmary (LLDSU) files by filtering out the detailed raw data information of the physical objects and by packing the numbers. For the years of 1991 and 1992, there are about 110 $\mu\mu$ DRE tapes of 200 Mb each, while the LLDSU files only occupy about 85 Mb in total.

When improved reconstruction software or calibration constants become available, the whole process is redone starting from the MDRE tapes.
Chapter 3

The Z Chambers of the Muon Spectrometer

Measurements of the $\theta$-angle and the transverse momentum $p_t$ determine the momentum $p$ of a muon. A good $\theta$-angle determination is essential to the muon pair analysis for acceptance calculations and for asymmetry measurements. Besides, for the inclusive muon analysis a reliable $\theta$-angle is needed in order to determine the transverse momentum of a muon with respect to a jet [37]. In $L_3$, the $\theta$-angle of a muon is mainly determined with the $Z$ chambers of the muon spectrometer, which measure up to 8 points in $z$ along the muon track.

In the first two sections of this chapter two essential ingredients for the position measurements with the $Z$ chambers are introduced — the drift time and the cell-map function. Then the method to reconstruct tracks in the $r$-$z$ plane is described in section 3, and the single wire resolution of the $Z$ chambers is checked in section 4 using real data. Following these, the simulation of the response of the $Z$ chambers to traversing charged particles is outlined in section 5. In section 6, the $\theta$-angles determined in the muon chamber region are compared with the $\theta$-angles at the interaction vertex for simulated muons.

3.1 Drift Time

A muon, created in an $e^+e^-$ collision at time $t = 0$, reaches a $Z$ chamber cell and creates drift electrons at time $t = t_{\text{TOF}}$. After a time period of $t_{\text{drift}}$, the avalanche initiated by the earliest-arriving electron cluster reaches the anode wire and creates an electric pulse. Further after having passed an amplifier, a discriminator and about 50-m of cable length,
therefore a time period of \( t_0 \), this electric signal arrives at a TDC*. This TDC records a number, \( N_{TDC} \), indicating that a signal arrived at time \( t = t_{ROF} + t_{drift} + t_0 \) (in units of TDC bins). In order to calculate the drift distance of the earliest-arriving electron cluster, the drift time \( t_{drift} \) has to be derived from \( N_{TDC} \). This is done through the following steps:

- **time-of-propagation (\( t_0 \)) subtraction**: The \( t_0 \) is determined, channel by channel, by timing the propagation of artificially induced signals. This is done on a daily basis, or whenever there is any hardware modification to the Z chambers. \( t_0 \) is measured in TDC bins.

- **TDC bin width conversion**: After the \( t_0 \) subtraction the remaining value is converted from TDC bins to ns. One TDC bin corresponds to 2.448 ns in the data and 1 ns in Monte Carlo simulation.

- **time-of-flight subtraction**: In the \( L_3 \) reconstruction package, at the time the Z chamber reconstruction is performed, the Z hits are not yet associated with accurate \( \phi \) information. The time-of-flight can be approximated by assuming that the incident particle travelled at the speed of light from the interaction point to the middle of the Z chamber cell and the signal travelled again at the speed of light from the middle of the anode wire to the end of the wire to reach the electronics.

### 3.2 Cell-map Function

In order to derive from the drift time the drift distance, and thus one spatial point, the drift path and drift velocity must be known. In the ideal case when the electric field is uniform, the drift paths are straight lines, the drift velocity is a constant, and therefore the relationship between the drift distance and the drift time is a linear one. In the case of the Z chamber cells, the electric field is far from being uniform, the drift paths are curves which differ in shape (see figure 3.1). The drift velocity changes from point to point and as a consequence the relationship between the drift distance and the drift time is a complicated one.

![Figure 3.1 The drift paths inside a Z chamber cell.](image)

*LeCroy LRS 1879 time digitizer*
3.2 Cell-map Function

A cell-map function has been constructed in order to correlate the measured drift time with a spatial point on the track of the incident particle. This function is based on an analysis of the test beam data taken at CERN in 1986 and 1987 with a Z chamber prototype without magnetic field.

The test beam setup is schematically shown in figure 3.2. The Z chamber prototype was situated on a precision table which could be moved with a maximum displacement somewhat larger than the cell width of 45.9 mm in the direction perpendicular to the beam. Rotations could take place with a maximum rotation angle of \( \alpha = 45^\circ \) around the axis parallel to the anode wires. The beam trajectory was defined by two microstrip detectors, placed in front of and behind the P chamber prototype.

Measurements of the drift time \( t \) were taken at various drift distances \( d \) and at various inclination angles \( \alpha \) (see figure 3.2). As expected, the dependence of the drift time on the drift distance was clearly non-linear. This non-linearity appears already at \( \alpha = 0^\circ \), and becomes more pronounced as the inclination angle \( \alpha \) increases (see figure 3.3). Using a linear dependence, i.e. a constant drift velocity, leads to a resolution of 1200 \( \mu \)m for track residuals.

The resolution was substantially improved to \(~500 \mu \)m by fitting the drift-distance and

---

\( ^\dagger \)This point does not necessarily coincide with the point where the earliest-arriving primary electron cluster was created.
Figure 3.3 The cell-map function for three inclination angles of $\alpha = 0^\circ$, $20^\circ$ and $40^\circ$.

drift-time correlation to third order Legendre polynomials\footnote{Legendre polynomials were chosen in order to reduce the correlations between the final parameters [38].} in two regions below and above the kink at $t = 445$ ns (see figure 3.3). The following four steps were taken to obtain a cell-map function from the test beam data without magnetic field:

- The drift time $t$ is converted into a new variable $s$ according to:

\[
s = \begin{cases} 
  -1 + t / 222.5 & t \leq 445 \text{ ns} \\
  -1 + (t - 445) / 635 & t > 445 \text{ ns} 
\end{cases}
\]

- Three polynomials are defined as functions of $s$:

\[
\begin{align*}
  P_1 &= s \\
  P_2 &= (3s^2 - 1) / 2 \\
  P_3 &= (5s^3 - 3s) / 2
\end{align*}
\]

- The drift-distance drift-time correlation is parametrized for each inclination angle $\alpha$ using the polynomials:

\[
d = \begin{cases} 
  B_1 + B_2 \times P_1 + B_3 \times P_2 & t \leq 445 \text{ ns} \\
  B_4 + B_5 \times P_1 + B_6 \times P_2 + B_7 \times P_3 & t > 445 \text{ ns} 
\end{cases}
\]

Seven smoothly $\alpha$-dependent variables $B_1$ through $B_7$ are obtained.

- The dependence of the variables $B$ on $\alpha$ is parametrized according to:

\[
B_i = A_i^1 + A_i^2 \times \alpha + A_i^3 \times \alpha^2 \quad (i = 1, 2, \cdots, 7)
\]
The $3 \times 7$ A's are sufficient for parametrizing the drift-time to drift-distance conversion function, i.e. the cell-map function, for all inclination angles with an averaged resolution of $\sim 500 \mu m$.

Two spatial points correspond to one drift distance: one on the left and one on the right hand side of the anode wire. In order to determine the $z$ coordinate of a hit, this left-right ambiguity has to be solved and the inclination angle $\alpha$, or the slope of the associated track, has to be known. This information is only available after fitting a track through a set of hits, i.e. after the pattern recognition has been performed.

### 3.3 Pattern Recognition

In the $r$-$z$ plane, particle trajectories are not bent by the magnetic field; the effect due to multiple scattering in the material between the MI and MO $Z$ chambers is also small. Therefore the task of the pattern recognition is to find straight lines passing the maximum numbers of hits.

The number of hits included in one fit may vary from a minimum of 3 to a maximum of 8, one hit from each plane. Shown in figure 3.4 is a track having 7 hits. On the first MI plane there is one background hit (the left-right circle pair). On the second MO plane there is one hit missing. In order to handle a variable amount of hits with a concise computer program, a reduction routine has been developed to find the best left-right combination for $N$ hits [40]. In this routine, the least significant $N$ bits of a binary number $M$ are allocated.

![Figure 3.4](image-url)  
*Figure 3.4* Side view of one octant showing one track (dashed line) having 7 hits. On the lowest plane, there is one background hit (the left-and-right circle pair).
to the $N$ hits. The value of the $i$-th bit of $M$ corresponds to the left-right position assigned to the $i$-th hit. When $M$ takes values from 0 through $2^N - 1$, all the $2^N$ possible ambiguity combinations are covered with only 2 nested loops. The best ambiguity combination is then found with the smallest $\chi^2$. The routine can handle any amount of hits and can also be easily generalized to deal with an arbitrary number of ambiguities for each hit.

Using the cell-map function and the reduction routine, the pattern recognition proceeds through the following steps [41]:

1. A search is made to find sets of 4 MI hits, one from each plane. For each set of hits, a rough track slope is calculated from the coordinates of the corresponding anode wires. Using this track slope as the initial value, the reduction routine finds the best ambiguity combination for this set of hits and the corresponding new slope. A Z segment is defined if the fit has a $\chi^2$ smaller than a preset value.

Each MI segment is extrapolated to the MO region. A hit is picked up in each MO plane if it is the closest to and within 10 mm in $z$ of this segment. The segment is then refit including these MO hits.

2. The best segment is defined as the one having the largest number of hits and the smallest $\chi^2$. Those segments which share any MI hit(s) with the best segment are disregarded. In a similar way, the second-best segment is defined among the remaining segments and also here the hit-sharing segment(s) are disregarded, etc.

If certain segments are very close to each other in the $r$-$z$ plane, i.e. within the MI region, the RMS distances in $z$ between them are less than 10 cm, only the best segment is kept while all others are disregarded. The best segment is the one with the largest number of hits and with the smallest distance in $z$ to the interaction point when extrapolated to the beam line. This procedure is needed in order to handle events with a large amount of background hits.

3. Similar to the above two steps, 3-MI-hit segments are constructed out of the so far unused MI hits with another preset $\chi^2$ value. In contrast, all MO hits are available for being picked up.

4. If so far no segment has been found, the preset $\chi^2$ values are loosened and steps 1, 2 and 3 are repeated.

5. Steps 1-4 are repeated starting from MO hits. Similarly, all MI hits are available for being picked up.

6. If only 2 planes in MI and 2 planes in MO have hits, they are renamed as MI planes and steps 1-5 are applied. This is to deal with regions where broken or disconnected wires lie up in $\theta$.

7. The RMS distance in $z$ is checked between every MI and MO segment pair. A two-segment Z track is constructed out of each pair of segments if the RMS distance
between them is less than 10 cm. A one-segment Z track is defined for every unmatched segment. For each two-segment Z track, the slope and intercept are calculated by a refit including all the hits used by the two component segments; for each one-segment Z track, the parameters are taken directly from the segment fitting.

After the Z and P chamber reconstruction, the \( \theta \)-angle of a muon track is determined by combining the Z track with a track obtained with the P chambers in the \( r-\phi \) plane [37]. Due to hardware problems, about 1.3% of the muon tracks do not have a reconstructed Z track\(^8\). The \( \theta \)-angle of these tracks are determined with hits in the calorimeters at a later stage (see chapter 4).

### 3.4 Single Wire Resolution of the Z Chambers

For the reconstruction of Z tracks, the cell-map function is crucial. The result of the reconstruction can be used to check the performance of and to improve the cell-map function. Taking the distance to the anode wire, \( d_{\text{fit}} \) obtained from the segment fitting as the true value for the drift distance, the difference between this value and the drift distance, \( d_{\text{map}} \) given by the cell-map function can be defined as the residual. It is expected that the residual distribution peaks around zero, independent of drift time and track slope.

The straight line approach as described in section 3.3 was adopted to achieve high track finding efficiency, and a drift time dependent correction was introduced to incorporate the influence of the magnetic field [42] such that the residuals scatter around zero as can be seen in figure 3.5 a. In figure 3.5 b, a two-Gaussian fit is made to the residual distribution. The main Gaussian contains about 86% of the hits and has a \( \sigma \) of about 670 \( \mu \)m. This value of the \( \sigma \) includes a small contribution of \( \sim 30 \mu m \) [35] from the multiple scattering in between the MI and MO Z chambers and a contribution of \( \sim 400 \mu m \) from the alignment accuracy between different Z chamber planes. The second Gaussian has a \( \sigma \) of 3.76 mm. It is due to hits with large drift distances. For these hits, the cell-map function does not work very well.

### 3.5 Simulation of the Z Chamber Response

An outgoing charged particle can hit up to two cells in one Z chamber plane (see figure 2.15). However for the reconstruction of a track in the \( r-z \) plane, one well-simulated hit from each Z chamber plane is sufficient (see section 3.3). Therefore, the major task for the Z chamber simulation is to find the corresponding drift-time when a cell is traversed by a

\(^8\)In cases when no Z track is found due to lack of hits, but the P chamber reconstruction does require Z information for the combined fit, a default Z track is taken with a slope of \( \sim 0.4 (\theta = 111.8^\circ) \), and intercept 0. This track is chosen such that the calibrations concerning the \( p_t \) fit take the average values.
charged particle in such a way that the track of the particle intersects the $z$ axis within the cell. This process is the reverse of calculating drift distance using a cell-map function.

Normally, the inverse of the cell-map function is constructed and used to calculate the drift time [37]. However with our Z chambers, the drift time is obtained by solving the equation:

\[
\text{cell-map as a function of drift time} = \text{drift distance}.
\]

This is done with an iteration method. The complete procedure is as follows:

- From the tracking of the particle, the inclination angle $\alpha$ and the drift-distance $d$ are known. With $\alpha$ as input to the cell-map function, a drift-distance $d_k$ is calculated, assuming the drift-time to be $t_k = 445$ ns which is the position of the kink in the curves of figure 3.3.

- Comparing $d$ and $d_k$ establishes whether the drift-time is $t \leq t_k$ or $t > t_k$. A root-finder [43] is then used to find $t$, according to the cell-map function, in the interval $[0, 445]$ or $[445, 1715]$ to an accuracy of about 15 ns. Here $t = 1715$ ns is the maximum drift-time under normal circumstances; the 15 ns accuracy corresponds to the intrinsic, i.e. test beam, resolution of the Z chambers.

- The drift-time is smeared using a Gaussian with a $\sigma$ of 14 ns, as the $z$ coordinates of the sense wires are only known to an accuracy of about 400 $\mu$m with respect to each other.
3.6 Resolution of the $\theta$-Angle Determination

- The time-of-flight is added to the drift-time, and the final time information is converted into the number of TDC bins.

This procedure is done on a track-by-track basis for a given cell. In case one cell might record more than one hit, i.e. when more than one track hit the same cell, or a track spirals through a cell, the possible hits are first sorted in ascending order of drift time; then successive hits are combined, if the time intervals between them are shorter than 100 ns.

The performance of the cell-map function and the reconstruction procedure on simulated data are checked. A single wire resolution of about 600 $\mu$m is obtained. This resolution is similar to what is achieved with real data.

3.6 Resolution of the $\theta$-Angle Determination

For muons originating from the interaction vertex, the $\theta$-angles determined in the muon chamber region correspond to the production angles at the interaction vertex, smeared by the multiple scattering in the inner subdetectors and by the finite resolution of the Z chambers.

For small scattering angles, the total deflection due to multiple scattering can be approximated by a Gaussian with a width [44]:

$$\Delta \theta_{\text{MS}} = \frac{0.0136}{p} \cdot (1 + 0.038 \cdot \ln l) \cdot \sqrt{l},$$

where $p$ is the muon momentum in GeV, and $l$ is the distance traversed in the inner subdetectors measured in units of radiation length. For a 50 GeV muon traversing at $\theta = 60^\circ$, $l = 156 X_0$ [20], therefore:

$$\Delta \theta_{\text{MS}} = 4 \text{ mrad}.$$

The resolution of the Z chambers is estimated, according to the single wire resolution, to be

$$\Delta \theta_{\text{SW}} = 1.5 \text{ mrad}$$

assuming the muon traversed at $\theta = 60^\circ$ and only 4 hits from the MI (or MO) Z chambers are included in the fit. If in total 6 hits, 3 from MI and 3 from MO, are used in the fit, this contribution is reduced to

$$\Delta \theta_{\text{SW}} = 0.14 \text{ mrad}.$$

Thus the production $\theta$-angles at the interaction vertex can be approximated by the ones determined with the muon chambers to an accuracy of about 4 mrad. This is confirmed by a Monte Carlo study which compares the muon chamber reconstructed $\theta$-angles with that given by the generator at the interaction vertex for simulated muons (see figure 3.6).
Figure 3.6 The differences between the muon chamber reconstructed $\theta$-angles and that given by the generator at the interaction vertex for simulated muons at 45.6 GeV. The distribution is a Gaussian with a $\sigma$ of about 4 mrad.
Chapter 4

Muon Identification with the Calorimeters

Minimum ionizing particles (MIPs) produce distinct signals in ECAL and HCAL as illustrated schematically in figure 4.1. This chapter describes the muon identification with these calorimeters. The use of the calorimeter information in addition to the muon spectrometer information enables us to determine the cross sections with small systematic errors.

Figure 4.1 A muon in a $e^+e^- \rightarrow \mu^+\mu^-\gamma$ event creates a track in the TEC, a small bump in the ECAL and a long and thin minimum ionizing track in the HCAL. Also shown is the 37 GeV ECAL signal from the photon.
4.1 Muon Identification with the ECAL

4.1.1 Procedure

The energy deposits and the spatial locations of traversing particles in the ECAL are reconstructed with a cluster algorithm. Clusters are formed from arrays of neighbouring crystals each having an energy above 10 MeV. A cluster is subdivided into bumps centred around a local energy maximum. In principle, one bump corresponds to one traversing particle.

A typical bump from a traversing muon consists of a few crystals, with a total energy deposit of ~260 MeV. The ECAL information alone is not sufficient for muon identification. It is used to confirm or veto the presence of a muon when there is only a TEC track or an identified HCAL MIP (see next section), but no information from the muon spectrometer. In order to increase the rejection power, a "cone" of 100 mrad in $\phi$ around a TEC track, or in space around an HCAL MIP, is defined. All ECAL bumps inside this cone are added. The criteria for the confirmation of a muon by an ECAL MIP are:

- the total number of crystals inside this cone is less than 15,
- the total energy deposit inside this cone is less than 1 GeV.

These criteria suppress background contamination from hadronic $\tau$ decays. In order to reject electronic noise, bumps with energies lower than 100 MeV are disregarded.

4.1.2 Efficiency Determination

Muons radiating hard bremsstrahlung photons ($E_\gamma > 0.8$ GeV) will not pass the selection criteria. Also, muons that pass through the dead regions will not be recognized. Therefore, the usefulness of the MIP finding procedure is determined by the reliability of the Monte Carlo simulation for these effects. Using muons that are identified with the muon spectrometer, it is found that the total number of crystals around a muon track, $N_{\text{crys}}$, and the total energy deposit, $E_{\text{ECAL}}$, can be well simulated. In figure 4.2, the distributions of these two quantities are compared between the 1992 data and the Monte Carlo simulation. Very good agreement is observed. The distribution of muons over three categories: muons having identified ECAL MIPs (all bins except the first bins in figure 4.2 a and b), muons with hard bremsstrahlung photons (the overflows) and muons going through the dead regions (the first bins) is given in table 4.1. The percentage of muons having identified ECAL MIPs, i.e. the efficiency of this MIP finding procedure, is shown in figure 4.3 as function of the $\theta$ and $\phi$ angles. The large dips in the $\phi$ distribution are due to crystals with malfunctioning readout. Also here the data is very well simulated.
4.1 Muon Identification with the ECAL

![Graphs showing the total number of crystals and total energy deposit inside a 100 mrad cone around a muon. The dots correspond to the 1992 data, the histograms are from Monte Carlo simulation normalized to the data.]

Figure 4.2 a) The total number of crystals and b) the total energy deposit inside a 100 mrad cone around a muon. The dots correspond to the 1992 data, the histograms are from Monte Carlo simulation normalized to the data.

<table>
<thead>
<tr>
<th>Sample</th>
<th>MIP Identified</th>
<th>Hard Photon</th>
<th>No Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991 pre-scan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8477 muons</td>
<td>90.6% ± 0.3%</td>
<td>7.4% ± 0.3%</td>
<td>2.0% ± 0.2%</td>
</tr>
<tr>
<td>104549 MC muons</td>
<td>90.16% ± 0.09%</td>
<td>7.70% ± 0.08%</td>
<td>2.14% ± 0.04%</td>
</tr>
<tr>
<td>1991 scan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4607 muons</td>
<td>91.6% ± 0.4%</td>
<td>6.9% ± 0.4%</td>
<td>1.5% ± 0.2%</td>
</tr>
<tr>
<td>104355 MC muons</td>
<td>90.86% ± 0.09%</td>
<td>7.71% ± 0.08%</td>
<td>1.43% ± 0.04%</td>
</tr>
<tr>
<td>1992</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32969 muons</td>
<td>89.71% ± 0.17%</td>
<td>7.19% ± 0.14%</td>
<td>3.09% ± 0.10%</td>
</tr>
<tr>
<td>102929 MC muons</td>
<td>89.43% ± 0.10%</td>
<td>7.61% ± 0.08%</td>
<td>2.96% ± 0.05%</td>
</tr>
</tbody>
</table>

Table 4.1 Breakdown of muon signals in the ECAL.

The small difference in the MIP finding efficiency between data and Monte Carlo is corrected on a track-by-track basis when Monte Carlo events are used to calculate the detector acceptance, as will be discussed in chapter 5.

The performance of the crystals in the gap regions between the two Ferris wheels of the muon spectrometer in θ, and between the octants and chambers in φ can not be evaluated using the above procedure. We therefore assume that the performance of these crystals is similar to those in the checked region. Since the gap regions only contribute about 9% to the total angular coverage, a small deviation from this assumption can be tolerated. A
Figure 4.3  The ECAL MIP finding efficiency as function of the $\theta$ (a) and $\phi$ angles (b). The dots correspond to the 1992 data, the histograms are Monte Carlo simulation.

large difference would show up as a significant change in the cross section when those gap regions are included or removed in the analysis. This is not the case, as will be shown in chapter 5.

4.2  Muon Identification with the HCAL

4.2.1  Procedure

The interactions between particles and the HCAL are reconstructed as clusters formed from geometrically connected hits. A typical muon cluster is a long and thin track which passes through the interaction point if extrapolated backwards (see figure 4.1). This feature determines the muon identification with the HCAL. An algorithm has been developed to define an HCAL cluster as a MIP. It consists of the following steps:

1) Coordinate assignment

One HCAL cluster usually contains two types of hits: $\phi$-hits and $z$-hits. The former have accurate $r$ and $\phi$ coordinates while the latter only have accurate $z$ coordinates. In order to parametrize a track in space, three dimensional information is needed for each hit. A $\phi$-hit ($z$-hit) is assigned the averaged $z$ ($\phi$) coordinate of all its neighbouring $z$-hits ($\phi$-hits). The $r$ of each $z$-hit is updated accordingly.

2) Definition of the geometrical centre of the cluster

Besides the interaction point, the geometrical centre of the cluster is taken as the second space point to define a straight track. Its $\theta$-angle, $\theta_{cl}$, is determined with a histogram method as follows:
4.2 Muon Identification with the HCAL

Figure 4.4 Illustration of the histogram method used to determine the $\theta$-angle of the geometrical centre of a HCAL cluster. $\theta_0$ is the centre of the shaded bin; $\theta_1$ is the averaged $\theta$-angle of all the hits within 4.6° around $\theta_0$ (all the 3 bins); $\theta_{cl}$ is the averaged $\theta$-angle of all the hits within 1.7° around $\theta_1$ (only the 2 hatched bins).

- An array of 3° bins is defined covering the $\theta$ angle region of $15^\circ \leq \theta \leq 165^\circ$, and the $\phi$-hits are distributed over these bins. The bin containing the maximum number of hits defines $\theta_0$ (see figure 4.4);

- The average $\theta$ of all the hits within 4.6° around $\theta_0$ defines $\theta_1$;

- The average $\theta$ of all the hits within 1.7° around $\theta_1$ is calculated and taken as $\theta_{cl}$.

The values of 3°, 4.6° and 1.7° are chosen according to the granularity of the HCAL barrel. The $\phi$-angle, $\phi_{cl}$, is determined in a similar way.

3) Selection

A rectangular shaped volume is defined around the straight line connecting the interaction point and the centre of the cluster. A $\phi$-hit ($\phi$-hit) is considered to be inside the volume if it is within 80 (90) mm, in the $r$-$\phi$ ($r$-$\phi$-$z$) plane, from the straight line. The percentage, and the largest $r$ coordinate, $r_{\text{max}}$, of hits inside the volume are determined.

Due to a loss of detection efficiency in the transition regions between two rings of HCAL modules, there are problems with the definition of $r_{\text{max}}$ in these regions. Therefore, a search is made for an associated muon filter hit within 200 mrad in $\phi$ around the HCAL cluster.

Finally, a HCAL cluster is defined as a MIP, if it satisfies the following conditions:
• the total number of hits is less than 45,
• the percentage of hits inside the volume is larger than 65%,
• the \( r_{\text{max}} \) is larger than 1550 mm or there is an associated muon filter hit.

The distributions of these three quantities are compared between the 1992 data and the Monte Carlo simulation in figure 4.5 a, b and c for muons identified with the muon spectrometer. The energy deposits of muons in the HCAL are shown in figure 4.5 d. Good agreement is observed for all these distributions.

The small shoulder in the \( N_{\text{hit}} \) distribution at \( N_{\text{hit}} = 15 \) corresponds to edge clusters which miss most of the \( \phi \) chambers. Due to a lack of hits in the outer layers, 3.1% of the clusters have \( r_{\text{max}} < 1550 \text{ mm} \). These clusters are mostly located in the transition regions between the central modules and their neighbours (0.12 < |\( \cos \theta \)| < 0.27). About 76% of these clusters have associated muon filter hits and can therefore still be recognized as MIPs.

### 4.2.2 Efficiency Determination

Similar to the ECAL case, the efficiency of the HCAL MIP finding procedure is determined using muons identified with the muon spectrometer. The result is listed in table 4.2 and shown in figure 4.6 as function of the \( \theta \) and \( \phi \) angles. Here the data from the 1991 and 1992 physics runs are combined. Again the gap region between the two Ferris wheels of the muon spectrometer is assumed to behave in the same way as the checked region both in data and in the Monte Carlo simulation.

As can be seen from figure 4.6 a, the discrepancy in the MIP finding efficiency between data and Monte Carlo is small, except for the transition regions between the central modules and their neighbours in \( \theta \) and the gap regions between modules in \( \phi \). In the analysis described in chapter 5, the transition regions in \( \theta \) are only used for the \( \theta \) angle determination but not for muon identification. For other regions in \( \theta \) and for the whole region in \( \phi \), the MIP finding efficiency is averaged. The discrepancy between the averaged data and Monte

<table>
<thead>
<tr>
<th>Sample (1991+1992)</th>
<th>MIP Identified</th>
<th>Bad Cluster</th>
<th>No Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Include 0.12 &lt;</td>
<td>( \cos \theta )</td>
<td>&lt; 0.27</td>
<td></td>
</tr>
<tr>
<td>51043 muons</td>
<td>97.11% ± 0.07%</td>
<td>0.94% ± 0.04%</td>
<td>1.95% ± 0.06%</td>
</tr>
<tr>
<td>116680 MC muons</td>
<td>98.12% ± 0.04%</td>
<td>0.45% ± 0.02%</td>
<td>1.43% ± 0.03%</td>
</tr>
<tr>
<td>Exclude 0.12 &lt;</td>
<td>( \cos \theta )</td>
<td>&lt; 0.27</td>
<td></td>
</tr>
<tr>
<td>42212 muons</td>
<td>98.25% ± 0.06%</td>
<td>0.30% ± 0.03%</td>
<td>1.45% ± 0.06%</td>
</tr>
<tr>
<td>96480 MC muons</td>
<td>98.60% ± 0.04%</td>
<td>0.07% ± 0.01%</td>
<td>1.33% ± 0.04%</td>
</tr>
</tbody>
</table>

Table 4.2 Breakdown of muon signals in the HCAL.
4.2 Muon Identification with the HCAL

Figure 4.5  a) The total number of hits in a muon HCAL cluster, b) the percentage of HCAL hits inside the volume around the muon track, c) the $r_{\text{max}}$ of a muon HCAL cluster and d) the energy deposit of a muon in the HCAL. The dots correspond to the 1992 data, the histograms are from Monte Carlo simulation. The simulation is normalized to the data according to the total number of muons used.

Figure 4.6  The HCAL-MIP finding efficiency as function of the $\theta$ (a) and $\phi$ angles (b).
Carlo efficiencies is corrected on a track-by-track basis. The imperfection of this treatment is included in the systematic error.
Chapter 5

Muon Pair Cross Sections around the Z Pole

In this chapter the muon pair cross section measurements are presented. In the first sections it is shown that by making use of the calorimeter information for the selection of events, the acceptance and efficiency determination for muon pair production becomes almost independent of our knowledge of the geometrical structure and performance of the individual detector components concerned. The cross section measurements obtained in this way, and their systematic errors, are presented and discussed in the last part of the chapter. The data samples were collected during the 1991 and 1992 runs with a total luminosity of about 33 pb⁻¹.

5.1 Definition of "Muons"

5.1.1 AMUI Muons

The reconstruction of the trajectory of a muon outside the support tube needs data from at least two layers of P chambers. The trajectory is back-tracked through the inner subdetectors to the interaction region to optimize the determination of the momentum, the transverse and longitudinal distance of closest approach to the beam axis (DCA) and the θ and φ angles. Tracks reconstructed in such a manner are referred to as "AMUI" muons which can be either "doublet"s or "triplet"s depending on whether two or three P chamber segments are used.

For each muon track the transverse DCA, Dₓ and the longitudinal DCA, Dᵧ are calculated. The comparison between the 1992 data and the Monte Carlo simulations for these
two variables is shown in figure 5.1. Good agreement is observed for the $D_r$ distribution. The discrepancy for the $D_z$ distribution is due to a large amount of tracks in the data reconstructed with $z$ hits from only the MI or MO $Z$ chambers. Visually scanning [45] muon pair events reveals that the quality of AMUI muons is well reflected by these DCAs. A very large value of $D_r$ indicates that the fitted track is assigned too low a momentum and a wrong $\phi$ angle. A very large value of $D_z$ leads to an inaccurate $\theta$ angle and causes an extra uncertainty of about 1% to the momentum determination. In order to avoid errors in the analysis due to badly reconstructed tracks, tracks with $D_r > 80$ mm are removed as AMUIs. Tracks with $D_z > 40$ mm or no $z$ information at all are kept, but their $\theta$ angles are corrected according to the associated calorimeter information.

The momenta of AMUI muons are mainly determined by the measurements in the muon spectrometer. In order to separate muons with collinear radiation from genuine low energy muons, and to make the cross section measurements less sensitive to Monte Carlo simulation of the final state or bremsstrahlung radiation, the energies of all hard collinear photons ($E_\gamma > 1$ GeV) are added to the muon momentum, if the opening angle between the muon and the photon is smaller than 100 mrad. Gaussian fits to the momentum spectra show momentum resolutions of $\sim 2.8\%$ for triplets and $\sim 20\%$ for doublets (see figure 5.2).

### 5.1.2 MIP Muons

Muons which are not registered in more than one layer of the P chambers cannot be reconstructed as AMUIs. However, most of these muons can be identified as minimum
5.1 Definition of "Muons"

![Graphs showing momentum resolutions for triplet and doublet AMUI muons in the 1992 data sample.](image)

**Figure 5.2** Momentum resolutions of: a) triplet and b) doublet AMUI muons in the 1992 data sample. The superimposed curves are Gaussian fits with $\sigma$'s of 0.028 and 0.196 respectively.

Ionizing particles (MIPs) using the calorimeter information, together with the individual track segment reconstructed by a P chamber. Within cones of 200 mrad, items reconstructed by individual detector components are matched to form objects. An object is defined as a MIP muon, if one of the combinations listed in table 5.1 is satisfied.

The $\theta$ angle of a MIP muon is taken in order of preference from the associated muon chamber Z segment or from the HCAL and ECAL clusters. The $\phi$ angle is taken in order of preference from the associated TEC track, the muon chamber P segment, or from the HCAL and ECAL clusters.

For MIP muons with associated TEC tracks, the corresponding TEC measured momenta are sometimes used in the selection procedure. The coarse resolution of these measurements

<table>
<thead>
<tr>
<th>Category</th>
<th>MUCH</th>
<th>Back up</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P segment(s)</td>
<td>Z segment(s)</td>
</tr>
<tr>
<td>1</td>
<td>Y</td>
<td>——</td>
</tr>
<tr>
<td>2</td>
<td>Y</td>
<td>——</td>
</tr>
<tr>
<td>3</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>4</td>
<td>N</td>
<td>——</td>
</tr>
<tr>
<td>5</td>
<td>N</td>
<td>——</td>
</tr>
</tbody>
</table>

**Table 5.1** MIPs as combinations of information from individual detector components. Here "Y" and "N" represent "yes" and "no" respectively, while "—-" means "no requirement". In the HCAL column, "a cluster" should not have more than 45 hits.
Figure 5.3 AMUI momentum divided by TEC momentum. The superimposed curve is a Gaussian fit with a $\sigma$ of 1.42.

(~140% at 45 GeV*, see figure 5.3) has a very small effect on the final cross section measurements because it affects less than 8% of the total event sample.

5.2 Event Selection

Events having

- two AMUI muons (AMUI $\oplus$ AMUI), or
- one AMUI muon and one MIP muon (AMUI $\oplus$ MIP), or
- two MIP muons (MIP $\oplus$ MIP)

are considered as muon pair candidates. For events with more than two muon candidates, two muon tracks are chosen which have the largest numbers of P and Z segments found in the muon chambers, and have the most of the following three items: TEC tracks, HCAL and ECAL MIPs, and have the smallest values of $\sqrt{D_r^2 + D_z^2}$.

In order to suppress cosmic ray contamination and background from inclusive muon production, the following general conditions are imposed on all events:

*This momentum resolution corresponds to $\sigma(1/p_t) = 0.33$/GeV for 45.6 GeV muons at an averaged $\theta$-angle of 66°. A better calibration of the TEC yields $\sigma(1/p_t) = 0.22$/GeV. However, this calibration was not available at the time when the LLDSUs were produced.
5.2 Event Selection

G.1 At least one of the two muons must be more than 0.5° in \( \phi \) away from the octant centres and more than 1° in \( \phi \) away from centres of the gaps between octants. In these regions the scintillator coverage is poor;

G.2 The event should not have more than 5 (3) TEC tracks, if it has more than 1 (less than 2) AMUI muon(s);

G.3 The total energy deposit in the HCAL should not be more than 15 GeV;

G.4 At least one of the two muons must have an associated "in-time" scintillator hit within 3 ns of the bunch crossing time after correction for the time of flight;

G.5 The event must have at least one TEC track or, the time difference between the two scintillator hits associated with the muons must be smaller than 3 ns.

In addition to the above general conditions, the following cuts are applied depending on the event classification in order to suppress background from \( e^+e^- \rightarrow \tau^+\tau^- \) and from \( e^+e^- \rightarrow e^+e^-\mu^+\mu^- \):

- AMUI \( \oplus \) AMUI subsample
  
  1.1 At least one of the two muons must have a measured momentum higher than 66% of \( E_{\text{beam}} \);
  
  1.2 The acollinearity angle, i.e. the complement of the opening angle between the two muons, is required to be smaller than 50°.

- AMUI \( \oplus \) MIP subsample
  
  2.1 The AMUI muon must have a measured momentum higher than 66% of \( E_{\text{beam}} \);
  
  2.2 The acollinearity angle between the two muons must be smaller than 50°;
  
  2.3 There should be at least two TEC tracks.

- MIP \( \oplus \) MIP subsample
  
  3.1 At least one of the two muons must have a TEC measured momentum higher than 20% of \( E_{\text{beam}} \);
  
  3.2 The acollinearity angle between the two muons must be smaller than 4°. This cut value is chosen such that the amount of \( \tau \) background is minimal while keeping the systematic uncertainty small;
  
  3.3 There should be at least two TEC tracks.

After applying the above criteria, 30,819 events are obtained from the data. Among these selected events, 81.0%, 11.3% and 7.7% belong to the AMUI \( \oplus \) AMUI, AMUI \( \oplus \) MIP and MIP \( \oplus \) MIP subsamples respectively. The angular distributions of the muons in
Figure 5.4 The $\cos \theta$ (left) and $\phi$ (right) distributions of the muons in the selected events as compared between the 1992 data (dots) and Monte Carlo simulation (histograms, including $e^+e^- \rightarrow \tau^+\tau^-$ background). The $\phi$ distributions are folded onto one octant.

the selected events are shown in figure 5.4. Also shown are the corresponding Monte Carlo predictions.

It can be seen in figure 5.4 that, after the inclusion of events with MIP muons, the overall angular distributions agree better with the Monte Carlo expectations. The remaining
imperfections at $|\cos \theta| = 0.72$ and in the $\phi$ distribution will lead to systematic uncertainties, which are discussed in section 5.7.

It has to be noted that the event selection is set up such that it does not depend critically on information from any one of the TEC, the calorimeters or the muon spectrometer. The use of these subdetectors as backups to each other allows a high overall efficiency with a small systematic error. However, our analysis lacks redundancy in one aspect: all events are required to have at least one in-time scintillator hit (cut G.4, see above). It is therefore crucial to have good control over the uncertainties introduced by the scintillator geometry and efficiency.

5.3 Efficiencies

An event has to be triggered and properly detected by a combination of TEC, ECAL, SCNT, HCAL and MUCH before it can be reconstructed and selected. In order to calculate the cross sections, a detailed Monte Carlo simulation is needed to determine the detector acceptance. Nevertheless, discrepancies in the detection and trigger efficiencies exist between data and Monte Carlo, and they have to be taken into account to avoid systematic errors.

In this section, we describe how we simulate and correct for inefficiencies of: 1) the muon chambers, 2) the calorimeters, 3) the scintillators, 4) the trigger and, 5) the TEC track finding procedure.

5.3.1 Muon Chamber Efficiency

Reconstruction of muon tracks is only possible when reliable information from at least two layers of P chambers is available. Any inefficiency of the muon chambers and of the reconstruction software will result in the transition from an AMUI muon to a MIP muon, or even the loss of the track. When only AMUI $\oplus$ AMUI events are used to determine the cross sections, the muon chamber inefficiency introduces an event loss of about 8% for the 1992 data. After the inclusion of AMUI $\oplus$ MIP and MIP $\oplus$ MIP events into the cross section analysis, the final event loss due to the muon chamber inefficiency is $\sim 0.4%$.

Similar to the analysis described in [46], the muon chamber inefficiency is determined by counting the percentages of muons (all the AMUI and MIP muons in the selected events) which have no P segments in the layers they traversed. The inefficiencies, averaged over all the octants and over the whole running period in 1992, are found to be 6.4%, 6.1% and 1.5% for MI, MM and MO chambers respectively. These inefficiencies are predominantly due to dead cells in the P chambers. To correct for the effect of these inefficiencies, the Monte Carlo acceptance calculation is repeated with a certain number of P segments removed. The amount of removed segments correspond to the measured inefficiencies of the corresponding angular regions in $\phi$, separately for the two Ferris wheels. Consequently,
Figure 5.5 The $\phi$ dependence of the detection efficiencies in the MI (bottom), MM (middle) and MO (top) layers seen in the 1992 data (dots) and as simulated with Monte Carlo (histograms).

triplet AMUIs can become doublets or even MIPs, doublets can become MIPs, and so on. In figure 5.5, the $P$ segment finding efficiencies in the three layers are plotted for the 1992 data. Also shown are the Monte Carlo counterparts. As expected, good agreement is observed.

As shown in [46] and [47], with the above described segment-removing method, the muon chamber inefficiency is simulated to a relative accuracy of 10%. Therefore, the uncertainty on the final cross section measurements introduced by the uncertainty of the muon chamber inefficiency simulation is estimated to be $0.4\% \times 10\% = 0.04\%$.

5.3.2 Calorimeter Efficiencies

When using Monte Carlo events to determine the detector acceptance, discrepancies in the calorimeter efficiency are corrected on an event by event basis.

Each accepted Monte Carlo event is counted with a certain weight $W$, which is the product of two weight factors, $W_1$ and $W_2$, assigned to the two muons respectively. The
weight factors obtain their values in the following way:

\[ W_i = \begin{cases} 
1 & \text{if muon } i \text{ has P segment(s),} \\
\epsilon_{\text{ECAL}} & \text{if it is identified by ECAL only,} \\
\epsilon_{\text{HCAL}} & \text{if it is identified by HCAL only,} \\
\epsilon_{\text{ECAL}} \cdot \epsilon_{\text{HCAL}} & \text{if it is identified by ECAL and HCAL combined,}
\end{cases} \]

where

\[
\epsilon_{\text{ECAL}} = \frac{\text{percentage of data AMUIs which have ECAL MIP}}{\text{percentage of MC AMUIs which have ECAL MIP}} \\
\epsilon_{\text{HCAL}} = \frac{\text{percentage of data AMUIs which have HCAL MIP}}{\text{percentage of MC AMUIs which have HCAL MIP}}.
\]

From tables 4.1 and 4.2, the values of \( \epsilon_{\text{ECAL}} \) and \( \epsilon_{\text{HCAL}} \) can be calculated. These values are listed in table 5.2. The listed errors are due to statistics.

<table>
<thead>
<tr>
<th>Period</th>
<th>1991 pre-scan</th>
<th>1991 scan</th>
<th>1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_{\text{ECAL}} )</td>
<td>1.005 ± 0.004</td>
<td>1.008 ± 0.005</td>
<td>1.003 ± 0.002</td>
</tr>
<tr>
<td>( \epsilon_{\text{HCAL}} )</td>
<td>0.996 ± 0.001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2 Values of \( \epsilon_{\text{ECAL}} \) and \( \epsilon_{\text{HCAL}} \) for the three running periods.

The impact of these efficiency corrections on the final cross section measurements is expected to be small, since only about 5% of the identified muons have no P segments and of these muons about 50% are in the gap region between the two Ferris wheels. Varying the values of \( \epsilon_{\text{ECAL}} \) and \( \epsilon_{\text{HCAL}} \) by amounts that are equal to the corresponding statistical errors, the final cross sections change within ± 0.07%. Therefore, a systematic error of 0.07% is assigned to the final cross section measurements to cover the uncertainty introduced by these calorimeter efficiency corrections.

### 5.3.3 Scintillator Efficiency

The event selection relies on the scintillator response to suppress cosmic ray contamination. Imperfections in the Monte Carlo description of the scintillator geometry introduce an uncertainty in the acceptance calculation. Furthermore, in the regions within the scintillator coverage, the counters may fail to provide in-time hits due to uranium noise from the hadron calorimeter, and due to a possible malfunction of the electronics. Imperfections in the Monte Carlo simulations of these effects are also error sources. The geometry effect is estimated to be negligible [48]; the effects from noise contamination and from a possible electronics malfunction are treated below.

The sample of events with more than one good TEC track contains a total of 28,307 events (\( N_{\text{tot}} \)). Of these events, 25,554 (\( N_2 \)) have two SCNT hits, 2,753 (\( N_1 \)) have only
one hit. Among those events having two hits, 597 events have one hit with wrong timing ($N_{2}^{\text{bad}}$). Therefore, the probability to have one SCNT hit with wrong timing is

$$\eta = \frac{N_{2}^{\text{bad}}}{2 \cdot N_{2}} = (1.17 \pm 0.05)\%,$$

and the probability to lose one event due to noise contamination or a possible electronics malfunction is:

$$\epsilon = \frac{N_{2}}{N_{\text{tot}}} \cdot \eta^{2} + \frac{N_{1}}{N_{\text{tot}}} \cdot \eta = (0.13 \pm 0.01)\%.$$

Since the Monte Carlo simulation produces $\eta = 0$, the final cross sections are scaled up by 0.13%. The 0.01% uncertainty introduced by this correction is neglected.

### 5.3.4 Trigger Efficiencies

Muon pair events are mainly triggered by two independent triggers, the muon trigger and the TEC trigger. Since the performance of the trigger system is not simulated in our Monte Carlo, the trigger efficiencies determined from real data are needed to correct the number of selected events.

For the AMUI ⊕ AMUI subsample, the muon trigger alone is almost 100% efficient. Within this subsample, the muon trigger efficiency is determined by comparing the number of events triggered by both the muon trigger and the TEC trigger to the total number of TEC triggered events. The TEC trigger efficiency is obtained in a similar way. The results, together with the combined trigger efficiency for this subsample, are listed in table 5.3. It can be concluded that the trigger system does not introduce any event loss for the AMUI ⊕ AMUI subsample.

For the AMUI ⊕ MIP subsample and especially for the MIP ⊕ MIP subsample which mainly covers the edge and gap regions of the muon spectrometer, the efficiency of the muon trigger drops drastically — it is about 80% for the AMUI ⊕ MIP subsample and 45% for the MIP ⊕ MIP subsample. The TEC trigger becomes important as a backup. In

<table>
<thead>
<tr>
<th>Period</th>
<th>Trigger Efficiency</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>MUCH</strong></td>
<td><strong>TEC</strong></td>
<td>Combined</td>
</tr>
<tr>
<td>1991 pre-scan</td>
<td>100.00% ± 0.03%</td>
<td>85.0% ± 0.6%</td>
<td>100.00%</td>
</tr>
<tr>
<td>1991 scan</td>
<td>99.97% ± 0.03%</td>
<td>89.1% ± 0.5%</td>
<td>100.00%</td>
</tr>
<tr>
<td>1992</td>
<td>99.85% ± 0.03%</td>
<td>92.1% ± 0.2%</td>
<td>99.99%</td>
</tr>
</tbody>
</table>

**Table 5.3** The trigger efficiencies relevant to the AMUI ⊕ AMUI subsample. The uncertainties on the combined trigger efficiencies are all of the order of 0.004% and therefore not listed.
order to be less dependent on one single trigger, we correct for the trigger efficiencies in the following way: we count the events triggered by the muon trigger with weight 1, and scale the number of the remaining TEC triggered events up according to the TEC trigger efficiency.

We cannot simply take the TEC trigger efficiencies determined with the AMUI ⊗ AMUI subsample because of its poor coverage in \( \phi \). In order to properly determine the TEC trigger efficiencies, we use large angle e\(^+\)e\(^-\) → e\(^+\)e\(^-\) (Bhabha) events, which are mainly triggered by another independent trigger, the energy trigger. These events are selected with the following criteria:

- There should be two ECAL bumps with energies higher than 30 GeV;
- The acollinearity angle between these two bumps must be smaller than 10\(^\circ\);
- There should be fewer than four TEC tracks.

From \( N_E \) Bhabha events triggered by the energy trigger, \( N_T \) events are also triggered by the TEC trigger. The TEC trigger efficiency can be written as:

\[
\epsilon_T = \frac{(1 + \eta) \cdot N_T}{N_E},
\]

where \( \eta \) is a small correction (= 1.55% [26]) taking the contamination from e\(^+\)e\(^-\) → \( \gamma \gamma \) into account. The obtained TEC trigger efficiency as function of cos \( \theta \) and \( \phi \) is shown in figure 5.6. As a cross check, the TEC trigger efficiency as determined with the AMUI ⊗ AMUI subsample (|cos \( \theta | \leq 0.8 \)) is also shown. The large dips in the \( \phi \) distribution are due to three problematic outer TEC sectors (No. 2, 4 and 11). The averaged efficiencies for two groups of (outer) TEC sectors: sectors in front of the gaps and sectors away from the gaps, are listed in table 5.4.

![Figure 5.6](image)

**Figure 5.6** The TEC trigger efficiency in 1992 as determined with Bhabha (dots) and AMUI ⊗ AMUI (histogram) events as functions of: a) cos \( \theta \) and b) the \( \phi \) angle.
<table>
<thead>
<tr>
<th>Sample</th>
<th>( \cos \theta ) range</th>
<th>Sectors in front of gaps</th>
<th>Sectors away from gaps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1991 pre-scan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e^+e^- )</td>
<td>0.00 — 0.74</td>
<td>69.4% ± 1.2%</td>
<td>92.9% ± 0.5%</td>
</tr>
<tr>
<td>( \mu^+\mu^- )</td>
<td>0.00 — 0.74</td>
<td>70.0% ± 1.3%</td>
<td>92.9% ± 0.5%</td>
</tr>
<tr>
<td></td>
<td>0.00 — 0.80</td>
<td>70.1% ± 1.3%</td>
<td>92.0% ± 0.5%</td>
</tr>
<tr>
<td>( e^+e^- )</td>
<td>0.80 — 0.85</td>
<td>31.5% ± 3.3%</td>
<td>29.4% ± 2.2%</td>
</tr>
<tr>
<td></td>
<td>1991 scan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e^+e^- )</td>
<td>0.00 — 0.74</td>
<td>80.7% ± 1.3%</td>
<td>89.5% ± 0.7%</td>
</tr>
<tr>
<td>( \mu^+\mu^- )</td>
<td>0.00 — 0.74</td>
<td>83.5% ± 1.1%</td>
<td>91.5% ± 0.6%</td>
</tr>
<tr>
<td></td>
<td>0.00 — 0.80</td>
<td>83.8% ± 1.1%</td>
<td>91.5% ± 0.6%</td>
</tr>
<tr>
<td>( e^+e^- )</td>
<td>0.80 — 0.85</td>
<td>70.2% ± 4.3%</td>
<td>72.5% ± 3.0%</td>
</tr>
<tr>
<td></td>
<td>1992</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e^+e^- )</td>
<td>0.00 — 0.74</td>
<td>88.6% ± 0.4%</td>
<td>94.0% ± 0.2%</td>
</tr>
<tr>
<td>( \mu^+\mu^- )</td>
<td>0.00 — 0.74</td>
<td>86.9% ± 0.5%</td>
<td>94.3% ± 0.2%</td>
</tr>
<tr>
<td></td>
<td>0.00 — 0.80</td>
<td>87.1% ± 0.5%</td>
<td>94.4% ± 0.2%</td>
</tr>
<tr>
<td>( e^+e^- )</td>
<td>0.80 — 0.85</td>
<td>82.1% ± 0.8%</td>
<td>86.2% ± 0.7%</td>
</tr>
</tbody>
</table>

Table 5.4  The TEC trigger efficiency as determined with Bhabha and AMUI @ AMUI events. Because of the gap between the ECAL barrel and endcaps in the region \( 0.74 \leq |\cos \theta| \leq 0.80 \), the Bhabha events are divided into two samples.

Beyond \( |\cos \theta| = 0.8 \), the TEC trigger efficiency can only be determined with Bhabha events. The results are also listed in table 5.4. The big increases in the TEC trigger efficiencies from 1991 to 1992 are due to relaxations in the trigger conditions. The amount of events affected by these efficiencies is about 0.8% of the total sample.

As shown in figure 5.6 a, the TEC trigger efficiency as determined with the AMUI @ AMUI sample is rather flat along the \( \cos \theta \) axis. This justifies the use of the efficiencies determined with Bhabha events within \( |\cos \theta| \leq 0.74 \) for the larger angular region of \( |\cos \theta| \leq 0.8 \).

By comparing the values determined with Bhabha events with those determined with the AMUI @ AMUI sample in the same \( |\cos \theta| \) region of \( 0 \leq |\cos \theta| \leq 0.74 \), the TEC trigger efficiencies as obtained with Bhabha events are estimated to have been determined to better than 2%. Since the amount of events affected by the TEC trigger inefficiency corrections is only about 8% of the total sample, a systematic error of 0.16% is assigned to cover the uncertainty in the cross section measurements introduced by these corrections.
5.3.5 TEC Track Finding Efficiencies

About 5.8% of AMUI ⊕ AMUI events (4.7% of the total sample) have only one in-time scintillator hit. These events are mainly located in the two 6° gaps between the scintillators at around $\phi = 0°$ and 180° (see figure 2.4). They pass the selection because they have at least one TEC track. Therefore, this amount of events has to be corrected for the efficiency to find at least one TEC track in these $\phi$ regions.

Similarly, the numbers of AMUI ⊕ MIP and of MIP ⊕ MIP events have to be corrected for the efficiency to find at least two TEC tracks.

These TEC track finding efficiencies are again determined with barrel Bhabha events. The results, normalized to those produced by the Monte Carlo simulation, are listed in tables 5.5 and 5.6. These efficiencies are assumed to be valid in the whole angular range up to $|\cos \theta| = 0.85$.

<table>
<thead>
<tr>
<th>Period</th>
<th>Relative efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991 pre-scan</td>
<td>98.3% ± 0.7%</td>
</tr>
<tr>
<td>1991 scan</td>
<td>96.0% ± 0.8%</td>
</tr>
<tr>
<td>1992</td>
<td>98.0% ± 0.3%</td>
</tr>
</tbody>
</table>

*Table 5.5* The relative efficiencies to find at least one TEC track for horizontal Bhabha events. These values are normalized to those produced by the Monte Carlo simulation.

<table>
<thead>
<tr>
<th>Period</th>
<th>Relative efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sectors in front of gaps</td>
</tr>
<tr>
<td></td>
<td>All Events</td>
</tr>
<tr>
<td>1991 pre-scan</td>
<td>94.6% ± 0.7%</td>
</tr>
<tr>
<td>1991 scan</td>
<td>90.9% ± 0.8%</td>
</tr>
<tr>
<td>1992</td>
<td>96.7% ± 0.3%</td>
</tr>
</tbody>
</table>

*Table 5.6* The relative efficiencies for the TEC to find at least two tracks for Bhabha events. These values are normalized to those produced by the Monte Carlo simulation.

By comparing with values determined with AMUI ⊕ AMUI events, the TEC track finding efficiencies as obtained with Bhabha events are estimated to have been determined to better than 1%. Since the amounts of events affected by these TEC track finding efficiency corrections are about 4.7% and 19% respectively, the total systematic error introduced by
these one- and two-track efficiency corrections is estimated to be 0.2%.

5.4 Acceptance

Due to the geometry and imperfection of the detector, only a certain fraction of the produced $\mu^+\mu^-(\gamma)$ events can be recorded, reconstructed and identified. In order to determine the total cross section of $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$, we need to calculate this fraction, defined as the acceptance.

For this purpose, Monte Carlo $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$ events are generated, in the full solid angle at $\sqrt{s} = 91.250$ GeV using the KORALZ-3.8 program, and subjected to full detector simulation (see subsection 1.2.4). These events are then reconstructed and processed with the same selection procedure used to analyze the data. In the end, the acceptance for the on-peak point ($\epsilon_a$) and its statistical uncertainty ($\Delta\epsilon_a$) can be calculated according to the total number of simulated events, $N_{\text{tot}}$, and the number of events which pass the final selection, $N_{\text{sel}}$: \[ \epsilon_a = \frac{N_{\text{sel}}}{N_{\text{tot}}}, \quad \Delta\epsilon_a = \sqrt{\frac{\epsilon_a(1-\epsilon_a)}{N_{\text{tot}}}}. \]

The results for the three running periods are listed in table 5.7. The listed errors are statistical only. The subsample distribution of the selected $\mu^+\mu^-(\gamma)$ events for the 1992 running period is given in table 5.8.

<table>
<thead>
<tr>
<th>Period</th>
<th>$N_{\text{tot}}$</th>
<th>$N_{\text{sel}}$</th>
<th>$\epsilon_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991 pre-scan</td>
<td>94521</td>
<td>65246</td>
<td>69.03% ± 0.15%</td>
</tr>
<tr>
<td>1991 scan</td>
<td>94521</td>
<td>65205</td>
<td>68.98% ± 0.15%</td>
</tr>
<tr>
<td>1992</td>
<td>94521</td>
<td>65076</td>
<td>68.85% ± 0.15%</td>
</tr>
</tbody>
</table>

Table 5.7 On-peak acceptance $\epsilon_a$ and its statistical uncertainty $\Delta\epsilon_a$ as determined with Monte Carlo simulation.

<table>
<thead>
<tr>
<th></th>
<th>$N_{\text{sel}}$</th>
<th>% of the selected sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMUI ⊕ AMUI</td>
<td>51752</td>
<td>79.5%</td>
</tr>
<tr>
<td>AMUI ⊕ MIP</td>
<td>8549</td>
<td>13.1%</td>
</tr>
<tr>
<td>MIP ⊕ MIP</td>
<td>4775</td>
<td>7.3%</td>
</tr>
</tbody>
</table>

Table 5.8 Subsample distribution of the 65076 events selected out of a Monte Carlo sample of 94521 $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$ events. This sample is simulated according to the 1992 running conditions.

It is worth mentioning that, although the muon chamber induced inefficiency for the AMUI ⊕ AMUI subsample increased from 3.7% in the 1991 pre-scan period to 7.8%
in 1992, the acceptance only dropped by a relative amount of about 0.3%. This drastic reduction in the sensitivity of the event selection to the muon chamber inefficiency is solely due to the inclusion of the calorimeter information.

At off-peak energies, the effect of hard initial state photon radiation in the Z exchange is more pronounced. Therefore, more events have muon(s) outside the fiducial volume resulting in a decrease in the acceptance. Since the measurements for the off-peak points are clearly limited by the statistics, the energy dependence of the acceptance is estimated using Monte Carlo events at the generator level without detailed detector simulation.

For each energy point, including the on-peak point, 100,000 e⁺e⁻ → μ⁺μ⁻(γ) events were generated, and then subjected to the cosθ, minimal energy and acollinearity cuts. The numbers of events that pass the selection criteria, normalized to the on-peak point, are listed in table 5.9. The acceptances for the off-peak points are corrected accordingly.

<table>
<thead>
<tr>
<th>E_{CM} (GeV)</th>
<th>88.480</th>
<th>89.470</th>
<th>90.230</th>
<th>91.250</th>
<th>91.970</th>
<th>92.970</th>
<th>93.720</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_{sel}/N^{peak}_{sel}</td>
<td>0.985</td>
<td>0.991</td>
<td>0.994</td>
<td>1.000</td>
<td>0.996</td>
<td>0.993</td>
<td>0.986</td>
</tr>
</tbody>
</table>

Table 5.9 Energy dependency of the acceptance as determined with the KORALZ-3.8 generator.

5.5 Background Contamination

Background in the selected muon pair samples can be classified into two groups: events from e⁺e⁻ collisions and cosmic rays.

5.5.1 Background from e⁺e⁻ Collisions

The dominant background contamination from e⁺e⁻ collisions is due to the following process:

\[ e^+e^- \rightarrow \tau^+\tau^- (\gamma) . \]

The contamination rate η_{ττ} can be estimated, using Monte Carlo events, according to:

\[ \eta_{\tau\tau} = \frac{1}{\epsilon_a} \cdot \frac{\sigma_{\tau\tau}}{\sigma_{\mu\mu}} \cdot \frac{N_{\text{sel}}}{N_{\text{tot}}} , \]

where \( \epsilon_a \) is the detector acceptance for e⁺e⁻ → μ⁺μ⁻(γ), \( N_{\text{tot}} \) and \( N_{\text{sel}} \) are the numbers of simulated and selected Monte Carlo e⁺e⁻ → τ⁺τ⁻(γ) events respectively. \( \sigma_{\mu\mu} \) and \( \sigma_{\tau\tau} \) are the production cross sections of the processes e⁺e⁻ → μ⁺μ⁻(γ) and e⁺e⁻ → τ⁺τ⁻(γ) respectively.
Based on a full detector simulated Monte Carlo sample of 105,000 $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ events, the background fraction from this process is found to be $\eta_{\tau\tau} = (1.23 \pm 0.04)\%$ for all the three running periods assuming $\sigma_{\tau\tau} = \sigma_{\mu\mu}$. The error is due to statistics only. The breakdown of the selected $\tau^+\tau^- (\gamma)$ background events for the 1992 running period is given in table 5.10.

<table>
<thead>
<tr>
<th>$N_{\text{sel}}$</th>
<th>Relative contamination $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMUI $\oplus$ AMUI</td>
<td>449</td>
</tr>
<tr>
<td>AMUI $\oplus$ MIP</td>
<td>276</td>
</tr>
<tr>
<td>MIP $\oplus$ MIP</td>
<td>166</td>
</tr>
<tr>
<td>All Combined</td>
<td>891</td>
</tr>
</tbody>
</table>

Table 5.10 Breakdown of the $\tau^+\tau^-$ background for the 1992 running period as predicted by a Monte Carlo sample of 105,000 $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ events.

As shown in table 5.10, half of the $\tau^+\tau^-$ background events are MIP related. The accuracy of the $\tau^+\tau^-$ background subtraction is dominated by the consistency of the MIP finding performances with data and with Monte Carlo simulation. In order to check this consistency, AMUI $\oplus$ MIP events are specially selected imposing all selection requirements except for the muon energy cut. In figure 5.7 the muon energy distribution is compared between the 1992 data and the Monte Carlo simulation. In between $E_\mu / E_{\text{beam}} = 23\%$ and 66\%, there are 391 data events, while the Monte Carlo predicts 372 events with 280 events from $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$. From this, it is concluded that the relative accuracy of our Monte Carlo simulation of $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ is better than 10\%. Therefore, a systematic error of 0.12\% is assigned to the final cross section measurements to cover the imperfection of the $\tau^+\tau^-(\gamma)$ background subtraction.

Background from $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ can be neglected (only 5 events passed the selection out of 40,000 with a production cross section of 1.05 nb). Background from other processes such as $e^+e^- \rightarrow$ hadrons have also been checked and found to be negligible.

### 5.5.2 Cosmic Ray Background

The residual cosmic ray background in the selected sample is estimated by varying the DCA requirement on the TEC tracks from 4 mm to 8 mm. As can be seen in figure 5.8 which shows the distribution of the smallest DCA of TEC tracks in the 1992 data, cosmic ray events can be assumed to be uniformly populated along the DCA axis. Therefore, the number of cosmic rays located inside the region $\text{DCA} \leq 4$ mm can be expected to be the same as the number of cosmic rays located in the region $4 \text{ mm} < \text{DCA} \leq 8$ mm. By visually scanning the events with large DCAs, the numbers of cosmic ray background events in
Figure 5.7 The reconstructed muon momentum for AMUI @ MIP events, $E_\mu$, normalized to the beam energy, as compared between the 1992 data (dots) and Monte Carlo simulations of $e^-e^+\rightarrow \mu^+\mu^- (\gamma)$ (blank histogram), $e^-e^+\rightarrow \tau^+\tau^- (\gamma)$ (shaded histogram) and $e^-e^+\rightarrow e^-e^+\mu^+\mu^-$ (hatched histogram). The relative normalization is done according to the luminosity.

Figure 5.8 The smallest DCA of TEC tracks in the 1992 sample. The events in the region indicated by "scan" were visually inspected.

the normal samples are estimated to be $N_{\cos}^{\text{peak}} = 3 \pm 0, 2 \pm 2$ and $28 \pm 8$ for the three on-peak points respectively, with the errors being the numbers of ambiguous events. This corresponds to a cosmic ray background of less than 0.15%. For the off-peak points in the 1991 scan period, the numbers of cosmic ray background events are calculated using the
expression:

\[ N_{\cos} = N_{\cos}^{\text{peak}} \cdot \frac{T}{T^{\text{peak}}}, \]

where \( T \) denotes the amount of running time.

Due to the uncertainty of the cosmic ray background subtraction, a systematic error of 0.04% is assigned to the final cross section measurements. This value is averaged over the three running periods, it includes both the statistical uncertainty and the uncertainty due to the ambiguous events.

### 5.6 Cross Section Calculation

In previous sections, we have described the event selection to obtain the data samples and determined the efficiencies of the relevant detector components and trigger system. The acceptances have also been calculated and the background contamination estimated. We can now proceed to calculate the total cross section for \( e^+e^- \rightarrow \mu^+\mu^- (\gamma) \):

\[ \sigma_{\mu\mu} = \frac{N_{\text{cor}} - N_{\cos}}{\varepsilon_{\text{dr}} \cdot (1 + \eta_{\text{te}}) \cdot L}, \]  \hspace{1cm} (5.1)

where \( N_{\text{cor}} \) is the total number of selected events corrected for the trigger, scintillator and TEC efficiencies. \( L \) is the integrated luminosity. The results for the three running periods are given in Table 5.11. The listed errors are due to the statistics only.

In figure 5.9, these measurements are compared with theoretical expectations from a model independent approach for \( M_Z = 91.195 \text{ GeV} \), \( \Gamma_Z = 2.495 \text{ GeV} \) and \( \Gamma_H \approx 83.5 \text{ MeV} \).

<table>
<thead>
<tr>
<th>Period</th>
<th>( E_{\text{CM}} ) (GeV)</th>
<th>( \mathcal{L} ) (nb(^{-1}))</th>
<th>Events selected</th>
<th>Cross section (nb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>91 pre-scan</td>
<td>91.254</td>
<td>5035.25</td>
<td>5199</td>
<td>1.503 ± 0.021</td>
</tr>
<tr>
<td>91 scan</td>
<td>88.480</td>
<td>779.40</td>
<td>137</td>
<td>0.26 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>89.468</td>
<td>849.96</td>
<td>284</td>
<td>0.49 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>90.228</td>
<td>793.29</td>
<td>469</td>
<td>0.87 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>91.222</td>
<td>2929.92</td>
<td>2785</td>
<td>1.39 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>91.967</td>
<td>699.92</td>
<td>568</td>
<td>1.19 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>92.966</td>
<td>758.24</td>
<td>364</td>
<td>0.71 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>93.716</td>
<td>829.85</td>
<td>276</td>
<td>0.49 ± 0.03</td>
</tr>
<tr>
<td>92</td>
<td>91.294</td>
<td>20600.62</td>
<td>20737</td>
<td>1.463 ± 0.010</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>33276.45</td>
<td>30819</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.11** Cross section measurements at various C.M. energies.
5.7 Systematic Errors

Figure 5.9 The muon pair cross section measurements (dots) as compared to theoretical expectation (curve) from a model independent approach. Also shown are the differences between them. The errors are due to statistics only.

These parameters are the results of a fit to the measurements presented here, combined with other cross section measurements from L3 (see section 8.1).

5.7 Systematic Errors

Besides the systematic errors introduced by the efficiency corrections, there are systematic uncertainties arising from the event selection and from the acceptance calculation. These uncertainties are estimated by varying the cut values and the edges of the fiducial volume in both $\theta$ and $\phi$, within ranges larger than the corresponding resolutions.
Systematic Errors Introduced by Event Selection

The following example illustrates the method to determine the systematic uncertainties introduced by various selection requirements (see section 5.2).

Shown in figure 5.10 is the distribution of the maximum reconstructed muon momentum, $E_{\text{max}}$, normalized to the beam energy $E_{\text{beam}}$, for AMUI $\oplus$ AMUI events from 1992. The events are selected imposing all selection requirements except for the cut on $E_{\text{max}} / E_{\text{beam}}$ (cut 1.1). This distribution is compared with the Monte Carlo simulations including background from $e^+e^- \rightarrow \tau^+\tau^- (\gamma)$ (hatched histogram) and from $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ (shaded histogram). Also shown is the nominal cut position. When the cut position is changed, the numbers of selected data events, of Monte Carlo $\mu^+\mu^- (\gamma)$ events and of the $\tau^+\tau^- (\gamma)$ background events change. Apart from statistical fluctuations, the cross section may also change due to effects which are not perfectly reproduced by the Monte Carlo simulation. The 1992 analysis is repeated with the $E_{\text{max}}$ cut placed at a few positions different from the nominal position. From the variations on the cross section (see figure 5.11), a systematic error of 0.10% on the cross section measurements is estimated due to cut 1.1.

The statistical errors in figure 5.11 are calculated in the following way. Let $N_0$ and $\sigma_0$ be the total number of selected events and the measured cross section at the actual cut value;

![Figure 5.10](image)

Figure 5.10  The maximum reconstructed muon momentum for AMUI $\oplus$ AMUI events, $E_{\text{max}}$, normalized to the beam energy. These events are selected by applying all selection requirements except for cut 1.1.
Figure 5.11  Relative change in the 1992 cross section as a function of the $E_{\text{max}} / E_{\text{beam}}$ cut for AMUI @ AMUI events (cut 1.1). The solid line corresponds the central value, the dashed lines indicate the systematic error assigned.

let $N$ and $\sigma$ be the counterparts when the cut is moved to another position, we have:

$$\sigma_0 = \frac{N_0}{A_0}, \quad \sigma = \frac{N}{A}$$

and

$$\Delta \left( \frac{\sigma - \sigma_0}{\sigma_0} \right) = \frac{A_0}{A} \cdot \frac{\Delta N}{N_0} = \frac{N_0}{N} \cdot \frac{\Delta N}{N_0} = \frac{\Delta N}{N}$$

since $\sigma \approx \sigma_0$. Here $A$ is equivalent to the denominator in equation 5.1. When $N_0$ is fixed, the statistical error on $N$ is:

$$\Delta N = \left\{ \begin{array}{ll} \sqrt{N - N_0} & \text{if } N > N_0, \\ \sqrt{(N_0 - N) \cdot N / N_0} & \text{if } N \leq N_0. \end{array} \right.$$ 

So we have

$$\Delta \left( \frac{\sigma - \sigma_0}{\sigma_0} \right) = \left\{ \begin{array}{ll} \sqrt{N - N_0} / N & \text{if } N > N_0, \\ \sqrt{(N_0 - N) \cdot N / N_0} & \text{if } N \leq N_0. \end{array} \right.$$ 

In figures 5.12 and 5.13, the relative changes in the 1992 cross section are shown as functions of cut values for all other selection requirements. Among these requirements, the total HCAL energy deposit cut ($E_{\text{HCAL}}$, cut 5.3) and the maximum TEC momentum cut ($E_{\text{TEC}} / E_{\text{beam}}$, cut 3.1) each introduces an uncertainty of 0.15%, while the uncertainties introduced by other cuts are smaller, 0.10% or 0.05% (cut 3.2). The complete list of the estimated systematic errors due to event selection is given in table 5.12.

Systematic Errors Introduced by Acceptance Calculation

As shown in figure 5.4, there are discrepancies between the data and Monte Carlo simulation in the angular distributions, which also lead to systematic errors.

The systematic error due to edges of the fiducial volume in $\theta$ is estimated by repeating the 1992 analysis with one extra requirement: at least one of the two muons must satisfy
Figure 5.12  Relative change in the 1992 cross section as a function of: a) the $E_{\text{HCAL}}$ cut (cut G.3); b) the acollinearity cut for AMUI ⊗ AMUI and AMUI ⊗ MIP events (cuts 1.2 and 2.2); c) the $E_\mu / E_{\text{beam}}$ cut for AMUI ⊗ MIP events (cut 2.1); d) the maximum TEC momentum cut for MIP ⊗ MIP events (cut 3.1) and e) the acollinearity cut for MIP ⊗ MIP events (cut 3.2).
|cos θ| ≥ x with x = 0.05, 0.10, 0.15 and 0.20, or |cos θ| ≤ x with x = 0.80, 0.75, · · ·, 0.60. From the variations on the cross section (see figure 5.14), a systematic error of 0.20% is assigned.

The systematic error due to gaps of the fiducial volume in φ is estimated by repeating the analysis with the gaps (see cut G.1 in section 5.2) artificially enlarged from 0.5° to 0.5° + Δφ in the octant centres, and from 1° to 1° + Δφ in between octants. From the variations on the cross section (see figure 5.15), a systematic error of 0.30% is assigned.

In summary, a total systematic error of 0.85% is assigned to the cross section measurements, to be compared with the 0.70% statistical error for 1992. This total systematic error includes a 0.60% contribution from the luminosity determination. The various contributions

---

**Table 5.12** Systematic errors on the cross sections from the event selection.

<table>
<thead>
<tr>
<th>Cut(s)</th>
<th>Systematic Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of TEC tracks (cut G.2)</td>
<td>0.11</td>
</tr>
<tr>
<td>Total HCAL energy deposit (cut G.3)</td>
<td>0.15</td>
</tr>
<tr>
<td>Max. muon momentum (cut 1.1)</td>
<td>0.10</td>
</tr>
<tr>
<td>Acollinearity (cuts 1.2 and 2.2)</td>
<td>0.10</td>
</tr>
<tr>
<td>Muon momentum (cut 2.1)</td>
<td>0.10</td>
</tr>
<tr>
<td>Max. TEC momentum (cut 3.1)</td>
<td>0.15</td>
</tr>
<tr>
<td>Acollinearity (cut 3.2)</td>
<td>0.05</td>
</tr>
<tr>
<td>Total</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Figure 5.14 Relative change in the 1992 cross section as a function of the $|\cos \theta_{\mu}|$ cut.

Figure 5.15 Relative change in the 1992 cross section when the gaps in $\phi$ are artificially enlarged by half width of $\Delta \phi$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Systematic Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event Selection (see table 5.12)</td>
<td>0.30</td>
</tr>
<tr>
<td>Acceptance (statistics)</td>
<td>0.15</td>
</tr>
<tr>
<td>Acceptance (edges in $\theta$)</td>
<td>0.20</td>
</tr>
<tr>
<td>Acceptance (gaps in $\phi$)</td>
<td>0.30</td>
</tr>
<tr>
<td>Background Subtraction</td>
<td>0.13</td>
</tr>
<tr>
<td>Muon Chamber Inefficiency</td>
<td>0.04</td>
</tr>
<tr>
<td>Calorimeter Inefficiencies</td>
<td>0.07</td>
</tr>
<tr>
<td>Scintillator Inefficiency</td>
<td>0.01</td>
</tr>
<tr>
<td>Trigger Inefficiencies</td>
<td>0.16</td>
</tr>
<tr>
<td>TEC Track Finding Inefficiencies</td>
<td>0.20</td>
</tr>
<tr>
<td>Subtotal</td>
<td>0.60</td>
</tr>
<tr>
<td>Luminosity Determination [26]</td>
<td>0.60</td>
</tr>
<tr>
<td>Total</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 5.13 Summary of all contributions to the total systematic error on the cross section measurements.

to this total error are listed in table 5.13.
Chapter 6

Muon Pair Charge Asymmetries around the Z Pole

This chapter presents the muon pair forward-backward charge asymmetry analysis. It starts with a discussion of the angular distribution of the muon pairs, then describes the event selection and the methods to extract the asymmetries from the data. Finally, the results are presented and the systematic errors discussed.

6.1 Angular Distribution of Muon Pairs

An essential point in the asymmetry analysis is the use of angle $\theta_c$:

$$\cos \theta_c = \frac{\sin(\theta_+ - \theta_-)}{\sin \theta_+ + \sin \theta_-}, \quad (6.1)$$

where $\theta_+$ and $\theta_-$ denote the $\theta$-angles of the $\mu^+$ and $\mu^-$ in the lab frame respectively. This angle is in fact the scattering angle of the $\mu^-$ in the reduced C.M. system of the muon pair assuming that the initial state photons have zero transverse momenta.

In terms of $\theta_c$, the differential cross section for muon pair production has a "Born" form:

$$\frac{d\sigma(s)}{d\cos \theta_c} = \sigma(s) \cdot \left\{ \frac{3}{8} \cdot (1 + \cos^2 \theta_c) + A_{FB}(s) \cdot \cos \theta_c \right\}. \quad (6.2)$$

This equation holds also when hard initial state radiation is allowed. When the cut on the acollinearity angle of the $\mu^+\mu^-$ pair is not less than $6^\circ$, the additional asymmetry introduced by the initial-final state interference through a non-Born shaped contribution is negligible (see subsection 1.2.3).
6.2 Data Sample

The asymmetry measurement is essentially the determination of the charges and \( \theta \) angles of the outgoing muons. It does not involve absolute normalization. To increase statistics, runs with detector problems other than the muon spectrometer are included in this analysis. In order to suppress background contamination and to minimize the effect of a wrong charge determination, events are required to meet the following conditions:

- There should be two oppositely charged AMUI muons identified in the angular region defined by \( 0.05 \leq | \cos \theta_{\pm} | \leq 0.80 \);
- The event should not have more than 5 TEC tracks;
- The total energy deposit in the HCAL should not be more than 15 GeV;
- At least one of the two muons should have an associated scintillator hit which after the time-of-flight correction is within 3 ns with respect to the bunch crossing time;
- The event must have at least one TEC track or, the difference in the timings of the two associated scintillator hits must be smaller than 3 ns;
- At least one of the two muons must have a measured momentum higher than 66\% of \( E_{\text{beam}} \);
- At least one of the two muons should have a muon chamber reconstructed DCA \( D_r \) smaller than 30 mm;
- The acollinearity angle between the muon pair should be smaller than 15°.

In figure 6.1, the acollinearity angle distribution is compared between the 1992 data and Monte Carlo simulation. Very good agreement is observed.

After applying these selection criteria, 26,135 events are selected from the 1991 and 1992 data samples with a total luminosity of 36 pb\(^{-1}\). Although more luminosity is used than for the cross section analysis, fewer events are selected because the events are required to have two AMUI muons. The total number of events (\( N_{\text{TOT}} \)), and the numbers of forward (\( N_F \)), backward (\( N_B \)) events at each energy point are listed in table 6.1. An event is specified as "forward" if \( \theta_e \) is smaller than 90°, and "backward" otherwise.

The fiducial volume cut of \( 0.05 \leq | \cos \theta_{\pm} | \leq 0.80 \) is consistent with \( 0.05 \leq | \cos \theta_e | \leq 0.80 \) because the boost due to initial state photon radiation is very small. Shown in figure 6.2 is the difference between \( \cos \theta_e \) and \( \cos \theta_\pm \). The distribution is a Gaussian with a \( \sigma \) of about 0.003. The long tails correspond to events with relatively large acollinearity angle between the muons.
Figure 6.1 The acollinearity angle distribution as compared between the 1992 data (points) and Monte Carlo simulation (histogram). The simulation is normalized to the data according to the total number of events.

<table>
<thead>
<tr>
<th>Period</th>
<th>(E_{\text{CM}}) (GeV)</th>
<th>(N_{\text{TOT}})</th>
<th>(N_F)</th>
<th>(N_B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>91 pre-scan</td>
<td>91.254</td>
<td>4492</td>
<td>2287</td>
<td>2205</td>
</tr>
<tr>
<td>91 scan</td>
<td>88.480</td>
<td>108</td>
<td>48</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>89.468</td>
<td>218</td>
<td>89</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td>90.228</td>
<td>381</td>
<td>180</td>
<td>201</td>
</tr>
<tr>
<td></td>
<td>91.222</td>
<td>2360</td>
<td>1197</td>
<td>1163</td>
</tr>
<tr>
<td></td>
<td>91.967</td>
<td>487</td>
<td>254</td>
<td>233</td>
</tr>
<tr>
<td></td>
<td>92.966</td>
<td>295</td>
<td>170</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>93.716</td>
<td>221</td>
<td>121</td>
<td>100</td>
</tr>
<tr>
<td>92</td>
<td>91.294</td>
<td>17573</td>
<td>8901</td>
<td>8672</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>26135</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1 The total number of events \(N_{\text{TOT}}\), and the numbers of forward \(N_F\) and backward \(N_B\) events at various C.M. energies.

6.3 Determination of the Asymmetries

Based on equation 6.2, three methods have been developed to extract the asymmetry from the data. They are briefly described in the following.
6.3.1 Counting Method and Extrapolation to Full Solid Angle

According to the definition of the asymmetry (see equation 1.8), one can divide the \( \cos \theta_c \) range into bins, count the number of events in each bin \( (N_i) \), then derive the asymmetry \( A'_{\text{FB}} \) within \( 0.05 \leq |\cos \theta_c| \leq 0.80 \):

\[
A'_{\text{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{N_F' - N_B'}{N_F' + N_B'}.
\]

\( \sigma_F \) and \( \sigma_B \) are directly proportional to the acceptance corrected total number of events \( N_F' \) and \( N_B' \):

\[
\sigma_F \propto N_F' = \sum \epsilon(\theta^F_c)^{-1} \cdot N_i \quad (0.05 \leq \cos \theta^F_c \leq 0.80),
\]

\[
\sigma_B \propto N_B' = \sum \epsilon(\theta^B_c)^{-1} \cdot N_j \quad (-0.05 \leq \cos \theta^B_c \leq -0.80).
\]

\( \epsilon(\theta^F_c) \) and \( \epsilon(\theta^B_c) \) are the acceptances for events with \( \theta_c = \theta^F_c \) in the forward hemisphere or \( \theta^B_c \) in the backward hemisphere (see figure 6.3). The statistical error on \( A'_{\text{FB}} \) can be calculated from

\[
\Delta A'_{\text{FB}} = \frac{2}{N^2} \sqrt{N_F^2 \cdot \sum \frac{N_i}{\epsilon(\theta^F_c)^2} + N_B^2 \cdot \sum \frac{N_j}{\epsilon(\theta^B_c)^2}},
\]

where \( i \) and \( j \) run over the forward and backward bins respectively.

For comparison with measurements from other methods, other experiments, and the Standard Model expectations, \( A'_{\text{FB}} \) is extrapolated to the full \( \theta_c \)-angle range using the Born
6.3 Determination of the Asymmetries

![Graph showing acceptance as a function of \( \cos \theta_c \)]

**Figure 6.3** The acceptance in 1992 as function of \( \cos \theta_c \), obtained using full detector Monte Carlo simulation of \( e^+e^- \rightarrow \mu^+\mu^- (\gamma) \) events.

The formula of the differential cross section:

\[
A_{FB} = \frac{3 + (K_1^2 + K_1 K_2 + K_2^2)}{4(K_1 + K_2)} \cdot A'_{FB}, \\
\Delta A_{FB} = \frac{3 + (K_1^2 + K_1 K_2 + K_2^2)}{4(K_1 + K_2)} \cdot \Delta A'_{FB}.
\]

\( K_1 = 0.05 \) and \( K_2 = 0.80 \) are the \( \cos \theta_c \) boundaries.

Due to the smearing from \( \cos \theta_{e^-} \) to \( \cos \theta_e \), the \( K \)'s have an uncertainty of about 0.003. This introduces a multiplicative systematic error of about 0.3% on the \( A_{FB} \)'s, which is negligible. By varying \( \epsilon(\theta_e') \) and \( \epsilon(\theta_{e^-}') \) within their errors, the systematic error introduced by uncertainties on the acceptance determination on the \( A_{FB} \)'s is found to be also negligible.

### 6.3.2 Fitting the Angular Distribution

The second method to determine the asymmetry is to fit the "Born" formula (see equation 6.2) to the acceptance corrected \( \cos \theta_e \) distribution. This is done with a \( \chi^2 \) fit.

In figure 6.4, the acceptance corrected differential cross sections are shown for three different C.M. energies: \( \sqrt{s} = 89.468 \), 91.294 and 93.716 GeV. The superimposed curves are the fits of equation 6.2 to these distributions. The fitted asymmetries are listed in the figure. The errors include the uncertainty on the acceptance determination.
\(e^+ e^- \rightarrow \mu^+ \mu^- (\gamma)\)

\[\sqrt{s} = 89.468 \text{ GeV}\]
\[A_{FB} = -0.19 \pm 0.07\]

\[\sqrt{s} = 91.294 \text{ GeV}\]
\[A_{FB} = 0.007 \pm 0.008\]

\[\sqrt{s} = 93.716 \text{ GeV}\]
\[A_{FB} = 0.09 \pm 0.07\]

**Figure 6.4** The acceptance corrected \(\cos \theta_c\) distributions for three different C.M. energies. The superimposed curves and the corresponding asymmetries are the results of fitting equation 6.2 to these distributions.
6.3.3 Unbinned Maximum Likelihood Fit

The differential cross section normalized with the total cross section is the theoretical probability density for events to distribute along the \( \theta_c \) axis according to:

\[
P(\cos \theta_c) = \frac{3}{8} \cdot (1 + \cos^2 \theta_c) + A_{FB} \cdot \cos \theta_c.
\]

Therefore, a likelihood function can be constructed out of \( N \) selected events

\[
\mathcal{L} = \prod_{i=1}^{N} \left\{ \frac{3}{8} \cdot (1 + \cos^2 \theta_{c}^{i}) + A_{FB} \cdot \cos \theta_{c}^{i} \right\}
\]

as a function of \( A_{FB} \) assuming charge-symmetric acceptance [46]. The asymmetry \( A_{FB} \) can than be obtained by maximizing \( \mathcal{L} \), which is equivalent to minimizing

\[
-\log \mathcal{L} = -\sum_{i=1}^{N} \log \left\{ \frac{3}{8} \cdot (1 + \cos^2 \theta_{c}^{i}) + A_{FB} \cdot \cos \theta_{c}^{i} \right\}.
\]

6.4 Asymmetries and the Systematic Errors

The charge asymmetry varies with the C.M. energy, as can be seen from equation 1.9 and in figure 6.4. The results for all energy points from the three running periods, obtained with the above described methods, are given in table 6.2. The listed errors are statistical only.

Within the statistical errors, the three methods yield identical results. While the counting method has an extra systematic uncertainty due to the extrapolation, the unbinned maximum likelihood fitting method does not require acceptance correction. Therefore, the results from the unbinned maximum likelihood fitting method are chosen to be used in the following

<table>
<thead>
<tr>
<th>Period</th>
<th>( E_{CM} ) (GeV)</th>
<th>Counting</th>
<th>( \cos \theta_c ) Distribution</th>
<th>Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>91 pre-scan</td>
<td>91.254</td>
<td>0.019 ± 0.016</td>
<td>0.027 ± 0.016</td>
<td>0.031 ± 0.015</td>
</tr>
<tr>
<td>91 scan</td>
<td>88.480</td>
<td>-0.13 ± 0.11</td>
<td>-0.10 ± 0.10</td>
<td>-0.17 ± 0.10</td>
</tr>
<tr>
<td></td>
<td>89.468</td>
<td>-0.22 ± 0.07</td>
<td>-0.19 ± 0.07</td>
<td>-0.18 ± 0.07</td>
</tr>
<tr>
<td></td>
<td>90.228</td>
<td>-0.05 ± 0.06</td>
<td>-0.04 ± 0.05</td>
<td>-0.04 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>91.222</td>
<td>0.014 ± 0.023</td>
<td>0.017 ± 0.021</td>
<td>0.022 ± 0.021</td>
</tr>
<tr>
<td></td>
<td>91.967</td>
<td>0.05 ± 0.05</td>
<td>0.05 ± 0.05</td>
<td>0.05 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>92.966</td>
<td>0.15 ± 0.06</td>
<td>0.14 ± 0.06</td>
<td>0.13 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>93.716</td>
<td>0.09 ± 0.07</td>
<td>0.09 ± 0.07</td>
<td>0.08 ± 0.07</td>
</tr>
<tr>
<td>92</td>
<td>91.294</td>
<td>0.011 ± 0.008</td>
<td>0.007 ± 0.008</td>
<td>0.012 ± 0.008</td>
</tr>
</tbody>
</table>

Table 6.2 Asymmetry measurements at various C.M. energies.
Figure 6.5 The muon pair charge asymmetry measurements (points) as compared to a theoretical expectation (curve) from a model independent approach. Also shown are the differences between them. The errors are due to statistics only.

discussions and in chapter 8 when extracting the Z parameters from our measurements. In figure 6.5, these values are compared with a theoretical expectation from a model independent approach for $M_Z = 91.195$ GeV, $\Gamma_Z = 2.495$ GeV, $\tilde{g}_Y^l = -0.041$ and $\tilde{g}_Y^\ell = -0.500$. These parameters are the results of a fit to the measurements presented in this and the previous chapters, combined with other measurements from $L_3$ (see section 8.2).

The systematic uncertainty on the asymmetries comes from five sources: wrong charge assignment, inaccurate $\theta$ angle determination, charge dependence of the acceptance, background events and the assumption that the differential cross section has a "Born" form.
6.4 Asymmetries and the Systematic Errors

Charge Confusion

The effect of charge confusion on the asymmetry measurements is checked by selecting events which passed all cuts except the opposite-charge requirement. With the 1992 data sample, out of a total of 17,659 events, 86 are same-charge events. Of these 86 events, 12 are triplet-triplet events, 26 are triplet-doublet events, 48 are doublet-doublet events. In figure 6.6 which shows the distribution of the \( Q \cdot E_{\text{beam}} / E_{\mu} \) of the second muon versus that of the first muon, these same-charge events fall into the first and third quadrants. From the relative ratio of the opposite- and same-charge events, the average probability for one muon track (triplets and doublets alike) to have a wrong charge assignment is estimated to be 0.24\%. This charge-confusion rate can be translated to a systematic error of \((2 \pm 4) \times 10^{-5}\) on the asymmetry measurements.

![Figure 6.6](image)

*Figure 6.6* \( Q \cdot E_{\text{beam}} / E_{\mu} \) of the second muon versus that of the first muon for events selected without the opposite-charge requirement for the 1992 data.

The \( \theta \) Angle Determination

The finite resolution in the \( \theta \) angle determination (see section 3.6) may introduce an uncertainty on the asymmetry measurements. In order to check this effect, Monte Carlo asymmetries are calculated using different \( \theta \) angles: one for the reconstructed \( \theta \) angles and
one for the $\theta$ angles given by the generator. The difference between these two asymmetries is found to be $1 \times 10^{-5}$. This value is taken as the uncertainty introduced by the finite resolution in the $\theta$ angle determination on the asymmetry measurements.

It has to be noted that a possible systematic error introduced by the acollinearity angle cut is implicitly included in the above treatment. In fact, in the vicinity of the nominal position of the acollinearity cut ($15^\circ$), the asymmetry varies very little with the acollinearity cut. This can be seen in figure 6.7 which shows the measured $A_{FB}$ for 1992 as a function of the acollinearity cut.

![Figure 6.7](image)

**Figure 6.7** The 1992 muon pair asymmetry as a function of the acollinearity cut. The nominal cut value is $15^\circ$.

### The Charge Dependence of the Acceptance

The acceptance can be charge dependent if the momentum resolutions for $\mu^+$ and for $\mu^-$ are different in different hemispheres. In a simple case when the $\mu^+$ momentum resolution is the same in the forward and backward hemispheres but the $\mu^-$ momentum resolution is good in the forward hemisphere and bad in the backward hemisphere, more events with $\mu^-$ in the forward hemisphere will be able to pass the $E_{\text{max}}$ cut thus introducing a bias in the event sample. By varying the $E_{\text{max}}$ cut (see figure 5.10), the systematic error introduced by a possible charge dependence of the acceptance on the asymmetry measurements is estimated to be 0.0006.
6.4 Asymmetries and the Systematic Errors

Background Events

The 0.78% background from $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ events (see section 5.5) does not modify the asymmetries since these events are expected to have the same asymmetries as muon pair events. In fact, a possible effect of the $\tau$ background on the asymmetry measurements has been included in the systematic error introduced by a possible charge dependence of the acceptance.

The additional asymmetry introduced by the 0.05% cosmic ray background, whose asymmetry is $0.014 \pm 0.018$ [46], is calculated to be $5 \times 10^{-6}$. This value is taken as systematic error.

"Born" Approximation

It has been evaluated in [12] that the "Born" approximation, when using variable $\theta_c$ as defined in equation 6.1, introduces an uncertainty of less than 0.001 on the asymmetry determination.

Total Systematic Error

In summary, a conservative total systematic error of 0.0013 is assigned to the asymmetry measurements. The individual contributions to this total error are listed in table 6.3. Compared to the statistical errors, the systematic error is negligible.

<table>
<thead>
<tr>
<th>Source</th>
<th>Systematic error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge confusion</td>
<td>$2.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\theta$ angle determination</td>
<td>$1.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>Charge dependence of the acceptance</td>
<td>$6.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>Background events</td>
<td>$5.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>&quot;Born&quot; approximation</td>
<td>$&lt; 1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Total</td>
<td>$1.3 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 6.3 Contributions to the total systematic error on the asymmetry measurements.
Chapter 7

Study of Photon Radiation

Photon emission from the initial and final state leptons is assumed to be a well-known QED process and can in principle be accurately simulated by the KORALZ-3.8 Monte Carlo program. Deviations in the photon yields and angular distributions in Z decays from these predictions would indicate the existence of an exotic photon source. The acoplanarity distribution of muon pair events with large acollinearity angle between the muons but without any detected isolated photon may reveal the abscondence of exotic weakly interacting neutral particles. In addition, acollinear events with missing momenta along the beam direction carry information about $e^+e^-$ annihilations at energies below the nominal C.M. energies and can thus be used to study $e^+e^-$ annihilations in the so far untested range of C.M. energies of $70 < \sqrt{s} < 85$ GeV.

In this chapter, we first check how well the various photon yields are described by the KORALZ-3.8 Monte Carlo program (sections 7.1 and 7.2), then examine the possible existence of an exotic neutral particle (sections 7.2 and 7.3) and end with the determination of the properties of $e^+e^- \rightarrow \mu^+\mu^-$ at $\sim 80$ GeV (section 7.3).

In order to suppress the contamination from $e^+e^- \rightarrow \tau^+\tau^-$, only events with two AMUI muons are used in these analyses.

7.1 Collinear and Isolated Photon Yield

Photon Identification

Photon identification and measurements of the angles and energy are performed with the TEC and ECAL. For photons with energies greater than 1.5 GeV, the energy resolution is:

$$\frac{\Delta E}{E} = 2.0\% .$$
In the case of muon pair production, we can regard all ECAL clusters with scaled energy \( x = E_{\text{cluster}} / E_{\text{beam}} \) greater than 0.02 as photon candidates, since the clusters created by muons are typically 0.26 GeV or \( x = 0.006 \).

**Definitions of Collinear and Isolated Photons**

The angle between a photon and the closest muon, \( \alpha_{\mu\gamma} \), is a good measure of the separation between them. In figure 7.1, the \( \alpha_{\mu\gamma} \) distribution is plotted for the 1992 data. Also shown is the Monte Carlo expectation which has been normalized to the data according to the luminosity. As can be seen, both the shape and height of the \( \alpha_{\mu\gamma} \) distribution are very well reproduced by the simulation. For the analyses described in this and the next sections, we use the data at the Z peak, and consider photons with \( \alpha_{\mu\gamma} \leq 100 \) mrad as collinear photons, those with \( \alpha_{\mu\gamma} > 100 \) mrad as isolated photons.

A collinear ECAL cluster may include the energy deposit from the muon. In order to determine the energy of the collinear photon, all the ECAL clusters are added together if they are within a cone of 100 mrad around the muon track, then an average muon energy deposit of 0.26 GeV is subtracted from the energy sum. When the muon hits a dead crystal or goes through a gap between two crystals, this approach fails. However, this is rarely the case (see table 4.1).

**Figure 7.1** The angle between a photon and the closest muon as compared between the 1992 data (dots) and the KORALZ-3.8 Monte Carlo simulation (histogram). The Monte Carlo is normalized to the data according to the luminosity.
7.1 Collinear and Isolated Photon Yield

Event Selection

For the analysis described in this and the next sections, the event selection criteria remain essentially the same as described in section 5.2 for events with two AMUI muons. Two changes are made in order to keep events with hard and isolated photons: 1) the cut of 66\% \(E_{\text{beam}}\) on the maximum momentum of the muons is replaced by a cut of 60\% \(\sqrt{s}\) on the total event energy — the energies of the muons plus that of the isolated photons and 2) the cut on the acollinearity angle is dropped.

Comparison between Data and Monte Carlo

The \(x\) distribution for the most energetic collinear photons from the 1992 data sample is compared with the KORALZ-3.8 Monte Carlo simulation in figure 7.2. The \(\alpha_{\text{xy}}\) distribution for the most energetic isolated photons is shown in figure 7.3 in three photon energy regions: \(0.02 < x \leq 0.1, 0.1 < x \leq 0.3\) and \(x > 0.3\).

These distributions include background from \(\mu\) bremsstrahlung in the detector material, and they are not corrected for the inefficiencies in the photon identification arising from dead crystals and photon conversions. Nevertheless, good agreement is observed, indicating that at least for the most energetic photon, the data is well simulated by the KORALZ-3.8 Monte Carlo program.

![Graph](image)

**Figure 7.2** The scaled energy \(x\) of the most energetic collinear photon of the 1992 data (dots) and of the KORALZ-3.8 Monte Carlo simulation (histogram). The Monte Carlo is normalized to the data according to the luminosity.
$e^+e^- \rightarrow \mu^+\mu^- n\gamma$

Figure 7.3 The angle $\alpha_{\mu\gamma}$ between the most energetic isolated photon and the closest muon as compared between the 1992 data (dots) and the KORALZ-3.8 Monte Carlo simulation (histogram). The Monte Carlo is normalized to the data according to the luminosity.

7.2 Multiple Hard Photon Radiation

As shown in the previous section, the most energetic photon, collinear and isolated alike, is well described by the KORALZ-3.8 program. In this section, we check the production of multiple isolated hard photons. For the discussions described in this section and in subsection 7.3.1, we include the 1990 data sample.
7.2 Multiple Hard Photon Radiation

![Graph showing photon distribution](image.png)

\[ e^+ e^- \rightarrow \mu^+ \mu^- n \gamma \]
\[ \sqrt{s} = 91.25 \text{ GeV} \]

**Figure 7.4** The scaled energy \( x \) of the second most energetic isolated photon for events with at least two isolated photons. The dots correspond to the 1990 — 1992 data, the histograms are Monte Carlo expectations from KORALZ-3.8 (shadowed histogram) and from KORALZ-4.0 (blank histogram). The Monte Carlo expectations are normalized to the data according to the luminosity.

Shown in figure 7.4 is the energy distribution of the second most energetic isolated photon for events with at least two isolated photons. Obviously, the KORALZ-3.8 program underestimates the number of data events with multiple isolated hard photons in the final state. Due to the inclusion of high order diagrams, the KORALZ-4.0 program describes the spectrum better but still cannot explain the total rate and the clustering of 3 events (one from each year) which have \( x \approx 0.6 \) for the second most energetic isolated photons and are well separated from the others. In the invariant mass spectrum of the two most energetic isolated photons for events with at least two isolated photons, these three events cluster around 60 GeV as can be seen in figure 7.5.

Taking the shape of the \( M_{\gamma\gamma} \) distribution from the KORALZ-4.0 prediction shown in figure 7.5, and simulating \( 10^6 \) experiments with the number of events in each experiment equal to the total number of data events (83) with \( M_{\gamma\gamma} > 2.5 \) GeV, we estimate the probability for observing three or more clustered events within \( 55 < M_{\gamma\gamma} \leq 65 \) GeV to be about \( 2 \times 10^{-3} \). Therefore, a fluctuation cannot be ruled out. More data are needed to ascertain the origin of these events.

---

*When the 1993 data is added, the probability for observing three or more clustered events within \( 55 < M_{\gamma\gamma} \leq 65 \) GeV increases.*
Figure 7.5 The invariant mass, $M_{\gamma\gamma}$, of the two most energetic isolated photons as compared between the 1990 — 1992 data (dots) and the KORALZ-4.0 (histogram) Monte Carlo simulation. The Monte Carlo is normalized to the data according to the luminosity.

7.3 Muon Pair Production around 80 GeV

This section describes the analysis of muon pair events which have large acollinearity angles between the muons but have no detected isolated photon. Muon pair events with missing photons along the beam pipe are a source of these events. However, exotic events with weakly interacting neutral particles can also exhibit such a signature.

7.3.1 Search for Weakly Interacting Neutral Particle

Events with two AMUI muons are searched for, requiring that at least one of the muons have a measured energy higher than 30% of the beam energy. Further, the acoplanarity angle between the muons must be larger than 20° and there should be no ECAL, nor HCAL clusters with energies higher than 1 GeV and with opening angles larger than 8° with respect to the muons. In the 1990 - 1992 data sample, no events are found. Shown in figure 7.6 is the acoplanarity distribution as compared between the 1992 data and Monte Carlo simulation for events without detected isolated hard photons. Good agreement is observed.

Replacing the acoplanarity angle cut with an acollinearity angle requirement of larger than 5°, we find 74 events with missing invariant mass $M_{\text{miss}} > 20$ GeV in the 1990 - 1992 data sample. Although there is some clustering around $M_{\text{miss}} = 57$ GeV as can be seen in figure 7.7, the total number of events is completely consistent with a background expectation of 75 events from $e^+e^- \rightarrow \tau^+\tau^-$ and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$. 
7.3 Muon Pair Production around 80 GeV

7.3.2 Muon Pair Production around 80 GeV

Muon pair events with missing photons along the beam pipe are predominantly events with hard initial state radiation. The rate for these events is proportional to the muon pair
cross section at the effective energy $\sqrt{s}$ in the C.M. frame of the two muons. The angular distribution of these events in the C.M. frame of the two muons reflects the muon pair asymmetry at the effective energy. These events may therefore be used to study the muon pair production in $e^+e^-$ annihilations at energies $\sim 80$ GeV.

In this analysis, the essential point is to determine the energy of the missing photon. A method is introduced to calculate this energy assuming the photon has escaped at $\theta = 0^\circ$ or $180^\circ$. This method requires only the $\theta$-angle measurements of the muons but not their momenta.

**Energy of the Missing Photon**

For a coplanar event where the photon escapes at $\theta = 0^\circ$ or $180^\circ$ (see figure 7.8), we have three equations due to momentum and energy conservations (neglecting $m_\mu$):

$$
\begin{align*}
    p_1 \cdot \sin \theta_1 - p_2 \cdot \sin \theta_2 &= 0 \\
    p_1 \cdot \cos \theta_1 + p_2 \cdot \cos \theta_2 \pm E_\gamma &= 0 \\
    p_1 + p_2 + E_\gamma &= 2 \cdot E_{\text{beam}}
\end{align*}
$$

where $p_1$ and $p_2$ are the absolute values of the muon momenta, $E_\gamma$ the energy of the initial state photon, $E_{\text{beam}}$ the beam energy. The "-" sign is taken when the initial state photon escaped in the negative $z$ direction. From these equations $E_\gamma$ can be calculated:

$$
E_\gamma = \frac{|\sin(\theta_1 + \theta_2)|}{|\sin(\theta_1 + \theta_2)| + \sin \theta_1 + \sin \theta_2} \cdot \sqrt{s} \tag{7.1}
$$

![Figure 7.8 Schematic lay-out of an event with the photon escaping along the beam pipe.](image)

In practice, the initial state photons do not always escape exactly at $\theta = 0^\circ$ or $180^\circ$ and the $\theta$ angles of the muons can only be determined with a finite resolution. In order to check the validity of the above approximation, two studies have been carried out:
7.3 Muon Pair Production around 80 GeV

- Events with energetic isolated photons are specially selected requiring that the isolation angles between the photons and muons be larger than 600 mrad. For each event, the coordinate system is rotated in such a way that the photon points exactly to $\theta = 0^\circ$. The photon energy is then calculated with equation 7.1 using the new $\theta$ angles of the muons. By comparing the calculated photon energy with the measured value, an estimation of the resolution of equation 7.1 can be obtained. The result is 2.7% for the data, 2.3% for the KORALZ-3.8 Monte Carlo, for $E_\gamma > 5$ GeV (see figure 7.9 a and b). This check is indirect, and it overestimates the resolution of equation 7.1 because the direction of the photon is accurately known.

- Since the Monte Carlo program can reliably simulate the photon detection and the $\theta$ angle determination for muons, KORALZ-3.8 Monte Carlo events with photons going close to the beam pipe can be used to directly check the resolution. By comparing the photon energies calculated with equation 7.1 with the ones given by the generator, the resolution is found to be 3.9% for $E_\gamma > 5$ GeV as shown in figure 7.9 c.

In conclusion, the resolution for $E_\gamma$ is thought to be $\sim 4\%$ when $E_\gamma$ is higher than 5 GeV.

**Effective C.M. Energy**

Let $p'_1$, $p'_2$ and $p'_\gamma$ be the 4-momenta of the two muons and the initial state photon respectively. Due to 4-momentum conservation, the following equation holds:

$$p'_1 + p'_2 + p'_\gamma = (0, i \cdot 2E_{\text{beam}}).$$

From this equation, the effective C.M. energy of the two muons can be deduced:

$$\sqrt{s'} = \sqrt{-|p'_1 + p'_2|^2} = \sqrt{-|(0, i \cdot 2E_{\text{beam}}) - p'_\gamma|^2} = \sqrt{s} \cdot \sqrt{1 - \frac{E_\gamma}{E_{\text{beam}}}}.$$

From the above formula, it can be estimated that for an event with a 10 GeV initial state photon, the error of 4% on $E_\gamma$ introduces an uncertainty of about 0.45 GeV on the $\sqrt{s'}$, taking $E_{\text{beam}} = 45.6$ GeV.

**Event Selection and Background Contamination**

Besides the five general cuts described in section 5.2, the following additional requirements are also applied for the event selection:

- There should be two and only two AMUI muons;
- At least one of the muons should have a measured momentum higher than 50% $E_{\text{beam}}$;
Figure 7.9 The energy $E_γ$ calculated according to equation 7.1 for: a) the detected isolated photon in the 1992 data, b) the detected isolated photon in the KORALZ-3.8 Monte Carlo and c) the missing photon in the KORALZ-3.8 Monte Carlo. This energy is normalized to the measured (a and b) or generated (c) photon energy. Gaussian fits to these distributions yield σ’s of 2.7%, 2.3% and 3.9% for a), b) and c) respectively.

- The acoplanarity angle between the muons should be smaller than 5°;
- There should be no ECAL, nor HCAL clusters, with energies higher than 1 GeV and with opening angles more than 8° with respect to the muons;
- The calculated energy $E_γ$ for the missing photon should be higher than 5 GeV.

To obtain sufficient statistics, we group the events into two bins: $70 \text{ GeV} \leq \sqrt{s'} < 80 \text{ GeV}$ and $80 \text{ GeV} \leq \sqrt{s'} < 86 \text{ GeV}$, and average the $\sqrt{s'}$ over all the events in each bin.
Muon Pair Cross Sections around 80 GeV

To simplify the calculation, only the data at the peak is used. In Table 7.1, the numbers of radiative events ($N^R$) selected out of the 1991 and 1992 data sample are listed for the two $\sqrt{s}$ bins. Also given are the numbers of Monte Carlo events ($N^R_{\mu\mu}$) selected out of a sample of 94,521 KORALZ-3.8 $e^+e^- \to \mu^+\mu^-(\gamma)$ events which are simulated according to the 1992 hardware conditions†. The relative background contaminations from $e^+e^- \to \tau^+\tau^-$, $\eta_{\tau\tau}$, are determined by applying the selection criteria to a Monte Carlo sample of 105,000 $e^+e^- \to \tau^+\tau^-$ events‡. The last two columns are the ZFITTER calculated and the measured (improved) Born cross sections ($\sigma^B_{ZFITTER}$ and $\sigma^B$). The listed errors are statistical only.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>$N^R$</th>
<th>$N^R_{\mu\mu}$</th>
<th>$\eta_{\tau\tau}$ (%)</th>
<th>$\sigma^B_{ZFITTER}$ (nb)</th>
<th>$\sigma^B$ (nb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>83.7 ± 0.4</td>
<td>32</td>
<td>68</td>
<td>11.9</td>
<td>0.062</td>
<td>0.058 ± 0.013</td>
</tr>
<tr>
<td>75.6 ± 0.7</td>
<td>12</td>
<td>30</td>
<td>3.0</td>
<td>0.025</td>
<td>0.022 ± 0.007</td>
</tr>
</tbody>
</table>

Table 7.1 Improved Born cross sections of $e^+e^- \to \mu^+\mu^-$ at $\sqrt{s} = 80$ GeV.

The measured Born cross sections are calculated according to:

$$\sigma^B(s') = \frac{N^R(s')}{[L \cdot G(s'/s) \cdot e^R(s') \cdot (1 + \eta_{\tau\tau}) \cdot (1 - \delta_{\text{QED}})$$

where $L$ (≈ 28565.8 nb) is the total integrated luminosity, $e^R(s')$ the detection efficiency for events which change from $s$ to $s'$ through initial state photon radiation. $G(s'/s)$ is the radiator function (see subsection 1.2.3), and it is obtained from Monte Carlo simulation:

$$G(s'/s) = \frac{N^R_{\mu\mu}(s')}{N^R_{\mu\mu}(s)} \frac{\sigma^B_{ZFITTER}(s') \cdot N^R_{\mu\mu}(s)}{\sigma^B_{ZFITTER}(s) \cdot e^R(s') \cdot (1 - \delta_{\text{QED}})}$$

Here, subscript "$\mu\mu$" indicates "Monte Carlo", $N_{\mu\mu}(s)$ is the total number of events in the Monte Carlo sample and $\sigma_{\mu\mu}(s)$ is the cross section for these Monte Carlo events ($\sigma_{\mu\mu}(s) = 1.48$ nb). The detection efficiency $e^R(s')$ is assumed to be the same for data and for Monte Carlo simulation.

The statistical errors on $\sigma^B(s')$ is calculated according to:

$$\Delta \sigma^B(s') = \sigma^B(s') \cdot \sqrt{\frac{1}{N^R(s')} + \frac{1}{N^R_{\mu\mu}(s')} - \frac{1}{N_{\mu\mu}(s)}}$$

Since the determination of $\sigma^B(s')$ relies heavily on the Monte Carlo simulation, a systematic uncertainty of 6% on $\sigma^B(s')$ is assigned to account for imperfections in the

†Only the KORALZ-3.8 Monte Carlo events have been passed through the full detector simulation and reconstruction. For the analysis under study, the imperfection of this program is included in the systematic error assigned.
‡Background contamination from $e^+e^- \to e^+e^-\mu^+\mu^-(\gamma)$ and from cosmic rays are negligible.
simulation. This estimate is based on the fact that there are 165 events having photons with energies higher than 5 GeV and opening angles larger than 600 mrad with respect to the muons in the 1992 data sample, while the Monte Carlo simulation predicts 175 events.

**Muon Pair Asymmetries around 80 GeV**

Besides the selection criteria described above, the following additional requirements are applied in order to reject events with wrongly determined muon charges:

- The two muons should have opposite charges;
- At least one of the muons should have a muon chamber reconstructed transverse DCA $D_t$ smaller than 30 mm.

In order to increase the statistics, we use events from all $\sqrt{s}$ points in this analysis. The results for the two $\sqrt{s'}$ bins, based on the 1991 and 1992 date sample, are listed in table 7.2 with $N_F$ being the number of forward events which have $\cos \theta_c > 0$, $N_B$ the number of backward events. $A_{FB}$ is the improved Born asymmetry determined through a maximum likelihood fit (see subsection 6.3.3) and corrected for the effect of final state radiation. Again, the listed errors are statistical only.

Compared to the statistical errors, the systematic uncertainty on $A_{FB}$ is very small and therefore neglected.

<table>
<thead>
<tr>
<th>$\sqrt{s'}$ (GeV)</th>
<th>$N_F$</th>
<th>$N_B$</th>
<th>$A_{FB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$83.7 \pm 0.4$</td>
<td>11</td>
<td>28</td>
<td>$-0.48 \pm 0.15$</td>
</tr>
<tr>
<td>$75.6 \pm 0.7$</td>
<td>4</td>
<td>15</td>
<td>$-0.73 \pm 0.17$</td>
</tr>
</tbody>
</table>

*Table 7.2* Improved Born asymmetries of $e^+e^- \rightarrow \mu^+\mu^-$ at $\sqrt{s} = 80$ GeV.
Chapter 8

Determination of the Z Parameters

In this chapter, the cross section and asymmetry measurements presented in previous chapters are interpreted in a model independent way, and in terms of the Standard Model. The results are extracted with $\chi^2$ fits using the MINUIT package [49]. The $\chi^2$ values are calculated from the measurements and their errors, including correlations and the uncertainties introduced by the luminosity determinations, and from the theoretical expectations from ZFITTER. Effects arising from uncertainties on the LEP energy determinations have also been taken into account [18].

Within ZFITTER, measurable quantities like the hadronic and leptonic cross sections of the Z decays, and the leptonic forward-backward charge asymmetries can be calculated in three different ways according to:

1) the Standard Model with $M_Z, m_t$ and $m_H$ as inputs;

2) a model independent approach using $M_Z, \Gamma_Z, \Gamma_{ll} \ (l = e, \mu$ and $\tau)$ and the hadronic partial width $\Gamma_{\text{hadron}}$ as inputs [9];

3) a variant of 2) using $M_Z, \Gamma_Z, \tilde{g}_V^l$ and $\tilde{g}_A^l \ (l = e, \mu$ and $\tau)$ and $\Gamma_{\text{hadron}}$ as inputs [9, 10].

Two additional input parameters are $\alpha$ and $\alpha_s$. The availability of the two model independent approaches makes it possible to check the Standard Model in a way independent of the model itself.

In section 8.1, the mass and widths of the Z are determined with the first model independent approach using only the cross section measurements presented in chapter 5. The results are compared with the corresponding Standard Model predictions. This is
repeated including cross section measurements of other decay channels of the Z from L3. A value for the number of light neutrino species is also obtained.

In section 8.2, the mass, widths and effective coupling constants of the Z are determined with the second model independent approach. This is done using cross section and asymmetry measurements presented in chapters 5 and 6, without and with measurements of other decay channels and the τ polarization measurement from L3.

In section 8.3, a limit is given in terms of the Standard Model on the mass of the top quark based on the measurements presented in chapters 5 and 6, together with other results from L3.

In section 8.4, our results on the partial width and effective coupling constants of Z decaying into muon pairs are compared with those from other LEP experiments. The measurements presented in chapters 5 and 6 are converted into lowest order values. These values, together with the "medium energy" cross section and asymmetry measurements presented in chapter 7, are joined by the "low energy" measurements from previous experiments and compared with the Standard Model predictions.

8.1 Mass and Widths of the Z

As can be seen from equation 1.5, the cross section of the muon pair production can be parametrized in terms of \( M_Z \), \( \Gamma_Z \) and the partial widths \( \Gamma_{ee} \) and \( \Gamma_{\mu\mu} \). In this parametrization the interference term, a contribution of about 0.2% in the range of \( \sqrt{s} = M_Z \pm 1 \text{ GeV} \), is fixed to its Standard Model value. Imposing lepton universality: \( \Gamma_{ee} = \Gamma_{\mu\mu} = \Gamma_{ll} \), the cross section measurements presented in chapter 5 are sufficient to determine:

\[
\begin{align*}
M_Z &= 91,213 \pm 37 \text{ MeV} \\
\Gamma_Z &= 2,475 \pm 63 \text{ MeV} \\
\Gamma_{ll} &= 83 \pm 2 \text{ MeV}
\end{align*}
\]

with \( \chi^2 / N_{\text{DOF}} = 11/6 \). The values for the Z widths are in good agreement with the Standard Model predictions for \( M_Z = 91.213 \text{ GeV} \):

\[
\begin{align*}
\Gamma_Z &= 2,498 \text{ MeV} \\
\Gamma_{ll} &= 84 \text{ MeV}
\end{align*}
\]

Including cross section measurements of \( e^+e^- \rightarrow \text{hadrons} \), \( e^+e^- \rightarrow e^+e^- (\gamma) \) and \( e^+e^- \rightarrow \tau^+\tau^- (\gamma) \) [50] leads to the values listed in table 8.1. When the lepton universality assumption is removed, the leptonic partial widths can be determined separately (see table 8.1). These values for the Z widths are again in good agreement with the Standard Model predictions and confirm the hypothesis of lepton universality.
8.1 Mass and Widths of the Z

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Universality</th>
<th>No Universality</th>
<th>Standard Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ (MeV)</td>
<td>91 195 ± 9</td>
<td>91 195 ± 9</td>
<td>2 496</td>
</tr>
<tr>
<td>$\Gamma_Z$ (MeV)</td>
<td>2 495 ± 10</td>
<td>2 495 ± 10</td>
<td>2 496</td>
</tr>
<tr>
<td>$\Gamma_{\text{hadron}}$ (MeV)</td>
<td>1 748 ± 10</td>
<td>1 749 ± 11</td>
<td>1 743</td>
</tr>
<tr>
<td>$\Gamma_{ee}$ (MeV)</td>
<td>83.4 ± 0.5</td>
<td>83.4 ± 0.5</td>
<td>83.9</td>
</tr>
<tr>
<td>$\Gamma_{\mu\mu}$ (MeV)</td>
<td>83.1 ± 0.8</td>
<td>83.9</td>
<td>83.9</td>
</tr>
<tr>
<td>$\Gamma_{\tau\tau}$ (MeV)</td>
<td>84.1 ± 0.9</td>
<td>83.7</td>
<td>83.7</td>
</tr>
<tr>
<td>$\Gamma_{\ell\ell}$ (MeV)</td>
<td>83.5 ± 0.5</td>
<td>83.9</td>
<td>83.9</td>
</tr>
<tr>
<td>$\chi^2 / N_{\text{DOF}}$</td>
<td>52/60</td>
<td>50/58</td>
<td>50/58</td>
</tr>
</tbody>
</table>

Table 8.1  Z parameters obtained from model independent fits to the cross section measurements of $e^+e^- \rightarrow \text{hadrons}$, $e^+e^-(\gamma)$, $\mu^+\mu^-(\gamma)$ and $\tau^+\tau^-(\gamma)$. The Standard Model expectations are calculated with ZFITTER using $M_Z = 91.195$ GeV, $m_t = 170$ GeV and $m_H = 300$ GeV.

As shown in equations 1.6 and 1.7, within the framework of the Standard Model, $\Gamma_Z$ can be rewritten as:

$$\Gamma_Z = \Gamma_{\text{hadron}} + \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{invisible}} = \Gamma_{\text{hadron}} + (3 + \delta_\tau) \cdot \Gamma_{\ell\ell} + N_\nu \cdot \Gamma_{\nu\nu}$$

and is therefore sensitive to the number of light neutrino ($m_\nu < M_Z/2$) species, $N_\nu$, equivalent to the number of fermion families. Here, $\delta_\tau = -0.0023$ accounts for the effect of the tau mass [8] and, it is assumed that all contributions to the "invisible" width are due to $e^+e^- \rightarrow \nu_l \bar{\nu}_l (\gamma) (l = e, \mu, \tau, \cdots)$. In figure 8.1, the cross section measurements presented in chapter 5, and the hadron cross section measurements from $L_3$ [50] are compared with expectations from the Standard Model for different numbers of light neutrino species. Clearly the case with three neutrino species is favoured.

$N_\nu$ can be expressed as:

$$N_\nu = \frac{\Gamma_{\text{invisible}}}{\Gamma_{\nu\nu}} = \frac{\Gamma_{\text{invisible}}}{\Gamma_{\ell\ell}} \cdot \left( \frac{\Gamma_{\ell\ell}}{\Gamma_{\nu\nu}} \right)_{\text{SM}} .$$

Using the results presented in table 8.1, $N_\nu$ is found to be $N_\nu = 2.98 \pm 0.05$. This value is almost independent of the unknown parameters of the Standard Model, since most higher-order corrections involving $m_H$ and $m_t$ cancel in the ratio:

$$\left( \frac{\Gamma_{\ell\ell}}{\Gamma_{\nu\nu}} \right)_{\text{SM}} = 0.5015 .$$

Here, subscript "SM" indicates that the corresponding quantity is calculated according to the Standard Model. The above value of the $N_\nu$ can be compared to the result of $N_\nu = 3.14 \pm 0.25$ obtained from the $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ cross section measurement of $L_3$ [51].
Figure 8.1  Muon pair (left) and hadron (right) cross section measurements as presented in chapter 5 and in [50]. The curves are the Standard Model expectations for 2 (dashed), 3 (solid) and 4 (dotted) types of light neutrino species. Also shown are the deviations of our measurements from the Standard Model expectations for 3 neutrino species. The curves are calculated with ZFITTER.

8.2 Effective Coupling Constants of the Z

Introducing effective coupling constants $\bar{g}_V^l$ and $\bar{g}_A^l$, the muon pair cross section becomes a function of $(\bar{g}_V^{l2} + \bar{g}_A^{l2})$ with $l = e, \mu$. Furthermore, the forward and backward charge asymmetry in the muon pair production is sensitive to the product of $\bar{g}_V^e$ and $\bar{g}_V^\mu$. Assuming lepton universality, the effective axial and vector coupling constants can be determined, together with $M_Z$ and $\Gamma_Z$, from the measurements presented in chapters 5 and 6:

\[
M_Z = 91212 \pm 37 \text{ MeV} \\
\Gamma_Z = 2481 \pm 62 \text{ MeV} \\
\bar{g}_V^l = -0.047 \pm 0.007 \\
\bar{g}_A^l = -0.497 \pm 0.006
\]
8.2 Effective Coupling Constants of the Z

with $\chi^2 / N_{DOF} = 16/14$. Here the signs of the coupling constants are determined from the $\tau$ polarization measurement [52] and from neutrino scattering experiments [53]. Our results are in good agreement with the Standard Model predictions for $m_t = 170$ GeV:

\[
\begin{align*}
\bar{g}_V^f & = -0.036 \\
\bar{g}_A^f & = -0.501 .
\end{align*}
\]

Including cross section and asymmetry measurements of other channels, together with measurement of the $\tau$ polarization from $L_3$, one obtains the results listed in table 8.2. When the lepton universality assumption is removed, the $\bar{g}_V^f$ and $\bar{g}_A^f$ for each lepton family $f$ can be determined separately (see also table 8.2). These values for the $\bar{g}_V$ and $\bar{g}_A$ are again in good agreement with the Standard Model predictions and support the assumption of lepton universality. The significantly better result for $\bar{g}_V^f$, as compared to $\bar{g}_V^{e,\mu}$, is due to the inclusion of the $\tau$ polarization measurement which can be obtained from the distribution of the $\tau$ decay products observed in the detector. In figure 8.2, the new results, together with the 68% confidence contours of the fits, are compared with the Standard Model predictions for three different values of the top mass.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Universality</th>
<th>No Universality</th>
<th>Standard Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ (MeV)</td>
<td>$91.195 \pm 9$</td>
<td>$91.195 \pm 9$</td>
<td>2 496</td>
</tr>
<tr>
<td>$\Gamma_Z$ (MeV)</td>
<td>$2.495 \pm 10$</td>
<td>$2.495 \pm 10$</td>
<td>1 743</td>
</tr>
<tr>
<td>$\Gamma_{\text{hadron}}$ (MeV)</td>
<td>$1.748 \pm 10$</td>
<td>$1.749 \pm 11$</td>
<td>1 743</td>
</tr>
<tr>
<td>$\bar{g}_V^e$</td>
<td></td>
<td>$-0.036 \pm 0.009$</td>
<td></td>
</tr>
<tr>
<td>$\bar{g}_V^\mu$</td>
<td></td>
<td>$-0.06 \pm 0.03$</td>
<td></td>
</tr>
<tr>
<td>$\bar{g}_V^\tau$</td>
<td></td>
<td>$-0.038 \pm 0.008$</td>
<td></td>
</tr>
<tr>
<td>$\bar{g}_A^e$</td>
<td></td>
<td>$-0.500 \pm 0.002$</td>
<td></td>
</tr>
<tr>
<td>$\bar{g}_A^\mu$</td>
<td></td>
<td>$-0.497 \pm 0.004$</td>
<td></td>
</tr>
<tr>
<td>$\bar{g}_A^\tau$</td>
<td></td>
<td>$-0.502 \pm 0.003$</td>
<td></td>
</tr>
<tr>
<td>$\bar{g}_V^f$</td>
<td>$-0.041 \pm 0.004$</td>
<td></td>
<td>$-0.036$</td>
</tr>
<tr>
<td>$\bar{g}_A^f$</td>
<td>$-0.500 \pm 0.001$</td>
<td></td>
<td>$-0.501$</td>
</tr>
</tbody>
</table>

$\chi^2 / N_{DOF}$: 88/108 (86/104)

Table 8.2 The effective coupling constants obtained from model independent fits to the cross section and asymmetry measurements of $e^+e^- \to \text{hadrons}$, $e^+e^- (\gamma)$, $\mu^+\mu^- (\gamma)$ and $\tau^+\tau^- (\gamma)$, and to the $\tau$ polarization measurement. The Standard Model expectations are calculated with ZFITTER using $M_Z = 91.195$ GeV, $m_t = 170$ GeV and $m_H = 300$ GeV.
Figure 8.2  Correlation between the effective coupling constants. The black dot indicates the result of the fit under the assumption of lepton universality, while the solid contour corresponds to the 68% confidence level. The dashed contours are for the three lepton species separately. Also shown are the Standard Model predictions for $M_Z = 91.195$ GeV, $m_H = 300$ GeV and for three different values of $m_t$.

8.3 Limit on the Top Mass

In the framework of the Standard Model, weak interactions contribute to the radiative corrections which transform the lowest order coupling constants into the effective ones. While these corrections are rather insensitive to the mass of the Higgs boson, this is not the case for the mass of the top quark, $m_t$, and consequently a prediction for $m_t$ can be derived from the measurements.

Including our $\alpha_s$ measurement from hadronic event topologies and tau decays, $\alpha_s = 0.123 \pm 0.006$ [26] as a constraint, the fit to measurements of the hadronic cross section, leptonic cross section and forward-backward asymmetry, average tau polarization, $b\bar{b}$ forward-backward asymmetry [54] and of the $Z \to b\bar{b}$ partial width [55] yields:

$$M_Z = 91\,195 \pm 9 \text{ MeV}$$
$$m_t = 171^{+31}_{-37} \pm 18 \text{ GeV},$$

with the second error on $m_t$ indicating the shift in its central value when varying the mass of the Higgs boson, $m_H$, from 60 to 1000 GeV around the central value of 300 GeV. This value of $m_t$ is consistent with the result of $m_t = 174 \pm 16$ GeV obtained from a direct search for top quark production in $p\bar{p}$ collisions [56].
8.4 Comparison with Results from Other Experiments

In figure 8.3, our results on the partial width $\Gamma_{\mu\mu}$ and effective coupling constants $\bar{g}_V^\mu$ and $\bar{g}_A^\mu$ of Z decaying into muon pairs are compared with those from other LEP experiments [57]. Good agreement is observed.

The measurements presented in chapters 5 and 6 include all corrections, while those presented in chapter 7 are the improved Born values. Results from earlier experiments at lower C.M. energies have traditionally been presented in the lowest order Born approximation. In order to check the consistency between these measurements, the results presented in chapters 5 through 7 are converted into lowest order values (see table 8.3). The conversion is done according to:

$$\sigma_0 = \left\{ \begin{array}{c} \sigma - (\sigma - \sigma_0)_{\text{SM}} \\ \sigma^B - (\sigma^B - \sigma_0)_{\text{SM}} \end{array} \right\} \quad \text{and} \quad A_{FB}^0 = \left\{ \begin{array}{c} A_{FB} - (A_{FB} - A_{FB}^0)_{\text{SM}} \\ A_{FB}^B - (A_{FB}^B - A_{FB}^0)_{\text{SM}} \end{array} \right\}$$

Expressions $(\sigma - \sigma_0)_{\text{SM}}$, $(\sigma^B - \sigma_0)_{\text{SM}}$, $(A_{FB} - A_{FB}^0)_{\text{SM}}$ and $(A_{FB}^B - A_{FB}^0)_{\text{SM}}$ obtain their values from the Standard Model. These expectations are calculated with the ZFITTER package using $M_Z = 91.195$ GeV, $m_t = 170$ GeV and $m_H = 300$ GeV. The total errors which include the statistical and systematic uncertainties and the uncertainty caused by the luminosity determination are kept unchanged.

In figures 8.4 and 8.5, our values of the lowest order total cross section and charge asymmetry for $e^+e^- \rightarrow \mu^+\mu^-$, together with those made by earlier experiments at lower energies [58], are compared with the Standard Model predictions. It is interesting to see

![Graph](image)

**Figure 8.3** Our results on the partial width $\Gamma_{\mu\mu}$ and effective coupling constants $\bar{g}_V^\mu$ and $\bar{g}_A^\mu$ of Z decaying into muon pairs compared with those from other LEP experiments.
<table>
<thead>
<tr>
<th>$E_{\text{CM}}$ (GeV)</th>
<th>$\sigma_0$ (nb)</th>
<th>$A^0_{\text{FB}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.6 ± 0.7</td>
<td>0.020 ± 0.007</td>
<td>−0.73 ± 0.17</td>
</tr>
<tr>
<td>83.7 ± 0.4</td>
<td>0.056 ± 0.013</td>
<td>−0.45 ± 0.15</td>
</tr>
<tr>
<td>88.480</td>
<td>0.34 ± 0.02</td>
<td>−0.12 ± 0.10</td>
</tr>
<tr>
<td>89.468</td>
<td>0.68 ± 0.03</td>
<td>−0.15 ± 0.07</td>
</tr>
<tr>
<td>90.228</td>
<td>1.23 ± 0.04</td>
<td>−0.01 ± 0.05</td>
</tr>
<tr>
<td>91.222</td>
<td>1.91 ± 0.03</td>
<td>0.05 ± 0.02</td>
</tr>
<tr>
<td>91.254</td>
<td>2.012 ± 0.025</td>
<td>0.058 ± 0.015</td>
</tr>
<tr>
<td>91.294</td>
<td>1.961 ± 0.016</td>
<td>0.039 ± 0.008</td>
</tr>
<tr>
<td>91.967</td>
<td>1.41 ± 0.05</td>
<td>0.09 ± 0.05</td>
</tr>
<tr>
<td>92.966</td>
<td>0.67 ± 0.04</td>
<td>0.19 ± 0.06</td>
</tr>
<tr>
<td>93.716</td>
<td>0.40 ± 0.03</td>
<td>0.17 ± 0.07</td>
</tr>
</tbody>
</table>

Table 8.3 Lowest order cross section and asymmetry measurements at various C.M. energies. The listed errors include both the statistical and the systematic uncertainties.

that data obtained over a period of about 15 years from four different e$^+e^-$ accelerators located in Germany, the United States, Japan and Switzerland agree perfectly well with the Standard Model. In fact, the precise measurement of the muon pair charge asymmetry at the PETRA and PEP energies of $\sqrt{s} = 30$ GeV already made it possible to predict the mass of the Z being around 100 GeV before the actual discovery of the Z, showing that the presence of a high mass particle could be sensed at low energy. At LEP a similar situation occurs, where the existence of the high mass top quark can be felt by measurements of $\Gamma_Z$ and the effective weak couplings of the Z to leptons and to the b quarks. To determine precisely the effective Z to muon couplings, thus contributing to the verification of the Standard Model, has been the main aim of this thesis.
Figure 8.4  The lowest order total cross section for $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$ compared between experimental measurements (points) and the Standard Model expectations (solid curve) for $M_Z = 91.195$ GeV. The dotted curve corresponds to contribution from the $\gamma$ exchange.
Figure 8.5: The lowest order forward-backward charge asymmetry of e⁺e⁻ → μ⁺μ⁻ (ν(ν)) compared between experimental measurements (points) and the Standard Model expectations (solid curve) for M_Z = 91.195 GeV. The dotted curve corresponds to contribution from the γ exchange.

\[ A_{FB} \]

\[ e^+ e^- \rightarrow \mu^+ \mu^- (\nu \bar{\nu}) \]

\[ \sqrt{s} \text{ (GeV)} \]

PEP
- HRS
- MAC
- MARK II

PETRA
- CELLO
- JADE
- MARK I
- TOPAZ
- PLUTO
- TASSO

TRISTAN
- LEP
- AMY
- VENUS

This thesis
- Other

\[ 0 \]

0.4

0.8
Appendix A

Single Wire Resolution of the P Chambers

The doublet momentum resolution, derived from equation 2.2, is dominated by the uncertainty in the \((\alpha_1-\alpha_2)\) measurement*. The 21% doublet resolution corresponds to \(\Delta(\alpha_1-\alpha_2) = 1.7\) mrad. This uncertainty in the \((\alpha_1-\alpha_2)\) measurement has a negligible contribution from multiple scattering, and is dominated by the single wire resolution of the P chambers, including the inaccuracy of the cell-map function and the fluctuation introduced by the electronics.

With \(N\) drift distance measurements, \(d_1, d_2, \cdots, d_N\) from \(N\) consecutive wires whose radial coordinates are \(r_1, r_2, \cdots, r_N\), a line segment can be fitted. Its slope is given by:

\[
S = \frac{\sum (d_i - \bar{d}) \cdot (r_i - \bar{r})}{\sum (r_i - \bar{r})^2}
\]

with

\[
\bar{r} = \frac{1}{N} \sum r_i \quad \text{and} \quad \bar{d} = \frac{1}{N} \sum d_i.
\]

In order to simplify the formulae, the Lorentz angle is assumed to be 0° (instead of 19°), the drift paths are therefore perpendicular to the wire plane. The slope is directly related to the local tangent angle, \(\alpha\), of the muon track:

\[
S = \tan \alpha.
\]

Neglecting errors on the \(r\)'s, the error on \(S\) is dominated by uncertainties on the drift distance measurements:

\[
\Delta S = \sqrt{\sum \frac{\Delta^2 d_i \cdot (r_i - \bar{r})^2}{\sum (r_i - \bar{r})^2}}.
\]

*The contribution due to alignments in the tangential or radial directions is negligible.
On an average basis, $|\Delta d_i|$ can be regarded as a constant $\Delta d$, which is the single wire resolution of the P chambers. Therefore, $\Delta S$ becomes:

$$
\Delta S = \frac{\Delta d}{\sqrt{\sum (r_i - \bar{r})^2}} = \frac{\Delta d}{\Delta R} \sqrt{\frac{1}{12} \cdot N \cdot (N + 1) \cdot (N - 1)}
$$

with $\Delta R = 9$ mm being the spacing between neighbouring wires. This error can be translated to the uncertainty on the $\alpha$ measurement:

$$
\Delta \alpha = \cos^2 \alpha \cdot \Delta S
$$

Taking the typical $\alpha$ angles $\alpha_1 \approx \alpha_2 = 11.25^\circ$, and $N = 14$ for MI, $N = 22$ for MM, we have:

$$
\Delta \alpha_1 = 0.0071 \cdot \Delta d \quad \text{(A.1)}
$$

$$
\Delta \alpha_2 = 0.0036 \cdot \Delta d \quad \text{(A.2)}
$$

and

$$
\Delta (\alpha_1 - \alpha_2) = \sqrt{\Delta^2 \alpha_1 + \Delta^2 \alpha_2} = 0.008 \cdot \Delta d
$$

Here, the $\Delta \alpha$'s are in mrad, while $\Delta d$ is in $\mu$m. As shown in figure A.1, equations A.1 and A.2 have been verified by a Monte Carlo study. Therefore it can be concluded that the single wire resolution of the P chambers is:

$$
\Delta d = \frac{\Delta (\alpha_1 - \alpha_2)}{0.008} = 210 \mu m
$$

**Figure A.1** Result of a Monte Carlo study showing the uncertainties on the $\alpha$'s as measured with 14 consecutive wires in MI or 22 wires in MM when the single wire resolution is 100, 150, 200, 250 and 300 $\mu$m respectively. The superimposed lines correspond to equations A.1 and A.2.
This value is almost identical with the result from a more sophisticated study which determines the P chamber single wire resolution from the "sagitta" distribution for every three neighbouring hits [34].

It has to be noted that both the method described here and the one presented in [34] are not sensitive to systematic shifts in the $t_0$, nor to the effect caused by cross-talk between neighbouring wires.
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Summary

This thesis presents a test of the Standard Model through measurements of the muon pair production in $Z$ decays. The experiment was performed with $L_3$, one of the four large LEP detectors at CERN, Geneva. The Standard Model postulates the existence of a sixth quark, the top quark, and the Higgs particle. Although the top quark can not be directly observed with $e^+e^-$ machines at present, it modifies the effective weak couplings of the $Z$ to, inter alia, leptons.

In chapter 1, the relevant theoretical background is given concerning the cross section and asymmetry measurements of muon pairs as function of the total centre of mass energy. From these measurements, the effective $Z$ to lepton coupling constants $\tilde{g}_V$ and $\tilde{g}_A$ can be determined.

Chapter 2 describes the LEP collider and the $L_3$ detector. The LEP collider provides high luminosity electron and positron beams at precisely determined energies. The electrons and positrons are guided to collide in the center of the $L_3$ detector. From the inside out, a muon track is sampled by the central track detector, the electromagnetic calorimeter, the scintillation counters, the hadron calorimeter, the muon filter and the muon spectrometer. The electromagnetic calorimeter and the muon spectrometer provide high resolution energy measurements for leptons.

Chapter 3 is dedicated to the reconstruction and simulation of tracks in the $Z$ chambers of the muon spectrometer. The single wire resolution is checked and found to be $\sigma = 670$ $\mu$m thanks to the improved reconstruction software and the adjustments to the "cell-map" function. The simulation of the $Z$ chamber response to a traversing muon is described and the resolution of the $\theta$-angle determination is estimated.

In chapter 4, the procedures of the muon identification with the calorimeters are discussed. Using the calorimeters as complement to the muon spectrometer, the acceptance and efficiency determinations are almost independent of our knowledge of the geometrical structure and performance of any individual detector component. Also, the acceptance is increased.

In chapter 5, the determination of the muon pair cross sections is described. Muons
are defined as AMUIs or MIPs depending on whether they are identified with the muon chambers or with the calorimeters. Events having two AMUI muons, or one AMUI muon and one MIP muon, or two MIP muons are considered as muon pair candidates. After applying the selection criteria, over 30,000 events are selected out of the 1991 and 1992 data samples, with 81%, 11% and 8% belonging to the three categories respectively. The numbers of selected events are corrected for inefficiencies of the trigger system, the TEC track finding, the scintillators and the MIP finding with the calorimeters. The detector acceptances and the background contaminations are determined with Monte Carlo simulations. Despite a substantial increase in the muon chamber inefficiency from 3.7% in the first period in 1991 to 7.8% in the third period in 1992, the acceptance changes only by a relative amount of ± 0.3%. On the Z resonance, the total cross section is determined to be σ_{μμ} = 1.39 ± 0.03, 1.50 ± 0.02 and 1.46 ± 0.01 nb at √s = 91.222, 91.254 and 91.294 GeV respectively. The relative systematic error on these measurements is estimated to be 0.8%, including the 0.6% contribution from the analysis presented in this chapter.

The muon pair charge asymmetries are obtained in chapter 6 using only events with two AMUI muons. The results are AFB = 0.02 ± 0.02, 0.031 ± 0.015 and 0.012 ± 0.008 at √s = 91.222, 91.254 and 91.294 GeV respectively. The systematic error on these measurements is estimated to be 0.0013.

Chapter 7 is devoted to studies of photon radiation in muon pair production. Events with missing photons along the beam pipe are used to study the muon pair production in e+e− annihilations at energies around 80 GeV. Measurements of the improved Born values of the cross section and asymmetry are obtained. The results are σ_{μμ}^B = 0.022 ± 0.007 and 0.058 ± 0.013 nb, AFB = −0.73 ± 0.17 and −0.48 ± 0.15 at √s = 75.6 ± 0.8 and 83.7 ± 0.4 GeV respectively. A systematic error of 6% is assigned to the cross section measurements, while the systematic error on the asymmetry measurements is neglected.

In chapter 8, the mass, the total and partial widths of the Z, and the effective coupling constants are determined with model independent approaches using the measurements presented in chapters 5 and 6. Together with measurements of other Z decay channels and that of the τ polarization from L3 and assuming lepton universality, the results are: M_Z = 91.195 ± 9 MeV, Γ_Z = 2.495 ± 10 MeV, Γ_{hadron} = 1.748 ± 10 MeV, Γ_τ = 83.5 ± 0.5 MeV, g_A = −0.041 ± 0.004 and g_μ = −0.500 ± 0.001. These results are in good agreement with the Standard Model expectations for M_Z = 91.195 GeV, m_t = 170 GeV and m_H = 300 GeV. Using these Z widths, the number of light neutrino species is found to be N_ν = 2.98 ± 0.05. Within the framework of the Standard Model, a limit is obtained on the mass of the top quark: m_t = 171^{+36}_{-42} GeV. In the last section, the measurements presented in chapters 5 through 7 are converted into lowest order values, they are then joined by the "low energy" measurements from previous experiments and compared with the Standard Model predictions. Good agreement is observed.
Samenvatting

Dit proefschrift beschrijft een test van het Standaard Model aan de hand van metingen aan muonparen ontstaan uit het verval van Z deeltjes. Het experiment werd uitgevoerd met $L_3$, één van de vier grote LEP detectoren te CERN, Genève. Het Standaard Model postuleert het bestaan van een zesde quark, de top quark, en het Higgs deeltje. Hoewel de top bij de bestaande $e^+e^-$ versnellers niet direct waargenomen kan worden, beïnvloedt hij de effectieve zwakke koppelingen van het Z deeltje aan o.a. leptonen.

In hoofdstuk 1 wordt de relevante theoretische achtergrond geschetst die noodzakelijk is voor een beter begrip van de werkzame doorsnede en asymmetrie metingen van muonparen als functie van de totale botsingsenergie. Gebruik makend van deze gemeten grootheden kunnen de effectieve zwakke axiale en vector koppelingen constanten $g_V$ en $g_A$ bepaald worden.

De LEP versneller en de $L_3$ detector worden in hoofdstuk 2 beschreven. De LEP versneller levert botsende bundels electronen en positronen met een hoge luminositeit en een nauwkeurig gedefinieerde totale botsingsenergie. De botsingen treden op in het hart van de $L_3$ detector. Vanuit het interactie punt gezien laat een muonspoor informatie achter in de centrale spoordetector, de electromagnetische calorimeter, de scintillatie tellers, de hadron calorimeter, het muonfilter en de muonspectrometer. De electromagnetische calorimeter en de muonspectrometer leveren de nauwkeurige energie metingen voor respectievelijk de electronen en photonen, en de muonen.

Hoofdstuk 3 behandelt de reconstructie en simulatie van sporen in de Z kamers van de muonspectrometer. De draad resolutie van de Z kamers is $\sigma = 670 \, \mu m$ dankzij een verbetering van het reconstructie programma en een aanpassing van de drifttijd-driftafstandsrelatie, de zogenaamde "cellmap" functie. Vervolgens wordt de simulatie van muonsporen in de Z kamers beschreven en de nauwkeurigheid van de $\theta$ hoek meting bepaald.

In hoofdstuk 4 wordt de methode voor de herkenning van muonen met de calorimeters beschreven. Door gebruik te maken van de calorimeters als complement van de muonspectrometer worden de acceptantie en efficiëntie bepalingen voor muonen nagenoeg onafhankelijk van een nauwkeurige kennis wat betreft geometrie en werking van de individuele detector componenten. Een belangrijk bijkomstig voordeel van deze nieuwe
methode is de verbetering van de acceptantie.

De bepaling van de werkzame doorsnede voor muonparen wordt in hoofdstuk 5 beschreven. Muionsporen worden gedefinieerd als AMUI's of MIP's afhankelijk van het feit of zij als zodanig door de muonspectrometer of door de calorimeters worden herkend. Drie categorieën worden onderscheiden als muonpaar kandidaten nl. twee AMUI's, één AMUI en één MIP, en twee MIP's. Meer dan 30,000 gebeurtenissen worden uit de meetgegevens van de jaren 1991 en 1992 geselecteerd, waarvan respectievelijk 81%, 11% en 8% behoren tot de drie bovengenoemde categorieën. De gekozen aantallen gebeurtenissen dienen gecorrigeerd te worden voor de efficiënties van de volgende hard- en software onderdelen: het trigger systeem, de TEC spoorherkenning, de scintillatoren, en de procedure voor het vinden van een MIP met de calorimeters. De detector acceptanties en de verontreiniging van enige achtergrond processen worden bepaald met Monte Carlo simulaties. Ondanks een aanzienlijke verhoging van de muonkamer inefficiëntie van 3.7% in de eerste periode van 1991 naar 7.8% in de derde periode van 1992, is de relatieve verandering van de acceptantie slechts ± 0.3%. De totale werkzame doorsnede op de Z resonantie worden gemeten als \( \sigma_{\mu \mu} = 1.39 \pm 0.03, 1.50 \pm 0.02 \) en \( 1.46 \pm 0.01 \) nb bij waarden voor de totale botsingsenergie van respectievelijk \( \sqrt{s} = 91.222, 91.254 \) en 91.294 GeV. De relatieve systematische fout in deze metingen wordt geschat op 0.8%, inclusief de 0.6% bijdrage van de analyse zoals gepresenteerd in dit hoofdstuk.

Hoofdstuk 6 beschrijft de bepaling van de muonpaar ladings-asymmetrie gebruik makend van de gebeurtenissen met twee AMUI muonen. Op de Z resonantie zijn de resultaten; \( A_{FB} = 0.02 \pm 0.02, 0.031 \pm 0.015 \) en \( 0.012 \pm 0.008 \) bij respectievelijk \( \sqrt{s} = 91.222, 91.254 \) en 91.294 GeV. De systematische fout voor deze metingen wordt geschat op 0.0013.

Hoofdstuk 7 is gewijd aan de photon emissie in muonpaar productie. Gebeurtenissen met missende photonen in de richting van de bundelpijp worden gebruikt voor het bestuderen van muonpaar productie in e+e- annihalaties bij botsingsenergien rond de 80 GeV. Metingen van de Born gecorrigeerde waarden voor de werkzame doorsneden en asymmetriën worden bepaald: \( \sigma_{\mu \mu}^B = 0.022 \pm 0.007 \) en \( 0.058 \pm 0.013 \) nb, \( A_{FB}^B = -0.73 \pm 0.17 \) en \( -0.48 \pm 0.15 \) bij respectievelijk \( \sqrt{s} = 75.6 \pm 0.8 \) en 83.7 ± 0.4 GeV. Een systematische fout van 6% wordt toegekend aan deze werkzame doorsnede bepalingen, de systematische fout van de asymmetrie bepaling kan verwaarloosd worden.

In hoofdstuk 8 worden op model onafhankelijke wijzen de massa, de totale en partiële breedte en de effectieve zwakke koppelingen constanten van de Z bepaald m.b.v. de metingen gepresenteerd in de hoofdstukken 5 en 6. Onder de aanname van lepton universaliteit en gebruik makend van de metingen van andere vervalskanalen van de Z en de \( \tau \) polarisatie meting van \( L_3 \) zijn de resultaten: \( M = 91,195 \pm 9 \) MeV, \( \Gamma = 2,495 \pm 10 \) MeV, \( \Gamma_{hadron} = 1,748 \pm 10 \) MeV, \( \Gamma_{\nu} = 83.5 \pm 0.5 \) MeV, \( \bar{g}_v = -0.041 \pm 0.004 \) en \( \bar{g}_A = -0.500 \pm 0.001 \). Deze resultaten zijn in goede overeenstemming met de verwachting van het Standaard Model voor \( M = 91.195 \) GeV, \( m_t = 170 \) GeV en \( m_{\nu} = 300 \) GeV. Gebruik makend van de bepaalde Z breedten blijkt binnen het kader van het Standaard Model het aantal neutrino soorten
$N_v = 2.98 \pm 0.05$ te zijn. De massa van de top quark wordt bepaald: $m_t = 171.36_{-42}^{+36}$ GeV. In het laatste gedeelte van het hoofdstuk worden onze metingen van $Z$ naar muonparen en van de "lage energie" experimenten getoond en vergeleken met de Standaard Model voorspellingen. De overeenkomst van de metingen met de theorie is uitstekend te noemen.
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Curriculum Vitae

The author of this thesis was born in Echeng, China on April 25, 1966. He received his high school education at the First High School of Echeng County. After an entrance examination, he started his study of physics at Wuhan University in 1982. In 1986 he obtained his Bachelor of Science degree in experimental solid state physics and was selected as a candidate to be sent abroad. While preparing to go abroad, he continued his study in solid state physics at Wuhan University. Since 1988 he has been with the Dutch National Institute for Nuclear and High Energy Physics (NIKHEF) working for the L3 experiment. He was first a visitor and started his Ph.D. research in 1990.

During his high school and university years, he won many math and physics contests. In 1990, he went to Milan to support Ruud Gullit. In 1992, he watched the Winter Olympics in Albertville.