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TITLE : Ultimate gradient accelerators: physics and prospects
TIME : 27 & 28 February, 1, 2, & 3 March from 11.00 to 12.00hrs
PLACE : Auditorium

ABSTRACT

As introduction, the needs and ways for ultimate acceleration gradients are discussed briefly.

The Plasma Wake Field Acceleration is analyzed in the most important details. The structure of specific plasma oscillations and "high energy driver beam SP-plasma" interaction is presented, including computer simulation of the process.

Some practical ways to introduce the necessary mm-scale bunching in driver beam and to arrange sequential energy multiplication are discussed. The influence of accelerating beam particle - plasma binary collisions is considered, also.

As applications of PWFA, the use of proton super-colliders beams (LHC and Future SC) to drive the "multi particle types" accelerator, and the arrangements for the electron-positron TeV range collider are discussed.
PLASMA WAKE FIELD ACCELERATION

CERN Academic Training Lectures
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Reasons to look for extremely high accelerating gradients.

For deeper understanding of Nature we need to study smaller and smaller scale in space and time (living cells - 1 mcm; molecules, atoms - 1 Angstrom; nuclei - 10 fermi; hadrons - $10^{-14}$ cm; fundamentals - down to $10^{-16}$ cm;...).

This leads unavoidably to the necessity to use higher and higher energies per “elementary - unstructured - constituent”:

$$\Delta = \frac{\hbar c}{E} \implies \frac{10^{-14}}{E_{\text{GeV}} \text{ cm}}$$

The current frontier is

$$10^{-16} \text{ cm} \Rightarrow 100 \text{ GeV} \quad \text{(for colliders!!)}$$

- in energy & momentum transfers
  and in generating of masses.

The aim now is 1 TeV and higher.

Hence - the need for growing and growing scale of High Energy Physics complexes.
To shrink the scale (and cost) of HEP facilities, we need advances in acceleration physics and in technologies.

Speaking over-simplistic, this way includes higher magnetic fields for cyclic accelerators and higher accelerating gradients for linear accelerators. In these lectures, I intend to discuss the second line only.
The ways to extreme gradients.

Suggestions and attempts to higher gradients were made by many people through decades and for many times.

For us, at Novosibirsk, the main drive in the direction, since 1960s, was *Linear Collider* hope for hundreds of GeV electron-positron collisions.

When we first presented publicly (1978) the VLEPP project, which incorporates 100 MeV/m accelerating gradient for linacs, this "high gradients problem" was considered by the majority as a main one in linear collider business.

Additionally to VLEPP project, we have discussed at that time the possibility to use the huge energy, stored at existing and - especially - planned proton beams of super-accelerators. This option was called the "proton klystron", and the way was shown to excite by such beams 1 cm wave length range linear accelerating structures up to limit of gradients.

Since that time in our Lab and in many other labs this level was proved practical for GHz (cm wave range) range, short pulse normal conducting accelerator structures - just have enough RF power and use proper technology for structures!
But it was (more or less) evident from the beginning - 100 MeV/m is close to the ultimate limit for metallic accelerating structures: electric field at the surface is additionally few times higher, and it starts to produce “cold currents”, and occasional discharges degrade the surface instead of improving it (training process fails).

Hence, to reach much higher gradients we need to shift from solid materials shaping of electromagnetic fields to plasma based structures.

(At really high electric fields, the exposed surface in any case converts into something close to plasma.)
Inside plasma, plasma electrons and electromagnetic fields oscillate coherently and the limit for maximal electric field looks differently. More or less universal limit (under "resonant excitation") can be evaluated such a way:

*electric field energy density should be less than (rest frame) energy density of plasma electrons.*

In equations:

\[
\frac{E_{\text{ultim}}^2}{8\pi} = \frac{1}{2} n_e mc^2,
\]

hence

\[
E_{\text{ultim}} = \sqrt{\frac{4\pi e^2 n_e}{r_e}}.
\]

Later on, you shall feel additionally the "naturality" of this limit.
To reach very high electric fields (in vacuum or in plasma) - as a stand-alone goal - is not a problem at all. For example, while passing a laser focus - hence, at the micron distances - particles are accelerated at GeV/cm rate. But it is not an “accelerator”, which is a device to accelerate particles at macroscopic distances up to “unlimited” energy!

To arrange high accelerating (longitudinal, parallel to beam particles velocity!) electric field in plasma devices of very different type, number of approaches were proposed (through decades!) and are under consideration now. Among them:
- smokotron;
- laser beam excitation;
- two laser beams beat-wave acceleration;
- plasma wake field acceleration (PWFA).

For the first three approaches I myself, very personally, do not see any way to reach high gradient and perfect phasing for arranging a “high energy accelerator”.

The last one - PWFA - up to now was based, in considerations and experiments, on driver electron beams of modest energy - the most natural transition from electron beams of klystrons, exciting usual linacs.

But, as we shall see later, it is not the way to extreme gradients: we need to use as high energy - "rigid" - beams, as economically possible.

This is the way to overcome plasma and beam-plasma (local) instabilities and to replace efficiently the accuracy of solid surfaces, which shape fields in usual linacs.

And a very important component of the whole approach is to arrange a proper focusing channel to keep the beams "very carefully" aligned.
Basics of PWFA.

Initial assumptions:

* Plasma should be arranged in advance - driver beam ionization is too weak.
  And ionization should be close to 100%.

* In good approximation, we can take into account plasma electrons "movability" only - ions (protons) can be considered as of infinite mass.
  The electron temperature can be considered negligibly low.

* For the beginning, driver and accelerating beams are considered "absolutely rigid".
We shall try to excite non-propagating axially symmetric plasma oscillations - "the excited plasma tube". The phase velocity along the axis is determined by velocity of driver beam - in all our considerations it will be the light velocity "c".

Local resonant frequency of plasma oscillations is the "electron plasma frequency":

\[
\omega_e = \sqrt{\frac{4\pi e^2 n_e}{m_e}}
\]

As was shown above, the limiting accelerating field in plasma - under resonant excitation - is

\[
E_{\text{ultim}} = \sqrt{\frac{4\pi e^2}{r_e} \cdot n_e}
\]
The real accelerating field should be, say, 3 times lower. Hence, the plasma density needed to achieve accelerating field should be not less than

\[ n_e = \frac{9}{4\pi} \cdot \frac{r_e}{e^2} \cdot E_{\text{acc}}^2. \]

Hence, to achieve 1 GeV/m we need to use plasma density $10^{15}$. 
Let us evaluate number of particles, which we need for to excite in plasma accelerating field needed.

The balance of energies - lost by driver particles and stored in “plasma tube” per 1 cm:

\[
\frac{E^2}{8\pi} \cdot \pi \cdot \frac{\lambda^2}{(2\pi)^2} \cdot 2 = eN_{\text{need}} \cdot \frac{E}{2}.
\]

Hence

\[
N_{\text{need}} = \frac{1}{8\pi} \cdot \frac{E \lambda^2}{e}.
\]

For 1 GeV/m=30 kGs and 1 mm wave length:

\[
N_{\text{need}} = 8 \cdot 10^9
\]

(of course, the estimate assumes all the particles passing in a proper oscillation phase).
To minimize peak driver current, it is worth to distribute the driver particles among the microbunches, separated by distance $\lambda$. This train, while traveling at speed of light, should excite plasma oscillations resonantly.

Hence:

$$\lambda = \frac{c}{\omega_e} = \sqrt{\frac{\pi}{r_e n_e}}.$$ 

For plasma density $1 \times 10^{15}$, it is equal to $\lambda = 1 \text{mm}$.

If the driver particles are distributed in a 10 microbunch train of length $= 1 \text{ cm}$, the mean current in the train would be

$$\frac{8 \times 10^9 \cdot 3 \times 10^{10}}{6 \times 10^{18}} \times 40 \text{A}. $$

The necessary peak current in microbunches should be 10 to 15 times higher.

This estimate is correct for linear regime, when accelerating field is very much lower than the ultimate one; if it comes closer - the efficiency is going down.
A very important problem is related to the transversal forces acting on driver and accelerated beam particles in the excited plasma channel. These (de)focusing forces are strongly dependent on the phase of plasma oscillations; they are minimal ("almost zero") at maximums of accelerating (decelerating) fields and grow at channel radius up to fraction (around one third) of longitudinal force maximum. And, of course, its action is higher if the beam energy is lower. Later on, we would consider this crucial item in more details.

This problem makes life a lot more complicated.
The structure and properties of PWFA oscillations.
Results of computer simulation of PWFA process.
Focusing problems.

One of the most important component of the whole PWFA approach is the excitation and the influence of transversal forces.

The maximum of radial force, acting on driver or accelerating beam particles (the sum of $eE_{rad}$ and $e\beta B_{az}$) is proportional to the electric longitudinal field - accelerating or decelerating. In linear regime of plasma oscillation, this effective transversal field is about $1/3$ of the longitudinal field maximum. The rise radius is about $\lambda/2\pi$.

So, the equation of transversal single particle motion will be:

$$\frac{d^2 r}{ds^2} + \frac{2\pi}{3} \frac{\text{grad}E_{eV} \cdot \kappa}{E_{eV} \cdot \lambda} \cdot r = 0,$$

where $s$ is the longitudinal coordinate, and $\kappa$ is the fraction of transversal force maximum at the current phase.
If the coefficient is negative (defocusing), as it will be seen in practical examples, the accelerating field about 1 GeV/m leads to incurable defocusing and, hence, to the loss of particles at this phases - no matter how strong, but realizable, is external focusing system.

If this coefficient is positive, effective "plasma beta-function" will be

\[
\beta_{pl} = \sqrt{\frac{3}{2\pi} \cdot \frac{E_{eV} \cdot \lambda}{\text{grad}E_{eV} \cdot \kappa}}.
\]

To direct the drive beam properly, we need appropriate external focusing - by quadrupoles (the drive beam energy is high!). So, drive bunches - #2 and the following - as well, as witness bunch, travel under combined action of homogeneous focusing of plasma oscillations and alternating focusing of quadrupoles.

(Let us do not forget - the plasma focusing is strongly dependent on the position in the train and on local phase, being homogeneous for given particle. But the result of its action, which defines the plasma beta-value, varies strongly with beam energy variation.)
Hence, we need to take care on stability of incoherent transversal oscillations of all the useful particles of very different energy and very different plasma focusing - in a time!

This incoherent problem might be analyzed more or less safely just straightforward, and we shall play with while considering “practical” application options. The positive result of plasma focusing - it directs the following particles to the same channel as preceding ones.

Much less clear at the moment is coherent stability problem in its accelerator-type part. Partial help to prevent coherent instability comes from the fact, that all the bunches, coherent oscillations of which are under action of coherent oscillations of preceding bunches via plasma excitation, have at “any” moment different frequencies. It is not excluded, that to prevent coherent instability, the external focusing should be strong enough (at least, not negligibly small), in comparison to plasma focusing.
The phase structure of accelerating and transversal forces is quite complicated. At the initial, linear stage, their maximums are shifted almost at $\pi/2$. At the developed stage, which is of the main interest, this shift is going down and it is necessary to play with driver and witness bunches phasing and lengths very carefully.
"Transversal" microbunching.

To find the way to introduce a proper microbunch structure in the bunches of driver beam (and, also, to prepare a proper witness microbunch to be accelerated) is one of the crucial elements of the whole approach. The problem looks non-trivial, because we need 0.1 mm range length of each "very high" energy microbunch and its positioning should take into account plasma frequency variation (non-linear regime!), hence - microbunches should be non-equidistant. This positioning should prevent parts of driver microbunches from entering the accelerating phases, what would eat the plasma oscillation energy instead of to pump it in, and the plasma defocusing phases.

The way proposed to solve this problem is to use very low emittance beams and transversal cutting. The general layout looks as following. At some part of the beam channel with high beta value, say, in vertical direction, we arrange local RF structure acting on the traveling bunch with the vertical force linearly depending on the position along the bunch (zero action at the bunch center).
The resulting transversal vertical momentum should much higher than due-to-emittance internal momentums in the bunch. Upon passing long enough free space, the different head-to-tail constituents of each bunch will be positioned differently in vertical direction. At this stage a target-cutter is placed, the holes of which are absolutely transparent but the other parts of the target destroy beam components completely. At the same place a vertically focusing lens is placed (focal length is 2 times smaller than RF section to target distance). At the same distance after the target the same RF structure is located, which compensates the vertical momentums of bunch components. Hence, at the exit of this section, each driver bunch will be transformed in a seria of microbunches - properly shaped and properly positioned!

We will consider an example of microbuncher while discussing Novosibirsk experiment.
Pre-microbunching.

To maximize excitation efficiency of driver beam, it is worth to try to shift as much particles as possible in the useful phases of each driver bunch, using RF energy modulation - prior the final microbunching via sliced target. It is not very easy for wave length around 1 mm.

Let us consider an energy modulation option, based on crossing of driver beam by parallel traveling linearly polarized RF-beam at small angle $\theta_{RF}$ with diffraction limited cross-section. When these beams are traveling at such an angle, the phase velocity relative to the driver particles (slipping velocity) becomes

$$\frac{\theta_{RF}^2}{2} c,$$

Bringing particle-to-RF shift $\lambda/\pi$ at the traveling distance of order of

$$L_{RF} = \frac{\lambda}{\pi \theta_{RF}^2}.$$. 
After such a shift the beams should be separated transversally. For this to occur, the separation should overcome the diffraction RF-beam size (assumed to be greater then driver beam diameter):

$$\theta_{RF} L_{RF} = \sqrt{\lambda L_{RF}}$$

- practically the same as previous limitation.

The resulting amplitude of the driver bunch energy modulation, upon a single interaction with the RF-wave of amplitude $E_0$ and of structure described, will be about

$$\Delta E_{\text{mod}} = eE_0 \sqrt{\lambda L_{RF}}.$$ 

To be effective, the modulation should be few times (say, by factor $k_{RF}$) greater than the energy spread:

$$k_{RF} \cdot \delta E_{\text{dr}} = eE_0 \sqrt{\lambda L_{RF}},$$

and this will give sinusoidal energy modulation with the amplitude $\Delta E_{\text{mod}}$ along the bunch.
Hence, the required peak RF-power should be:

\[ P_{\text{peak}} = \frac{E_0^2}{4\pi} \cdot \lambda L_{\text{RF}} \cdot c = \frac{k_{\text{RF}}^2}{4\pi} \cdot \frac{(mc^2)^2}{e^2} \cdot c \cdot \gamma_{\text{dr}}^2 \cdot \left( \frac{\delta E}{E} \right)^2 \]

The total energy \( E_{\text{pulse}} \) in such RF-bunch will be:

\[ E_{\text{pulse}} = \frac{1}{4\pi} \cdot \frac{mc^2}{r_e} L_{\text{bunch}} \gamma^2 k_{\text{RF}}^2 \left( \frac{\delta E}{E} \right)^2 \]

The best layout for such modulation looks to use open optical (confocal) cavity of dissipation time greater than duration of a driver train, with mirror-to-mirror distance two times higher than bunch-to-bunch distance.

If we would use \( k_{\text{mod}} \) of such cavities (all of them could be packed very locally), the total energy stored in these cavities will be proportionally lower.

(Let us remind, the beams passing the modulator practically do not absorb RF power, and the only losses are at cavity mirrors.)
As a result, the average RF power needed will be

\[ P_{av} = E_{pulse} f, \]

or:

\[ P_{av} = \frac{1}{8\pi} \cdot \frac{mc^2}{r_e} \cdot L_{bunch} \cdot \gamma^2 \cdot \frac{1}{k_{cav}} \cdot k_{RF}^2 \left( \frac{\delta E}{E} \right)^2 \cdot f, \]

where \( f \) is the macrobunch repetition rate.

In many cases such power looks quite acceptable.

Upon this stage, the energy modulation should be transformed via "bending section" in to density modulation along each bunch, thus rising the future microbunches intensities.
Sequential acceleration.

It is very interesting and important to find the way to reach much higher energies than the driver energy. This energy multiplying is very similar to the usual high voltage klystron driving of linac and to the “two-beam accelerator”.

The goal looks this way:
- using sequence of \( N \) driver bunches of energy \( E_{dr} \)
- to reach almost proportionally higher energy

\[
E_{acc} \Rightarrow NE_{dr}.
\]

The way in principle looks quite obvious. One driver bunch, via plasma excitation, transfers energy to the accelerating - “witness” - bunch up to exhaustion; then the next bunch shall replace the previous one. Then the process shall be repeated \( N \) times.
The problem is - how to arrange this process in a most efficient and cost effective way. In my understanding, the best way is to arrange “spiral delay-line” for driver beam train. In this case, we can use a single straight tunnel to host all the beams.

If we use a spiral of actual curvature radius $R$ and outer radius $r$, the delay $\delta L$ of a train at the spiral length $L_0$ will be:

$$\delta L = \frac{L_0 \cdot r}{2 \cdot R}.$$

At the length of drive energy loss, which is about

$$L_0 = \frac{E_{dr}}{\text{grad}E_{dr}},$$

the delay should be equal to distance between the train bunches $\delta L_b$.

The spiral curvature is defined by driver energy $E_{dr}$, also. Hence, the requirement for “r” will be:

$$r = 2 \cdot \frac{\text{grad}E}{B_{sp}} \cdot \delta L_b$$

where $B_{sp}$ is the spiral magnetic field used.
As you will see in examples below, this requirement is quite moderate, at least, formally. All the tolerances are to be analyzed carefully!

When such a spiral delay-line is arranged along the whole linear accelerator, the driver beam, bunch by bunch, transfers the energy to the same microbunch under acceleration.
The influence of particles-plasma scattering.

Not similar to usual accelerators, at PWFA the accelerating channel is filled with plasma particles. Hence, additionally to the coherent fields, beams particles are influenced by pair collisions, also. To minimize this influence, it is worth to use “100% ionized” hydrogen plasma, which consists of protons and electrons, and we shall have in mind this option, only. There are several collision processes of possible importance.

a). Nuclear interaction of hadrons of driver/accelerating beams.

The hadron-hadron total cross-section does not exceed

\[ \sigma_{\text{hadron}} \leq 1 \cdot 10^{-25} \text{cm}^{-2} \]

Hence, the probability to loose particles in 1 km path (in our consideration - per 1 TeV) is less then

\[ \sigma_{\text{hadron}} \cdot n_{\text{pl}} \cdot 1\text{km} = 10^{-25} \cdot 10^{15} \cdot 10^5 = 10^{-5} \]

So, it is not a limitation in “any” practical conditions.

b). The bremsstrahlung cross-section even for electrons and positrons is of the same order, so, it is also not important.
Hence, the single scattering processes are not a problem.

The most important process - if we do care of emittance growth - is the Coulomb multiple scattering for accelerating particles. The advance of mean square of the particle angle due to this process is

$$\delta \Theta^2_0 = \frac{200}{E_{\text{MeV}}^2} \cdot \frac{x}{X_0} = 5 \cdot 10^{-24} \cdot \frac{\delta L_{\text{acc}} n_{\text{pl}}}{E_{\text{MeV}}^2},$$

where $x/X_0$ is fraction of radiation length passed. Consequently, emittance differential will be

$$\delta \varepsilon_{\text{acc}} = 5 \cdot 10^{-24} \cdot \frac{n_{\text{pl}}}{\text{grad} E} \cdot \frac{\beta_{\text{acc}}(E)}{E^2} \cdot \delta(E),$$

where $E$ - the current energy in MeV.
As a result, current contribution to the final emittance growth will be
\[
\delta \varepsilon_{\text{fin}} = 5 \cdot 10^{-24} \cdot \frac{n_{\text{pl}}}{E_{\text{fin}} \, \text{grad}E} \cdot \frac{\beta_{\text{acc}}(E)}{E} \cdot \delta E.
\]

The final emittance, if influenced by plasma collisions only, will make up:
\[
\varepsilon_{\text{fin}} = 5 \cdot 10^{-24} \frac{n_{\text{pl}}}{E_{\text{fin}} \, \text{grad}E} \cdot \int_{E_{\text{in}}}^{E_{\text{fin}}} \frac{\beta_{\text{acc}}(E)}{E} \, dE.
\]

This result shows one of the most important limitation for electron-positron collider based on ultimate gradient PWFA.
Plasma arrangements.

Plasma along the whole pass of beams should be prepared in advance. As was mentioned above, to minimize the plasma binary collisions influence, especially on accelerating particles for collider use, the plasma is worth to be hydrogen one, and ionization degree should be close to 100%. Let us evaluate the energy needed for such arrangement:

\[ E_{\text{source}} = \pi R_{\text{pl}}^2 n_{\text{pl}} L_{\text{acc}} \eta^{-1} E_{\text{ion}}, \]

where

- \( E_{\text{ion}} \) - ionization potential,
- \( \eta \) - efficiency of energy transfer from ionization source,
- \( R_{\text{pl}} \) - radius of plasma channel.

Example:

\[ E_{\text{source}} = \pi \cdot (1\,\text{mm})^2 \cdot 10^{15} \cdot 10^5 \cdot 30 \cdot 10\,\text{eV} \cdot 2 \cdot 10^{-19} = 200 \frac{\text{J}}{\text{TeV}} \]

At repetition rate "f", the power for ionization will be proportional to "f".
"LINEAR COLLIDER LUMINOSITY"

\[ c := 3 \cdot 10^{10} \quad m_e := 10^{-27} \quad e := 4.8 \cdot 10^{-10} \quad r_e := 3 \cdot 10^{-13} \]

1. SR at collision.

\[ \delta E - \text{SR energy losses per cm}; \]

\[ L_{\text{rad}} - \text{length of full radiation losses}; \]

\[ |H| := |H_{\text{beam}}| + |E_{\text{beam}}|; \]

\[ \sigma_1, \sigma_x, \sigma_z - \text{longitudinal, horizontal and vertical bunch sigmas}; \]

\[ N - \text{number of particles per bunch}; \]

\[ E_{\text{spread}} - \text{relative energy spread after collision (+-).} \]

\[ \gamma := \frac{E}{m_e c^2} \]

\[ \delta E(\gamma, H) := r_e^2 \gamma^2 H^2 \]

\[ L_{\text{rad}}(\gamma, H) := \frac{m_e c^2}{r_e^2 \gamma H^2} \]

\[ H_{\text{bunch}}(N, \sigma_1, \sigma_x) := \frac{e \cdot N}{\sigma_1 \cdot \sigma_x} \]

\[ E_{\text{spread}}(N, \gamma, \sigma_1, \sigma_x) := \frac{0.4 \cdot r_e^3 \gamma N^2}{\sigma_1 \cdot \sigma_x^2} - \text{very flat.} \]

\[ E_{\text{spread}}(N, \gamma, \sigma_1, \sigma_x) := \frac{0.4 \cdot r_e^3 \gamma N^2}{\sigma_1 \left(\sigma_x + \sigma_z\right)^2} - \text{in general.} \]
2. Luminosity limitation due to beam-beam transversal instability.

\[ L(\gamma, N, f, \sigma) = \frac{2 \cdot \pi \cdot \gamma \cdot N \cdot f}{r_e \cdot \sigma} \]

\[ \sigma_z(\gamma, N, \sigma, \sigma_x) = \frac{r_e \cdot N \cdot \sigma}{8 \cdot \pi^2 \cdot \sigma_x \cdot \gamma} \]

Example:

\[ L(2 \cdot 10^6, 10^{11}, 10^2, 10^{-1}) = 4.189 \cdot 10^{33} \text{ cm}^{-2} \text{ s}^{-1} \]

\[ E_{\text{spread}}(10^{11}, 2 \cdot 10^6, 10^{-1}, 2 \cdot 10^{-4}) = 0.054 \]

\[ \sigma_z(2 \cdot 10^6, 10^{11}, 10^{-1}, 2 \cdot 10^{-4}) = 9.499 \cdot 10^{-8} \]

3. "Geometrical" luminosity.

\[ L_g(N, f, \sigma_x, \sigma_z) = \frac{N^2 \cdot f}{4 \cdot \pi \cdot \sigma_x \cdot \sigma_z} \]

Example:

\[ L_g(10^{11}, 10^2, 2 \cdot 10^{-4}, 10^{-7}) = 3.979 \cdot 10^{33} \]
4. To cure transverse coherent instability in accelerator:

\[ \delta E := \frac{E_{\text{head}} - E_{\text{tail}}}{2\cdot E} \]

\( \beta_f \) - current \( \beta \)-function accelerator;

\( r_d, L_d \) - radius and spacing of diaphragms.

\[ \delta E(N, \gamma, \beta_f, r_d, L_d) := \frac{r_e \cdot N \cdot \beta_f^2}{\gamma r_d^2 \cdot L_d} \]

Example:

\[ \delta E(2 \cdot 10^{11}, 2 \cdot 10^4, 100, 0.3, 1.5) = 0.222 \]
5. Minimal vertical size at IP, $\sigma_z$
limited by overfocusing instability and maximal "affordable" energy spread $\delta_{mE}$ due to beamstrahlung.

\[
\sigma_z(\gamma, \delta_{mE}, \sigma_1) := \frac{1}{8 \cdot \pi^2} \left( \frac{\delta_{mE} \cdot \sigma_1^3}{r_e \cdot \gamma^3} \right)^{0.5}
\]

Example:
\[
\sigma_z(2 \cdot 10^6, 0.1, 0.1) = 8.175 \cdot 10^{-8}
\]

The horizontal size limitation:

\[
\sigma_x(N, \gamma, \sigma_1, \delta_{mE}) := N \cdot \left( \frac{r_e^3 \cdot \gamma}{\sigma_1 \cdot \delta_{mE}} \right)^{0.5}
\]

Example:
\[
\sigma_x(10^{11}, 2 \cdot 10^6, 0.1, 0.1) = 2.324 \cdot 10^{-4}
\]
6. If vertical size is limited externally and horizontal size is limited by beamstrahlung:

\[ L_z(N, \gamma, \sigma_1, \sigma_{z0}, \delta, mE, f) = \frac{N \cdot \sigma_1^{-0.5} \cdot \delta mE^{-0.5} \cdot f}{4 \cdot \pi \cdot r e^{1.5} \cdot \gamma^{-0.5} \cdot \sigma_{z0}} \]

Example ( \( \sigma_{z0} = 1 \text{nm} \)):

\[ L_z\left(2 \cdot 10^9, 2 \cdot 10^6, 0.1, 1 \cdot 10^{-7}, 0.2, 10^4\right) = 9.686 \cdot 10^{33} \]

(of course, without beam-disruption enhancement and travelling focus).

\[ L_z\left(5 \cdot 10^9, 0.6 \cdot 10^6, 0.03, 0.4 \cdot 10^{-6}, 0.2, 10^4\right) = 6.054 \cdot 10^{33} \]
**ELECTRON-POSITRON COLLIDER**

(limits)

\[ \beta_{pl} = \sqrt{\frac{E \cdot \lambda}{\text{grad}E}} \quad \text{grad}E = \frac{2 \cdot \pi \cdot e}{3 \cdot r_e \cdot \lambda} \quad n_{pl} = \frac{\pi}{r_e \cdot \lambda^2} \]

\[ \delta E = \frac{0.1 \cdot e^3 \cdot \gamma \cdot N^2}{\sigma_{long} \cdot \beta_{fin} \cdot \varepsilon_{fin}} \quad \text{for round beams at collision} \]

\[ \delta E = \frac{0.4 \cdot e^3 \cdot \gamma \cdot N^2}{\sigma_{long} \left( \sigma_x + \sigma_z \right)^2} \quad \text{for flat (any!) beams at collision} \]

If witness average beta-value is determined by half of *plasma maximal focusing*, the final emittance will be:

\[ \varepsilon_{fin} = 5 \cdot 10^{-24} \cdot \frac{n_{pl}}{E_{fin} \text{MeV} \cdot \text{grad}E \text{MeV}} \int_{E_{in}}^{E_{fin}} \frac{\lambda}{\text{grad}E \text{MeV} \cdot E} dE \]

\[ = 10^{-23} \cdot \left( \text{grad}E \text{MeV} \right)^{-1.5} \cdot n_{pl} \cdot \sqrt{\frac{E_{fin} \text{MeV} - E_{in} \text{MeV}}{E_{fin} \text{MeV}}} \]

\[ = 10^{-10} \cdot \left( \text{grad}E \text{MeV} \right)^{-1.5} \cdot \lambda^{-1.5} \cdot E_{fin} \text{MeV}^{-0.5} \]

\[ = 10^{-10} \cdot 10^{-1.5} \cdot 0.1^{-1.5} \cdot (10^6)^{-0.5} = 1 \cdot 10^{-13} \]
\[ L_{\text{lim}} = \frac{1}{4\pi r_e^{1.5}} \cdot N \cdot f \cdot \frac{\sigma_{\text{long}}^{0.5}}{\beta_0} \cdot \frac{\delta E}{E} \cdot \frac{\text{finMeV}}{E_0} \]

\[ L_{\text{lim}} = 3.5 \cdot 10^{22} \cdot \frac{\sigma_{\text{long}}}{\beta_0} \cdot \text{gradE MeV}^{0.75} \cdot \lambda^{0.75} \cdot \text{E finMeV}^{-0.25} \cdot \left( \frac{\delta E}{E} \right)^{0.5} \cdot N \cdot f \]

\[ 10^{41} \cdot \frac{\sigma_{\text{long}}}{\beta_0} \cdot \text{gradE MeV}^{0.75} \cdot \lambda^{0.75} \cdot \text{E finMeV}^{-1.25} \cdot \left( \frac{\delta E}{E} \right)^{0.5} \cdot P_{MW^0} \]

\[ P_{MW} \quad \text{- mean witness power.} \]
For 1 TeV + 1 TeV: \( \sigma_{\text{long}} = \beta_{0^\circ} \)

\[
\varepsilon_{\text{fin}} = \frac{10^{-10}}{0.1 \cdot 1.5 \cdot 10^1 \cdot 1.5 \cdot (10^6)^{0.5}} = 1 \cdot 10^{-13}
\]

\[
L_{\text{lim}} = 3.5 \cdot 10^{22} \cdot 0.1 \cdot 0.75 \cdot 10^{-0.25} \cdot 0.2 \cdot 0.5 \cdot 2 \cdot 10^9 \cdot 10^4 = 9.899 \cdot 10^{33}
\]

\[
P_{\text{acc}} = \frac{2 \cdot 2 \cdot 10^9}{6 \cdot 10^{18}} \cdot 10^4 \cdot 10^6 = 6.667 \quad \text{MW}
\]
If it would be possible to reach $\beta_{\text{fin}} = 1 \text{ mm}

(very optimistic)

$L_{\gamma\gamma} = \frac{\left(\frac{1}{3} \cdot 2 \cdot 10^9\right)^2}{4 \cdot \pi \cdot 0.1 \cdot 10^{-13}} \cdot 10^4 = 3.537 \cdot 10^{34}$

Over-optimistic(!!):

$L_{\gamma\gamma} = \frac{\left(\frac{1}{3} \cdot 5 \cdot 10^9\right)^2}{4 \cdot \pi \cdot 0.1 \cdot 10^{-13}} \cdot 10^4 = 2.21 \cdot 10^{35}$

$L_{e\gamma} = \frac{5 \cdot 10^9 \cdot \frac{1}{3} \cdot 5 \cdot 10^9}{4 \cdot \pi \cdot 0.1 \cdot 10^{-13}} \cdot 10^4 = 6.631 \cdot 10^{35}$
Let us discuss in a very preliminari way the linear collider option, based on PWFA approach. The general layout, I have in mind in discussion, looks a following way

An 11 GeV electron “driver” accelerator produces trains of bunches. Each train contains 100 of bunches, 1 cm, 10^{11} in each bunch, separated by 1 or several of the accelerator wavelengths. Each bunch, by the using of pre-buncher and transversal cutter, is transformed in to 10 microbunches of 0.2 mm length. The last bunch of a train (#100) is directed in to plasma of appropriate density; all the others enter in to spiral isochronous delay-line. Traveling through plasma with properly arranged quadrupole lenses structure inside - 0.5 cm long lenses per each 10 cm, the fine-cutted bunch excites acceleration field of 1 GeV per meter.
With some short delay (half of plasma wave length to accelerate electrons. full plasma wave length to accelerate positrons), the driver bunch is folowed by "witness" (bunch under acceleration) - the 0.1 mm short bunch of the "same" energy, containing 2 or 5 times $10^9$ particles.

(The whole process we have seen on transparencies but let us look again.)

Upon the last microbunch of driver bunch #100 - passing 10 m section - is decelerated to 1 GeV, the bunch is carefully replaced by the previous one - #99, and the microbunch under acceleration, of energy +10 GeV, is positioned the same way relative to the new bunch. This cycle is repeated for 100 times bringing the witness to 1 TeV - upon 1 km of acceleration.
Such super-cycles are repeated at, say, 10 kHz frequency.

The same arrangements and processing are made for a counter beam.

So, we got a 1 TeV + 1 TeV collider (electron-positron, photon-photon, electron-photon).

Upon some play with corresponding numbers, we arrive to the table of parameters (draft and optional, of course!):
Open questions ("non-technical").

Stability of high amplitude free plasma oscillations.

"Drive beam - plasma" coherent stability.

Phasing related structures and tolerances.
LHC based PWFA: focusing limits

$$\beta_{av}(\Delta p_l, \beta_{pl}, F)$$

For driver beam 7 TeV initial:

head

<table>
<thead>
<tr>
<th>Energy (TeV)</th>
<th>$(100, 10^6, 5000)$</th>
<th>$(100, 1800, 5000)$</th>
<th>$10^4$ cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>(100 m -good)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For witness (beam under acceleration):

<table>
<thead>
<tr>
<th>Energy (TeV)</th>
<th>$(100, 70, 500)$</th>
<th>(100, 65, 500)</th>
<th>(100, $10^5$, 500)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>80</td>
<td>-371</td>
<td>1000</td>
</tr>
<tr>
<td>7</td>
<td>210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>630</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Acceptance for 0.7 TeV:

$$\frac{0.01^2}{80} = 1.25 \cdot 10^{-6} \text{ cm} \cdot \text{rad}$$

- not bad.

All accepted at lowest energy is accelerated without due to emittance losses!
LHC based PWFA

1. LHC beam characteristics should be adjusted to the needs of PWFA (different than for collider option):

   the ejected bunch should have around $5 \cdot 10^{10}$ p/cm for full 1 GeV/m - hence bunch compression;

   for easier microbunching we need few times smaller emittance - at least, in one (vertical) direction;

   to ease sequential acceleration - to make bunch to bunch distance as small as possible, hence, to arrange macrobunches;

   it is worth to eject one macrobunch in a time (?);

   some bunches should be used for production of secondary particles to accelerate via PWFA.

2. To reduce the cost of accelerating device (?) - let the lenses be placed at 1m separation.

3. At such device it is easy to accelerate low emittance beams - starting from around 100 GeV (p+, p- cooled, e+, e-, muons, etc.);

   for direct secondary beams - pions and kaons - we need to care on maximal acceptance, hence, the strongest plasma focusing possible.
MICRO-BUNCHING

Transversal momentum spread in driver beam of energy $E$ and emittance $\varepsilon_{\text{dr}}$ is

$$\delta p_{\text{tr}} = E \cdot \frac{\varepsilon_{\text{dr}}}{\beta}$$

For effective micro-bunch cutting gradient of transversal kick along the bunch should be (much) greater then:

$$\delta p_{\text{grad}} = k \cdot \frac{\left(\delta p_{\text{tr}}\right)}{\lambda_{\text{pl}}}$$

$$\delta p_{\text{grad}} = \frac{k \cdot E \cdot \varepsilon_{p}}{\lambda_{\text{pl}} \cdot \beta}$$

Transversal RF wave-length should be $2 \cdot \pi \cdot \sigma_{\text{long}}$ or greater.
Consequently, integrated transversal RF field amplitude should be (much) greater, than:

\[
eV_{RF} = \frac{k \cdot E \cdot \sigma_{long}}{\lambda_{pl}} \left[ \frac{\varepsilon \, dr}{\beta} \right]
\]

\[
eV_{RF}(E_{dr}, \sigma_{long}, \lambda_{pl}, \varepsilon_{dr}, \beta, k) = \frac{k \cdot E \cdot \sigma_{long}}{\lambda_{pl}} \left[ \frac{\varepsilon \, dr}{\beta} \right]
\]

\[
eV_{RF} = \frac{1}{2} k_{eff} k_{mcb} \cdot E_{drive} \left[ \frac{\varepsilon_{drive}}{\beta_{chop}} \right]
\]

- more reasonable formula!!

For 10 GeV electron driver:

\[
\frac{1}{2} \cdot 10 \cdot 10 \cdot 10^{10} \cdot \sqrt{\frac{10^{-7}}{10^{5}}} = 5 \text{ MV} \quad \text{very modest req.}
\]
\[ F = \frac{E_{\text{eV}^a} \lambda}{300 \cdot B_{\text{max}} \cdot \Delta_1} \]

\[ \dot{\beta}_{\text{pl}} = \frac{3 \cdot E_{\text{eV}^\lambda}}{2 \pi \cdot E_{\text{eV}_{\text{cm}}^\kappa}} \]

\[
C(\Delta_{\text{pl}}, \beta_{\text{pl}}, F) = \begin{bmatrix}
\cos\left(\frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}}\right) & \beta_{\text{pl}} \cdot \sin\left(\frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}}\right) & 1 & 0 \\
-1 & \frac{1}{F} & 1 & 0 \\
-\sin\left(\frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}}\right) & \cos\left(\frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}}\right) & \left(\frac{1}{F}\right) & 1 \\
-\sin\left(\frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}}\right) & \cos\left(\frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}}\right) & 0 & 1
\end{bmatrix}^{\frac{1}{2}}
\]

1. \(C(1, 0.01i, 1) = 3.613 \cdot 10^8\)

2. \(\beta_{\text{pl}} = 0.2, 0.21, 4.00\)

---

\[ C(1, \beta_{\text{pl}}, 1) \]

\[ C(1, \beta_{\text{pl}}, 10) \]

* Leaders and followers
* Transcend, cherish, stability!
\text{Ez}_0 \quad F_{\text{mid}} \quad \text{Driver beam}

\begin{array}{|c|c|c|}
\hline
\text{Ez max [r=0]} & 1.29e+03 \quad \text{MeV/m} & \text{Ez min [r=0]} & -1.44e+03 \quad \text{MeV/m} \\
\text{Plasma density} & 1.00e+15 \quad \text{cm}^{-3} & \text{Beam radius} & 1.50e-01 \quad \text{mm} \\
\text{N max [r=0]} & 1.63e+14 \quad \text{cm}^{-3} & \text{Linear beam dens.} & 2.30e+11 \quad \text{cm}^{-1} \\
\text{Beam length [front]} & 3.00e+01 \quad \text{mm} & \text{Beam length [back]} & 1.00e-01 \quad \text{mm} \\
\text{sigm} & 0.00e+00 & \text{sigmaEz} & 0.00e+00 \\
\text{Ezw} & 0.00e+00 & \text{MeV/m} & \\
\hline
\end{array}

\text{t + cos}
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ez max [r=0]</td>
<td>9.01e+02 MeV/m</td>
</tr>
<tr>
<td>Plasma density</td>
<td>1.00e+15 cm⁻³</td>
</tr>
<tr>
<td>N max [r=0]</td>
<td>1.63e+14 cm⁻³</td>
</tr>
<tr>
<td>Beam length [front]</td>
<td>3.00e+01 mm</td>
</tr>
<tr>
<td>sigmaEr</td>
<td>0.00e+00</td>
</tr>
<tr>
<td>Ez [0]</td>
<td></td>
</tr>
<tr>
<td>Beam radius</td>
<td>1.50e-01 mm</td>
</tr>
<tr>
<td>Linear beam dens.</td>
<td>2.30e+11 cm⁻¹</td>
</tr>
<tr>
<td>Beam length [back]</td>
<td>1.00e-01 mm</td>
</tr>
<tr>
<td>sigmaEz</td>
<td>0.00e+00</td>
</tr>
</tbody>
</table>

\[ i + \cos - de \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_z$ max [r=0]</td>
<td>$9.69 \times 10^2$</td>
<td>$MeV/m$</td>
</tr>
<tr>
<td>Plasma density</td>
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<td>$cm^{-3}$</td>
</tr>
<tr>
<td>$N_{max}$ [r=0]</td>
<td>$1.42 \times 10^{14}$</td>
<td>$cm^{-3}$</td>
</tr>
<tr>
<td>Beam length [front]</td>
<td>$1.00 \times 10^{-1}$</td>
<td>$mm$</td>
</tr>
<tr>
<td>$\sigma_{E_r}$</td>
<td>$7.70 \times 10^{-4}$</td>
<td>$mm$</td>
</tr>
<tr>
<td>$E_zw$</td>
<td>$9.43 \times 10^2$</td>
<td>$MeV/m$</td>
</tr>
<tr>
<td>$E_z$ min [r=0]</td>
<td>$-1.03 \times 10^3$</td>
<td>$MeV/m$</td>
</tr>
<tr>
<td>Beam radius</td>
<td>$1.50 \times 10^{-1}$</td>
<td>$mm$</td>
</tr>
<tr>
<td>Linear beam dens.</td>
<td>$2.00 \times 10^{11}$</td>
<td>$cm^{-1}$</td>
</tr>
<tr>
<td>Beam length [back]</td>
<td>$1.00 \times 10^{-1}$</td>
<td>$mm$</td>
</tr>
<tr>
<td>$\sigma_{E_z}$</td>
<td>$7.47 \times 10^{-2}$</td>
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<tr>
<td>Parameter</td>
<td>Value 1</td>
<td>Units</td>
</tr>
<tr>
<td>---------------------------</td>
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<tr>
<td>$E_z(0)$</td>
<td>1.08e+03</td>
<td>$MeV/m$</td>
</tr>
<tr>
<td>Plasma density</td>
<td>1.00e+15</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$N_{max}[r=0]$</td>
<td>1.63e+14</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>Beam length [front]</td>
<td>1.00e-01</td>
<td>mm</td>
</tr>
<tr>
<td>sigmaEr</td>
<td>-7.79e-03</td>
<td></td>
</tr>
<tr>
<td>$E_{sw}$</td>
<td>-1.07e+03</td>
<td>$MeV/m$</td>
</tr>
</tbody>
</table>

- $E_z(0)$: Electric field at $r=0$.
- Plasma density: Density of the plasma.
- $N_{max}[r=0]$: Maximum density at $r=0$.
- Beam length [front]: Beam length at front.
- sigmaEr: Error in $E_z(0)$.
- $E_{sw}$: Electric field at $r=0$.
Ez[0]  Fmidl  Driver beam + witness

\[ E_z[0] \quad F_{\text{midl}} \quad \text{Driver beam + witness} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_z ) max ( r=0 )</td>
<td>8.80e+02</td>
<td>MeV/m</td>
</tr>
<tr>
<td>Plasma density</td>
<td>1.00e+15</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>N max ( r=0 )</td>
<td>-1.63e+14</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>Beam length [front]</td>
<td>7.00e-02</td>
<td>mm</td>
</tr>
<tr>
<td>sigmaEr</td>
<td>-2.69e-02</td>
<td></td>
</tr>
<tr>
<td>Ezw</td>
<td>-9.11e+02</td>
<td>MeV/m</td>
</tr>
<tr>
<td>( E_z ) min ( r=0 )</td>
<td>-9.10e+02</td>
<td>MeV/m</td>
</tr>
<tr>
<td>Beam radius</td>
<td>1.50e-01</td>
<td>mm</td>
</tr>
<tr>
<td>Linear beam dens.</td>
<td>-2.30e+11</td>
<td>cm(^{-1})</td>
</tr>
<tr>
<td>Beam length [back]</td>
<td>7.00e-02</td>
<td>mm</td>
</tr>
<tr>
<td>sigmaEz</td>
<td>-2.44e-02</td>
<td></td>
</tr>
</tbody>
</table>
LINEAR “WAVES” IN PLASMA

\[ \text{rot } \vec{H} = -\frac{4\pi n_0 e}{c} \vec{v} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}, \quad \frac{\partial \delta n}{\partial t} + n_0 \text{div } \vec{v} = 0 \]

\[ \text{rot } \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t} \quad \frac{\partial \vec{v}}{\partial t} = -\frac{e}{m} \vec{E} \]

\((n = n_0 + \delta n — \text{density of plasma electrons, } \delta n \ll n_0)\)

(Plasma ions are immovable)

\[ \text{rot } \frac{\partial \vec{H}}{\partial t} = -c \text{ rot rot } \vec{E} = -\frac{4\pi n_0 e}{c} \frac{\partial \vec{v}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \]

\[ \text{rot rot } \vec{E} + \frac{\omega_p^2}{c^2} \vec{E} + \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0, \quad \text{where } \omega_p^2 = \frac{4\pi n_0 e^2}{m}. \]

Always \( \vec{E} = \vec{E}_1 + \vec{E}_2: \quad \text{rot } \vec{E}_1 = 0, \quad \text{div } \vec{E}_2 = 0. \]

<table>
<thead>
<tr>
<th>LANGMUIR “WAVE”</th>
<th>( \omega = \omega_p, \quad \frac{\partial^2 \vec{E}_1}{\partial t^2} = -\omega_p^2 \vec{E}_1. )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELECTROMAGNETIC WAVE</td>
<td>( \omega = \sqrt{\omega_p^2 + k^2 c^2}, )</td>
</tr>
</tbody>
</table>

\[ \Delta \vec{E}_2 - \frac{1}{c^2} \frac{\partial^2 \vec{E}_2}{\partial t^2} - \frac{\omega_p^2}{c^2} \vec{E}_2 = 0 \]

(since rot rot \( \vec{E} = \nabla \text{div } \vec{E} - \Delta \vec{E} \)).

"electric field energy to kinetic energy oscillations"
Density perturbations are caused by LANGMUIR "WAVES":

\[
\frac{\partial^2 \delta n}{\partial t^2} = -n_0 \text{div} \frac{\partial \vec{v}}{\partial t} = \frac{n_0 e}{m} \text{div} \vec{E} = -\omega_p^2 \delta n
\]

(perturbations oscillate, but don’t move away!)

**NOTE:** Movable ions \(\Rightarrow\) \(\omega_p^2 = 4\pi n_0 e^2 \left(\frac{1}{m} + \frac{1}{M}\right)\);

\(T_e \neq 0 \implies \omega = \omega_p \left(1 + \frac{3k^2 T_e}{8\pi n_0 e^2}\right),\)

\[
v_{gr} = \frac{\partial \omega}{\partial k} = \frac{3k T_e \omega_p}{4\pi n_0 e^2} \sim \frac{v_{Te}^2}{c} \text{ for } k = \frac{\omega_p}{c} \]

\(T_e = 10 \text{ eV} \implies v_T e \sim 5 \cdot 10^{-3} c\)

Charged particle of velocity \(\approx c\) excites plasma wave if

\[
v_{ph} = \frac{\omega}{k} = c
\]

Hence ONLY LANGMUIR "WAVE" with the spatial period \(2\pi c/\omega_p\) is excited

(For EM Wave \(v_{ph}\) always \(> c\))
APPLICABILITY OF LINEAR THEORY

Let \( l \) and \( \tau \) be characteristic length (radius) and time of the system, \( v \) — velocity of plasma electrons

\[
\delta n \ll n_0, \quad |\vec{v}| \ll c,
\]

\[
\left| \frac{\partial \vec{v}}{\partial t} \right| \gg |(v \nabla)\vec{v}| \implies \frac{1}{\tau} \gg \frac{v}{l}
\]

\[
|\vec{E}| \gg \frac{v}{c} |\vec{H}| \implies \frac{v}{c} \ll \frac{\delta n}{n_b}
\]

\((E \sim 4\pi e \delta n l \text{ — field of the wave})\)
\((H \sim 4\pi e n_b l \text{ — beam field})\)

Bunch sequence:

\(\tau \sim 1/\omega_p, \quad l \sim c/\omega_p, \quad v \sim c \delta n/n_0\)

Linear response if \(\delta n \ll n_0\)
Plasma Wake Field Acceleration - an old hope for very high acceleration gradients.

Good experiments at Argonne and KEK - but at "normally low" gradients
Fig. 2

$\Delta E \text{ (keV)}$

DELAY TIME (psec)

Representative Error
Options:

** normal conducting short-pulse short-wave "usual" linac;
up to around 100 MeV/m,
limited by break-down electric field at surface

\[\rightarrow\text{Frontier energy linear collider (e}^+\text{e}^-,\ e\gamma,\ \gamma\gamma)!\]

** super-conducting (pulsed) "usual" linac;
up to 25 MeV/m,
limited by cold emission electric
and/or critical magnetic fields at surface.

\[\rightarrow\text{"Crystal clear" e}^+\text{e}^-\text{ linear collider (Top Factory, ...)!}\]

"No-surface" = plasma based acceleration
(laser-driven; beam-driven)

????
How to arrange multiple acceleration using single driver accelerator?

Spiral delay line!

R - spiral curvature

$$\delta L = \frac{r}{2R_0} L_0 \quad \rightarrow \quad r = 2R \frac{\delta L}{L_0} \quad \rightarrow \quad r = 2 \frac{E_{\text{acc}}}{H_{\text{bend}}} \delta L$$
Focusing problems of e$^+e^-$ collider

\[ F_{\text{lens}} = \frac{E_{\text{ev}} \cdot 2 \Delta_{\text{lens}}}{300 B_{\text{max}} A_{\text{lens}}} \]

\[ \beta_{\text{pl}} = \sqrt{\frac{3 E_{\text{ev}} \cdot \lambda}{2 \pi \cdot \text{grad} E_{\text{ev}/cm} \cdot \varepsilon(\gamma)}} \]

\[ \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix} \left( \begin{pmatrix} \cos \frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}} & -\frac{1}{\beta_{\text{pl}}} \sin \frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}} \\ \beta_{\text{pl}} \sin \frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}} & \cos \frac{\Delta_{\text{pl}}}{\beta_{\text{pl}}} \end{pmatrix} \right) \]

\[ \cos \mu = \frac{1}{2} \text{Tr} \left[ \left( \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \right] \]

\[ M_F, \quad M_D \]

\[ \beta_{\text{max}} = \frac{M_F \cdot 1 \cdot 2}{\sin \mu} \quad \beta_{\text{min}} = \frac{M_D \cdot 1 \cdot 2}{\sin \mu} \]

\[ \beta_{\text{av}} = \frac{\beta_{\text{max}} + \beta_{\text{min}}}{2} \]
For collider: $\beta_{av}(\Delta p, \beta_p, F)$

**Driver:**
- 1-st mcbunch:
  - $\beta_{av}(10, 10^4, 70) = 140 \text{ cm}$
  - $\beta_{av}(10, 60, 70) = 55 \text{ cm}$
- Last mcbunch
  - Start:
    - $\beta_{av}(10, 7, 70) = 7.5 \text{ cm}$
  - End:
    - $\beta_{av}(10, 2.2, 7) = 6 \text{ cm}$
    - $\beta_{av}(10, 1000, 7) = 20 \text{ cm}$

**"Witness":**
- 10 GeV
  - Start:
    - $\beta_{av}(10, 10, 70) = 20 \text{ cm}$
- 100 GeV
  - $\beta_{av}(10, 60, 700) = 60 \text{ cm}$
- 1000 GeV
  - $\beta_{av}(10, 200, 7000) = 200 \text{ cm}$
10 GeV: $e^-$
$\lambda_{RF} = 10 \text{ cm}$

$e^-$: 10 GeV
$\lambda_{RF} = 10 \text{ cm}$

1 TeV/km

$L = \left( \frac{1}{2\pi} \cdot \frac{1}{10} \cdot N_{\text{DRIVE}} \right)^2 \cdot f_{\text{superbunch}} \cdot \frac{1}{4\pi \sigma_{\text{hor}} \sigma_{\text{vert}}}$
Ez[0]  Fmidl  Driver beam + witness

<table>
<thead>
<tr>
<th>Ez max [r=0]</th>
<th>1.06e+03  MeV/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma density</td>
<td>1.00e+15 cm$^{-3}$</td>
</tr>
<tr>
<td>N max [r=0]</td>
<td>1.63e+14 cm$^{-3}$</td>
</tr>
<tr>
<td>Beam length [front]</td>
<td>1.00e-01 mm</td>
</tr>
<tr>
<td>sigmaEr</td>
<td>1.03e-04</td>
</tr>
<tr>
<td>Ezw</td>
<td>1.04e+03  MeV/m</td>
</tr>
<tr>
<td>Ez min [r=0]</td>
<td>-1.09e+03 MeV/m</td>
</tr>
<tr>
<td>Beam radius</td>
<td>1.50e-01 mm</td>
</tr>
<tr>
<td>Linear beam dens.</td>
<td>2.30e+11 cm$^{-1}$</td>
</tr>
<tr>
<td>Beam length [back]</td>
<td>1.00e-01 mm</td>
</tr>
<tr>
<td>sigmaEz</td>
<td>1.31e-02</td>
</tr>
</tbody>
</table>
LHC based PWFA

1. LHC beam characteristics should be adjusted to the needs of PWFA (different than for collider option):
   - the ejected bunch should have around $5 \cdot 10^{10}$ p/cm$^2$ for full 1 GeV/m - hence bunch compression;
   - for easier microbunching we need few times smaller emittance - at least, in one (vertical) direction;
   - to ease sequential acceleration - to make bunch to bunch distance as small as possible, hence, to arrange macrobunches;
   - it is worth to eject one macrobunch in a time (?);
   - some bunches should be used for production of secondary particles to accelerate via PWFA.

2. To reduce the cost of accelerating device (?) - let the lenses be placed at 1m separation.

3. At such device it is easy to accelerate low emittance beams - starting from around 100 GeV (p+, p- cooled, e+, e-, muons, etc.);
   - for direct secondary beams - pions and kaons - we need to care on maximal acceptance, hence, the strongest plasma focusing possible.
Ez[0]  \quad F_{\text{midl}}  \quad \text{Driver beam + witness}

- Ez max \[r=0\] = \text{8.86e+02 MeV/m}
- Plasma density = \text{1.00e+15 cm}^{-3}
- N max \[r=0\] = \text{-1.63e+14 cm}^{-3}
- Beam length [front] = \text{7.00e-02 mm}
- \sigma_{E_r} = \text{-2.59e-02 mm}
- Ezw = \text{-9.94e+02 MeV/m}
- Ez min \[r=0\] = \text{-1.00e+03 MeV/m}
- Beam radius = \text{1.50e-01 mm}
- Linear beam dens. = \text{-2.30e+11 cm}^{-1}
- Beam length [back] = \text{7.00e-02 mm}
- \sigma_{E_z} = \text{-1.51e-02 mm}
\[ \text{Ez}[0] \quad \text{Fmid} \quad \text{Driver beam + witness} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ez max ([r=0])</td>
<td>9.80e+02 MeV/m</td>
</tr>
<tr>
<td>Plasma density</td>
<td>1.00e+15 cm(^{-3})</td>
</tr>
<tr>
<td>N max ([r=0])</td>
<td>-1.84e+14 cm(^{-3})</td>
</tr>
<tr>
<td>Beam length [front]</td>
<td>7.00e-02 mm</td>
</tr>
<tr>
<td>sigmaEr</td>
<td>1.39e-03</td>
</tr>
<tr>
<td>Ezw</td>
<td>9.57e+02 MeV/m</td>
</tr>
<tr>
<td>Ez min ([r=0])</td>
<td>-1.09e+03 MeV/m</td>
</tr>
<tr>
<td>Beam radius</td>
<td>1.50e-01 mm</td>
</tr>
<tr>
<td>Linear beam dens.</td>
<td>-2.60e+11 cm(^{-1})</td>
</tr>
<tr>
<td>Beam length [back]</td>
<td>7.00e-02 mm</td>
</tr>
<tr>
<td>sigmaEs</td>
<td>3.70e-02</td>
</tr>
</tbody>
</table>
LHC based PWFA: focusing limits

\[ \beta_{av}(\Delta p_l, \beta_{pl}, F) \]

For driver beam 7 TeV initial:

head

7 TeV \( (100, 10^6, 5000) \) \( 10^4 \) cm
\( (100, 1800, 5000) \) \( 1.7 \cdot 10^3 \)

7 TeV \( (100, 210, 5000) \) 210

0.7 TeV \( (100, 70, 500) \) 80
\( (100, 65, 500) \) -37i !!
\( (100, 10^5, 500) \) 1000

For witness (beam under acceleration):

0.7 TeV \( (100, 70, 500) \) 80

7 TeV \( (100, 210, 5000) \) 210

70 TeV \( (100, 630, 50000) \) 630

Acceptance for 0.7 TeV:

\[ \frac{0.01^2}{80} = 1.25 \cdot 10^{-6} \text{ cm \cdot rad} \]

- not bad.

All accepted at lowest energy is accelerated without due to emittance losses!
For LHC:

\[ a \in (3.38, 10) \]

\[ b \in (5, 10) \]

- put 120 -

- punches!!

So for LHC at 15MV/m of transverse RE

Help at x = 30 cm we need 200 m long section.

\( p \) under sub-punching up to 1 cm (sigma):\)

\[ a \in (3.8, 10) \]

\[ b \in (5.2, 10) \]

- the best hope for LHC.

At distance \( L \) of 1 km (transverse separation of micro-punches will be:

\[ \frac{\lambda \cdot \sqrt{b}}{v \cdot \sqrt{L}} \]

\[ \frac{q^3}{b} \cdot \frac{J}{\rho} \cdot K \cdot \rho \cdot \mu \cdot \gamma \cdot q^3 \cdot \rho \]

\[ cm \]

\[ \Sigma \cdot 4.4 = (0.1, 10, 5.1, 10) \]

\[ b \]
Accelerating

\[ 10^{10} \times 10^{10} \times 10^{10} \times 10^{10} \times 10^{10} \times 10^{10} \times 10 \]

Dri

chopper

LHC beam

Potential: proton energy multiplication \( K_{\text{acc}} \)

Intensity \( \rightarrow 1.5 \times 10^{-2} \cdot \frac{1}{K_{\text{acc}}} \cdot N_{\text{LHC}} \)

Multiple-energy acceleration \( \{ e^-, e^+; \pi^+, \mu^+, K^\pm \} \)

Acceleration of unstable particle

\[
\frac{N_f}{N_{in}} = \left( \frac{E_{in}}{E_f} \right) \frac{\Delta E}{c \cdot \tau_0}
\]

1 GeV/m is good even for \( K^\pm \)!
SPIRAL DELAY-LINES

If \( r \ll R \):

\[
L = L_0 \left(1 + \frac{r}{2 \cdot R}\right)
\]

The difference \( L - L_0 \) in this case is equal to:

\[
\delta L = \frac{L_0 \cdot r}{2 \cdot R}
\]

The radius \( r \), needed to give \( \delta L \) delay at the length \( L_0 \), is equal to:

\[
r = 2 \cdot R \cdot \frac{\delta L}{L_0}
\]

\[
r(R, \delta L, L_0) = 2 \cdot R \cdot \frac{\delta L}{L_0}
\]

\[
r(4000, 2, 7000) = 2.286 \text{ meter}
\]

\[
r = 2 \cdot \frac{E_{\text{acc}}}{H_{\text{bend}}} \cdot \delta L
\]

\[
2 \cdot \frac{3 \cdot 10^4}{10^5} \cdot 3 = 1.8 \text{ m (LHC)}
\]

So, for LHC preferred bunch-bunch distance is about 3 meter!

For 10 GeV, 10 cm driver:

\[
2 \cdot \frac{3 \cdot 10^4}{1.5 \cdot 10^4} \cdot 0.1 = 0.4
\]
Proton Klystron

Stored energy of SPS \( \sim 3 \text{ MJ} \)
(or Main Ring)

\[ 3 \text{ MJ} \rightarrow \text{50 km, 100 MeV/m} \rightarrow 5 \text{ TeV} \]
\[ \lambda \approx 5 \text{ cm} \]
(in principle)

\( \left( \text{p}^\pm; \text{e}^\pm; \text{p}^\mp; \text{S}^\pm \right) \)
incl. polarized

- H.e. proton beam - not electron beam
  (because of SR losses)

- Peak power (single-turn ejection, no long. comp)
  \( \rightarrow 100 \text{ GW RF} \)
Principle idea - quite simple

\[ E_{\text{acc.}} \approx E_p \]
\[ N_{\text{acc.}} \approx 0.1N_p \]
I. 
- Unbunched Pb.
- Linac sect. $E_{lin} > \Delta E$
- Energy modulated unbunched Pb beam

II. 
- Basic Pb accel.
- Bunched Pb beam

- Proper super-bunching
- Proper delays — several rings in main tunnel (slightly cluttered)
1. Short superbunch

\[ E_o \approx 10^2 \frac{eN_p}{\lambda^2} = 1.5 \cdot 10^{-11} \frac{N_p}{\lambda_{cm}} \quad \text{[MV/cm]} \]

2. Long superbunch

\[ E_o \approx 3 \cdot \frac{I_{amps}}{\sqrt{\lambda_{cm}}} \quad \text{[MV/cm]} \]

- for SPS \( \lambda = 5 \text{cm} \) \( \rightarrow \) \( E_o = 30 \text{ MeV/m} \)
- with 3-fold prebunching
  \[ E_o = 100 \text{ MeV/m} \]

- **Strong focusing**
  - for excitation beam and accelerated beam
    - simultaneously

- \( v_1 \neq v_2 \) problem - phase-shift insertions
  - becomes easier at \( \gamma \rightarrow \infty \)

---

100 ÷ 150 MeV/m - the possible maximum if electric field arises slowly

see Balakin's talk
M-energy modulator section;
DMA—dispersion magnets array.

\[ x = \gamma \frac{AE}{E_0} \]

\[ L = \int \gamma \frac{H}{Ip} ds \]

DMA optics arrangement for tunable dispersion

**Bunched beam current:** \[ I(t) = \sum_{n=-\infty}^{\infty} I_n e^{-in\omega_0 t}, \quad \omega_0 = \frac{2\pi}{\lambda} \]

For the simplest "one-cascade klystron" bunching:

\[ I_n = I_o \int_n (n \cdot \frac{u}{\sigma_E} \cdot \frac{2\pi}{\lambda} \cdot \sigma_E L) \exp \left[ -\frac{1}{2} \left( n \frac{2\pi}{\lambda} \sigma_E L \right)^2 \right] \]

\[ u = \hat{U}/E_0 \] —normalized amplitude of the modulator voltage;

\[ \sigma_E = \left( \frac{\langle E^2 \rangle}{E_0^2} \right)^{1/2} \] —normalized rms beam energy spread;

\[ \sigma_E L \] —dispersion path-length difference.
M - energy modulator
DMA - dispersion magnet arrays
BP - by-pass
PTL - proton transfer lines
PDL - pion decay line
ED - energy doubler
MCR - muon cooling ring
ESR - electron (positron) storage ring
UBD - used beam dumps
PWM - phasing wiggler magnets
NDB - no-dispersion bends
1, 2, ... 6 - fractions of the SPS one-turn extracted proton beam, synchronized in PTL to provide manifold energy build-up
Acceleration using PK

- any stable particles
  - if $\gamma_{\text{initial}}$ high enough
  - $e^+, e^-$ (initial accelerator - few GeV)
    - no SR limitations
      - be careful - SR in lenses! - low emittance
    - $p$ (fraction of initial $p$ beam)
    - $\bar{p}$ - stored, cooled and accelerated
      - (in high energy booster?)
    - nuclei (?)

- polarized beams
  - easy to overcome depolarizing effects

- unstable particles

  energy gain needed: \[ \frac{dE}{ds} \bigg|_0 = \frac{mc}{c^2} \cdot \ln \left( \frac{E_f}{E_i} \right) \cdot \ln \left( \frac{N_i}{N_f} \right) \]

  - $\mu^+$ - no problem - but to cool with ionization cooling (before accel)
  - $\pi^+ \geq 40$ MeV/m

  \[ K^+ \geq 300 \text{ MeV/m} \text{ - very special efforts (Bal.t.)} \]
• Colliding beams of unstable particles (and narrow intense \( \gamma \)-beams)

\[
\text{SPS} \rightarrow \text{PK + superlinac} \rightarrow \text{high field ring}
\]

\[
\begin{align*}
L_{\pi\pi} & \approx 3 \cdot 10^{27} \text{ cm}^{-2} \text{s}^{-1} \\
L_{\pi\rho} & \approx 3 \cdot 10^{28} \text{ cm}^{-2} \text{s}^{-1} \\
L_{m\mu} & \approx 3 \cdot 10^{31} \text{ cm}^{-2} \text{s}^{-1} \quad \text{(ionization cooling)}
\end{align*}
\]

• \( e^+e^- \) linear collider (SPS → PK + superlinac)

\[
L_{ee} \approx 10^{31} \text{ cm}^{-2} \text{s}^{-1}
\]

But — when VBA instead of SPS !!!
LAYOUT OF THE VEPP-2M COMPLEX

3 MEV LINAC
PES
RF

900 MeV
CMD-2
CMD-2
VEPP-2M
WIGGLER
SND

200 MEV SYNCHROTRON

e⁻⁻⁻⁻-> e⁺⁺⁺⁺ CONVERSOR

e⁻⁻⁻⁻⁻⁻ e⁺⁺⁺⁺⁺⁺ 200 ÷ 700 MEV
The BEP storage ring design and construction were specialized for two goals:

- To have a large acceptance for high efficiency of positron production for VEPP-2M experiments.
- To produce and study a high intensity beam \((N \sim 10^{12})\) with low emittance as a source for the future linear colliders.

$$+ \text{PWFA}$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>(E) Mev</td>
<td>120 - 850</td>
</tr>
<tr>
<td>Circumference</td>
<td>(P) m</td>
<td>22.35</td>
</tr>
<tr>
<td>Accelerating voltage frequency</td>
<td>(f_0) MHz</td>
<td>26.83</td>
</tr>
<tr>
<td>Maximum current</td>
<td>(I) A</td>
<td>(2 (N = 10^{12}))</td>
</tr>
<tr>
<td>Momentum compaction factor</td>
<td>(\alpha) -</td>
<td>0.05</td>
</tr>
<tr>
<td>Betatron tunes</td>
<td>(\nu_x) -</td>
<td>3.61</td>
</tr>
<tr>
<td></td>
<td>(\nu_z) -</td>
<td>3.18</td>
</tr>
<tr>
<td>RMS beam length</td>
<td>(\sigma_s) cm</td>
<td>10</td>
</tr>
<tr>
<td>RMS energy spread</td>
<td>(\sigma_E) -</td>
<td>(5 \cdot 10^{-4})</td>
</tr>
<tr>
<td>Vertical emittance</td>
<td>(\epsilon_z) (cm \cdot rad)</td>
<td>(10^{-8})</td>
</tr>
<tr>
<td>Horizontal emittance</td>
<td>(\epsilon_x) (cm \cdot rad)</td>
<td>(6 \cdot 10^{-6})</td>
</tr>
</tbody>
</table>
Layout of VEPP-2M complex

3MeV LINAC ILU

Synchrotron B-3M
250 MeV

$e^-, e^+$ convertor

900 Mev Booster BEP

CMD

RF

VEPP-2M Wiggler

SND

200-700 MeV
LAYOUT OF VEPP–2
EXPERIMENT

RF wavelength: .................. 10 cm
RF power: ......................... 200 kW (Q=10⁴)
Number of resonant
cavities: ......................... 2 × 10
Plasma density: .................. $10^{14} + 5 \cdot 10^{15}$ cm⁻³
Plasma length: .................. up to 1 m
Total length: ..................... 10 m

DESIGNED BEAM PARAMETERS

Number of particles: $10^{12}$  \( \sigma_z = 3 \text{ cm} \)
Particle energy: 700 MeV  \( \sigma_z = 0.1 \text{ cm} \)
The experiment requires a plasma with a density \( n \) in a range of \( 10^{15} \text{ cm}^{-3} \).
An appropriate length of the plasma column should be about 1 m with a diameter less of 1 cm.
To provide the conditions for the resonant excitation by a modulated beam with 10 driving electron bunches one has to have the longitudinal profile of the plasma density sufficiently uniform and reproducible from a shot to shot.
An accuracy of 2\% is sufficient for that.
We suppose that relevant technique for creating such a plasma, for initial experiments, is a discharge with the hollow cathode and anode in argon in a specially configurated magnetic field with the radial minimum B:
The preliminary experiments are performing with modules of 20 cm length:

First results are optimistic enough and show that energy cost of $0.1 \times 10^{15} \text{cm}^{-3}$ plasma column is about 2 Jouls (for $10^{-4}$ s pulse duration).
Fig. 1

Quartz tube

Magnet rings

L = 100 cm

Fig. 2

Anodes

Cathodes

> 50%
Open questions ("non-technical").

Stability of high amplitude free plasma oscillations.

"Drive beam - plasma" coherent stability.

Phasing related structures and tolerances.

Proper "acceleration - focusing" games.

Final focusing - chromaticity corrections!