SELECTED TOPICS ON VIOLATION OF CP INVARIANCE

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Extended Version of Lectures
given by L. Van Hove for the
Academic Training Programme
May-June 1967

GENEVA
1967
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1. **INTRODUCTION**

Before the discovery of the decay $K_\pi \rightarrow (\pi^+ \pi^-)$ \(^1\), all known elementary particle interactions were successfully classified according to strength of coupling and conserved quantities as shown in Table 1 (we disregard gravitational interaction).

We want to make a few remarks about this table:

1) The distinction between superstrong and medium-strong interactions is introduced in view of the relative success of SU(3) symmetry, more exactly its success as a broken symmetry. It is experimentally often difficult to separate clearly the effects of these two interactions.

ii) The relative strength of weak and strong interactions is not defined, as long as we do not specify the value of a "characteristic mass" (e.g. the vector boson mass) of weak interactions. If this mass is of the order of 1 GeV, the relative strength turns out to be $\sim 10^{-5}$.

iii) The relation

$$Q = I_3 + \frac{B + S}{2}$$

implies that $I_3$ and $S$ are simultaneously conserved, or not conserved, as all known interactions conserve $B$ and $Q$.

iv) $C, P, T$ are defined as operators which transform observables in the usual way (for example

$$P^{-1} \Psi P = -\bar{\Psi}$$

$$P^{-1} V_o P = V_o$$

where $(V_o, \Psi)$ is a polar vector, etc.)
Conservation of, for example, $P$ by $H_{st}$ means:

$$[P, H_{st}] = 0.$$  

It is then possible to choose $H_{st}$ eigenstates as simultaneous eigenstates of $P$, and this implies that the mean value of any quantity odd under $P$ on an $H_{st}$ eigenstate vanishes.

v) $B$ (baryonic number) is defined as the difference between the number of the baryons and of the antibaryons, $L_8 (L_8')$ as the difference between the number of the leptons $e^+, \nu_e (\mu^+, \nu_\mu)$ and the number of antileptons $e^-, \bar{\nu}_e (\mu^-, \bar{\nu}_\mu)$. The separated conservation of $L_8$ and $L_8'$ has been tested both at Brookhaven $^2$ and at CERN $^3$.

vi) If CPT is conserved, CP conservation is equivalent to T conservation, that is:

$$[CP, H] = 0 \quad \text{implies} \quad [T, H] = 0$$

if

$$[CPT, H] = 0.$$  

vii) On theoretical grounds one believes that CPT conservation is more likely to hold for any interaction than separate C, P, or T conservations. In fact, while it is easy to build local field-theoretical Hamiltonians which are not P or C or T invariant, CPT invariance is implied in any field theory by Lorentz invariance plus causality [CPT Theorem $^4$].

It is well known that CPT invariance implies:

a) The equality of the masses of particles and antiparticles.

b) The equality of the lifetimes of particles and antiparticles.
c) The equality of the branching ratios for the decays of particles and antiparticles into corresponding (by means of CPT) sets of states which are closed under final-state interactions (if we restrict ourselves, for example, to strong final-state interactions this equality is valid up to electromagnetic corrections; if we consider also electromagnetic final-state interactions, it is valid up to weak corrections).

Some experimental tests of the validity of CPT are the following:

a) The best number available comes from the knowledge of \( K_L^0 - K_S^0 \) mass difference. Using the experimental fact that \( |K_L^0|K_S^0 > \ll 1 \), one can prove\(^5\) that:

\[
|M_{K^0} - M_{\bar{K}^0}| \lesssim 2|m_L - m_S|
\]

so that from \( |m_L - m_S|/m_L \approx 0.6 \times 10^{-14} \) we see that here CPT invariance is valid for \( M_{K^0} - M_{\bar{K}^0} \) with an accuracy of the order of \( 1.2 \times 10^{-14} \).

Also a comparison between \( \pi^+ \) and \( \pi^- \) masses allows one to set an upper limit \( |m_{\pi^+} - m_{\pi^-}|/2m_{\pi^+} \lesssim 10^{-4} \).\(^6\)

b) \( \pi^+ \) and \( \pi^- \) lifetimes have been compared in several different experiments\(^7,8\); the over-all result is:

\[
[T(\pi^+)/T(\pi^-)] - 1 = (0.47 \pm 0.22) \times 10^{-2}.
\]

The agreement with CPT requirement, although not very good, is within three standard deviations.

Also \( K^+ \) and \( K^- \) lifetimes have been compared: the best value is\(^9\):

\[
[T(K^+)/T(K^-)] - 1 = (-0.5 \pm 1.0) \times 10^{-3}.
\]

\(^*)\) The numbers quoted are the ones of A.H. Rosenfeld et al.\(^6\). The experimental references are quoted therein.
A typical feature of Table 1 is its hierarchy structure: as the coupling constant decreases, less and less symmetry is left; so there are universally conserved quantities (Q, B, L_e, L_μ, T, CPT), quantities which are conserved by all interactions but weak ones (I_3, S, C, P), and quantities which are conserved only by strong (I) or only by superstrong [SU(3)] interactions.

Let us discuss how the discovery of CP violation in K decay could be fitted into this scheme; three main classes of theories have been proposed to explain the new phenomenon:

a) the first class tries to explain K^0_L → π^+π^- without assuming true CP violation, but no theory of this type is able at present to fit experimental data.

b) The second class of theories introduces a CP violation either in the weak interactions or in a new "very weak" interaction which, according to whether its strangeness selection rule is ΔS < 2 or ΔS ≤ 2, may have a strength 10^{-3} times smaller than H_{wk}', or 10^{-3} times smaller than H_{wk}.

c) The third class introduces a CP violation in K^0_L → π^+π^- by means of a second order effect, assuming C violation either in H_{med.st.} or in H_{e.m.}. This last class of theories breaks the hierarchy of increasing symmetry as the coupling strength increases, as CP is conserved by weak but not, for example, by electromagnetic interactions. Both classes (b) and (c) contain many schemes which are easily reconciled with present experimental evidence.

2. EXPERIMENTAL FACTS CONCERNING THE K^0-\bar{K}^0 SYSTEM

Let us define K^0 as the eigenstate of (H_{st} + H_{e.m.}) which is also the eigenstate of the strangeness S with eigenvalue +1

S |K^0> = |K^0>
and define $|\bar{K}^0>$ as:

$$|\bar{K}^0> = CP|K^0>.$$  

$K^0$ and $\bar{K}^0$ are orthogonal states, and we see at once that they cannot be eigenstates of the total Hamiltonian, as transitions $K^0 \rightarrow \bar{K}^0$ are allowed, for example by second-order weak interactions.

Let us define $K^0_S$ and $K^0_L$ as the two linear combinations of $K^0$, $\bar{K}^0$, which have exponential decay laws under the weak interactions:

$$|K^0_S(t)> = \exp[i(m_S + i\gamma_S)t] |K^0_S(0)>$$

$$|K^0_L(t)> = \exp[i(m_L + i\gamma_L)t] |K^0_L(0)>,$$

where $m_L$, $m_S$ are the masses and $(1/2\gamma_S)$, $(1/2\gamma_L)$ the lifetimes. The available experimental information on these quantities is:

$$m_S \approx m_L = 497.9 \pm 0.4 \text{ MeV}$$

$$|m_S - m_L| = 0.48 \pm 0.017 \ (\tau_S)^{-1}$$

$$\tau_S = \frac{1}{2\gamma_S} = (0.87 \pm 0.09)10^{-10} \text{ sec}$$

$$\tau_L = \frac{1}{2\gamma_L} = (5.68 \pm 0.26)10^{-8} \text{ sec}.$$  

Assume now that CP is conserved by all interactions, including the weak ones. The two eigenstates $K^0_L$, $K^0_S$ would then also be CP eigenstates, that is they would be identical with:

$$K^0_1 = \frac{1}{\sqrt{2}} (K^0 + \bar{K}^0) \quad \text{CP} = +1$$

$$K^0_2 = \frac{1}{\sqrt{2}} (K^0 - \bar{K}^0) \quad \text{CP} = -1$$

and only $K^0_1$ ($K^0_2$) would be allowed to decay into a final state with CP = +1 (CP = -1).
Consider the decay of $K^0_S$, $K^0_L$ into $\pi^+\pi^-$. We remark that for the $(\pi^+\pi^-)$ system the CP operator is identical to the permutation operator, as $P$ interchanges, in the centre-of-mass system, the spatial coordinates of $\pi^+$ and $\pi^-$, and $C$ their charges. Then, if Bose statistics hold,

$$\text{CP} \mid \pi^+\pi^- > = \mid \pi^+\pi^- >.$$

For a $\pi^0\pi^0$ system, even if Bose statistics are not valid, one has from angular momentum conservation

$$\text{CP} \mid \pi^0\pi^0 > = \mid \pi^0\pi^0 >$$

for a $\pi^0\pi^0$ system generated by the decay of a K meson. Hence, under CP conservations in the weak interactions, only $K^0_L$ can decay into $\pi^+\pi^-$ and $\pi^0\pi^0$.

From an experimental point of view it is now known from Christenson et al.\textsuperscript{1}) that not only the short-lived $K^0_S$ but also the long-lived $K^0_L$ decays into $\pi^+\pi^-$. In addition recent CERN-Nimrod\textsuperscript{9}) and Princeton\textsuperscript{10}) experiments have shown the existence of $K^0_L \rightarrow \pi^0\pi^0$ ($K^0_S \rightarrow \pi^0\pi^0$ is well known to exist).

The $K^0_L \rightarrow 2\pi$ decays are fully described by two complex numbers, for which one can adopt the definition:

$$\eta_{++} = \frac{< \pi^+\pi^- \mid H \mid K^0_L >}{< \pi^+\pi^- \mid H \mid K^0_S >}$$

and

$$\eta_{00} = \frac{< \pi^0\pi^0 \mid H \mid K^0_L >}{< \pi^0\pi^0 \mid H \mid K^0_S >}.$$

Our present experimental knowledge on their absolute values is the following:
\[ |\eta_{00}| = \left( 4.3 \pm 1.1 \right) 10^{-3} \]
\[ |\eta_{0\pi}| = \left( 4.9 \pm 0.5 \right) 10^{-3} \]
\[ |\eta_{+\pi}| = \left( 1.94 \pm 0.09 \right) 10^{-3} \]

The main trouble with the measurement of \( \eta_{00} \) is the possibility of a substantial background from the unknown, CP allowed reaction \( K_L^0 \rightarrow \pi^0 \gamma \gamma \): both the fact that the same number (within the experimental errors) has been found with two completely different techniques, which would have to be affected in a different way by this background, and the fact that the measured \( \gamma \)-ray spectrum agrees better with the \( K_L^0 \rightarrow \pi^0 \pi^0 \rightarrow 4\gamma \) spectrum than with the theoretically expected \( K_L^0 \rightarrow \pi^0 \gamma \gamma \rightarrow 4\gamma \) one, make it unlikely that the background from this reaction exceeds 10% of the found events\(^\text{12} \).

Up to now we have no direct information about the phase \( \varphi_{+-} \) of \( \eta_{+-} \): in fact all the \( K_S^0-K_L^0 \) interference experiments have been performed by means of a beam containing a regenerated \( K_S^0 \) component and this adds to \( \varphi_{+-} \) an unknown "regeneration phase" \( \varphi_\rho \); this latter phase can be expressed by means of the \( K^0 \) and \( \bar{K}^0 \) forward-scattering amplitudes on the nuclei, which are used as regenerators, as follows:

\[ \varphi_\rho = \arg i [\bar{f}(0) - f(0)]. \]

Some information on this number can be obtained by comparing \( \text{Im } f(0) \), \( \text{Im } \bar{f}(0) \) (obtained, by means of isospin invariance, from the known \( K^+ \) and \( K^- \) total cross-sections) and \( |\bar{f}(0) - f(0)| \), which is proportional to the regeneration amplitude.

Rubbia and Steinberger find in this way\(^\text{13} \):

\[ \cos \varphi_\rho = 1.06 \pm 0.09 \]
and
\[ \varphi_{+-} = 1.47 \pm 0.30 \]

while Bott-Bodenhausen et al.\textsuperscript{14} find:
\[ \cos \varphi_\rho = 0.94 \pm 0.15 \]
\[ \varphi_{+-} = 1.22 \pm 0.36. \]

The two independent determinations agree rather well, and also agree with the hypothesis that \( f(0) \) and \( \bar{f}(0) \) are mainly imaginary.

Considerable interest is being devoted to the search for CP, or T, violation in other decays. Amongst the \( K \) decays one of the most favourable cases is that of \( K_{\mu3} \): if we assume a current \times current weak semileptonic interaction (with or without intermediate boson), the mesonic fields appear in the matrix element as follows:

\[ < \pi(p_\pi) \mid J_\mu \mid K(p_K) > \]

where \( J_\mu \) is the strangeness-changing part of the weak current, which conserves parity.

From Lorentz invariance we can write this matrix element as:

\[ f_+(q^2) \left( p_K + p_\pi \right)_\mu + f_-(q^2) \left( p_K - p_\pi \right)_\mu \]

where \( f_+ \) and \( f_- \) are two unknown form factors, and \( q^2 = (p_K - p_\pi)^2 \): if the final state interaction (which is of electromagnetic order) is neglected, T invariance imposes that \( f_+ \) and \( f_- \) are relatively real (i.e. have the same phase). If T invariance does not hold one can have a non-vanishing mean value for quantities odd under T, in particular for the transverse muon polarizations

\[ P = \vec{\sigma}_\mu \cdot \left( \vec{p}_\pi \wedge \vec{p}_\mu \right) \]
where all vectors are measured in the $K$ rest system. Calculation shows that $P$ is proportional to $\text{Im} (f^*_+ f^*_-)$. 

A very precise measurement of $P$ has been performed by Young et al.\textsuperscript{15}, who find

$$P = 0.003 \pm 0.014$$

from which the relative phase of $f_+$ and $f_-$ is estimated to be less than $2.9^\circ$: to appreciate the sensitivity of this experiment we remember that the experiment of Overseth et al.\textsuperscript{16} on $\Lambda \to p \pi^-$ decay set an upper limit of $8^\circ$ on the relative phase of the $s$ and $p$ wave decay amplitudes, while the polarized neutron decay experiment by Burgy et al.\textsuperscript{17} set an upper limit of $10^\circ$ on the relative phase of the axial and vector contributions to $\beta$ decay. Although such numbers are rather small, they do not exclude the possibility of a $T$ violation, of the order of magnitude of the ratio between $K_L^0 \to 2\pi$ and $K_S^0 \to 2\pi$ amplitudes.

3. TENTATIVE EXPLANATIONS OF $K_L^0 \to 2\pi$ WITHOUT CP VIOLATION

Several models have been proposed to retain CP violation despite the existence of $K_L^0 \to 2\pi$ decays. The most interesting one was proposed independently by Bell and Perring\textsuperscript{18} and Bernstein, Cabibbo and Lee\textsuperscript{19} and postulates the existence of a very weak, long-range external field $V(x)$ which couples with opposite sign to particles and antiparticles: this field could be produced by the whole galaxy, in which matter is probably more common than antimatter, and the apparent CP-violating decay would be produced by a CP non-invariant surrounding space.

It is easy to show\textsuperscript{20} that in this hypothesis $K_S^0$ and $K_L^0$ may be written as:
\[ K_S^0 = (1 + \sqrt{1 + |\delta|^2}) (K_L^0 + \delta K_2^0) \]
\[ K_L^0 = (1 + \sqrt{1 + |\delta|^2}) (K_2^0 - \delta K_1^0) . \]

\( \delta \) is a parameter which depends on the external field as follows

\[ \delta \approx \frac{V}{(m_1 - m_2) + l(Y_1 - Y_2)} , \]

where \( V \) is the average over the space of the potential \( V(x) \). Calculation shows that, if \( V(x) \) is not a scalar field, \( \delta \) depends on the K energy in the rest frame of the galaxy\(^*\), in such a way that the rate \( K_L^0 \rightarrow \pi^+ \pi^- \)

would have to show a dependence on K energy of the form \( E^{2J} \), where \( J \) is the spin of the cosmological field.

Several experiments have been performed\(^**\) at different energies, and any such dependence is clearly ruled out.

The possibility \( J = 0 \) is also ruled out: indeed, this model with \( J = 0 \) predicts \( \eta_{+\pm} = \eta_{00} \), and \( \tan \varphi_{\pm} = -2(m_L - m_S) \gamma_S \); both predictions are in disagreement with experience\(^***\).

\(^*\) This apparent breakdown of Lorentz invariance is obviously due to the presence of the external source of the field \( V(x) \).

\(^**\) For instance V. Fitch et al.\(^21a\) find

\[
\text{Rate } K_2^0 \rightarrow \pi^+ \pi^- / \text{Rate } K_2^0 \rightarrow \text{all charged channels} = 1.97 \pm 0.8 \times 10^{-3}
\]

with a mean \( K^0 \) momentum of 1.5 GeV while X. de Bouard et al.\(^21b\) find 2.24 ± 0.45 with 10.7 GeV particles.

\(^***\) This possibility is not very appealing from a theoretical point of view, as it is difficult to couple a scalar field to particles and antiparticles with different signs.
Another proposal has been advanced, that the particle which decays into $2\pi$ is not the same particle of $K_L^0$, that is, there are three neutral $K$. But both mass and lifetime of the long-lived particle which decays into $2\pi$, and those of the usual $K^0_L$, agree completely, so that this explanation is very unlikely. Other explanations have been tried, to retain CP: a breakdown of Bose statistics (which however does not explain $K_L^0 \to \pi^0\pi^0$) and the possible existence of a light particle $X$, with $CP = -1$, such that the observed $K_L^0 \to 2\pi$ is, in reality, the double process $K_2^0 \to K_3^0 + X$, $K_4^0 \to 2\pi$ \cite{footnote23}. This model is ruled out by $K^0_S - K^0_L$ interference experiments. Still other possibilities have been tried \cite{footnote24}. It is clear, however, that at present the only convincing explanation of $K_L^0 \to 2\pi$ is in terms of CP violation.

4. POSSIBILITY OF STRONG OR ELECTROMAGNETIC VIOLATIONS OF CP

Another interesting possibility to explain the CP violating decay $K_L^0 \to 2\pi$ is to attribute it to a CP violation in the electromagnetic part $H_{\text{e.m.}}$ or in the strong part $H_{\text{st}}$ of the total Hamiltonian. $K_L^0 \to 2\pi$ would then occur through a second-order effect of type $H_{\text{wk}} H_{\text{e.m.}}$ or $H_{\text{wk}} H_{\text{st}}$: an estimate of the order of magnitude ($\alpha/\pi \approx 3 \times 10^{-3}$) makes the first possibility more appealing, but the second is also possible.

The latter possibility has been investigated by Prentki and Veltman\cite{footnote25}, Okun\cite{footnote26} and Lee and Wolfenstein\cite{footnote27}, while the former (CP violation in electromagnetic interactions), has been studied by
Bernstein, Feinberg and especially Lee*): both classes of models attribute CP violation to C violation, P conservation having been tested in low-energy nuclear physics up to the level of weak interactions*). Regarding violation by H_{e.m.}, a very important result is that low-energy nuclear processes are almost powerless to reveal any T violation (i.e. also any C violation if we believe in CPT and P conservation). The reason is the following. In nuclear physics one is always concerned with the matrix element of the electromagnetic current J_\mu between nuclear states, which to a good approximation can be written as the sum of matrix elements of J_\mu between states of single, almost free nucleons

< N(p + q) \mid J_\mu \mid N(p) > .

Such a matrix element can be written, by relativistic invariance, and for nucleons satisfying the free nucleon Dirac equation, as

\bar{u}(p + q) \left[ iF_1(q^2)\gamma_\mu + \frac{1}{m_p} F_2(q^2)(2p + q)_\mu + \frac{1}{m_p} F_3(q^2) q_\mu \right] u(p).

The requirement of hermiticity imposes

F_1 = F_1^* \quad F_2 = F_2^* \quad F_3 = -F_3^*

while T invariance would require all form factors to be real.

These relations are well known: however, we want to sketch their derivation. In a metric convention in which the three spatial components of a vector are real and the fourth is imaginary, hermiticity and T invariance impose

\begin{align*}
J_\mu^+ &= \epsilon_\mu J_\mu \\
J_\mu^T &= T J_\mu T^{-1} = -\epsilon_\mu J_\mu
\end{align*}

\begin{align*}
\epsilon_1 &= 1 \quad i = 1, 2, 3 \\
\epsilon_4 &= -1
\end{align*}

*) Some P violating effects have been observed for instance by F. Böhm and E. Kankeleit* but can be explained as due to weak interactions (a weak four-nucleon vertex, for instance).
Let us write the current operator in terms of fields and gradients of fields:

\[ J_\mu = i F_1 \bar{u} \gamma_\mu u + \frac{i}{m} F_2 \left[ (\partial_\mu \bar{u})u - \bar{u}(\partial_\mu u) \right] + \]
\[ + \frac{i}{m} F_3 \left[ (\partial_\mu \bar{u})u + \bar{u}(\partial_\mu u) \right]. \]

It is easy to compute \( J^+\) and \( J^T \)

\[ J^+ = -i F_1^* \bar{u} \gamma_\mu \gamma_5 u - \frac{i}{m} F_2^* \epsilon_\mu \left[ \bar{u}(\partial_\mu u) - u(\partial_\mu \bar{u}) \right] \]
\[ - \frac{i}{m} F_3^* \epsilon_\mu \left[ (\partial_\mu \bar{u})u + u(\partial_\mu \bar{u}) \right] = \]
\[ = i \epsilon_\mu F_1^* \bar{u} \gamma_\mu u + \frac{i}{m} F_2^* \epsilon_\mu \left[ (\partial_\mu \bar{u})u - \bar{u}(\partial_\mu u) \right] \]
\[ - \frac{i}{m} F_3^* \epsilon_\mu \left[ (\partial_\mu \bar{u})u + \bar{u}(\partial_\mu u) \right] \]
\[ J^T = -i F_1^* \epsilon_\mu \bar{u} \gamma_\mu u - \frac{i}{m} F_2^* \epsilon_\mu \left[ (\partial_\mu \bar{u})u - \bar{u}(\partial_\mu u) \right] \]
\[ - \frac{i}{m} F_3^* \epsilon_\mu \left[ (\partial_\mu \bar{u})u + \bar{u}(\partial_\mu u) \right] \]

from which our relations follow.

A check of \( T \) invariance (and not of hermiticity!) can therefore be performed only by looking at the term

\[ \frac{1}{m} F_3(q^2) q_\mu, \]

which is forbidden if \( T \) invariance holds. But current conservation by itself forbids this term, if both nucleons are on the mass shell, i.e. satisfy the free nucleon Dirac equation. Indeed, one then has

\[ 0 = \langle N(p + q)| \partial_\mu J_\mu |N(p) \rangle = \frac{1}{m} F_3(q^2) q^2 \bar{u}(p + q)u(p). \]
Hence any effect of T violation in nuclear physics must be due to
nucleon binding and must consequently be of the order of the ratio
of the nucleon binding energy to the nucleon mass ($\sim 10^{-2}$). Of course
in elementary particles physics one can consider the matrix element
of $J_\mu$ between states of very different masses, as for instance:

$$< N^* | J_\mu | p > \quad \text{or} \quad < A | J_\mu | E^0 >,$$

where $N^*$ is any nucleon isobar; such possible tests of T-invariance
have been extensively studied\textsuperscript{3)}, but no decisive experimental informa-
tion is available up to now.

In the same way we can consider the matrix element of $J_\mu$
between two boson states, e.g. $\pi^+(p)$ and $\pi^+(p + q)$. We can write

$$< \pi^+(p + q) | J_\mu | \pi^+(p) > = f_+(q^2) \left( 2p + q \right)_\mu + f_-(q^2) q_\mu,$$

where hermiticity implies $f_+ = f_+^*$, $f_- = -f_-^*$, and T imposes that
both $f_+$ and $f_-$ are real: again, however, $f_-$ is zero by current
conservation, if both $\pi$ are on the mass shell.

In a similar way, for strong interactions, it can be shown\textsuperscript{31)}
that the coupling of a neutral pseudoscalar meson to a baryon antibaryon
pair is T and C invariant, if it is P invariant. If a symmetry group
[isospin or SU(3)] holds, this property is true for the whole meson
multiplet (triplet or octet).

From an experimental point of view the best proof of C conservation
in strong interaction has been obtained from $p\bar{p}$ annihilation into mesons.

\textsuperscript{3}) $\Sigma^0 \rightarrow A e^+ e^-$ decay, which involves the matrix element $< A | J_\mu | E^0 >$
in the approximation of one photon exchange has been studied by
Bernstein, Feinberg and Lee\textsuperscript{2a}). Pion electroproduction through
a resonance [especially $N^* (1/2, \frac{1}{2})(1520)$], which involves the
matrix element $< N^* | J_\mu | p >$, was considered by N. Christ and
T.D. Lee\textsuperscript{30}).
Two main experiments have been performed: the most significant results are the following:

a) Pranzini et al.\textsuperscript{32)} combining data on all annihilation channels of $\bar{\nu}_p$ in pions find that, if $g$ is an effective coupling constant of the $C$ violating Hamiltonian, then $g/E_{\text{st}} < 10^{-2}$.

For the strange channels (in particular $\bar{K}^0 K^+ \pi^-$ and $K^0 K^- \pi^+$) the upper limit on this constant turns out to be

$$\frac{g}{E_{\text{st}}} < 2 \times 10^{-2}.$$  

b) A further experiment at CERN\textsuperscript{33)}, which studies particularly the strange $K^+ \bar{K}^0 \pi^-$ and $K^- \bar{K}^0 \pi^+$ channels, has been able to push the experimental limit to $10^{-2}$ also for the strange channels.

The importance of looking at strange channels depends on the already quoted theorem\textsuperscript{31}): indeed $C$ violation might possibly appear in strange particle couplings with a strength of the order of $10^{-1}$, characteristic of medium-strong interactions, while it might only appear in pure $\pi$-nucleon couplings at the level of isospin symmetry breaking.

The above numbers are not easy to interpret because we are in the unpleasant situation of not knowing clearly what "large" or "small" means. However, they seem to rule out the possibility of a large $C$ violation in strong interactions.

A reaction, which can show a $C$ violation in strong or electromagnetic interactions without distinguishing between them, is the decay $\pi^0 \to 3\gamma$. However, kinematical factors depress this decay very seriously, in such a way that, even if $C$ violation were maximal, the ratio $I(\pi^0 \to 3\gamma)/I(\pi^0 \to 2\gamma)$ would probably not exceed $10^{-6}$\textsuperscript{34}). The experimental limit is, at present $5 \times 10^{-6}$\textsuperscript{35}), so that we are not yet able to arrive at any conclusion.
A very interesting consequence of T violation at any level (strong, electromagnetic or weak) could be the existence of a static electric dipole moment of any particle with non-vanishing spin. Both P and T forbid this, so that the effect should involve the weak interactions to provide for P violation. If T were violated to a considerable degree by strong or electromagnetic interactions, an estimate of the order of magnitude would give for the electric dipole moment of the neutron (one chooses the neutron for this test for obvious experimental reasons):

\[ d = e \ell \approx 6 m_p \approx 4 \times 10^{-19} \text{ cm}. \]

This estimate is rough and over-optimistic; better ones, in the framework of some model, have been done\(^{36}\), and obtain numbers of the order of \( e \times (10^{-20} - 10^{-21} \text{ cm}) \).

The experimental limit is at present \( e \times 10^{-20} \text{ cm} \), but further work is in progress (see note added in proof, p.39).

5. \( \eta \) DECAYS

We now examine the \( \eta \) decays, which are so far the most studied process for testing C invariance of the electromagnetic and strong interactions. We recall that the \( \eta \) meson has mass 549 MeV and width \( < 10 \text{ keV} \). The known decays are

\[
\begin{align*}
\eta & \to \gamma \gamma \\
\eta & \to \pi^0 \gamma \gamma \\
\eta & \to \pi^0 \pi^0 \pi^0 \\
\eta & \to \pi^+ \pi^- \pi^0 \\
\eta & \to \pi^+ \pi^- \gamma.
\end{align*}
\]
The $3\pi$ decays violate G parity* and are therefore believed to be of electromagnetic nature. Also the other known decays, which all involve photons, are electromagnetic.

We now examine the isospin and C properties of the $3\pi$ decay states of $\eta$. For a $3\pi$ state, $G = C(-1)^I = -1$; hence (using $C = +1$ for $\eta$), we have the following possibilities

<table>
<thead>
<tr>
<th>C violating decays</th>
<th>C conserving decays</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I = 0$</td>
<td>$I = 1$</td>
</tr>
<tr>
<td>$C = -1$</td>
<td>$C = +1$</td>
</tr>
<tr>
<td>$I = 2$</td>
<td>$I = 3$</td>
</tr>
<tr>
<td>$C = -1$</td>
<td>$C = +1$</td>
</tr>
</tbody>
</table>

Two of the three pion states ($I = 1, 3$) allow completely symmetric isospin combinations of three pions so that the spatial wave function must also be completely symmetric, and no relative angular momentum is needed. On the contrary the $I = 0$ three-pion state is completely antisymmetric in isospin, and the spatial wave function must therefore also be completely antisymmetric. The simplest dependence of the decay matrix element on the $\pi$ energies $s, t, u$ compatible with this requirement is

$$
\frac{(u-t)(u-s)(s-t)}{M^3}
$$

where $M$ is a reference mass. This $s, t, u$ dependence depresses the contribution of this amplitude considerably, if we assume, as usual, $M$ to be of the order of the $\rho$-mass. As recently shown by Y. Fujii and C.L. Shaw³⁷), more complicated matrix elements give a more limited depression. On the same grounds the partial antisymmetry required by the $I = 2$

---

* One defines $G = C e^{i\pi I_2}$; if the strong interactions conserve $C$ and $I$, they conserve also $G$. It is easy to show by means of angular momentum wave functions that $e^{i\pi I_2} |\pi_0^+\rangle = - |\pi_0^-\rangle$, so that $G |\pi_0^+\rangle = - |\pi_0^-\rangle$. More generally a $|n \pi^+\rangle$ state has $G = (-)^n$, and for it $G$ can be rewritten as $C(-)^n$, so that $C$ of a $|n \pi^+\rangle$ state with $I = I_0$ is $C = (-)^{n+I_0}$. For $\eta, C = 1, I = 0$, hence $G = 1$. 
state depressed its contribution to a smaller degree. However, even if
C is maximally violated, I = 0, 2 states are not likely to contribute
substantially to the rates.

One further remark must be made on the I = 3 state. If we believe
in the usual assumption of the validity of a |ΔI| ≤ 1 selection rule
for the electromagnetic interaction [which is in fact not very well
founded experimentally] we see that I = 3 final state is not allowed
at first order in e^2, so that the ratio R = Π(3π^0)/Π(π^+π^-π^0) can be
calculated from I = 1 Clebsch-Gordan coefficients:

\[ R = 1.5 \times \text{phase space factor} \approx 1.7. \]

Recent experiments have given a smaller value, of the order of 0.6 \(^{38}\)
Such a severe disagreement would force us to introduce a large (≈ 30%)
fraction of I = 3 amplitude; whether this will oblige us to abandon
the |ΔI| ≤ 1 selection rule for electromagnetic interactions is not
yet clear.

We now proceed to discuss the test of C invariance in the
η → π^+π^-π^0 decay: if C is conserved, no asymmetry is possible in
the final state between π^+ and π^−, so that the asymmetry parameter A

\[ A = \frac{N_+ - N_-}{N_+ + N_-} \]

vanishes, where N_+ is the number of events with E(π^+) > E(π^-) and N_- the
number with E(π^-) > E(π^+) [E = pion energy in the η rest system].

The best experimental value, obtained in a CERN-ETH experiment, is
A = (0.3 ± 1) × 10^{-2} \(^{39}\). Depending on the model used, theoretical
estimates of how large A could be if C is violated vary from several
per cent to a fraction of 1%. The asymmetry would be generated by
the interference between a C violating amplitude A_ and a C conserving
one A_+, the interference term in |A_+ + A_-|^2 being

\[ 2 \text{Re}(A_+A_-^*) \].

\(^{*)}\) Experimental tests for this rule in connection with elementary
particles can be found in photoproduction and similar processes,
but they are not very accurate. The main evidence comes from
low-energy nuclear physics; however, as long as one considers
the nucleus as a collection of nucleons, the result is trivial
because the nucleons have I = 1/2.
Because of centrifugal barriers in $A_-$ (i.e. in the I = 0 and 2 states), we cannot hope that $\text{Re} \left( A_+ A_-^* \right) / \left[ |A_+|^2 + |A_-|^2 \right]$ would be larger than $\sim 10^{-4}$. In addition, if the decay interaction is a purely local one, CPT invariance implies that $A_+$ and $A_-$ are relatively imaginary, so that if $A_+$ is real,

$$A_-^* = -A_- \quad \text{and} \quad \text{Re} \left( A_+ A_-^* \right) = 0.$$ 

Non-vanishing interference only occurs because of final-state interaction, and we have to introduce a further factor $\sin(\delta_+ - \delta_-)$ where $\delta_+$ and $\delta_-$ are the final-state interaction phase shifts in the $C = +1$ and $C = -1$ decay states. The evaluation of this last factor is largely a matter of taste. The usual method is to assume the C violating decay to be dominated by a C violating $\eta\rho\pi$ vertex.

*) Many physicists have proposed a $\rho\pi\eta$ coupling as the source of C non-invariance. For the I = 0 decay state this coupling can be written

$$\tilde{H} = g_{\eta\rho\pi} \frac{i}{2} \left[ \eta(\partial_{\mu} \eta) - (\partial_{\mu} \eta)i \right]$$

and is isospin conserving. Some consequences of this assumption are studied for instance by Y. Fujii and G. Marx[40a], M. Nauenberg[40b] and S. Glashow and C. Sommerfeld[40c] and, with particular reference to the calculation of final state-interaction in $\eta \rightarrow 3\pi$ decay by Y. Fujii and G.L. Shaw[37]). This coupling allows also $\eta \rightarrow \pi^0\pi^0\pi^0$; the estimated rate is however rather low, as this is a second order effect, which involves a C violating and an electromagnetic vertex.
Two other $\eta$ decays have been investigated. The first is $\eta \to \pi^+ \pi^- \gamma$. For the $\pi^+\pi^-$, final states with $C = +1$ or $-1$ are possible. The $C = -1$ states, which conserve $C$ in the decay [because $C(\eta) = -C(\gamma) = 1$ as shown by the high rate for $\eta \to \gamma \gamma$ transition], correspond to odd $\pi^+\pi^-$ relative angular momentum, and they are mainly P states. The $C = +1$ states, which violate $C$, correspond to even $\pi^+\pi^-$ angular momentum; the most favourable state is an $L = 2$ state, as $\eta \to \pi^+ \pi^- \gamma$ with $\pi^+\pi^-$ in an S state is a $0 \to 0$ electromagnetic transition, and is therefore forbidden by Lorentz invariance and the fact that the real photons are transversal. Again a centrifugal factor depresses the $C$ violating amplitude, and again the interference term between $C = 1$ and $C = -1$ amplitudes vanishes, if there is no final-state interaction (these points are analogous to what happens in the $3\pi$ decays). The present upper limit for the asymmetry $A$ between $\pi^+$ and $\pi^-$ (defined as for the $3\pi$ decays) is

$$A = (1.5 \pm 2.5) \times 10^{-2}.\)$$

It was obtained in the CERN-ETH experiment mentioned above.

The last $\eta$ decay to be discussed is $\eta \to \pi^0 e^+ e^-$. This decay is allowed by $C$ conserving electromagnetic interactions only in second order in the fine structure constant, because the first order diagram is clearly $C$ violating.

An order of magnitude estimation has been calculated by Bernstein, Fainberg and Lee, who get a very large branching ratio, of the order of 30%. This result contains, as usual, an "interaction range", about which we know nothing, so that it can be easily wrong by an order of magnitude. The experimental limit of
\[
\frac{\Gamma(\pi^0 e^+ e^-)}{\Gamma(\text{all})} < 0.23 \times 10^{-2}
\]

has been published in 1966 \(^{42}\), and more recent work is lowering it further. Also here, however, one can have theoretical excuses for finding no violation, the simplest being the assumption that the C violating part of \( J_\mu \) obeys a \(|\Delta I| = 0\) selection rule, so that a \( \eta \rightarrow \pi^0 \) transition is forbidden.

For all such cases, the best practical attitude may be to attach to theoretical estimates no more than the crudest significance, and to realize that CP violation has been found, until now, at the very low level of a few parts per thousand. The same high level of accuracy may well be a naive but safe guess for the search for other violation effects.

6. **THE NEUTRAL K SYSTEM**

In this section we want to review the general description of the \( K_L^0 - K_S^0 \) system, and to see what the various symmetries imply \(^*\).

We recall the definition of \( K^0, \bar{K}^0 \) as simultaneous eigenstates of \( (H \_{\text{st}} + H \_{\text{e.m.}}) \) and \( S \) (strangeness), with eigenvalues \((m_{K^0}, +1)\) and \((m_{\bar{K}^0}, -1)\). Any coherent superposition of \( K^0, \bar{K}^0 \) can be described by a time-dependent two component state vector \(^{**}\)

\[
\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}
\]

in such a way that the system at the time \( t \) is represented by

\[
|\psi(t)\rangle = a(t) \ |K^0\rangle + b(t) \ |\bar{K}^0\rangle.
\]

In the absence of weak interactions \( a(t) \) and \( b(t) \) are of the form

\(^*\) This kind of analysis has been performed many times; see for instance: Lee and Wolfenstein\(^{27}\), Bell and Steinberger\(^{53}\), Lee and Wu\(^{20}\), and Wu and Yang\(^{43}\).

\(^{**}\) We work from now on in the K centre-of-mass system; then the time \( t \) is the proper time.
\[ a(t) = a(0) e^{-iK^0 t} \]

\[ b(t) = b(0) e^{-iK^0 t} \]

and \(|\psi(t)|^2 = |a(t)|^2 + |b(t)|^2\) is independent of \(t\). If we introduce weak interactions, we allow the neutral \(K\) to decay into different systems \((2\pi, 3\pi, \pi^+ \pi^- \nu, \pi^0 \pi^- \pi^+ \nu)\) so that \(|\psi(t)|^2\) is a decreasing function of time, which we normalize to \(|\psi(0)|^2 = 1\).

Following the classical Weisskopf and Wigner method\(^{44}\), which we review briefly in the Appendix, we can express the time derivative of \(\psi(t)\) in terms of \(\psi(t)\) by a linear equation of the form

\[ i \frac{d\psi}{dt} = (M - i\Gamma) \psi(t), \]

where \(M\) and \(\Gamma\) are two Hermitian \(2 \times 2\) matrices, whose expression can be written down in the usual perturbation theory, if we disregard third and higher order powers of \(H_{\text{int}}\) (in this approximation they are also time independent):

\[ \Gamma_{11} = \pi \sum_{\alpha} \left| \langle K^0 | H_{\text{int}} | \alpha > \right|^2 \delta(K^0 o - m^\alpha) \]

\[ \Gamma_{22} = \pi \sum_{\alpha} \left| \langle \bar{K}^0 | H_{\text{int}} | \alpha > \right|^2 \delta(K^0 o - m^\alpha) \]

\[ \Gamma_{12} = \Gamma_{21}^* = \pi \sum_{\alpha} \left< K^0 | H_{\text{int}} | \alpha > \right> < \alpha | H_{\text{int}} | \bar{K}^0 > \delta(K^0 o - m^\alpha) \]

\[ M_{11} = m_{K^0} + \left< K^0 | H_{\text{int}} | K^0 > \right> + \sum_{\alpha} \frac{\left| \langle K^0 | H_{\text{int}} | \alpha > \right|^2}{m_{K^0} - m^\alpha} \]

\[ M_{22} = m_{\bar{K}^0} + \left< \bar{K}^0 | H_{\text{int}} | \bar{K}^0 > \right> + \sum_{\alpha} \frac{\left| \langle \bar{K}^0 | H_{\text{int}} | \alpha > \right|^2}{m_{\bar{K}^0} - m^\alpha} \]

\[ M_{12} = M_{21}^* = \left< K^0 | H_{\text{int}} | \bar{K}^0 > \right> + \sum_{\alpha} \frac{< K^0 | H_{\text{int}} | \alpha > < \alpha | H_{\text{int}} | \bar{K}^0 >}{m_{K^0} - m^\alpha} . \]

(1)
In writing down these formulae we have assumed $m_K^0 = m_{K^0}$. If this is not true, one should include the difference $m_{K^0} - m_{K^0}^0$ in the perturbation $H_{wk}$. It will be found back in $M_{22}$ where it replaces $m_{K^0}$ by $m_{K^0}^0$.

The set of states $|\alpha\rangle$, over which we have to sum, is in principle any complete set of states to which $K^0$ and $\bar{K}^0$ are coupled by $H_{wk}$. We can assume the states $|\alpha\rangle$ to be eigenstates of $(H_{st} + H_{e.m.})$ which we choose to be outgoing wave states, that is we include the final-state interactions in their definition.

These states satisfy the equation:

$$(H_{st} + H_{e.m.})|\alpha\rangle = m_\alpha^0 |\alpha\rangle,$$

which defines $m_\alpha^0$.

The principal value $P$ is obtained, as usual, by considering in the sum only states with energy $m_\alpha^0$ such that $|m_\alpha^0 - m_{K^0}^0| > \epsilon$, and by taking the limit $\epsilon \to 0$.

We note also that $\Gamma_{11}$ and $\Gamma_{22}$ are, by definition, real and positive. It can be easily shown that, more generally, the matrix $\Gamma$ is positive definite, that is

$$\sum_{ij=1,2} c_i^* \Gamma_{ij} c_j$$

is real and positive for any pair of complex numbers $c_1$, $c_2$ not simultaneously vanishing.

Also, $M_{11}$ and $M_{22}$ are real, and they are positive because $m_{K^0}$ is positive and much larger than the other terms. However, $M_{11} - m_{K^0}$ and $M_{22} - m_{K^0}$ can have any sign.

The problem to be solved is to diagonalize the matrix $(M - i\Gamma)$, that is to find its eigenvectors and its eigenvalues: once this has been done, and we have the eigenvectors

$$|K_S^0\rangle = \begin{pmatrix} a_S \\ b_S \end{pmatrix}, \quad |K_L^0\rangle = \begin{pmatrix} a_L \\ b_L \end{pmatrix}$$
and the eigenvalues:

\[(M - i\Gamma) |K_S^0 > = (m_s - i\gamma_s) |K_S^0 >\]

\[(M - i\Gamma) |K_L^0 > = (m_L - i\gamma_L) |K_L^0 >,\]

we can write down the explicit time evolution of any state \( |\psi(0) > \), by decomposing it into:

\[|\psi(0) > = c_s |K_S^0 > + c_L |K_L^0 >,\]

and obtaining:

\[|\psi(t) > = e^{-i\gamma_s t} c_s |K_S^0 > + e^{-i\gamma_L t} c_L |K_L^0 >.\]

We can also write down explicitly \( < K_S^0 |K_L^0 > \), which can be different from 0 if CP is violated:

\[< K_S^0 |K_L^0 > = a_L a_S^* + b_L b_S^*.\]

It is convenient to write the eigenvectors of \((M - i\Gamma)\) as

\[|K_S^0 > = \left( \begin{array}{c} a_S \\ b_S \end{array} \right) = \frac{1}{\sqrt{2(1 + |\epsilon_1|^2)}} \left( \begin{array}{c} 1 + \epsilon_1 \\ 1 - \epsilon_1 \end{array} \right)\]

\[|K_L^0 > = \left( \begin{array}{c} a_L \\ b_L \end{array} \right) = \frac{1}{\sqrt{2(1 + |\epsilon_2|^2)}} \left( \begin{array}{c} 1 + \epsilon_2 \\ -1 - \epsilon_2 \end{array} \right)\]

and to introduce

\[\epsilon = \frac{1}{2} (\epsilon_1 + \epsilon_2)\]

\[\delta = \frac{1}{2} (\epsilon_1 - \epsilon_2).\]

If both \(\epsilon\) and \(\delta\) are small, their explicit expressions in terms of \(\Gamma_{ij}, M_{ij}\) and the eigenvalues are
\[ \epsilon \approx \frac{1}{2} \left( \frac{\text{Im} (\Gamma_{12}) + i \text{Im} (M_{12})}{(\gamma_S - \gamma_L) + i(m_S - m_L)} \right) \]  

and \[ <K_S^0|K_L^0> \approx 2[\text{Re } \epsilon - i \text{ Im } \delta]. \] We shall now show that CPT invariance implies \( \delta = 0 \).

7. **Consequences of CPT Invariance**

We have already defined \( \bar{K}^0 \) as

\[ |\bar{K}^0> = \text{CP}|K^0>. \]

As in general the operator \( T \), when applied to a state of a spin zero particle at rest, cannot change it into another state, we have also

\[ T|K^0> = e^{i\beta}|K^0>. \]

As the relative phase of \( K^0 \) and \( \bar{K}^0 \), which are orthogonal states, has no physical meaning, we can choose it in such a way that also the equality

\[ |\bar{K}^0> = \text{CPT}|K^0> \]

holds.

In fact let us start from two states \( K^0, \bar{K}^0 \) such that

\[ |\bar{K}^0> = \text{CP}|K^0> \]

but

\[ \text{CPT}|K^0> = e^{2i\alpha}|\bar{K}^0>, \alpha \neq 0. \]
We can define two new states $|K'\rangle$, $|\bar{K}'\rangle$ as follows:

$$|K'\rangle = e^{+i\alpha}|K\rangle$$

$$|\bar{K}'\rangle = e^{+i\alpha}|\bar{K}\rangle,$$

and for these new states both

$$CPT|K'\rangle = |\bar{K}'\rangle$$

and

$$CPT|K'\rangle = |\bar{K}'\rangle$$

hold.

To discuss the implications of CPT invariances let us briefly recall the properties of unitary and anti-unitary operators.

A unitary operator $U$ is a linear operator such that, for any pair of states $\varphi$, $\psi$ in Hilbert space, the following equality holds:

$$< \varphi | \psi > = < \varphi' | \psi' >$$

where

$$|\varphi'\rangle = U|\varphi\rangle \quad \text{and} \quad |\psi'\rangle = U|\psi\rangle.$$  

This is equivalent to the condition:

$$U^*U = 1,$$

where $U^*$ is the Hermitian conjugate of $U$. We recall that linearity of $U$ means

$$U(a|\varphi\rangle + b|\psi\rangle) = a|\varphi'\rangle + b|\psi'\rangle.$$  

This linearity is closely related to the identity $< \varphi | \psi > = < \varphi' | \psi' >$, because $< \varphi | \psi >$ is linear in $|\psi\rangle$. Examples of unitary operators in elementary quantum mechanics are, for instance, rotations and translations, and in general any operator of the form

$$0 = e^{i\alpha P},$$
where \( P \) is a Hermitian operator \( (P = P^\dagger) \) and \( \alpha \) a real number. Other examples are given by the discrete symmetry operators \( P, C \), which have also the property

\[
P^2 = C^2 = 1.
\]

On the contrary the time-reversal transformation \( T \) cannot be represented by a unitary operator: to justify this statement let us consider, as an example, the transition probability between a state \( \phi \) of a system at time \( 0 \) and a state \( \psi \) of the same system at time \( t \): this can be written in terms of the matrix element

\[
\langle \phi | 0(t) | \psi \rangle,
\]

where \( 0(t) \) is the time evolution operator of the system: if the Hamiltonian of the system is time independent, we can write it simply as:

\[
0(t) = e^{iHt}.
\]

A sensible definition of the time reversal operator \( T \) must satisfy the conditions:

i) to change \( t \) into \(-t\)

ii) to change \( H \) in the operator \( H' \) obtained by applying time reversal to momenta and spins contained in \( H \) (time-reversal invariance of \( H \) means \( H = H' \)).

As \( t \) is only a parameter in quantum mechanics, and therefore commutes with every operator, the simplest way of reaching our goal is to change \( i \) into \(-i\) in \( \exp(iHt) \), that is to perform a complex conjugation. Then our operator should satisfy the so-called anti-linearity property

\[
T a | \psi \rangle = a^\ast T | \psi \rangle,
\]

and therefore it cannot be a linear, nor in particular a unitary operator. One selects for \( T \) an anti-unitary operator.
We define an anti-unitary operator $A$ by means of the following conditions:

i) It must be anti-linear, i.e.

$$A(a|\varphi> + b|\psi>) = a^*|\varphi'> + b^*|\psi'>$$

where $|\varphi'> = A|\varphi>$ and $|\psi'> = A|\psi>$. 

ii) It must satisfy $<\psi|\varphi> = <\varphi'|\psi'>$. Anti-linearity is closely connected with the latter condition, because $<\psi|\varphi>$ is linear in $\varphi$ and anti-linear in $\psi$. [Remember $<\psi|\varphi> = <\varphi|\psi>^*$.] 

We now discuss the effect of $A$ on an operator $0$ describing an observable. We define $0'$, the transformed operator of $0$ by means of $A$, by the condition that for all $|\varphi>$

$$0'|\varphi'> = |(0\varphi)'>$$

where $|\varphi'> = A|\varphi>$, $|(0\varphi)'> = A|0\varphi>$. This gives $0'A = A0$, i.e.

$$0' = A0A^{-1}.$$ 

For matrix elements one has

$$<\varphi|0|\psi> = <\varphi'|0'|\psi'>^*$$

where again $|\varphi'> = A|\varphi>$, $|\psi'> = A|\psi>$. 

Just as $T$, CPT is an anti-linear operator: to assume CPT invariance means to assume $H = H'$, where $H' = (CPT)H(CPT)^{-1}$, and $H$ is the total Hamiltonian of our system. As mentioned above, CPT invariance of $H$ can be derived from Lorentz invariance and locality in axiomatic field theory (CPT theorem).

In what follows we shall actually assume that $H_{st} + H_{e.m.}$ and $H_{wk}$ are separately invariant under CPT, while the CPT theorem in principle states only that

$$CPT(H_{st} + H_{e.m.} + H_{wk})(CPT)^{-1} = H_{st} + H_{e.m.} + H_{wk}.$$
The question has been raised as to whether one could have

\[ \text{CPT}(H_{\text{st}} + H_{\text{e.m.}}) (\text{CPT})^{-1} = H_{\text{st}} + H_{\text{e.m.}} + \Delta H_1 \]

\[ \text{CPT} H_{\text{wk}} (\text{CPT})^{-1} = H_{\text{wk}} - \Delta H_2 \]

and

\[ \Delta H_1 = \Delta H_2 \]

from the invariance of the total Hamiltonian. At this point we have to be rather careful about the precise meaning given to the various parts of the total Hamiltonian. A first possibility occurs if we are able, in the framework of a field theory, to write down an explicit form of the Hamiltonian in terms of local fields, as we think to be the case, for example, for $\mu$ decay. In this case, any term of the Hamiltonian which is local and is a scalar under the Lorentz transformations is CPT invariant according to the CPT theorem, so that $\Delta H_1 = \Delta H_2 = 0$. Another possibility is that one cannot write down explicit expressions for the various parts of the Hamiltonians in terms of field; then we can distinguish the parts only be means of selection rules. For instance, let us use the fact that $H_{\text{st}} + H_{\text{e.m.}}$ is a part of the total Hamiltonian which conserves parity $P$ and strangeness $S$. Then $\Delta H_1 = \Delta H_2$ has to conserve $P$ and $S$. Re-applying CPT to the above equations and using $(\text{CPT})^2 = 1$, one finds $(\text{CPT}) \Delta H_1 (\text{CPT})^{-1} = -\Delta H_1$. One can then redefine $H_{\text{st}} + H_{\text{e.m.}}$ as $H_{\text{st}} + H_{\text{e.m.}} + \frac{1}{2} \Delta H_1$ and $H_{\text{wk}}$ as $H_{\text{wk}} - \frac{1}{2} \Delta H_1$, both operators being now CPT invariant. For example,

\[ (\text{CPT})H_{\text{wk}} (\text{CPT})^{-1} = H_{\text{wk}} - \Delta H_2 + \frac{1}{2} \Delta H_1 = H_{\text{wk}} - \frac{1}{2} \Delta H_1 \]

We see that, again, we are led to a situation where we can put $\Delta H_1 = 0$.

Thus, our assumption of separate CPT conservation by $H_{\text{st}} + H_{\text{e.m.}}$ and $H_{\text{wk}}$ is not more stringent in an essential way than the assumption of conservation by the complete Hamiltonian, and we use it hereunder.
We want now to show that CPT invariance implies

\[ \epsilon_1 = \epsilon_2 \]

i.e.

\[ \delta = 0 \]

in the notation of page 24. As a first result we have the equality \( m^0_\alpha = m^0_{\beta} \), as \( H_{st} + H_{o,m} \) commutes with CPT, so that we can use the formulae (1), page 22, for \( \Gamma_{ij} \) and \( M_{ij} \). As a second step, we see by CPT invariance of \( H_{wk} \) that:

\[ \langle K^0 | H_{wk} | \alpha \rangle = \langle \bar{K}^0 | H_{wk} | \alpha' \rangle^* = \langle \alpha' | H_{wk} | \bar{K}^0 \rangle \]

(3)

where

\[ |\alpha'\rangle = \text{CPT} |\alpha\rangle, \]

and we have used the hermiticity of \( H_{wk} \).

The set of states \( |\alpha'\rangle \) is again a set of \( (H_{st} + H_{o,m}) \) eigenstates, which have the same eigenvalues \( m^0_{\alpha} \). In fact by our assumptions

\[
\text{CPT}(H_{st} + H_{o,m}) |\alpha\rangle = |[(H_{st} + H_{o,m}) \alpha]\rangle = \\
= (H_{st} + H_{o,m})' |\alpha\rangle = (H_{st} + H_{o,m}) |\alpha'\rangle
\]

so that the eigenvalue equation:

\[
(H_{st} + H_{o,m}) |\alpha\rangle = m^0_{\alpha} |\alpha\rangle
\]

becomes

\[
(H_{st} + H_{o,m}) |\alpha'\rangle = m^0_{\alpha} |\alpha'\rangle.
\]

The new set of states \( |\alpha'\rangle \) is exactly on the same footing as \( |\alpha\rangle \), apart from the fact that it contains incoming states instead of outgoing ones. The new set is complete and orthonormal, similar to the set \( |\alpha\rangle \), so from Eq. (1), one obtains, by using Eq. (3)
\[ \Gamma_{11} = \Gamma_{22} \]

and

\[ M_{11} = M_{22} \]

as of course CPT invariance implies also

\[ < K^0 | H_{\text{wk}} | K^0 > = < \bar{K}^0 | H_{\text{wk}} | \bar{K}^0 > \ . \]

On the contrary it is easy to see that CPT invariance imposes no restriction on the off-diagonal elements. In fact let us examine for instance

\[ \Gamma_{12} = \pi \sum_{\alpha} < K^0 | H_{\text{wk}} | \alpha > < \alpha' | H_{\text{wk}} | \bar{K}^0 > . \]

Using CPT invariance of \( H_{\text{wk}} \) we can write it

\[ \Gamma_{12} = \pi \sum_{\alpha'} < \bar{K}^0 | H_{\text{wk}} | \alpha' > * < \alpha' | H_{\text{wk}} | K^0 > * , \]

where, as usual, \( | \alpha' > = \text{CPT} \ | \alpha > . \)

As \( H_{\text{wk}} \) is Hermitian, we get

\[ \Gamma_{12} = \pi \sum_{\alpha'} < K^0 | H_{\text{wk}} | \alpha' > < \alpha' | H_{\text{wk}} | \bar{K}^0 > , \]

which, because of the completeness of the sets \( | \alpha > \) and \( | \alpha' > \), is identical to the original form of \( \Gamma_{12} \).

Under the assumption of CPT invariance the matrix \( (M - i\Gamma) \) is then of the form:

\[ \)

*) This is already enough to assure the vanishing of the approximate expression for \( \delta \), we gave on page 25.
\[
\begin{pmatrix}
    c_{11} & c_{12} \\
    c_{21} & c_{11}
\end{pmatrix},
\]

and the eigenvalues are determined by the secular equation:

\[(c_{11} - \lambda)^2 = c_{21} c_{12}\]

which implies:

\[\lambda = c_{11} \pm \sqrt{c_{21} c_{12}}.\]

Let us choose \(\lambda_S = c_{11} - \sqrt{c_{21} c_{12}}\) and \(\lambda_L = c_{11} + \sqrt{c_{21} c_{12}}\), and define \(X = \sqrt{c_{21} c_{12}}\). Then:

\[
\begin{pmatrix}
    X & c_{12} \\
    c_{21} & X
\end{pmatrix}
\begin{pmatrix}
    a_S \\
    b_S
\end{pmatrix} = 0
\]

\[
\begin{pmatrix}
    -X & c_{12} \\
    c_{21} & -X
\end{pmatrix}
\begin{pmatrix}
    a_L \\
    b_L
\end{pmatrix} = 0
\]

so that \(a_S/b_S = a_L/b_L\) or, in the previous notations

\[
\frac{1 + \varepsilon + \delta}{1 - \varepsilon - \delta} = \frac{1 + \varepsilon - \delta}{1 - \varepsilon + \delta},
\]

which implies \(\delta = 0\).

As another application of CPT invariance, we discuss the phase properties of the matrix elements \(2\pi|H_{\nu k}|K^0\rangle\) and \(2\pi|H_{\nu k}|\bar{K}^0\rangle\). Without spoiling the relations \(|K^0\rangle = \text{CP}|\bar{K}^0\rangle = \text{CPT}|K^0\rangle\), we are free to fix the relative phase of the \(K^0-\bar{K}^0\) system with respect to a non-strange system, so we are free to choose, as most authors do:

\[
<2\pi(I = 0)|H_{\nu k}|K^0\rangle = A_0 \exp[i\delta_0(m_K^2)]
\]

where \(\delta_0(m_K^2)\) is the strong interaction \(\pi-\pi\) phase shift in the \(S\)-wave with isospin \(I = 0\) at a centre-of-mass energy equal to the kaon mass, and \(A_0\) is a real parameter. Then, if CPT invariance holds,
\[ <2\pi(I = 0)|H_{wk}|\bar{K}^c> \text{ is again } A_0 e^{i\delta_0(m_K^2)} . \]

In the same way we can define a parameter \( A_2 \) such that
\[ < 2\pi(I = 2)|H_{wk}|\bar{K}^c > = A_2 e^{i\delta_2(m_K^2)} , \]
where \( \delta_2 \) is analogous to \( \delta_0 \) for the \( I = 2 \) channel. If CPT is conserved, one has
\[ < 2\pi(I = 2)|H_{wk}|\bar{K}^c > = A_2^* e^{i\delta_2(m_K^2)} . \]

If CP is violated, this amplitude can be different from the previous one, that is
\[ \text{Im } A_2 \neq 0 . \]

For future use we define also
\[ \epsilon' = \frac{i}{\sqrt{2}} \frac{\text{Im } A_2}{A_0} e^{i[\delta_2(m_K^2) - \delta_0(m_K^2)]} . \]

It is easy to prove that, if \( \epsilon \) and \( \epsilon' \) are small parameters, the approximate equalities
\[ \eta_+ = \epsilon + \epsilon' \quad \eta_{cc} = \epsilon - 2\epsilon' \]
hold\(^{43}\).

8. CONSEQUENCES OF CP INVARIANCE

Let us see briefly what would be the consequence of CP invariance of \( H_{st} + H_{e.m.} \) and \( H_{wk} \). Defining \( |\alpha'> = CP|\alpha> \), and using the CP invariance of \( H_{st} + H_{e.m.} \), we see again that the set \( |\alpha'> \) is a set of \( H_{st} + H_{e.m.} \) eigenstates, with the same eigenvalues \( m_0 \). As also CP interchanges \( K^c \) and \( \bar{K}^c \), we have again the two consequences we had from CPT:
\[ \Gamma_{11} = \Gamma_{22}, \quad M_{11} = M_{22} . \]

*) One uses the property CPT \[ 2\pi(I) = \exp (2i\delta) 2\pi(I) > \text{ which holds for outgoing wave states } |2\pi(I) > \.]
But this time CP is a unitary, and not an anti-unitary operator, so we obtain restrictions also on the non-diagonal elements of \((M - i\Gamma)\). Let us consider, for instance, \(\Gamma_{12}\)

\[
\Gamma_{12} = \pi \sum_{\alpha} <K^0|H_{wk}\alpha > <\alpha|H_{wk}|\bar{K}^0 > \delta (m_{K^0} - m_{\alpha}^0).
\]

Applying CP invariance, we now have

\[
\Gamma_{12} = \pi \sum_{\alpha'} <\bar{K}^0|H_{wk}\alpha' > <\alpha'|H_{wk}|K^0 > \delta (m_{\bar{K}^0} - m_{\alpha'}^0) = \Gamma_{21}.
\]

We knew already, from the hermiticity of \(\Gamma\), that \(\Gamma_{12} = \Gamma_{21}^*\), so the result is that \(\Gamma_{12}\) must be real, if CP invariance holds.

In exactly the same way one finds also that \(M_{12}\) is real.

The matrix \((M - i\Gamma)\) is then of the form:

\[
\begin{pmatrix}
 a & b \\
 b & a
\end{pmatrix}
\]

\(a\) real, \(b\) complex.

The eigenvalue problem then gives immediately the well known result

\[
K_S^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{K^0 + \bar{K}^0}{\sqrt{2}}, \quad K_L^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{K^0 - \bar{K}^0}{\sqrt{2}}.
\]

9. SOME MODELS OF CP VIOLATION

We have already discussed some particular models which could explain the existence of the \(K_L^0 \to 2\pi\) decays, in particular some models which try to save CP conservation by assuming a long range force, and models attributing \(K_L^0 \to 2\pi\) to a C violating part in the strong or electromagnetic interactions. The two further models to be discussed now are best introduced as further assumptions about the matrix \((M - i\Gamma)\) or its eigenvectors.
9.1 The superweak model\textsuperscript{27,45)}

We have seen that CP invariance imposed on the elements of the matrix \( M - iI \) two restrictions which are not imposed by CPT, namely

\[
M_{12} = M_{21}^* \quad \text{and} \quad \Gamma_{12} = 
\Gamma_{21}^* .
\]

One of the most economical ways of introducing CP violation would be to abandon one of these restrictions, in particular to take

\[
M_{12} \neq M_{21}^* \quad \text{but} \quad \Gamma_{12} = \Gamma_{21}^* .
\]

This assumption is more economical than the other one: in fact, if \( \Gamma_{12} \neq \Gamma_{21}^* \), some matrix element \( \langle K^0 | H_{\text{wk}} | \alpha \rangle \) must violate CP invariance, and one would therefore expect that this violation would also manifest itself in \( M_{12} \), so that we would need additional assumptions to explain why it is not so.

On the contrary \( M_{12} \) can have an imaginary part if \( \langle K^0 | H_{\text{wk}} | \bar{K}^0 \rangle \) is different from zero and is not real; so we can postulate the existence of a "superweak" interaction, which allows transitions with \( \Delta S = 2 \), and is CP violating. This new interaction would be so weak that its second order effects are completely negligible in comparison with second order effects of the normal, CP conserving weak interactions. A specific model is the one of Wolfenstein\textsuperscript{45)}, which, in the framework of the current \( \times \) current weak non-leptonic interactions, introduces a small \( \Delta S = -\Delta Q \) part of the current, out of phase with respect to the \( \Delta S = \Delta Q \) part. In this sense, his model is similar to the Sachs one\textsuperscript{46)}.

In the Wolfenstein model there is no sizeable CP violating effect in the \( K^0 \) and \( \bar{K}^0 \) decays, so that the parameter \( \epsilon' \) is \( \sim 0 \). \( K_L^0 \) can decay into \( 2\pi \) only because it contains a small mixture of \( K_L^0 = (K^0 + \bar{K}^0)/\sqrt{2} \), namely

\[
K_L^0 = N(K_2^0 + \epsilon K_1^0), \quad K_2^0 = \frac{1}{\sqrt{2}} (K^0 - \bar{K}^0), \quad N = \frac{1}{\sqrt{1 + |\epsilon|^2}} .
\]
It is then clear that in this model

\[ \eta_{+-} = \eta_{00} = \epsilon . \]

This formula, together with the expression (2) for \( \epsilon \), which becomes in our case

\[ \epsilon \approx \frac{i}{2} \frac{\text{Im}(M_{\ell \ell})}{(\gamma_S - \gamma_L) + i(m_S - m_L)} , \]

and the experimentally known values of \( |\eta_{+-}|, \gamma_S, \gamma_L, |m_S - m_L| \), allows us to estimate the order of magnitude of the coupling constant of this new "superweak" interaction. As in this model \( \epsilon \) is of the order of \( 10^{-3} \), and \( (\gamma_S - \gamma_L) \) and \( m_S - m_L \) are both effects which arise from second order in weak interactions, we see that the first order matrix element of the superweak interaction must be three orders of magnitude smaller than second order matrix elements of usual weak interaction. Then it is so small that no experimentally observable effects are expected to be detected outside the K system. On the contrary a charge asymmetry in the leptonic decays of the long-lived \( K_L^0 \) is allowed, and would have to be:

\[ \frac{\Gamma(K_L^0 \to \pi^+ \ell^- \bar{\nu}) - \Gamma(K_L^0 \to \pi^- \ell^+ \nu)}{\Gamma(K_L^0 \to \pi^+ \ell^- \bar{\nu}) + \Gamma(K_L^0 \to \pi^- \ell^+ \nu)} = -2 \text{Re} \, \epsilon = -2 \text{Re} \, \eta_{+-} \approx -3 \times 10^{-3} \] (4)

Also the phases of \( \eta_{+-} \) and \( \eta_{00} \) are predicted; they would have to be equal and such that

\[ \text{tg} \, [\arg(\eta_{+-})] = \left( \frac{m_S - m_L}{\gamma_S - \gamma_L} \right) \approx 1 . \]

Unfortunately, this very simple model, which would explain also the failure of the search for CP and P violation in processes which do not involve neutral kaons, and also the lack of transverse \( \mu \) polarization in \( K_{\mu3} \) decays, is in disagreement with the recent measurement of \( |\eta_{00}| \) which is found to be definitely different from \( |\eta_{+-}| \) (see above).
9.2 Models with "phase-mismatch"

A very interesting question has been recently considered, in particular by Mathur, namely: what are the states CPT $|K_S^0>$, CPT $|K_L^0>$? As in our definitions CPT $|K^0> = |K^0>$, CPT $|\bar{K}^0> = K^0$, also CPT $|K_S^0>$, CPT $|K_L^0>$ must be linear superpositions of $K^0$ and $\bar{K}^0$. They can be general superpositions, but it is interesting to study the simple assumption that $|K_S^0>$ and $|K_L^0>$ are eigenstates of CPT, i.e. that

$$\text{CPT} |K_S^0> = e^{i\alpha_S} |K_S^0>$$
$$\text{CPT} |K_L^0> = e^{i\alpha_L} |K_L^0>$$

with $\alpha_S, \alpha_L$ real because CPT preserves the norm of each state. If we write down explicitly:

$$|K_S^0> = \left[ (1 + \epsilon)|K^0> + (1 - \epsilon)|\bar{K}^0> \right] \cdot \left[ 2(1 + |\epsilon|^2) \right]^{-1/2}$$

the first equation above gives

$$e^{i\alpha_s}[(1 + \epsilon)|K^0> + (1 - \epsilon)|\bar{K}^0>] = (1 + \epsilon^*)|K^0> + (1 - \epsilon^*)|\bar{K}^0>,$$

which implies:

$$\alpha_S = 0 \quad \epsilon^* = -\epsilon.$$ 

In exactly the same way we find for

$$|K_L^0> = \frac{(1 + \epsilon)|K^0> - (1 - \epsilon)|\bar{K}^0>}{\sqrt{2(1 + |\epsilon|^2)}}$$

(we have assumed CPT invariance)

that one has

$$\alpha_L = \pi \quad \text{and again} \quad \epsilon^* = -\epsilon.$$ 

So we obtain the very important result
Re $\epsilon = 0$,

which implies the orthogonality

$$< K_S^0 | K_L^0 > = 0.$$

This is obvious, as now $K_S^0$ and $K_L^0$ are eigenvectors of CPT with eigenvalues $\pm 1$. The orthogonality of $K_S^0$ and $K_L^0$ implies some very interesting consequences: for instance their decay states will also be orthogonal because the time evolution preserves orthogonality. Hence the total decay rate of any initial neutral $K$ system is the sum of two exponentials, without any interference term. This does not contradict the observed interference in the $(\pi^+\pi^-)$ decay mode; it would be compensated by an out of phase interference in the $(\pi^0\pi^0)$ channel.

In fact, to exemplify, let us assume that $K_S^0$ decays into a $2\pi$ state with pure $I = 0$. Then the orthogonality condition taken in the $2\pi$ channel, which reads

$$< \pi\pi \text{ from } K_S^0 | \pi\pi \text{ from } K_L^0 > = 0,$$

implies that $K_L^0$ decays only into the $I = 2$ $2\pi$-state. It is easy to show that this implies

$$\eta_{oo} = -2\eta_{+-},$$

so that no interference in the total $K_L^0$, $K_S^0 \rightarrow 2\pi$ decay rate is observable. We remark also that this relation is completely compatible with the present data. As a last remark on this class of models we want to recall that it implies the vanishing of charge asymmetry in the leptonic decays

$$K_L^0 \rightarrow \pi^+ \ell^- \bar{\nu} \quad \text{and} \quad K_L^0 \rightarrow \pi^- \ell^+ \nu,$$

also if the $\Delta S = \Delta Q$ rule is not valid. In fact, for a fixed kinematical configuration, let us call $f$, $g$, $\bar{f}$ and $\bar{g}$ the amplitudes for the decays:
\[ K^0 \rightarrow \pi^- \ell^+ \nu = f \]
\[ K^0 \rightarrow \pi^+ \ell^- \bar{\nu} = g \]
\[ \bar{K}^0 \rightarrow \pi^+ \ell^- \bar{\nu} = \bar{f} \]
\[ \bar{K}^0 \rightarrow \pi^- \ell^+ \nu = \bar{g} \, . \]

Then, neglecting the final state interaction between \( \pi \) and \( \ell \) (which is electromagnetic), CPT implies\(^{43}\) \( \bar{f} = -f^* \), \( \bar{g} = -g^* \), while the \( \Delta S = \Delta Q \) rule would imply \( g = \bar{g} = 0 \), and CP invariance would require \( f = -\bar{f} \), \( g = -\bar{g} \).

The amplitudes for the decay of \( K^0_L \rightarrow \pi^+ \ell^- \bar{\nu} \), \( K^0_L \rightarrow \pi^- \ell^+ \nu \) are:

\[ A(K^0_L \rightarrow \pi^+ \ell^- \bar{\nu}) = (1 + \epsilon)g - (1 - \epsilon)f \]
\[ A(K^0_L \rightarrow \pi^- \ell^+ \nu) = (1 + \epsilon)f - (1 - \epsilon)g \, . \]

Using the above implications of CPT and our result \( \text{Re} \, \epsilon = 0 \), we easily see that

\[ |A(K^0_L \rightarrow \pi^+ \ell^- \bar{\nu})|^2 = |A(K^0_L \rightarrow \pi^- \ell^+ \nu)|^2 \]

for any kinematical configuration.

The models just discussed can be characterized by a "phase-mismatch" between the \( K^0, \bar{K}^0 \) states as they appear in production processes and in kaon decay. Indeed, since \( \epsilon \) is purely imaginary, one can write with a real phase \( \varphi \)

\[ \sqrt{2} \, |K^0_S > = e^{i\varphi} |K^0 > + e^{-i\varphi} |\bar{K}^0 > , \]
\[ \sqrt{2} \, |K^0_L > = e^{i\varphi} |K^0 > - e^{-i\varphi} |\bar{K}^0 > ; \]

except for the extra phases \( e^{\pm i\varphi} \) in front of the states \( |K^0 >, |\bar{K}^0 > \), these equations are identical with those obtained for the case of CP invariance, see section 8, p. 34.
Notes added in proof (15 September, 1967):

1. New upper bounds have been published for the electric dipole moment of the neutron (see p. 16). They are:

   \[ d = ( -2 \pm 3 ) \cdot 10^{-22} \text{ cm}, \quad \text{P.D. Miller et al., Phys. Rev. Letters 12, 381 (1967).} \]

   \[ d = (+2.4 \pm 3.9) \cdot 10^{-22} \text{ cm}, \quad \text{C.G. Shull and R. Nathans, Phys. Rev. Letters 12, 384 (1967).} \]

2. At the International Conference on Electron and Photon Interactions, Stanford, 5-9 September, 1967, Dorfan and Saal reported the results of experiments done at SLAC and Brookhaven, respectively, which measured a positive asymmetry in the leptonic $\pi^0\nu$ decay of $K^0_L$. They give for the asymmetry (4) on p. 36 a positive number of the same order of magnitude as mentioned in Eq. (4). This result is in disagreement with the phase-mismatch models.
APPENDIX

THE WEISSKOPF-WIGNER METHOD

Let us discuss the problem of two degenerate or "quasi-degenerate" eigenstates \( K^0 \) and \( \bar{K}^0 \), of a Hamiltonian \( H_0 \), or more generally \( |1> \) and \( |2> \) (we will see later what "quasi-degenerate" means), in the presence of an interaction \( H_I \) which allows them to decay into a common set of decay states \( |\alpha> \). The method used is a straightforward extension of the Weisskopf-Wigner method which dealt with the case of a single state \(|1>\). We write down the general time-dependent wave function of the system as:

\[
|\psi(t)> = a_1(t)|1> + a_2(t)|2> + \sum_\alpha a_\alpha(t)|\alpha>
\]

The decay states \(|\alpha>\) have the same spatial three-momentum of the initial state (e.g., zero momentum in the rest system), but not necessarily the same energy. These states can be chosen to be eigenstates of \( H_0 \): if we introduce an index \( \rho \), which runs through 1, 2, and all \( \alpha \), we can write

\[
H_0 |\rho> = m_\rho |\rho> ,
\]

where \( m_\rho \) is the "effective mass" (we assume we are working in the rest system). Let us write down the Schrödinger equation

\[
i \frac{d}{dt} |\psi(t)> = H|\psi(t)> , H = H_0 + H_I
\]

from which we get*:

\[
i \frac{d}{dt} a_\rho(t) = \sum_\rho' <\rho|H|\rho'> a_{\rho'}(t)
\]

\[
= m_\rho a_\rho(t) + \sum_\rho' <\rho|H_I|\rho'> a_{\rho'}(t).
\]

*) We have chosen the set \(|1>, |2>, |\alpha>\) to be a set of normalized and orthogonal states.
We go over to the so-called interaction representation: i.e., we define a new set of coefficients

\[ b_\rho(t) = a_\rho(t) e^{i \rho t}. \]

In this representation the Schrödinger equation becomes

\[ i \frac{\partial}{\partial t} b_\rho(t) = \sum_{\rho'} \bar{W}(\rho, \rho') e^{i \Delta(\rho, \rho') t} b_{\rho'}(t) \]

where

\[ \bar{W}(\rho, \rho') = < \rho | H_I | \rho' > \]

\[ \Delta(\rho, \rho') = m_\rho - m_{\rho'} . \]

Since \( H_I \) is small, the equation can now be solved by perturbation theory in terms of the initial values \( b_\rho(0) \): the zeroth, first and second order solutions in \( H_I \) are:

\[ b_\rho^{(0)}(t) = b_\rho(0) \]

\[ b_\rho^{(1)}(t) = b_\rho(0) - i \sum_{\rho'} \int_0^t dt' e^{i \Delta(\rho, \rho') t'} \bar{W}(\rho, \rho') b_{\rho'}(0) \]

\[ b_\rho^{(2)}(t) = b_\rho^{(1)}(t) + \sum_{\rho', \rho''} \int_0^t dt' \int_0^{t'} dt'' e^{i \Delta(\rho, \rho') t'} e^{i \Delta(\rho', \rho'') t''} \bar{W}(\rho, \rho') \bar{W}(\rho', \rho'') b_{\rho''}(0), \]

which become, after the integrations are performed:
\[ b^{(1)}_{\rho}(t) = b_{\rho}(0) - \sum_{\rho'} \frac{e^{-i\Delta(\rho,\rho')t}}{\Delta(\rho,\rho')} W(\rho,\rho') b_{\rho'}(0) \]

\[ b^{(2)}_{\rho}(t) = b_{\rho}(0) - \sum_{\rho'} \frac{e^{-i\Delta(\rho,\rho')t}}{\Delta(\rho,\rho')} W(\rho,\rho') b_{\rho'}(0) + \]

\[ + \sum_{\rho'} \sum_{\rho''} \left( \frac{e^{-i\Delta(\rho,\rho')t}}{\Delta(\rho,\rho')} - \frac{e^{-i\Delta(\rho,\rho'')t}}{\Delta(\rho,\rho'')} \right) \frac{W(\rho,\rho') W(\rho',\rho'')}{\Delta(\rho',\rho'')} b_{\rho''}(0). \]

We remark that these expressions are not singular when masses \( m_{\rho} \) become equal. For example, when \( m_{\rho'} = m_{\rho''} \); i.e., \( \Delta(\rho',\rho'') = 0 \), the difference in brackets gives a zero which compensates the vanishing of \( \Delta(\rho',\rho'') \) in the denominator. It is good at this stage to use a familiar mathematical trick to avoid divergences in individual terms when some values of \( m_{\rho} \) become equal. One substitutes

\[ \Delta(\rho,\rho') \rightarrow \Delta(\rho,\rho') + i\epsilon, \]

thereby making all denominators \( \neq 0 \), and one postpones taking the limit \( \epsilon = 0 \) at the end of the calculation. The sign of \( \epsilon \) is fixed by the requirement that the time-dependent exponentials like \( \exp[it \{ \Delta(\rho,\rho') + i\epsilon \}] \), over which one has to integrate, do not blow up even when \( t \rightarrow \infty \). Hence \( \epsilon > 0 \) for \( t > 0 \) (one would take \( \epsilon < 0 \) for \( t < 0 \)). We now specify the initial conditions by stating the initial state is composed of \( |1\rangle \) and \( |2\rangle \) only

\[ b_{1}(0) \neq 0, \quad b_{2}(0) \neq 0, \quad b_{\alpha}(0) = 0. \]

We also assume for simplicity \( W_{12} = W_{21} = 0 \) (this can always be achieved by redefinition of \( |1\rangle \) and \( |2\rangle \)). Using the fact that

\[ \frac{e^{i\Delta t}}{\Delta} \rightarrow \text{it} \quad \text{for} \quad \Delta \rightarrow 0 \]
we obtain for \( b_i^{(2)}(t) \) \((i = 1, 2)\)

\[
b_i^{(2)}(t) - b_i(0) = -it \frac{W_{ii}}{\Delta(i,i)} b_i(0) + \\
\sum_{\alpha} \sum_{j=1,2} \left( \frac{e^{i\Delta(i,\alpha)t} - 1}{\Delta(i,\alpha)} - it \frac{W(i,\alpha) W(\alpha,j)}{\Delta(\alpha,j)} \right) b_j(0). \tag{A.1}
\]

The validity of this approximation requires that

\[ |t[\Delta(i,2) + i\epsilon]| \ll 1. \]

This condition requires only

\[ |t \Delta(i,2)| \ll 1 \tag{A.2} \]

because \( \epsilon \) can always be chosen arbitrarily small. We further neglect in Eq. (A.1) the first term in the bracket compared to the second one. This is valid if \( t \) satisfies

\[ |t \Delta(i,\alpha)|_{av} \gg 1, \tag{A.3} \]

where \( |\Delta(i,\alpha)|_{av} \) gives the average order of magnitude of \( |\Delta(i,\alpha)| = |m_i - m_{\alpha}| \) for a decay state \( \alpha \), as in this case this term oscillates quickly, and gives no average contribution. Finally, since Eq. (A.1) now contains a term linear in \( t \), one has to expect that the perturbation series will blow up for large \( t \); this is indeed the case because one can see by explicit calculation that the fourth, sixth, ... order approximations grow with \( t \) as \( t^2, t^3, \ldots \). The condition for validity of the second order approximation [Eq. (A.1)] is that

\[ |t \sum_{\alpha} \frac{W(i,\alpha) W(\alpha,j)}{\Delta(\alpha,j)}| \ll 1. \tag{A.4} \]
We note that conditions (A.2), (A.3) and (A.4) are compatible if
\[ |\Delta(12)| \quad \text{and} \quad \sum_{\alpha} \frac{W(i,\alpha)W(j,\alpha)}{\Delta(a,j)} \ll |\Delta(i,\alpha)|_{av}, \]
i.e. when the interaction $H_I$ and the mass splitting $\Delta(12) = m_1 - m_2$ are small enough.

With the approximations made, Eq. (A.1) becomes
\[ b_i(t) - b_i(0) = -i\epsilon \sum_{j} o_{ij} b_j(0) \]  
(A.5)

with
\[ o_{ij} = \langle i|H_I|j \rangle + \sum_{\alpha} \frac{\langle i|H_I|\alpha \rangle < \alpha|H_I|j \rangle}{\Delta(\alpha,j) + i\epsilon}. \]

For the calculation of the limit $\epsilon \to 0$ in the sum $\sum_{\alpha}$ one uses (we take $\epsilon > 0$ as must be done for $t > 0$)
\[ \lim_{\epsilon \to 0} \frac{1}{x + i\epsilon} = P\left(\frac{1}{x}\right) - i\pi\delta(x) \]

\[ P\left(\frac{1}{x}\right) = \text{principal part of } \frac{1}{x}. \]

We now come to the last point, which consists in extending Eq. (A.5) to other times $t_0 > 0$
\[ b_i(t_0 + t) - b_i(t_0) = -it \sum_{j} o_{ij} b_j(t_0) \]  
(A.6)

where $t > 0$ is again supposed to verify Eqs. (A.2), (A.3) and (A.4).
To obtain the more general Eq. (A.6) one can take over the above proof of Eq. (A.5), replacing everywhere in the proof the initial time 0 by $t_0$. The only complication is that now we cannot put $b_{a}(t_0) = 0$, so that extra terms containing $b_{a}(t_0)$ will occur in Eq. (A.1). These terms represent mathematically the physical circumstance that the decay products already present at $t_0 > 0$ can, through the weak interaction $H_I$, react back on the $K^0$ and $K^0$ amplitudes $b_1$, $b_2$ and contribute to them in the time interval from $t_0$ to $t_0 + t$. The essential point is that this contribution will not grow linearly in $t$; it will have an oscillatory character in $t$ and remain as small as, let us say, the contribution of the first term in the bracket of Eq. (A.1); it can therefore be neglected compared to the second term which grows linearly in $t$, this neglect being valid as soon as condition (A.3) is satisfied. This is the reason why Eq. (A.6) is valid for $t_0 > 0$ under our approximations.

It is worth remarking that the situation would be completely different if one had $t_0 < 0$ and $t > 0$. Then $b_{a}(t_0)$ would describe the decay products in a decay taking place "backward in time", i.e. from time 0 to an earlier time $t_0 < 0$. Their effect on $b_1$, $b_2$ between time $t_0$ and the later time $t_0 + t > t_0$ would certainly give contributions growing linearly with $t$, because between $t_0$ and $t_0 + t$ the "backward decay" products would in fact move coherently to reconstruct an increasing amplitude of $K^0$ or $K^0$.

Having accepted Eq. (A.6) for $t_0 > 0$ and $t > 0$ subject to the conditions (A.2), (A.3) and (A.4), we remember that the coefficients $O_{ij}$ are small, so that Eq. (A.6) can be replaced with good approximation by a differential equation

$$\frac{db_1(t_0)}{dt_0} = -1 \sum_j O_{ij} b_j(t_0), \quad t_0 > 0.$$
The corresponding equation for the original amplitudes $a_i$ is

$$\frac{da_i(t)}{dt} = -i \sum_j \left( m_i \delta_{ij} + o_{ij} \right) a_j(t), \quad t > 0$$

which is the equation we set out to prove.

* * *
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