HQQET: An Effective Theory Approach to Heavy Quarkonia Decays

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Abstract
We discuss systems containing a heavy quark and a heavy antiquark in the infinite mass limit of QCD. Studying the limit of equal velocities for both heavy particles, we formulate an effective theory approach to heavy quarkonia-like systems. The method is well suited to processes in which the two heavy quarks annihilate, such as electromagnetic and strong decays of charmonium and bottomonium and weak decays of $B_c$. 

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1 Introduction

In the past five years considerable progress has been made in heavy quark physics by studying systems with a single heavy quark in the infinite mass limit of QCD [1]. The mass $m$ of the heavy quark sets a scale large compared to the intrinsic scale $\Lambda_{QCD}$ of the light QCD-degrees of freedom, and the appearance of such disparate mass scales allows us to use an effective theory treatment of systems with a single heavy quark. This effective theory, the so-called heavy quark effective theory (HQET) has the interesting property of additional symmetries which are not present in full QCD. These symmetries, in addition to the usual machinery of effective theory, are a powerful tool in heavy hadron physics, which allows us to make QCD based and in many cases even model independent statements. The progress in this field is well documented in more or less extensive review articles [2].

Almost all the applications considered so far deal with the one heavy particle sector of QCD. In HQET particle and antiparticle number are separately conserved and all applications deal with either a single heavy quark or a single heavy antiquark. First attempts to consider states containing two heavy (anti)quarks or a quark and an antiquark revealed some difficulties [3] when one calculates QCD radiative corrections. The anomalous dimensions of operators having matrix elements with states containing two heavy quarks turn out to be complex, at least if interpreted in the naïve way. In addition, the imaginary parts behave as $1/v$, where $v$ is the relative three velocity of the heavy quarks. Subsequent investigations [4] showed that the imaginary parts of the anomalous dimensions yield phase factors which have to be interpreted as the non-abelian analogue of the Coulomb phase well known from electrodynamics. These phases are an infrared contribution, which has to be absorbed into the states. After redefinition of the states the anomalous dimensions are real, well behaved as the relative velocity becomes small, and hence are the true short distance contribution.

Heavy quarkonia have to show up in the sector of HQET containing a heavy quark and a heavy antiquark. The velocities of the two heavy particles in such a state differ only by a small amount of the order $\Lambda_{QCD}/m$, and hence one wants to describe heavy quarkonia in the limit, where the two heavy quarks move with the same velocity, which is then identified with the velocity of the heavy quarkonium. However, such a limit is ill defined for static quarks due to the divergent phases mentioned above. A proper treatment of these phases lies at the heart of the formulation of a Heavy Quarkonium Effective Theory (HQ$Q\bar{Q}$ET), since these phases are related to the binding mechanism of the heavy quarkonium states. We shall see in what follows that HQ$Q\bar{Q}$ET may not simply be related to the two particle sector of HQET, because the divergent phases prevent us from taking the strict infinite mass limit.

In this paper we shall clarify the underlying assumptions necessary to formulate HQ$Q\bar{Q}$ET based on the static limit. In the next section we shall give a theoretical description of the method, which has been applied to inclusive heavy quarkonia decays already in [5] and compare to related ideas of Bodwin, Braaten and Lepage (BBL) [6]. In section 3 we discuss how to calculate QCD and higher order non-perturbative corrections in HQ$Q\bar{Q}$ET.

2 Formulation of HQ$Q\bar{Q}$ET

Our starting point is the Lagrangian of QCD, which we expand in inverse powers of $1/m$ [7, 8]. The part of the Lagrangian involving the heavy quark is unique up to terms of order
$1/m$ and is given by
\[ \mathcal{L} = \bar{h}_v^{(+)}(i v D)h_v^{(+)} + \left( \frac{1}{2m} \right) \bar{h}_v^{(+)} i \not\!D P_\perp i \not\!D h_v^{(+)} + \mathcal{O}(1/m^2), \]
while the corresponding expansion for the field $Q$ of full QCD is given by
\[ Q = \exp(-i m v x) \left( 1 + \frac{1}{2m} P_\perp \not\!D + \mathcal{O}(1/m^2) \right) h_v^{(+)}(x), \]
where $P_\pm = (1 \pm \not\!v)/2$. The superscript $(+)$ indicates that the field $h_v^{(+)}$ describes a static quark moving with the velocity $v$; correspondingly we introduce also the field $h_w^{(-)}$, which describes an antiquark moving with the velocity $w$. The Lagrangian and the expansion of the full QCD field for the antiquark field is obtained from (1) and (2) by the replacement $v \to -w$.

Let us first consider only the static, mass independent term of the expansions (1) and (2) and write the Lagrangian for a two-particle system consisting of a static quark and a static antiquark as
\[ \mathcal{L} = \bar{h}_v^{(+)}(i v D)h_v^{(+)} - \bar{h}_w^{(-)}(i w D)h_w^{(-)}. \]
Based on such a Lagrangian we may now consider the matrix elements involving two heavy particle states; as an example we shall study
\[ G = \langle A|\tilde{Q}(x)\Gamma Q(x)|0\rangle, \]
where $A$ is a state containing a heavy quark and a heavy antiquark moving with velocities $v$ and $w$ respectively. In the static limit this matrix element becomes
\[ G_{\text{static}} = \langle \tilde{A}|\bar{h}_v^{(+)}(x)\Gamma h_w^{(-)}(x)|0\rangle \exp[im(v+w)x], \]
where the tilde denotes the static limit of the state. Logarithmic dependences on the heavy quark mass may be calculated using renormalization group improved perturbation theory in the framework of HQET. The one-loop QCD radiative corrections to a current of this type have been calculated in [3] and [9]. It turns out that in a naive calculation the anomalous dimension seem to acquire an imaginary part, which for $v \to w$ develops a divergence of the general structure
\[ \text{Im } \gamma = f(\alpha_s) \frac{1}{\sqrt{(vw)^2 - 1}} \]
where $f(\alpha_s)$ is known up to two loops [9]
\[ f(\alpha_s) = \frac{4}{3} \alpha_s \left( 1 + \frac{\alpha_s}{4\pi} \left[ \frac{31}{3} - \frac{10}{9} n_f \right] + \cdots \right). \]

The real part of the anomalous dimension vanishes as $v \to w$ due to current conservation, and the solution of the renormalization group equation with a purely imaginary anomalous dimension yields for $v \to w$ a phase factor of the form
\[ \exp(i \phi(vw)) = \exp \left\{ i \frac{1}{\sqrt{(vw)^2 - 1}} \int_{\alpha_s(m)}^{\alpha_s(\mu)} d\alpha \frac{f(\alpha)}{\beta(\alpha)} \right\} , \quad \beta(\alpha(\mu)) = \frac{\partial}{\partial \mu} \alpha(\mu) \]
which is ill behaved in the limit $v \to w$.

This divergence of the imaginary part prevents us from taking the limit $v \to w$ for two heavy static particles. On the other hand, this is exactly the limit in which we want to describe heavy quarkonium states. From this we conclude that the purely static Lagrangian (3) is not appropriate for the description of heavy quarkonium states.

In fact, the phases are the non-abelian counterpart of the Coulomb phases well known in QED. They are related to the long range part of the one gluon (one photon) exchange potential, which decreases too slowly and thus leads to infrared problems. Consequently, these phases are an infrared effect and have to be absorbed into the states [4].

In the channels, where this potential is attractive, bound states may occur, and these phases are directly related to the binding mechanism. This may be explicitly seen for the abelian case using eikonal methods, which correspond to the heavy mass limit [10]. The binding of a heavy quarkonium is clearly an infrared effect and has to be reproduced by the dynamics of a properly constructed effective theory for quarkonia. In such a state the two velocities differ only by a small amount of order $1/m$ which is a hint that we need to go beyond the static limit to describe quarkonium states.

In order to see how higher-order terms of the Lagrangian cure the problem, we make use of the fact that the full QCD Lagrangian is independent of the arbitrarily chosen velocity vectors $v$ and $w$ [11]. The only kinematic quantity entering in full QCD are the true momenta of the particle $p = mv + k$ and the antiparticle $p' = mw - k'$, where $k$ and $k'$ are the residual momenta of the two heavy quarks. Thus we are led to define
\[
\mathcal{V} = v + \frac{iD}{m} \quad \text{and} \quad \mathcal{W} = w + \frac{iD}{m} \tag{9}
\]
corresponding to $p/m$ and $p'/m$ respectively. These combinations are invariant under infinitesimal reparametrizations of the velocities $v \to v + \delta v$ and $w \to w + \delta w$ [11], since under such a reparametrization we have $D \to D - m\delta v$ for the quark and $D \to D - m\delta w$ for the antiquark moving with the velocity $-w$. In a full QCD calculation the singularity corresponding to the one appearing in (6) occurs in the imaginary part of the vertex function as
\[
\text{Im } \Gamma(p, p') = f(\alpha_s) \frac{m^2}{\sqrt{(pp')^2 - m^4}} \ln \left( \frac{m}{\lambda} \right), \tag{10}
\]
where $\lambda$ now is an infrared regulator, revealing the infrared origin of the singularity. The vertex function of full QCD depends on the full quark momenta $p$ and $p'$, which are split into a large piece $mv$ ($mw$) and a residual part $k$ ($k'$) as we switch to the effective theory. This is, however, an artificial procedure, and the singularity may be reproduced either in the dependence on the velocity or through the residual momenta.

In heavy quarkonia the velocities of its heavy constituents are almost equal, $vw \sim 1$, and one would rather go to the limit $v = w$ and reproduce the divergence of the imaginary part as a singularity in the dependence on the residual momenta. This is achieved formally by reinserting the full momenta for the velocities in the divergent phase, i.e. by the replacement $v \to \mathcal{V}$ and $w \to \mathcal{W}$, and we obtain from (8)
\[
\exp(i \phi(vw)) \tilde{h}^{(+)}_v \Gamma h^{(-)}_{-w} \quad \to \quad \tilde{h}^{(+)}_v \exp(i \phi(\mathcal{V} \mathcal{W})) \Gamma h^{(-)}_{-w} \tag{11}
\]
If we now consider the limit \( v \to w \) we have also \( \mathcal{V} \to \mathcal{W} \), but now the phase depends on the residual momenta, which are represented by the covariant derivatives acting on the heavy quark fields

\[
\exp(i\phi(\mathcal{V}\mathcal{W})) \to \exp(i\phi(\mathcal{V}^2)) = \exp \left\{ i \frac{1}{\sqrt{v'^2 - 1}} \int_{\alpha_{s}(m)}^{\beta_{s}} d\alpha f(\alpha) \right\},
\]

(12)

which means that we have rewritten the singular phases in such a way that they now depend on the difference of the residual momenta rather than on the difference of the velocities.

However, the difference between \( v \) and \( \mathcal{V} \) is a term of higher order in \( 1/m \), which was added in such a way that \( \mathcal{V} \) is a reparametrization invariant quantity. In order to construct a leading order Lagrangian capable of reproducing the (infrared) phase factors and also generate binding of the two heavy quarks, we rewrite the static Lagrangian (3) in a reparametrization-invariant form; in this way we obtain as the leading order Lagrangian

\[
\mathcal{L}_0 = \frac{m}{2} \bar{h}_v^+(\mathcal{V}^2 - 1) h_v^+ + \frac{m}{2} \bar{h}_w^-(\mathcal{W}^2 - 1) h_w^- = \bar{h}_v^+(iD) h_v^+ + \bar{h}_w^+(D) h_w^- + \bar{h}_w^- (D h_w^-) + \bar{h}_v^+ (D h_v^+),
\]

(13)

where we have replaced \( w \to -w \) in the last step. In the Lagrangian (13) we now may perform the limit \( v \to w \) without encountering a problem in the calculation of QCD radiative corrections; we now have already to leading order the scale of the heavy mass \( m \), and ultraviolet contributions show up as \( \ln(\Lambda/m) \) (\( \Lambda \) being the ultraviolet cut-off) and infrared contributions as \( \ln(m/\lambda) \) (\( \lambda \) being the infrared cut-off). In this way a clean separation between (calculable) short distance effects and (non-perturbative) infrared contributions is achieved. The diverging phases now show up as a singularity in the residual relative momentum, hence as an infrared effect.

We also expect that the Lagrangian (13) ensures the existence of bound states, corresponding to the “unperturbed” heavy quarkonia states. However, this also shows that – unlike for heavy-light systems – there is no infinite mass limit for quarkonia; the “unperturbed” states described by (13) will still be mass dependent.

The Lagrangian (13) is the minimum that is needed to shift the phases from the velocity dependence into the residual momenta. The spin dependent terms appearing as well in order \( 1/m \) do not contribute to the infrared behaviour. This is well known from QED, where all infrared contributions are independent of the spin of the radiating particle; only the total charge plays a role. Spin symmetry thus remains unbroken to leading order, and spin symmetry breaking effects may be calculated as perturbations without encountering infrared problems.

The leading order Lagrangian (13) resembles very much the one of non-relativistic QCD (NRQCD) as formulated by Caswell and Lepage [12]. They suggest an expansion in \( v/c \), where \( v \) is the typical relative velocity of the two heavy quarks bound in the quarkonium. Although it does not seem that the two approaches are completely equivalent [5], they have many common features.

The binding of the two heavy quarks will generate a small non-perturbative scale \( \Lambda \), which now in general depends on the heavy mass. Although the bottomonium and the charmonium
are not Coulombic systems, the case of a $\alpha/r$ potential is instructive. Neglecting any running of $\alpha$, the size of such a Coulombic system is $R_{Bohr} = 1/(\alpha m)$, which is large compared to the Compton wavelength $\lambda_Q = 1/m$. However, the small scale $1/R_{Bohr}$ depends on the mass such that it does not approach a finite limit as $m \to \infty$, even for running $\alpha$.

The Lagrangian (13) is the starting point of an effective theory treatment of heavy quarkonium decays. The processes which may be considered in this type of approach are decays of heavy quarkonia, in which the two heavy quarks annihilate. The annihilation process is governed by a large energy scale set by the heavy quark mass $m$, while the binding of the quarkonium introduces a small scale $\Lambda$. The appearance of these disparate mass scales allows for an effective theory treatment, yielding an expansion in powers of $\Lambda/m$ of the relevant amplitudes of full QCD.

Heavy Quark Spin symmetry implies that the “unperturbed” heavy quarkonia states fall into degenerate spin symmetry quartets. For a given orbital angular momentum $\ell$ and radial excitation quantum number $n$, the four states (in the spectroscopic notation $2S+1\ell_J$)

$$[n^1\ell_\ell, n^2\ell_{\ell-1}, n^3\ell_\ell, n^4\ell_{\ell+1}]$$

form such a spin symmetry quartet. An exception are the $S$ waves ($\ell = 0$), for which the three polarization directions of the $n^3S_1$ and the $n^1S_0$ form the spin symmetry quartet. The consequences of this symmetry for transitions from an excited quarkonium to the ground states have been investigated recently [13].

In order to exploit the consequences of the spin symmetry for the transition matrix elements we shall use the trace formalism. We denote with $|Y\rangle$ the spin symmetry quartet consisting of the spin singlet and the spin triplet for a given orbital angular momentum $\ell$. The coupling of the heavy-quark spins may be represented by the matrices

$$H_Y(v) = \begin{cases} P_+\gamma_5 & \text{for the spin singlet} \\ P_+\gamma_5 & \text{for the spin triplet} \end{cases}$$

Using these representations one may then analyse the spin structure of matrix elements for processes involving quarkonia. As a simple example we study a matrix element like (4) with $|A\rangle$ being a quarkonium state $|Y(J,J_z,n,l)\rangle$. In the heavy mass limit we have

$$\langle Y(J,J_z,n,l,S)|\bar{h}_{v_\ell}^{(+)}\Gamma h_{w}^{(-)}|0\rangle = a(n,l)\text{Tr}\{\bar{H}_Y\Gamma\}$$

where $a(n,l)$ is independent of the spin coupling of the heavy quarks. A simple consequence of spin symmetry is that the basis of the 16 Dirac matrices may be reduced to four, which are the generalization of the Pauli matrices [5, 14]. Furthermore, from (16) we have for the ground state quarkonia

$$\langle \bar{Y}(1,J_z,0,0,1)|\bar{h}_{v_\ell}^{(+)}\gamma_5 h_{w}^{(-)}|0\rangle = \langle \bar{Y}(0,0,0,0,0)|\bar{h}_{v_\ell}^{(+)}\gamma_5 h_{w}^{(-)}|0\rangle.$$ 

In a simple wave function picture this means that the wave functions at the origin of the two ground state quarkonia are equal in the heavy mass limit.
3 Higher Order Correction in HQ\(\tilde{Q}\)ET

In a similar way as in HQET the corrections in HQ\(\tilde{Q}\)ET fall into two classes. The first type of corrections are the QCD radiative corrections, which in general may be calculated systematically. The second class are the recoil corrections appearing as a power series in \(\lambda/m\); they originate from matrix elements of higher dimensional operators induced by the expansion of the currents and the Lagrangian.

Let us first consider the QCD radiative corrections, which may be calculated systematically using the Feynman rules of the effective theory. Based on our choice for the leading order term \(\mathcal{L}_0\), which is the sum of \(\mathcal{L}_{\text{static}}\) and the first order term \(\bar{h}(iD)^2 h/(2m)\), one derives for the propagator of the heavy particle with velocity \(v\) and residual momentum \(k\)

\[
H(k) = P_+ \frac{i}{vk + \frac{1}{2m}k^2 + i\epsilon};
\]

the corresponding expression for the antiparticle is obtained by the replacement \(v \rightarrow -v\). This expression contains all orders in \(1/m\); however, the reason why we had to include these higher-order terms was that this removes the divergent phase occurring in the limit of small relative velocity. This phase may be absorbed as a long-distance effect into the states, which thus have to evolve according to the dynamics dictated by \(\mathcal{L}_0\) given in (13). The “true” short distance contribution is well behaved as the relative velocity of the heavy particles vanishes. Furthermore, it may be expanded again in powers of \((1/m)^n\) without encountering a problem. This is at least true at the one-loop level, where (18) is the propagator of NRQCD [12], and it has been shown in [6] that the coefficients of the ultraviolet divergences at the one-loop level may be expanded in \(1/m\).

On the other hand, if we start directly from the static limit using the propagator of HQET

\[
H_{\text{stat}}(k) = P_+ \frac{i}{vk + i\epsilon},
\]

imaginary parts will show up, which become ill-defined as velocities of heavy quarks coincide, such as

\[
\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \delta(vk) \delta(v'k) \rightarrow \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} (\delta(vk))^2 \quad \text{as} \quad v \rightarrow v'
\]

at the one-loop level, which are contributions to the divergent phase. However, these are absorbed into the states as being a long distance contribution which is generated by the infrared dynamics of \(\mathcal{L}_0\) and consequently they have to be dropped here.

With this additional prescription, namely to shift the terms diverging as the two velocities become equal into the states, we may as well obtain a \(1/m\) expansion of the short distance contribution by calculating directly with (19), i.e. within the HQET framework. This should coincide with what is obtained in using (18) and subsequent expansion in powers of \(1/m\). The diverging terms reappear in the states, since by an appropriate choice of the leading order Lagrangian (as in (13)) the corresponding terms are reproduced by the residual momenta, but now they are buried in the non-perturbative infrared physics of the states. The price to pay is a mass dependence of the states, which is not accessible via a \(1/m\) expansion.

This simplified prescription allows us to apply the full machinery of HQET to the calculation of the short-distance corrections in HQ\(\tilde{Q}\)ET, and at the one-loop level the results
obtained for inclusive heavy quarkonia decays coincide with what is obtained in NRQCD [6] and subsequent expansion in powers of $1/m$ [5]. A proof whether this is true also in higher orders in the loop expansion lies beyond the scope of the present article.

The other type of corrections are the power corrections, i.e. the higher order corrections in the $1/m$ expansion. These are induced by the expansion of the field and of the Lagrangian and are included as perturbations. It has been noticed that the terms of order $1/m^2$ and higher of the Lagrangian and the fields depend on the convention: As in any effective theory it is possible to perform local field redefinitions, thereby shifting certain terms from the Lagrangian into the fields and vice versa [15]. However, any physical matrix element remains unchanged, if such a field redefinition is performed. For our purpose a convenient definition of the higher order terms in the Lagrangian is the one, in which all terms, which would vanish by the use of the static equation of motion derived from (3), are shifted into the definition of the fields. In this convention the Lagrangian up to and including $1/m^2$ terms takes the form

$$\mathcal{L} = \mathcal{L}_{\text{static}} + \mathcal{L}_I$$

where

$$\mathcal{L}_{\text{static}} = \bar{h}^{(+)}(+i D)h^{(+)} - \bar{h}^{(-)}(i D)h^{(-)}$$

$$\mathcal{L}_I = \left( \frac{1}{2m} \right) L_1 + \left( \frac{1}{2m} \right)^2 L_2$$

$$= \left( \frac{1}{2m} \right) (K_1 + G_1) + \left( \frac{1}{2m} \right)^2 (K_2 + G_2) + \mathcal{O}(1/m^3) .$$

Since now only a single velocity $v$ appears, we omit the subscript $v$ from the field operators in the following. Furthermore, we have defined

$$K_1 = K_1^{(+)} + K_1^{(-)} \quad K_1^{(\pm)} = \bar{h}^{(\pm)}[(i D)^2 - (i v D)^2]h^{(\pm)}$$

$$G_1 = G_1^{(+)} + G_1^{(-)} \quad G_1^{(\pm)} = (-i)\bar{h}^{(\pm)}\sigma_{\mu\nu}(i D^\mu)(i D^\nu)h^{(\pm)}$$

$$K_2 = K_2^{(+)} + K_2^{(-)} \quad K_2^{(\pm)} = \bar{h}^{(\pm)}[(i D_\mu), [(i v D), (i D^\nu)]h^{(\pm)}$$

$$G_2 = G_2^{(+)} + G_2^{(-)} \quad G_2^{(\pm)} = (-i)\bar{h}^{(\pm)}\sigma_{\mu\nu}\{(i D^\mu), [(i v D), (i D^\nu)]\}h^{(\pm)} .$$

The corresponding expansion of the field $\bar{Q}_v^{(+)}$ reads

$$\bar{Q}_v^{(+)}(x) = \left( 1 + \frac{1}{2m} P_- i \Slash{D} - \frac{1}{8m^2}(i v D)P_- i \Slash{D} \right. \right.$$

$$- \left. \frac{1}{8m^2} (i D)^2 - (i D^\mu)^2 - i \sigma_{\mu\nu} i D^\mu i D^\nu \right) + \mathcal{O}(1/m^3) \bar{h}^{(+)}(x) .$$

In fact this is the form that has been obtained from QCD by a sequence of Foldy-Wouthuysen transformations in [8].

The corrections to currents involving heavy quarks are then obtained by inserting the expansion (24) for the full fields of QCD, while the corrections to the states are implemented via time-ordered products of the expansion of the currents with higher-order terms of the Lagrangian. However, here we have to take into account the fact that the leading order dynamics of the states already contains the first order kinetic energy contribution $K_1$ and hence this piece must not to be included as a perturbation.
The higher order terms of the heavy mass expansion for heavy quarkonia are parametrized then by matrix elements involving the “unperturbed” quarkonia states that are described by $L_0$ and operators expressed in terms of the static fields $h^{\pm}_v$. Due to the mass dependence of the states flavour symmetry is broken, which means that these matrix elements are different for charmonium and bottomonium states. These matrix elements are genuinely non-perturbative quantities and spin symmetry may be used to count the number of independent parameters. Accessing the actual values of these parameters requires input beyond HQ$\bar{Q}$ET; they may be extracted from experiment, calculated using some model framework, or eventually obtained from a lattice calculation.

4 Conclusions

The heavy mass limit has been used with great success for systems involving a single heavy quark. In this limit the mass gap between the particles and the antiparticles becomes infinitely large and, as a consequence, particle and antiparticle numbers are separately conserved. HQET has been formulated in the one (anti)particle sector, and this is sufficient for almost all applications considered so far.

In order to describe heavy quarkonia one has to deal with the particle-antiparticle sector. Treating both heavy constituents in the static limit is only possible if the two velocities $v$ and $v'$ are very different, i.e., if $vv' - 1 = \mathcal{O}(1)$. The naively calculated QCD radiative corrections exhibit logarithmically divergent and purely imaginary contributions, which behave as $1/\sqrt{(vv')^2 - 1}$. Interpreting these pieces in the standard way as contributions to the anomalous dimension yields then phases, which are ill-behaved as $v \to v'$.

It has been shown that these phases may be removed by a redefinition of the states [4]. The phases are thus a property of the states and and hence an infrared effect. After redefinition of the states, the “true” short distance contributions, i.e., the anomalous dimensions become real.

In a heavy quarkonium state the velocities of the two heavy constituents differ only by a small amount of the order $1/m$, and it is appropriate to choose the same velocity for both heavy particles, which is then identified with the velocity of the heavy quarkonium. However, such a limit may not be taken from the expression obtained in the static limit with $v \neq v'$.

In the present paper we have shown that one may formulate a Heavy Quarkonium Effective Theory (HQ$\bar{Q}$ET), despite of these difficulties. The key-point is that the leading order term has to be chosen in such a way that the diverging phases are generated by its dynamics. This forced us to include also the first order kinetic energy term into the leading order Lagrangian (13), which determines the evolution of the states. Consequently the “unperturbed” states have to depend on the mass and heavy flavour symmetry is lost. In other words, from this point of view it is likely that there is no mass independent, static limit of a heavy quarkonium state, since the purely static Lagrangian does not generate the diverging phases and probably also does not generate bound states.

However, the Lagrangian (13) still has an unbroken heavy quark spin symmetry, and hence the “unperturbed” heavy quarkonia have to fall into spin symmetry quartets, since all four orientations of the two heavy quark spins will lead to degenerate states. This symmetry also allows to reduce the number of independent non-perturbative parameters.
This effective theory framework allows us also the calculation of corrections. The main result of the present analysis is that the short distance corrections may be calculated using the methods of HQET. However, if the velocities of the two heavy quarks become equal, divergent terms will appear, which are infrared contributions and which have to be shifted into the states. The appropriately chosen $L_0 (13)$ contains these divergent terms as piece of its infrared dynamics. The remaining expressions are the “true” short distance contributions, which remain well behaved as $v \rightarrow v'$. This prescription has been applied at the one-loop level and yields the same $1/m$ expansion as a NRQCD calculation with a subsequent $1/m$ expansion.

The second type of corrections corresponds to the $1/m$ expansion of the Lagrangian and the fields. Including these contributions proceeds along the same lines as in HQET, with the only difference that no time-ordered products with the first order kinetic energy operator $K_1$ are present, since this is included already in the definition of the “unperturbed” states.

HQ$\overline{Q}$ET may be applied to all processes in which the two heavy particles annihilate, such as inclusive and exclusive decays of charmonium and bottomonium, (electromagnetic and strong) and also to weak decays of $B$. Some applications of this idea have been investigated in [5] and using related methods in [6], but HQ$\overline{Q}$ET opens a wide field of further applications.

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