1.1 Triple Higgs coupling studies in an EFT framework

The measurement of the triple Higgs coupling is one of the major goals of the future colliders. The direct measurement at lepton colliders relies on the production of Higgs boson pairs in two main channels, $e^+e^- \rightarrow ZHH$ which is dominant at centre-of-mass energies below 1 TeV and maximal at around 500 GeV, and $e^+e^- \rightarrow HH\nu\bar{\nu}$ that becomes dominant for high-energy colliders. This direct measurement requires to be at least at a centre-of-mass energy of 500 GeV, and is hence only possible at future linear colliders such as the International Linear Collider (ILC) operating at 500 GeV or 1 TeV [1], or the Compact LInear Collider (CLIC) operating at 1.4 TeV (stage 2) or 3 TeV (stage 3) [2]. The SM triple Higgs coupling sensitivity is estimated to be $\delta\kappa = (\lambda_{HHH}/\lambda_{HHH}^{SM} - 1) \sim 28\%$ at the 500 GeV ILC with a luminosity of 4 ab$^{-1}$ [3,4] and $\delta\kappa \sim 13\%$ at the CLIC when combining the 1.4 TeV run with 2.5 ab$^{-1}$ of data and the 3 TeV run with 5 ab$^{-1}$ of data [5].

Still, circular-lepton-collider projects such as the Circular Electron Positron Collider (CEPC) [6] or the FCC-ee [7,8], that run at energies below 500 GeV (not to mention the ILC or the CLIC running at lower energies), can provide a way to constraint the triple Higgs coupling [9]. Since the work of Ref. [10] that proposed for the first time to use precision measurements to constrain the triple Higgs coupling, in particular the measurements in single Higgs production at lepton colliders, there have been studies of the combination of single and double Higgs production observables not only at lepton but also at hadron colliders [11–14]. The analyses use the framework of the Standard Model Effective Field Theory (SMEFT). According to the latest ECFA report [15] the combination of HL-LHC projections [16] with ILC exclusive single-Higgs data gives $\delta\kappa = 26\%$ at 68% CL, while with FCC-ee (at 250 or 365 GeV) it goes down to $\delta\kappa = 19\%$ and with CEPC we get $\delta\kappa = 17\%$. We will present in more details the results of Refs. [12,13] that demonstrate how important the combination of the LHC results with an analysis at lepton colliders is, and show the potential of the FCC-ee*.

Fig. E.1 (left) displays the latest experimental results available at the 13 TeV LHC for the search of non-resonance Higgs pair production and the 95% CL limits on the triple Higgs coupling, which have been presented in Ref. [17]. The results constraint $\delta\kappa$ in the range $[-6.0 : 11.1]$. We can compare them to the projections at the HL-LHC with 3 ab$^{-1}$ presented in the HL-HE LHC report [16] in an SMEFT framework, using a differential analysis in the channel $pp \rightarrow HH$. Compared to the projection in Ref. [12], which also included single Higgs data in the channels $pp \rightarrow W^\pm H, ZH, t\bar{t}H$, there is a substantial improvement thanks to the

*Julien Baglio thanks Christophe Grojean for his very useful inputs in this subsection.
experimental differential analysis. We have $-0.5 \leq \delta \kappa_\lambda \leq 0.5$ at 68% CL and $-0.9 \leq \delta \kappa_\lambda \leq 1.3$ at 95% CL. The degeneracy observed in Ref. [12] with a second minimum at $\delta \kappa_\lambda \sim 5$ is now excluded at 4$\sigma$.

The combination with data from lepton colliders removes the second minimum even more drastically and only the SM minimum is left at $\delta \kappa_\lambda = 0$ [13], in particular when data from 250 GeV and 350-365 GeV centre-of-mass energies is combined [13]. This is shown in Fig. E.2 where two setups are compared, the combination of HL-LHC data with circular lepton colliders (FCC-ee or CEPC) data on the left-hand side, and the combination of HL-LHC data with the ILC data on the right-hand side. In both cases the lepton-collider data consists of measurements in the channels $e^+e^- \to W^+W^-, ZH, \ell\nu\ell\nu H$. The second minimum disappears completely even with a relatively low integrated luminosity of $L = 200$ fb$^{-1}$ at 350 GeV combined to the data at 250 GeV. Note that the FCC-ee (or CEPC), thanks to its much higher luminosity in the 250 GeV run, is doing significantly better than the ILC.

1.2 Probing heavy neutral leptons via Higgs couplings

Since the confirmation of neutrino oscillations in 1998 by the Super-Kamiokande experiment [18], it has been established that at least two neutrinos have a non-zero mass [19]. This experimental fact cannot be accounted for in the SM and requires new physics. One of the simplest extensions is the addition of new heavy, neutral leptons that are gauge singlets and mix with the active neutrinos to generate the light neutrino masses. An appealing model, allowing for these new fermionic states to be in the GeV to a few TeV range while having Yukawa couplings of order one, is the inverse seesaw (ISS) model [20–22], in which a nearly conserved lepton-number symmetry [23,24] is introduced, naturally explaining the smallness of the mass of the lightest neutrino states while allowing for large couplings between the heavy neutrinos and the Higgs boson, leading to a rich phenomenology. In this view the very precise

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Fig. E.1: Left: Latest experimental bounds on the triple Higgs coupling from the ATLAS collaboration at the 13 TeV LHC, combining $b\bar{b}b\bar{b}$, $b\bar{b}\tau^+\tau^-$, and $b\bar{b}\gamma\gamma$ final states. Taken from Ref. [17]. Right: Minimum negative-log-likelihood distribution of $\kappa_\lambda$ at the HL-LHC with 3 ab$^{-1}$ of data including differential observables in Higgs pair production, with ATLAS (blue), CMS (red), and ATLAS+CMS (black) projected results. Figure taken from Ref. [16].

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study of the Higgs sector at lepton colliders can offer a unique opportunity to test low-scale seesaw mechanisms such as the ISS.

**Heavy neutral leptons in the GeV regime**

We begin with the GeV regime. In these low-scale seesaw models the mixing between the active and the sterile neutrinos leads to modified couplings of neutrinos to the $W$, $Z$, and Higgs bosons. This naturally leads to the idea of using precision measurements of the Higgs boson branching fractions into gauge bosons in order to test the mass range $M_N < M_H$ where $M_N$ is the mass of the heavy neutrino states, $M_H$ is the mass of the Higgs boson. As $H \rightarrow NN$ is allowed, the invisible Higgs decay width is modified and hence the branching fraction $\text{BR}(H \rightarrow W^+W^-)$ is modified via the modified total decay width $\Gamma_H$. According to an analysis in 2015 [25] the FCC-ee could be the most competitive lepton collider to test this option, as demonstrated in Fig. E.3. In particular, the experimental sensitivity to $\text{BR}(H \rightarrow W^+W^-)$ is expected to be $0.9\%$ at the FCC-ee, compared to $1.3\%$ at the CEPC operating at 240 GeV [26] and $6.4\%$ at the ILC operating at 250 GeV [1]†.

**Probing heavy neutral leptons in the multi-TeV regime**

Since the coupling of the heavy neutral leptons to the Higgs boson can be quite large in low-scale seesaw models for masses $M_N$ of a few TeV, it is also very appealing to use again Higgs properties to probe a mass regime of $M_N \sim \mathcal{O}(1−10\text{ TeV})$.

Off-diagonal couplings of the Higgs boson to heavy neutral leptons will induce charged-lepton-flavour-violating (cLFV) decays [28]. In particular, simplified formulae were provided in Ref. [29], showing that cLFV Higgs decays exhibit a different functional dependence on seesaw parameters than cLFV radiative decays. They thus provide complementary observables to search for heavy neutral leptons. In a typical low-scale seesaw model like the ISS, the predicted branching fractions can be as large as $\text{BR}(H \rightarrow \tau\mu) \sim 10^{-5}$ and it could even reach $\text{BR}(H \rightarrow \tau\mu) \sim 10^{-2}$ in a supersymmetric model [30] thus being well within the reach of a Higgs

†The latest analysis at the ILC, using a luminosity of 500 fb$^{-1}$, states that a precision of 4.1% can be achieved [27].
Fig. E.3: Estimated sensitivities on the heavy sterile neutrino properties from the decay \( H \to W^+W^- \), assuming 10 years of data taking. The black line denotes the bound from the LHC coming from \( H \to \gamma\gamma \) with up to 2015 data. Taken from Ref. [25].

factory like the FCC-ee. However, Higgs observables are uniquely sensitive to diagonal couplings as well and this was studied in particular in Refs. [31,32] using the triple Higgs coupling and in Ref. [33] using a direct physical observable, the production cross section \( \sigma(e^+e^- \to W^+W^-H) \). Taking into account all theoretical and experimental constraints that were available, the three studies have found sizeable effects.

In the triple Higgs coupling studies the one-loop corrections to \( \lambda_{HHH} \), defined as the physical triple Higgs coupling after electroweak symmetry breaking, are studied. The calculation is performed in the on-shell scheme and compares the SM prediction to the prediction in low-scale seesaw models (specifically the ISS in Ref. [32]). Representative one-loop diagrams involving the new heavy neutral leptons are given in Fig. E.4 and details of the calculation and analytical formulae can be found in the original articles. The results are given in terms of deviations with respect to the tree-level value \( \lambda_{HHH}^0 \) and to the renormalised one-loop value in the SM \( \lambda_{HHH}^{1,\text{SM}} \) of the triple Higgs coupling,

\[
\Delta^{(1)} \lambda_{HHH} = \frac{1}{\lambda^0} \left( \lambda_{HHH}^1 - \lambda^0 \right),
\]

\[
\Delta^{\text{BSM}} = \frac{1}{\lambda_{HHH}^{1,\text{SM}}} \left( \lambda_{HHH}^1 - \lambda_{HHH}^{1,\text{SM}} \right). \tag{1.1}
\]

with \( \lambda_{HHH}^1 \) being the one-loop renormalised triple Higgs coupling in the low-scale seesaw model considered. The constraints from low-energy neutrino observables are implemented via the \( \mu_X \)-parametrisation, see Ref. [29] for more details as well as the appendix A of Ref. [32] for terms beyond the lowest order in the seesaw expansion. All relevant theoretical and experimental bounds are taken into account and the most stringent constraint comes from the global fit to electroweak precision observables and lepton universality tests [34].

Fig. E.5 displays the results of the analysis in the plane \( M_R - |Y_\nu| \) where \( M_R \) is the seesaw scale and \( |Y_\nu| \) is the magnitude of the Yukawa coupling between the heavy neutral leptons and...
E (Triple) Higgs-coupling imprints at future lepton colliders

Fig. E.4: Representative Feynman diagrams for the one-loop corrections to $\lambda_{HHH}$ involving the neutrinos in the ISS model.

Fig. E.5: Contour maps of the heavy neutral lepton correction $\Delta^{BSM}$ to the triple Higgs coupling $\lambda_{HHH}$ (in %) as a function of the heavy neutral lepton parameters $M_R$ (in TeV) and $|Y_\nu|$ at a fixed off-shell Higgs momentum $q_{H^*} = 500$ GeV (left) and $q_{H^*} = 2500$ GeV (right). The details of the spectrum are given in Ref. [32]. The grey area is excluded by the constraints on the model and the green lines on the right figure are contour lines that correspond to our approximate formula while the black lines correspond to the full calculation.

The triple Higgs coupling $\lambda_{HHH}$ is a viable new (pseudo-) observable for the neutrino sector in order to constraint mass models, and might also be used in the context of the FCC-ee in an indirect way in $e^+e^- \rightarrow ZH$ at the 2-loop order, given the expected sensitivity the FCC-ee is supposed to reach in this channel. Studies remain to be done in this context.

The study presented in Ref. [33] considered a more direct observable, the production
Fig. E.6: ISS neutrino contributions to the process $\ell^+\ell^- \to W^+W^-H$ in the Feynman-'t Hooft gauge. Mirror diagrams can be obtained by flipping all the electric charges and the indices $i, j$ run from 1 to 9.

Fig. E.7: Left: Contour map of the neutrino corrections $\Delta_{\text{BSM}}$ at the 3 TeV CLIC, using a $-80\%$ polarised electron beam, as a function of the seesaw scale $M_R$ and $|Y_{\nu}|$. Right: Pseudo-rapidity distributions of the $W^+$ (black), $W^-$ (red) and Higgs (blue) bosons. The solid curves stand for the SM predictions while the dashed curves stand for the ISS predictions, for the benchmark scenario described in the text. Figures taken from Ref. [33].

cross section $\sigma(e^+e^- \to W^+W^-H)$ at lepton colliders. The setup is the same as in Refs. [32] albeit with an updated global fit using NuFIT 3.0 [37] to explain neutrino oscillations. The representative diagrams in the Feynman-'t Hooft gauge are displayed in Fig. E.6 with the contributions of the heavy neutral leptons in the $t$-channel.

The deviation $\Delta_{\text{BSM}}$ stands now for the comparison between the total cross section $\sigma(e^+e^- \to W^+W^-H)$ calculated in the ISS model and in the SM, $\Delta_{\text{BSM}} = (\sigma_{\text{ISS}} - \sigma_{\text{SM}})/\sigma_{\text{SM}}$. Using the CLIC baseline for the polarisation of the beams [2] with an unpolarised positron beam, $P_{e^+} = 0$, and a polarised electron beam, $P_{e^-} = -80\%$, the contour map at 3 TeV in the same $M_R - |Y_{\nu}|$ plane is presented in the left-hand side of Fig. E.7. Again the grey area is excluded by the constraints that mostly originate from the global fit [34]. The process $e^+e^- \to W^+W^-H$ exhibits sizeable negative deviations, of at least $-20\%$. Note that the full results can be approximated within $1\%$ for $M_R > 3$ TeV by the simple formulae presented in Ref. [33]. Compared to the left-hand side of Fig. E.5 the coverage of the parameter space is here much larger. Optimised cuts can also be chosen to enhance the deviation, such as the cuts $|\eta_{H/W}\ell| < 1$ and $E_H > 1$ TeV (see the right-hand side of Fig. E.7 for the $\eta$ distributions) that push the corrections down to $-66\%$ while keeping an ISS cross section at a reasonable level: 0.14 fb, to be compared to 1.23 fb before cuts. This has been studied for a benchmark
scenario with $|Y_{\nu}| = 1$ and heavy neutrinos in the range 2.4-8.6 TeV. The results means that this observable has a great potential that needs to be checked in a detailed sensitivity analysis. In the context of the FCC-ee, a similar observable could be chosen to test the effects of heavy neutral leptons in the same mass range, albeit at the one-loop level, namely the production cross section $\sigma(e^+e^- \rightarrow ZH)$.

1.3 Conclusions

This contribution has presented the current status of the triple Higgs coupling measurements at the LHC and the prospects for future lepton colliders. As combined studies in an EFT framework using precision measurements in single Higgs observables, as well as direct Higgs pair production, have shown, lepton colliders are able to completely remove the degeneracy in the measurement of the triple Higgs coupling beyond the 4$\sigma$ level, and the combination of data collected at a centre-of-mass energy of 250 GeV with data collected at energies of at least 350 GeV is of crucial importance with very high precision measurements in single Higgs physics. Opportunities offered by the Higgs sector to test neutrino mass models at future lepton colliders have also been presented. The FCC-ee is very competitive to test the heavy-sterile-neutrino option in the GeV regime. As far as the TeV regime for the heavy-neutrino scale is concerned, studies in the literature have shown that the CLIC and ILC at high energies could offer new avenues in the Higgs sector via precision measurements of the triple Higgs coupling as well as of the production cross section of a pair of $W$ bosons in association with a Higgs boson. In the same spirit the FCC-ee may well offer new opportunities in the same mass-regime via precision calculations at one- and two-loops for the $ZH$ production cross section, that remain to be studied.

References


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