Chapter A

Introduction and overview

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This report includes a collection of studies devoted to a discussion of (i) the status of theoretical efforts towards the calculation of higher-order Standard Model (SM) corrections needed for the FCC-ee precision measurement programme, (ii) the possibility of making discoveries in physics by means of these precision measurements, and (iii) methods and tools that must be developed to guarantee precision calculations of the observables to be measured. This report originates from presentations at the 11th FCC-ee Workshop: Theory and Experiments, 8–11 January 2019, CERN, Geneva [1], with 117 registered participants and 42 talks on theory.

1 The FCC-ee electroweak factory

In the 2018 report [2], we focused on theoretical issues of the FCC-ee Tera-Z, which will be a $e^+e^-$ collider working at the $Z$ resonance energy region. However, the FCC-ee collider project will work in several energy regions, making it a complete electroweak factory, covering the direct production of all massive bosons of the SM and the top quark. This plan is summarised in Table A.1.1.

Table A.1.1: Run plan for FCC-ee in its baseline configuration with two experiments. The WW event numbers are given for the entirety of the FCC-ee running at and above the WW threshold.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Run duration (years)</th>
<th>Centre-of-mass energies (GeV)</th>
<th>Integrated luminosity (ab$^{-1}$)</th>
<th>Event statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCC-ee-Z</td>
<td>4</td>
<td>88–95</td>
<td>150</td>
<td>$3 \times 10^{12}$ visible $Z$ decays</td>
</tr>
<tr>
<td>FCC-ee-W</td>
<td>2</td>
<td>158–162</td>
<td>12</td>
<td>$10^8$ WW events</td>
</tr>
<tr>
<td>FCC-ee-H</td>
<td>3</td>
<td>240</td>
<td>5</td>
<td>$10^6$ ZH events</td>
</tr>
<tr>
<td>FCC-ee-tt</td>
<td>5</td>
<td>345–365</td>
<td>1.7</td>
<td>$10^6$ $t\bar{t}$ events</td>
</tr>
</tbody>
</table>

The exceptional precision of the FCC-ee comes from several features of the programme.

1. Extremely high statistics of $5 \times 10^{12}$ $Z$ decays, $10^8$ WW, $10^6$ ZH, and $10^6$ $t\bar{t}$ events.

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2. High-precision (better than 100 keV) absolute determination of the centre-of-mass energies at the Z pole and WW threshold, thanks to the availability of transverse polarisation and the resonant depolarisation. This is a unique feature of the circular lepton colliders, $e^+e^-$ and $\mu^+\mu^-$. At higher energies, WW, ZZ, and $Z\gamma$ production can be used to constrain the centre-of-mass energy with precisions of 2 and 5 MeV, at the ZH cross-section maximum and at the $t\bar{t}$ threshold, respectively. At all energies, $e^+e^- \rightarrow \mu^+\mu^-$ events, which occur at a rate in excess of 3 kHz at the Z pole, provide, by themselves, in a matter of minutes, the determination of the centre-of-mass energy spread, the residual difference between the energies of $e^+$ and $e^-$ beams and (relative) centre-of-mass energy monitoring with a precision that is more than sufficient for the precision needs of the programme.

3. The clean environmental conditions and an optimised run plan allow a complete programme of ancillary measurements of currently precision-limiting input quantities for the precision EW tests. This is the case for the top quark mass from the scan of the $t\bar{t}$ production threshold; of the unique, direct, measurement of the QED running coupling constant at the Z mass from the $Z-\gamma$ interference; of the strong coupling constant by measurements of the hadronic-to-leptonic branching fractions of the Z, the W, and the $\tau$ lepton; and, of course, of the Higgs and Z masses themselves.

For the reader’s convenience, we also include Table A.1.2 from the CDR, showing some of the most significant FCC-ee experimental accuracies compared with those of the current measurements. More on the experimental precision of the FCC-ee can be found in volumes 1 and 2 of the CDR documents [3,4]. The experimenters are working hard to reduce systematic uncertainties by devising dedicated methods and ancillary measurements; the task of the theoretical community will be to ensure that the SM predictions will be precise enough so as not to spoil the best foreseeable experimental accuracies, i.e., the statistical uncertainties.

If future theory uncertainties match the FCC-ee experimental precision, the many different measurements from the FCC-ee will provide the capability of exhibiting and deciphering signs of new physics. Here are two examples: the EFT analysis searching for signs of heavy particles physics with SM couplings shows the potential to exhibit signs of new particles up to around 70 TeV; with a very different but characteristic pattern, observables involving neutrinos would show a significant deviation if these neutrinos were mixed with a heavy counterpart at the level of one part in 100 000, even if those were too heavy to be directly produced.

Table A.1.2 shows that the FCC-ee has the potential to achieve (at least) a 20–100 times higher precision or better in electroweak precision measurements over the present state-of-the-art situation. This includes such input quantities as the Z, Higgs, and top masses, and the strong and QED coupling constants at the Z scale. This extremely favourable situation will require leap-jumps in the precision of the theoretical computations for Standard Model phenomena, for all quantities given in Table A.1.2. The theory calculation must also be able to include the improved input parameters [2,5], which, in the particular case of the FCC-ee, will be measured within the experimental programme.

The quantities listed in Table A.1.2 are called electroweak precision observables (EWPO) and encapsulate experimental data after extraction of well-known and controllable QED and QCD effects, in a model-independent manner. They provide a convenient bridge between real data and the predictions of the SM, or of the SM plus new physics. Contrary to raw experimental data (like differential cross-sections), EWPOs are also well-suited for archiving and long-term use. Archived EWPOs can be exploited over long periods of time for comparisons with steadily
Table A.1.2: Measurement of selected electroweak precision observables (EWPOs) at the FCC-ee, compared with the current precision. The systematic uncertainties are initial estimates and might improve on further examination. This set of measurements, together with those of the Higgs properties, achieves indirect sensitivity to new physics up to a scale $\Lambda$ of 70 TeV in a description with dimension-6 operators, and possibly much higher in some specific new physics models.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Current value</th>
<th>± Error stat.</th>
<th>FCC-ee syst.</th>
<th>Comment, dominant experimental error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$ (keV)</td>
<td>91186700</td>
<td>± 2200</td>
<td>100</td>
<td>From $Z$ line shape scan, beam energy calibration</td>
</tr>
<tr>
<td>$\Gamma_Z$ (keV)</td>
<td>2495200</td>
<td>± 2300</td>
<td>7</td>
<td>From $Z$ line shape scan, beam energy calibration</td>
</tr>
<tr>
<td>$R_Z^Z$ $(\times 10^3)$</td>
<td>20767</td>
<td>± 25</td>
<td>0.06</td>
<td>Ratio of hadrons to leptons, acceptance for leptons</td>
</tr>
<tr>
<td>$\alpha_s(m_Z) (\times 10^4)$</td>
<td>1196</td>
<td>± 30</td>
<td>0.1</td>
<td>From $R_Z^Z$</td>
</tr>
<tr>
<td>$R_b$ $(\times 10^6)$</td>
<td>216290</td>
<td>± 660</td>
<td>0.3</td>
<td>&lt;60 Ratio of $b\bar{b}$ to hadrons, stat. extrapolated from SLD</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}^0 (\times 10^3)$ (nb)</td>
<td>41541</td>
<td>± 37</td>
<td>0.1</td>
<td>Peak hadronic cross-section, luminosity measurement</td>
</tr>
<tr>
<td>$N_\nu (\times 10^3)$</td>
<td>2991</td>
<td>± 7</td>
<td>0.005</td>
<td>1 Z peak cross-sections, luminosity measurement</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}$ $(\times 10^6)$</td>
<td>231480</td>
<td>± 160</td>
<td>3</td>
<td>2–5 From $A_{\mu\mu}^{0\nu}$ from $A_{\mu\mu}^{0\nu}$ at $Z$ peak, beam energy calibration</td>
</tr>
<tr>
<td>$1/\alpha_{\text{QED}}(m_Z)(\times 10^3)$</td>
<td>128952</td>
<td>± 14</td>
<td>4</td>
<td>Small From $A_{FB}^{0\nu}$ off peak</td>
</tr>
<tr>
<td>$A_{FB}^{0\nu}$</td>
<td>992</td>
<td>± 16</td>
<td>0.02</td>
<td>1-3 $b$ quark asymmetry at $Z$ pole, from jet charge</td>
</tr>
<tr>
<td>$A_{FB}^{0\nu\tau}$ $(\times 10^4)$</td>
<td>1498</td>
<td>± 49</td>
<td>0.15</td>
<td>&lt;2 $\tau$ polarisation and charge asymmetry, $\tau$ decay physics</td>
</tr>
<tr>
<td>$m_W$ (MeV)</td>
<td>80350</td>
<td>± 15</td>
<td>0.5</td>
<td>From WW threshold scan, beam energy calibration</td>
</tr>
<tr>
<td>$\Gamma_W$ (MeV)</td>
<td>2085</td>
<td>± 42</td>
<td>1.2</td>
<td>0.3 From WW threshold scan, beam energy calibration</td>
</tr>
<tr>
<td>$\alpha_s(m_W)(\times 10^4)$</td>
<td>1170</td>
<td>± 420</td>
<td>3</td>
<td>Small From $R^W$</td>
</tr>
<tr>
<td>$N_\nu (\times 10^3)$</td>
<td>2920</td>
<td>± 50</td>
<td>0.8</td>
<td>Small Ratio of invisible to leptonic, in radiative $Z$ returns</td>
</tr>
<tr>
<td>$m_{\text{top}}$ (MeV/$c^2$)</td>
<td>172740</td>
<td>± 500</td>
<td>17</td>
<td>Small From $t\bar{t}$ threshold scan, QCD errors dominate</td>
</tr>
<tr>
<td>$\Gamma_{\text{top}}$ (MeV/$c^2$)</td>
<td>1410</td>
<td>± 190</td>
<td>45</td>
<td>Small From $t\bar{t}$ threshold scan, QCD errors dominate</td>
</tr>
<tr>
<td>$\lambda_{\text{top}}/\lambda_{\text{top}}^{\text{SM}}$</td>
<td>1.2</td>
<td>± 0.3</td>
<td>0.10</td>
<td>Small From $t\bar{t}$ threshold scan, QCD errors dominate</td>
</tr>
<tr>
<td>$t\bar{t}Z$ couplings</td>
<td>± 30%</td>
<td>0.5 – 1.5%</td>
<td>Small</td>
<td>From $E_{\text{CM}} = 365$ GeV run</td>
</tr>
</tbody>
</table>
improving theoretical calculations of the SM predictions, and for validations of the new physics models beyond the SM. They are also useful for the comparison and combination of results from different experiments. However, removing trivial but sizeable QED or QCD effects from EWPOs might induce additional sources of uncertainty. The work needed is well-known concerning QED, more significant conceptual work may need to be done for QCD.

Let us summarise briefly the mandatory improvements of the calculations of QED effects in EWPOs according to recent work [6]:

1. improved calculation of the additional light fermion pair emissions (for Z boson mass and width);
2. better calculation of the final-state radiation effects in the presence of cut-offs (for \( R^Z_\ell \));
3. implementation of a new QED matrix element in the Monte Carlo (MC) event generator for low-angle Bhabha processes (for the luminosity determination in view of the measurement of \( \sigma^0_{\text{had}} \) and other cross-sections);
4. \( \mathcal{O}(\alpha^2) \) calculation for \( e^+e^- \rightarrow Z\gamma \) (for the determination of \( N_v \));
5. improved MC simulation of \( \tau \) decays (for the effective weak mixing angle and tau branching ratio measurements);
6. QED effects at the W pair production threshold (for measurement of the W mass and width);
7. initial–final-state interference (e.g., for the forward–backward charge asymmetry of lepton pairs around the Z peak).

For more on the related subject of the separation of QED effects from weak quantities at the FCC-ee precision and generally on the improvements in the definition of EWPOs, see recent discussions in Ref. [2]. A similar systematic discussion of the QCD effects in EWPOs is in progress, see Ref. [2] and Section B.2 in this report.

For the FCC-ee data analysis, owing to the rise of non-factorisable QED effects above the experimental uncertainties, direct use of MC programs might become the standard for fitting EWPOs to the data, even at the Tera-Z stage [2,6,7]. New MC event generators will have to provide built-in provisions for an efficient direct fitting of EWPOs to data, which are not present in the LEP legacy MCs. Section C.3 of Ref. [2] describes possible forms of future EWPOs at FCC-ee experiments and specifies the new required MC software. It is emphasized there that, owing to non-factorisable QED contributions, the multiphoton QED effects will have to be factorised at the amplitude level. Additional quantities available in tau and heavy flavour physics will reach \( 10^{-5} \) precision and are likely to need similar attention.

Very precise determinations of \( M_W \) at the FCC-ee will rely on the precise measurement of the cross-section of the \( e^+e^- \rightarrow W^+W^- \) process near the threshold. A statistical precision of 0.04\% of this cross-section translates into 0.6 MeV experimental uncertainty on \( M_W \), compared with the current 3 MeV theoretical uncertainty for \( M_W \). Therefore, improved theoretical calculations are required for the generic \( e^+e^- \rightarrow 4f \) process near the WW threshold with an improvement of one order of magnitude. The most economical solution will be to combine the \( \mathcal{O}(\alpha^1) \) calculation for the \( e^+e^- \rightarrow 4f \) process with the \( \mathcal{O}(\alpha^2) \) calculation for the doubly resonant \( e^+e^- \rightarrow W^+W^- \) subprocess. The former calculation is already available [8]. The latter will need
to be developed; inclusion of the resummed QED corrections will be mandatory. For details, see Chapter B and Ref. [9].

In the case of the FCC-ee-H, $M_H$ will be obtained from the $e^+e^- \rightarrow HZ$ process with a precision better than 10 MeV [3,10]. Theory uncertainties (mainly owing to final-state radiation effects) will be subdominant. The main focus will be on calculations of Higgs boson branching ratios and self-couplings. See Chapters B and E.

The anticipated experimental uncertainty on the $m_t$ measurement at FCC-ee-tt [2] is $\mathcal{O}(20)$ MeV. On the theory side, there are several sources of uncertainties: (i) the perturbative uncertainty for the calculation of the threshold shape with higher-order QCD corrections; (ii) the threshold mass definition translated into the $\overline{\text{MS}}$ scheme; and (iii) the precision of $\alpha_s$. Combining these three sources of uncertainty, a theoretical uncertainty close to the experimental one and less than 50 MeV for $m_t$ appears feasible. In addition, a very accurate determination of the efficiency of experimental acceptances and selection cuts is needed. This task will require the inclusion of higher-order corrections and resummation results in a Monte Carlo event generator; next-to-leading-order (NLO) QCD corrections for off-shell $t\bar{t}$ production, and matching between these contributions, complement previous semi-analytic results.

In this report, we are especially interested in the discussion of input parameters and of EWPOs connected with W, H, and top production physics. These are masses of heavy SM particles, their couplings, and also $\alpha_{\text{QED}}$ and $\alpha_{\text{QCD}}$, which, as running quantities, must be adjusted carefully at the considered high-energy regions. These issues will be discussed in this report.

2 What this theory report brings: an overview

The report is divided into four basic chapters. Both the workshop and this report are mainly devoted to precision theoretical calculations. It is a most important subject because the value of most of the FCC-ee experimental analyses relies on the precision of the Standard Model and BSM predictions.

In Chapter B, the status and prospects for measurements and determination of $\alpha_{\text{QED}}$ and $\alpha_s$ at the FCC-ee are given, but also issues of QED and QCD resummations, an EFT radiative correction approach to W boson production, heavy quarkonia, analysis of the weak mixing angle from data (important, as it definitely has non-perturbative effects different from those in $\alpha$), QCD vertex functions beyond two loops, EFT and QED in flavour physics, top pair production and mass determination, and a summary of SM precision predictions for partial Higgs decay widths.

In Chapter C, numerical and analytical methods for precision multiloop calculations are presented and recent advances in the field are discussed. The chapter is an addition to the 2018 report [2]. We mentioned already that Monte Carlo generators are very important, as they link pure experimental data with theory. Generators for precision $e^+e^-$ simulations, $\tau$, top, and W boson physics, heritage projects, and the need for proper software preservation with Monte Carlo generators are also discussed in Chapter C.

Chapter D consists of only one contribution. SMEFT theory is a bridge between SM physics and the analysis of extended gauge models. The chapter is connected with this issue and a specific code is presented. For another discussion, see the talk by J. de Blas [12].

*Examples show that estimations of higher-order corrections can differ from actual calculations by factors of three to five [7,11].

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In Chapter E, finally, three contributions are collected, about Higgs models that go beyond the Standard Model theory.

References


