THE STANDARD ELECTROWEAK THEORY
AND ITS EXPERIMENTAL TESTS

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Lectures given at the
LVth Les Houches Summer School, France
30 June - 26 July, 1991
1. INTRODUCTION

These lectures on electroweak interactions start with a short summary of the Glashow–Weinberg–Salam theory [1] and then cover in detail the main subjects of present interest in phenomenology: the Higgs sector and the open problem of the experimental investigation on the origin of the Fermi scale of mass $G_F^{-1/2}$; the structure of radiative corrections; the strategy for the experimental verification of the theory; and finally a discussion of the status of LEP physics. The important domain of flavour-changing processes and of CP violation is not reviewed here (see the course given by G. Martinelli).

The modern electroweak theory inherits the phenomenological successes of the $(V-A)\otimes(V-A)$ four-fermion low-energy description of weak interactions [2], and provides a well-defined and consistent theoretical framework including weak interactions and quantum electrodynamics in a unified picture.

As an introduction, in the following we recall some salient physical features of the weak interactions. The weak interactions derive their name from their intensity. At low energy the strength of the effective four-fermion interaction of charged currents is determined by the Fermi coupling constant $G_F$. For example, the effective interaction for muon decay is given by

$$\mathcal{L}_{\text{eff}} = (G_F/\sqrt{2})[\bar{\nu}_\mu \gamma^\mu(1-\gamma_5)\mu][\bar{e} \gamma^\nu(1-\gamma_5)e_e]$$

(1.1)

with [3]

$$G_F = 1.16637(2) \times 10^{-5} \text{ GeV}^{-2}$$

(1.2)

In natural units $\hbar = c = 1$, $G_F$ has dimensions of $(\text{mass})^{-2}$. As a result, the intensity of weak interactions at low energy is characterized by $G_F E^2$, where $E$ is the energy scale for a given process ($E \approx m_\mu$ for muon decay). Since

$$G_F E^2 = G_F m_\mu^2 (E/m_\mu)^2 \approx 10^{-5} (E/m_\mu)^2$$

(1.3)

where $m_\mu$ is the proton mass, the weak interactions are indeed weak at low energies (of order $m_\mu$). The quadratic increase with energy cannot continue for ever, because it would lead to a violation of unitarity. In fact, at large energies the propagator effects can no longer be neglected, and the current-current interaction is resolved into current-$W$ gauge boson vertices connected by a $W$ propagator. The strength of the weak interactions at high energies is then measured by $g_W$, the $W-\mu-\nu_\mu$ coupling, or, even better, by $\alpha_W = g_W^2/4\pi$ analogous to the fine-structure constant $\alpha$ of QED. In the standard electroweak theory, we have

$$\alpha_W = \sqrt{2} G_F m_\mu^2 / \tau = \alpha/\sin^2 \theta_W \equiv 1/30$$

(1.4)
That is, at high energies the weak interactions are no longer so weak.

The range \( r_W \) of weak interactions is very short: it is only with the experimental discovery of the \( W \) and \( Z \) gauge bosons that it could be demonstrated that \( r_W \) is non-vanishing. Now we know that

\[
r_W = \frac{\hbar}{m_W c} \approx 2.5 \times 10^{-16} \text{ cm},
\]

(1.5)
corresponding to \( m_W \approx 80 \text{ GeV} \). This very large value for the \( W \) (or the \( Z \)) mass makes a drastic difference, compared with the massless photon and the infinite range of the QED force. The experimental limits on the photon mass \([3]\) are listed in the following. From a laboratory experiment, one obtains \( m_\gamma < 10^{-14} \text{ eV} \) by a method based on the vanishing of the electric field inside a cavity with conducting walls, predicted by Gauss' law. In fact, the exact \( r^{-2} \) behaviour of the electric field corresponds to \( m_\gamma = 0 \). From the observed distribution of planetary magnetic fields (the field should be damped by an extra factor \( e^{-m_\gamma r} \) if \( m_\gamma \neq 0 \)) the Pioneer probe to Jupiter obtained \( m_\gamma < 6 \times 10^{-18} \text{ eV} \). Finally, indirect evidence from galactic magnetic fields indicates that \( m_\gamma < 3 \times 10^{-27} \text{ eV} \). Thus, on the one hand, there is very good evidence that the photon is massless. On the other hand, the weak bosons are very heavy. A unified theory of electroweak interactions has to face this striking difference.

Another apparent obstacle in the way of electroweak unification is the chiral structure of weak interactions: in the massless limit for fermions, only left-handed quarks and leptons (and right-handed antiquarks and antileptons) are coupled to \( W \)’s. This clearly implies parity and charge-conjugation violation in weak interactions.

The universality of weak interactions and the algebraic properties of the electromagnetic and weak currents [the conservation of vector currents (CVC), the partial conservation of axial currents (PCAC), the algebra of currents, etc.] have been crucial in pointing to a symmetric role of electromagnetism and weak interactions at a more fundamental level. The old Cabibbo universality for the weak charged current [4]:

\[
J^{\text{weak}}_\alpha = \bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \mu + \bar{\nu}_\tau \gamma_\alpha (1 - \gamma_5) \tau + \cos \theta_c \, \bar{u} \gamma_\alpha (1 - \gamma_5) d + \sin \theta_c \, \bar{u} \gamma_\alpha (1 - \gamma_5) s + \ldots,
\]

(1.6)
suitably extended, is naturally implied by the standard electroweak theory. In this theory the weak gauge bosons couple to all particles with couplings that are proportional to their weak charges, in the same way as the photon couples to all particles in proportion to their electric charges (\( d' = \cos \theta_c \, d + \sin \theta_c \, s \) is the weak-isospin partner of \( u \) in a doublet).

Another crucial feature is that the charged weak interactions are the only known interactions that can change flavour: charged leptons into neutrinos.
or up-type quarks into down-type quarks. On the contrary, there are no flavour-changing neutral currents at tree level. This is a remarkable property of the weak neutral current, which is explained by the introduction of the GIM mechanism [5] and has led to the successful prediction of charm.

The natural suppression of flavour-changing neutral currents, the separate conservation of $e, \mu$ and $\tau$ leptonic flavours, the mechanism of CP violation [6] through the phase in the quark-mixing matrix, are all crucial features of the Standard Model. Many examples of new physics tend to break the selection rules of the standard theory. Thus the experimental study of rare flavour-changing transitions is an important window on possible new physics.

In the following sections we shall see how these properties of weak interactions fit into the standard electroweak theory.

2. GAUGE THEORIES

In this section we summarize the definition and the structure of a gauge Yang–Mills theory [7],[8]. We will list here the general rules for constructing such a theory. Then in the next section these results will be applied to the electroweak theory.

Consider a Lagrangian density $L[\phi, \partial_\mu \phi]$ which is invariant under a $D$ dimensional continuous group of transformations:

$$\phi' = U(\theta^A)\phi \quad (A = 1, 2, ..., D). \quad (2.1)$$

For $\theta^A$ infinitesimal, $U(\theta^A) = 1 + ig \sum \theta^A T^A$, where $T^A$ are the generators of the group $\Gamma$ of transformations (2.1) in the (in general reducible) representation of the fields $\phi$. Here we restrict ourselves to the case of internal symmetries, so that $T^A$ are matrices that are independent of the space–time coordinates. The generators $T^A$ are normalized in such a way that for the lowest dimensional non-trivial representation of the group $\Gamma$ (we use $t^A$ to denote the generators in this particular representation) we have

$$\text{tr}(t^A t^B) = 1/2 \delta^{AB}. \quad (2.2)$$

The generators satisfy the commutation relations

$$[T^A, T^B] = ig C_{ABC} T^C. \quad (2.3)$$

In the following, for each quantity $V^A$ we define

$$V = \sum_A T^A V^A. \quad (2.4)$$
If we now make the parameters $\theta^A$ depend on the space–time coordinates $\theta^A = \theta^A(x, \mu)$, $\mathcal{L}[\phi, D_\mu \phi]$ is in general no longer invariant under the gauge transformations $U[\theta^A(x, \mu)]$, because of the derivative terms. Gauge invariance is recovered if the ordinary derivative is replaced by the covariant derivative:

$$D_\mu = \partial_\mu + igV_\mu,$$

(2.5)

where $V^A_\mu$ are a set of $D$ gauge fields (in one-to-one correspondence with the group generators) with the transformation law

$$V'_\mu = UV_\mu U^{-1} - (1/ig)(\partial_\mu U)U^{-1}.$$

(2.6)

For constant $\theta^A$, $V$ reduces to a tensor of the adjoint (or regular) representation of the group:

$$V'_\mu = UV_\mu U^{-1} = V_\mu + ig[\theta, V_\mu],$$

(2.7)

which implies that

$$V'^C_\mu = V^C_\mu - gC_{ABC}V^A_\mu V^B_\mu,$$

(2.8)

where repeated indices are summed up.

As a consequence of Eqs. (2.5) and (2.6), $D_\mu \phi$ has the same transformation properties as $\phi$:

$$(D_\mu \phi)' = U(D_\mu \phi).$$

(2.9)

Thus $\mathcal{L}[\phi, D_\mu \phi]$ is indeed invariant under gauge transformations. In order to construct a gauge-invariant kinetic energy term for the gauge fields $V^A$, we consider

$$[D_\mu, D_\nu]\phi = ig(\partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu])\phi \equiv igF_{\mu\nu}\phi,$$

(2.10)

which is equivalent to

$$F^A_{\mu\nu} = \partial_\mu V^A_\nu - \partial_\nu V^A_\mu - gC_{ABC}V^B_\mu V^C_\nu.$$

(2.11)

From Eqs. (2.1), (2.9) and (2.10) it follows that the transformation properties of $F^A_{\mu\nu}$ are those of a tensor of the adjoint representation

$$F'_{\mu\nu} = UF_{\mu\nu}U^{-1}.$$

(2.12)

The complete Yang–Mills Lagrangian, which is invariant under gauge transformations, can be written in the form

$$\mathcal{L}_{YM} = -\frac{1}{g^2} \sum_A F^A_{\mu\nu} F^{A\mu\nu} + \mathcal{L}[\phi, D_\mu \phi].$$

(2.13)
For an Abelian theory, as for example QED, the gauge transformation reduces to \( U[\theta(x)] = \exp[i e Q \theta(x)] \), where \( Q \) is the charge generator. The associated gauge field (the photon), according to Eq. (2.6), transforms as

\[
V'_\mu = V_\mu - \partial_\mu \theta(x). \tag{2.14}
\]

In this case, the \( F_{\mu\nu} \) tensor is linear in the gauge field \( V_\mu \) so that in the absence of matter fields the theory is free. On the other hand, in the non-Abelian case the \( F_{\mu\nu}^A \) tensor contains both linear and quadratic terms in \( V_\mu^A \), so that the theory is non-trivial even in the absence of matter fields.

3. THE STANDARD MODEL OF THE ELECTROWEAK INTERACTIONS

In this section, we summarize the structure of the standard electroweak Lagrangian and specify the couplings of \( W^\pm \) and \( Z \), the intermediate vector bosons (IVBs).

For this discussion we split the Lagrangian into two parts by separating the Higgs boson couplings:

\[
\mathcal{L} = \mathcal{L}_{\text{symm}} + \mathcal{L}_{\text{Higgs}}. \tag{3.1}
\]

We start by specifying \( \mathcal{L}_{\text{symm}} \), which involves only gauge bosons and fermions:

\[
\mathcal{L}_{\text{symm}} = -\frac{1}{4} \sum_{A=1}^{3} F^A_{\mu\nu} F^{A\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi}_L i \gamma^\mu D_\mu \psi_L
\]

\[
+ \bar{\psi}_R i \gamma^\mu D_\mu \psi_R. \tag{3.2}
\]

This is the Yang–Mills Lagrangian for the gauge group \( SU(2) \otimes U(1) \) with fermion matter fields. Here

\[
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \text{and} \quad F^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu
\]

\[
- g \epsilon_{ABC} W^B_\mu W^C_\nu \tag{3.3}
\]

are the gauge antisymmetric tensors constructed out of the gauge field \( B_\mu \) associated with \( U(1) \), and \( W^A_\mu \) corresponding to the three \( SU(2) \) generators; \( \epsilon_{ABC} \) are the group structure constants [see Eqs. (2.3)] which, for \( SU(2) \), coincide with the totally antisymmetric Levi-Civita tensor (recall the familiar angular momentum commutators). The normalization of the \( SU(2) \) gauge coupling \( g \) is therefore specified by Eq. (3.3).

The fermion fields are described through their left-hand and right-hand components:

\[
\psi_{L,R} = [(1 \mp \gamma_5)/2] \psi, \quad \bar{\psi}_{L,R} = \bar{\psi}[(1 \pm \gamma_5)/2], \tag{3.4}
\]
with \( \gamma_5 \) and other Dirac matrices defined as in the book by Bjorken–Drell [9]. In particular, \( \gamma_3^t = 1, \gamma_4^t = \gamma_5 \). Note that, as given in Eq. (3.4),
\[
\tilde{\psi}_L = \psi_L^\dagger \gamma_0 = \psi_L^\dagger [(1 - \gamma_5)/2] \gamma_0 = \psi_L^\dagger [(1 + \gamma_5)/2] \gamma_0 = \tilde{\psi}_L (1 - \gamma_3)/2.
\]
The matrices \( P_\pm = (1 \pm \gamma_5)/2 \) are projectors. They satisfy the relations \( P_\pm P_\mp = P_\pm, P_\pm^2 = 0, P_+ + P_- = 1 \).

The sixteen linearly independent Dirac matrices can be divided into \( \gamma_5 \)-even and \( \gamma_5 \)-odd according to whether they commute or anticommute with \( \gamma_5 \). For the \( \gamma_5 \)-even, we have
\[
\tilde{\psi}_L \Gamma_E \psi = \tilde{\psi}_L \Gamma_E \psi_R + \tilde{\psi}_R \Gamma_E \psi_L \quad (\Gamma_E \equiv 1, i\gamma_5, \sigma_{\mu\nu}), \tag{3.5}
\]
whilst for the \( \gamma_5 \)-odd,
\[
\tilde{\psi}_L \Gamma_O \psi = \tilde{\psi}_L \Gamma_O \psi_R + \tilde{\psi}_R \Gamma_O \psi_L \quad (\Gamma_O \equiv \gamma_\mu, \gamma_\mu \gamma_5). \tag{3.6}
\]

In the Standard Model the left and right fermions have different transformation properties under the gauge group. Thus, mass terms for fermions (of the form \( \psi_L \psi_R + \text{h.c.} \)) are forbidden in the symmetric limit. In particular, all \( \psi_R \) are singlets in the minimal Standard Model. But for the moment, by \( \psi_R \) we mean a column vector, including all fermions in the theory that span a generic reducible representation of \( SU(2) \otimes U(1) \). The standard electroweak theory is a chiral theory, in the sense that \( \psi_L \) and \( \psi_R \) behave differently under the gauge group. In the absence of mass terms, there are only vector and axial vector interactions in the Lagrangian that have the property of not mixing \( \psi_L \) and \( \psi_R \). Fermion masses will be introduced, together with \( W^\pm \) and \( Z \) masses, by the mechanism of symmetry breaking. The covariant derivatives \( D_\mu \psi_{L,R} \) are explicitly given by
\[
D_\mu \psi_{L,R} = \left[ \partial_\mu + ig \sum_{A=1}^3 t_{L,R}^A W_\mu^A + ig^\prime \frac{1}{2} Y_{L,R} B_\mu \right] \psi_{L,R}, \tag{3.7}
\]
where \( t_{L,R}^A \) and \( 1/2 Y_{L,R} \) are the \( SU(2) \) and \( U(1) \) generators, respectively, in the reducible representations \( \psi_{L,R} \). The commutation relations of the \( SU(2) \) generators are given by
\[
[t_{L,R}^A, t_{L,R}^B] = i \epsilon_{ABC} t_{L,R}^C \quad \text{and} \quad [t_{L,R}^A, t_{L,R}^B] = i \epsilon_{ABC} t_{L,R}^C. \tag{3.8}
\]
We use the normalization (2.2) [in the fundamental representation of \( SU(2) \)]. The electric charge generator \( Q \) (in units of \( e \), the positron charge) is given by
\[
Q = t_L^3 + 1/2 \ Y_L = t_R^3 + 1/2 \ Y_R. \tag{3.9}
\]
Note that the normalization of the $U(1)$ gauge coupling $g'$ in (3.7) is now specified as a consequence of (3.9).

All fermion couplings to the gauge bosons can be derived directly from Eqs. (3.2) and (3.7). The charged-current (CC) couplings are the simplest. From

$$g(t^1W^1_\mu + t^2W^2_\mu) = g \left\{ \left[ (t^1 + it^2)/(\sqrt{2}) \right] (W^1_\mu - iW^2_\mu)/\sqrt{2} + h.c. \right\} = g \left\{ \left[ (t^+W^-_\mu)/\sqrt{2} \right] + h.c. \right\},$$

(3.10)

where $t^\pm = t^1 \pm it^2$ and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$, we obtain the vertex

$$V_{\bar{\psi}W} = g\bar{\psi}\gamma_\mu \left[ \left( t^+_\mu/\sqrt{2} \right) (1 - \gamma_5)/2 + \left( t^+_\mu/\sqrt{2} \right) (1 + \gamma_5)/2 \right] \times \psi W^-_\mu + h.c.$$ 

(3.11)

In the neutral-current (NC) sector, the photon $A_\mu$ and the mediator $Z_\mu$ of the weak NC are orthogonal and normalized linear combinations of $B_\mu$ and $W^3_\mu$:

$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W^3_\mu,$$

$$Z_\mu = - \sin \theta_W B_\mu + \cos \theta_W W^3_\mu.$$ 

(3.12)

Equations (3.12) define the weak mixing angle $\theta_W$. The photon is characterized by equal couplings to left and right fermions with a strength equal to the electric charge. Recalling Eq. (3.9)for the charge matrix $Q$, we immediately obtain

$$g \sin \theta_W = g' \cos \theta_W = e,$$

(3.13)

or equivalently,

$$\tan \theta_W = g'/g.$$ 

(3.14)

Once $\theta_W$ has been fixed by the photon couplings, it is a simple matter of algebra to derive the $Z$ couplings, with the result

$$\Gamma_{\bar{\psi}W} = g/(2 \cos \theta_W) \bar{\psi} \gamma_\mu$$

$$\times \left[ \left( t^3_\mu (1 - \gamma_5) + t^3_\mu (1 + \gamma_5) - 2Q \sin^2 \theta_W \right) \psi Z_\mu \right],$$

(3.15)

where $\Gamma_{\bar{\psi}W}$ is a notation for the vertex. In the minimal Standard Model, $t^3_\mu = 0$ and $t^3_\mu = \pm 1/2$.

In order to derive the effective four-fermion interactions that are equivalent, at low energies, to the CC and NC couplings given in Eqs. (3.11) and (3.15), we anticipate that large masses, as experimentally observed,
are provided for $W^\pm$ and $Z$ by $\mathcal{L}_{\text{higgs}}$. For left–left CC couplings, when the momentum transfer squared can be neglected with respect to $m_W^2$ in the propagator of Born diagrams with single $W$ exchange, from Eq. (3.11) we can write

$$\mathcal{L}_{\text{eff}}^{CC} \simeq (g^2/8m_W^4)[\bar{\psi} \gamma_\mu (1 - \gamma_5)t_\mu^+ \psi][\bar{\psi} \gamma^\nu (1 - \gamma_5)t_\nu^- \psi]. \quad (3.16)$$

By specializing further in the case of doublet fields such as $\nu_e - e^-$ or $\nu_\mu - \mu^-$, we obtain the tree-level relation of $g$ with the Fermi coupling constant $G_F$ measured from $\mu$ decay [see Eq. (1.2)]:

$$G_F/\sqrt{2} = g^2/8m_W^2. \quad (3.17)$$

By recalling that $g \sin \theta_W = \epsilon$, we can also cast this relation in the form

$$m_W = \mu_{\text{Born}}/\sin \theta_W, \quad (3.18)$$

with

$$\mu_{\text{Born}} = (\pi \alpha/\sqrt{2}G_F)^{1/2} \simeq 37.2802 \text{ GeV}, \quad (3.19)$$

where $\alpha$ is the fine-structure constant of QED ($\alpha \equiv e^2/4\pi = 1/137.036$).

In the same way, for neutral currents we obtain in Born approximation from Eq. (3.15) the effective four-fermion interaction given by

$$\mathcal{L}_{\text{eff}}^{NC} \simeq \sqrt{2} G_F \rho_0 \bar{\psi} \gamma_\mu [\ldots] \psi \bar{\psi} \gamma^\nu [\ldots] \psi, \quad (3.20)$$

where

$$[\ldots] \equiv t_\mu^+(1 - \gamma_5) + t_\nu^-(1 + \gamma_5) - 2Q \sin^2 \theta_W \quad (3.21)$$

and

$$\rho_0 = m_W^2/m_Z^2 \cos^2 \theta_W. \quad (3.22)$$

All couplings given in this section are obtained at tree level and are modified in higher orders of perturbation theory. In particular, the relations between $m_W$ and $\sin \theta_W$ [Eqs. (3.18) and (3.19)] and the observed values of $\rho (\rho = \rho_0$ at tree level) in different NC processes, are altered by computable electroweak radiative corrections, as discussed in Section 7.

The gauge-boson self-interactions can be derived from the $F_{\mu\nu}$ term in $\mathcal{L}_{\text{symm}}$, by using Eq. (3.12) and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$. Defining the three-gauge-boson vertex as in Fig. 1, we obtain ($V \equiv \gamma, Z$)

$$\Gamma_{W^- W^+ \nu} = ig_{W^- W^+ \nu}[g_{\mu\nu}(q-p)_\lambda + g_{\mu\lambda}(p-r)_\nu + g_{\nu\lambda}(r-q)_\mu], \quad (3.23)$$

with

$$g_{W^- W^+ \gamma} = g \sin \theta_W = \epsilon \quad \text{and} \quad g_{W^- W^+ Z} = g \cos \theta_W. \quad (3.24)$$
We now turn to the Higgs sector [10] of the electroweak Lagrangian. Here we simply review the formalism of the Higgs mechanism applied to the electroweak theory. In the next section we shall make a more general and detailed discussion of the physics of the electroweak symmetry breaking. The Higgs Lagrangian is specified by the gauge principle and the requirement of renormalizability to be

$$\mathcal{L}_{\text{Higgs}} = (D_a \phi)^\dagger (D^a \phi) - V(\phi^\dagger \phi) - \bar{\psi}_L \Gamma^a \psi_R \phi - \bar{\psi}_R \Gamma^a \psi_L \phi^\dagger, \quad (3.25)$$

where $\phi$ is a column vector including all Higgs fields; it transforms as a reducible representation of the gauge group. The quantities $\Gamma$ (which include all coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz and gauge groups. The potential $V(\phi^\dagger \phi)$, symmetric under $SU(2) \times U(1)$, contains, at most, quartic terms in $\phi$ so that the theory is renormalizable. Spontaneous symmetry breaking is induced if the minimum of $V$ — which is the classical analogue of the quantum mechanical vacuum state (both are the states of minimum energy) — is obtained for non-vanishing $\phi$ values. Precisely, we denote the vacuum expectation value (VEV) of $\phi$, i.e. the position of the minimum, by $v$:

$$\langle 0 | \phi(x) | 0 \rangle = v \neq 0. \quad (3.26)$$

The fermion mass matrix is obtained from the Yukawa couplings by replacing $\phi(x)$ by $v$:

$$M = \bar{\psi}_L \mathcal{M} \psi_R + \bar{\psi}_R \mathcal{M}^\dagger \psi_L, \quad (3.27)$$

with

$$\mathcal{M} = \Gamma \cdot v. \quad (3.28)$$

In the minimal Standard Model, where all left fermions $\psi_L$ are doublets and all right fermions $\psi_R$ are singlets, only Higgs doublets can contribute to fermion masses. There are enough free couplings in $\Gamma$, so that one single
complex Higgs doublet is indeed sufficient to generate the most general fermion mass matrix. It is important to observe that by a suitable change of basis we can always make the matrix $\mathcal{M}$ Hermitian, $\gamma_5$-free, and diagonal. In fact, we can make separate unitary transformations on $\psi_L$ and $\psi_R$ according to

$$\psi'_L = U \psi_L, \quad \psi'_R = V \psi_R$$

and consequently

$$\mathcal{M} \rightarrow \mathcal{M}' = U^\dagger \mathcal{M} V .$$

This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}_{\text{symm}}$. For quarks, the Cabibbo–Kobayashi–Maskawa [4],[11] unitary transformation relates the mass eigenstates $d, s$ and $b$ to the CC eigenstates $d', s', b'$, i.e. the states coupled by $W$ emission to $u, c, t$, respectively. The NC is then automatically diagonal in flavour at tree level (GIM mechanism [5]). In the case of leptons, if the neutrinos are massless then clearly there is no mixing.

If only one Higgs doublet is present, the change of basis that makes $\mathcal{M}$ diagonal will at the same time diagonalize also the fermion–Higgs Yukawa couplings. Thus, in this case, no flavour-changing neutral Higgs exchanges are present. This is not true, in general, when there are several Higgs doublets. But one Higgs doublet for each electric charge sector i.e. one doublet coupled only to $u$-type quarks, one doublet to $d$-type quarks, one doublet to charged leptons would also be all right [12], because the mass matrices of fermions with different charges are diagonalized separately. For several Higgs doublets it is also possible to generate CP violation by complex phases in the Higgs couplings [13]. In the presence of six quark flavours, this CP-violation mechanism is not necessary. In fact, at the moment, the simplest model with only one Higgs doublet seems adequate for describing all observed phenomena.

We recall that the Standard Model, with $N$ fermion families with the observed quantum numbers, is automatically free of $\gamma_5$ anomalies [14] owing to cancellation of quarks with lepton loops.

We now consider the gauge-boson masses and their couplings to the Higgs. These effects are induced by the $(D_\mu \phi)^\dagger (D^\mu \phi)$ term in $\mathcal{L}_{\text{Higgs}}$ [Eq. (3.25)], where

$$D_\mu \phi = \left[ \partial_\mu + ig \sum_{A=1}^3 t^A W^A_\mu + ig' (Y/2) B_\mu \right] \phi .$$
Here $t^A$ and $1/2Y$ are the $SU(2) \otimes U(1)$ generators in the reducible representation spanned by $\phi$. Not only doublets but all non-singlet Higgs representations can contribute to gauge-boson masses. The condition that the photon remains massless is equivalent to the condition that the vacuum is electrically neutral:

$$Q|v\rangle = (t^3 + 1/2 Y)|v\rangle = 0.$$  \hfill (3.32)

The charged $W$ mass is given by the quadratic terms in the $W$ field arising from $\mathcal{L}_{\text{Higgs}}$, when $\phi(z)$ is replaced by $v$. We obtain

$$m_W^2 W^+_{\mu} W^{-\mu} = g^2 [(t^+ v / \sqrt{2})]^2 W^+_{\mu} W^{-\mu},$$  \hfill (3.33)

whilst for the $Z$ mass we get [recalling Eq. (3.12)]

$$1/2 m_Z^2 = (g \cos \theta_W + g' \sin \theta_W)^2 |t^3 v|^2 = (g^2 / \cos^2 \theta_W)|t^3 v|^2.$$  \hfill (3.34)

where the factor of $1/2$ on the left-hand side is the correct normalization for the definition of the mass of a neutral field. By using Eq. (3.32), relating the action of $t^3$ and $1/2Y$ on the vacuum $v$, and Eqs. (3.14), we obtain

$$1/2 m_Z^2 Z_\mu Z^\mu = |[g \cos \theta_W t^3 - g' \sin \theta_W (Y/2)]v|^2 Z_\mu Z^\mu.$$  \hfill (3.35)

For Higgs doublets

$$\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ v \end{pmatrix},$$  \hfill (3.36)

we have

$$|t^+ v|^2 = v^2, \quad |t^3 v|^2 = 1/4 v^2,$$  \hfill (3.37)

so that

$$m_W^2 = 1/2 g^2 v^2, \quad m_Z^2 = 1/2 g^2 v^2 / \cos^2 \theta_W.$$  \hfill (3.38)

Note that by using Eq. (3.17) we obtain

$$v = 2^{-3/4} G_F^{-1/2} = 174.1 \text{ GeV}.$$  \hfill (3.39)

It is also evident that for Higgs doublets

$$\rho_0 = m_W^2 / m_Z^2 \cos^2 \theta_W = 1.$$  \hfill (3.40)

This relation is typical of one or more Higgs doublets and would be spoiled by the existence of Higgs triplets etc. In general,

$$\rho_0 = \sum_i [(t_i)^2 - (t_i^3)^2 + t_i] v_i^2 / \sum_i 2 (t_i^3)^2 v_i^2.$$  \hfill (3.41)
for several Higgses with VEVs $v_i$, weak isospin $t_i$, and $z$-component $t_i^z$. These results are valid at the tree level and are modified by calculable electroweak radiative corrections, as discussed in Sections 7 and 8.

If only one Higgs doublet is present, then the fermion-Higgs couplings are in proportion to the fermion masses. In fact, from the Yukawa couplings $g_{a_1 f_1} f_L^a (\phi f_R + h.c.)$, the mass $m_f$ is obtained by replacing $\phi$ by $v$, so that $m_f = g_{a_1 f_1} v$.

With only one complex Higgs doublet, three out of the four Hermitian fields are removed from the physical spectrum by the Higgs mechanism and become the longitudinal modes of $W^+, W^-$, and $Z$. The fourth neutral Higgs is physical and should be found. If more doublets are present, two more charged and two more neutral Higgs scalars should be around for each additional doublet.

Finally, the couplings of the physical Higgs $H$ to the gauge bosons can be simply obtained from $L_{\text{Higgs}}$, by the replacement

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v + (H/\sqrt{2}) \end{pmatrix},$$

(3.42)

[so that $(D_\mu \phi)^\dagger (D^\mu \phi) = 1/2(\partial_\mu H)^2 + ...]$, with the result

$$L[H, W, Z] = g^2(v/\sqrt{2}) W^+W^-H + (g^2/4) W^+W^-H^2$$
$$+[(g^2vZ\mu)/(2\sqrt{2}\cos^2 \theta_W)] H$$
$$+[(g^2/(8\cos^2 \theta_W)] Z\mu Z^\mu H^2.$$  

(3.43)

We have thus completed our summary of the standard electroweak theory and of the $W^\pm, Z$ couplings.

**4. THE HIGGS SECTOR**

**4.1. The Higgs and Beyond: The Problem of the Fermi Scale**

The gauge symmetry of the Standard Model was difficult to discover because it is well hidden in nature. The only observed gauge boson that is massless is the photon. The graviton is still unobserved, even at the classical level of gravitational waves; the gluons are presumed massless but unobservable because of confinement, and the $W$ and $Z$ weak bosons carry a heavy mass. Actually the main difficulty in unifying weak and electromagnetic interactions was the fact that e.m. interactions have infinite range ($m_e = 0$), whilst the weak forces have a very short range, owing to $m_{W,Z} \neq 0$.

The solution of this problem is in the concept of spontaneous symmetry breaking, which was borrowed from statistical mechanics.
Consider a ferromagnet at zero magnetic field in the Landau-Ginzburg approximation. The free energy in terms of the temperature $T$ and the magnetization $M$ can be written as

$$ F(M, T) \simeq F_0(T) + 1/2 \mu^2(T)M^2 + 1/4 \lambda(T)(M^2)^2 + \ldots \quad (4.1) $$

This is an expansion which is valid at small magnetization, and which is the analogue in this context of the renormalizability criterion; $\lambda(T) > 0$ is assumed for stability; $F$ is invariant under rotations, i.e. all directions of $M$ in space are equivalent. The minimum condition for $F$ reads

$$ \frac{\partial F}{\partial M} = 0, \quad [\mu^2(T) + \lambda(T)(M^2)]M = 0. \quad (4.2) $$

There are two cases. If $\mu^2 > 0$, then the only solution is $M = 0$, there is no magnetization, and the rotation symmetry is respected. If $\mu^2 < 0$, then another solution appears, which is

$$ |M_0|^2 = -\mu^2/\lambda. \quad (4.3) $$

The direction chosen by the vector $M_0$ is a breaking of the rotation symmetry. The critical temperature $T_{\text{crit}}$ is where $\mu^2(T)$ changes sign:

$$ \mu^2(T_{\text{crit}}) = 0. \quad (4.4) $$

It is simple to realize that the Goldstone theorem holds. It states that when spontaneous symmetry breaking takes place, there is always a zero-mass mode in the spectrum. In a classical context this can be proven as follows. Consider a Lagrangian

$$ \mathcal{L} = |\partial_t \phi|^2 - V(\phi) \quad (4.5) $$

symmetric under the infinitesimal transformations

$$ \phi \to \phi' = \phi + \delta \phi, \quad \delta \phi_i = i\delta \theta_{ij}\phi_j. \quad (4.6) $$

The minimum condition on $V$ that identifies the equilibrium position (or the ground state in quantum language) is

$$ \left( \frac{\partial V}{\partial \phi_i} \right)(\phi_i = \phi_i^0) = 0. \quad (4.7) $$

The symmetry of $V$ implies that

$$ \delta V = \left( \frac{\partial V}{\partial \phi_i} \right)\delta \phi_i = i\delta \theta \left( \frac{\partial V}{\partial \phi_i} \right)\theta_{ij}\phi_j = 0. \quad (4.8) $$

By taking a second derivative at the minimum $\phi_i = \phi_i^0$ of the previous equation, we obtain

$$ \frac{\partial^2 V}{\partial \phi_i \partial \phi_j}(\phi_i = \phi_i^0)\theta_{ij} = 0 + \left( \frac{\partial V}{\partial \phi_i} \right)(\phi_i = \phi_i^0)\theta_{ik} = 0. \quad (4.9) $$
The second term vanishes owing to the minimum condition, Eq. (4.7). We then find
\[ \delta^2 V / \delta \phi_a \partial \phi_i \left( \phi_i = \phi^0_i \right) \partial \phi_i^0 = 0 . \] (4.10)
The second derivatives \( M^2_{ab} = (\delta^2 V / \delta \phi_a \partial \phi_i)(\phi_i = \phi^0_i) \) define the squared mass matrix. Thus the above equation in matrix notation can be read as
\[ M^2 t\phi^0 = 0 , \] (4.11)
which shows that if the vector \( t\phi^0 \) is non-vanishing, i.e. there is some generator that shifts the ground state into some other state with the same energy, then \( t\phi^0 \) is an eigenstate of the squared mass matrix with zero eigenvalue. Therefore, a massless mode is associated with each broken generator.

When spontaneous symmetry breaking takes place in a gauge theory, the massless Goldstone mode exists, but it is unphysical and disappears from the spectrum. It becomes, in fact, the third helicity state of a gauge boson that takes mass. This is the Higgs mechanism. Consider, for example, the simplest Higgs model described by the Lagrangian
\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu}^2 + \left( 1/\mu^2 + ie A_\mu \right) \phi^2 + \frac{1}{4} \lambda \phi^2 . \] (4.12)
Note the 'wrong' sign in front of the mass term for the scalar field \( \phi \), which is necessary for the spontaneous symmetry breaking to take place. The above Lagrangian is invariant under the \( U(1) \) gauge symmetry
\[ A_\mu \rightarrow A'_\mu = A_\mu - (1/\epsilon) \partial_\mu \theta(x) , \quad \phi \rightarrow \phi' = \phi \exp[i\theta(x)] . \] (4.13)
Let \( \phi^0 = v \neq 0 \), with \( v \) real, be the ground state that minimizes the potential and induces the spontaneous symmetry breaking. Making use of gauge invariance, we can make the change of variables
\[ \phi(x) \rightarrow (1/\sqrt{2}) \left[ \rho(x + v) + v \right] \exp[i\zeta(x)/v] , \]
\[ A_\mu(x) \rightarrow A_\mu - (1/\epsilon v) \partial_\mu \zeta(x) . \] (4.14)
Then \( \rho = 0 \) is the position of the minimum, and the Lagrangian becomes
\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu}^2 + \frac{1}{2} \phi^2 \rho^2 A_\mu^2 + \frac{1}{4} \epsilon^2 \rho^2 A_\mu^2 + e^2 \rho v A_\mu^2 + \mathcal{L}(\rho) . \] (4.15)
The field \( \zeta(x) \), which corresponds to the would-be Goldstone boson, disappears, whilst the mass term \( \frac{1}{4} \epsilon^2 v^2 A_\mu^2 \) for \( A_\mu \) is now present; \( \rho \) is the massive Higgs particle.
The Higgs mechanism is realized in well-known physical situations. For a superconductor in the Landau-Ginzburg approximation the free energy can be written as

\[
F = F_0 + \frac{1}{2} B^2 + \left( (\nabla - 2ieA_\phi)^2 \right) / 4m - \alpha |\phi|^2 + \beta |\phi|^4 .
\]  

(4.16)

Here \( B \) is the magnetic field, \( |\phi|^2 \) is the Cooper pair \( (e^- e^-) \) density, \( 2e \) and \( 2m \) are the charge and mass of the Cooper pair. The 'wrong' sign of \( \alpha \) leads to \( \phi \neq 0 \) at the minimum. This is precisely the non-relativistic analogue of the Higgs model of the previous example. The Higgs mechanism implies the absence of propagation of massless phonons (states with dispersion relation \( \omega = kv \) with constant \( v \)). Also the mass term for \( A \) is manifested by the exponential decrease of \( B \) inside the superconductor (Meissner effect).

Thus the Higgs effect is not an abstract device, but it is endowed with a precise physical reality. In the electroweak theory it is absolutely necessary in order to give masses to the \( W \)'s and \( Z^0 \) and to the fermions, as well as to ensure the correct high-energy behaviour required by renormalizability. However a more profound physical reality could be hidden behind or accompany the Higgs formalism. In fact, the Higgs mechanism is at present without experimental support. Actually the clarification of the physical origin of the electroweak symmetry breaking is one of the most important problems for experimental particle physics in the next decade. There are arguments indicating that the minimal Standard Model with fundamental Higgs fields cannot be the whole story and that some kind of new physics must necessarily appear near the Fermi scale. The most famous argument of this type is based on the so-called 'hierarchy problem', which we now describe. There is no unification of the fundamental forces in the Standard Model, because a separate gauge group and coupling is introduced for each interaction. On the other hand, the structural unity implied by the common, restrictive property of gauge invariance strongly suggests the possibility that all the observed interactions actually stem from a unified theory at some more fundamental level. The idea of unification at energies of order \( m_{\text{GUT}} \approx 10^{15} \text{ GeV} \), below the energy scale where quantum gravity becomes effective at masses of the order of the Planck mass, \( m_P \approx 10^{19} \text{ GeV} \), has been much studied in recent years. However, the question remains whether unification without the inclusion of quantum gravity is really plausible. It is clear that the inclusion of gravity must induce major changes in the physics of the Standard Model at energies of order \( m_P \) and possibly even below.

Thus, at least because of the fact that gravity is not included in the Standard Model, new physics must necessarily emerge at some large energy
scale $\Lambda$ (equivalent to some small distance scale). Then the problem is to understand what order of magnitude can reasonably be expected for $\Lambda$. In particular, we can ask whether it is natural to expect that $\Lambda$ may be as large as $m_P$ or $m_{\text{QCD}}$. In other words, is it possible that the Standard Model holds without any new physical input up to the energy scale of quantum gravity? The answer is probably negative, because then we could not naturally explain the enormous value of $m_P/m_P$, i.e. the ratio between the Planck and the Fermi scales of masses.

To develop this point further, we recall that in the Standard Model the fermion and vector-boson masses are all specified in terms of the VEV of the Higgs field $v$, according to Eqs. (3.27) and (3.38). The value of $v$ is determined by the curvature scale of the Higgs potential $V$:

$$V(\phi) = -\frac{1}{2}\mu^2\phi^\dagger\phi + (\lambda/4)(\phi^\dagger\phi)^2,$$

according to

$$v = \mu/\sqrt{\lambda}.$$  \hfill (4.17)

The observed values of the masses require for $v$ (and, therefore, roughly for $\mu$ as well) that $v \approx 10^2$ GeV [see Eq. (3.39)].

If $\Lambda \approx (10^{15} - 10^{17})$ GeV, then we face the problem of justifying the presence of two so largely different mass scales in a single theory (the so-called hierarchy problem). In general, if $\Lambda$ is very large in comparison with $\mu$, then, even if we set by hand a small value for $\mu$ at the tree level, the radiative corrections would make $\mu$ increase up to nearly the order of $\Lambda$. This problem is particularly acute in theories with scalars, as in the Standard Model, because the degree of divergence of mass corrections is quadratic, whilst the same divergences are only logarithmic for spin-1/2 fermions.

One general way out would be that the limit $\mu \to 0$ corresponds to an increase of the symmetry of the theory. In fact, the observed value of $\mu^2$ can be decomposed as

$$\mu^2 = \mu_0^2 + \delta \mu^2,$$  \hfill (4.19)

where $\mu_0$ is the tree-level value and $\delta \mu^2$ arises from the loop quantum corrections. If no new symmetry is induced when $\mu \to 0$ and no other non-renormalization theorem is operative, then a small value for $\mu_0^2/\Lambda^2$ can only arise from an unbelievably precise cancellation between $\mu_0^2/\Lambda^2$ and $\delta \mu^2/\Lambda^2$. If, however, $\mu = 0$ leads to an additional symmetry, then $\delta \mu^2$ must be proportional to $\mu_0^2$, because for $\mu_0 = 0$ both the tree diagrams and the loop corrections must respect the symmetry. Then, if one starts
from a small value of $\mu^2/\Lambda^2$, the radiative corrections would preserve the smallness of $\mu^2/\Lambda^2$.

For fermions, chiral symmetry is added when $m_f \to 0$, because the axial currents are also conserved in this limit, as their divergence is proportional to the fermion mass. Chiral symmetry and the logarithmic degree of divergence for fermion masses considerably alleviate the hierarchy problem in theories with no fundamental scalars.

In the Standard Model no additional symmetry is gained for $\mu_0 = 0$. This is also seen from the explicit formula for $\delta \mu^2$ at the one-loop level, which shows that $\delta \mu^2$ is not proportional to $\mu_0^2$:

$$\mu^2/\Lambda^2 = (\mu_0^2/\Lambda^2) + (1/128\pi^2)(d/dv)^2 \sum_J (2J + 1)(-1)^J m_J^2(v) + \ldots,$$

(4.20)

where terms which vanish with $\Lambda \to \infty$ are indicated by the dots. The sum over $J$ includes both particles and antiparticles (counted separately) of spin $J$ and mass $m_J$ (expressed as a function of $v$). Thus we are forced to the conclusion that in the Standard Model the natural value for $\delta \mu^2/\Lambda^2$ is of order one or so. Therefore, as $\Lambda$ can be interpreted as the energy scale where some essentially new physical ingredient becomes important, we are led to expect that the validity of the present framework cannot be extended beyond $\Lambda \approx (1-10)$ TeV.

The problem of explaining the Fermi scale is seen to be closely connected to the Higgs mechanism and to the consequent presence of scalars, which makes the problem of testing the Higgs sector particularly crucial.

One possible solution is that the Higgses are really scalar fundamental fields, but naturalness is restored by supersymmetry.

Supersymmetry (SUSY for short) [15] relates bosons and fermions, so that in a multiplet that forms one representation of supersymmetry there is an equal number of bosonic and fermionic degrees of freedom. This implies that SUSY generators are spin-1/2 charges $Q_\alpha$. SUSY leads to an extension of the Poincaré algebra. Besides the obvious algebraic relations between $Q_\alpha$ and the Poincaré generators, which specify the spinorial transformations of $Q_\alpha$ under Lorentz transformations and its invariance under translations, the essentially new relation is the anticommutator

$$\{Q_\alpha, Q_\beta\} = -2(i\gamma_{\mu})_{\alpha\beta} P^\mu,$$

(4.21)

where $P^\mu$ is the energy–momentum four-vector, which generates space–time translations. If all fundamental symmetries are gauge symmetries, then also SUSY is presumably a local symmetry. This immediately leads to
the realm of gravity. In fact, the product of two local SUSY transformations is a translation with space-time-dependent parameters, as follows from Eq. (4.21). But a translation with space-time-dependent parameters is a general coordinate transformation. As ordinary gravity can be seen to arise from gauging the Poincaré group, a similar gauging of the Poincaré algebra enlarged by SUSY generators leads to an extended version of gravity, called supergravity. In fact, supersymmetry and supergravity play a crucial role in most present attempts at constructing a sensible theory of quantum gravity, including superstring theories [16] that at present represent the most advanced and promising project for a theory of gravity (and of all particle interactions).

Theorists like SUSY for several reasons. SUSY is the maximum symmetry compatible with a non-trivial S-matrix in a local relativistic field theory. The powerful constraints between couplings and masses implied by SUSY drastically reduce the degree of singularity of field theory as deduced from power counting. In some cases a finite field theory is even obtained. For example, \( N = 4 \) extended SUSY Yang-Mills theories are finite in four dimensions. In general, more powerful non-renormalization theorems are deduced for SUSY theories. This property may solve many naturalness problems of the Standard Model. In particular, the hierarchy problem can be solved in SUSY theories because the mass divergences of bosons and fermions become the same, and the quadratic divergences of scalars are reduced to logarithmic divergences as for spin-1/2 fermions. Thus in SUSY theories, we automatically obtain

\[
(d/du)^2 \sum_J [(2J + 1)(-1)^{2J} m_J^2(\nu)] = 0 ,
\]

because of a cancellation between bosons and fermions. Of course the cancellation is only exact in the limit of unbroken SUSY. But we know that SUSY must be broken because the SUSY partners of ordinary particles have not been observed. In broken SUSY, the \( \Lambda \) appearing in Eq. (4.20) can be identified with the mass scale that is typical of SUSY partners of ordinary states. Thus if \( \Lambda \) is of order \( G_F^{-1/2} \) or at most \( O(1 \text{ TeV}) \), then the hierarchy problem would be solved in a natural way. This is the only known way out of the hierarchy problem compatible with fundamental scalar Higgs fields.

We have seen that most theorists working on quantum gravity and superstrings tend to consider SUSY as 'established' at \( m_P \) and beyond. For economy, we are naturally led to also try to use SUSY at low energy to solve some of the problems of the Standard Model, including the hierarchy problem. It is therefore very important that it was indeed shown [17] that models where SUSY is softly broken by gravity do offer a viable alternative. We stress again that the supersymmetric way is very appealing to
theorists. In fact, it would represent the ultimate triumph of a continuous line of progress obtained by constructing field theories with an increasing degree of exact and/or broken symmetry and applying them to fundamental interactions. Also the value of the ratio of knowledge versus ignorance would be remarkably large in the case of SUSY: the correct degrees of freedom for a description of physics up to gravity would have been identified, the Hamiltonian would be essentially known, and the theory would, to a large extent, be computable up to $m_P$.

The alternative main avenue to physics beyond the Standard Model is compositeness or, more generally, the existence of new strong forces. For example, the electroweak symmetry could be broken by condensates of new fermions attracted by a new force with $\Lambda_{\text{new}} \approx m_P \approx G_F^{-1/2}(\Lambda_{\text{QCD}})$, as in technicolour theories [18]. Or the Higgs scalar can be a composite of new fermions bound by a new force [19]. In the last two cases there are unsolved problems related to the fermion masses. Or the $SU(2) \otimes U(1)$ gauge symmetry can be a low-energy fake [20]. At high energies $\gtrsim m_P$, the $W$ and $Z^0$ would be resolved into their constituents. Another possibility is that the Higgs mass becomes large [21] [$O(1 \text{ TeV})$]. Then, as we shall see, the weak interactions become strong and a spectroscopy of tightly bound weak-interaction resonances appears (e.g. $WW, WZ$, or $ZZ$ states).

However, it is fair to say that the compositeness alternative is not at all so neat and clearly formulated as the supersymmetric option. On the contrary, in many respects the compositeness way is not well defined at all and leads to many unsolved problems.

Of course, the two avenues are not necessarily mutually exclusive, and theoretical frameworks where both appear have been considered [22].

Searching for the standard Higgs particle appears to be a good way to organize the experimental solution of the symmetry-breaking problem.  

The search for the Higgs is being pursued at LEP 1 and will continue at LEP 200. Indeed all previous limits on the Higgs mass $m_H$ have been dwarfed by only a few months of LEP operation. For the standard Higgs we have at present the following results [23]:

\begin{align*}
\text{ALEPH} : & \quad m_H > 51 \text{ GeV} \\
\text{OPAL} : & \quad m_H > 47.3 \text{ GeV} \\
\text{L3} : & \quad m_H > 47.5 \text{ GeV} \\
\text{DELPHI} : & \quad m_H > 42 \text{ GeV}
\end{align*}

These limits are obtained from negative searches of the process $e^+e^- \rightarrow HZ^* (\rightarrow ff)$.
As is well known, the value of the Higgs mass is not predictable even in the minimal Standard Model with a single Higgs doublet. What is certainly true is that the Higgs boson cannot be too heavy or the perturbative theory becomes sick and breaks down [24]. If \( m_H \geq O(1 \text{ TeV}) \) the perturbative rates for \( VV \to VV \) scattering \( (V \equiv W, Z) \) violate the unitarity limit [21] for \( \sqrt{s} \gg m_W \). More important than this, in non-asymptotically free gauge theories there are Landau poles where the coupling constant blows up according to the renormalization group improved perturbation theory (unless the renormalized coupling is not vanishing so that the theory is a free theory, i.e. trivial). This phenomenon is also present in QED, but it would only occur beyond the Planck scale of mass, so that the problem can be solved at such large energies by embedding the theory in a larger context (e.g. grand unification). The coupling of the quartic term \( \lambda(\phi^+\phi)^2 \) in the Higgs potential increases with \( m_H^2 \) \( (m_H^2 \sim \lambda/G_F) \); see Eqs. (3.39) and (4.18). In addition, for a given \( m_H \), \( \lambda \) increases logarithmically with energy because the theory is not asymptotically free in the Higgs sector. Thus the position of the Landau pole depends on \( m_H \). Imposing that the Landau pole be far enough for the theory to make sense up to a scale \( \Lambda \), gives a bound [25] on the standard Higgs mass which is plotted in Fig. 2, taken from Ref. [26]. We see that for a light Higgs, i.e. \( m_H \leq 180-200 \text{ GeV} \), the perturbative regime is valid up to \( M_{\text{GUT}} \) or \( M_P \). For a heavier Higgs the value of \( \Lambda \) decreases until eventually \( \Lambda \sim m_H \) for \( m_H \sim 1 \text{ TeV} \), the theory is valid up to \( \Lambda \sim 1 \text{ TeV} \).

We can understand these results by the following crude simplification [24]. The renormalization group equation for the quartic coupling \( \lambda \), in the limit of neglecting gauge and Yukawa couplings, becomes:

\[
\frac{d\lambda(t)}{dt} = \frac{3}{4\pi^2} \lambda^2(t),
\]

with \( t = \ln \Lambda/v \), where \( v \) is the Higgs vacuum expectation value and \( \Lambda \) is the scale where \( \lambda \) is evaluated. The coefficient \( \beta_0 = 3/4\pi^2 \) is obtained from one-loop corrections to the quartic coupling in the \( \lambda(\phi^+\phi)^2 \) theory. The normalization of \( v \) and \( \lambda \), in physical terms, is here chosen such that

\[
\lambda \equiv \lambda(v) = \sqrt{2} \ G_F \ m_H^2
\]

(4.25)

\[
v = (2\sqrt{2} \ G_F)^{-1/2} \approx 174 \text{ GeV}.
\]

(4.26)

When \( \lambda(t) \) is large the gauge and Yukawa couplings can be neglected with the exception of the top Yukawa coupling, which can become large if \( m_t \geq v : g_{\text{top}} = m_t/v \). By solving (4.24) one obtains

\[
\lambda(t) = \frac{\lambda}{1 - 3/4\pi^2 \lambda t},
\]

(4.27)
The minus sign in the denominator, typical of non-asymptotically free theories, implies the increase of \( \lambda(t) \) with the scale \( \Lambda \) up to an infinite value which is obtained for \( 3/4 \pi^2 \lambda t = 1 \). In order to avoid the Landau pole the condition

\[
\frac{3}{4 \pi^2} \lambda t = \frac{3}{4 \pi^2} \sqrt{2} G_F m_H^2 \quad \text{in} \quad \Lambda / \nu < 1
\]

must be imposed. This condition is equivalent to:

\[
m_H \leq \frac{893 \text{ GeV}}{\sqrt{\ln \Lambda / 174 \text{ GeV}}}
\]

or \( m_H < 144, 165, 675 \text{ GeV} \) for \( \Lambda = 10^{19}, 10^{15}, 10^3 \text{ GeV} \) respectively.

We see that this simple model reproduces the quantitative features of the bounds on \( m_H \) in Fig. 2 fairly well. The curves in Fig. 2 are obtained by a more refined renormalization group treatment of the problem, with inclusion of gauge and top effects. The obvious criticism to the above approach is that a perturbative evaluation of the \( \beta \) function is not justified in the vicinity of the Landau pole. Thus it is very interesting that the va-

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![Figure 2: Combined limits (from Ref. [26]) on \( m_H \) and \( m_t \) from vacuum stability and avoiding the Landau pole up to a scale \( \Lambda \).](image-url)
lidity of the bound has been confirmed by recent computer simulations of
the electroweak theory on the lattice [27]. The precise value of the upper
limit on \( m_H \) depends on the exact definition of \( \Lambda \) and on where one
fixes the line between acceptable and not acceptable. In fact the lattice results
nicely extrapolate the perturbative evaluation and impose limits on \( m_H \)
such that:

\[
m_H \leq (8-10)m_W \approx 0.6-0.8 \text{ TeV}.
\]  

(4.30)

It is thus fair to conclude that, to the best of our knowledge, the internal
consistency of the Standard Model demands that the Higgs mass is below
1 TeV.

In Fig. 2 there is also a forbidden region at large \( m_t \) and small \( m_H \). This
boundary is determined by the requirement of vacuum stability [24],[28].

At tree level the scalar potential can be written in the form:

\[
V(\varphi) = -\mu^2|\varphi|^2 + \frac{\mu^2}{2}\varphi^4.
\]  

(4.31)

The quantum corrections can be computed by expanding in the number of
loops. At one loop one obtains:

\[
V(\varphi) = -\mu^2|\varphi|^2 + \frac{\mu^2}{2}\varphi^4 + \gamma|\varphi|^4 \left( \ln \frac{|\varphi|^2}{v^2} - \frac{1}{2} \right)
\]  

(4.32)

with

\[
\gamma = \frac{3 \sum_{\text{vectors}} m_i^4 + \sum_{\text{scalars}} m_i^4 - 4 \sum_{\text{fermions}} m_i^4}{64\pi^2 v^4}
\]  

(4.33)

It is simple to check that also in the corrected form \( v \) is an extremum of
\( V(\varphi) \). In the minimal Standard Model with one Higgs doublet and three
fermion families one obtains

\[
\gamma = \frac{3m_W^4 + 6m_t^4 + m_H^4 - 12m_t^4}{64\pi^2 v^4}
\]  

(4.34)

The extra factor of three in front of \( m_t^4 \) is of course due to colour.

For the realization of spontaneous symmetry breaking and stability of
the theory one requires that i) the extremum at \( \varphi = v \) is a minimum, i.e.
\( V(\varphi) < 0 \) and ii) \( V(\varphi) \to +\infty \) for \( |\varphi| \to \infty \), so that the Hamiltonian is
bound from below.

At small \( m_t, m_t < 80 \text{ GeV} \), the first requirement leads to the Linde–
Weinberg limit \( m_H^2 > 2\gamma v^2 \), or \( m_H \geq 7 \text{ GeV} \). This limit is by now void,
because of the experimental lower bounds on both \( m_t \) and \( m_H \). For the
second requirement to be fulfilled, \( m_H \) must increase with \( m_t \) in order
to prevent $\gamma$ from becoming too negative [28]. At large $|\varphi|$ the one-loop evaluation of $V(\varphi)$ is not sufficient, and one needs a resummation of the large logarithms $\log \varphi^2/\langle \varphi \rangle^2$. The results are shown in Fig. 2 [26]. The above limits are only valid in the minimal Standard Model with one Higgs doublet. Note that in case that there are two or more Higgs doublets the limits refer to some average mass. Thus for the lightest Higgs the lower limit can be easily evaded but the upper limit is a fortiori valid. The bottom line is that either the Higgs is found below $\approx 1$ TeV or new physics beyond the Standard Model should appear. At least one should see the onset of a new non-perturbative regime where the weak interactions become strong.

In conclusion there are solid arguments for new physics near the Fermi scale of mass $G_F^{-1/2}$. Either a fundamental scalar Higgs exists and naturalness is restored by supersymmetry, or new strong forces will manifest themselves, drastically changing the framework of the Standard Model beyond $O(1$ TeV). A new non-perturbative regime will set up, with new resonances, and the physics will become less predictable above that energy. An important point is that all conceivable possibilities are very complex. Each of them implies a rich new spectrum of states and phenomena: the whole spectrum of superpartners in SUSY; new hadrons, excited vector bosons, etc., in the composite alternative. The new physics is in all cases distributed over a large interval of energies. The low-lying fringes of the new spectroscopy, or at least their virtual effects, should already be accessible to LEP 1 and LEP 200. A lot of discoveries are expected at the LHC, to be followed by more at the SSC.

4.2. Search for the Standard Higgs

It is clear that no other accelerator is better than LEP for finding a Standard Model Higgs with mass $m_H \leq m_Z$. We have already mentioned the present lower limits on $m_H$ obtained at LEP 1 [23]. In the next few years of continued LEP 1 operation one can presumably improve the limits up to $m_H \leq 60$ GeV. Beyond that the increase of energy is absolutely necessary. At LEP 200, which will be operational at the beginning of 1994, the range $m_H = 50$–90 GeV can be explored. The LEP 200 process for observing the Higgs is $e^+ e^- \rightarrow ZH$ (with a real $Z$ in the final state and not a virtual one as in the analogous process at LEP 1). The observation of the Standard Model Higgs with $m_H \leq 80$ GeV is considered an easy problem at LEP 200 with project energy and luminosity [29]. For $m_H \sim 80$–95 GeV the problem is considerably more difficult because of the small cross-section and of the $H/Z$ confusion due to the overlapping of masses. This case was studied recently by Kunst and Stirling [30]. The total cross-sections for the signal ($e^+ e^- \rightarrow HZ$) and the main background ($e^+ e^- \rightarrow ZZ$) are
shown in Fig. 3 for $\sqrt{s} = 200$ GeV. The signal cross-section is small (~ 0.5 pb without branching ratios).

In the channel $\ell^+\ell^- + \text{jets}$, $\ell = e, \mu$, with $\int L \, dt = 500 \, \text{pb}^{-1}$, the ratio of the numbers of events for signal and background [30] is 20.4/2.0, 17.3/28, 15.1/5.1 for $m_H = 85, 91.1, 95$ GeV respectively, with $m_Z = 4m_H \pm 1$ GeV, where $m_Z$ is the reconstructed mass of $Z \rightarrow \text{jets}$. A moderate help can be obtained from cuts in $\cos \theta (\theta = \mu^+\text{beam angle})$ as the angular distributions are different. The conclusion of Ref. [30] is that, close to $m_Z$, high energy ($\sqrt{s} \approx 200$ GeV) and large luminosities ($\int L \, dt \geq 500 \, \text{pb}^{-1}$) are needed.

Beyond LEP 200 the future of $e^+e^-$ accelerators is probably in linear colliders. The search for the Higgs at linear $e^+e^-$ colliders with $\sqrt{s} = 1–2$ TeV was discussed at the La Thuile Workshop [31] and elsewhere. An international Workshop has been organized by ICFA to study the physics potential of an $e^+e^-$ collider with $\sqrt{s} = 0.5$ TeV and $L = 10^{33} \, \text{cm}^{-2} \, \text{s}^{-1}$. The results have been presented in Finland in September 1991. For the intermediate-mass Higgses with $m_Z \leq m_H \leq 2m_Z$ (the region which is difficult at the LHC and SSC) $e^+e^-$ linear colliders are good. It turns out

![Graph](image)

**Figure 3:** Total cross-sections for $e^+e^- \rightarrow ZZ$ and $e^+e^- \rightarrow ZH$ for $m_H = 80, 85, 90,$ and 95 GeV as functions of the $e^+e^-$ c.m. energy (from Ref. [30]).
(Fig. 4) that $\sqrt{s} = 0.5$ TeV is just the energy where the cross-sections of $e^{+}e^{-} \rightarrow HZ$ and $e^{+}e^{-} \rightarrow \nu\bar{\nu}H$ (via $W-W$ fusion) are equal for $m_H \approx 100$ GeV [31]. For $m_H$ in this region, $\geq 10^8$ events/year can be expected at $\sqrt{s} \geq 0.5$ TeV with $L = 10^{33}$ cm$^{-2}$ s$^{-1}$. The $\gamma-\gamma$ background can be controlled [32].

The search for the minimal Standard Model Higgs [33] at the LHC and the SSC has been discussed in great detail at the Aachen Workshop [34] as well as at previous ones on LHC [31],[35] and SSC [36] physics. This is a good reference problem, but not necessarily the central issue of physics at the LHC. After all the Higgs might be found at LEP. Such a discovery there would not at all mean that the LHC is no longer necessary. In fact, we have seen that one expects some new physics at the weak scale to accompany the Higgs. The minimal Standard Model might well be wrong for the Higgs sector. For example, the Higgs sector of supersymmetric models involves at least two Higgs doublets [15],[33]. The couplings of the lightest SUSY Higgs are not as in the minimal Standard Model. However, it would in many cases be impossible to prove at LEP that the Higgs candidate is the particle predicted by the minimal Standard Model. The Higgs search is a

![Graph](image_url)

**Figure 4**: Cross-section for $e^{+}e^{-} \rightarrow H + \ldots$ as a function of energy for different mechanisms (Ref. [31]); WW and ZZ indicate the corresponding fusion channels (e.g. WW stands for $e^{+}e^{-} \rightarrow H\nu\bar{\nu}$), while ZH means $e^{+}e^{-} \rightarrow ZH$ (via Z exchange).
good reference problem in the sense that experiments must be good enough to see the standard Higgs in order to prove adequate for the solution of the electroweak symmetry-breaking question. The discovery of the Higgs is in fact a very difficult experimental problem, because the Higgs is heavy and, its couplings being proportional to masses, it is essentially not coupled to light particles (the most common ones). Heavy real or virtual states must be excited in order to produce the Higgs, so that the cross-sections are relatively small. In addition, below the $WW$ or $ZZ$ threshold, the dominant decay into the heaviest accessible pair of quarks is swamped by the QCD background. The case of the Standard Model Higgs was studied by a dedicated group at the Aspen Workshop, convened by Z. Kunszt and J. Stirling.

The problem was restarted from scratch. Calculations of the total width (Fig. 5) and of the branching ratios (Fig. 6) were updated by Z. Kunszt and J. Stirling. The inclusion of the effects from the running of the $b$-quark mass makes the $b \bar{b}$ partial width smaller, and the rare decay branching ratios below the $t \bar{t}$, $WW$, and $ZZ$ thresholds larger. In particular the $H \rightarrow \gamma \gamma$ branching ratio was found to be larger by a welcome factor of 2 with respect to previous calculations. The production occurs mainly through

![Figure 5: The total width (from Ref. [34]) of the standard Higgs as a function of $m_H$ (and $m_t$).](image)

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Figure 6: The branching ratios (from Ref. [34]) of the standard Higgs.

For $m_H > 200$ GeV, the WW and ZZ channels are dominant.

There is little dependence on $m_t$.

gluon-gluon fusion ($gg \rightarrow H$) via a quark loop (dominated by virtual $t$ exchange) or through $WW$ fusion plus a small $ZZ$ contribution $q\bar{q} \rightarrow Hq\bar{q}$).

For $m_t \geq 90$ GeV the gluon-fusion process is dominant up to very heavy Higgs masses: $m_H \geq 600$ GeV for $m_t \simeq 90$ GeV, or $m_H \geq 1$ TeV for $m_t \simeq 180$ GeV (Fig. 7).

The intermediate-mass Higgs is the most difficult case. It is assumed that a light Higgs with mass $m_H \leq m_Z$ will be discovered at LEP 1 or LEP 200.

The intermediate Higgs range is defined by $m_Z \leq m_H \leq 2m_Z$, i.e. below the threshold for $H \rightarrow ZZ$. This region would be hopeless if $H \rightarrow t\bar{t}$ were allowed. Now it is known from CDF results that indeed $m_t \geq m_Z$, so that the dominant decay of the intermediate Higgs is $H \rightarrow b\bar{b}$. This implies that the accessible decay modes $H \rightarrow ZZ^* \rightarrow 4\ell^\pm$ and $H \rightarrow \gamma\gamma$ have a much larger branching ratio. High luminosity, $L \simeq 10^{34}$ cm$^{-2}$ s$^{-1}$, is absolutely necessary for detecting the intermediate Higgs at the LHC.

The first very important conclusion which was obtained [34] is that with $\int L \, dt \simeq 10^8$ pb$^{-1}$ and both $e$ and $\mu$ detection, it is possible to observe the
intermediate Higgs for $m_H \geq 130 \text{ GeV}$ through the chain $H \rightarrow ZZ^* \rightarrow 4\ell^\pm$ ($\ell = e, \mu$) [37]. The signal rate before cuts is 100–700 events per year as seen from Fig. 8 (the dip at $m_H \sim 160 \text{ GeV}$ corresponds to the opening of the threshold for $WW$ decay, which is not practicable because of the $t\bar{t} \rightarrow WWb\bar{b}$ background). A thorough study of backgrounds was done. Particular attention was devoted to the $Zb\bar{b}$ channel (the leptons from $b\bar{b}$ can be hard and isolated enough to mimic the $Z^*$). The dominant process $gg \rightarrow Zb\bar{b}$ was studied by van Eijk and Kleiss in Ref. [34]. Detailed simulations of the $t\bar{t}, Zb\bar{b}, Z^*Z^*$, and $Z^*\gamma$ backgrounds were performed. The signal is already visible over the background without isolation cuts (Fig. 9a), but becomes much more prominent with these (Fig. 9b).

Much work was devoted to the problem of closing the window $m_Z \leq m_H \leq 130 \text{ GeV}$. This is a particularly hard task. The main line of attack is based on the process $pp \rightarrow H(\rightarrow \gamma\gamma)X$, first discussed in Ref. [37] and then widely studied [31],[36]. This process was further analysed at the Aachen Workshop. I refer the reader to the article by C. Seez et al. for a detailed discussion [34]. The conclusion was that this channel is extremely difficult, but feasible with a very good detector. The signal rate is $0.5 \times 10^3$ events
Figure 8: The cross-section times branching ratio (from Ref. [34]) for pp → H(→ Z*Z* → 4ℓ±)X at the LHC and SSC.

Figure 9: The signal and background for the intermediate-mass Higgs (H → ZZ* → 4ℓ±, ℓ = e, μ), a) without and b) with isolation cuts (from Ref. [34]).
Figure 10: The signal for $H \rightarrow \gamma \gamma$ at the LHC and SSC (from Ref. [34]).

per $\int L \, dt \approx 10^5 \text{ pb}^{-1}$ (Fig. 10). The intrinsic background from $q\bar{q} \rightarrow \gamma \gamma$ and $gg \rightarrow \gamma \gamma$ already poses a formidable problem. A superb electromagnetic calorimeter is required, and vertex localization is very important for the $\gamma \gamma$ invariant-mass reconstruction. In Table 1 we show the comparison of signal versus intrinsic background for $m_H = 80$–$150$ GeV and $\int L \, dt \approx 10^5 \text{ pb}^{-1}$.

<table>
<thead>
<tr>
<th>$m_H$ (GeV)</th>
<th>$\Delta M$ (GeV)</th>
<th>Signal</th>
<th>Background</th>
<th>$S/\sqrt{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>1.0</td>
<td>570</td>
<td>11800</td>
<td>5.2</td>
</tr>
<tr>
<td>100</td>
<td>1.5</td>
<td>1180</td>
<td>13700</td>
<td>10.1</td>
</tr>
<tr>
<td>150</td>
<td>2.0</td>
<td>830</td>
<td>5600</td>
<td>11.1</td>
</tr>
</tbody>
</table>

The reducible background from jets misidentified as photons demands a large rejection factor $r_{2j} = r_{1j}^2 > 10^6$, where $r_{2j}$ and $r_{1j}$ correspond to double- and simple-jet misidentification, respectively. The possibility of a position detector, located some 2 m away, in order to see the separation between the two $\gamma$'s from $\pi^0$ decays, was suggested as a main device for the discrimination of the jet background.
Figure 11: The signal rate for $p + p \rightarrow (H \rightarrow \gamma\gamma)W(\ell\nu)X$ or $p + p \rightarrow (H \rightarrow \gamma\gamma)Z(\ell\ell)X$ at the LHC and SSC (from Ref. [34]).

An additional possibility, at small $m_H$, is provided by the associated production of $HW$ followed by $H \rightarrow \gamma\gamma$: $pp \rightarrow H(\rightarrow \gamma\gamma)W(\ell\nu)X$. This process was studied at the Aachen Workshop by Kleiss, Kunst, and Stirling [38], and, from the experimental point of view, by Di Lella et al. [34]. The good thing about this process is that the sum of the irreducible background from $W\gamma\gamma$ and of the reducible one from $b\bar{b}g$, $b\bar{b}g$, $b\gamma\gamma$, $Wjj,\ldots,$ with misidentifications, is very small in comparison with the signal. The bad thing is that the signal rate is also very small (Fig. 11)[38]. The resulting number of events for signal and background after cuts are collected in Table 2. It is concluded [34] that this channel is very difficult but could provide a useful way of confirming the signal from $pp \rightarrow H(\rightarrow \gamma\gamma)X$.

<table>
<thead>
<tr>
<th>$m_H$ (GeV)</th>
<th>Signal</th>
<th>Background</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Irreducible</td>
<td>Reducible</td>
</tr>
<tr>
<td>75</td>
<td>17</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>22</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>130</td>
<td>18</td>
<td>2</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>

31
A similar process, studied in Refs. [39],[40], is pp → t̅tH followed by $H → γγ$ and the leptonic decay of at least one $W$ from the $t, t̅$ disintegrations. The signal at the LHC is larger by a factor of about 2 with respect to the $WH$ channel, while the background is the same. Taken together the two associated production channels $WHX$ and $t̅tHX$, followed by $H → γγ$, offer a quite promising possibility.

The possibility of detecting the Higgs via $H → τ⁺τ⁻$, as proposed in Ref. [41], was also considered in detail. The conclusion is negative: this channel turns out to be hopeless for the standard Higgs [34]. As we shall see in Section 4.4 for particular values of the parameters, it could be of use for the SUSY Higgs $A$.

Turning to the case of a heavy Higgs, $m_H > 2m_Z$, the golden channel is $H → ZZ → 4ℓ±$ [42], while $H → WW → ℓℓνν$ is much more difficult, particularly because of the $t̅t → WWbb$ background. The rate for $H → ZZ → 4ℓ±$ is displayed in Fig. 12 as a function of $m_H$ and $m_τ$ [34]. Detailed studies and simulations of the irreducible background from $q̅q$, $gg → ZZ$ (which is the dominant one in this case) and of the reducible background from $t̅t$, Zbb, and $Z +$ jets were performed [34]. The reducible background is in all cases small after cuts. With $\int L\, dt \simeq 10^{5}\, \text{pb}^{-1}$ and $ℓ = e, μ$, the
Figure 13: Signal versus background for $H \rightarrow ZZ \rightarrow 4\ell^\pm$ for $m_H = 0.6$–0.8 TeV (from Ref. [34]).

discovery range at the LHC extends up to $m_H = 800$ GeV (Fig. 13) (with $\int L dt \simeq 10^4$ pb$^{-1}$ the corresponding value would go down to 400 GeV). The ultimate discovery range at the LHC could be improved, perhaps, up to $m_H \simeq 1$ TeV by using $H \rightarrow ZZ \rightarrow \ell\ell\nu\nu$, but the possibility of extracting the signal from the background from $b\bar{b}$, $Zb\bar{b}$, etc., is not demonstrated. Alternatively one could try to use $H \rightarrow WW \rightarrow \ell\nu jj$ or $H \rightarrow ZZ \rightarrow \ell\nu jj$ with jet tagging. Jet tagging was first studied in Ref. [43] and further considered at the Aachen Workshop by M. Seymour [34]. At large $m_H$, a substantial fraction of the Higgs events is produced by $WW$ fusion. As is well known, the idea of tagging is to detect the near forward and backward quark jets, with $E_T \sim O(1$ TeV) and $p_T \sim O(m_W)$ left out after $W$ emission. Studies done at the Workshop indicate that jet tagging may indeed be possible, perhaps even at $L > 10^{33}$ cm$^{-2}$ s$^{-1}$.

In conclusion, as was stated by D. Froidevaux in Ref. [34], at the LHC, with $10^5$ pb$^{-1}$, the process $H \rightarrow ZZ \rightarrow 4\ell^\pm$ with real or virtual $Z$, allows the range $m_H = 130$–800 GeV to be covered. The same range is obtained at the SSC with $10^4$ pb$^{-1}$. For $m_H = 80$–130 GeV the channels $H \rightarrow
\( \gamma \gamma H V \rightarrow \gamma \gamma t l \nu \) and \( t l H \rightarrow t l \gamma \gamma \) are extremely difficult but feasible. The ratio \( S/\sqrt{B} \) is actually better at the LHC with \( 10^{6} \) pb\(^{-1} \) than at the SSC with \( 10^{4} \) pb\(^{-1} \), but the operation at a luminosity 10 times larger is more demanding for the detector. The ultimate discovery range at the LHC could perhaps be extended up to 1 TeV by using \( H \rightarrow ZZ \rightarrow t l l l \nu \nu \) or \( H \rightarrow WW \rightarrow e\mu j j \) with jet tagging, but this is not established.

### 4.3. Longitudinal \( W^{\pm} \) and \( Z \)

The \( V \) states (\( V \equiv W^{\pm}, Z \)) with helicity zero (longitudinal \( V \), denoted by \( V_L \)) are absent in the symmetric limit where the \( V \) are massless. It is thus clear that the longitudinal modes are directly related to the symmetry-breaking mechanism. If the Higgs is not found in the LHC discovery range, then the \( VV \) interactions become strong and the perturbative cross-section violates unitarity [21] for \( m_H, \sqrt{s} \geq O(1 \text{ TeV}) \). This is due to the growth of the \( V_L V_L \rightarrow V_L V_L \) scattering amplitudes, which become dominant in that regime. If the Higgs is not found at the LHC, the study of the interactions among \( V_L \) becomes the most direct way of attacking the symmetry-breaking problem [44]. In a theory with spontaneous symmetry breaking, no matter if the breaking is dynamical (e.g., due to condensates) or induced by either elementary or composite Higgses, the longitudinal \( V \) arise from the Goldstone bosons with the corresponding quantum numbers. In fact, at large energies, when contributions of order \( m_V/\sqrt{s} \), arising from mass terms, can be neglected, the amplitudes for \( V_L V_L \) scattering approach those for the corresponding Goldstone bosons (\( \sqrt{s} \) being the \( V_L V_L \) centre-of-mass energy). For example,

\[
A(W^{\pm}_L Z_L \rightarrow W^{\pm}_L Z_L) = A(w^{\pm} z \rightarrow w^{\pm} z) + O\left(\frac{m_V}{\sqrt{s}}\right), \quad (4.35)
\]

where \( v_L \) is the Goldstone boson which corresponds to \( V_L \). This 'equivalence theorem' [45], valid to all orders of perturbation theory, is also used as a handy method for practical computations.

At low momenta, the Goldstone boson couplings are fixed by the symmetry. As a consequence, there are low-energy theorems [46] that specify the Goldstone boson amplitudes at threshold. An effective Lagrangian formalism can be based on the low-energy theorems. This provides a framework for an extrapolation near threshold of the amplitudes which satisfy the low-energy theorems. At \( \sqrt{s} \gg m_V \) but not too large, one may think to combine the equivalence theorem and the low-energy limit and to apply the effective Lagrangian results directly to \( V_L V_L \) scattering. Such smooth extrapolations can provide reasonable approximations only for \( \sqrt{s} \ll 4\pi G_F^{1/2} \), provided that no resonances are met on the way. For example, in the Standard Model.
the regime of low-energy theorems is no longer valid for $\sqrt{s} \simeq m_H$, because $m_H$ is a resonance in the VV channel. At large $\sqrt{s}$ ($\sqrt{s} \gg m_H$), the Higgs contribution cancels [21] the bad high-energy behaviour—obtained by extrapolating the trend derived from the low-energy theorems—which eventually would violate unitarity. For a light Higgs, the high-energy $V_L V_L$ scattering amplitudes remain small, of order $G_F m_H^2$ [21]. In the absence of the Higgs, some other mechanism, which one would like to discover, should intervene to quench the singular high-energy behaviour.

An analogy with QCD can be established: $W_L^\pm$ and $Z_L$ are analogous to $\pi^\pm$ and $\pi^0$ in QCD because $V_L$ are eaten up Goldstone bosons of $SU(2) \otimes U(1)$, while the pions are the (pseudo)-Goldstone bosons of $SU(2) \otimes SU(2)$ chiral symmetry. The pions obey Weinberg's low-energy theorems [47], which are embodied in the formalism of chiral Lagrangians [48]. The chiral Lagrangian regime would hold up to $\sqrt{s} \ll 4\pi F_\pi \simeq 1.2$ GeV were it not for the presence of vector mesons $\rho, \omega$ that induce drastic differences already at $\sqrt{s} \sim m_\rho$. In the case of $W_L^\pm$ and $Z_L$, $F_\pi$ is replaced by $G_F^{-1/2}$.

Two broad possibilities emerge and have been amply discussed in the literature. On the one hand, the situation of the Standard Model can be stretched up to large $m_H$, where a very broad enhancement is present in the scalar channel with $I = 0$ ($I$: weak isospin). On the other hand, the QCD picture can be mimicked with vector resonances with $I = 1$ ($\rho$) or $I = 0$ ($\omega$). This is for example the case of models based on $SU(N_{TC})$ technicolour [18] or scaled-up QCD (i.e. $N_{TC} = 3$) or the 'BESS' model of Casalbuoni, Gatto et al. [49], which is a non-renormalizable Lagrangian model with no Higgs (eliminated as in the non-linear $\sigma$ model [50]) extended to include an extra $SU(2)$, which leads to heavy vector $\rho$-like states (with $I = 1$).

A more general approach that can generate a QCD-like or a Higgs-like model, or other cases as well, was adopted by Dobado, Herrero and Terron [51] (for related work, see also Ref. [52]). Higher-order terms in the momenta are added to the lowest-order effective Lagrangian. While the lowest-order effective Lagrangian is fixed by the low-energy theorems in terms of a single-energy parameter, $G_F^{1/2}$, the next-order couplings depend on two arbitrary parameters. By varying those constants one can switch from one type of physics to another. Some procedure of unitarization is implemented in order to extend the model at large $\sqrt{s}$ (in a purely phenomenological way) so that the model formally makes sense also in the presence of resonances.

Extensive studies based on the various models listed above were performed for the Aachen Workshop [34], together with detailed experimental simulations, in order to evaluate the capabilities of the LHC in this domain of physics. The general procedure is to compute the $VV$ scattering
amplitudes in a given model, to compare the results with the Standard Model prediction for some large but still admissible Higgs mass, to check whether the deviations would be measured at the LHC, and to disentangle the different models.

The processes that are best suited for an experimental investigation are those with no $tar{t} \rightarrow WWb\bar{b}$ background: $ZZ, W^\pm Z, \text{and } W^\pm W^\pm$ (equal charges!) final states. Different qualitative behaviours are expected in these channels, depending on the dynamics of $V_L V_L$ scattering: in the Higgs-like regime, sizeable effects are expected in the $ZZ$ channel and not in the $W^\pm Z$ or $W^\pm W^\pm$ reactions. Conversely a $\rho$-like resonance would show up in the $WZ$ channel and not elsewhere.

For equal-sign $WW$ final states, the production rate of $W^-W^-$, with $M_{WW} > 0.8$ TeV, is about one third of that of $W^+W^+$ (because $u$ quarks are more abundant than $d$ quarks at large $x$ in the proton). The background from $W^\pm t\bar{t}$, from $q\bar{q} \rightarrow W^\pm W^\pm q\bar{q}$ via gluon exchange, and from QCD jets has been evaluated by Barger et al. [53]. In models with no $I = 2$ resonances, as those studied by Dobado et al. [51], there is little activity in the channel $pp \rightarrow W^\pm W^\pm X \rightarrow \ell^\pm \nu\ell^\pm \nu X$, and the signal is small with respect to the background (Fig. 14). For $\int L dt \simeq 10^5$ pb$^{-1}$ the rate is of the order of 10 events per year at $M_{WW} > 0.8$ TeV. The situation is no better at the SSC with $10^4$ pb$^{-1}$. This does not necessarily mean that this process is not interesting, because the actual dynamics could be different from that of the models studied in the literature. If a doubly-charged resonance exists, it would show up in this channel.

In the $ZZ \rightarrow 4\ell^\pm$, $\ell = e, \mu$, channel the signal from $W_L W_L \rightarrow Z_L Z_L$ plus $Z_L Z_L \rightarrow Z_L Z_L$ was computed in the model by Dobado et al. [51], compared with the irreducible background from the Standard Model processes $q\bar{q}, gg \rightarrow ZZ$. For $\int L dt \simeq 10^5$ pb$^{-1}$, $M_{ZZ} > 0.5$ TeV, $p_T^Z > 10$ GeV, $|y_Z| < 2.5$, the background amounts to about 220 events (for $m_t = 100$ GeV), while the signal is of about 15 events in a Higgs-like picture, and half of that in a scaled-up QCD model. The corresponding numbers at the SSC, with $10^4$ pb$^{-1}$ and the same cuts, are 73 (background), 10 (Higgs-like), and 5 (QCD-like) events. Without jet tagging it is difficult to separate the $VV \rightarrow VV$ signal from the irreducible background, particularly because the latter is only computable with limited accuracy and both the signal and the background have a structureless mass distribution. (Recently, the next-to-leading QCD corrections to $q\bar{q} \rightarrow ZZ$ have been computed [54].)

The prospects are much more promising for models with resonances, as for example a $\rho^2$-like particle observable in $W^\pm Z$ final states or an $\omega$-like object visible in $Z\gamma$. At the Aachen Workshop the $WZ$ channel was studied in full detail in $SU(N_{TC})$ models realized in the effective Lagrangian
Figure 14: Like-sign WW invariant-mass distribution for various strongly-interacting models and for the total background. Rates for $W^+W^+$ and $W^+W^-$ are added [51]. The short-dashed line is for the QCD-rescaled case and the lower solid line is for the Higgs-like model. The upper solid curve corresponds to the unitarized-low energy theorems results [53]. The Standard Model rates for $M_H = 1$ TeV are also displayed for comparison [53] (dot-dashed line). The long-dashed lines are the predictions for the total background in the cases $m_t = 100$ GeV (upper line) and $m_t = 200$ GeV (lower line), respectively [53].

approach [51] and in the BESS model [49],[55]. In scaled-up QCD

$$m_{\rho_{TC}} \approx \frac{\nu}{F_{\pi}} m_{\rho} \approx 2 \text{ TeV} \quad (4.36)$$

$$\Gamma(\rho_{TC} \rightarrow VV) \approx \frac{m_{\rho_{TC}}^2}{96\pi v^2} \approx 450 \text{ GeV}. \quad (4.37)$$
Figure 15: WZ invariant-mass distribution for the signal and background processes with the optimal cuts (a 2.5 rapidity cut has been chosen) [51]. Rates are for $W^+Z + W^-Z$ and for $L = 4 \times 10^6 \text{ pb}^{-1}$. The results of the signal are for three possible cases in SU($N_{TC}$) theories corresponding to: $m_p = 1.0 \text{ TeV}$, $1.5 \text{ TeV}$, and $2.0 \text{ TeV}$, respectively. The lower solid histogram represents the WZ fusion contribution to the signal. The dotted histogram is the $q\bar{q}'$ annihilation contribution to the signal via $\rho-\omega$ mixing. The total background is the dashed histogram, and the total signal + background is the upper solid histogram.
For $SU(N_{TC})$ with $N_{TC} \neq 3$ one takes [18]:

$$m_{\rho_{TC}} \simeq 2 \text{ TeV} \sqrt{\frac{3}{N_{TC}}},$$

$$\Gamma(\rho_{TC} \rightarrow VV) \simeq 450 \text{ GeV} \left(\frac{3}{N_{TC}}\right)^{3/2}.$$ (4.38)

Thus for $N_{TC} \simeq 12$ one has $m_{\rho_{TC}} \simeq 1 \text{ TeV}$ and $\Gamma_{\rho_{TC}} \simeq 55 \text{ GeV}$, while for $N_{TC} = 5$, $m_{\rho_{TC}} \simeq 1.5 \text{ TeV}$ and $\Gamma_{\rho_{TC}} \simeq 185 \text{ GeV}$. The results for these three representative cases ($m_{\rho_{TC}} = 1, 1.5, 2 \text{ TeV}$) are summarized in Fig. 15. The full process under consideration is $pp \rightarrow W^\pm ZX \rightarrow \ell^\pm \nu_\ell \ell^- \nu_\ell$, $\ell = e, \mu$. The $W^+Z$ rate is about twice the $W^-Z$ rate. The resonant signal in $W^\pm Z_L$ is produced either by $WZ$ fusion or by $q\bar{q}$ annihilation with $\rho_{TC}$ coupled via $W$-$\rho_{TC}$ mixing. The irreducible background is from the standard processes $q\bar{q} \rightarrow WZ, W\gamma \rightarrow WZ$, and $WZ \rightarrow WZ$ (with no $\rho_{TC}$ exchange). With optimized cuts the following S/B ratios were obtained at the LHC [51] (in number of events per $10^5 \text{ pb}^{-1}$): 660/53 for $m_{\rho_{TC}} = 1 \text{ TeV}, 50/11$ for $m_{\rho_{TC}} = 1.5 \text{ TeV}$, and 20/13 for $m_{\rho_{TC}} = 2 \text{ TeV}$. At the SSC with $10^4 \text{ pb}^{-1}$ (with different cuts optimized to the SSC case) the corresponding numbers are: 263/24, 36/8, and 24/16, respectively. The resonant is visible in the mass and $p_T$ distributions. The invariant mass distributions in the LHC case are shown in Fig. 15. Detailed simulations presented by Rodrigo et al. [34] show that the signal clearly emerges at large $p_T$ over the complete background, also including the reducible one with 3-lepton events from $t\bar{t}$ production (Fig. 16).

The production of a different type of $p$-like resonance in the $WZ$ channel was studied in the context of the BESS model [49],[55]. The values of the free parameters $m_{\rho_{TC}}, \gamma'$ were chosen in such a way as to have $m_{\rho_{TC}} = 1, 1.5, 2, 2.5 \text{ TeV}$ with widths $11-44$, 84-355, 353, 455 GeV, respectively. The $\rho_{TC}^\pm$ is coupled to $WZ$ and also to $q\bar{q}$ via the $W^+\rho_{TC}^-$ mixing (an additional direct coupling with quarks could be switched on by letting a parameter called $b$ be different from zero). The background is the same as in the previous discussion. In Fig. 17a,b, we report the $\rho_{TC}$ distributions for the case $\rho_{TC} = 2.0 \text{ TeV}$, with $b = 0$. Here again the LHC with $\int L \, dt \simeq 10^5 \text{ pb}^{-1}$ is compared with the SSC with $\int L \, dt \simeq 10^4 \text{ pb}^{-1}$. Even if $m_{\rho_{TC}} \simeq 2 \text{ TeV}$ is large enough to provide a special advantage to the SSC, we see that the S/B ratios in the two cases are comparable [684/310 (LHC) and 1010/462 (SSC)]. The discovery range at both LHC and SSC extends up to $\sim 2.5 \text{ TeV}$.

Summarizing (see the report by M. Lindner, S. Dimopoulos et al. in Ref. [34]): $W^\pm W^\pm$ is small, below the background, in models with no $I = 2$ resonances. The $ZZ$ channel is in principle good for the Higgs-like case, but
Figure 16: Signal and background for $\rho_{TC}^\pm$, with $m_{\rho_{TC}^\pm} \simeq 1\text{--}2$ TeV. The solid line is the total signal in $W_L^\pm Z_L$ final states (obtained by adding boson fusion and $q\bar{q}'$ annihilation). The dashed line is the fusion contribution alone. The stars indicate the total background. (From Ref. [34].)

Figure 17: Transverse-momentum distribution of signal and background at the LHC (a) and the SSC (b), for $\rho_{TC}$ production in BESS (from Refs. [49] and [55]), with $m_{\rho_{TC}} = 2$ TeV and $b = 0$. The upper histogram is the signal from $q\bar{q}$ annihilation, the centre one that from fusion, and the lower one from the total background.
it is very difficult to disentangle a non-resonant signal from the continuum. The \( W^+ Z \) (or \( Z\gamma \)) is good for \( \rho \)-like (\( \omega \)-like) resonances. A \( pT_C \) resonance with \( m_{pT_C} < 2.5 \text{ TeV} \) can be detected in the \( WZ \) channel at the LHC with \( \int L \, dt \approx 10^8 \text{ pb}^{-1} \) or at the SSC with \( \int L \, dt \approx 10^9 \text{ pb}^{-1} \). In conclusion if there are resonances with \( \Gamma \ll M \), they can be detected. Otherwise a structureless signal is difficult to be established both at the LHC and at the SSC.

### 4.4. Supersymmetric Higgses

As we discussed in Section 4.1, many theorists consider that fundamental scalar Higgses are most likely to be accompanied by supersymmetry in order to make the theory natural when looked down from very high energy scales such as \( M_{\text{GUT}} \) or \( M_{\text{Pl}} \). However in all supersymmetric extensions of the Standard Model at least two Higgs doublets are necessary [15],[33], giving their masses one to the up fermions and the other to the down ones. Thus in supersymmetric models there are at least three neutral and two charge-conjugated charged physical Higgses. In the MSSM the spectrum of physical Higgses is specified by two parameters: the mass of one of the neutral Higgses and \( t \beta = v_u/v_d \), the ratio of the VEVs of the Higgses that give mass to up fermions, \( v_u \), and to down fermions, \( v_d \). In the MSSM, \( t \beta \) is always larger than 1 (while in a generic two-doublet model there is no such restriction). Also, values of \( t \beta > m_t/m_b \) are not allowed. The neutral Higgses are denoted by \( h, A, \) and \( H \): \( h \) is the lightest Higgs (\( J^{CP} = 0^+ \)), \( A \) is the Higgs with opposite CP (\( 0^- \)), and \( H \) is the heavy Higgs with quantum numbers \( 0^+ \). At tree level, in terms of the parameters \( t \beta \) and \( m_A \), one has [15],[33],

\[
m_{H^\pm}^2 = m_A^2 + m_W^2 \tag{4.39}
\]

\[
m_{h,H}^2 = \frac{1}{2} \left[ (m_A^2 + m_Z^2) \pm \sqrt{(m_A^2 + m_Z^2)^2 - (2m_A m_Z \cos 2\beta)^2} \right] \tag{4.40}
\]

so that \( m_H^2 \geq m_W \); \( m_h \leq m_Z, m_A \); \( m_H > m_Z, m_A \). From LEP data analysed in terms of tree level formulae one would obtain \( t \beta \geq 1.6, m_A \geq 40 \text{ GeV}, m_h \geq 33 \text{ GeV} \). But for large \( m_t \) there is the possibility that radiative corrections could induce rather large shifts in the Higgs masses [57]. In particular \( m_h \) could exceed \( m_Z \). The results of Refs. [57] imply that for \( m_t \sim 130 \text{ GeV} \), the shift of \( m_h \) due to the radiative corrections is of a few GeV and becomes rapidly larger with increasing \( m_t \) (the effect increases as \( m_t^4 \)).

The case of the MSSM is a particularly important and interesting one. The first observation is that if the MSSM is true, then most probably a
Higgs will be found at LEP. We repeat that the lucky event of the discovery of a Higgs particle at LEP does not in any sense diminish the physics case for the LHC. This is obviously true if the observed properties of the light Higgs depart from the behaviour of the standard Higgs and are consistent with the MSSM. But this is also true if the accessible information obtained at LEP on the light Higgs is compatible with the Standard Model. In fact for most of the parameter space the properties of the light Higgs are close enough to the Standard Model for LEP not to be able to clearly distinguish the two cases. It is only the experimental investigation of the LHC energy domain that can possibly clarify the issue. In particular the question of the search for the SUSY Higgses at the LHC is an important one.

The production cross-sections of SUSY Higgses at the LHC/SSC are often larger than for the Standard Model Higgs of the same mass. This is because of the addition of s-quark loops in the gluon-fusion mechanism and also because of the larger couplings to $b\bar{b}$ for $tg\beta$ large. However the couplings to $\gamma\gamma$, $WW$, and $ZZ$ are typically suppressed with respect to the Standard Higgs. For example the modes $A \rightarrow WW$ or $ZZ$ are forbidden for the $A$ boson. But, for large $tg\beta$, the channels $A \rightarrow \tau\tau$ or $H \rightarrow \tau\tau$ are promising: the $\tau\tau$ mode which is no good for the Standard Higgs can be viable for SUSY Higgses. Similarly the mode $H \rightarrow ZZ \rightarrow 4\ell^{\pm}$ is good for the heavy SUSY Higgs provided that $m_H < 2m_t$ and $tg\beta$ is small. At large $m_A$, $h \rightarrow \gamma\gamma$ is a good channel while $H \rightarrow \gamma\gamma$ is viable at small $m_A$.

In conclusion, for the neutral Higgses of the MSSM the detection is in general a hard problem. A separate analysis, as complicated as the one for the Standard Higgs, would be necessary for each set of values of $tg\beta$, $m_{A,H}$, and $m_t$. While much more work is needed on this subject [58], in much of the parameter space detection is possible for at least one of the Higgses. However, there is a region at intermediate values of $m_A$ and $tg\beta$ where, for $m_t$ large, no known method would allow detection of any of the Higgses neither at LEP or at LHC/SSC so that more studies are demanded.

The case of the charged Higgs was also considered at the Aachen Workshop, especially by M. Fecini [[34]. The charged Higgs could be observed if it is present in $t$ decays: $t \rightarrow H^{\pm}b$. As the dominant $H^{\pm}$ decay would be $H^{\pm} \rightarrow \tau^{\pm}\nu$, the signature would be a measurable violation of $\tau-\mu$ universality in $t\bar{t}$ events. For $m_t \approx 200$ GeV, $m_{H^{\pm}}$ could be detected in the range $m_{H^{\pm}} = 100-150$ GeV, while for $m_t = 150$ GeV, $m_{H^{\pm}}$ would be visible up to $m_H \sim 100$ GeV (Fig. 18).
Figure 18: Sensitivity to violations of the $\tau \leftrightarrow \mu$ universality induced by a charged Higgs (from M. Fetcini).

4.5. Conclusion on the Higgs

The main goal of experiments on particle physics in the near future is the clarification of the electroweak symmetry-breaking problem. The solution must be within the TeV energy region: the origin of the weak scale cannot lie too far from $G_F^{-1/2} \sim 293$ GeV. Probably a whole universe of new physics will open up. Examples are offered by the supersymmetric model, which provides a well-defined extension of the Standard Model, more natural than the Standard Model itself. Other possibilities are less well defined. Apart from possible completions of the Standard Model in the direction of extending the electroweak group (new $W'$ and $Z'$), all alternatives to fundamental scalar Higgses and supersymmetry involve new strong forces and a breakdown of the perturbative regime in the TeV energy region.

A common feature of all conceivable ways beyond the Standard Model is the prediction of a rich spectroscopy of new states and new phenomena. This means that one expects discoveries over a wide range of energies.
Actually it would be a great thing for the LHC and SSC if the low-lying fringes of the new spectroscopy were already found at LEP 1 and 200. Far from decreasing the physics motivations in favour of the LHC and SSC, the discovery at LEP of some new physics or at least of some departures from the Standard Model would make the argument for the LHC even stronger.

The results obtained at the Aachen Workshop clearly demonstrate that the discovery potential of the LHC with $L \approx 10^{34}$ cm$^{-2}$ s$^{-1}$ is perfectly adequate to the goal of solving the problems of the electroweak symmetry breaking and of the origin of the weak scale of mass. It is also evident, from the detailed comparison made in the previous sections, that the LHC with $L = 10^{34}$ cm$^{-2}$ s$^{-1}$ is very much comparable with the SSC with $L = 10^{33}$ cm$^{-2}$ s$^{-1}$ (although the task is more demanding for the detectors at high luminosity). For standard Higgses, we have seen that the discovery range extends up to (0.8-1) TeV in both cases. In $WW$, $WZ$, $ZZ$ scattering resonances, such as the $\rho$-like or $\omega$-like ones, vector bosons of technicolour are visible up to 2-2.5 TeV at the LHC and the SSC, while in both cases non-resonant amplitudes are very difficult to study. One also finds [34] that new $W'$ and $Z'$ can be found up to 4.5-5 TeV at the LHC, and up to 5-6 TeV at the SSC. Gluinos and squarks can be observed up to 1.5 TeV at the LHC and up to 1.5-2 TeV at the SSC.

5. BASIC RELATIONS FOR PRECISION TESTS

In the standard electroweak theory, there are a number of basic relations that one wants to verify experimentally with the best possible precision. The same quantity $\sin^2 \theta_W$ appears in all these relations.

First, $\sin^2 \theta_W$ can be measured from the value of $m_W$. Starting from the tree-level relations [Eqs. (3.13) and (3.17)], $\sin^2 \theta_W = \epsilon^2 / g^2$ and $G_F / \sqrt{2} = g^2 / 8 m_W^2$, we obtain

$$\sin^2 \theta_W = \left(\pi\alpha / \sqrt{2} G_F\right) / [m_W^2 (1 - \Delta r)]$$

$$= \left(37.2802 \text{ GeV}^2\right) / [m_W^2 (1 - \Delta r)]\ ,$$

(5.1)

where $\Delta r \neq 0$ owing to the effect of radiative corrections.

Then $\sin^2 \theta_W$ is also related to the ratio of the vector boson masses. At tree level [see Eq. (3.22)],

$$\sin^2 \theta_W = 1 - m_W^2 / \rho_0 m_Z^2\ ,$$

(5.2)

where $\rho_0 = 1$ in the Standard Model with only doublets of Higgs bosons. In general, this relation is also modified by radiative corrections:

$$\sin^2 \theta_W = 1 - m_W^2 / \rho_{\text{mass}} m_Z^2\ ,$$

(5.3)
with $\rho_{\text{mass}} = \rho_0(1 + \delta \rho_{\text{mass}})$. However, as we shall discuss in detail later, Eq. (5.2) with $\rho_0 = 1$ is often adopted as a definition of $\sin^2 \theta_W$ at all orders. Clearly in this case, $\rho_{\text{mass}} = 1$ by definition.

Finally, $\sin^2 \theta_W$ can be obtained for neutral-current couplings. At tree level, the four-fermion interaction from $Z$ exchange is given [see Eqs. (3.20) and (3.21)] by

$$M_{1f} = \left[ \sqrt{2} \frac{G_F m_Z^2}{D(s)} \rho_0(J_3^f - 2\sin^2 \theta_W J_{1m}^f)(J_3^f - 2\sin^2 \theta_W J_{1m}^f) \right]$$

(5.4)

where $D(s)$ is the $Z$ propagator, and $J_3^f$ and $J_{1m}^f$ are the weak isospin and electromagnetic currents for the fermion $f$. Excluding pure QED corrections, electroweak radiative corrections modify $M_{1f}$ according to

$$M_{1f} = \left[ \sqrt{2} \frac{G_F m_Z^2}{D(s)} \rho_{1f}(J_3^f - 2k_i \sin^2 \theta_W J_{1m}^i)(J_3^f - 2k_f \sin^2 \theta_W J_{1m}^f) + \ldots \right]$$

(5.5)

where $\rho_{1f} = \rho_0(1 + \delta \rho_{1f})$; $k_a = 1 + \delta k_a(a = i, f)$ are in general different for different fermions and depend on the scheme adopted (for example, $\delta k_a$ depend on the definition of $\sin^2 \theta_W$). The ellipsis indicates possible additional small non-factorizable terms.

6. INPUT PARAMETERS

For LEP physics [59], a self-imposing set of input parameters is given by $\alpha_s, \alpha, G_F, m_Z, m_f$ and $m_H$. Clearly the Fermi coupling $G_F = 1.16637(22) \times 10^{-5} \text{GeV}^{-2}$ is conceptually more complicated than $\alpha_{\text{weak}} = 2 \pi \left( \frac{2}{4\pi} - 3 \right)$ (which would more naturally accompany $\alpha = 1 / 127.036$ and $\alpha_s$) or $\sin^2 \theta_W$ or $m_W$, but is preferred for practical reasons because it is known with all the desirable accuracy. Similarly, $m_Z$ has now been measured at LEP with remarkable precision. This preliminary task of LEP in view of precision tests of the Standard Model has already been accomplished to a nearly final degree of accuracy (see Section 10 for details on the lineshape measurement).

The LEP results on $m_Z$, as summarized at the Geneva Conference [60], are reported in Table 1.1. The resulting relative precision is impressive: $\delta m_Z / m_Z = 2.3 \times 10^{-4}$.

---

1 When writing this talk I decided to update the experimental data by also including results that were not yet available when it was given.
TABLE 3: Results from LEP

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$m_Z$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>91.182 ± 0.009</td>
</tr>
<tr>
<td>DELPHI</td>
<td>91.177 ± 0.010</td>
</tr>
<tr>
<td>L3</td>
<td>91.181 ± 0.010</td>
</tr>
<tr>
<td>OPAL</td>
<td>91.161 ± 0.009</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>$91.175 ± 0.005 \text{(stat.)} ± 0.020 \text{ (LEP)}$</td>
</tr>
</tbody>
</table>

Among the quark and lepton masses, $m_f$, the main unknown is the top quark mass. Our ignorance of $m_t$ is at present a serious limitation for precise tests of the electroweak theory because the radiative corrections are relatively large for large $m_t$ and depend quadratically on $m_t$ [59]. This fact can be used to put stringent constraints on $m_t$ from the existing electroweak measurements, in particular an upper bound on $m_t$, to be discussed in detail later. As for lower bounds on $m_t$ the best results arise from the failure to observe the t quark at $e^+e^-$ and hadron colliders. LEP and SLC lead to a model-independent bound $m_t \geq 45 \text{ GeV}$. From CDF one learns that $m_t \geq 89 \text{ GeV}$, provided that the t quark semi-leptonic branching ratio is as predicted by the Standard Model.

The Higgs mass $m_H$ is largely unknown. One of the most impressive performances of LEP up to now has been the dwarfing of all previous lower bounds on $m_H$. The LEP ones are summarized in Eq. (4.23). The basis for the theoretical upper bound on $m_H$ was reviewed in Section 4.1. As a result calculation of radiative corrections is now done for $50 \text{ GeV} < m_H < 1 \text{ TeV}$. The sensitivity of the radiative corrections to variations of $m_H$ in the range $50 \text{ GeV} < m_H < 1 \text{ TeV}$ is not large. In a sense, this level of accuracy fixes the goal for precision tests of the Standard Model because the clarification of the symmetry-breaking sector of the theory is the main target of present-day experiments.

Finally, for electroweak calculations involving hadrons, the value of the QCD coupling $\alpha_s$ must also be specified. The best value of $\alpha_s$ at the Z mass, obtained from experiments at energies lower than $m_Z$ (in particular from deep inelastic scattering) is given by [61] $\alpha_s(m_Z) = 0.11 ± 0.01$. At LEP one finds [61] something like $\alpha_s(m_Z) \simeq 0.12 ± 0.01$. (Note that I am more conservative than usual on errors, which are dominated by theoretical uncertainties.) The QCD corrections to processes involving quarks are typically of order $\alpha_s/\pi$. As a consequence the stated error on $\alpha_s$ leads to a few per mille relative uncertainty on the corresponding predictions.
7. LARGE CONTRIBUTIONS TO RADIATIVE CORRECTIONS

A set of important quantitative contributions to the radiative corrections arise from large logarithms [e.g. terms of the form \( \frac{\alpha}{\pi} \ln (m_Z/m_f) \)]\(^n\) where \( f \) is a light fermion, and from quadratic terms in \( m_t \), i.e. terms proportional to \( G_F m_t^2 \).

The sequences of leading and close-to-leading logarithms are fixed [62] by well-known and consolidated techniques (\( \beta \) functions, anomalous dimensions, penguin-like diagrams, etc.). For example, large logarithms dominate the running of \( \alpha \) from \( m_* \), the electron mass, up to \( m_Z \), with the result that [63]

\[
\frac{\alpha(m_Z)}{\alpha} = 1/(1 - \delta \alpha) .
\] (7.1)

At present, the best value of \( \delta \alpha \), obtained by extracting the relevant effective light-quark masses from the data on \( \mu^+ \mu^- \rightarrow \) hadrons, is given by [63]

\[
\delta \alpha = 0.0061 \pm 0.0009 .
\] (7.2)

Large logarithms of the form \( [\alpha/\pi \ln (m_Z/\mu)]^n \) also enter, for example, in the relation between \( \sin^2 \theta_W \) at the scales \( m_Z \) (LEP, SLC) and \( \mu \) (e.g. the scale of low-energy neutral-current experiments).

The quadratic dependence on \( \alpha \) [64] (and on other possible widely broken isospin multiplets from new physics) arises because, in spontaneously broken gauge theories, heavy loops do not decouple. On the contrary, in QED or QCD, the running of \( \alpha \) and \( \alpha_s \) at a scale \( Q \) is not affected by heavy quarks with mass \( M \gg Q \). According to an intuitive decoupling theorem [65], diagrams with heavy virtual particles of mass \( M \) can be ignored at \( Q \ll M \) provided that the couplings do not grow with \( M \) and that the theory with no heavy particles is still renormalizable. In spontaneously broken gauge theories, one important difference is in the longitudinal modes of weak gauge bosons. These modes are generated by the Higgs mechanism, and their couplings grow with masses (as is also the case for the physical Higgs couplings). The upper limit on \( m_t \) from radiative corrections arises from this phenomenon. Other subtler sources of non-decoupling are related, for example, to the presence of chiral anomalies [14] (which may not be completely cancelled if heavy particles are removed). Another very important consequence is that precision tests of the electroweak theory may be sensitive to new physics even if the new particles are too heavy for their direct production. With the value of \( m_t \) being continuously pushed up by experiment, the quantitative importance of the terms of order \( G_F m_t^2 \) is increasingly large. Both the large logarithms and the \( G_F m_t^2 \) terms have a
simple structure and are to a large extent universal, i.e. common to a wide class of processes. Their study is important for an understanding of the pattern of radiative corrections. One can also derive approximate formulae (e.g. improved Born approximations, see Section 9), which can be useful in cases where a limited precision may be adequate.

8. DETERMINATION OF $\sin^2 \theta_W$ FROM $m_Z$

Once $\alpha, \alpha_s, G_F, m_Z, m_f$ and $m_H$ have been chosen as input parameters, $\sin^2 \theta_W$ is a derived quantity. We now consider the calculation of $\sin^2 \theta_W$ beyond the tree approximation. A precise definition of $\sin^2 \theta_W$ must be specified before its value can be computed.

At tree level the relation between $\sin^2 \theta_W$ and $m_Z$ is obtained from Eqs. (5.1) and (5.2) (with $\Delta r = 0$):

$$\sin^2 \theta_W \cos^2 \theta_W = \left(\frac{\pi \alpha}{\sqrt{2} G_F}\right)/(\rho_0 m_Z^2).$$

(8.1)

Beyond the tree level, the quantity $\Delta r$ [66] is introduced by radiative corrections:

$$\sin^2 \theta_W \cos^2 \theta_W = \left(\frac{\pi \alpha}{\sqrt{2} G_F}\right)/[\rho_0 m_Z^2(1 - \Delta r)],$$

(8.2)

where $\Delta r \equiv \Delta r(\alpha, \alpha_s, G_F, m_Z, m_f, m_H)$ is of course different for different definitions of $\sin^2 \theta_W$. In the following, some particularly interesting definitions of $\sin^2 \theta_W$ will be discussed. We now set $\rho_0 = 1$. Let us first consider the usual definition [67]:

$$\sin^2 \theta_W \equiv s_W^2 = 1 - (m_W^2/m_Z^2).$$

(8.3)

From now on, the symbol $s_W^2$ will always refer to this specific definition of $\sin^2 \theta_W$. Clearly, given $m_Z$ from LEP, $s_W^2$ is directly equivalent to $m_W$. In this case $\Delta r$ specifies the relation between $m_Z$ and $m_W$:

$$(1 - m_W^2/m_Z^2) m_W^2 = \left(\frac{\pi \alpha}{\sqrt{2} G_F}\right)/(1 - \Delta r).$$

(8.4)

The value of $\Delta r$ as a function of the input parameters has been studied in great detail. In particular, the following results are obtained [63], [66], [68]:

$$1/(1 - \Delta r) = \alpha(m_Z)/\alpha \cdot 1/[1 + (c_W^2/s_W^2) \delta \rho] + \text{small}$$

$$= 1/(1 - \delta \alpha) \cdot 1/[1 + (c_W^2/s_W^2) \delta \rho] + \text{small},$$

(8.5)

where $c_W^2 = 1 - s_W^2$; $\delta \alpha$ was defined in Eqs. (7.1) and (7.2), and $\delta \rho \rightarrow \delta \rho_t$, for large $m_t$ with $\delta \rho_t$ given by [63], [69]

$$\delta \rho_t = (3G_F m_t^2/8\pi^2 \sqrt{2}) + (3G_F m_t^2/8\pi^2 \sqrt{2})^2(19 - 2\pi^2)/3 + O((G_F m_t^2)^3).$$

(8.6)
By ‘small’ in Eq. (8.5), we mean terms (which at one-loop accuracy are known) without large logs and/or leading powers of \(G_F m_t^2\); \(\delta \rho_t\) is the dominant term, for large \(m_t\), of the famous \(\rho\)-parameter first studied in Refs. [64]. The two-loop term was obtained in Ref. [69] and the geometric series resummation was advocated in Ref. [68].

Going back to the \(Z\)-exchange amplitude near the resonance and the parameters \(\rho_f\), \(k_i\), and \(k_f\) defined in Eq. (5.5) with the definition of \(\sin^2 \theta_W \equiv s_W^2\), we obtain [63]

\[
\rho_f = 1 + \delta \rho + \text{‘small’}
\]  

(8.7)

and

\[
k_f = 1 + (c_W^2 / s_W^2) \delta \rho + \text{‘small’}
\]  

(8.8)

(here \(f \neq b\); the \(b\)-quark will be reconsidered in the following), where the ‘small’ terms are non-universal (i.e. process-dependent). Note that \(k_f\) contains additional ‘large’ terms with respect to those included in \(\Delta r\) and \(\rho_f\). As these ‘large’ terms are also universal, this suggests that \(k_f s_W^2\) could be a better effective \(\sin^2 \theta_W\) than \(s_W^2\) for physics at the \(Z\) pole. Before going into this matter, we will add some comments on \(\delta \rho\). For any weak iso-spin fermion doublet (e.g. new heavy quarks or leptons), the quantity \(3m_t^2\) in \(\delta \rho_t\) (at one loop) becomes [63],[66]

\[
3m_t^2 = N_c [m_u^2 + m_d^2 - 2m_s^2 m_d^2 / (m_u^2 - m_d^2) \ln (m_u^2 / m_d^2)],
\]  

(8.9)

where \(N_c\) is the number of colours. For negligible \(m_d,u\), \(3m_t^2 = N_c m_d^2\), whilst for \(m_d = m_d + \epsilon\): \(3m_t^2 = N_c \frac{4}{3} \epsilon^2\) to leading order in \(\epsilon\) (the result vanishes for unsplit doublets). Similarly, many more kinds of broken weak iso-spin multiplets can contribute to \(3m_t^2\) [66],[70] (squarks and sleptons [71], charged Higgses [72], etc. The upper limit on \(m_t\) \(\lesssim 200\) GeV (see Section 11) together with the direct lower limit on the t-quark mass, \(m_t \gtrsim 89\) GeV, leave little space for heavy multiplets, except for nearly degenerate ones.

The contribution of the neutral Higgs mass to \(\delta \rho\) is only logarithmic at one loop, (see, e.g., Eq. (12.7) in Section 12). There are no \(m_H^2\) terms but only logs, because the ‘custodial’ \(SU(2)\) symmetry is not broken in the Higgs sector [21]. Power terms appear only at two loops [73], but their effect is sizeable only for \(m_H \gtrsim 1\) TeV.

We now consider a different class of definitions of \(\sin^2 \theta_W\). Assume that we fix \(k_f = 1\) [defined by Eq. (5.5)] for one given \(Z\) vertex (e.g. \(k_f = 1\) in \(Z \rightarrow \epsilon^+ \epsilon^-\) for on-shell \(Z\)). Then it follows from the previous discussion that \(\delta \kappa\) is ‘small’ for all neutral-current processes (for \(f \neq b\)) near the \(Z\) pole. Let us introduce the notation
\[ \sin^2 \theta_W \text{(from } Z \rightarrow e^+ e^-) \equiv \hat{s}_W^2 \] 

Equation (8.10)

In this case, Eq. (5.5) becomes

\[ M_f = \frac{\sqrt{2}}{2} G_F m_Z^2 / D(s) \]

\[ \rho_f (J_3^f - 2 J_{em}^f) J_3^f - 2 \hat{s}_W J_{em}^f) + \text{small}, \] 

Equation (8.11)

where \( \rho_f \) is given by Eq. (8.7) in terms of \( \delta \rho \), which is specified in Eqs. (8.6). In the present case,

\[ \hat{s}_W\hat{c}_W = \frac{(\pi \alpha / \sqrt{2} G_F) / [m_Z^2 (1 - \Delta^f)]}{\Sigma} , \]

Equation (8.12)

and we find [63]

\[ \hat{s}_W \approx \hat{s}_W^2 + \hat{c}_W^2 \delta \rho + \text{small} , \]

Equation (8.13)

In other words, apart from 'small' terms, we have

\[ \hat{s}_W^2 \approx 1 - \left( \frac{m_Z^2}{\rho m_Z^2} \right) \]

Equation (8.14)

---

Figure 10: The behaviour of \( \hat{s}_W^2 \) and \( \hat{s}_W^2 \) [defined by Eqs. (8.3) and (8.10), respectively] as functions of \( m_e \), for \( m_e = 91 \) GeV, \( m_H = 100 \) GeV. The solid [dashed] lines are computed to \( O(\alpha^2) [O(\alpha)] \). From Ref. [63].
with $\rho \approx 1 + \delta \rho$. It is interesting to note that $s_{\nu\nu}'$ is less dependent than $s_{W}$ on $m_{t}^{2}$. In fact [63],[66]

$$\Delta r \approx \delta \alpha \cdot \left( c_{W}^{2} / s_{W}^{2} \right) \delta \rho,$$

$$\Delta \bar{r} \approx \delta \alpha \cdot \delta \rho,$$  \hspace{1cm} (8.15)

(8.16)

so that the amplifying factor $c_{W}^{2} / s_{W}^{2}$ in front of $\delta \rho$ is missing in $\Delta \bar{r}$. Note that given $m_{Z}$, $s_{W}^{2}$ is known with an accuracy of about $\pm 0.002$ when $m_{t}$ varies between 60 and 210 GeV (see Fig. 19). There is a class of definitions of $\sin^{2} \theta_{W}$ in the literature that correspond to $s_{W}^{2}$ if 'small' terms are neglected: $s_{W}^{2}$ of Hollik [74]; $s_{W}^{2} m_{Z}^{2}$ of Lynn and Kennedy [75]; $(\sin^{2} \theta_{W})_{N_{0}}$ [76],[77], etc. Note that because of Eqs. (7.1) and (8.16) one can also write:

$$s_{W}^{2} c_{W}^{2} = \left[ \pi \alpha(m_{Z})/\sqrt{2} G_{F} \right] / \rho m_{t}^{2}.$$  \hspace{1cm} (8.17)

9. IMPROVED BORN APPROXIMATION

For precision tests of the electroweak theory, the complete one-loop radiative corrections (plus higher-order and possibly exponentiated purely photonic corrections) are mandatory and are indeed available for the processes of practical relevance. However, in many cases a less accurate estimate can be enough. For this purpose, it is useful to know [63] that formulae as simple as those of the Born approximation can be written in a way that takes all 'large' corrections into account. As a result, for $e^{+}e^{-} \rightarrow f\bar{f}$, with $f \neq e, b$, the amplitude for $\gamma$ and $Z$ exchange near the resonance can be written in the form

$$M_{f\bar{f}} = Q_{f} Q_{\bar{f}} \left\{ 4 \pi \alpha(m_{Z}) / s \right\} J_{em}^{\ast} J_{em}^{f},$$

$$+ \sqrt{2} G_{F} \rho / m_{t}^{2} / \left( s - m_{Z}^{2} + i \Gamma_{Z}/m_{Z} \right) \not{\!} J^{\ast} \not{\!} J,$$  \hspace{1cm} (9.1)

where $\rho = 1 + \delta \rho$, $\delta \rho$ being given by Eq. (8.6), and

$$J_{em}^{f} = \gamma_{\mu} J^{\ast} = \gamma_{\mu} \left[ i f_{\gamma} (1 - \gamma_{5}) - 2 Q_{f} s_{W}^{2} \right],$$  \hspace{1cm} (9.2)

with $Q_{f}$ and $f_{\gamma}$ being respectively the electric charge and the third component of the weak isospin (e.g. for $f = \mu^{-}, q = -1, l_{\mu} = -1/2$). The important features are the replacement of $\alpha$-fixed with $\alpha$-running, the inclusion of the $s$-dependence for the total width $\Gamma_{Z}$ in the resonant denominator (see Section 10), the presence of the factor of $\rho$ multiplying $G_{F}$, and the use in the $Z$ couplings of the effective $\sin^{2} \theta_{W} = s_{W}^{2}$, introduced in the previous section. Clearly for $f = e$, i.e. for Bhabha scattering, the
t-channel exchange is also to be included. The improved Born approximations include the real parts of self-energies (sometimes [75] called ‘oblique’ corrections). All large logs and all $G_F m_t^2$ terms are included. What are left out are ‘small’ corrections from imaginary parts of self-energies, vertices, and boxes.

For $f = b$ there are additional large terms from the vertex corrections [78] of the type shown in Fig. 20. The longitudinal $W$ modes are, also in this case, responsible for the presence of quadratic mass terms. We can simply modify the improved Born approximation in order to include these terms as well. The recipe [63],[79] is as follows: in Eqs. (9.1) and (9.2), replace $\rho$ by

$$\sqrt{\rho_b}, \quad \rho_b \equiv \rho \left(1 - \frac{4}{3} \delta\rho\right), \quad (9.3)$$

and $s_W^2$ by

$$s_W^2 \left(1 + \frac{2}{3} \delta\rho\right) \equiv s_W^2 k_s. \quad (9.4)$$

Note that whilst many sorts of heavy particles can contribute to $\delta\rho$, only the $t$-quark (or a new $t'$) contributes to $\delta\rho_b$, because the $W$ nearly always turns a $b$-quark into a $t$ (or a $t'$) quark.

As a final example, the improved Born approximation for the inclusive widths $\Gamma[Z \to f\bar{f}(\gamma,g)]$ (including photons and gluons in the final state), is given by

$$\Gamma[Z \to f\bar{f}(\gamma,g)] = N_c (G_F \rho m_Z^2 / 4\pi \sqrt{2}) [1 + (1 - 4|Q_F| s_W^2)^2], \quad (9.5)$$

where

$$N_c = \begin{cases} 1 & [1 + (3\alpha/4\pi) Q_f^2] \quad \text{(leptons)} \\ 3 & [1 + (3\alpha/4\pi) Q_f^2] [1 + \alpha_s(m_Z^2)/\pi + ...] \quad \text{(quarks)} \end{cases} \quad (9.6)$$

In the same approximation the total width is given simply by

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}(\gamma,g)] + \text{rare decays}, \quad (9.7)$$
where the rare decays [80] (e.g. \( Z \rightarrow H \mu^+ \mu^- \) for a relatively light Higgs) are numerically unimportant in the Standard Model. For \( \Gamma(Z \rightarrow bb) \), replace \( \rho \) by \( \rho_b \) and \( \hat{s}_W^b \) by \( \hat{s}_W^b k_b \), where \( \rho_b \) and \( k_b \) are given in Eqs. (9.3) and (9.4). Note that in the Standard Model for \( f \neq b \), Eq. (9.5) can also be written in the form [by using Eq. (8.18)]:

\[
\Gamma(Z \rightarrow f\bar{f}(\gamma, \rho)) = N_c[a(m_Z)m_Z/(48 \hat{z}_W^2 \hat{c}_W^2)](1+(1-4Q_F^2 \hat{s}_W^2)^2). (9.8)
\]

10. THE Z LINE-SHAPE

A considerable amount of work has deservedly been devoted to the theoretical study of the Z line-shape [81]. The present experimental accuracy on \( m_Z \) obtained at LEP is \( \delta m_Z = \pm 21 \text{ MeV} \) (recall Table 3, Section 6). The error on \( m_Z \) can eventually go down to \( \delta m_Z = \pm 5-10 \text{ MeV} \) if the transverse polarization of the beams measured by adequate polarimeters will be used to calibrate the energy by spin-resonance methods [82]. Similarly, a measurement of the total width to an accuracy \( \delta \Gamma = \pm 10 \text{ MeV} \) has by now been achieved. The prediction of the Z line-shape in the Standard Model to such an accuracy has posed a formidable challenge to theory, which has been successfully met. For the inclusive process \( e^+e^- \rightarrow f\bar{f}X \), with \( f \neq e \) (for simplicity, we leave Bhabha scattering aside) and \( X \) including \( \gamma \)'s and gluons, the physical cross-section can be written in the form of a convolution [81]:

\[
\sigma(s) = \int_{z_0}^1 dz \hat{\sigma}(zs)G(z, s), \quad (10.1)
\]

where \( \hat{\sigma} \) is the reduced cross-section, and \( G(z, s) \) is the radiator function that describes the effect of initial-state radiation; \( \hat{\sigma} \) includes the purely weak corrections, the effect of final-state radiation (of both \( \gamma \)'s and gluons), and also non-factorizable terms (initial- and final-state radiation interferences, boxes, etc.) which, being small, can be treated in lowest order and effectively absorbed in a modified \( \hat{\sigma} \). The radiator \( G(z, s) \) has an expansion of the form [81]

\[
G(z, s) = \delta(1 - z) + \alpha/\pi(a_{11}L + a_{10}) + (\alpha/\pi)^2(a_{22}L^2 + a_{11}L + a_{20}) + \ldots + (\alpha/\pi)^n \sum_{i=0}^n a_{ni}L^i, \quad (10.2)
\]

where \( L = \ln s/m_Z^2 \simeq 24.2 \) for \( \sqrt{s} \approx m_Z \). All first- and second-order terms are known exactly. The sequence of leading and next-to-leading logs can be exponentiated (closely following [83] the formalism of structure functions in
QCD). For $m_Z \approx 91$ GeV, the convolution displaces the peak by $+110$ MeV, and reduces it by a factor of about 0.74. The exponentiation is important in that it amounts to a shift of about 14 MeV in the peak position.

A model-independent analysis [84] of the reduced cross-section $\bar{\sigma}$ leads to the following general expression (here $m \equiv m_Z$):

$$\bar{\sigma}(s) = \left[12\pi\Gamma, \Gamma_f/(|D(s)|)^2\right][s/m^2 + R_f(s - m^2)/m^2 + \Gamma/m I_f + ...]
+ [4\pi\alpha_s(m^2)Q_f^2 N_c/3s] ,$$

(10.3)

with $N_c$ given by Eq. (9.6) and

$$D(s) = s - m^2 + im \Gamma[s/m^2 + \epsilon(s - m^2)/m^2] + ... .$$

(10.4)

This form of the resonant term was obtained by starting from a general renormalizable field theory (the Standard Model being a particular case). Near the resonance, $(s - m^2)/m^2 \approx \Gamma/m$ is of order $\alpha$ (or $\alpha_W$). As only a perturbative calculation of $\bar{\sigma}$ is possible in $\alpha$ or $\alpha_W$, at the same level of accuracy we can expand vertices, propagators, etc., in $(s - m^2)/m^2$ near the resonance. For example, for the inverse propagator, we can write the expansion

$$D(s) = s - m^2 + \Pi(s)
= s - m^2 + Re \Pi(m^2) + (s - m^2)Re \Pi'(m^2) + i Im \Pi(m^2) +
i(s - m^2)Im \Pi'(m^2) + ...
= [1 + Re \Pi'(m^2)](s - m^2 + im \Gamma[s/m^2 + \epsilon(s - m^2)/m^2 + ...]),$$

(10.5)

with

$$m\Gamma = Im \Pi(m^2)/[1 + Re \Pi'(m^2)],$$

(10.6)

and $Re \Pi(m^2) = 0$ because of the specific definition of $m \equiv m_Z$ that is adopted. (Recently a different definition of $m_Z$ was discussed [85]. The present prescription could lead to problems related to gauge invariance at the two-loop level. Within the one-loop framework it is perfectly all right and is generally adopted because of its calculational simplicity.) The overall factor $1 + Re \Pi'(m^2)$ is reabsorbed in the numerator. The parameter $\epsilon$ measures the deviation from the scaling behaviour of the $s$-dependent width; $\epsilon$ is noticeably different from zero only if, in the final state of $Z$ decays, there are important channels with massive particles. For example, for $Z \rightarrow AA$, $\epsilon_A \approx (4m_A^2/m^2)B(Z \rightarrow AA)$. The following identity is valid:

$$D(s) = s - m^2 + im \Gamma[s/m^2 + \epsilon(s - m^2)/m^2 + ...]
= [1 + i\gamma(1 + \epsilon)](s - m^2 + im\Gamma),$$

(10.7)
with $\gamma = \Gamma/m$ and:

$$m = m[1 - (\gamma^2/2)(1 + \epsilon) + \ldots], \quad (10.8)$$

$$\Gamma = \Gamma[1 - (\gamma^2/2)(1 + 3\epsilon) + \ldots]. \quad (10.9)$$

As the factor $1 + i\gamma(1 + \epsilon) \simeq \exp i\gamma(1 + \epsilon)$ cannot be observed from the absolute square of $D(s)$, we see that, on the one hand, at $\epsilon = 0$ the replacement of the constant $\Gamma$ by the $s$-dependent width $s\Gamma/m^2$ leads to a variation of the apparent mass by

$$\delta m = -\frac{1}{2} \gamma^2 m \simeq -34 \text{ MeV}, \quad (10.10)$$

which is clearly an important effect [86],[87]. On the other hand, the additional effect from $\epsilon$,

$$\delta m_\epsilon = -\frac{1}{2} \gamma^2 \epsilon m \simeq -34\epsilon \text{ MeV}, \quad (10.11)$$

is certainly small because $\epsilon$ cannot exceed a few percent or so at most. Note that the effect of $\epsilon$ cannot be disentangled from $m$ in the fit, so that it leads to an ambiguity in the determination of $m$. In conclusion, $\epsilon$ can be safely neglected at the price of allowing an error of a few MeV on $m$ and $\Gamma$.

Of the two parameters $R_f$ and $I_f$ which appear in Eq. (10.3) for $\sigma$ (in addition to the inclusive partial widths $\Gamma_e$ and $\Gamma_f$), $I_f$, like $\epsilon$, depends only on the spectrum of particles below the $Z$. In fact, $I_f$ is determined by absorptive parts of vertices and boxes. Thus large deviations from the Standard Model value cannot occur for $I_f$. In the Standard Model, $I_f$ is very small [for $\mu^+\mu^-, I_\mu \simeq -(1-2) \times 10^{-7}$], and as it is multiplied by $\Gamma/m$, it can be neglected. The main contribution to $R_f$ is already present at the Born level and arises from $\gamma-Z$ interference. Higher-order corrections are relatively important, especially for muons. New physics, for example a heavy $Z'$, can modify $R_f$ because the non-resonating background is changed. In the Standard Model, $R_f$ is small [for muons, $R_\mu \simeq (4.5-6) \times 10^{-2}$, for hadrons $R_h \simeq (0.75-1) \times 10^{-3}$]. Determining $R_f$ from a fit is difficult. In first approximation, it can be fixed at its Standard Model value. Realistic deviations from the Standard Model would not appreciably affect the determination of $m$ and $\Gamma$.

In conclusion, a model-independent analysis of the reduced cross-section proves that a modified Breit–Wigner with $s$-dependent width, plus photon exchange and interference is a perfectly adequate basis for the experimental study of the line shape. Whilst in the Minimal Standard Model all the parameters $\Gamma, \Gamma_e, \Gamma_f, R_f$ and $I_f$ can be computed from $m_Z$, given $m_\pi$ and $m_H$, the general expression of $\sigma$ will, in principle, allow a model-independent measurement of the $Z$ parameters. Starting from Eq. (10.3) for
approximate analytic solutions of the convolution integral can be found [84], [88]–[90]. The resulting compact analytic expressions are sufficiently precise for most applications.

11. PRECISION TESTS OF THE STANDARD ELECTROWEAK THEORY

It is clear that the set of input parameters specified in Section 6 can be separated into two parts. On the one hand, $\alpha_s, G_F, m_Z, m_{\text{had}}$, are well known and the ambiguities associated with these quantities on the radiative corrections are quite small. We can add $\alpha_s$ to this class, in that, if it is true that the experimental error on $\alpha_s$ is relatively large, it only enters as a small correction to electroweak processes involving hadrons and is practically irrelevant for purely leptonic processes. Also, by fitting the electroweak data one cannot obtain a better value for $\alpha_s(m_Z)$ than that derived from QCD tests. On the other hand $m_t$ and $m_H$ are largely unknown. Thus, for each relevant observable, one can only express the prediction of the Standard Model as a function of $m_t$ and $m_H$, obtained by using the best available calculations of radiative corrections, with $\alpha_s, G_F, m_Z$ and $m_{\text{had}}$ fixed at their experimental values with the corresponding errors. By comparing these predictions with experiment, one can check their mutual consistency and derive constraints on $m_t$ and $m_H$.

Actually the sensitivity on $m_H$ is so small that for all the measured quantities the ambiguity due to varying $m_H$ in the range $50 \text{ GeV} < m_H < 1 \text{ TeV}$ is below the present experimental error, so that for practical purposes, at the present stage of accuracy, the relevant predictions can be plotted as functions of $m_t$ in the form of a band of values determined by $\delta m_B, \delta m_\alpha (\delta \alpha_s)$ (see Figs. 21 to 27).

Note that from this point of view $\sin^2 \theta_W$ is not a primary quantity. It is not part of the set of input parameters. It is a derived quantity that one could even decide not to introduce at all. I stress this point in order to make it clear that all disputes over which is the better definition of $\sin^2 \theta_W$ beyond the tree level are completely secondary. First of all it is always true that physical results are independent of definitions. Differences in physical results obtained from a different definition of input parameters (scheme dependence) can at most occur by terms of higher order, due to the truncation of the perturbative series at a given order. But, for $\sin^2 \theta_W$, its precise definition is only necessary to compute it from the input parameters, but cannot matter for the prediction of observables because, with the choice specified above, $\sin^2 \theta_W$ is not taken as an input parameter of the theory.

The widespread use of expressing the experimental result for each given observable in terms of the corresponding value of $\sin^2 \theta_W$ (within a specified
Figure 21: Labelled by $m_Z$ is the theoretical prediction for $s_W^2$ obtained from $m_Z$ as a function of $m_t$ for $m_H = 50 - 1000$ GeV (the uncertainty due to $\delta m_Z = 21$ MeV leads to the double boundaries). The error bands implied by the CDF and UA2 measurements of $m_W/m_Z$ and by the data on $R_\nu$ are also shown.

has been studied in great detail [66]. The result for $s_W^2$, obtained starting from the average LEP value for $m_Z$ (see Table 3), as a function of $m_t$, is plotted in Fig. 21, where the uncertainties for $50$ GeV < $m_H$ < $1$ TeV and $\delta m_Z = \pm 21$ MeV are also visible. We see that $m_t$ is the main unknown in the calculation of $m_W/m_Z$ from $m_Z$, followed in importance by the ambiguity from varying the Higgs mass in the above range, while the remaining uncertainty from the experimental error on $m_Z$ is very small.

When the available direct experimental information on $m_W/m_Z$ is added, the sensitivity of $s_W^2$ to $m_t$ provides a very strong constraint on $m_t$. The ratio $m_W/m_Z$ is directly measured at hadron colliders but can also be obtained (assuming the validity of the Standard Model) from the ratio $R_\nu = \sigma^{NC}/\sigma^{CC}$ of neutral current (NC) to charged current (CC) cross-sections in neutrino–nucleus deep inelastic scattering. The value of $m_W/m_Z$ has been measured at hadron colliders [92]. From CDF and UA2 we have the results reported in Table 4.
Table 4:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$m_W/m_Z$</th>
<th>$s_W^2 = 1 - m_W^2/m_Z^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>0.8768±0.0046</td>
<td>0.231±0.008</td>
</tr>
<tr>
<td>UA2</td>
<td>0.8841±0.0042</td>
<td>0.2184 ± 0.0077</td>
</tr>
<tr>
<td>Average</td>
<td>0.8807±0.0031</td>
<td>0.2246±0.006</td>
</tr>
</tbody>
</table>

By combining $m_W/m_Z$ with the LEP value for $m_Z$ one obtains $m_W = 80.30±0.28$ GeV. The corresponding average value of $s_W^2$ is also shown in Fig. 21 as a horizontal band, obviously independent of $m_t$, in the $s_W^2-m_t$ plane.

As is well known, the value of $s_W^2$ extracted from $R_\nu$ is also nearly independent of $m_t$ in the interesting range of values for the top mass. This fact arises from a largely accidental cancellation [93], specific to this process and to the Standard Model, between two different sources of $m_t$ dependence, as discussed in the following.

The ratio $R_\nu = \sigma_{NC}/\sigma_{CC}$ for $\nu-N$ scattering is given in terms of $s_W^2$ by [see Eqs. (5.4) and (5.5)]:

$$R_\nu = \rho_{\nu N}^2 \left( \frac{1}{2} - k_{\nu N} s_W^2 + \frac{5}{9} (k_{\nu N} s_W^2)^2 (1 + r) \right) + \ldots ,$$  \hspace{1cm} (11.2)

where $r = (\sigma^0/\sigma^*)_{CC} \approx 0.4$ is also measured. The tree approximation (with $\rho_0 = 1$) is recovered for $\rho_{\nu N} = k_{\nu N} = 1$. Some large logarithms from definition for it) is no longer adequate at the present level of sophistication. For comparing the constraining power of different experiments it would be more appropriate to quote the range of $m_t$ implied by each of them [91] (see Table 6). In fact, $\sin^2 \theta_W$ is just one particular observable of the theory. Outside the domain of precision tests, with appropriate definitions of $\sin^2 \theta_W$, as was seen in Section 9, one can write simple improved Born approximations that include the main contributions of radiative corrections (e.g., large logarithms and terms of order $G_F m_t^2$). While, for precision tests, the use of as complete as possible radiative corrections is mandatory, these approximate formulae are very useful for our understanding of the pattern of radiative corrections and for everyday-life estimates of rates and experimental sensitivities.

We go back to the definition of $s_W^2$ given in Eq. (8.3). Clearly in this case the observables $s_W^2$ and $m_W$ are directly equivalent given that $m_Z$ is among the input parameters. In the Standard Model, $s_W^2$ can be computed from the input parameters by the relation [see Eq. (8.4)]:
\[
\frac{s_{\nu}^2 c_{\nu}^2}{m^2_Z} = \left(1 - \frac{m_{}\nu^2}{m^2_Z}\right) \frac{m_{\nu}^2}{m^2_Z} = \frac{\pi \alpha}{\sqrt{2} G_F} \frac{1}{m^2_Z} \frac{1}{1 - \Delta r}
\]  

(11.1)

where \(c_{\nu}^2 = 1 - s_{\nu}^2\) and \(\Delta r \equiv \Delta r(\alpha_s, G_F, m_\nu, m_f, m_H)\) is the effect of radiative corrections. The quantity \(\Delta r\) as a function of the input parameters the radiative corrections to \(\sigma^{CC}\) are also included in \(\rho_{\nu N}\). But for the sake of this argument we are only considering the \(G_F m^2_\nu\) terms. For fixed \(R_\nu = (\text{the experimental value})\) and \(s_{\nu}^2 \sim 0.23\) there is a strong cancellation in the Standard Model between the \(m_\nu\) dependence of \(\rho_{\nu N} \approx 1 + 6 \rho\) and of \(k_{\nu N} \approx 1 + c_{\nu}^2/s_{\nu}^2 \delta \rho\) [Eqs. (8.7),(8.8)], so that as a result \(\delta s_{\nu}^2 \approx 0.26 \rho\), where \(\delta \rho\) is given in Eq. (8.6). For realistic values of \(m_\nu\) the resulting contribution of the quadratic \(m_\nu\) terms is no longer dominant.

The most precise experimental results on \(R_\nu\) were obtained by the CHARM [94] and CDHS [95] collaborations at CERN. The original results on \(\sin^2 \theta_\nu\) were given for fixed \(m_\nu\) and \(m_H\). CHARM obtained \(s_{\nu}^2 = 0.236 \pm 0.005 \text{ (exp)} \pm 0.005 \text{ (th)}\) for \(m_\nu = 45\) GeV and \(m_H = 100\) GeV, while the CDHS result was \(s_{\nu}^2 = 0.2275 \pm 0.005 \text{ (exp)} \pm 0.005 \text{ (th)}\) for \(m_\nu = 60\) GeV and \(m_H = 100\) GeV. The theoretical error arises from hadronic uncertainties and the effect of the charm threshold. An average at \(m_\nu = 60\) GeV and \(m_H = 100\) GeV gives \(s_{\nu}^2 = 0.232 \pm 0.006\) (where the error \(6 \times 10^{-3}\) is obtained as \(6 \times 10^{-3} = \sqrt{(5/\sqrt{2})^2 + 5^2} \times 10^{-3}\)). The corresponding combined result at different values of \(m_\nu\) and \(m_H\) can also be obtained from the known form of the radiative corrections. The result is shown [96] in Fig. 21.

There are many more, less precise experimental results on \(s_{\nu}^2\) from low-energy neutral current data, most of them being well known since a long time [97],[98]. These additional data are all consistent among them and with the results in Fig. 21.

We now consider the implications for the standard electroweak theory of the LEP results on the \(Z\) partial widths and asymmetries. In Figs. 22 to 24, we compare the data on the \(Z\) widths (collected in Table 5) with the predictions of the Standard Model, obtained with the programme ZSHAPE [99] which includes a state-of-the-art set of electroweak radiative corrections.
Figure 22: $\Gamma_Z$ vs. $m_t$ as predicted by the Standard Model for $m_H = 50 - 1000$ GeV and $\alpha_s(m_Z) = 0.11 - 0.13$ compared with the LEP result.

Figure 23: $\Gamma_h$ vs. $m_t$ as predicted by the Standard Model for $m_H = 50 - 1000$ GeV and $\alpha_s(m_Z) = 0.11 - 0.13$ compared with the LEP result.
Figure 24: \( \Gamma_\ell \) Vs. \( m_\ell \) as predicted by the Standard Model for \( m_H = 50 - 1000 \text{ GeV} \) compared with the LEP result.

Additional important information is provided by the measurement of a number of asymmetries. In particular we refer to the forward–backward asymmetries for charged leptons (\( A_{FB}^l \)) and for the \( b \)-quark (\( A_{FB}^b \)) and the \( \tau \) polarisation asymmetry (\( A_{pol}^\tau \)). The value of those asymmetries, combined over the LEP experiments, are given in the following [60]. For \( A_{FB}^b \) one has:

\[
A_{FB}^b(\sqrt{s} = m_Z) = 0.0163 \pm 0.0036
\]

(11.3)

obtained from the average value \( y = \bar{a}^2/\bar{a}^2 = 0.0048 \pm 0.0012 \) quoted by V. Nikolaenko [105] and J. Carter [60], by using the relation:

\[
A_{FB}^b = \frac{3y}{(1+y)^2} + 0.002.
\]

(11.4)

which corresponds to the definitions of ZFITTER [102]. For \( A_{FB}^b \) the result, after correction for the \( B \rightarrow \bar{B} \) mixing effect, is given by:

\[
A_{FB}^b(\sqrt{s} = m_Z) = 0.126 \pm 0.022.
\]

(11.5)

Actually the measured value includes the effect from QCD corrections. We prefer to consider the value of \( A_{FB}^b \) with the QCD correction [106] removed:

\[
(A_{FB}^b)_{\text{measured}} \approx (A_{FB}^b)_{E-W} \left(1 - \frac{0.79 \alpha_s(m_Z)}{\pi}\right)
\]

\[
\approx 0.97 (A_{FB}^b)_{E-W}.
\]

(11.6)
Table 5: Results from LEP. The average also includes systematic errors as given by J. Carter [60] at the Geneva Conference. The average value of $\Gamma_{\nu\nu}$ corresponds to $N_\nu = 2.99 \pm 0.05$, which is the best determination of the number of light neutrinos from LEP. Totally equivalent predictions are obtained by other complete calculations of the line shape [100]–[106]. The predicted widths are plotted as a function of $m_t$ and compared with the data. The ambiguities corresponding to $m_H = 50$–1000 GeV and $\alpha_s(m_Z) = 0.11$–0.13 are also indicated. Note that the leptonic width (obtained by assuming $e-\mu-\tau$ universality) gives a particularly strong constraint on $m_t$, because of its insensitivity to $\alpha_s(m_Z)$, the rather low central value and the small experimental error.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ALEPH</th>
<th>DELPHI</th>
<th>L3</th>
<th>OPAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_Z$ (MeV)</td>
<td>$2484 \pm 17$</td>
<td>$2465 \pm 20$</td>
<td>$2501 \pm 17$</td>
<td>$2492 \pm 16$</td>
</tr>
<tr>
<td>$\Gamma_h$ (MeV)</td>
<td>$1744 \pm 15$</td>
<td>$1726 \pm 19$</td>
<td>$1747 \pm 16$</td>
<td>$1739 \pm 17$</td>
</tr>
<tr>
<td>$\Gamma_t$ (MeV)</td>
<td>$83.1 \pm 0.7$</td>
<td>$83.4 \pm 0.8$</td>
<td>$83.5 \pm 0.7$</td>
<td>$83.0 \pm 0.7$</td>
</tr>
<tr>
<td>$R = \Gamma_h/\Gamma_t$</td>
<td>$21.00 \pm 0.20$</td>
<td>$20.70 \pm 0.29$</td>
<td>$20.93 \pm 0.22$</td>
<td>$20.95 \pm 0.22$</td>
</tr>
<tr>
<td>$\Gamma_{inv}$ (MeV)</td>
<td>$491 \pm 13$</td>
<td>$488 \pm 17$</td>
<td>$501 \pm 14$</td>
<td>$504 \pm 15$</td>
</tr>
<tr>
<td>$\Gamma_e$ (MeV)</td>
<td>$83.8 \pm 0.9$</td>
<td>$82.4 \pm 1.2$</td>
<td>$82.5 \pm 0.9$</td>
<td>$82.9 \pm 1.0$</td>
</tr>
<tr>
<td>$\Gamma_\mu$ (MeV)</td>
<td>$81.4 \pm 1.4$</td>
<td>$86.9 \pm 2.1$</td>
<td>$86.0 \pm 1.6$</td>
<td>$83.2 \pm 1.5$</td>
</tr>
<tr>
<td>$\Gamma_\tau$ (MeV)</td>
<td>$82.4 \pm 1.8$</td>
<td>$82.7 \pm 2.4$</td>
<td>$85.8 \pm 1.9$</td>
<td>$82.7 \pm 1.9$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average</th>
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</thead>
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<tr>
<td>$\Gamma_Z$ (MeV)</td>
<td>$2487 \pm 10$</td>
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<td>$\Gamma_h$ (MeV)</td>
<td>$1740 \pm 9$</td>
</tr>
<tr>
<td>$\Gamma_t$ (MeV)</td>
<td>$83.2 \pm 0.4$</td>
</tr>
<tr>
<td>$R = \Gamma_h/\Gamma_t$</td>
<td>$20.92 \pm 0.11$</td>
</tr>
<tr>
<td>$\Gamma_{inv}$ (MeV)</td>
<td>$496.2 \pm 8.8$</td>
</tr>
<tr>
<td>$\Gamma_e$ (MeV)</td>
<td>$83.0 \pm 0.5$</td>
</tr>
<tr>
<td>$\Gamma_\mu$ (MeV)</td>
<td>$83.8 \pm 0.8$</td>
</tr>
<tr>
<td>$\Gamma_\tau$ (MeV)</td>
<td>$83.3 \pm 1.0$</td>
</tr>
</tbody>
</table>

Dropping the E-W subscript, in the following we shall use:

$$A_{FB}(\sqrt{s} = m_Z) = 0.130 \pm 0.022$$  \hspace{1cm} (11.7)

and the corresponding theoretical prediction. Finally, for $A_{pol}$ one has:

$$A_{pol}(\sqrt{s} = m_Z) = 0.134 \pm 0.035 .$$  \hspace{1cm} (11.8)

The experimental results on the asymmetries are compared with the Standard Model predictions in Figs. 25–27.
When all the data are combined I find the following range for the top-quark mass:

\[ m_t = 140 \pm 35 \text{ GeV} \]  

(11.9)

Figure 25: \( A_{FB} \times 10^3 \) vs. \( m_t \) as predicted by the Standard Model for \( m_H = 50 - 1000 \text{ GeV} \) compared with the LEP result.

Figure 26: \( A_{FB} \) vs. \( m_t \) as predicted by the Standard Model for \( m_H = 50 - 1000 \text{ GeV} \) compared with the LEP result.
Figure 27: $A_{FB}^T$ vs. $m_t$ as predicted by the Standard Model for $m_H = 50$-1000 GeV compared with the LEP result.

The central value of $m_t$ is smaller, $\simeq 120$ GeV, for light Higgs ($m_H = 50$-100 GeV), while, it is larger ($\simeq 160$ GeV) for heavy Higgs ($m_H = 0.5$-1 TeV). The above result on $m_t$ is in agreement with other recent analyses of the data [107]. The exact range of $m_t$ depends on all sorts of details, e.g. the set of data which are included, the weight given to the CDF bound on $m_t$, the range assumed for $\alpha_s(m_Z)$ (here $\alpha_s(m_Z) = 0.12 \pm 0.01$), and so on. The ranges of $m_t$ implied by each experiment are listed in Table 6. We repeat that $m_t$ is a more adequate figure of merit for experiments than $\sin^2 \theta_W$ (defined in one way or another) because the constraints on $m_t$ are the obvious goal of precision tests of the electroweak theory at present.

Table 6: Values of $m_t$ (in GeV) implied by different measurements [$\alpha_s(m_Z) = 0.12 \pm 0.01$]

<table>
<thead>
<tr>
<th>$m_H$</th>
<th>50–100 GeV</th>
<th>$m_H = 0.5$–1 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_W/m_Z$</td>
<td>150 ± 50</td>
<td>180 ± 50</td>
</tr>
<tr>
<td>$R_u$</td>
<td>100 ± 60</td>
<td>140 ± 50</td>
</tr>
<tr>
<td>$\Gamma_Z$</td>
<td>90 ± 60</td>
<td>150 ± 50</td>
</tr>
<tr>
<td>$\Gamma_h$</td>
<td>100 ± 75</td>
<td>170 ± 50</td>
</tr>
<tr>
<td>$\Gamma_t$</td>
<td>80 ± 50</td>
<td>90 ± 60</td>
</tr>
<tr>
<td>$A_{FB}$</td>
<td>140 ± 60</td>
<td>180 ± 60</td>
</tr>
<tr>
<td>$A_{FB}^T$</td>
<td>130 ± 120</td>
<td>170 ± 120</td>
</tr>
<tr>
<td>Average</td>
<td>295 ± 100</td>
<td>330 ± 100</td>
</tr>
</tbody>
</table>

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12. TOWARD A MODEL-INDEPENDENT ANALYSIS OF THE DATA

In the present section we propose a different way [108] of analysing the data, which does not necessarily assume the validity of the Standard Model from the start and takes into account recent theoretical studies on the parametrization of possible effects of new physics on precision experiments [109][110] to [114]. Given the set of input parameters as specified above, we start from the basic observables $m_W/m_Z$, $\Gamma_\ell$ and $A_{FB}$. We assume charged lepton universality, which is supported by the data at the present level of accuracy, so that $\Gamma_\ell$ and $A_{FB}$ refer to the corresponding average data. From these three quantities we can isolate the corresponding dynamically significant corrections $\Delta r_W$, $\Delta \rho$ and $\Delta k'$, which contain the small effects one is trying to disentangle, and are defined in the following. First we introduce $\Delta r_W$ as obtained from $m_W/m_Z$ by the relation (see Eq. (11.1)):

$$
\left(1 - \frac{m_W^2}{m_Z^2}\right) \frac{m_W}{m_Z} = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F}\frac{\Gamma_\ell}{m_Z^2(1 - \Delta r_W)}.
$$

(12.1)

Here $\alpha(m_Z) = \alpha/(1 - \delta \alpha)$ is fixed to the conventional value 1/128.8 so that the effect of the running of $\alpha$ due to known physics is extracted from $(1 - \Delta r) = (1 - \Delta \alpha)/(1 - \Delta r_W)$. We take $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$. A possible departure from these values would then be included in $\Delta r_W$. In order to define $\Delta \rho$ and $\Delta k'$ we consider effective vector and axial-vector couplings $g_V$ and $g_A$ for the on-shell $Z$ to charged leptons, defined by the formulae:

$$
\Gamma_\ell = \frac{G_F m_Z^2 (g_V^2 + g_A^2)}{6\pi\sqrt{2}}
$$

(12.2)

$$
A_{FB}(\sqrt{s} = m_Z) = \frac{\frac{3}{2} g_V^2}{(g_V^2 + g_A^2)^2}.
$$

(12.3)

Note that $\Gamma_\ell$ stands for the inclusive partial width $\Gamma(Z \to \ell \ell + \text{photons})$. We could extract from $(g_V^2 + g_A^2)$ the factor $(1 + 3\alpha/4\pi + \ldots)$ which is induced in $\Gamma_\ell$ from final state radiation, but we prefer the simpler definition of Eq. (12.2). The asymmetry in Eq. (12.3) is obtained from the data after deconvolution of initial-state radiation. Contributions from box diagrams and imaginary parts of vertex functions and propagators are also absorbed in the definition of $g_A$ and $g_V$ (of course, this can only be possible for a single channel, which we chose to be the charged-lepton channel). In terms of $g_V$ and $g_A$, $\Delta \rho$ and $\Delta k'$ are given by [111]:

$$
\frac{g_A}{2} = -\frac{1}{2}\left(1 + \frac{\Delta \rho}{2}\right)
$$

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\[ \frac{g_V}{g_A} = 1 - 4s_W^2 = 1 - 4(1 + \Delta k')s_W^2. \]  

\[ (12.4) \]

In Eqs. (12.4) \( s_W^0 \) is a particular effective \( \sin^2 \theta_W \) for on-shell \( Z \) [68], [73], [77], [115] while \( s_W^0 \) is the corresponding quantity before non-pure-QED corrections, given by:

\[ s_W^2 = \frac{\pi a(m_Z)}{\sqrt{2} G_F m_Z^2} \]

\[ (12.5) \]

with \( c_W^2 = 1 - s_W^2 \) (\( s_W^2 = 0.23145 \) for \( m_Z = 91.175 \) GeV).

In the Standard Model, for sufficiently large \( m_t \), \( \Delta \gamma \), \( \Delta \rho \) and \( \Delta k' \) are all dominated by quadratic terms in \( m_t \) of order \( G_F m_t^2 \) [63] and in this limit one has \( \Delta \gamma \sim -c_W^2/s_W^2 \Delta \rho \sim (c_W^2 - s_W^2)/s_W^2 \Delta k' \) [68]. As new physics can more easily be disentangled if not masked by large conventional \( m_t \) effects, it is convenient to keep \( \Delta \rho \) while trading \( \Delta \gamma \) and \( \Delta k' \) for two quantities with no contributions of order \( G_F m_t^2 \). We thus introduce the following linear combinations [111]:

\[ \epsilon_1 = \Delta \rho \]

\[ \epsilon_2 = c_W^2 \Delta \rho + \frac{s_W^2 \Delta \gamma}{(c_W^2 - s_W^2)} - 2s_W^2 \Delta k' \]

\[ \epsilon_3 = c_W^2 \Delta \rho + (c_W^2 - s_W^2) \Delta k' \]

\[ (12.6) \]

Clearly \( \epsilon_2 \) and \( \epsilon_3 \) no longer contain terms of order \( G_F m_t^2 \) but only logarithmic terms in \( m_t \). The leading terms for large Higgs mass, which are logarithmic, are mainly contained in \( \epsilon_1 \) but are also present in \( \epsilon_3 \). In the Standard Model one has the following 'large' asymptotic contributions [68],[64],[116]:

\[ \epsilon_1 = \frac{3G_F m_t^2}{8\pi} - \frac{3G_F m_W^2}{4\pi} \tan^2 \theta_W \ln \left( \frac{m_H}{m_Z} \right) + \ldots \]

\[ \epsilon_2 = -\frac{G_F m_t^2}{2\pi^2} \ln \left( \frac{m_t}{m_Z} \right) + \ldots \]

\[ \epsilon_3 = \frac{G_F m_W^2}{12\pi^2} \ln \left( \frac{m_H}{m_Z} \right) - \frac{G_F m_t^2}{6\pi^2} \ln \left( \frac{m_t}{m_Z} \right) + \ldots \]

\[ (12.7) \]

In passing, we note the following interesting alternative expression for the leptonic width in terms of \( \bar{s}_W^0 \) defined in Eqs. (12.2)-(12.5) and \( \epsilon_3 \) (valid in linear approximation in \( \epsilon_1 \) and \( \epsilon_3 \)):

\[ \Gamma_L = \frac{\alpha(m_Z) m_Z}{48\bar{s}_W^0 c_W^2} \left( 1 + \frac{\epsilon_3}{c_W} \right) \left[ 1 + (1 - 4s_W^2)^2 \right] \]

\[ (12.8) \]

The definitions in Eqs. (12.1-12.6) are quite general and do not commit us to any particular model. The epsilon are useful in that they provide a very
efficient parametrization of the most important input data with respect to the sensitivity to new physics so that they represent a convenient starting point for a model-independent analysis of the data. One can then formulate a hierarchy of simple and rather general assumptions valid in large classes of models which are needed in order to relate the epsilons to a progressively larger set of observables.

Starting from the hadron collider result on $m_W/m_Z$ given in Table 4, by combining it with the LEP value for the $Z$ mass, $m_Z = 91.175 \pm 0.021$ GeV, and using Eq. (12.1), one obtains:

\[ \Delta r_W = (-2.2 \pm 1.7) \times 10^{-2} \]  \hspace{1cm} (12.9)

similarly from the LEP results on the charged-lepton partial width and the forward-backward asymmetry given in Table 6 and Eq. (11.3), respectively, one finds:

\[ g_A^2 = 0.2495 \pm 0.0012 \]  \hspace{1cm} (12.10)

\[ g_V/g_A = 0.074 \pm 0.008 \text{ or } \\ \hat{g}_W^2 = 0.2315 \pm 0.0020 \]  \hspace{1cm} (12.11)

One can now use Eq. (12.4) to derive the results for $\Delta \rho$ and $\Delta k'$:

\[ \epsilon_1 = \Delta \rho = (-0.19 \pm 0.49) \times 10^{-2} \]  \hspace{1cm} (12.12)

\[ \Delta k' = (0.02 \pm 0.87) \times 10^{-2}. \]  \hspace{1cm} (12.13)

Finally the corresponding results for $\epsilon_2$ and $\epsilon_3$ are obtained from Eq. (12.6):

\[ \epsilon_2 = (-1.10 \pm 0.88) \times 10^{-2} \]  \hspace{1cm} (12.14)

\[ \epsilon_3 = (-0.14 \pm 0.67) \times 10^{-2}. \]  \hspace{1cm} (12.15)

We can make contact with the notation of Refs. [86], [110] by setting $\Delta \rho = \alpha T, \epsilon_2 = -\alpha U/(4\hat{g}_W^2)$ and $\epsilon_3 = \alpha S/(4\hat{g}_W^2)$. We then obtain:

\[ S = -0.18 \pm 0.85, \quad T = -0.26 \pm 0.67, \quad U = 1.4 \pm 1.1. \]  \hspace{1cm} (12.16)

Note, however, that in Refs. [109] and [110] $S$ and $T$ are defined in the specific context of dominance of vacuum polarization corrections from new physics, while the present definitions are general. Also, in Refs. [109],[110] $S$ and $T$ are defined as deviations from the theoretical predictions of the Standard Model for given values of $m_t$ and $m_H$, while the values in Eq. (12.16) are unsubtracted.

The predictions of the Standard Model for $\epsilon_1, \epsilon_2$ and $\epsilon_3$ have been studied in detail in Ref. [108]. We refer the reader to this paper for a description of the calculations, a comparison of results from several available programmes.
Figure 28: Theoretical predictions for $\epsilon_1, \epsilon_2$ and $\epsilon_3$ in the Standard Model as functions of $m_t$ for $m_H = 50 - 1000$ GeV.

of radiative corrections and a discussion of the theoretical errors on the epsilons. Here we only quote the main results. In Fig. 28 the predicted values of the epsilons are shown together on the same scale. Note that indeed $\epsilon_2$ and $\epsilon_3$ are much flatter than $\epsilon_1$ in $m_t$, (this is especially true for $\epsilon_3$, while some dependence is still visible in $\epsilon_2$, due mainly to logarithmic terms in $m_t$ which are three times larger for $\epsilon_2$ than for $\epsilon_3$, as is seen from Eq. (12.7)). In Figs. 29 to 31 the results are displayed and compared with the present experimental values. Only the comparison of $\Delta \rho$ from theory and experiment implies a strong constraint on $m_t$, while for $\epsilon_2$ and $\epsilon_3$ one observes consistency with the Standard Model with no important constraints on $m_t$ and $m_H$. However, $\epsilon_2$ and $\epsilon_3$ impose interesting bounds on new physics, as we shall see.

In the Standard Model, the knowledge of $\epsilon_1$ and $\epsilon_3$ allows one to determine all other observables measured at the $Z$ pole and related to charged leptons, such as $A^\mu_p$ and $A^{LR}$. As mentioned in the introduction, this is also true in a very large class of models where new physics only contributes at $q^2 = m_Z^2$ through either vacuum polarization amplitudes and/or vertex corrections of the form $\Delta V_p(z \rightarrow \ell + \ell) = \bar{u}(\Delta g_{AT} + \Delta g_V)\gamma_\mu u$. It is therefore worthwhile to establish this connection in terms as general as possible.
Figure 29: $\epsilon_1 \times 10^2$ vs. $m_t$, as predicted by the Standard Model for $m_H = 50$-$1000$ GeV compared with the LEP result.

Figure 30: $\epsilon_2 \times 10^2$ vs. $m_t$ as predicted by the Standard Model for $m_H = 50$-$1000$ GeV compared with the LEP result.
Figure 31: $c_3 \times 10^2$ vs $m_t$ as predicted by the Standard Model for $m_H = 50$-1000 GeV compared with the LEP result.

Together with Eq. (12.2), but in place of Eq. (12.3), one could have used $A'_{pol}$ to define the effective on-shell vector and axial vector $Z$ couplings to charged leptons, via the relation:

$$A_{pol}' = \frac{2g'_{A}g_{V}}{g'_{A}^{2} + g'_{V}^{2}}$$  \hspace{1cm} (12.17)

We have adopted different symbols for the effective couplings because $g_{V}$ and $g_{A}$, defined from $A_{FB}$, certainly cannot be confused with $g'_{V}$ and $g'_{A}$ obtained from $A'_{pol}$, to the level of accuracy at which the LEP experiments are aiming. On the contrary $A'_{pol}$ and $A_{LR}$ can be considered as equivalent in this respect. In fact we have checked that the relative differences between the two asymmetries are smaller than $10^{-4}$ in the Standard Model.

The set of $c_i'$, related to $g'_{A}$ and $g'_{V}$ in the same way as the set of $c_i$ to $g_{A}$ and $g_{V}$ as in Eq. (12.4), are also sizeably different. One can easily understand the corresponding pattern of corrections. The same effective $\sin^2 \theta_W$ would describe $A_{FB}$ and $A'_{pol}$ when only quadratic and logarithmic terms in $m_t$ and $m_H$ are included, because such terms only arise from vacuum polarization diagrams, which affect the asymmetries in the same way. Thus the effective values of $\sin^2 \theta_W$, defined from one or the other asymmetry, can only differ by a constant term in $m_t$ and $m_H$. In fact numerically we find:

$$\langle \delta \theta_W \rangle_{pol} = (\delta \theta_W)_{FB} + \delta : \delta = (1.3 \pm 0.2) \times 10^{-3}$$  \hspace{1cm} (12.18)
The main contribution to $\delta$ is from the constant $+0.002$ that appears in Eq. (11.4). This in turn arises from the imaginary part of the photon vacuum polarization diagrams [117]. Clearly, by the same argument that leads to the independence of $\delta$ from $m_t$ and $m_H$, one obtains that the same shifts also apply to all cases of new physics in oblique or universal vertex corrections. Starting from the experimental value of $A_{\mu}^\prime$ obtained by LEP [Eq. (11.8)] (see also Fig. 27), which corresponds to (from Eq. (12.17) using $g_\gamma'/g_A' = [1 - 4(\delta_{\gamma}' p_{\mu})_{pol}]$:

$$(\delta_{\gamma}' p_{\mu})_{pol} = 0.2332 \pm 0.0045,$$  \hfill (12.19)

we can apply the correction in Eq. (12.18) and obtain $(\delta_{\gamma}' p_{B}) = 0.2319 \pm 0.0045$ and hence a new value for $\Delta k'$:

$$\Delta k' = (0.2 \pm 1.9) \times 10^{-3}$$  \hfill (12.20)

We now proceed to consider quantities that also involve quarks and are still measured at the $Z$ pole. In the Standard Model, to obtain a prediction for these quantities a range of values for $\alpha_s(m_Z)$ has to be assumed. Also, the large quadratic terms in $m_t$ from the $Z \to b\bar{b}$ vertex are to be taken into account. Beyond the Standard Model, it is obvious that a stronger form of universality must be assumed in order to directly transfer into the quark sector the information embodied by the $\xi_i$'s measured in the charged-lepton sector. Quark-lepton universality is automatic in the case of oblique corrections, while it is in general violated if vertex corrections are important.

Particularly important quantities, (nearly) independent of $\alpha_s(m_Z)$, are $\Gamma_3/\Gamma_h$ and $A_{FB}^\prime$. Similar quantities from charm or light quarks are not equally interesting both in terms of experimental precision and because of the large $m_t$ term [78] in the $Z \to b\bar{b}$ vertex corrections. For example, the data on $\Gamma_3/\Gamma_h$ in principle allow us to set a bound on $m_t$ which is independent of assumptions on the absence of exotic contributions to $\xi_i$. However, such a limit is not very interesting at the moment because a comparatively large error is introduced by ambiguities associated with the semileptonic branching ratio or by other uncertainties in the case of DELPHI that uses a purely hadronic $b$-selection criterion. On the contrary, $A_{FB}^\prime$ is almost unaffected by the presence of large $m_t$-dependent vertex corrections. Schematically the reason is that $A_{FB}^\prime = 3\eta_\tau \eta_\tau$ with $\eta = g_\gamma/g_A/(g_A^2 + g_\gamma^2)$, so that $\delta A_{FB}^\prime = 3(\eta_\tau \delta \eta_\tau + \eta_\tau \delta \eta_\tau)$. We see that the sensitivity on $\eta_\tau$, which contains the $Z \to b\bar{b}$ vertex correction, is strongly suppressed by the small factor $\eta_\tau$. This is confirmed by an accurate numerical calculation of $A_{FB}^\prime$. We define
a new quantity \((\bar{s}_W^2)_b\) by the identity:

\[
A^B_{FB} = 3 \frac{1 - 4(\bar{s}_W^2)_b}{1 + \left(1 - 4(\bar{s}_W^2)_s\right)} \frac{\beta \left(1 - \frac{3}{4}(\bar{s}_W^2)_s\right)}{\beta^2 + \frac{3-\alpha_s^2}{\beta^2} \left(1 - \frac{3}{4}(\bar{s}_W^2)_s\right)^2}.
\] (12.21)

where \(\beta = \sqrt{1 - 4m_t^2/m_Z^2}\). We can explicitly evaluate the relation, as a function of \(m_t\) and \(m_H\), between \((\bar{s}_W^2)_s\) and the similar quantity \((\bar{s}_W^2)_{FB}\) previously defined from the charged lepton asymmetry \(A^L_{FB}\). We obtain:

\[
(\bar{s}_W^2)_b = (\bar{s}_W^2)_{FB} = \delta_b, \quad \delta_b = (0.9 \pm 0.15) \times 10^{-3}
\] (12.22)

(valid for \(m_t = 90-300\) GeV, \(m_H = 50-1000\) GeV). As a consequence, if quark-lepton universality is assumed, one can use the present combined LEP result on \(A^B_{FB}\) given in Eq. (11.7) which is equivalent, by Eq. (12.21), to:

\[
(\bar{s}_W^2)_b = 0.2259 \pm 0.0040
\] (12.23)

and obtain an independent input on \(\Delta k'\). In fact we derive, from Eqs. (12.22) and (12.23), the value \((\bar{s}_W^2)_{FB} = 0.2259 \pm 0.0040\), and then, from Eq. (12.4), the result:

\[
\Delta k' = (-2.4 \pm 1.7) \times 10^{-2}
\] (12.24)

While flavour universality of new physics is the crucial assumption that is needed to relate different measured quantities at the \(Z\) pole, some hypotheses on the absence of new sources of substantial \(q^2\) dependence have to be formulated in order to add low energy measurements to the picture. Actually, in most of the relevant cases both flavour universality and \(q^2\) in-dependence have to be combined in order to make contact with important experiments, such as \(\nu - \bar{\nu}\) and \(N\) deep inelastic scattering and atomic parity violation. In models where oblique corrections, which directly possess flavour universality, are dominant, sizeable \(q^2\)-dependent effects from new physics are absent if terms involving second and higher derivatives with respect to \(q^2\) can be neglected in vacuum polarization form factors. This is a good approximation in models with no decoupling, where first-order derivatives lead to effects of order 1 in the limit when the scale \(\Lambda\) of new physics becomes very large. On the other hand, in models with decoupling, such as those considered in Ref. [114], first-order derivatives are of order \(v^2/\Lambda^2\), where \(v\) is a parameter of the order of the electroweak scale (typically the Higgs vacuum expectation value). In this case the effect of second-order derivatives, of order \(m_Z^2/\Lambda^2\), is not relatively negligible. Note, however, that in these models both effects are quite small unless the scale \(\Lambda\) of the new physics is very close to the domain of energies of present experiments.
If one assumes flavour and lepton-quark universality and no additional \( q^2 \)-dependence, the available data on neutrino–nucleus deep inelastic scattering and on parity violation in Cs atoms lead to further constraints on \( \epsilon_1 \) and \( \epsilon_3 \), while they have no direct effect on \( \epsilon_2 \). The present data [94], [95] on \( R_\nu \) and \( R_\rho \), the ratios of neutral- to charged-current processes in deep inelastic neutrino scattering on nuclei, imply the following constraints [98], [112]:

\[
R_\nu : \epsilon_1 - 0.34\epsilon_3 = (-0.07 \pm 0.45) \times 10^{-2}
\]
\[
R_\rho : \epsilon_1 - 0.02\epsilon_3 = (-1.34 \pm 0.95) \times 10^{-2}
\]  \( (12.25) \)

with our definition of epsilons for comparison, we also report the result from the ratio of neutrino to antineutrino scattering on electrons [118], which gives [98], [112] \( \epsilon_3 - 0.74\epsilon_1 = (0.13 \pm 2.12) \times 10^{-2} \).

Similarly, the results on parity violation in Cs [119], [120] lead to a value of \( \epsilon_3 \), while the sensitivity to \( \epsilon_1 \) is accidentally almost exactly cancelled because of the particular ratio of protons to neutrons in Cs [112]. Neglecting the \( \epsilon_1 \) contribution, one finds in this case the general result:

\[
Q_W = -72.84 \pm 0.13 - 102\epsilon_3.
\]  \( (12.26) \)

The present experimental value [119],[120]:

\[
(Q_W)_{exp} = -71.04 \pm 1.81
\]  \( (12.27) \)

implies the result:

\[
\epsilon_3 = (-1.8 \pm 1.8) \times 10^{-2}
\]  \( (12.28) \)

Note that in Eqs. (12.25), (12.28) the quoted values for \( \epsilon_1 \) and \( \epsilon_3 \) are inclusive of all standard and possibly non-standard effects.

In a large variety of different models the epsilon parameters are suitable for a discussion of the possible effects of new physics on the various observables. There is an extensive and still rapidly growing literature on the subject. A summary is given in Ref. [108].

We now start from the model-independent determination of the epsilons given in Eqs. (12.12), (12.14), (12.15) and progressively make various stages of assumptions that allow us to combine an increasing large set of data. We first assume that there are no new physics effects that can invalidate the connection with \( A_{pol}^\nu \) (e.g. peculiar four-fermion interactions which could affect \( A_{FB}^\nu \) but not \( A_{pol}^\nu \)). As a second step we assume flavour and lepton-quark universality and we include hadronic quantities measured at the \( Z \)-peak. In particular we consider \( A_{FB}^\nu \), which is precisely measured and relatively unaffected by \( \alpha_s(m_Z^2) \). Then, in addition, we assume the absence
of additional sources of $q^2$ dependence beyond the Standard Model and include neutrino-nucleus deep inelastic scattering and parity violation in Cs atoms. The inclusion of the data on $A^\pi_{pol}$ allows us to combine the values of $\Delta k'$ given in Eqs. (12.13) and (12.20), thus obtaining:

$$\Delta k' = (0.04 \pm 0.81) \times 10^{-2} \quad \text{(12.29)}$$

We can then evaluate $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ by using the known values of $\Delta r_W$ and $\Gamma_Z$ and the above value of $\Delta k'$. This gives:

$$\begin{align*}
\epsilon_1 &= \Delta \rho = (-0.19 \pm 0.49) \times 10^{-2} \\
\epsilon_2 &= (-1.11 \pm 0.87) \times 10^{-2} \\
\epsilon_3 &= (-0.12 \pm 0.63) \times 10^{-2} .
\end{align*} \quad \text{(12.30)}$$

This stage is permissible in all models, including the case of a new $Z'$ from an extra $U(1)$ provided the universality of charged leptons is maintained.

We now add hadronic quantities measured at the $Z$ pole. The $b$-asymmetry $A^B_{FB}$ leads to the determination of $\Delta k'$ given in Eq. (12.24), which can be combined with those leading to Eq. (12.29). The resulting value is:

$$\Delta k' = (-0.40 \pm 0.70) \times 10^{-2} \quad \text{(12.31)}$$

At this stage one could also include the hadronic width $\Gamma_h$ and/or the total width $\Gamma_Z$, for $\alpha_s(m_Z)$ varying in a specified interval, for example in the range measured at LEP, as given is Section 11. The presence of large vertex corrections in the $Z \rightarrow b\bar{b}$ vertex makes the relation with the epsilons strongly $m_t$ dependent (as is also the case for $\Gamma_b/\Gamma_h$). For these reasons we restrict our attention to $A^B_{FB}$ at this stage. For the epsilons we then obtain:

$$\begin{align*}
\epsilon_1 &= \Delta \rho = (-0.25 \pm 0.49) \times 10^{-2} \\
\epsilon_2 &= (-0.06 \pm 0.86) \times 10^{-2} \\
\epsilon_3 &= (-0.41 \pm 0.58) \times 10^{-2} \quad \text{(12.32)}
\end{align*}$$

The next step, valid if the effects of a large $q^2$ difference can be described as in the Standard Model, is to include the low-energy data, in particular neutrino-nucleus deep inelastic scattering and parity violation in Cs atoms. By combining the low-energy results given in Eqs. (12.25) to (12.28) with the rest of the data one finds:

$$\begin{align*}
\epsilon_1 &= \Delta \rho = (-0.07 \pm 0.36) \times 10^{-2} \\
\epsilon_2 &= (-0.80 \pm 0.81) \times 10^{-2} \\
\epsilon_3 &= (-0.28 \pm 0.51) \times 10^{-2} \quad \text{(12.33)}
\end{align*}$$
Figure 32: Data on $\epsilon_1$ and $\epsilon_3$. The $1\sigma$ ellipses and their projections on the $\epsilon_1$, $\epsilon_3$ axes are shown. The Standard Model prediction is also displayed for reference purposes (the four solid lines are for different values of $m_H$, $m_H = 50$, 100, 500, 1000 GeV, and the dots mark values of $m_t$ in the range $m_t = 50$-270 GeV). Cases a) to d) correspond to different input data. a) is obtained from $\Gamma_\ell$ and $A_{FB}^\ell$. In b) the data on $A_{FB}^\ell$ have also been taken into account. The result for $A_{FB}^\ell$ is added in c). All data, including low energy experiments (Eq. (12.33)), are included in d). The predictions of QCD-like versions of technicolour (with $N_{TC} = 4$ and one technifamily) are also shown for comparison. Case e) corresponds to the all data but for $A_{FB}^\ell$. 

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At each stage, in the $\epsilon_1$-$\epsilon_3$ plane, the experimental result is compared in Figs. 32a to 32d with the predictions of the Standard Model for different values of $m_t$ and $m_H$. It is well known [112] that the data on atomic parity violation in Cs push the value of $\epsilon_3$ on the negative side [see Eq. (12.28)]. We see that this tendency toward negative values of $\epsilon_3$ is also supported by the very recent data on $A_{FB}^b$ [see Eq. (12.32)]. The effect of $A_{FB}^b$ can be appreciated from Fig. 32e, which is a fit to the same data as Fig. 32d but with $A_{FB}^b$ removed.

As seen from the overall summary in Fig. 32d, the central value of $\epsilon_3$ is negative and about 1σ away from the Standard Model prediction. Clearly, models where an additional positive contribution to $\epsilon_3$ is predicted are a fortiori discouraged, as is the case for the class of technicolour models [108], [109]--[113], [121] leading to the band of values (corresponding to $N_{TC} = 4$ and one technifamily) also shown in Fig. 32d. The values of $\epsilon_2$ are consistent with the Standard Model, with a still rather large error (Fig. 33).

Summarizing, we have shown that it is possible and indeed useful to introduce the parameters $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ (or equivalently $S, T$ and $U$) following a general definition independent of assumptions as the dominance of oblique corrections as were made in previous discussions. The presence of three parameters is related to the fact that $m_W/m_Z, \Gamma_t$ and $A_{FB}^b$ carry

![Graph showing $\epsilon_2$ vs. $m_t$](image)

**Figure 33**: $\epsilon_2$ vs. $m_t$ in the Standard Model for different values of $m_H (m_H = 50, 100, 500, 1000 \text{ GeV})$ with the experimental result in Eq. (12.33), obtained from all the data which are available at present.
qualitatively different information and are the most precisely measured observables defined at the W/Z mass scale (beyond the input parameter $m_z$); $\epsilon_2$ and $\epsilon_3$ are good indicators of the presence of new physics effects, because the uncertainties due to our ignorance of $m_t$ are concentrated in $\epsilon_1$. The epsilons have been studied in the Standard Model as functions of $m_t$ and $m_H$, and the associated theoretical errors were estimated; $\epsilon_3$ turns out to be particularly interesting, being independent of $m_t$ with very good accuracy, sensitive to Higgs sector, and likely to collect large contributions in models with no decoupling. Present data on $A_{FB}$ and on parity violation in atomic cesium favour values of $\epsilon_3$ smaller than in the Standard Model (although compatible with it).

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