NUCLEAR ANTISHADOWING AND
THE QUARKONIUM NUCLEON ELASTIC SCATTERING AMPLITUDE

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Abstract

If nucleon antishadowing in photon and lepton quarkonium \((QQ)\nu\) production is the result of the coherent interaction of the quarkonium \((QQ)\nu\) systems coupled to the photon in nuclear matter, the quarkonium nucleon elastic amplitude must have a large positive real part in the forward direction. This can be achieved by requiring that the cross-sections, at high energy, become beam-independent, \(\sigma_{NN} \approx \sigma_{Q\nu}\) and, in particular, \(\sigma_{NN} \approx \sigma_{Q\nu} \approx \sigma_{Q\nu}\).

Recently, Kharenev and Satz [1] made the important observation that shadowing effects in nuclei are necessarily related to the presence or absence of coherence phenomena in the interactions of photons and hadrons in nuclear matter. This proposal, if proved to be quantitatively adequate to explain data on the reactions of hadrons, photons and leptons with nuclei, may lead to a revision of accepted ideas on changes of nuclear structure functions in nuclei (for a review see [2]).

Bialas and Czyz [3], following the arguments of [1], have shown, using the formalism of nuclear optics [4], that the predicted shadowing and antishadowing effects are indeed calculable. Using the example of photoproduction of heavy quarkonia, they have presented results showing the observed qualitative features of shadowing and antishadowing seen in \(\psi\) leptonproduction [5].

Coherence and antishadowing, in the case of photoproduction of vector mesons \((V)\) on nuclei of atomic number \(N\),

\[
\gamma + A \rightarrow V + \text{anything},
\]

depends critically on the real part of the forward vector meson-nucleon amplitude, \(A_{NN}(E)\), where \(E\) is the vector meson energy in the rest frame of \(N\), the imaginary part of \(A_{NN}(E)\) playing no role because of the total cross-section unitarity constraint [3]. Implicitly, one assumes that the hadronic system coupled to the photon inside nuclear matter behaves essentially as a vector meson. Antishadowing occurs when \(Re A_{NN}(E)\) is positive. In Ref. [3] reasonable values, in the range 1.05-1.15, for the ratio \(\rho_{NN}^{V}\) of normalized vector meson production cross-section on nuclei over the production cross-section on nucleons, were obtained assuming for \(\rho_{V}\),

\[
\rho_{V}(E) = \Re A_{NN}(E) / \Im A_{NN}(E),
\]

values of the order of 0.5-1 (for \(\sigma_{NN} = 10 \text{ mb} \)).

For quantitative estimates of \(\rho_{NN}^{V}\) and to test the importance of the mechanism of [1], one then needs to know \(\rho_{V}(E)\) \((V = s\bar{s}, c\bar{c}, b\bar{b} \text{ and } t)\) at intermediate and high energies. Experimentally and theoretically, not much is known.

In principle, the amplitude \(A_{NN}(E)\) for quarkonium vector mesons (of the second or third generation) nucleon scattering should have a simple structure. It is crossing-even \((QQ = Q\bar{Q})\) and exotic (no planar resonances and no quark to be exchanged). In other words, it is a Pomeron-like amplitude. As one now believes that the Pomeron corresponds to an asymptotically growing cross-section and the real part of the amplitude is essentially the \(\ln E\) derivative of the imaginary part, we expect \(\rho_{V} > 0\), as required by the calculation of [3].

However, the situation is not as simple as that.

First, if one takes at face value the results of [3], one not only needs a positive \(\rho_{V}\) but a large value for it. \(\rho_{V} \gg -0.5\). Such large positive values for \(\rho\) were never observed at intermediate and high energies. If we treat the Pomeron as a Regge pole with intercept \(\alpha = 1 + \epsilon \approx 0.08\) [6], this gives \(\rho_{V} \approx \pi/2\epsilon \approx 0.14\), too small a number. If we choose a Pomeron of the Froissart type, \((\ln E)^{2} / \rho_{V} \approx \pi/\ln(E/E_{0})\), then we have no problem provided \(E > E_{0}\).
One should also note that at intermediate energies, $E \approx 10$–30 GeV, all known hadronic cross-sections, including exotic reactions $(pp, K^+p)$, decrease with energy. This implies $\rho < 0$, as observed.

Second, we can always add a crossing-even real function, a constant, to the real part of the amplitude without affecting the imaginary part. This means that such a constant cannot be generated by derivatives of the imaginary part. This has to do with the need of one subtracted dispersion relation for the crossing-even amplitude and, in the case of the $2\gamma N \rightarrow 2\gamma N$ amplitude, with the Thompson limit of Compton scattering. In fact, such a contribution, a negative one, seems to be needed in all crossing-even amplitudes [7,8]. This negative real part contribution helps, via unitarity, to explain the decrease of the total cross-section in the intermediate energy region [8].

Finally, the only experiment where $\rho_0(E)$ was determined, was a delicate one [9] that measured the interference between the photoproduction amplitude, $\gamma N \rightarrow \gamma A \rightarrow e^+e^-A$, and the Bethe-Heitler amplitude. A large and negative value for $\rho_0$ ($E \approx 6$ GeV) was obtained: $\rho_0 = -0.48 \pm 0.23, -0.45$.

Apparently, we then have a difficult task ahead of us if we want a full explanation of antishadowing as a coherent effect due to the real part of the $A_{NN}$ amplitude. At low and intermediate energies, $E \leq 30$ GeV, we have a negative and large $\rho_N$, at $E \geq 30$ GeV we need a positive and large $\rho_N$.

The model we propose here is an extension of the very simple Pomeron + Reggeon model successfully applied in Ref. [10] to $pp$ and $pp$ forward elastic scattering in the 15 < $E$ < 2 x 10$^6$ GeV energy region. Here, as explained above, we only need the crossing-even Pomeron amplitude:

$$\frac{ImA(E)}{E} = \sigma(E) - A + B \ln E/F_0^2, \quad (3)$$

and

$$\rho(E) = \frac{Re A(E)}{Im A(E)} = \frac{1}{A} \ln \frac{A}{B} \ln \frac{E}{E_0}, \quad (4)$$

At energies $E < F_0$ the cross-section decreases with $E$, and $\rho(E)$ is negative; (3) and (4) thus roughly simulate what is experimentally observed at $E < 12$ GeV. Above $E_0$, the cross-section rises and $\rho(E)$ becomes positive. The maximum value of $\rho(E)$ occurs for $E = E_M$, with

$$\ln \frac{E_M}{E_0} = \left(\frac{A}{B}\right)^{1/2}, \quad (5)$$

and

$$\rho(E_M) = \frac{\pi}{2} \left(\frac{B}{A}\right)^{1/2}. \quad (6)$$

In the $pp$ ($pp$) case we have $A = 38, 12$ mb, $B = 0.31$ mb, $E_0 = 36.2$ GeV [10] and the maximum value of $\rho$ occurs for $\sqrt{E} - 2$ TeV and $\rho_M = 0.14$.

For the quarkonium nucleon amplitude we shall then use Eqs. (3) and (4). The question now is how to fix the parameters $A, B, E_0$. The simplest idea is to make use of the naive quark-model relations, $\sigma_{NN} \approx 2/3\sigma_{NN}, \sigma_{p\bar{p}} = 2\sigma_{NN} + 2\sigma_{p\bar{p}}$, etc., which are quite successful at low and intermediate energies. Doing this amounts to writing

$$\sigma_{NN}(E) = \lambda \sigma_{NN}(E),$$

where $\lambda_{NN}$ is a quark-model factor and $\sigma_{NN}$ is given by (3). But then one immediately sees that $A$ and $B$ in (3) are affected by the same multiplicative factor such that (6) does not change. The maximum of $\rho$ will remain 0.14, which is too small, and we are back to the problem we started from.

At this point, instead of (7), we adopt an old proposal of Froissart [11], recently revived by Martin [12]. The asymptotic term in the cross-section, the term in $(ln E)^2$, is assumed to be universal, and to reflect the high-energy limit of the physics of the underlying theory (QCD). The non-dominant term, presumably related to the size of the interacting hadrons, is assumed to obey the constituent-quark model relations.

This means that $A$ will be beam-particle-dependent,

$$A = A_0 - \lambda_{NN} A, \quad (8)$$

but $B = 0.31$ mb and $E_0 = 36.2$ GeV remain the same, independently of the colliding particles. It is clear that this prescription $\rho_N$ will be more negative below $E_0$, and more positive above, than in nucleon-nucleon scattering.

We can test this idea in the case of the already-mentioned measurement of $\rho_0$ in photoproduction at $E \approx 6$ GeV [9]. From experiment [13] or quark model, $\sigma_{NN} \approx 10$ mb at 6 GeV. This corresponds to $\lambda_{NN} \approx 9$ mb and $\rho_0(6$ GeV) = $-0.175$, consistent with data [9]. Note that with (7) we should have obtained $\rho_0(6$ GeV) = $-0.045$, a value outside the experimental error interval.

For the heavier quarkonium nucleon amplitudes we make use of the fact that, as the $(Q\bar{Q})$ states are much smaller than the nucleon, the cross-section is controlled by the vector meson radius $r_N$,

$$\sigma_{NN}^2 = A_0 \sim r_N^2 \quad (9)$$

with the Coulomb-like potential,

$$r_N \sim m_N^{-1}. \quad (10)$$

We shall then write

$$A_0 = A_0 \left(\frac{m_N}{m_N}ight)^2 \quad (11)$$

For instance, for the cross-section $\sigma_{NN}$, from (3) and (11), we obtain 1.4 mb, at 12 GeV, and 1.9 mb at 200 GeV, in agreement with experiment [14]. Incidentally, the $1/m_N^2$ dependence of the total cross-section is also observed in the $f$-dominated Pomeron [15].

Our results are presented in the table. Two observations can be made:

1) The vector meson nucleon total cross-sections, in particular $\sigma_{NN}$ and $\sigma_{p\bar{p}}$, rapidly approach each other as the energy increases.
ii) Large positive values for \( \rho_v \), in the studied energy region for \( \psi \) leptoproduction, \( 40 \lesssim E \lesssim 240 \text{ GeV} \) [5] and above, are easily obtained in the model.

It should be clear that our assumptions, (8) with a fixed asymptotic \((\ln E)^2\) term and (9), are presumably very crude. For instance, it is difficult to accept the limit \( \rho_v \to 0 \), \( A_v \to 0 \), and a constant \( B(\ln E/E_0)^2 \) term. A more elaborate model may require some \( \rho_v \) dependence of \( B \). However, if large positive real parts are required at intermediate and high energy, it is essential that \( B \) decreases with \( \rho_v \) very slowly in comparison with \( A_v \). Assumption (9) at high energies is not very critical.

It is difficult at the moment to draw conclusions. The ideas of [1] have to be tested in a more systematic and quantitative way. In particular, antishadowing in deep inelastic scattering at relatively low energy [16] has to be understood in the framework of [1].

If the proposal of [1] for quarkonium leptoproduction, with the interpretation of [3], is shown to be valid, then the problem can be turned around. Photon and virtual photon interactions in nuclei may become a way of studying the various quarkonia nucleon elastic scattering forward amplitudes and their asymptotic behaviours.

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References

Use of formulas (3) and (4), for \( \sigma_{VN}(E) \) and \( \rho_{V}(E) \), in the case of different vector mesons at various energies. Only the parameter \( A_{V} \) changes in going from one vector meson to the other. The parameters \( B \) and \( E_{0} \) remain fixed with the \( pp (\bar{p}p) \) values: \( B = 0.31 \text{ mb} \) and \( E_{0} = 36.2 \text{ GeV} \). For \( E \gtrsim 40 \text{ GeV} \), the real parts of the amplitudes, in the cases of \( \psi N, T N \) and \( (\bar{t}t) N \), become positive and large (\( \rho(E) \gtrsim 0.5 \)).

<table>
<thead>
<tr>
<th>Vector meson mass (GeV)</th>
<th>( A_{V} ) (mb)</th>
<th>( \sigma_{V}(E) ) mb, Eq. (3)</th>
<th>( \rho_{V}(E) ), Eq. (4)</th>
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<tbody>
<tr>
<td>( \phi ) (1.02)</td>
<td>9</td>
<td>9.4 9.9 12.0</td>
<td>-0.11 0.17 0.25</td>
</tr>
<tr>
<td>( \psi ) (3.1)</td>
<td>1</td>
<td>1.4 1.9 4.0</td>
<td>-0.78 0.87 0.78</td>
</tr>
<tr>
<td>( Y ) (2.5)</td>
<td>0.1</td>
<td>0.5 1.0 3.0</td>
<td>-2.23 1.66 0.96</td>
</tr>
<tr>
<td>( \bar{t}t ) (170)</td>
<td>( 3 \times 10^{-4} )</td>
<td>- 0.9 2.9</td>
<td>- 1.84 1.01</td>
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</tbody>
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