Standard Model of Flavour Mixing and CP Violation – Status Report and Perspectives

A. Ali*)
Theory Division, CERN
CH-1211 Geneva 23, Switzerland

To be published in the Festschrift in honour of Professor Abdus Salam, Salamfest, World Scientific Series on 20th Century Physics – Vol. 4.

CERN-TH.7123/93
December 1993

*) On leave of absence from DESY, Hamburg, FRG.
Standard Model of Flavour Mixing and CP Violation - Status Report and Perspectives

A. Ali
Theory Division, CERN
CH-1211 Geneva 23, Switzerland

1 Introduction

In the electroweak theory of Glashow, Salam and Weinberg, popularly known as the Standard Model (SM) [1], fermions and gauge bosons \((W^\pm, Z^0)\) get their masses through the Higgs mechanism. For the quarks, the gauge- and mass-eigenstates are not the same, leading to intergenerational couplings in the weak charged-current interactions. These flavour non-diagonal couplings are described by the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. The CKM matrix is one of the few aspects of the SM that remains to be fully tested. The complex phase residing in the CKM matrix is the only source of CP violation in the SM. So far CP violation has been observed only in the decays of the neutral kaons in laboratory experiments. However, its macroscopic implication is all too obvious, as it is regarded as the key ingredient responsible for the baryon asymmetry that we observe today in the universe. The CKM matrix elements are thus fundamental constants of nature. In their understanding lies the clue to our understanding of quark masses, mixings, CP violation, and eventually of such existential questions as the baryon asymmetry of the universe.

The primary aim of experiments in flavour physics is to determine the CKM matrix elements precisely. This is done by measurements of the charged current (CC) and the so-called flavour-changing neutral-current (FCNC) processes. The latter involve quark transitions such as \(s \to d, c \to u, b \to s\). FCNC transitions, being forbidden in the SM Lagrangian [3], are induced by higher-order CC weak interactions and hence they are intrinsically sensitive to much larger mass scales than \(m_W\). In the SM, FCNC transitions among down-type quarks involve, as virtual state, the top quark, and these transitions are at present the only source of information for the CKM matrix elements involving the \(t\)-quark, \(V_{ij}, j = d, s, b\). Precise knowledge of these matrix elements would determine the CKM parameters completely, hence also the CKM phase and with it all the CP asymmetries in the quark sector.

In this article, written for the Festschrift in honour of Professor Abdus Salam, I review the present status of the CKM matrix, which in the SM governs the flavour transitions of the left-chiral matter fields and CP violation. In theoretical discussions of these matters,
the constraints that unitarity imposes on the parameters of the CKM matrix play a central role. Such constraints are often expressed as triangle relations in the complex plane of the CKM parameter space, and there are altogether six of them. I present a state-of-the-art profile of perhaps the most interesting of the CKM triangles following from the constraint \( \Sigma_i V_{ii}V_{ii}^* = 0 \) [4, 5]. The constraints that follow from the recent measurement of the electromagnetic penguin decay \( B \to K^* + \gamma \) [6, 7] and the existing CC data [8] on another triangle following from the unitarity relation \( \Sigma_i V_{ii}V_{ii}^* = 0 \) are also quantified. As discussed by Altarelli [9], the SM has passed rather excruciating experimental tests in the electroweak sector. At the end of this review, the same message will also emerge from the flavour physics, although here the arguments in favour of the SM are not yet completely quantitative.

The profile of the CKM unitarity triangles will become increasingly sharp in future as precision experiments in \( B \) and \( K \) physics bring to bear the full force of their measurements. In anticipation of a major experimental surge foreseen for the rest of this and the next decade(s) in precision studies of \( B \) physics, I review a number of rare FCNC \( B \) decays and CP violation involving \( B \) hadrons. These measurements will provide a quantitative test of the SM in the flavour sector, in particular, its short-distance physics. Hopefully, these experiments will also serve as windows on non-SM physics, which inevitably lies there and which hopefully will bring us closer to an understanding of the large number of parameters in the SM, such as fermion masses and mixing angles.

## 2 Charged Weak Currents in the SM

We retrace here some of the essential steps leading to the introduction of the CKM matrix in the CC weak interactions. The SM particle content and their (weak) isospin and chiral properties of the fields may be seen in any standard textbook on particle physics. We shall concentrate here on the interaction term \( \mathcal{L}(f, \phi) \) involving the fermions and the Higgs fields in SM, which has the Yukawa form:

\[
\mathcal{L}(f, \phi) = \sum_{j=1}^{3} \left\{ (h_j)_{jk} \bar{q}_L^j \Phi^c \bar{d}_R^k \right\} + \sum_{j,k=1}^{3} \left\{ (h'_q)_{jk} \bar{q}_L^j \Phi^c \bar{d}_R^k + (h'q)_{jk} \bar{q}_L^j \Phi^c \bar{d}_R^k \right\},
\]

where

\[ \Phi^c = i\sigma_2 \phi^* = \begin{pmatrix} \phi^0^* \\ -\phi^- \end{pmatrix}, \]

and both \( \Phi \) and \( \Phi^c \) transform as a (weak) isospin doublet. Here \( q_L^j(q_R^j) \) and \( l_L^j(l_R^j) \) are, respectively, the \( SU(2)_L(U(1)_R) \) quark and lepton fields, respectively, and we have assumed three families of fermions. The Yukawa coupling constants \( (h_j)_{jk}, (h'_q)_{jk} \) and \( (h'q)_{jk} \) are arbitrary complex numbers and each term above is independently \( SU(2)_L \otimes U(1)_Y \)-invariant due to the fact that the \( SU(2)_L \) acts only on \( l_L^j \) and \( q_L \) and on the Higgs doublet \( \Phi \) and \( \Phi^c \), whose products are \( SU(2) \) scalars. After spontaneous symmetry breaking
\[ L(f, \phi)^{SSB} = \sum_{j=1}^{3} (m_{j})_{U}^{L} V_{L}^{R} \left( 1 + \frac{1}{v} \phi \right) \]
\[ - \sum_{j,k=1}^{3} \left\{ (m_{jk})_{U} u_{L}^{k} u_{R}^{k} + (m_{jk})_{D} d_{L}^{k} d_{R}^{k} \right\} \left( 1 + \frac{1}{v} \phi \right) + \text{h.c.}, \]

where
\[ (m_{j})_{U} = (h_{j})_{U} \frac{v}{\sqrt{2}}, \]
\[ (m_{jk})_{U} = -(h'q)_{jk} \frac{v}{\sqrt{2}}, \]
\[ (m_{jk})_{D} = -(h'q)_{jk} \frac{v}{\sqrt{2}}, \]

and \( v \) is the neutral-Higgs vacuum expectation value. Since in the SM there are no right-handed fields \( \nu_{R}^{i} \) \((i = 1, 2, 3)\), the neutrinos \( \nu^{i} \) remain massless and the charged lepton mass matrix \((m_{j})_{L}\) is diagonal. Hence, there are no family-changing leptonic interactions. This aspect of the SM is now considered as vulnerable. In particular, as data on the solar neutrino flux are consolidating, suggestions that neutrinos may have a tiny mass and that they mix with each other are gaining currency. Likewise, astrophysical data from COBE and elsewhere having a bearing on the dark matter problem of the universe are more comfortably accommodated if there exists a significant component of the so-called hot dark matter, which may be identified with massive neutrinos. These aspects are being discussed by J. Ellis [10].

In (3), \((m_{jk})_{U}\) and \((m_{jk})_{D}\) are the \((3 \times 3)\) quark mass matrices for the up- and down-type quarks, respectively. In order to write the Lagrangian in terms of the quark mass eigenstates one has to diagonalize the mass matrices. This can be done with the help of two unitary matrices for each of the two mass matrices. We denote them for the up-type quarks by \( V_{L}^{u} \) and \( V_{R}^{u} \) (likewise the ones for the down-type quarks are denoted by \( V_{L}^{d} \) and \( V_{R}^{d} \)):

\[ V_{L}^{u} m_{U} V_{R}^{u \dagger} \equiv (m_{\text{diag.}})_{U} \equiv \text{Diag.} (m_{u} , m_{c} , m_{t}) \]
\[ V_{L}^{d} m_{D} V_{R}^{d \dagger} \equiv (m_{\text{diag.}})_{D} \equiv \text{Diag.} (m_{d} , m_{s} , m_{b}) \]

with \( V_{L}^{u \dagger} V_{L}^{u} = 1 \), etc. The physical quark fields (mass eigenstates) are related to the corresponding fields in the Lagrangian by the transformation:

\[ \hat{u}_{L} = V_{L}^{u} u_{L} = V_{L}^{u} \begin{pmatrix} u_{L} \\ c_{L} \\ t_{L} \end{pmatrix} \]
Symbolically, the CKM matrix can be written as:

\[ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} = V_L^d \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}. \]  

Likewise, \( \tilde{u}_R = V^u_R u_R, \) \( \tilde{d}_R = V^d_R d_R. \) One can now rewrite the term \( \mathcal{L}(f, \Phi) \) in the SM Lagrangian in terms of the physical fields, obtaining

\[ \mathcal{L}(f, \Phi)^{SSB} = - \left( 1 + \frac{\Phi}{v} \right) \left\{ \sum_{i=1}^{6} m_{\psi_i} \bar{q}_i q_i + \sum_{j=1}^{3} m_{l_j} \bar{l}_j l_j \right\} \]  

where now it is understood that \( q_1 = \frac{1}{2} (u^L_L + u^R_R) = u, \) etc., and we have dropped the hat over the quark fields. The identification of the parameters \( m_{u_i}, m_{q_i} \) with the lepton and quark masses is now evident. Written in terms of the physical boson (\( W^\pm, Z^0, A_\mu \)) and fermion fields, it is easy to show that the neutral current (NC) part of \( \mathcal{L}(f, W, B) \) is manifestly flavour-diagonal. Thus, all flavour-changing transitions in the SM are confined to the charged current (CC) sector. This is an important feature of the SM, which we will confront with experiments. The absence of FCNC interactions in the SM Lagrangian is due to the choice of a single Higgs doublet and to the unitarity of the mixing matrix [3].

The charged current \( J_{\mu}^{CC} \) is derived from the SM Lagrangian using the mass eigenstates. The CC couplings in the SM involve only the left-handed fermions \( q^L_1 \) and \( l^L_1 \) and the charged current \( J_{\mu}^{CC}, \) which couples to the \( W^\pm, \) is:

\[ J_{\mu}^{CC} = (\bar{u}, \bar{e}, \bar{\tau}) L \gamma_\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \]  

where we have again dropped the hats and \( V_{CKM} \equiv V^u_L V^d_L \) is a \( (3 \times 3) \) unitary matrix in the flavour space, first written down by Kobayashi and Maskawa in 1973 [2]. It is a generalization of the Cabibbo rotation [2] for the three-quark-flavour \((u, d, s)\) case, invented to keep the universality of weak interactions, which took the form of a \((2 \times 2)\) matrix by the inclusion of a c-quark with the GIM construction [3], and is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In the SM, CC have a \((V - A)\) structure, they violate P and C maximally, conserve the electric charge and the lepton- and baryon-number separately, but otherwise there are no other known restrictions on them except that \( V_{CKM}^\dagger V_{CKM} = 1. \) In general, \( \mathcal{L}^{CC} \) violates CP due to the appearance of a non-trivial phase in \( V_{CKM}. \)

### 3 The CKM Matrix and Unitarity Triangles

Symbolically, the CKM matrix can be written as:

\[ V_{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \]
The nine quantities $V_{ij}$ have to be determined experimentally via the couplings $W^\pm q \bar{q}$. The CKM matrix elements obey unitarity constraints, which state that any pair of rows, or any pair of columns, of the CKM matrix are orthogonal. This leads to six orthogonality conditions. The three involving the orthogonality of columns are listed below, with the quark pair in the parenthesis $(jk)$ representing the product of the $j$'th and $k$'th columns:

$$
(ds) : \sum_i V_{id} V_{is}^* = 0,
$$

$$
(sb) : \sum_i V_{is} V_{id}^* = 0,
$$

$$
(db) : \sum_i V_{id} V_{id}^* = 0. \quad (9)
$$

Similarly, there are three more such orthogonality conditions on the rows:

$$
(uc) : \sum_j V_{uj} V_{cj}^* = 0,
$$

$$
(ct) : \sum_j V_{cj} V_{ij}^* = 0,
$$

$$
(ut) : \sum_j V_{uj} V_{ij}^* = 0. \quad (10)
$$

The six orthogonality conditions can be depicted as six triangles in the complex plane. This is shown in fig. 1, in which each triangle is marked by the pair of quarks, representing the pair of columns, or of rows, whose orthogonality this triangle represents. Another way to view them is through the FCNC transitions in which the pair of quarks depicted participate. The triangle labelled $(ds)$ represents the unitarity constraints on the transition $s \to d$, which, for example, one encounters in the $K^0 - \bar{K}^0$ transition. The two others in this group, namely $(sb)$ and $(db)$, are encountered in the FCNC transitions in $B$ decays, such as particle-antiparticle mixings $B^0 - \bar{B}^0$ and rare $B$ decays $B \to K^* + \gamma$ and $B \to \omega + \gamma$. These unitarity relations will, of course, be tested in CP violation experiments as well.

The $(uc)$, $(ct)$ and $(ut)$ triangles can be measured in FCNC transitions involving the $Q = +2/3$ quarks, i.e. $c \to u$, $t \to c$ and $t \to u$. The analogous FCNC processes in this case are $D^0 - \bar{D}^0$ and $T^0 - \bar{T}^0$ mixings, and decays such as $D \to \rho + \gamma$, $T \to D^* + \gamma$ and $T \to (D, D^*, \rho) + Z$ etc. However, due to the fast decays of the charmed hadrons, which do not have any CKM suppression, unlike the $K$ and $B$ decays, and the even faster decays expected for the top quark, such FCNC transitions will be very difficult to measure. Of course, the sides of all of these triangles can be measured in CC decays of the quarks, as indicated in fig. 1. However, for the $(ct)$ and $(ut)$ triangles this would require the CKM matrix elements $V_{id}$ and $V_{is}$, once again a dismal proposition in direct top quark decays due to the anticipated tiny magnitudes of these matrix elements (see below). The $(uc)$ triangle involves CKM matrix elements that have all been measured, but getting a sharp profile of this triangle imposes enormous demands on the accuracy of the matrix elements involved. It should be noted that not all the sides and angles in these triangles are independent. It can be shown that only four of the eighteen angles in the six CKM unitarity triangles are independent [5].
It is obvious that measuring all these triangles independently will keep the experimental flavour physics community busy for some time to come! The unitarity triangle that has, in our opinion, the best chance to be measured precisely is the triangle \((d\bar{b})\) following from the constraint:

\[
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 .
\]  
(11)

This is so because the three terms here are expected to be of comparable magnitude. This can be readily seen in the Wolfenstein parametrization of the CKM matrix. It was noticed some time ago by Wolfenstein [11] that the elements of this matrix exhibited a hierarchy in terms of \(\lambda\), the Cabibbo angle. In this parametrization the CKM matrix can be written approximately as

\[
V_{CKM} \simeq \begin{pmatrix}
1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\
\frac{1}{2}\lambda^2 & 1 - \frac{1}{2}\lambda^2 - iA^2\lambda^4\eta & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} .
\]  
(12)

Using this representation of the CKM matrix, the \((d\bar{b})\) unitarity relation can be written as:

\[
\frac{V_{ub}^*}{\lambda V_{cb}} + \frac{V_{td}}{\lambda V_{tb}} = 1 ,
\]  
(13)

which is a triangle relation in the complex plane (i.e. \(\rho-\eta\) space). This is illustrated in fig. 2.

4 An Update of the CKM Matrix

In what follows we give an update of the Wolfenstein parameters \(A\), \(\lambda\), \(\rho\) and \(\eta\). Having done this, we will use these updates to get the profile of the CKM unitarity triangle shown in fig. 2. We recall that \(|V_{us}|\) has been extracted with good accuracy from \(K \to \pi\nu\bar{\nu}\) and hyperon decays [8] to be

\[
|V_{us}| = \lambda = 0.2205 \pm 0.0018 .
\]  
(14)

This agrees quite well with the determination of \(V_{ud} \simeq 1 - \frac{1}{2}\lambda^2\) from \(\beta\)-decay:

\[
|V_{ud}| = 0.9744 \pm 0.0010 .
\]  
(15)

The parameter \(A\) is related to the CKM matrix element \(V_{cb}\), which can be obtained from semileptonic decays of \(B\) mesons. We shall restrict ourselves to the methods based on the heavy-quark effective theory (HQET). In the heavy quark limit it has been observed that all hadronic form factors describing the dominant semileptonic transitions \(B \to D\nu\) and \(B \to D^*\nu\) can be expressed in terms of a single function, the Isgur-Wise function [12]. It has been shown that the HQET-based method works best for \(B \to D^*\nu\) decays, since these are unaffected by \(1/m_b\) corrections [13, 14, 15]. Since \(O(1/m_b)\) corrections to the Isgur-Wise function \(\xi_A(v \cdot v' = 1)\) determining the rate for the decay \(B \to D^*\nu\) at the symmetry point are absent, deviation from the relation \(\xi_A(v \cdot v' = 1) = 1\) is dominantly of perturbative QCD origin (here \(v\) and \(v'\) denote 4-velocities of the \(B\) and \(D^*\), respectively). The differential decay rate can be expressed as [15]:

\[
\lim_{v \cdot v' \to 1} \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \frac{d\Gamma(B \to D^*\nu\nu)}{d(v \cdot v')} = \frac{G_F^2}{4\pi^3} M_{D^*}^3 (M_B - M_{D^*}) |V_{cb}|^2 \xi_A^2 .
\]
Figure 1: The six CKM unitarity triangles following from the orthogonality of the indicated pairs of columns, or of rows. The sides are drawn to the presently estimated lengths.
The QCD corrected function $\xi_A_1(v \cdot v' = 1)$ turns out to be essentially 1, namely $\xi_A_1(v \cdot v' = 1) \approx 0.99$. There also exist theoretical estimates of the $O(1/m_c^2)$ power corrections \cite{16}. Although these corrections are model-dependent and the present estimates of their contribution at the level of $O(\pm 3\%)$ may or may not be entirely realistic, in all likelihood, at the symmetry point $v \cdot v' = 1$, the semileptonic decays $B \to D^* \ell \nu$ are under theoretical control.

However, there are no data at the point $v \cdot v' = 1$. Recently, lattice-QCD calculations of the Isgur-Wise function have been reported in the $v \cdot v'$ range where data are available \cite{17}. The experimental data and the lattice version of the Isgur-Wise function are in reasonable agreement \cite{18}. One could also use a parametrization of the Isgur-Wise function $\xi_A_1(y)$ to extrapolate the data to the point $y = v \cdot v' = 1$, where the heavy quark symmetry holds. Four such parametrizations have been used in the experimental analysis. A fit of the data on $B \to D^* \ell \nu$, determines the slope and $|V_{cb}|$. With the branching ratios measured by the ARGUS and CLEO experiments \cite{19, 20}, and using $\tau_B = 1.49 \pm 0.04$ ps, the present world average of the $B$-hadron lifetime \cite{18}, the results of a recentupdate of $|V_{cb}|$ based on HQET are shown in fig. 3. The parametrizations used are also given here. While there is some spread in the extracted value, typical values from the ARGUS data are centred around $|V_{cb}| = 0.045$, which incidentally is also the central value quoted by the lattice-QCD analysis in \cite{17}, and those of the CLEO data around $|V_{cb}| = 0.037$ \cite{20}. The errors are still significant so that all these determinations are consistent with each other. Eventually, data will become quite precise so that the required extrapolation to the symmetry point will be much reduced and with it the attendant theoretical uncertainties.

In the fits for the CKM unitarity triangle shown later, the following value for $|V_{cb}|$ has been used:

$$|V_{cb}| = 0.042 \pm 0.005 .$$  \hspace{1cm} (16)
This gives for the CKM parameter $A$:

$$A = 0.86 \pm 0.10 .$$  \hspace{1cm} (17)

The other two CKM parameters $\rho$ and $\eta$ are constrained at present by the measurements of $|V_{ub}/V_{cb}|$, $|\varepsilon|$ (the CP-violating parameter in the kaon system), $x_d$ (the $B_d^0 - \overline{B}_d^0$ mixing ratio) and (in principle) $\varepsilon'/\varepsilon$ ($\Delta S = 1$ CP violation in the kaon system). We shall not discuss the constraints from $\varepsilon'/\varepsilon$, due to the various experimental and theoretical uncertainties surrounding it at present and refer to [21] for details, but take up the rest in turn and present fits in which the allowed region of $\rho$ and $\eta$ is shown.

First of all, $|V_{ub}/V_{cb}|$ can be obtained by looking at the endpoint of the inclusive lepton spectrum in semileptonic $B$ decays [22]. The leptons from the semileptonic transition $b \rightarrow c \ell \nu_\ell$ range out earlier than the ones from the transition $b \rightarrow u \ell \nu_\ell$. The extraction of $|V_{ub}/V_{cb}|$, however, requires a model for the inclusive lepton spectra. The result of a recent analysis of the available data (from ARGUS [23], CLEO 1.5 (old data) [24] and CLEO II (new data) [25]), for four different models [27, 28, 29, 30], is shown in fig. 4. The CLEO II data lie systematically below the CLEO 1.5 and ARGUS data, and the spread in the values of the CKM ratio $|V_{ub}/V_{cb}|$ due to models is noteworthy. Clearly, there is much room for improvement here. Unlike the semileptonic decay $B \rightarrow D^* + \ell \nu_\ell$, the $b \rightarrow u$ transitions are not determined by the HQET rules alone and some model building is inevitable. The present average by consensus (barring the ISGW model) is [18, 26, 31]:

$$|V_{ub}/V_{cb}| = 0.08 \pm 0.02 .$$  \hspace{1cm} (18)

This gives for the Wolfenstein parameters:

$$\sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09 .$$  \hspace{1cm} (19)

The value of $|V_{ub}/V_{cb}|$ has considerable effect on the allowed domains for the parameters $(\rho, \eta)$, and in determining the CP asymmetries in a number of $B$ decays. Further constraints on these parameters are obtained from $|\varepsilon|$. The experimental value of $|\varepsilon|$ is [8]:

$$|\varepsilon| = (2.26 \pm 0.02) \times 10^{-3} .$$  \hspace{1cm} (20)

Theoretically, $|\varepsilon|$ is essentially proportional to the imaginary part of the box diagram for $K^0 - \overline{K}^0$ mixing and is given by [32]

$$|\varepsilon| = \frac{G_F^2 f_K^2 M_K M_{\overline{K}}^2 B_K}{6\sqrt{2} \pi^2 \Delta M_K} A^2 \lambda^6 \eta \left( y_c \left[ \eta_{ct} f_3(y_c, y_t) - \eta_{cc} \right] + \eta_{ut} f_2(y_t) A^2 \lambda^4 (1 - \rho) \right).$$  \hspace{1cm} (21)

Here, the $\eta_i$ are QCD correction factors, $\eta_{cc} \approx 0.82$, $\eta_u \approx 0.62$, $\eta_{ct} \approx 0.35$ for $\Lambda_{QCD} = 200$ MeV [33], $y_t \equiv m_t^2/M_{\overline{W}}^2$, and the functions $f_2$ and $f_3$ are given by

$$f_2(x) = \frac{1}{4} + \frac{9}{4} \frac{1}{(1-x)} - \frac{3}{2} \frac{1}{(1-x)^2} - \frac{3}{2} \frac{x^2 \ln x}{(1-x)^3},$$

$$f_3(x, y) = \ln \frac{y}{x} - \frac{3y}{4(1-y)} \left( 1 + \frac{y}{1-y} \ln y \right).$$  \hspace{1cm} (22)
Figure 3: Measurements of the CKM angle $|V_{cb}|$ from data on the semileptonic decay $B^0 \to D^*\ell^-\bar{\nu}$ from ARGUS [19] and CLEO [20] and $B^- \to D^{-}\ell^-\bar{\nu}$ from ARGUS [19], using the Isgur-Wise function $\xi_{A_1}$. The parametrizations used are also indicated (from [26]).
Determinations of $|V_{ub}/V_{cb}|$ from the Inclusive Lepton Spectrum

| Model - Experiment | $|V_{ub}/V_{cb}|$ |
|--------------------|------------------|
| ARGUS              | 0.11 ± 0.012     |
| ACCMM CLEO 1.5     | 0.11 ± 0.016     |
| CLEO II            | 0.076 ± 0.008    |
| ARGUS              | 0.20 ± 0.023     |
| ISGW CLEO 1.5      | 0.18 ± 0.026     |
| CLEO II            | 0.101 ± 0.010    |
| ARGUS              | 0.11 ± 0.012     |
| KS CLEO 1.5        | 0.10 ± 0.014     |
| CLEO II            | 0.056 ± 0.006    |
| ARGUS              | 0.13 ± 0.015     |
| WSB CLEO 1.5       | 0.13 ± 0.018     |
| CLEO II            | 0.073 ± 0.007    |

Figure 4: Determinations of $|V_{ub}/V_{cb}|$ from the inclusive lepton energy spectrum in $B$ decays. The data are from ARGUS [23], CLEO 1.5 [24] and CLEO II [25]. The extracted values are shown on the r.h.s., and the models used ACCMM [27], ISGW [28], KS [29] and WSB [30] are indicated on the l.h.s. (from [26]).
The errors are large enough to cover most present lattice-based numbers. The present CDF bound from direct top searches gives $m_t > 113$ GeV [35]. While the exact range of $m_t$ from electroweak analysis depends on a number of details, in particular on the value of $\alpha_s$ [9], a recent analysis based on the combined LEP data gives [36]:

$$m_t = 164 \pm 27 \text{ GeV.} \quad (23)$$

The final parameter in the expression for $|\epsilon|$ is $B_K$, which involves the matrix element $\langle K^0 | (\bar{d} \gamma^\mu (1 - \gamma_5) s)^2 | K^0 \rangle$. The evaluation of this matrix element has been the subject of much work, reviewed in [4]. Although the entire range of $B_K$ is $1/3 \leq B_K \leq 1$, the $1/N$ and lattice approaches are generally considered more reliable. They give estimates, which can be stated as

$$B_K = 0.8 \pm 0.2. \quad (24)$$

We now turn to $B_s^0 - \bar{B}_s^0$ mixing. The value of $x_d$, which we use in our analysis, is

$$x_d = 0.68 \pm 0.09. \quad (25)$$

This is slightly different from the value $x_d = 0.71 \pm 0.07$ quoted in [18], but certainly consistent with it. The mixing parameter $x_d$ is calculated from the $B_s^0 - \bar{B}_s^0$ box diagram. Unlike the kaon system, where the contributions of both the $c$- and the $t$-quarks in the loop are important, this diagram is dominated by $t$-quark exchange:

$$x_d \equiv \frac{(\Delta M)_B}{\Gamma} = \tau_B \frac{G_F^2}{6\pi^2} M_W M_B \left( f_{B_s}^2 B_{B_s} \right) \eta_B \epsilon_d \epsilon_d^\ast |V_{td}^\ast V_{ub}|^2, \quad (26)$$

where, in the Wolfenstein parametrization, $|V_{td}^\ast V_{ub}|^2 = \lambda^2 \left( 1 - \rho \right)^2 + \eta^2$. Here, $\eta_B$ is the QCD correction. In ref. [37], this correction is analysed in great detail, including the effects of a heavy $t$-quark. They find that $\eta_B$ depends sensitively on the definition of the $t$-quark mass, and that, strictly speaking, only the product $\eta_B \epsilon_d \epsilon_d^\ast (y_t)$ is free of this dependence. In the fits presented here we use the value $\eta_B = 0.55$, following ref. [37].

For the $B$ system, the hadronic uncertainty is given by $f_{B_s}^2 B_{B_s}$, analogous to $B_K$ in the kaon system, except that in this case, also $f_{B_s}$ is not measured. And, just like $B_K$, the evaluation of $f_{B_s}^2 B_{B_s}$ has been the subject of much work, reviewed in [4]. Until very recently the scaling law

$$f_B(m_B) \sqrt{m_B} = \text{constant}, \quad (27)$$

was thought to be valid for the $B$ system. This led to rather small values for $f_{B_s}^2 B_{B_s}$ in the range

$$100 \text{ MeV} \leq f_{B_s} \sqrt{B_{B_s}} \leq 170 \text{ MeV.} \quad (28)$$

However, recent lattice calculations have indicated that there are significant scaling violations. These have led to larger estimates for $f_{B_s}^2 B_{B_s}$, although the final number is still in a state of flux. We will use the following values in our analysis

$$f_{B_s} = 180 \pm 50 \text{ MeV},$$
$$B_{B_s} = 1.0 \pm 0.2. \quad (29)$$

The errors are large enough to cover most present lattice-based numbers.
13

We would also be very helpful. We will return to a quantitative discussion of these points in the next sections.

Likewise, rare B decays $B \rightarrow p + 1$, $B \rightarrow \ell + \gamma$, $B \rightarrow p + \ell^+\ell^-$ etc.

A good measurement of $x_d$ (and hence $\tau_d$) will be very effective in narrowing down the allowed range for $p$. Likewise, rare B decays $B \rightarrow p + 1$, $B \rightarrow \omega + \gamma$, $B \rightarrow p + \ell^+\ell^-$ etc.

It is therefore qualitatively clear that a good measurement of $x_d$ (and hence $x_d/z_d$) will be very effective in narrowing down the allowed range for $p$. Likewise, rare B decays $B \rightarrow p + 1$, $B \rightarrow \omega + \gamma$, $B \rightarrow p + \ell^+\ell^-$ etc.

would also be very helpful. We will return to a quantitative discussion of these points in the next sections.

Table 1: Parameters used in the CKM fits.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Input Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>V_{cb}</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
</tr>
<tr>
<td>$\tau(B)$</td>
<td>1.49 ± 0.04 (ps)</td>
</tr>
<tr>
<td>$</td>
<td>\epsilon_K</td>
</tr>
<tr>
<td>$x_d$</td>
<td>0.68 ± 0.09</td>
</tr>
<tr>
<td>$m_t$</td>
<td>164 ± 27 GeV</td>
</tr>
<tr>
<td>$B_K^{-}$</td>
<td>0.8 ± 0.2</td>
</tr>
<tr>
<td>$f(B_d)$</td>
<td>1.0 ± 0.2</td>
</tr>
<tr>
<td>$f(B_d)$</td>
<td>180 ± 50 MeV</td>
</tr>
</tbody>
</table>

5 A Profile of the (db) Unitarity Triangle

We shall concentrate on the (db) triangle, shown in fig. 2, in this section. In order to find the allowed unitarity triangle, the computer programme MINUIT was used in ref. [4] to fit the CKM parameters $A$, $\rho$, and $\eta$ to the experimental values of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $|\epsilon|$, and $x_d$. These fits have since then been updated and the results are being included here. The parameters used in this analysis have been discussed in the previous section and their input values from experimental measurements or theoretical estimates are listed in table 1. The results of the fit themselves are shown in fig. 5. In all these graphs, the solid line has $\chi^2 = \chi^2_{\text{min}} + 1$, which corresponds to 39% confidence level region [8]. For comparison, we include the dashed line, which is the 90% C.L. region ($\chi^2 = \chi^2_{\text{min}} + 4.6$). (Note that, strictly speaking, these fits do not represent the true sense of confidence levels since some of the errors shown in table 1 are theoretical, and theoretical “errors” are not Gaussian! However, this method is not unreasonable, being quite similar to the practice of adding experimental statistical and systematic errors in quadrature.) It is clear that, as we pass from fig. 5(a) to fig. 5(c), the “most likely” unitarity triangle becomes more and more acute. However, it is also clear that there is a substantial overlap between the 90% C.L. regions. The “best values” of the parameters ($\rho$, $\eta$), corresponding to the triangles drawn in the figures, together with their $\chi^2$, are given in table 2.

We remark that the range of $\eta$ is well constrained by the very accurate measurement of $|\epsilon|$, and it depends only modestly on $x_d$ (and hence $f(B_d)$) or, as we are going to argue in the next section, on the ratio $x_s/x_d$. It should also be remarked that if the constraint from $|\epsilon|$ is removed from this analysis, then the present data are not enough to prove that indeed one has a triangle in the $\rho - \eta$ plane. The value of $\rho$ is strongly affected by the $x_d$ measurement, which in turn depends strongly on both $m_t$ and $f(B_d)$. It is therefore qualitatively clear that a good measurement of $x_s$ (and hence $x_s/x_d$) will be very effective in narrowing down the allowed range for $\rho$. Likewise, rare B decays $B \rightarrow \rho + \gamma$, $B \rightarrow \omega + \gamma$, $B \rightarrow \rho + \ell^+\ell^-$ etc. would also be very helpful. We will return to a quantitative discussion of these points in the next sections.
Figure 5: Allowed region in $\rho$-$\eta$ space for different values of the Standard Model parameters given in table 1. The three figures correspond to $m_t = 140$, 165, and 190 GeV, respectively. The solid line represents the region with $\chi^2 = \chi^2_{\min} + 1$; the dashed line denotes the 90% C.L. region. The triangles show the best fit (updated from [4].)
15 OCR Output

the Standard Model.

motivated the necessity for measuring $\omega$, we will now turn to an estimate of its size in
theoretical predictions, the results are clearly consistent with each other within ±10%. Having
these calculations are all very encouraging and though there is still some spread in theo-
by Narison gives [41]:

calculation of the pseudoscalar coupling constant ratios in the QCD sum rule approach
over a number of lattice calculations gives $g_s / g_p = 1.08 ± 0.06$ [40]. Likewise, a recent
Along the same lines, Abada et al. [39] quote $g_{\omega} / g_\pi = 1.06 ± 0.04; a recent average
ratio $g_{\omega} / g_\pi$.

$\tau_B \gamma M_B (f_{\omega} B) / f_{\pi} B$.

Mixing in the $B^0 - B^0$ system follows quite closely that of the $B_s^0 - B_s^0$ system. The $B^0 - B^0$
box diagram is again dominated by t-quark exchange, and the mixing parameter $x_s$ is

given by a formula analogous to that of eq. (26):

$x_s \equiv (\Delta M)_{B_s} / t_{B_s} = \tau_B \gamma M_B (f_{\omega} B) / f_{\pi} B (f_{\omega} B) / f_{\pi} B$.

Using the fact that $|V_{cb}| = |V_{ts}|$ (eq. (12)), it is clear that one of the sides of the unitarity
triangle, $|V_{td}/\lambda V_{cb}|$, can be obtained from the ratio of $x_d$ and $x_s$:

$\frac{x_d}{x_s} = \frac{\tau_B \gamma M_B (f_{\omega} B) / f_{\pi} B \cdot \gamma V_{td}^2}{\tau_B \gamma M_B (f_{\omega} B) / f_{\pi} B \cdot \gamma V_{ts}^2}$.

All dependence on the t-quark mass drops out, and we are left with the square of the ratio of CKM matrix elements, multiplied by a factor that reflects $SU(3)$ flavour-breaking effects. The only real uncertainty in this factor is the ratio of hadronic uncertainties – the other quantities will be either calculated or measured. Whether or not $x_s$ can be used to help constrain the unitarity triangle will depend crucially on the theoretical status of the ratio $f_{\omega} B / f_{\pi} B$.

Lattice calculations indicate that the ratio $f_{\omega} B / f_{\pi} B$ could be calculated much more accurately than either $f_B$ or $f_{\pi} B$, due to the cancellation of some systematic uncertainties. For example, ref. [38] gives

$f_{\omega} B = 188-246$ MeV,
$f_{\pi} B = 204-241$ MeV.

Along the same lines, Abada et al. [39] quote $f_{\omega} B / f_{\pi} B = 1.06 ± 0.04; a recent average
over a number of lattice calculations gives $f_{\omega} B / f_{\pi} B = 1.08 ± 0.06$ [40]. Likewise, a recent
calculation of the pseudoscalar coupling constant ratios in the QCD sum rule approach
by Narison gives [41]:

$f_{\omega} B / f_{\pi} B = 1.16 ± 0.05$.

These calculations are all very encouraging and though there is still some spread in theoretical predictions, the results are clearly consistent with each other within ±10%. Having motivated the necessity for measuring $x_s$, we will now turn to an estimate of its size in
the Standard Model.

6 $B^0_s - \bar{B}^0_s$ Mixing and the CKM Unitarity Triangle

Table 2: “Best Values” for the Wolfenstein parameters $\rho$ and $\eta$ following from the analysis based on fig. 5.

<table>
<thead>
<tr>
<th>$m_t$ (GeV)</th>
<th>$f_{B_d}$ (MeV)</th>
<th>$B_{B_d}$</th>
<th>$(\rho, \eta)$</th>
<th>$\chi^2_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>180 ± 50</td>
<td>1.0 ± 0.2</td>
<td>(-0.19, 0.29)</td>
<td>0.04</td>
</tr>
<tr>
<td>165</td>
<td>180 ± 50</td>
<td>1.0 ± 0.2</td>
<td>(0.14, 0.31)</td>
<td>0.19</td>
</tr>
<tr>
<td>190</td>
<td>180 ± 50</td>
<td>1.0 ± 0.2</td>
<td>(0.21, 0.27)</td>
<td>0.13</td>
</tr>
</tbody>
</table>
6.1 Standard model prediction for $x_s$

The SM expression for $x_s$ is given in eq. (30). Using eq. (12) to substitute for $V_{ts}V_{tb}$, we obtain

$$x_s = \tau_{B_s} \frac{G_F^2}{6\pi^2} M_W^2 M_{B_s} \left( f_{B_s}^2 B_{B_s} \right) \eta_{B_s} A^2 \lambda^4 y_t f_2(y_t).$$

(34)

The main uncertainties in this equation are again the $t$-quark mass and $f_{B_s}^2 B_{B_s}$. The lifetime of the $B_s$ meson has now been measured at LEP. We give below the LEP average for the $B_s^0$ and $B_s^0$ lifetimes [18]:

$$\tau_{B_s^0} = (1.26^{+0.22}_{-0.17}) \times 10^{-12} \text{ sec}$$

$$\tau_{B_s^0} = (1.48 \pm 0.10) \times 10^{-12} \text{ sec}.$$  

(35)

Within the measurement errors, both these values are consistent with the averaged value of $\tau_B$ used in the previous section. The mass of the $B_s^0$ mesons has also now been measured and its present world average is $\langle M_{B_s} \rangle = 5373.2 \pm 4.2$ MeV [18], thus removing one of the unknowns. We will also take the QCD correction $\eta_{B_s}$ to be equal to its $B_d$ counterpart, i.e. $\eta_{B_s} = 0.55$. Using the central value for $A$ in eq. (17), we obtain

$$x_s = \left(173\right) \frac{f_{B_s}^2 B_{B_s}}{(1 \text{ GeV})^2} y_t f_2(y_t).$$

(36)

The function $y_t f_2(y_t)$ is numerically equal to 2.03 for $m_t = 150$ GeV. As for $f_{B_s}^2 B_{B_s}$, as mentioned earlier, there is some controversy regarding its value. We use a value with a sufficiently large error:

$$f_{B_s} \sqrt{B_{B_s}} = 210 \pm 50 \text{ MeV}.$$  

(37)

This leads to

$$x_s = (7.6 \pm 3.5) y_t f_2(y_t),$$

(38)

The "central values" (taking $m_t = 150$ GeV) lies in the range:

$$x_s = 15.4 \pm 7.1,$$  

(39)

giving $x_s \simeq 8-24$. The Standard Model therefore predicts very large values for $x_s$. This is to be expected since, from eq. (31), one has $x_s/x_d \simeq |V_{ts}/V_{td}|^2$, which in our estimates lies between 11 and 30, apart from $SU(3)_{\text{flavour}}$-breaking effects. Using the central value for $x_d = 0.68$, used above, this gives $x_s \simeq 8-21$. Due to these large values, time-dependent measurements are necessary to obtain $x_s$. Present measurements at LEP and elsewhere yield a theoretically uninteresting lower bound: $x_s > 0.8$ at 90% C.L. [42]. Time-dependent measurements on $x_s$ at LEP will be exploring the lower fringe of the expected domain of $x_s$. Luckily, several proposals for doing dedicated $B$ physics in hadronic collisions, such as will be carried out at the LHC and HERA-B experiments, are capable of reaching the entire $x_s$ range we have presented.
Table 3: The “best values” of the CKM parameters ($\rho, \eta$) and the ratio $|V_{td}/V_{ts}|^2$, obtained by a minimum $\chi^2$ fit of the experimental data discussed in the text, for 3 different values of $m_t$. Note that $x_s = 15.0 \pm 3.0$ has been assumed for the fits. The resulting minimum $\chi^2$ values from the MINUIT fits are also given.

| $m_t$ (GeV) | ($\rho, \eta$) | $|V_{td}/V_{ts}|^2$ | $\chi^2_{\text{min}}$ |
|-------------|----------------|---------------------|-----------------|
| 100         | (0.01, 0.37)   | 0.056               | 0.03            |
| 140         | (0.02, 0.34)   | 0.056               | 0.24            |
| 180         | (0.03, 0.33)   | 0.057               | 0.93            |

6.2 CKM fits imposing constraints from $x_s$

In the previous section we have given our CKM fits, using the measured value of $x_d$ and its SM analysis, which depends on $m_t$, the coupling constant product $f_{B_s}^2 B_{B_d}$, and the QCD correction factor $\eta_B$. These fits show that the allowed region in $\rho-\eta$ is enormous, reflecting the large uncertainties in some of the input parameters. We have also argued that much of the uncertainty in the allowed region would disappear if, instead of $x_d$, one used the ratio $x_s/x_d$ to constrain the CKM parameters. The main point is that if this ratio were measured then one could restrict the allowed value of $\rho$. Unfortunately, $x_s$ has not been measured. However, the point can be illustrated by assuming a value for $x_s$, based on our SM estimates of this quantity in the previous section. We assume, for the sake of the CKM fits in this section:

$$x_s = 15.0 \pm 3.0 .$$ \hfill (40)

We now impose the constraints from this hypothetical measurement of $x_s$, together with the measured value for $x_d$, and the lattice QCD estimates of the coupling constant product [43]:

$$\frac{f_{B_s}^2 B_{B_d}}{f_{B_d}^2 B_{B_d}} = 1.19 \pm 0.10 .$$ \hfill (41)

The other constraints from the measured values of $|c|$, $|V_{cd}|$ and $|V_{ub}|$, which we have employed earlier, remain unchanged in these fits. The resulting CKM fits are shown in figs. 6(a)-(c), for the three assumed values of $m_t = 100, 140$ and $180$ GeV. Note that the allowed regions in the ($\rho, \eta$) for the three choices of $m_t$ are hardly distinguishable. The reason is that using the ratio $x_s/x_d$, the residual $m_t$ dependence is very mild. Our “best values” for the parameters ($\rho, \eta$), together with the value of the CKM matrix element ratio $|V_{td}/V_{ts}|^2$, and the $\chi^2$ are given in table 3. It is clear that the measurement of $x_s$ would greatly help in reducing the uncertainty in the unitarity triangle.

7 Rare B-Decays and the CKM Matrix Elements

FCNC $B$ and $K$ decays also provide valuable information on the CKM matrix. We shall concentrate here on the so-called $|\Delta B| = 1$, $\Delta Q = 0$ decays, leading to transitions such
Figure 6: Allowed region in $\rho$-$\eta$ space for different values of the Standard Model parameters, assuming $x_S = 15.0 \pm 3.0$. Figs. (a)-(c) have $m_t = 100$, 140 and 180 GeV, respectively. The solid line represents the region with $\chi^2 = \chi^2_{\text{min}} + 1$; the dashed line denotes the 90% C.L. region. The triangles show the best fit (from [4].)
as $b \to (s,d) + X$, where $X = g, \gamma, \ell^+\ell^-$, and $\nu\bar{\nu}$, as well as the purely leptonic decays $B_s \to \ell^+\ell^-$ and $B_d \to \ell^+\ell^-$. We shall consider the radiative $B$ decays in some detail since the first such decay has recently been measured by the CLEO collaboration with a branching ratio $BR(B \to K^+ + \gamma) = (4.5 \pm 1.0 \pm 0.9) \times 10^{-5}$ [6]. This measurement, together with the upper bound on the inclusive decay rate $BR(B \to \gamma + X) < 5.4 \times 10^{-4}$, also reported by CLEO [7], provide an upper and lower bound on the CKM matrix element ratio $|V_{ts}|/|V_{cb}|$ [44].

We briefly discuss the SM theoretical framework for inclusive and exclusive radiative $B$ decays dominated by the electromagnetic penguins. The inclusive decays are grouped as $B \to X_f + \gamma$, where we use the flavour of the light quark $f = s, d$ in the transition $b \to f$ to characterize the hadronic system recoiling against the photon. Including lowest-order QCD corrections and gluon bremsstrahlung, these decays are described at the parton level by the transitions $b \to f\gamma$ and $b \to f\gamma g$. In calculating the inclusive decay widths, we shall follow the work reported in [44]-[47]. The exclusive decays that are of interest may be grouped in an analogous way into $b \to s$ transitions:

- $B_u \to K^* + \gamma$, $B_d \to K^* + \gamma$,
- $B_s \to \phi + \gamma$,

which we shall also term CKM-allowed, according to the dominant CKM matrix element dependence of their decay rates, and $b \to d$ transitions:

- $B_d \to \rho + \gamma$, $B_d \to \omega + \gamma$, $B_u \to \rho + \gamma$,
- $B_s \to K^* + \gamma$,

which will be called CKM-suppressed.

### 7.1 Effective Hamiltonian for $B \to X_s + \gamma$ and $B \to X_d + \gamma$

The framework to incorporate (perturbative) short-distance QCD corrections in a systematic way is that of an effective low-energy theory with five quarks. It is obtained by integrating out the heavier degrees of freedom, i.e. the top quark and $W^\pm$ bosons.

Before using the unitarity properties of the CKM matrix, the effective Hamiltonian relevant for the processes $b \to f\gamma$ and $b \to f\gamma g$ has the form\(^2\)

$$H_{eff}(b \to f) = \frac{-4G_F}{\sqrt{2}} \left( V_{tb}V_{tb}^* \sum_{j=3}^8 C_j(\mu) O_j(\mu) + V_{cb}V_{cb}^* \sum_{j=1}^8 C_j'(\mu) O_j'(\mu) + V_{ub}V_{ub}^* \sum_{j=1}^8 C_j''(\mu) O_j''(\mu) \right), \quad (42)$$

and $j$ runs through a complete set of operators with dimension up to six; the $C_j(\mu)$ are their Wilson coefficients evaluated at the scale $\mu$.

\(^2\)The treatment here follows closely the one given in [48].
We recall here that \( O_1^{(t)} \) and \( O_2^{(t)} \) represent the colour-singlet and colour-octet four-fermion operators, respectively, obtained from the SM charged-current Lagrangian written in the charge retention form:

\[
O_1' = (\bar{e}_{L\alpha} \gamma^\mu b_{L\beta}) (\bar{f}_{L\alpha} \gamma^\mu c_{L\beta}) \quad \text{and} \quad O_2' = (\bar{u}_{L\alpha} \gamma^\mu b_{L\beta}) (\bar{f}_{L\alpha} \gamma^\mu u_{L\beta}) ,
\]

\[
O_2' = (\bar{e}_{L\alpha} \gamma^\mu b_{L\beta}) (\bar{f}_{L\alpha} \gamma^\mu c_{L\beta}) \quad \text{and} \quad O_2' = (\bar{u}_{L\alpha} \gamma^\mu b_{L\beta}) (\bar{f}_{L\alpha} \gamma^\mu u_{L\beta}). \tag{43}
\]

The remaining operators are the same for all three CKM prefactors, i.e. \( O_j' = O_j' = O_j \) for \( j \neq 1, 2 \). At tree level, the only contribution to \( b \to f \gamma \) comes from the magnetic moment operator

\[
O_7 = \frac{e}{16\pi^2} \bar{f} \sigma^{\mu\nu} (m_b R + m_f L) b F_{\mu\nu}, \tag{44}
\]

where \( L = (1 - \gamma_5)/2 \) and \( R = (1 + \gamma_5)/2 \), and \( e \) is the QED coupling constant. The four-fermion operators \( O_3, ..., O_8 \) and the QCD magnetic moment operator \( O_9 \), which is the gluonic counterpart of \( O_7 \), arise from penguin diagrams in the full theory (before integrating out \( W \) and \( t \)). These operators enter indirectly in \( b \to f \gamma \) decays due to operator mixing and through (virtual and bremsstrahlung) gluon corrections.

Taking into account the unitarity of the CKM matrix, only two of the three combinations \( \lambda_1 \equiv V_{tb} V_{ts}^* \) — or \( \xi_t \equiv V_{tb} V_{ts}^* \) for CKM-suppressed transitions — are independent. In the case of \( b \to s \) transitions, one finds \( |\lambda_u| \ll |\lambda_c|, |\lambda_t| \); therefore, when neglecting terms proportional to \( \lambda_u \), the effective Hamiltonian (42) can be brought into a form proportional to \( \lambda_t \approx -\lambda_c \):

\[
H_{eff}(b \to s \gamma) = -\left( \frac{4G_F}{\sqrt{2}} \lambda_t \sum_{j=1}^{8} C_j(\mu) O_j(\mu) \right). \tag{45}
\]

This makes it clear that measuring the transition \( B \to X_s + \gamma \), inclusively or through exclusive decay modes, amounts to measuring the CKM matrix element \( \lambda_t = V_{ts} V_{ts}^* \).

For \( b \to d \) transitions, where \( \xi_u, \xi_c \) and \( \xi_t \) are all of the same order of magnitude, it is most convenient to choose \( \xi_u \) and \( \xi_c \) as independent CKM factors during the matching for the Wilson coefficients at \( \mu = m_W \) and during the renormalization group evolution to \( \mu \approx m_s \). Since thereby terms of order \( O(m_u^2/m_W^2) \) and \( O(m_c^2/m_W^2) \) (or \( O(m_u^2/m_t^2) \) and \( O(m_c^2/m_t^2) \)) are neglected, the terms proportional to \( \xi_u \) and \( \xi_c \) are multiplied by just the same coefficient functions, i.e. \( C_j'(\mu) = C_j''(\mu) \) [although the corresponding operators are, of course, different, see (43)]. Finally, exploiting again the CKM unitarity constraint \( \xi_u + \xi_c = -\xi_t \), one ends up with

\[
H_{eff}(b \to d \gamma) = -\left( \frac{4G_F}{\sqrt{2}} \left( \xi_t \sum_{j=3}^{8} C_j(\mu) O_j(\mu) - \sum_{j=1}^{2} C_j(\mu) \left\{ \xi_c O_j'(\mu) + \xi_u O_j''(\mu) \right\} \right) \right), \tag{46}
\]

where the Wilson coefficients \( C_j(\mu) \) are precisely the same functions as in (45). Details of the full operator basis, the matching of the Wilson coefficients \( C_j \) at \( \mu \approx m_W \), and the complete leading-logarithmic renormalization group evolution to \( \mu \ll m_W \) can be found in literature [49].
7.2 A determination of $|V_{ts}|/|V_{cb}|$ from electromagnetic penguins

We now discuss the implications of the present measurement of $BR(B \to K^* + \gamma)$ and the eventual measurement of the branching ratio $BR(B \to X_s + \gamma)$ for the SM parameters. A much desired goal at present is to determine $m_t$ either directly or indirectly through the loop corrections in which the top quark dominates. Unfortunately, the measurement of $BR(B \to X_s + \gamma)$ would not be a big help here as the $m_t$-dependence of $BR(B \to X_s + \gamma)$ is rather mild and the QCD scale dependence more pronounced. (For details see [44].) In the SM context, the best use of the measurement of $BR(B \to X_s + \gamma)$, in our opinion, lies in the determination of the CKM matrix element ratio $\lambda_t/|V_{cb}|^2$. The unitarity constraint states that $(\lambda_t/|V_{cb}|)^2 \simeq (\lambda_c/|V_{cb}|)^2 = |V_{cb}|^2$. Since the CKM matrix element $|V_{cb}|$ is known from independent measurements, such as charmed hadron decays, the electromagnetic penguin decays $B \to X_s + \gamma$ provide a quantitative test of CKM unitarity.

In estimates of the inclusive branching ratio for $BR(B \to X_s + \gamma)$ one has to include the contribution of the QCD bremsstrahlung process $b \to s + \gamma + g$ and the virtual corrections to $b \to s + \gamma$, both calculated in $O(\alpha_s^n)$ in ref. [45]. Expressed in terms of the inclusive semileptonic branching ratio $BR(B \to X\ell\nu\ell)$, one finds:

$$BR(B \to X_s + \gamma) = \frac{\alpha |\lambda_t|^2}{\pi |V_{cb}|^2} \frac{|C_7(x_t, m_b)|^2 K(x_t, m_b)}{g(m_c/m_b)(1 - 2/3 g f(m_c/m_b))} \times BR(B \to X\ell\nu\ell), \quad (47)$$

where $x_t = m_t^2/m_W^2$ indicates the explicit $m_t$-dependence in $C_7(x_t, m_b) \equiv C_7(m_b)$, and $g(r) = 1 - 8r^2 + 8r^4 - r^8 - 24r^4\ln(r)$ is the phase-space function for $\Gamma(b \to c + \ell\nu\ell)$. The function $f(m_c/m_b)$ accounts for QCD corrections to the semileptonic decay and can be found, for example, in ref. [50]. It is a slowly varying function of $r$, and for a typical quark mass ratio of $r = 0.35 \pm 0.05$, it has the value $f(r) = 2.37 \pm 0.13$. The contributions from the decays $b \to u + \ell \nu\ell$ have been neglected in the denominator in (47) since they are numerically inessential ($|V_{tb}| \ll |V_{cb}|$). For the semileptonic branching ratio $BR(B \to X\ell\nu\ell)$ one can insert the measured value of about 11%. The Wilson coefficient $|C_7(x_t, m_b)|$ and the function $K(x_t, m_b)$, which is a $K$-factor in the sense of QCD corrections, have been computed in [44]. The resulting branching ratio $^3$ in the SM is estimated as [44]:

$$BR(B \to X_s + \gamma) = (3.0 \pm 0.5) \times 10^{-4} \quad (48)$$

for $m_t$ in the range $100 \, \text{GeV} \leq m_t \leq 200 \, \text{GeV}$. This is to be contrasted with the present upper limit on the exclusive radiative decay, $BR(B \to X_s + \gamma) < 5.4 \times 10^{-4}$ (at 90% C.L.) [7].

The QCD scale dependence of $C_7(x_t, \mu)$ entering through the renormalization of the Wilson coefficients is very pronounced in the presently available leading-logarithmic approximation, as first emphasized in [44] and also confirmed in a recent paper [51]. This uncertainty has been estimated by varying the scale parameter $\mu$ in the range $m_b/2 \leq \mu \leq 2m_b$ and introduces an additional theoretical error of $\pm 1 \times 10^{-4}$ in the above estimates for $BR(B \to X_s + \gamma)$.

---

$^3$obtained by setting the CKM matrix element ratio $|\lambda_t|^2/|V_{cb}|^2$ to 1, according to our present knowledge of the CKM matrix (readily seen e.g. in the Wolfenstein representation [11]).
Since $\alpha$ to a very good approximation, we have set $A_{\alpha} = |V_{ud}|$. 

$1.6 \times 10^{-5} < BR(B \to K^* + \gamma) < 7.4 \times 10^{-5}$. 

The CLEO measurements of the exclusive branching ratio $BR(B \to K^* + \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ yields, at 95% C.L., the following upper and lower limits [7]:

$1.6 \times 10^{-5} \leq BR(B \to K^* + \gamma) \leq 7.4 \times 10^{-5}$. 

Since $|V_{ts}| \approx 1$ to a very good approximation, we have set $\lambda_t = |V_{ts}|$. 

Figure 7: Inclusive branching ratio $BR(B \to X_s + \gamma)$ in the Standard Model as a function of the CKM matrix element ratio $(|V_{ts}|/|V_{cb}|)^2$, with the top-quark mass in the range $100 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$ and the QCD scale parameter $\mu$ in the range $2.5 \text{ GeV} \leq \mu \leq 10 \text{ GeV}$. The derived upper and lower (95% C.L.) bounds from the CLEO measurements [6, 7] are also shown (from [44]).

The dependence of the branching ratio $BR(B \to X_s + \gamma)$ on the CKM matrix element ratio $|V_{ts}|^2/|V_{cb}|^2$ is shown in fig. 7. The two curves starting from the origin delimit the SM expectations with $m_t$ and $\mu$ varied in the range $100 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$ and $2.5 \text{ GeV} \leq \mu \leq 10.0 \text{ GeV}$. An upper bound on the CKM matrix element ratio from the experimental upper limit on the inclusive branching ratio $BR(B \to X_s + \gamma)$ can be readily obtained from fig. 7. For the CLEO upper limit $BR(B \to X_s + \gamma) < 5.4 \times 10^{-4}$ (95% C.L.), the corresponding upper bound is:

$$|V_{ts}|/|V_{cb}| < 1.77.$$  

(49)

It has been argued in [44] that this bound can be improved theoretically, if the QCD-improved photon energy spectrum in the inclusive decay $B \to X_s + \gamma$ is used in the experimental analysis. The detailed calculations of the photon energy (equivalently hadron mass) spectrum in the decays $B \to X_s + \gamma$ are given in [45, 46]. In [44], where the first analysis for the determination of the CKM parameter $\lambda_t$ from the electromagnetic penguins was reported, a theoretically improved upper bound $BR(B \to X_s + \gamma) < 4.8 \times 10^{-4}$ (95% C.L.) was derived based on the CLEO upper bound, giving (see fig. 2):

$$|V_{ts}|/|V_{cb}| < 1.67.$$  

(50)

The CLEO measurements of the exclusive branching ratio $BR(B \to K^* + \gamma)$ = $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ yields, at 95% C.L., the following upper and lower limits [7]:

$1.6 \times 10^{-5} \leq BR(B \to K^* + \gamma) \leq 7.4 \times 10^{-5}$.  

(51)
To convert these experimental bounds into the ones on the CKM matrix element ratio, one has to calculate the exclusive branching ratio $BR(B \rightarrow K^* + \gamma)$. For that purpose we introduce the following exclusive-to-inclusive $B$-decay width ratio:

$$
R(K^*/X_s) \equiv \frac{\Gamma(B \rightarrow K^* + \gamma)}{\Gamma(B \rightarrow X_s + \gamma)}.
$$

The determination of this ratio has been taken up in a number of theoretical papers based on quark model wave-function type estimates [52], QCD sum rules of the older [53] and new-vintage [48, 54, 55], and lattice-QCD [56, 57]. Barring the earlier QCD sum rule results, most other estimates yield (5-15)%. We use the estimates from [44], yielding:

$$
R(K^*/X_s) = (13 \pm 3)\%.
$$

This can be used to predict the branching ratio for the exclusive decay $BR(B \rightarrow K^* + \gamma)$ [44]:

$$
BR(B \rightarrow K^* + \gamma) = (4.0 \pm 1.6 \pm 1.0) \times 10^{-5},
$$

where the first error reflects both the uncertainty in $R(K^*/X_s)$, given above, and in $m_t$ in the range $100 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$; the second error is due to the variation in $\mu$ in the range $2.5 \text{ GeV} \leq \mu \leq 10.0 \text{ GeV}$. The estimate (54) is in very comfortable agreement with the CLEO measurements, $BR(B \rightarrow K^* + \gamma) = (4.5 \pm 1.0 \pm 0.9) \times 10^{-5}$ [6]. This demonstrates that the SM is describing the electromagnetic penguin decays correctly — no mean achievement since we are discussing rates at the level of one per 20,000 $B$ decays!

It is easy to see that a more stringent upper bound on the ratio $|V_{ts}|/|V_{cb}|$ follows at present from the 95% C.L. upper limit on the inclusive branching ratio $BR(B \rightarrow X_s + \gamma)$. The measured branching ratio $BR(B \rightarrow K^* + \gamma)$, however, provides at present the lower bound on the matrix element ratio, since the corresponding experimental lower bound on $BR(B \rightarrow X_s + \gamma)$ is still being awaited. Using $R(K^*/X_s)^{\text{max}} = 16\%$ from the analysis in [44] and the 95% C.L. experimental lower bound from CLEO given in (51), one gets a 95% C.L. lower bound on the inclusive branching ratio:

$$
BR(B \rightarrow X_s + \gamma) \geq 1.0 \times 10^{-4}.
$$

With this lower bound on $BR(B \rightarrow X_s + \gamma)$ and the upper limit quoted in (50) above, one derives the following bounds on the CKM ratio [44]:

$$
0.50 \leq |V_{ts}|/|V_{cb}| \leq 1.67.
$$

This is the first determination of this ratio. The lower bound depends on $R(K^*/X_s)^{\text{max}}$, which is a model-dependent but now converging enterprise. While these estimates are not yet completely quantitative, the point that the electromagnetic penguins in $B$ decays provide a determination of the matrix element $|V_{ts}|/|V_{cb}|$, using the CKM unitarity constraints, has been demonstrated. Once the inclusive branching ratio $BR(B \rightarrow X_s + \gamma)$ has been measured, this model dependence can be largely removed and the route involving the exclusive decay $BR(B \rightarrow K^* + \gamma)$ that we have undertaken here will no longer be necessary. The determination of the CKM matrix element ratio from the electromagnetic penguins in $B$ decays obtained above can be compared with the indirect bounds due to
the CKM unitarity that can be worked out from the fits reported in the Particle Data Group [8]:

$$0.55 \leq |V_{us}|/|V_{cb}| \leq 1.68.$$  \hspace{1cm} (57)

The bounds (56) and (57) are quite comparable, although they have been derived by using very different considerations, which can be taken as a consistency check for the SM normalization of the electromagnetic penguins in $B$ decays. With improved data and theory these tests of the CKM unitarity will become much more precise.

### 7.3 Determining $V_{td}$ from electromagnetic penguins

Whereas in the branching ratio for $B \rightarrow X_s + \gamma$ the CKM matrix dependence was contained in an overall factor $|\lambda|^2$, the case of inclusive $B \rightarrow X_d + \gamma$ decays is somewhat more complicated with respect to the factorization of the CKM parameters. When evaluating the one-loop matrix elements for the parton process $b \rightarrow d\gamma g$, at least the four-quark operators $O_2'$ and $O_2''$ should be kept since their coefficients are of order unity (they contribute in penguin diagrams emitting both a gluon and a photon). These contributions bring to the fore a non-trivial dependence on the CKM matrix elements through terms proportional to $\xi_u$ and $\xi_c$ [see eq. (46)]. Explicit expressions for this dependence on the $\rho$ and $\eta$ parameter in the Wolfenstein representation of the CKM matrix have been evaluated in ref. [47]. The dependence of the branching ratio can be written explicitly as:

$$BR(B \rightarrow X_d + \gamma) = D_1|\xi_t|^2 \left(1 - \frac{1 - \rho}{(1 - \rho)^2 + \eta^2} D_2 - \frac{\eta}{(1 - \rho)^2 + \eta^2} D_3 + \frac{D_4}{(1 - \rho)^2 + \eta^2}\right).$$  \hspace{1cm} (58)

Here we have expressed $\xi_t$ and $\xi_c$ in terms of the Wolfenstein parameters $A$, $\lambda$, $\rho$ and $\eta$:

$$\xi_t = A \lambda^3 (1 - \rho + i\eta),$$  \hspace{1cm} (59)

$$\xi_c = -A \lambda^3.$$

The $m_t$-dependence of the coefficients $D_i$ is rather weak, as shown in table 4. To get the inclusive branching ratio as a function of $m_t$, we vary the CKM parameters $\rho$ and $\eta$ over the presently allowed range of the variables listed in table 1. We have described this procedure in detail earlier. The resulting branching ratio $BR(B \rightarrow X_d + \gamma)$ is shown in fig. 8 as a function of $m_t$, from which one obtains [59]:

$$BR(B \rightarrow X_d + \gamma) = (0.8-3.2) \times 10^{-5}$$  \hspace{1cm} (60)

for the top-quark mass in the range $100 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$. We remark that the $\mu$-dependence of the branching ratio $BR(B \rightarrow X_d + \gamma)$ is very significant, which can again be traced to the very significant QCD renormalization of the Wilson coefficients $C_i(m_W) \rightarrow C_i(\mu)$, in particular $C_7$.

A determination of the Wolfenstein parameters $\rho$ and $\eta$ will follow if one is able to measure the inclusive radiative branching ratios $BR(B \rightarrow X_s + \gamma)$ and $BR(B \rightarrow X_d + \gamma)$. That a number of unknowns, such as the top-quark mass and the QCD scale parameter, drop out from this ratio is obvious from the expressions given earlier. However, from the experimental point of view measuring these inclusive rates is a formidable proposition. It
is perhaps easier to measure the exclusive radiative decays, as CLEO has demonstrated on the example of the decay $B \rightarrow K^* + \gamma$. In the following we shall focus our attention on exclusive decays, such as $B_{u,d} \rightarrow K^* + \gamma$, $B_s \rightarrow \phi + \gamma$, and the corresponding CKM-suppressed modes listed at the beginning of this section, and determine them in a reasonable theoretical framework such as QCD sum rules.

For a generic radiative decay $B \rightarrow V + \gamma$ one defines a transition form factor $F_1(q^2)$ as:

$$\langle V, \lambda | \bar{f} \gamma^\mu q^\nu b | B \rangle = i \epsilon_{\mu\nu\rho\sigma} e^{(\lambda)}_{\nu} p_B^\rho p_V^\sigma 2 F_1^{B \rightarrow V}(q^2),$$

(61)

where $V$ is a vector meson ($V = \rho, \omega, K^*$ or $\phi$) with the polarization vector $e^{(\lambda)}$; and $B$ is the generic $B$-meson $B_{u,d}$ or $B_s$. The vectors $p_B, p_V$ and $q = p_B - p_V$ denote the four-momenta of the initial $B$-meson and the outgoing vector meson and photon, respectively.
With the above definition, the exclusive decay widths are given by \((B = B_u \text{ or } B_d)\):

\[
\Gamma(B \rightarrow K^* + \gamma) = \frac{\alpha}{32\pi^4} G_F^2 |\lambda_t|^2 |F_1^{B \rightarrow K^*}(0)|^2 C_7(x_t, m_b)^2 (m_b^2 + m_t^2) \frac{(m_B^2 - m_{K^*}^2)^3}{m_B^3},
\]

(62)

and by an analogous expression for \(\Gamma(B_s \rightarrow \phi + \gamma)\). The branching ratios of these exclusive decays can again be written in terms of the inclusive semileptonic branching ratio:

\[
BR(B \rightarrow K^* + \gamma) = \frac{6\alpha}{\pi} \frac{|\lambda_t|^2 |C_7(x_t, m_b)|^2 |F_1^{B \rightarrow K^*}(0)|^2}{|V_{cb}|^2 g(m_c/m_b)^2 (1 - 2/3\alpha_s f(m_c/m_b))} \frac{(1 - m_{K^*}^2/m_B^2)^3}{(1 - m_t^2/m_B^2)^3} \times (0.11),
\]

(63)

The ratio of the exclusive-to-inclusive radiative decay widths introduced earlier can now be expressed as \((B = B_u \text{ or } B_d)\):

\[
R(K^* / X_s) = \frac{(1 - m_{K^*}^2/m_B^2)^3 m_B^3 |F_1^{B \rightarrow K^*}(0)|^2}{(1 - m_t^2/m_B^2)^3 m_B^3 K(x_t, m_b)}. \]

(64)

The function \(K(x_t, m_b)\) is almost independent of \(m_t\) and has been estimated as \(K(x_t, m_b) \approx 0.83\), for \(m_t\) in the range 100 GeV \(\leq m_t \leq 200\) GeV [44]. In an analogous way, one defines the exclusive-to-inclusive ratio involving \(B_s\) decays.

Since the same short-distance-corrected coefficient function \(C_7(m_b)\) enters in the Hamiltonian for CKM-allowed (45) and CKM-suppressed (46) modes, the QCD scaling for the two-body decays \(b \rightarrow s + \gamma\) and \(b \rightarrow d + \gamma\) is identical and does not effect the ratio of the decay widths \(\Gamma(b \rightarrow d + \gamma)/\Gamma(b \rightarrow s + \gamma)\). The same applies for the exclusive decays such as \(B \rightarrow \rho + \gamma\) and \(B \rightarrow \omega + \gamma\), in which case also only the magnetic moment operator \(O_7\) contributes and the exclusive decay rates factorize in the CKM factors. From this observation a number of relations between the exclusive decay rates follow in the Standard Model [47]. This is exemplified by the decay rates for \(B_{u,d} \rightarrow \rho + \gamma\) and \(B_{u,d} \rightarrow K^* + \gamma\):

\[
\frac{\Gamma(B_{u,d} \rightarrow \rho + \gamma)}{\Gamma(B_{u,d} \rightarrow K^* + \gamma)} = \frac{|V_{td}|^2 |F_1^{B \rightarrow \rho}(0)|^2}{|V_{ts}|^2 |F_1^{B \rightarrow K^*}(0)|^2} \Phi_{u,d},
\]

(65)

where \(\Phi_{u,d}\) is a phase-space factor:

\[
\Phi_{u,d} = \frac{(m_t^2 + m_d^2)}{(m_t^2 + m_u^2)} \frac{(m_B^2 - m_{K^*}^2)}{(m_B^2 - m_{K^*}^2)}.
\]

(66)

The ratio (65) depends only on the CKM matrix elements and the ratio of form factors, while it is independent of the top-quark mass (and of the renormalization scale \(\mu\)).

The transition form factors have been evaluated in the light-cone QCD sum rule approach [48], yielding:

\[
\begin{align*}
F_1^{B \rightarrow K^* \gamma} & = 0.32 \pm 0.05, \\
F_1^{B \rightarrow \rho(\omega) \gamma} & = 0.24 \pm 0.04, \\
F_1^{B \rightarrow \phi \gamma} & = 0.29 \pm 0.05, \\
F_1^{B \rightarrow K^* \phi} & = 0.20 \pm 0.04.
\end{align*}
\]

(67)
(B_d, B_s) \rightarrow \omega + \gamma \text{ and the } \gamma \text{ decays, } (B_d, B_s) \rightarrow \gamma, \text{ which in the SM provide information on the CKM parameters in the QCD sum rule method. So, the SM dependence shown in fig. 9 should be useful to determine the indicated ratio of the CKM matrix elements.}

For the ratios of the form factors, which are needed to determine the ratios of the CKM matrix elements, the following estimates have been reported in [48]:

\[
\frac{F_{\mu B_\mu - (\rho, \omega) \gamma}}{F_{\mu B_\mu - K^* \gamma}} = 0.76 \pm 0.06, \tag{68}
\]

\[
\frac{F_{\mu B_{\mu S} - K^* \gamma}}{F_{\mu B_{\mu S} - \phi \gamma}} = 0.66 \pm 0.09, \tag{69}
\]

and

\[
\frac{F_{\mu B_{\mu S} - K^* \gamma}}{F_{\mu B_{\mu S} - K^* \gamma}} = 0.60 \pm 0.12. \tag{70}
\]

The eventual measurements of the CKM-suppressed radiative decays \((B_u, B_d) \rightarrow \rho + \gamma, B_d \rightarrow \omega + \gamma \text{ and } B_s \rightarrow K^* + \gamma\) could then be used to determine the CKM parameters. This is illustrated in fig. 9, where the ratio of the branching ratios \(BR(B_d \rightarrow \omega + \gamma)/BR(B \rightarrow K^* + \gamma)\) and \(BR(B_s \rightarrow K^* + \gamma)/BR(B \rightarrow K^* + \gamma)\) are plotted as a function of the CKM matrix element ratio \(|V_{ud}|/|V_{ts}|\). As a check of this method, we note that it predicts \(BR(B \rightarrow K^* + \gamma) = (4.8 \pm 1.5) \times 10^{-5} [48]\), which agrees well with the experimental measurement by CLEO. We expect that the estimates of the ratio of the exclusive branching ratio should, in any case, be more stable against variation of the parameters in the QCD sum rule method. So, the SM dependence shown in fig. 9 should be useful to determine the indicated ratio of the CKM matrix elements.

Apart from the radiative \(B\) decays, there is an entire class of FCNC semileptonic \(B\) decays, \(B \rightarrow X_f + \ell^+ \ell^-\), \(B \rightarrow X_f + \nu \bar{\nu}\), where \(f = d, s\), the purely leptonic decays \((B_d, B_s) \rightarrow \ell^+ \ell^-\) and the \(\gamma \gamma\) decays, \((B_d, B_s) \rightarrow \gamma \gamma\), which in the SM provide information

**Figure 9:** The ratios of the branching ratios \(BR(B_d \rightarrow \omega + \gamma)/BR(B \rightarrow K^* + \gamma)\) and \(BR(B_s \rightarrow K^* + \gamma)/BR(B \rightarrow K^* + \gamma)\) as a function of the CKM matrix element ratio \(|V_{ud}|/|V_{ts}|\). The two curves represent theoretical model uncertainties (from [48]).
on the CKM matrix elements and top-quark mass. Once again, these transitions can be classified as CKM-allowed and CKM-suppressed. Also, the ratios of the exclusive decay branching ratios, such as those involving the decays $B \rightarrow (\rho, \omega) + \ell^+ \ell^-$ and $B \rightarrow K^* + \ell^+ \ell^-$, should have dependence on the CKM matrix elements very similar to what we have shown for the corresponding decays in fig. 9. All of these remarks apply to the short-distance contribution. As is well known, for the FCNC semileptonic decays involving charged leptons, the long-distance contribution is quite important. So, an analysis where the long- and short-distance contributions have been treated coherently is required. The present best limit on the FCNC semileptonic decay is from the UA1 collaboration, giving $BR(B \rightarrow X + \mu^+ \mu^-) < 5.0 \times 10^{-5}$ at 90% C.L. [58], which lies about an order of magnitude away from the SM estimates. Since these and related matters have been reviewed recently in [59], we shall not discuss them any further. The expected rates for a number of FCNC $B$ decays that have been estimated in [59], together with the present experimental bounds and the measured rate for $BR(B \rightarrow K^* + \gamma)$, are given in table 5. We expect that some of these SM predictions will soon be tested in present experiments at CLEO and the Tevatron, but many more will be scrutinized in dedicated $B$ facilities expected to operate in the later part of this decade. With these measurements, the structure of the FCNC processes will be completely pinned down. There are already stringent constraints on models having enhanced FCNC couplings such as motivated by technicolor models.

8 CP Violation in the $B$ System

Mixing and CP violation in the $B$ system can be described in much the same way as in the kaon system. In the $B$ system, the states $|B^0\rangle$ and $|\bar{B}^0\rangle$ are eigenstates of the strong and electromagnetic interactions, but not of the weak interactions, which are responsible for their decay. Taking into account the weak interactions, one writes the $2 \times 2$ Hamiltonian (in the $B^0-\bar{B}^0$ basis)

$$H = M - \frac{i}{2} \Gamma,$$

where the mass matrix $M$ and the decay matrix $\Gamma$ are Hermitian. (Since neutral $B$'s do decay, $H$ itself is not Hermitian.) CPT invariance implies that the diagonal components of $H$ are equal, and if CP is conserved, $M$ and $\Gamma$ are real. Allowing for the possibility of CP violation, diagonalizing the Hamiltonian

$$H = \begin{bmatrix} m & M_{12} \\ M_{12}^* & m \end{bmatrix} - \frac{i}{2} \begin{bmatrix} \gamma & \Gamma_{12} \\ \Gamma_{12}^* & \gamma \end{bmatrix}$$

(72)

yields the eigenstates $B_L$ and $B_H$ ($L$ and $H$ denote the light and heavy states, respectively):

$$|B_L\rangle = \frac{p}{\sqrt{M_{12}^2 - \frac{i}{2} \Gamma_{12}}},$$

$$|B_H\rangle = \frac{q}{\sqrt{M_{12}^2 - \frac{i}{2} \Gamma_{12}}},$$

where

$$p = \frac{1 + \epsilon_B}{1 - \epsilon_B} \sqrt{\frac{M_{12}^2 - \frac{i}{2} \Gamma_{12}}{M_{12}^2 - \frac{i}{2} \Gamma_{12}}}.$$
Table 5: Estimates of the branching fractions for FCNC $B$ decays in the Standard Model for $m_t = 150$ GeV, $\mu = 5$ GeV and $f_{B_d} = 200$ MeV. Note that the CKM-suppressed decays depend quadratically on $|V_{td}|$, and the numbers correspond to $|V_{td}| = 9.2 \times 10^{-3}$. Numbers shown in the third column are present experimental upper limits except for $(B_d, B_u) \to K^* + \gamma$, which has been measured (updated from [59]).

<table>
<thead>
<tr>
<th>Decay modes</th>
<th>$BR$</th>
<th>Experimental upper limits (90% C.L.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(B_d, B_u) \to X_s \gamma$</td>
<td>$3.5 \times 10^{-4}$</td>
<td>$5.4 \times 10^{-4}$ [CLEO][7]</td>
</tr>
<tr>
<td>$(B_d, B_u) \to K^* \gamma$</td>
<td>$(3.5 - 6.5) \times 10^{-5}$</td>
<td>$4.5 \pm 1.0 \pm 0.9 \times 10^{-5}$ [CLEO][6]</td>
</tr>
<tr>
<td>$(B_d, B_u) \to X_d \gamma$</td>
<td>$1.5 \times 10^{-5}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to \rho + \gamma$</td>
<td>$(2.8 - 4.5) \times 10^{-8}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to X_s e^+ e^-$</td>
<td>$1.5 \times 10^{-5}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to X_d e^+ e^-$</td>
<td>$6.5 \times 10^{-7}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to X_s \mu^+ \mu^-$</td>
<td>$8.5 \times 10^{-6}$</td>
<td>$5.0 \times 10^{-6}$ [UA1][58]</td>
</tr>
<tr>
<td>$(B_d, B_u) \to X_d \mu^+ \mu^-$</td>
<td>$3.7 \times 10^{-7}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to K e^+ e^-$</td>
<td>$6.0 \times 10^{-7}$</td>
<td>$5.0 \times 10^{-7}$ [PDG][8]</td>
</tr>
<tr>
<td>$(B_d, B_u) \to K \mu^+ \mu^-$</td>
<td>$6.0 \times 10^{-7}$</td>
<td>$1.5 \times 10^{-4}$ [PDG][8]</td>
</tr>
<tr>
<td>$(B_d, B_u) \to K^* e^+ e^-$</td>
<td>$5.6 \times 10^{-8}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to K^* \mu^+ \mu^-$</td>
<td>$2.9 \times 10^{-6}$</td>
<td>$2.3 \times 10^{-5}$ [UA1][58]</td>
</tr>
<tr>
<td>$(B_d, B_u) \to X_s \nu \bar{\nu}$</td>
<td>$8.0 \times 10^{-5}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to X_d \nu \bar{\nu}$</td>
<td>$3.5 \times 10^{-5}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to K \nu \bar{\nu}$</td>
<td>$6.4 \times 10^{-8}$</td>
<td>-</td>
</tr>
<tr>
<td>$(B_d, B_u) \to K^* \nu \bar{\nu}$</td>
<td>$2.3 \times 10^{-5}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_s \to \gamma \gamma$</td>
<td>$1.5 \times 10^{-8}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_d \to \gamma \gamma$</td>
<td>$6.5 \times 10^{-10}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_s \to \tau^+ \tau^-$</td>
<td>$3.8 \times 10^{-7}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_d \to \tau^+ \tau^-$</td>
<td>$1.7 \times 10^{-8}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_s \to \mu^+ \mu^-$</td>
<td>$1.8 \times 10^{-9}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_d \to \mu^+ \mu^-$</td>
<td>$8.0 \times 10^{-11}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_s \to e^+ e^-$</td>
<td>$4.2 \times 10^{-14}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_d \to e^+ e^-$</td>
<td>$1.9 \times 10^{-15}$</td>
<td>-</td>
</tr>
</tbody>
</table>
In eq. (74), the symbol $\epsilon_B$ has been introduced to indicate the relation between this notation and that used normally in the kaon system. Analogously to the kaon system, a non-zero value of $\epsilon_B$ indicates CP violation in $B^0 - \bar{B}^0$ mixing. However, there is a big difference in the $B$ system — here it is found that $\Gamma_{12} \ll M_{12}$ (recall that $\Gamma_{12} \sim 2M_{12}$ for kaons). This implies that $\epsilon_B$ is much smaller than $\epsilon_K$, so that prospects for the observation of CP violation in the $\Delta B = 2$ transitions of $B^0 - \bar{B}^0$ mixing are virtually hopeless. Luckily for us, CP violation in $B$ decays ($\Delta B = 1$) can be large [60, 61]. (This is another difference between the $K$ and $B$ systems; in the kaon system, CP violation in $K$ decays ($\epsilon'/\epsilon$) is small.)

The most promising signal of CP violation in $B$ decays comes by considering a CP eigenstate final state $f$ to which both $B^0$ and $\bar{B}^0$ can decay. CP violation is manifested in a non-zero value of the time-dependent asymmetry [62]:

$$A_f(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to \bar{f})}. \quad (75)$$

The expression for this asymmetry is

$$A_f(t) = -\text{Im} \alpha_f \sin x_q \frac{t}{\tau_B}, \quad (76)$$

in which $x_q$, $q = d, s$ is the $B^0 - \bar{B}^0_q$ mixing parameter (eqs. (26), (34)), $\tau_B$ is the $B$ lifetime, and

$$\alpha_f = \frac{q}{p} \rho_f, \quad (77)$$

where $\rho_f$ is the ratio of the amplitudes for $\bar{B}^0_q$ and $B^0_q$ to decay to $f$,

$$\rho_f = \frac{|\langle f | H | \bar{B}^0_q \rangle|}{|\langle f | H | B^0_q \rangle|}. \quad (78)$$

If all amplitudes that contribute to the decay $B^0_q \to f$ have the same CKM phase $\phi_D$, then $\rho_f$ is a pure phase:

$$\rho_f = e^{-2i\phi_D}. \quad (79)$$

This is indeed the case, up to corrections from penguin diagrams [63], for all CP eigenstate final states. In addition, since $\Gamma_{12} \ll M_{12}$, $q/p$ is also a pure phase:

$$\frac{q}{p} = \frac{\sqrt{M_{12}^2}}{M_{12}} = e^{-2i\phi_M}. \quad (80)$$

Therefore

$$\alpha_f = e^{-2i(\phi_M + \phi_D)} \Rightarrow \text{Im} \alpha_f = -\sin 2(\phi_M + \phi_D). \quad (81)$$

Note that, although $\phi_M$ and $\phi_D$ each depend on the form chosen for the CKM matrix, the sum $\phi_M + \phi_D$ is convention-independent.

In the Wolfenstein parametrization (eq. (12)), only the matrix elements $V_{ub}$ and $V_{td}$ have large phases. In the following we will ignore all small CKM phases. It is not difficult to relate $\phi_M$ and $\phi_D$ to the CKM matrix elements. Like $x_q$, $M_{12}$ is calculable from the box
simultaneously. We illustrate this in figs. 10(a)–(c) by showing the region in sin 2α–sin 2β–sin 2γ shown in table 6 are correlated. That is, not all values in the ranges are allowed since the CP asymmetries all depend on ρ and η, the ranges for sin 2α, sin 2β and sin 2γ allowed in the Standard Model. The allowed ranges that correspond to each of the figures in figs. 5(a)–(c) are found in table 6. In this table we have assumed that the angle γ is measured in the triangle (fig. 2) as follows.

- **B_d** decays with \( b \rightarrow c \): For this class of decays, we have
  \[
  \alpha_f = \frac{V_{td}}{V_{td}^*}, \quad \text{Im} \alpha_f = -\sin 2\beta.
  \]  
  (84)

  The classic example here is \( B_d \rightarrow J/\Psi K_S \). For this particular final state, since \( J/\Psi K_S \) is a CP-odd state, there is an additional minus sign in the asymmetry.

- **B_d** decays with \( b \rightarrow u \): Here we have
  \[
  \alpha_f = \frac{V_{ub}V_{td}}{V_{ub}^*V_{td}^*}, \quad \text{Im} \alpha_f = \sin 2\alpha.
  \]  
  (85)

  One example of such a decay is \( B_d \rightarrow \pi^+\pi^- \).

- **B_s** decays with \( b \rightarrow u \): In this case
  \[
  \alpha_f = \frac{V_{ub}}{V_{ub}^*}, \quad \text{Im} \alpha_f = -\sin 2\gamma.
  \]  
  (86)

  This angle is the most difficult to measure. One possible decay mode is \( B_s \rightarrow \rho K_S \). Like the final state \( J/\Psi K_S \), \( \rho K_S \) is CP-odd, which changes the sign of the asymmetry. For other suggestions on measuring the angle γ and further references, see [5].

The asymmetries in eqs. (84), (85) and (86) can be expressed straightforwardly in terms of the CKM parameters ρ and η. The 90% C.L. constraints on ρ and η found previously can be used to predict the ranges of sin 2α, sin 2β and sin 2γ allowed in the Standard Model. The allowed ranges that correspond to each of the figures in figs. 5(a)–(c) are found in table 6. In this table we have assumed that the angle β is measured in \( B_d \rightarrow J/\Psi K_S \), and have therefore included the extra minus sign due to the CP of the final state.

Since the CP asymmetries all depend on ρ and η, the ranges for sin 2α, sin 2β and sin 2γ shown in table 6 are correlated. That is, not all values in the ranges are allowed simultaneously. We illustrate this in figs. 10(a)–(c) by showing the region in sin 2α–sin 2β.
Figure 10: Allowed values of the CP asymmetries $\sin 2\alpha$ and $\sin 2\beta$ for different values of the Standard Model parameters given in table 1. Figs. (a)-(c) have $m_t = 140$, 165 and 190 GeV, respectively (updated from [4].)
The next round of experiments in flavour physics will greatly help in improving the precision which inhibit drawing this conclusion at a completely quantitative level. We hope that the present measurements are entirely consistent with the SM, there are enough uncertainties in this analysis is rather rosy for the Standard Model! It is, however, also clear that while the processes, with the latter coming from $K$ and $B$ decays and neutral meson mixings. The results for the CKM-Wolfenstein parameters $\rho$ and $\eta$, shown in terms of the unitarity triangles, are summarized in fig. 5. The present status of the CKM matrix elements themselves is summarized in table 7. This table also includes the constraints from the electromagnetic $B$ penguins measured recently. Estimates of the CKM angles $\alpha$, $\beta$ and $\gamma$ are summarized through the correlations shown in fig. 10. The picture that emerges from this analysis is rather rosy for the Standard Model! It is, however, also clear that while the present measurements are entirely consistent with the SM, there are enough uncertainties which inhibit drawing this conclusion at a completely quantitative level. We hope that the next round of experiments in flavour physics will greatly help in improving the precision.

### Table 6: The allowed ranges for the CP asymmetries $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$, corresponding to the constraints on $\rho$ and $\eta$ shown in figs. 5. Values of $m_t$ and the coupling constant $f_{B_d} \sqrt{B_{B_d}}$ are stated. The range for $\sin 2\beta$ includes an additional minus sign due to the CP of the final state $J/\Psi K_S$ (updated from [4]).

<table>
<thead>
<tr>
<th>$m_t$ (GeV)</th>
<th>$f_{B_d} \sqrt{B_{B_d}}$ (MeV)</th>
<th>$\sin 2\alpha$</th>
<th>$\sin 2\beta$</th>
<th>$\sin 2\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>180 ± 50</td>
<td>-1 - 1</td>
<td>0.15 - 0.97</td>
<td>-1 - 1</td>
</tr>
<tr>
<td>165</td>
<td>180 ± 50</td>
<td>-1 - 1</td>
<td>0.16 - 0.98</td>
<td>-1 - 1</td>
</tr>
<tr>
<td>190</td>
<td>180 ± 50</td>
<td>-1 - 1</td>
<td>0.18 - 0.97</td>
<td>-1 - 1</td>
</tr>
</tbody>
</table>

space allowed by the data, for various values of $m_t$ and $f_{B_d} \sqrt{B_{B_d}}$. From these figures one sees that the smallest values of $\sin 2\beta$ occur only in a small region of parameter space around $\sin 2\alpha \simeq 0.2-0.6$. Excluding this small tail, one expects the CP asymmetry in $B_d \to J/\Psi K_S$ to be at least 30%, which is good news for the $B$-physics community interested in CP violation.

### 9 Summary

One of the most important remaining tests of the Standard Model involves the precise determination of the CKM matrix parameters. The knowledge of the values of these parameters might give us some insight into the mechanism of spontaneous symmetry breaking, particularly as regards the fermion mass matrices. For example, it is known that SM and some unorthodox models, such as Extended Technicolor, fulfil their charge in this respect very differently. New and crucial experiments are needed to settle many of these grey areas. Furthermore, this information would allow us to ascertain whether the CKM matrix is the sole source of CP violation, or if new physics is needed. The CKM triangles provide a convenient framework to interpret present and planned experiments. Presence of new physics, contributing to the various rates and asymmetries discussed in this report, will show itself in these triangles through deviations from the CKM expectations. In view of this we discussed most of the flavour physics in terms of these triangles, equivalently the CKM matrix elements.

We have given here an update of the CKM matrix taking into account CC and FCNC processes, with the latter coming from $K$ and $B$ decays and neutral meson mixings. The results for the CKM-Wolfenstein parameters $\rho$ and $\eta$, shown in terms of the unitarity triangles, are summarized in fig. 5. The present status of the CKM matrix elements themselves is summarized in table 7. This table also includes the constraints from the electromagnetic $B$ penguins measured recently. Estimates of the CKM angles $\alpha$, $\beta$ and $\gamma$ are summarized through the correlations shown in fig. 10. The picture that emerges from this analysis is rather rosy for the Standard Model! It is, however, also clear that while the present measurements are entirely consistent with the SM, there are enough uncertainties which inhibit drawing this conclusion at a completely quantitative level. We hope that the next round of experiments in flavour physics will greatly help in improving the precision.

33
in a number of crucial parameters. In the meanwhile, we have also to learn the mundane aspects – how to calculate coupling constants and form factors from first principles – and the sublime aspects – how to relate fermion masses and mixing parameters in a well-defined theoretical concept. Whether the SM will pass the impending experimental hurdles with flying colours and with its feathers intact can only be answered when the experimental jury has given its verdict. While this is being awaited, the romance between the Standard Model and experiments, which started in 1973 with the neutral current discovery in the Gargamelle bubble chamber, continues!

Acknowledgements
I owe my scientific career largely to the generosity of Professor Abdus Salam. He helped me at a crucial stage in my academic life, without which I would have definitely not been able to pursue a scientific career. I thank him sincerely for his timely help in rather difficult circumstances. I also take this opportunity to thank him for his scientific guidance and advice, which I have had the good fortune to enjoy in all these years.

The work reported here is based mostly on collaborations with Volodya Braun, Christoph Greub, David London, Thomas Mannel and Hubert Simma. I thank them for many valuable inputs and discussions. I also acknowledge the help of David Cassel and David London in making available updates of the CKM matrix elements presented here. Helpful discussions with Michael Danilov, John Ellis, Klaus Honscheid, Boris Kayser, Stephan Narison, Peter Schlein, Henning Schröder and Sheldon Stone are also thankfully acknowledged.
References


[22] H. Schröder (these Proceedings).
[26] These figures are updates made by Dave Cassel and are based on the ARGUS results reported in [19, 23] and the CLEO results reported in [20, 24, 25].


N.G. Deshpande et al., Z. Phys. C40 (1988) 369;


[60] For a review, see I. Bigi, V. Khoze, N. Uraltsev and A. Sanda, in CP Violation, ed.
C. Jarlskog (World Scientific)(1989) 175, and references quoted therein.

[61] R. Aleksan et al., in same Proc. as in ref. [4].