Effects of Tidal Forces on the Beam Energy in LEP


Abstract

The $e^+e^-$ collider LEP is used to investigate the $Z$ particle and to measure its energy and width. This requires energy calibrations with $\sim 20$ ppm precision achieved by measuring the frequency of a resonance which destroys the transverse beam polarization established by synchrotron radiation. To make this calibration valid over a longer period all effects causing an energy change have to be corrected for. Among those are the terrestrial tides due to the Moon and Sun. They move the Earth surface up and down by as much as $\sim 0.25$ m which represents a relative local change of the Earth radius of 0.04 ppm. This motion has also lateral components resulting in a change of the LEP circumference ($C_0=26.7$ km) by a similar relative amount. Since the length of the beam orbit is fixed by the constant RF-frequency the change of the machine circumference will force the beam to go off-center through the quadrupoles and receive an extra deflection leading to an energy change given by $\Delta C_0/C_0 \sim -\alpha_e \Delta E/E$. With the momentum compaction $\alpha_e = 1.85 \cdot 10^{-4}$ for the present LEP optics this gives tide-driven p.t.p. energy excursion up to about 220 ppm, corresponding to $\sim 18.5$ MeV for the $Z$ energy. A beam energy measurement carried out over a 24 hour period perfectly confirmed the effects expected from a more detailed calculation of the tides. A corresponding correction can be applied to energy calibrations.

I. INTRODUCTION

A significant improvement in the precision of the measurements providing the absolute energy calibration of the LEP beam was registered during the 1991 physics run with the availability of transverse beam polarization [1] and the implementation of the resonant depolarization method for the precise determination of the mean beam energy [2]. In particular a systematic uncertainty of $\sim 6.3$ MeV in the $Z$ mass and $\sim 4.9$ Mev in the $Z$ width were quoted [3] for the LEP results. The $Z$ mass error is dominated by the knowledge of the absolute energy scale, while the $Z$ width error stems from uncertainties in the differences between the various center-of-mass energies. The analysis of the 1991 beam energy data suggested that effects other than temperature changes in the dipoles contribute to the overall LEP energy reproducibility and the hypothesis that tidal forces might be responsible for it was anticipated in [4]. Fluctuations in the Energy Calibration data [2] were correlated to gravity variations in the Geneva area related to tidal forces. A rather strong correlation was found (Fig.1) despite the measurements having been taken over two months, [5]. Furthermore a tide related variation of the horizontal beam position was observed later [6], [7]. On the basis of this evidence the LEP Energy Working Group recommended the TidExperiment to be performed during the 1992 LEP run.

![Graph](image)

Figure 1: Correlation between relative beam energy deviations measured over the last two months of the 1991 LEP run and the corrections to the local gravity in the Geneva area ($1 \mu$gal = $10^{-6}$ cm s$^{-2}$).

II. GROUND MOTION AND ACCELERATORS

Alignment tolerances in modern and future accelerators have become more and more critical with the introduction of strong focusing magnetic elements to contain beam phase space and with increasing beam currents implying precise positional requirements to reduce interactions of wake fields with the beam environment [8],[9],[10]. Besides occasional motion from seasonal variations of water content in the soil, natural microseismic disturbances and other effects, Earth tides are the major example of periodic ground motion.
A. Earth Tides

The equilibrium between the gravitational attraction of Moon on Earth and the centrifugal forces results in a quadrupolar deformation of Earth's crust producing two daily bulges. Their amplitudes are modulated during the 29.53 days between two Moon-Earth-Sun conjunctions by the inclination of the ecliptic from its nodal plane (β = 3°28′ ± 1°20′) and by the 48 min 38 s difference between the periods of Lunar and Solar tides resulting in maxima at full and at new moon. These variations together with the ellipticities and oscillations of Earth's and Moon's orbits, equinox precession from Earth's oblateness and other components result in a wide spectrum of periodicities which makes the picture more complex [11]. Two important observables related to the above phenomena, the local gravity variations and the strain tensor describing the lateral motion on Earth's crust associated to that in the vertical direction, can be calculated with computer codes [12, 13].

B. Effects on Accelerators

Due to the strain the horizontal position of the magnetic elements in an accelerator changes periodically with time. The central orbit, defined as the orbit passing in average in the magnetic center of the quadrupoles, and for which the energy information from magnetic measurements is valid, becomes time-dependent. The same occurs to the energy $E_{op}(t)$ of particles circulating on the orbit defined by the operational RF frequency $f_{RF}$ used for the data taking, experiencing a time-dependent additional bending strength from off-axis passage in the quadrupoles and sextupoles. Beam energy changes $\Delta E(t) = E_{op}(t) - E_{idemo}$ are related to strain-driven differences $\Delta C = C_{op} - C_{idemo}$ between the lengths of the two orbits by the momentum compaction factor $\alpha$: 

$$
\left( \frac{\Delta E(t)}{E_{idemo}} \right)_{f_{RF}} = -\frac{1}{\alpha} \frac{\Delta C(t)}{C_e}
$$

where $E_{idemo}$ is the energy of particles on the operational orbit of length $C_e$ in absence of tides. The sign in Equ. (1) indicates that a positive strain (expanding ring) induces a reduction of the beam energy on the operational orbit with the usual notations. Monitoring of Earth's crust strain is quite complicated. A tidal effect simpler to measure is the time-dependent gravity variation $\Delta g(t)$ related to the strain via the coefficient

$$
\alpha_{str} = \frac{\Delta C(t)/C_e}{\Delta g(t)/g_o} < 0
$$

which measures the fraction of gravity change coupling into strain. Here $g_o$ is the unperturbed local gravity. A positive strain involves a negative $g-$variation and, from (1), a negative $E-$variation. Local gravity changes can be monitored with gravity-meters or be calculated to good accuracy by Geophysics codes and correspondent corrections $\Delta g_{corr} = -\Delta g$ applied. Tidal energy variations (1) can then be related to gravity changes:

$$
\left( \frac{\Delta E(t)}{E_{idemo}} \right)_{f_{RF}} = -\frac{\alpha_{str}}{\alpha} \frac{\Delta g(t)}{g_o}
$$

III. THE TIDAL EXPERIMENT

A. Resonant Depolarization

The mean beam energy is determined to great accuracy by measuring the spin tune $\nu_s$ of a polarized beam [2]. A time-varying radial magnetic field makes the spin to precess away from the vertical equilibrium position and a depolarizing resonance occurs at a frequency equal to the spin precession frequency:

$$
S_{dep} = \delta \nu_s S_{rev}.
$$

The depolarizer frequency at the resonance gives the fractional part of the spin tune and the beam energy:

$$
E_{beam} = \frac{m_e c^2}{\alpha_e} \nu_s = 0.4406486 \left( N_z \pm \frac{f_{dep}}{f_{rev}} \right)
$$

where $S_{rev}$ is the beam revolution frequency, $\alpha_e$ the gyromagnetic anomaly and $N_z$ an integer. An uncertainty $\Delta \nu_s = \Delta S_{dep}/S_{rev}$ on the fractional part of the spin tune sets the accuracy on the beam energy to:

$$
\frac{\Delta E}{E} = \frac{\Delta \nu_s}{\nu_s}.
$$

Offline data analysis provides a $\sim 2 \cdot 10^{-5}$ beam energy uncertainty ($\pm 1$ MeV at the Z resonance).

B. Quantitative predictions and Results

A strain amplitude $\Delta C/C_e \sim 4 \cdot 10^{-8}$ associated to a gravity variation $\Delta g/g_o \sim 2.5 \cdot 10^{-7}$ as predicted [12, 13] at LEP coordinates yields a strain coefficient $\alpha_{str} = -16\%$ and moves the Earth surface by up to about 25 cm. A similar relative change occurs in the transverse directions modifying the LEP circumference by $\Delta C \sim 1$ mm and producing, on a 90/60 lattice, ($\alpha_e = 1.85 \cdot 10^{-4}$) a relative beam energy variation of about 220 ppm ($\Delta E_{CM} \sim 18.5$ MeV at the Z resonance). A beam energy tidal dependence can be predicted from (2) where $\alpha_{str}$ is obtained by computer codes [12, 13]:

$$
\frac{1}{\Delta g_{corr}(t)} \frac{\Delta E(t)}{E_{idemo}} = -0.882 \text{ppm/ugal.}
$$

First indications [5, 7] that the beam energy is influenced by tidal forces where spread over a long time span.
To get a more accurate measurement of this correlation a dedicated experiment was carried out in which the beam energy was measured over a 24 hour time period of large tides. In Fig. 2 the measured energy points are plotted against the calculated variation of the gravitational acceleration. The linear fit through these data agrees within the errors with the prediction (5). The time evolution of energy measurements over the 24 hours experiment is shown in Fig. 3 and compared with the behavior predicted from strain evaluations (1).

IV. SUMMARY AND CONCLUSIONS

Fluctuations in the beam energy calibration data from the LEP 1991 run were correlated to local gravity variations from Earth tides. A dedicated experiment precisely measured the LEP electron beam energy over a 24 hour period to confirm the hypothesis of a dependence of the LEP beam energy on tidal Earth's crust deformations. The results agree well with the expected changes from detailed evaluations of tidal effects. This result provides a correction factor for past and future energy calibrations.

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VI. REFERENCES