FRACTION AND GLUONS

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ABSTRACT

I discuss here the dominant role played by gluons in determining energy dependence in multi-GeV hadron diffraction. We will look at the role played by gluons as interpreted by: (1) minijet models; (2) the Feynman-Ravndal “bootstrap”. We will then examine phenomenologically the question of whether the energy dependence of hadronic diffraction might change at multi-TeV energies.

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MINIJET PICTURE

In this model[1,2], hadrons interact via semi-hard collisions of their quark and gluon constituents. The probability of collision at impact parameter $\vec{b}$ is assumed to be

$$P_b(b, s) = W_b(b)\sigma_{ij}^{QCD}(s)$$

$$\sigma_{ij}^{QCD}(s) = \int d\vec{z} \frac{2^{\frac{5}{2}}}{s} F_{ij}(x_1x_2 s^{\frac{1}{2}})\sigma_{ij}(\vec{b})$$

Here, $i,j$ refer to colliding quarks or gluons. $W_b(b)$ is the impact parameter dependence of the interaction probability. The integral in (2) describes the dependence on the longitudinal fractional momenta $x_1, x_2$ of the protons $i,j$ mediating the collision. Consider $p-p$ and $\vec{p}-p$ collisions. For $i=j$ we take

$$W_b(b) = \frac{\alpha^2}{6\pi} (\mu_b^2 - \mu_i^2) K_3(\mu_b^2)$$

corresponding to dipole form factors for each nucleon (or $\vec{p}$).

$$\sigma_{ij}(\tau) = \int dx_1 dx_2 f_i(x_1)f_j(x_2)\delta(x_1x_2 - \tau)$$

Here $f_i$ are the usual structure functions and we take

$$\sigma_{ij}(\vec{b}) = K_{ij} \frac{g^2}{s^{\frac{1}{2}}} G(\vec{b} - m_0^2)$$

The colour factor: $K_{ij} = 1, 2\frac{g_s}{g}, g, 2g, g$. The energy dependence of the cross-section is determined by $m_0$. This is the critical energy scale defining the onset of the semi-hard collisions, which dictate the hadron interactions. For the gluon-gluon interaction, which dominates at high energy, the physics is most transparent when working with the gluon structure function $f_g(x) \sim (1 - x)^n/x^2$. The crucial quantity is $J$, which controls the evolution of the gluon structure at small $x$. In Regge language, $J$ will become the Pomeron intercept.

The factor in Eqs. (1) and (2) determining the energy dependence is

$$\int_{m_0^2}^{\infty} d\tau F_{gg}(\tau) \sim -\int_{m_0^2}^{\infty} d\tau \ln\frac{\tau}{\tau^*} \left(\frac{s}{m_0^2}\right)^{J-1}$$

for large enough $s$, to within logarithms. $F_{gg}$ counts the number of gluons in the colliding hadrons. The number increases rapidly at $x = m_0^2/\sqrt{s}$, and this is the origin of the rising cross-section. More quantitatively, at high enough energy,

$$P_{gg}(\mu_b, s) \sim W_{gg}(b) s^{J-1}$$

For an interaction probability $P_{gg} \ll 1$, the energy dependence of the amplitude for scattering is Pomeron-like $\sim s^{J-1}$.

When the number of gluons becomes large, $P_{gg}$ becomes unity for a critical impact parameter $b_c$ where

$$\text{const. } W_{gg}(\mu_b) s^{J-1} \sim 1$$

Then for $\mu_b$ large enough, const. $(\mu_b)^{2J} e^{-\mu_b b} s^{J-1} \sim 1$

$$b_c = \frac{J-1}{J} \ln s + \mathcal{O}(\ln\ln s)$$

The large number of gluons turns the nucleon into a black disk of radius $b_c$ and at very high energies

$$\sigma_{total} \rightarrow 2\pi \left(\frac{J-1}{J}\right)^2 \ln^2 s/s_0$$

as long as $J > 1$. For $J = 1$, $\sigma_{total} \sim \ln^2(s)$. It should be noted that up to Tevatron energies we can fit data with $J = 1$ or $J > 1$. Taking $f_g \sim x^{-1/2}(1-x)^3$ and $f_g \sim x^{-1}(1-x)^3$ leads to sub-leading $q\bar{q}$ and $qg$ contributions, which we parametrize with log $s$, constant, and $s^{-1/2}$ contributions, consistent with Regge analyses arguments. Fits to the data for $\sigma_{total}, \Re f(0)/\Im f(0)$, 'forward slope' and $d\sigma/dt$ using the eikonal

$$\chi(b, s) = \frac{1}{2} P(b, s) = \frac{1}{2}(P_{gg} + P_{qq} + P_{q\bar{q}})$$

are given in Ref. 2) (1992). The fits yield $J = 1 \sim 0.05$ (with $\epsilon = 0$ not out of the question, especially with the recent CDFD $\sigma_{total} = 71.5 \pm 3.0$ mb for $pp \rightarrow \sqrt{s} = 1.8$ TeV).

Support for this picture of high energy diffraction comes from: (a) measured minijet cross-sections[4], as shown in Fig. 1, and (b) fits to recent $\gamma p$ total cross-section measurements at HERA and lower energies made[5], using this minijet picture and vector dominance for the photon. It is to be noted that the cut-off equivalent to $m_0^2$ is taken by Halzen et al.[9] to be $3/2(m_0)^2s_{ee}$, corresponding to $(r^2)_{pion} \approx \frac{3}{5}(r^2)_{proton}$.

THE FEYNMAN-RVNDAL BOOTSTRAP

The minijet picture uses eikonal unitarization, which is assumed; in this connection it is interesting to look at a description of gluon-dominated high-energy interactions due originally to Feynman as reported by Ravndal[6]. Here the state vector for a proton in $p - p(\vec{p} - p)$ interaction

$$|\Psi\rangle = \int \sum_{n=0}^{\infty} C_n(x_1, \ldots, x_n) x_1 x_2 \ldots x_n dx_1 \ldots dx_n$$

where $C_n$ is the probability amplitude for finding $n$ partons each with momentum fraction $x_i$ ($i = 1, 2, \ldots, n$). Assume that diffractive interaction is due to independent wee partons with probability density

$$|C_n|^2 = \frac{|C_n|^2}{n!} \frac{c}{x_1} \frac{c}{x_2} \ldots \frac{c}{x_n}$$

Then since

$$\langle \psi | \psi \rangle = 1$$

(13a)
\[ 1 = \sum_{n=0}^{\infty} \frac{|C_n|^2}{n!} \ e^{s} \left( \int_{1/t}^{s} \frac{dx}{x^2} \right)^n, \]  
\tag{13b}

Feynman-Ravndal took \( J = 1 \), then \( |C_1|^2 = \rho \). The scattering amplitude, at any impact parameter \( b \), we write as \( S(s,b) = \langle \Psi | S | \Psi \rangle \).

A diffractive scattering of the proton is described by taking each parton with energy \( x s \) to scatter with amplitude \( \sigma(x s, b) \). Then

\[ S(s,b) = \langle \Psi | \sum_{n=0}^{\infty} C_n \sigma(x s, b) \sigma(x s, b) \ldots \sigma(x s, b) dx_1 \ldots dx_n \rangle \]

\[ = \sum_{n=0}^{\infty} \frac{|C_n|^2}{n!} \ e^{s} \left( \int \frac{dx}{x^2} \right)^n, \]  
\tag{14}

therefore,

\[ S(s,b) = e^{s} \int dx \ e^{s(x-b)} \int \sigma(x, b) \ e^{s(x-b)} \ e^{-x} \ e^{-x} \ e^{-x} \ e^{s(x-b)} \]  
\tag{15}

Our proton is made of partons. To be consistent with Eq. (11) we put \( \sigma(x s, b) = S(x s, b) \). This is the bootstrap condition. The solution of (15) is, for \( \epsilon = J = 1 \),

\[ S(s,b) = \frac{1}{1 + F(b) \rho^s}. \]  
\tag{16}

Taking \( F(b) = (\rho b)^2 K_b(\rho b) \) or \( e^{-s} \) yields \( \sigma_{\text{tot}} \to \log^2 s \) and \( F(b) = e^{-s} \rho^2 \), \( \sigma_{\text{tot}} \to \log s \). The forward \( d\sigma/dt \) slope determines \( \mu \) or \( a \). \( d\sigma/dt \) as determined here fits the data well up to near the dip region. It dips at \( |t| \) values which are slightly too small. Adding quark amplitudes as in the above minijet picture could fix this.

We have generalized to the case \( J > 1 \). Then we write, taking logs on both sides of Eq. (15):

\[ -\log S(s,b) \equiv \chi(s,b) = \rho s \int \frac{dx}{(s-x)^{1+\epsilon}} \left( 1 - e^{-x/s} \right), \]  
\tag{17a}

which on differentiating becomes

\[ \frac{d}{ds} \left( e^{-x/s} \right) = \frac{e^{-x/s}}{s^{1+\epsilon}} \left[ 1 - e^{-x/s} \right] \]  
\tag{17b}

which at large enough \( b \) leads to

\[ \chi(s,b) = F(b) \left( \frac{s}{\sigma_0} \right)^\alpha - \frac{c}{2(\epsilon + c)} F^2(b) \left( \frac{s}{\sigma_0} \right)^{2\alpha} + \frac{c}{12(\epsilon + c)} \left( 1 + \frac{3c}{\epsilon + c} \right) F^3(b) \left( \frac{s}{\sigma_0} \right)^{3\alpha} \ldots \]  
\tag{18a}

For \( \epsilon = 0 \),

\[ \chi(b,s) = \ln \left[ 1 + F(b) \left( \frac{s}{\sigma_0} \right)^\alpha \right]. \]  
\tag{18b}

When \( c \ll \epsilon \),

\[ \chi(b,s) \approx F(b) \left( \frac{s}{\sigma_0} \right)^\alpha - \frac{c}{3\alpha} F^2(b) \left( \frac{s}{\sigma_0} \right)^{2\alpha} + \frac{c}{12\alpha} F^3(b) \left( \frac{s}{\sigma_0} \right)^{3\alpha} - \cdots \]  
\tag{18c}

\[ \to F(b) \left( \frac{s}{\sigma_0} \right)^\alpha \text{ as } \epsilon \to 0 \]

In both approaches above, we see a behaviour that leads, in what we might call pole approximation, to

\[ A_{pale}(s,b) = 1 - S_{pale}(s,b) \approx F(b) \left( \frac{s}{\sigma_0} \right)^\alpha. \]  
\tag{19}

The question that QCD must answer it then why is \( \sigma_0 \sim 0.05 \) to 0.1? Since perturbative QCD leads to a value of \( \epsilon \) much larger than \( \sigma_0 \), it is clear that the answer lies in the non-perturbative domain.

**HIGHER ENERGIES**

It is possible that the non-perturbative terms drive the solution at higher energies through the usual integral equation

\[ T(s, t^2) = T_0(s, t^2) + \tilde{K} T(s, t^2) \]  
\tag{20}

where the perturbative ladder is cut-off at some \( k_F = Q_0 \). We model this writing

\[ A(s, t = 0) = C_0 s^\alpha + C_1 \left( 1 - \frac{Q_0}{s} \right)^2 \Theta(s - Q_0) s^{\alpha(0)}, \]  
\tag{21}

where \( C_1, \alpha(0) \) comes from the solution to the cut-off equation (20) (see Fig. 2). We show plots for an eikonalized version (Fig. 3) taking \( Q_0 = 2 \) GeV and \( Q_0 = 10 \) GeV. We add in lower trajectories for the lower energies. If the recently presented cosmic-ray data are definitive in terms of deducing \( \sigma_{\text{tot}} \) (see Fig. 4), then \( Q_0 \gtrsim 10 \) GeV, i.e., the non-perturbative Pomeron will dominate at least up to SSC energies.

**REFERENCES**
