One-loop corrections to three vector boson vertices in the Standard Model

E.N. Argyres¹,², G. Katsilieris², A.B. Lahanas³, C.G. Papadopoulos¹ and V.C. Spanos²

Abstract

We study the one-loop corrections to the three boson vertices $\gamma W^+ W^-$ and $ZW^+ W^-$, and we present expressions for the dipole $\Delta \kappa_V$ and quadrupole $\Delta Q_V$ form factors as a function of $Q^2, m_\text{top}$ and $m_{Higgs}$, where $Q^\mu$ is half of the 4-momentum carried by the neutral bosons. We also discuss the relevance of our results to future experiments, where reactions of the type $f_R \bar{f}_R \to W^+ W^-$ at high $Q^2$ will be analysed and we show that (I) it might be possible, given a good experimental sensitivity, to extract valuable informations for the mass of the top quark and (II) that any measured value of $\Delta \kappa_V$ above $10^{-2}$ will be a signal of new physics.

CERN-TH.6510/92
May 1992

¹CERN TH-Division, CH-1211, Geneva 23, Switzerland
²NRCPs, 'Democritos', Aghia Paraskevi GR-15310, Athens, Greece
³University of Athens, Physics Department, GR-15771, Athens, Greece
1 Introduction.

One of the most crucial, precise and tested predictions of the standard model of electroweak interactions is the magnitude and structure of the three-boson couplings. These couplings are a direct consequence of the gauge symmetry and specifically of the non-abelian nature of the underlying $SU(2)$ Lie algebra. Physically they are related to the static properties of the massive gauge bosons $W^\pm$, namely, the magnetic dipole $\mu_W$ and the electric quadrupole $Q_W$ moments.

In order to test the structure of the 3-boson coupling one has to parametrize it in the most general way, compare it with the experimental data and thus verify or rule out the Standard Model predictions. The most general parametrization leads to 9 form factors [1] among which $\kappa$, and $\lambda$, are the most important, compatible with C-, P- and T- invariance. They are related to the static quantities $\mu_W$ and $Q_W$

$$\mu_W = \frac{e}{2M_W^2}(1 + \kappa, + \lambda, )$$
$$Q_W = -\frac{e}{4M_W^2}(\kappa, - \lambda, )$$

The Standard model at tree order predicts $\kappa, = 1$ and $\lambda, = 0$.

So far there are no severe experimental bounds on $\kappa, $ and $\lambda, $. The most recent analysis on the observed rate of $W^\pm$ events in $pp$ collisions at $\sqrt{s} = 0.63, 1.8$TeV has yielded the following bounds at 95% CL [2]

$$-3.5 \leq \kappa, \leq 3.9$$
$$-3.6 \leq \lambda, \leq 3.5$$

However, forthcoming experiments. LEPII, LHC, SSC and HERA, may measure $\kappa, $ and $\lambda, $ with higher accuracy, of the order of $10^{-2}$ to $10^{-3}$[3].

The precision achievable by future experiments will provide the possibility to test not only the tree order predictions of the Standard Model, but also the radiative corrections, namely the one-loop 3-boson couplings. The value of this test might be of more than academic interest, if one considers the dependence of the one-loop result on the Standard Model unknown parameters, like the top-quark and the Higgs mass, which can then be more or less constrained by the measurement of 3-boson couplings.

In this paper we study the one-loop 3-boson vertices in the framework of the Standard Model and discuss the sensitivity of the results on the top-quark and Higgs mass. The paper is organized as follows. In section 2 we describe the structure of the 3-boson vertex at the order $\kappa, $ and $\lambda, $ are extracted from the one-loop calculations. In subsections 2.1, 2.2 and 2.3 we present the results for gauge boson graph contributions. Higgs and fermions respectively. Finally, in section 3, we discuss our results and their sensitivity to the top mass parameter.

2 Three boson vertices at one loop

The most general $W^+W^-V$ vertex, where V stands for $Z^0, $ and $\gamma, $ with the W’s on mass-shell and $V, $ an off-shell gauge boson, can be written as follows [4] (see fig.1 for notation).

$$\Gamma_{WV} = -ig_W\left[ f [g_{\mu\nu}\Delta_{\mu\nu} + 4 (g_{\mu\nu}\Delta_{\mu\nu} - g_{\mu\nu}\Delta_{\mu\nu})] + 2\Delta_{\mu\nu}(g_{\mu\nu}q_\delta - g_{\mu\nu}q_\delta) + \Delta_{\mu\nu}(g_{\mu\nu}q_\delta - Q^2g_{\mu\nu}q_\delta) + ... \right]$$

where

$$g_\mu = e, g_\delta = e\cos\theta_W$$
$$\Delta_{\mu\nu} = \kappa, + \lambda, - 1$$
$$\Delta_{\mu\nu} = -\lambda, (4)$$

and the ellipses denote C or P violating terms, as well as terms proportional to $Q^2$, which we do not present since we are interested in cases where the neutral bosons couple to massless fermions. The lowest order vertex in the Standard Model corresponds to $f = 1$. $\Delta_{\mu\nu} = \Delta_{\mu\nu} = 0$. At one loop the divergence of $f$ will be absorbed by the charge renormalization.

It is clear from Eq.(3) that in order to extract the one-loop contribution to $\Delta_{\mu\nu}$ and $\Delta_{\mu\nu}$ we need to calculate only the factors $a_i(Q^2)$ ($i = 1, 2, 3$), where

$$\Gamma_{WV}(1 - \text{loop}) = -ig_W\left[ a_1(Q^2)\Delta_{\mu\nu} + a_2(Q^2)q_\delta - g_{\mu\nu}\Delta_{\mu\nu} + a_3(Q^2)\Delta_{\mu\nu}q_\delta \right]$$

then the values of $\Delta_{\mu\nu}$ and $\Delta_{\mu\nu}$ are given by

$$\Delta_{\mu\nu} = \frac{1}{2}(a_1(Q^2) - 2a_2(Q^2) - 2Q^2a_3(Q^2))$$
$$\Delta_{\mu\nu} = \frac{\Delta_{\mu\nu}}{4}a_3(Q^2)$$

Notice that $\Delta_{\mu\nu}$, $\Delta_{\mu\nu}$ are both finite, whereas $a_2$, $a_3$ are separately divergent.

There are three kinds of graphs contributing to $\Gamma_{WV}$, namely
2. Gauge boson graphs.

1. (In this class of graphs we also include the ghost and
Goldstone boson loops associated with the gauge fixing of the local symmetry.)

2. Higgs scalar loop graphs.

3. Fermion loop graphs.

We will now proceed to discuss each of these contributions separately.

2.1 Gauge Boson Graphs.

For our calculations we find it convenient to work in the 't Hooft–Feynman gauge. Thus, unlike the unitary gauge, one has to consider Goldstone boson and ghost contributions. All relevant graphs are shown in fig. 2a.

The most difficult graph to calculate is that of fig. 2a(a), in which three gauge bosons meet in all three vertices. This calculation is presented in the Appendix.

The contributions of the graph of fig. 2a to the $\Delta \kappa^{WW}(Q^2)$ and $\Delta Q^{WW}(Q^2)$ form factors with $\gamma$ or $Z$ are as follows:

1. Graph 2a(a)

$$\Delta \kappa^{WW}(Q^2) = f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (8)$$

$$\Delta Q^{WW}(Q^2) = f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (9)$$

2. Graphs 2a(b)+2a(c)

$$\Delta \kappa^{WW}(Q^2) = f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (10)$$

$$\Delta Q^{WW}(Q^2) = 0 \quad (11)$$

3. Graph 2a(d)

$$\Delta \kappa^{WW}(Q^2) = -f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (12)$$

$$\Delta Q^{WW}(Q^2) = 0 \quad (13)$$

4. Graph 2a(e)

$$\Delta \kappa^{WW}(Q^2) = f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (14)$$

$$\Delta Q^{WW}(Q^2) = f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (15)$$

5. Graphs 2a(f) to 2a(j)

$$\Delta \kappa^{WW}(Q^2) = f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (16)$$

$$\Delta Q^{WW}(Q^2) = 0 \quad (17)$$

6. Graph 2a(k)

$$\Delta \kappa^{WW}(Q^2) = f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (18)$$

$$\Delta Q^{WW}(Q^2) = f_\gamma \left\{ \frac{G_FR^2}{4\pi^2} \int_0^1 dt \int_0^1 \frac{da}{L^2} \right\} \quad (19)$$

In the expressions above $R = M^2 = M_{Zn}^2$, and the functions $f_\gamma$, $f_\delta$, and $L_\gamma$ are defined as follows:

$$\beta = \frac{Q^2}{M^2} \left\{ 24t(1-7t)(1-\gamma^2)+10\gamma+24t^2+8t \right\}$$

$$\lambda = [as \ \beta \ with \ \ R = 0] \quad (20)$$

$$L_\gamma = \frac{Q^2}{M^2} \left\{ 24t(1-7t)(1-\gamma^2)-10\gamma+24t^2+8t \right\}$$

$$L_\gamma = [as \ L_\gamma \ with \ \ R = 0] \quad (23)$$
The graphs (f) to (j) of fig.2a have ultraviolet \(1\) singularities which, however, cancel in the sum. The prefactors appearing in the expressions are given by

\[
V = \gamma : \quad f_a = f_b = f_c = 1 \quad (24)
\]

\[
V = Z : \quad f_a = 1, \quad f_b = -(R - 1), \quad f_c = \frac{1}{2}(2 - R) \quad (25)
\]

Putting \(Q^2 = 0\) and summing up the contributions of all graphs of fig. 2a(a) to 2a(k) we recover exactly the results of reference [3] for the corresponding quantities. In their calculations they used the unitary gauge. This serves as a check of the correctness of our results.

At zero momentum transfer some of the graphs, namely 2a(a) and 2a(d), exhibit infrared singularities in \(\Delta \lambda^{VW}(Q^2)\) for \(V = \gamma\) which however cancel in the sum. At this point we should remark that for \(Q^2 \neq 0\) the graph 2a(a) has an infrared singularity due to the photon exchange which is not cancelled by anything else. This arises from the last term of \(\lambda\), multiplying \(\frac{M_H^2}{s}\), and is given by

\[
-\frac{\alpha Q^2}{\pi M_H^2} \int_0^1 dt \int_0^1 da \frac{1}{\left[ -\frac{s}{M_H^2} t (1-a) + t^2 + \frac{m^2}{M_H^2} (1-t) \right]} \quad (26)
\]

where we have introduced a small photon mass \(m\), for the exchanged photon. In the limit \(m\to 0\) this clearly exhibits an infrared singularity proportional to \(\log \left(\frac{M_H^2}{s}\right)\).

It is well known that these singularities are cancelled in real physical processes if one takes into account the soft photon emission. In principle, the cancellation of infrared singularities introduces a finite part which depends on the process under consideration. This does not, however, invalidate the calculation of the \(\Delta \lambda\) form factors since when right handed fermions are involved, in instance for the process \(f \to W^+W^-\), the main (s-channel) contribution of these couplings is proportional to

\[
\Delta \lambda = -\frac{\alpha}{\pi} \Delta \lambda_Z = \Delta \lambda - \Delta \lambda_Z + O(\frac{M_H^2}{s}) \quad (27)
\]

(see for instance eq.(7a) of reference [6]) where \(s = 4Q^2\) and the difference \(\Delta \lambda - \Delta \lambda_Z\) is free of infrared singularities, and thus independent of the specific process under consideration.

2.2 Higgs Boson Graphs

All graphs with at least a physical Higgs \(H\) circulating in the loop are shown in fig.2b. Notice that for the vertex \(ZW^+W^-\) the additional graphs fig.2b(d1),(d2), not present in the \(\gamma W^+W^-\) case, appear due to the existence of the couplings \(\hat{Z}F_{\mu H}, \hat{Z}H\). The graphs fig.2b(b),(c) yield vanishing contributions to \(\Delta \lambda(Q^2), \Delta Q(Q^2)\).

The total contribution of the graphs displayed in fig.2b to the photon form factors is given below:

\[
\Delta \lambda^{VW}(Q^2) = \frac{G_F M_W^2}{4\pi\sqrt{2}} \int_0^1 dt \int_0^1 da (2t^4 - (\mu^2 + 2)t^3 + (\mu^2 + 4)t^2 - \frac{8Q^2}{M_H^2} t (1-a)(1-t) \frac{1}{L_H} \quad (28)
\]

\[
\Delta Q^{VW}(Q^2) = \frac{G_F M_W^2}{4\pi\sqrt{2}} \int_0^1 dt \int_0^1 da \frac{4t^4(1-t)(1-a)}{L_H} \quad (29)
\]

In these expressions \(\mu^2 \equiv \frac{M_H^2}{s}\) where \(M_H\) is the mass of the Higgs and \(L_H\) is given by

\[
L_H = t^2 + \mu^2(1-t) - \frac{\mu^2}{M_H^2} t^4(1-a) \quad (30)
\]

For the \(ZW^+W^-\) form factors we have

\[
\Delta \lambda^{ZW}(Q^2) = \frac{G_F M_W^2}{4\pi\sqrt{2}} \left\{ \frac{1}{2} \int_0^1 dt \int_0^1 da (1-2R)t^4 - (\mu^2 + 2)(2-R)t^2 \right. \\
+ (8-2R-2R\mu^2 + 2\mu^2)t^3 - \frac{8Q^2}{M_H^2} (2-R)(1-a)(1-t) \frac{1}{L_H} \\
+ \frac{R}{2} \int_0^1 dt \int_0^1 da t^2(1-t) \left[ 2(1-t) + (R-\mu^2)\right] + \mu^2 \\
+ \frac{8Q^2}{M_H^2} t (1-a) - 6a(1-a) \frac{1}{L_H} \\
+ \frac{R}{2} \int_0^1 dt \int_0^1 da \frac{2a^2 t^2}{L_H} \right\} \quad (31)
\]

\[
\Delta Q^{ZW}(Q^2) = \frac{G_F M_W^2}{4\pi\sqrt{2}} \left\{ \frac{(2-R)}{2} \right. \int_0^1 dt \int_0^1 da \frac{4t^4(1-t)(1-a)}{L_H} \\
+ \frac{R}{2} \int_0^1 dt \int_0^1 da \frac{2a^2 t^2}{L_H} \right\} \quad (32)
\]

In these expressions \(R \equiv \frac{M_H^2}{s}\), \(L_H\) is as given before while the denominator \(L_H\) associated with the graphs depicted in fig.2b(d1),(d2) is given by

\[
L_H = (t-1)^2 + R\mu^2(1-a) - \frac{4Q^2}{M_H^2} t^4(1-a) \quad (33)
\]

The \(ZW^+W^-\) form factors for \(Q^2 = 0\) yield exactly the results of reference [5].
2.3 Fermion Graphs

The fermionic contribution to the dipole and quadrupole form factors stem from triangle graphs of the type shown in figure 2c, where a summation over all possible fermion loops is understood. For the calculations we write the relevant interaction Lagrangian as follows:

\[
L = -\frac{g}{\sqrt{2}} W_{\mu}^{-} f_{L} z_{\mu}^{a} f_{L} + (h.c.) + V_{\mu}(g_{L}^{a} f_{L} z_{\mu}^{a} f_{L} + g_{R}^{a} f_{R} z_{\mu}^{a} f_{R})
\]  

(34)

In the above expression \(V_{\mu}\) stands for either a photon or a Z-boson while \(g_{L}^{a}, g_{R}^{a}\) are the left and right handed couplings. The result we get from the graphs of fig.2c is

\[
\Delta Q_{UVW}(Q^2) = c_{1} \frac{G_{F} M_{W}^{2}}{157}\sqrt{2} C_{A} \left\{ \left( -\frac{\alpha}{\epsilon} \right) \int_{0}^{1} dt - \int_{0}^{1} dt_{L_{f}} \left( t_{L_{f}} + t_{L_{f}}(1 + t - \Delta) \right) \right\} 
\]

(35)

\[
\Delta Q_{UVW}(Q^2) = c_{1} \frac{G_{F} M_{W}^{2}}{157}\sqrt{2} C_{A} \left\{ \left[ 8\left( -\frac{\alpha}{\epsilon} \right) \int_{0}^{1} dt - \int_{0}^{1} dt_{L_{f}} \frac{t_{L_{f}}}{L_{f}} \right] \right\}
\]

(36)

where \(\epsilon \equiv \frac{\alpha_{s}}{M_{W}}\) and \(\Delta \equiv \frac{\alpha}{M_{W}}\), while \(L_{f}\) is given by

\[
L_{f} = t_{L_{f}}(1 + \frac{Q_{L}^{2}}{M_{W}^{2}}(1 - a)) - (1 + \epsilon - \Delta) + \epsilon
\]

(37)

The prefactor \(C_{A}\) is the colour factor equal to 3 for quark loops and to 1 for lepton loops, while \(c_{1} = 1, c_{2} = \tan \theta_{W}\). The left and right couplings of the fermions to \(\gamma\) and \(Z\) are given below:

\[
V = \gamma: \left( -\frac{g_{L}^{a}}{\epsilon} \right) = \left( -\frac{g_{R}^{a}}{\epsilon} \right) = Q_{f}
\]

(38)

\[
V = Z: \left( -\frac{g_{L}^{a}}{\sin \theta_{W} \cos \theta_{W}} \right) = \frac{1}{Q_{f} \sin^{2} \theta_{W}}
\]

\[
\left( -\frac{g_{R}^{a}}{\sin \theta_{W} \cos \theta_{W}} \right) = \frac{1}{Q_{f} \sin^{2} \theta_{W}}
\]

(39)

To specify our notation we give the charge \(Q_{f}\) and the weak isospin \(t_{L_{f}}\) for the various fermions in table 1.

<table>
<thead>
<tr>
<th>Fermions</th>
<th>(Q_{f})</th>
<th>(t_{L_{f}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u, c, t)</td>
<td>(\frac{2}{3})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(d, s, b)</td>
<td>(-\frac{1}{3})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(\nu_{\mu, \tau})</td>
<td>(0)</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(e, \mu, \tau)</td>
<td>(\pm\frac{1}{2})</td>
<td>(-\frac{1}{2})</td>
</tr>
</tbody>
</table>

Table 1: Fermion charges and weak isospins.

For our numerical calculations the values of the quark masses of the first two generations as well as those of all leptons are taken equal to zero. In this approximation the contribution of the first two fermion families vanishes owing to the triangle anomaly cancellation relation \(\sum_{f} t_{L_{f}}^{(f)} Q_{f} = 0\), which holds separately for each family. In view of this only the third family contribution need to be taken into account.

A subtlety arises at \(Q^2 = 0\) when the masses \(m_{t}, m_{b}\) of the top and bottom quarks respectively satisfy the relation

\[
M_{W} = m_{t} \pm m_{b}
\]

(40)

Although such top masses are not of practical interest due to the existing experimental bounds on the \(m_{t}\), it should not pass unnoticed that fermion loops exhibit singularities when Eq.(40) is satisfied. Presumably in a real measurable process such as \(e^{-} W^{+} \rightarrow e^{-} W^{+}\) for instance, see fig.3, these divergences will cancel in the cross section if one takes into account the process \(e^{-} W^{+} \rightarrow e^{-} b\) when the invariant mass of the \(b\) system is close to \(M_{W}\) or that of \(e^{-} W^{+} \rightarrow e^{-} W^{+} b\) in the case when the \(W^{+}\) mass is close to \(m_{t}\). This would be the mechanism analogous to that of cancelling the infrared divergences in QED processes. However, lacking the analogue of the Bloch-Nordsieck theorem in this case this is merely a conjecture.

3 Results and conclusions

In order to get numerical estimates of the form factors \(\Delta Q_{V}\) and \(\Delta Q_{W}\) we perform numerically the double integration over the Feynman parameters \(t, a\). More specifically we take the principal part on the first integration (say \(d a\)) since there exist simple poles inside the integration region. We have done it by using suitable numerical routines available in the NAG Fortran Library. Furthermore we have checked these results semi-analytically, i.e. by performing analytically the first integration and then numerically integrating over \(t\), where no pole-structure is present. Finally in one case, Eq.(8), we have checked also the \(t\)-integration with the exact analytic re-
sult. Several tests have been made for special values of the parameters in comparison with previous works.

The parameter values we use are:

\[ \alpha = \frac{1}{120}, \quad M_W = 80.66 \text{GeV}, \quad M_Z = 91.18 \text{GeV}, \quad m_t = 5 \text{GeV}, \quad \sin^2 \theta_W = 0.23 \]  

(41)

In fig. 4 we present the results for \( \Delta \kappa \) and \( \Delta Q_2 \) (solid line) as well as for \( \Delta \kappa = c(Q^2) \Delta x_2 \Delta Q_2 \) (dashed line) as a function of \( Q = \text{sign}(Q^2)|Q^2| \), where \( c(Q^2) = 5Q^2 \frac{\sqrt{Q^2}}{M_Z^2} \). As is well known in processes where real \( \bar{W} \) is produced unitarity may be violated because of the existence of longitudinal polarized \( \bar{W} \). This is not the case if a non-abelian gauge symmetry is present, since it induces delicate cancelations between different graphs. For instance, in the process \( f_f \to H + H^- \) with right-handed fermions the cancelation occurs between \( \gamma \) and \( Z \) exchange graphs and is based on the relation

\[ \kappa_1 - \kappa_2 = 0, \quad \lambda_1 - \lambda_2 = 0 \]  

(42)

which at tree order in the Standard Model is automatically satisfied. At one loop we see that \( \Delta \kappa \) is an increasing function of \( Q^2 \), and it grows like \( \log(Q^2) \). This essentially reflects the fact that the form factor is not by itself a gauge invariant object in the limit \( s \to \infty \), where the local \( SU(2) \times U(1) \) is restored. Nevertheless, the difference \( \Delta \kappa - c(Q^2) \Delta x_2 \Delta Q_2 \) vanishes for asymptotic values of \( Q^2 \), as unitarity (and gauge symmetry) requires. This means that for asymptotic values of \( Q^2 \), the relation Eq. (42) is valid at one loop as well [8]. However, this is not true for intermediate \( Q^2 \) values, as we will see below.

In fig. 3 we show the fermion contribution for \( \Delta \kappa \) (solid line: \( m_{\text{top}} = 150 \text{ GeV} \), short dashed line: \( m_{\text{top}} = 200 \text{ GeV} \) and \( \Delta \kappa - c(Q^2) \Delta x_2 \Delta Q_2 \) (dashed line: \( m_{\text{top}} = 150 \text{ GeV} \), dot-dashed line: \( m_{\text{top}} = 200 \text{ GeV} \)) and the Higgs contribution for \( \Delta \kappa \) (solid line: \( m_{H_{\text{top}}} = 100 \text{ GeV} \), short dashed line: \( m_{H_{\text{top}}} = 150 \text{ GeV} \) and \( \Delta \kappa - c(Q^2) \Delta x_2 \Delta Q_2 \) (dashed line: \( m_{H_{\text{top}}} = 100 \text{ GeV} \), dot-dashed line: \( m_{H_{\text{top}}} = 150 \text{ GeV} \)) as a function of \( Q \). The non-vanishing of the differences of \( \gamma \) and \( Z \) form factors is now evident for \( Q \) up to 1 TeV. This gives rise to the phenomenon of 'unitarity delay' first discussed in reference [9]. Finally in fig. 6 the results for \( \Delta Q_2 \) and \( \Delta Q_2 - c(Q^2) \Delta x_2 \Delta Q_2 \) are presented with the same values of the masses of the top quark and the Higgs scalar as in fig. 3.

Although we do not attempt to give a complete answer, we now focus on the question of whether future experiments like LEPII, NLC, \( e^+ e^- \sqrt{s} = 500 \text{ GeV} \), LHC and SSC, will be able to test the Standard Model predictions on the three boson couplings. For this we present in fig. 7 and 8 the fermion and Higgs contributions for \( \Delta \kappa \), \( \Delta Q_2 \) (solid lines) as well as for \( \Delta \kappa - c(Q^2) \Delta x_2 \Delta Q_2 \) (dashed lines), as a function of \( m_{\text{top}} \) and \( m_{H_{\text{top}}} \), respectively, for energies corresponding to \( \sqrt{s} = 200 \) and 500 GeV. Taking into account the limits presented in reference [10] for

LEPII and NLC.

\[ -0.21 \times 10^{-4} \leq \Delta \kappa \leq 0.22 \times 10^{-4} \text{ (LEPII)} \]
\[ -0.13 \times 10^{-2} \leq \Delta \kappa \leq 0.14 \times 10^{-2} \text{ (NLC)} \]

(43)

we have drawn two solid lines corresponding to the limits for LEPII(NLC) in the first graph of fig. 7(8). For LEPII, the Standard Model predictions are below the expected sensitivity[6] and thus no information can be extracted concerning the top mass. Nevertheless, the measurement of \( \Delta \kappa \) will be very important, since any measured value of it, above the expected sensitivity, will be a straightforward indication of new physics. In contrast for NLC, one can probe top masses of the order of 200-300 GeV, provided that the sensitivity will be of the order of \( 10^{-3} \). This phenomenon should be even stronger in LHC and SSC, but it requires further phenomenological analysis.

The top mass dependence of the one-loop cross section is not of course contained in \( \Delta \kappa \) only. Nevertheless the \( m_{\text{top}} \) contribution to \( f \) (see Eq. (3)) and to \( \Delta Q_2 \) is negligible compared to the \( \Delta \kappa \) one. This seems to be also the case in reference [12] where, using longitudinal \( W \), they essentially pick up the \( \Delta \kappa \) terms. For convenience we give the contributions of gauge boson and Higgs loops which are respectively \(-5 \times 10^{-3} \) and \( 3 \times 10^{-4} \), for \( \Delta \kappa - c(Q^2) \Delta x_2 \Delta Q_2 \) at \( \sqrt{s} = 500 \text{ GeV} \). As is evident from fig. 6 these contributions are much smaller than the fermionic ones. They are also independent of the top mass.

We conclude that three boson couplings provide a very rich field for both experiment and theory, in order to investigate the microcosm beyond the electroweak scale. Standard Model predictions can be tested, in forthcoming experiments, where large \( Q^2 \) will be available and very valuable information on the Standard Model parameters can be extracted. Furthermore, any measurement of these couplings, indicating a value of \( \Delta \kappa \) above \( 10^{-2} \) cannot be explained within the Standard Model and thus offers a possible signal of new physics.

E.N.A and C.G.P acknowledge support from EEC Program, SC1-CT91-0729.
Appendix.

In the 't Hooft-Feynman gauge the graph of fig. 2a when a Z is exchanged between the external W bosons is

\[ F_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \left( \epsilon^{\mu\nu\rho\sigma}V_{\mu}^{W*}(-g^{\rho\sigma}) V_{\nu}^{W*}(-g^{\rho\sigma}) V_{\mu}^{W*}(-g^{\rho\sigma}) \right) \]  

(A-1)

All kinematics and the flow of momentum are exhibited in fig. 1, \( V_{\mu}^{W*} \) are the 3 boson couplings at tree order. The momentum carried by the incoming photon is \( 2Q \) while the momenta of the \( W^+ \) and \( W^- \) gauge bosons are \( p = \Delta - Q, p' = -Q - \Delta \). From Eq. (A-1) we find only the coefficients of the tensor structures \( g_{\mu\nu\Delta\rho} \), \( Q_{\mu\nu}, Q_{\rho\sigma} \) and \( g_{\mu\nu}Q_{\rho\sigma} \). For instance if a term \( Q_{\mu\nu}Q_{\rho\sigma} \) appears in \( F_{\mu\nu} \) it can be ignored, since we are interested only in those terms that contribute to the quadrupole and dipole moments. Their knowledge also to know what \( g_{\mu\nu\Delta\rho} \) coefficient is. The external \( W^+ \) and \( W^- \) gauge bosons are put on their mass shell which means that \( \Delta \cdot \Delta = \Delta^2 + Q^2 = M_W^2 \). Besides, the \( F_{\mu\nu} \) vertex will be contracted by \( \epsilon^\mu_\nu \), \( \epsilon^\nu_\mu \) the polarisation vectors of the \( W^+ \) and \( W^- \) bosons. Therefore we have

\[ \epsilon^\mu_\nu(p) \cdot \Delta^\nu = \frac{\epsilon^\mu_\nu(p) \cdot (p - p')^\nu}{2} = -\epsilon^\nu_\mu(p) \cdot \frac{p'^\nu}{2} = \epsilon^\nu_\mu(p) \cdot Q^\nu \]  

(A-2)

Similarly, \( \epsilon^\nu_\mu(p') \cdot \Delta^\nu = -\epsilon^\nu_\mu(p') \cdot Q^\nu \). Therefore for the purpose of our calculations \( Q_\mu \rightarrow \Delta_\mu \) and \( Q_\nu \rightarrow -\Delta_\nu \). This facilitates the calculations a great deal.

The tensor structure of the numerator Eq. (A-1) is

\[ T_{\mu\nu} = (14 - 8n)k_\mu(k_\nu k_\sigma + 32\Delta_\mu Q_\nu Q_\sigma) + \frac{8(n - 14)}{k_\mu(k_\nu k_\sigma + 8k_\mu Q_\nu Q_\sigma) + (8n - 78)k_\mu Q_\nu Q_\sigma} + (g_{\mu\nu}(k_\rho Q_\sigma - k_\sigma Q_\rho) + (8n - 78)k_\mu Q_\nu Q_\sigma) + (g_{\mu\nu}(k_\rho Q_\sigma - k_\sigma Q_\rho) + (8n - 78)k_\mu Q_\nu Q_\sigma) + (g_{\mu\nu}(k_\rho Q_\sigma - k_\sigma Q_\rho) + (8n - 78)k_\mu Q_\nu Q_\sigma) + (g_{\mu\nu}(k_\rho Q_\sigma - k_\sigma Q_\rho) + (8n - 78)k_\mu Q_\nu Q_\sigma) \]  

where we have omitted terms not contributing to the quantities we are interested in, and \( n \) is the dimension of space-time in a dimensional regularization scheme. With these the \( F_{\mu\nu} \) vertex can be cast in the form

\[ F_{\mu\nu} = -ieg^2 \cos^2 \theta_W \frac{P_{\mu\nu}}{2\pi^2} \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \int_0^1 dy \frac{T_{\mu\nu}}{(k^2 - 2k \cdot l - A^2)^3} \]  

(A-3)

In Eq. (A-3) we have introduced the Feynman variables \( x, y \) and

\[ A^2 = (M_W^2 - Q^2)(x + y) + (M_Z^2 - \Delta^2)(1 - x - y) \]  

(A-4)

\[ l^\nu = Q^\nu(x - y) + \Delta^\nu(1 - x - y) \]  

(A-5)

Performing the \( k \) integration after a lengthy calculation we end up with

\[ F_{\mu\nu} = -ieg^2 \cos^2 \theta_W \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \int_0^1 dy \frac{T_{\mu\nu}}{(k^2 - 2k \cdot l - A^2)^3} \]

(A-6)

\[ s(Q^2, x, y) \] is the charge form factor whose explicit form need not be given. By defining new variables as \( y = \frac{t(1 - a)}{2t} \) and \( x = at \) and taking into account Eq. (6), we find

\[ F_{\mu\nu} = -ieg^2 \cos^2 \theta_W \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \int_0^1 dy \frac{T_{\mu\nu}}{(k^2 - 2k \cdot l - A^2)^3} \]

(A-7)

where

\[ L \equiv -\frac{Q^2}{M_W^2} a(t(1 - a) + t^2) + \frac{M_Z^2}{M_W^2} (1 - t) \]  

(A-8)

and

\[ b(a, t, Q^2) = \frac{Q^2}{M_W^2} \left[ 2a^2(1 - 3a - 7t)(1 - a) - 2t(1 - a) \right] + \frac{M_Z^2}{M_W^2} \left[ 2(1 - 3a)(1 - a) \right] \]

(A-9)
References


[12] W.Beenakker, 'Resummation of 1PI O($\alpha$) fermion loop corrections to $\epsilon\epsilon W^+$ at high energies', CERN-TH.6378/92.

Figure Captions

Figure 1. Kinematics of the three boson vertex.

Figure 2. The one loop Feynman graphs for gauge boson (a). Higgs (b) and fermion (c) contributions.

Figure 3. Feynman graphs for $tb$ and $Wb$ production.

Figure 4. The Q dependence of $\Delta x_V$ and $\Delta Q_V$ for gauge boson contribution, as explained in the text.

Figure 5. The Q dependence of $\Delta x_V$ for fermion and Higgs contributions, as explained in the text.

Figure 6. The Q dependence of $\Delta Q_V$ for fermion and Higgs contributions, as explained in the text.

Figure 7. The $m_{top}$ and $m_{W(130)}$ dependence of $\Delta x_V$ and $\Delta Q_V$ for $\sqrt{s} = 200 GeV$, as explained in the text.

Figure 8. The $m_{top}$ and $m_{W(130)}$ dependence of $\Delta x_V$ and $\Delta Q_V$ for $\sqrt{s} = 500 GeV$, as explained in the text.