Theoretical aspects of low-$Q^2$ physics at HERA

Gerhard A. Schuler

Theory Division, CERN
CH 1211 Geneva 23
Switzerland

Abstract

A general overview of the physics topics of low-$Q^2$ interactions at HERA is given, together with an introduction to the activities of the study group on low-$Q^2$ physics at HERA.

1 Introduction and motivation

Electron-proton collisions at low $Q^2$ offer a rich physics potential. First there is, of course, the wide spectrum of QCD studies ranging from perturbative to non-perturbative physics. Such studies are the basis of the main activities of the present study group and are summarized here in Ref. [1]. Second, besides its interest in its own right, photoproduction at HERA possesses many “applications”. These arise because most particles are photo-produced at HERA: hadron collisions have highest rates at low $Q^2$. Only cross sections of purely electromagnetic reactions such as $ep \rightarrow ep\gamma$ are larger. Reactions to be mentioned are the production of heavy quarks, of gauge bosons, of (light) non-strange hadrons, etc. For example, an increase by about a factor of ten of the world statistics of $D$ mesons is expected at HERA. The study of their rare decays is expected to yield, among others, improved limits on lepto-quarks complementing the ones accessible in searches of direct production [2]. The production of gauge bosons, also dominated by the low-$Q^2$ region, challenges the measurement of the $WW\gamma$ vertex, thereby constraining a possible anomalous $W$ magnetic moment. Because of their large rates, low-$Q^2$ processes also constitute major backgrounds to other physics areas. In either case, reliable estimates of photoproduction reactions are needed, ideally supplemented by suitable event generators. Next-to-leading-order calculations to many reactions are on the way, and also generators have become available in the course of this workshop [3–5].

QCD studies mainly concern the structure of the photon and the proton. In particular, the photon can be used up to equivalent fixed-target energies of about 30 TeV. Possible new effects in photon-nucleon interactions at high energies might be observable. An example would be the diffraction excitation of a high-energy photon into a photon and to a new sextet quark sector [6]. More conventionally, the photon structure function $F_2(x, Q^2)$ can be probed in a new kinematical domain. Photon structure functions are introduced in $e^+e^-$ collisions at high energies and thus far measured for $Q^2$ up to 390 GeV$^2$. At sufficiently inelastic $Q^2$, say $Q^2 > 1$ GeV$^2$, data are available at large $x$ only, $x \lesssim 0.1$. The structure function $F_2$ can be expressed as a sum of quark distribution functions in the photon, in complete analogy to the corresponding decomposition of the proton structure function $F_2(x, Q^2)$. The gluonic component of the photon, quantified through a gluon distribution function, is not directly accessible in two-photon collisions and is poorly constrained so far. Many reactions at HERA depend sensitively on the gluonic content of the photon and offer in turn the prospects for its measurement.

The parton distribution functions (PDF’s) of the proton are conventionally measured in deep inelastic lepton-nucleon scattering. Measurements in “low-$Q^2$ reactions” only recently have been entered global PDF analyses [7–9]. Such reactions are the Drell-Yan process, $pN \rightarrow \mu^+\mu^-X$, and prompt photon production, $p \rightarrow \gamma X$, in hadron-hadron collisions. Heavy-quark production in photo- and muon-proton experiments is a sensitive probe of the gluon density; $J/\psi$ production was recently used by the muon-proton experiment NMC to constrain the gluon [10]. The above-mentioned reactions all take place at HERA, thus complementing PDF measurements in deep inelastic scattering (DIS).

The comparison of individual production mechanisms with data will be the first tests of perturbative QCD, of the parton model, and of the factorization theorem at HERA. Measurements will include one-particle inclusive spectra, jet distributions in $p_T$ and rapidity, as well as distributions of jet jet invariant masses. In contrast to $e^+e^-$ collisions, these processes are produced in a highly non-trivial environment. More detailed tests will be those familiar with from studies in $e^+e^-$, $pp$ and deep inelastic $eN$ scattering: measurements of the strong coupling constant $\alpha_s$, of the triple-gluon coupling, of coherence effects [11] in multiple-parton reactions, etc.

Rather unique is the prospect of measuring the interplay between QCD at short and long distances at HERA. Reactions to be mentioned are the total photon-proton cross section, elastic and diffractive reactions such as $\gamma p \rightarrow ppX$, $pX$, or even two-photon reactions such as $\gamma\gamma \rightarrow XX$. It is worth noting that the photon radiates photons similar to an electron, despite its larger mass and its form factor. This results in two-photon rates at HERA that are comparable to the ones at LEP. Exclusive processes, such as $\gamma p \rightarrow J/\psi p+p$, $p+p$, etc., are large and expected to be measurable at HERA [1]. These low-momentum transfer, high-energy processes are ideally suited for the study of the Pomeron. Finally, the rise of the total photon-proton cross section is related to the small-x behaviour of proton structure functions.

2 Kinematical range of HERA

The kinematical range of electron-proton scattering at HERA ($ep \rightarrow 4X$, $x = s, t, u$) is shown in Fig. 1. The virtuality of the exchanged boson, $Q^2 = -(s - t)$, is denoted by $Q^2$. The invariant mass of the remaining system $X$ is denoted by $W$, $W^2 = m_X^2$. Also shown in Fig. 1 are lines of constant $x$ and $y$, related to $W$ and $Q^2$ by $W^2 - m_y^2 = (1 - x)Q^2/x = (1 - y)ps$, where $\sqrt{s}$ is the c.m. energy. One can see the enormous range in $Q^2$ that is covered at HERA, ranging from about $10^{-23}$ GeV$^2$ to $10^2$ GeV$^2$ in the case of $ep \rightarrow ep$. The lower limit of $Q^2$ increases with $W$: $Q^2 = m_y(W^2 - m_Y^2)/x^2$ for $W \ll \sqrt{s}$.

Inclusive $ep$ scattering is described by three structure functions $F_2, F_3$ (or $F_1$), and $F_2(x, Q^2)$. Several kinematical regions in $W$ and $Q^2$ have to be distinguished. First there is the truly DIS regime. It is approximately given by $Q^2 > 2 \ldots 10$ GeV$^2$ and $W > 2 \ldots 5$ GeV. With the last condition, a proper fragmentation of the hadronic system becomes possible. In the DIS region, the structure function $F_2$ can be expressed in terms of quark and gluon distributions functions for which reliable parameterizations exist.

Then there is the resonance region $W < 2$ GeV. The structure functions in the resonance region were fitted in Ref. [12] for $0.1 < Q^2 < 6$ GeV$^2$, and more recently at SLAC [13]. The parameterizations are well-behaved and can be used down to zero $Q^2$.

The region $2 < W < 5$ GeV has been fitted in Ref. [14] for $0.1 < Q^2 < 2$ GeV$^2$. More recently, parameterizations of the region $Q^2 < 10$ GeV$^2$ and $W > 1.75$ GeV have been obtained in Refs. [15] and [16] by simultaneously fitting data on the photoproduction cross section and on $F_2(x, Q^2)$ at low $Q^2$ (see next section). The ALLM parameterization [16] was shown to smoothly connect to standard DIS parameterizations at $Q^2 \sim 10$ GeV$^2$.

Finally, the region $Q^2 > 10$ GeV$^2$, $W < 2$ GeV can be described by extrapolating parameterizations either of the resonance region towards larger $Q^2$, or of the DIS region down to smaller $W$. Of course, eventually $ln(1 - x)$ terms have to be resummed ($x \rightarrow 1$ for $W \rightarrow m_p$ at fixed $Q^2$).

Besides sophisticated parameterizations of the respective kinematical regions, a “poor man’s prescription” is often sufficient. Standard DIS parameterizations are used throughout the whole region, but corrected so as to show the right behaviour as $Q^2 \rightarrow 0$ [14, 17]:

$$F_2(x, Q^2) \rightarrow F_2(x, Q^2) \left[1 - W^2(Q^2)\right]$$

(1)
$W$ [GeV]

$y = 10^{-1}$
$y = 10^{-2}$
$y = 10^{-3}$

$10.0$  $50.0$  $100.0$

$x = 10^{-3}$  $10^{-4}$  $10^{-5}$  $10^{-6}$  $10^{-7}$

Figure 1: Kinematic range of HERA. The region surrounded by the solid line is the phase space for $ep \to epp$. The horizontal line at $W = 0.938$ GeV is elastic $ep$ scattering. Shown are also selected lines $x$ constant $x$ and constant $y$.

or

$$F_1(x, Q^2) \to F_1(x, Q^2) [1 - \exp \left( -a^2 Q^2 \right)] ,$$

(2)

where

$$W^2(Q^2) = \frac{G_1^2 + \tau G_2^2}{1 + \tau}$$

(3)

and $G_1, G_2$ are the proton form factors, $\tau = Q^2/(4m_p^2)$, and $a^2 = 3.37$ GeV$^{-2}$.

### 3 Photon–proton total cross section and $\gamma p \to pp$

The study of photon–proton cross sections is interesting for the following reasons. First one gains insight into the non-perturbative high-energy behaviour of QCD. High-energy, small-$t$ **hadronic** processes were found to be controlled by single-Pomeron exchange [18]. While this idea is phenomenologically very successful, one still does not know what the Pomeron is and how it is generated from QCD. Further understanding is expected from the study of photon–proton cross sections. An obvious question is whether the photon–nucleon cross section shows the same rise with energy as that observed in hadronic collisions.

Recently, the observation of diffractive air-shower “exotics” [19] and an excess of muons in highly energetic cosmic air showers [20] has been reported. These observations, together with the largeUA1 real part, led to the speculation of an anomalous hadronic behaviour of the photon [6, 21, 22]. One consequence could be a large cross section for diffractive excitation of a high-energy photon into a $Z^0$ at HERA. Last but not least, the high-energy behaviour of the photon–proton cross sections is deeply related to the small-$x$ behaviour of the proton and photon PDFs. Saturation effects or even signs of hot spots within the proton and/or photon might become visible at HERA in $\sigma^{\text{sat}}(\gamma p)$ and $\sigma^{\text{sat}}(\gamma p \to pp)$.

At HERA, one expects photon–proton cross sections to be measurable in a range from about 100 to 250 GeV [1]. Current predictions of the total cross section differ substantially. The estimates can be divided into three categories, based on the parametrization of existing data, the Regge picture, or QCD minijets. In the first approach one extrapolates a fit to existing photoproduction data [23] which exist up to about 18 GeV photon–proton c.m. energy. The fit of the particle data group [21] was updated in Ref. [25] and predicts a cross section of $200 \pm 50 \, \text{pb}$ at $\sqrt{s_{pp}} = 250$ GeV.

A second approach is based on the Regge picture. One tries to fit simultaneously the data of $\sigma^{\text{t}}(W)$ and the proton structure function $F_2(x, Q^2)$ ($W^2 \equiv s_{pp} = (1 - x)Q^2/x + m_p^2$). Both quantities are related via

$$\sigma^{\text{t}}(W) = \frac{4\pi \alpha}{Q^2} F_2(x, Q^2) \bigg|_{Q^2=4m_p^2} .$$

(4)

Motivated by the Regge picture, the real photon cross section is decomposed into a diffractive and a non-diffractive piece, and correspondingly $F_2$ (at any $Q^2$) into a sea and a valence part. The diffractive (or sea) contribution is associated with the exchange of a Pomeron, while the non-diffractive (or valence) part represents the Regge contributions corresponding to $\rho, \omega, \phi$ and $a_2$ exchange. In detail, if

$$\sigma^{\text{t}}(W) = A_p W^{-\alpha} + A_v W^{-\beta} .$$

(5)
then $F_2$ behaves like

$$F_2(x, Q^2) = A_{max} x^{-1} + A_{sat} x^2.$$  \hspace{1cm} (6)

The fits of Refs. [15] and [16] differ by the ansatz to continue the real photon cross section towards non-zero $Q^2$. In Ref. [16], $F_2$ was fitted to

$$F_2 = C_B(t) x_B^{\alpha_B} (1 - x_B^{\alpha_B}),$$

$$F_3 = C_C(t) x_C^{\alpha_C} (1 - x_C^{\alpha_C}).$$  \hspace{1cm} (7)

The variables $x_B$, $x_C$, and $t$ in (7) were chosen to allow a smooth limit $Q^2 \to 0$. The form of $t$ and $x_B(x_C)$ is motivated by perturbation theory, e.g. $1/x_B = 1 + (W^2 - m^2)/(Q^2 + m^2)$, see Ref. [16]. Note, though, that the $Q^2$ dependence enters solely through

$$t = \ln \left( \frac{Q^2 + m^2}{Q^2} \right),$$

which logarithmic $\ln Q^2$ dependence is recovered at large $Q^2$. In contrast, in Ref. [15] extra power-like scaling violation terms are included.

$$F_2(x, Q^2) = 1.33 x^{0.55} (1 - x)^2 \left( \frac{Q^2}{Q^2 + 0.85} \right)^{0.41} + 0.17 x^{-0.08} (1 - x)^2 \left( \frac{Q^2}{Q^2 + 0.36} \right)^{1.08}.$$  \hspace{1cm} (8)

A third estimate [20] in the Regge picture actually specifies the transition of the photon to vector mesons, $\gamma \to V$, and uses the Regge picture for $V p \to V p$. The photoproduction cross section and the structure function $F_2$ are calculated on the basis of a non-diagonal generalization of the vector–meson dominance model. Simultaneously, correct dependences for $\sigma_{\gamma p}(V)$ and $F_2(x, Q^2)$ were achieved by introducing finite-width corrections for the $P$ meson. The meson–nucleon–nucleon cross sections were assumed to sum up in energy according to Lipkin’s formulas [27], $\sigma_{\gamma p}(s) = \sigma_{\gamma p}(s^0) + 0.5 s^{-0.2} + 0.5 s^{-0.5}$. In the limit of large $Q^2$, this leads to an $x$ dependence for $F_2$ similar to (6).

A third approach to calculating the total photon–proton cross section is based on jet cross sections [21, 22, 23, 24–31]. Most naively, the total cross section is given by the additive model defined by

$$\sigma(s) = \sigma_{\gamma p}(s) + \sigma_{\gamma p}(s) + \sigma_{\gamma p}(s).$$  \hspace{1cm} (9)

Here, $\sigma_{\gamma p}(s)$ describes the soft, i.e. non-perturbative, part of the cross section and has to be determined by comparing (9) to the measured cross section at some input energy. The soft cross section is assumed to be energy-independent, such that the rise with energy of (9) is solely driven by the QCD minijet cross sections as a consequence of the rapid increase of the number of gluons both in the proton and in the photon. The resolved photon contribution to the inclusive jet cross section $\gamma + p \to j + X$ gives rise to parton–parton scatterings such as $q + g \to q + g$, just as in the case of hadronic collisions. The direct cross section receives contributions from the $\gamma + g \to q + q\bar{q}$ and $\gamma + q \to q + g$ subprocesses. The minijet cross sections are cut-off by $p_{T, min}$, the minimal transverse momentum down to which the perturbative calculation is assumed to be valid, typically in the 1 GeV range. The small-$x$ behaviour of the gluon density in the photon then drives the high-energy behaviour of the cross section as follows:

$$x F_{2/3}(x) \sim x^{-1}, x F_{2/3}(x) \sim x^{-1} \Rightarrow \sigma_{\gamma p}(s) \sim x^{\alpha_B} (J_B), \sigma_{\gamma p}(s) \sim x^{\alpha_C}, \sigma_{\gamma p}(s) \sim x^{(J_B)}. \hspace{1cm} (10)$$

Table 1: Total photon–proton cross section in pb. The prediction of the minijet-based approach (“Impact picture”) is given for two different parametrizations of the photoproduction distributions, DG [22] and LAC 1 [33].

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$</th>
<th>Data [23]</th>
<th>Extrapolation [25]</th>
<th>Regge picture</th>
<th>Impact picture [33]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s_{NN}} = 10$ GeV</td>
<td>200 ± 30</td>
<td>180</td>
<td>170</td>
<td>145</td>
</tr>
<tr>
<td>$\sqrt{s_{NN}} = 250$ GeV</td>
<td>170-290</td>
<td>310-760</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The cross section estimate (9) is well suited for treatment in Monte Carlo event generators. Yet, Eq. (9) is not the total cross section but rather the total cross section times the jet multiplicity, because (9) is summed over all final states containing the jet(s) that define the cross section. Also, at the large parton multiplicities that arise at high energies, multiple particle scatterings take place and have to be corrected for.

Unitarization can be taken into account by using the impact parameter representation. Then the eikonal is taken to be additive as a soft and a QCD part, rather than the cross section itself, as done in (9). Care must be taken to correctly describe the probability for a photon to convert into a vector meson.

In Ref. [31], a systematic study of the photoproduction cross section based on minijets has been performed. In particular, the parameters of the model and their constraints have been investigated in order to come up with an estimate of the uncertainty of this approach. The main result is that current (photoproduction) data do not restrict the parameters (in particular $p_{T, min}$) well enough to allow for a discrimination of different PDFs of the proton. On the other hand, different small $x$ behaviours of the gluon distributions in the photon result in sufficiently different cross sections. This can be seen from Table 1, where the various predictions [15, 16, 25, 26, 31] of the total photon–proton cross section are compiled. There we also see that the minijet-based prediction is typically larger than the other estimates.

In Ref. [31], it is pointed out that the minijet-based approach predicts also the elastic reaction

$$\gamma + p \to p + p,$$  \hspace{1cm} (11)

when the impact parameter representation is used. In fact, no additional parameters are needed. Furthermore, not only the total elastic cross section is predicted, but also its slope in $t$, i.e. $d\sigma/dt$. Low-energy data on reaction (11) could therefore be used in Ref. [31] to further constrain the parameters of the minijet-based model.

A measurement of reaction (11) at HERA would yield very valuable information about the parton distributions at small $x$. Particularly useful is the $t$ dependence: combined with measurements of the total photoproduction cross section at different energies, also the gluon distribution function of the proton should be accessible.

An alternative estimate of reaction (11) was obtained in Ref. [31] based on single-Pomeran exchange and vector–meson dominance. The Pomeron was taken to be of the Donnachie–Landshoff type [13]. The results of this prediction are typically smaller than the minijet-based ones. A compilation of the various predictions is given in Table 2. In both approaches, the ratio of elastic to total cross section is predicted to increase with energy.
4.1 Weizsäcker–Williams approximation

Electron–proton cross sections at HERA mediated by photon exchange can be expressed in terms of virtual photon–proton cross sections:

\[
\frac{d^2\sigma_{\gamma p}(s)}{dy dQ^2} = A \sigma_{\gamma p}^T(W^2, Q^2) + B \sigma_{\gamma p}^W(W^2, Q^2).\]

(12)

Here \(\sigma_{\gamma p}^T(\sigma_{\gamma p}^W)\) are the cross section for transversely (longitudinally) polarized photons, and \(A\) and \(B\) are known functions of \(y\) and \(Q^2\), and the integration limits are specified by kinematics. A simple and convenient method to calculate the electron–proton cross section is the Weizsäcker–Williams approximation (WWA) [4]. Here \(\sigma_{\gamma p}\) is approximated by the product of the equivalent number of photons in the electron times the real photon–proton cross section:

\[
\frac{d^2\sigma_{\gamma p}(s)}{dy dQ^2} = \frac{\alpha}{2\pi} \frac{1}{Q^2} \frac{1 + (1 - y)\frac{Q^2}{\sqrt{s}}}{y} \sigma_{\gamma p}^W(W^2).\]

(13)

When using the WWA, one should always specify its range of validity. Typically, there exists a scale \(\Lambda^2\) [33] such that for \(Q^2 > \Lambda^2\), the virtual photon–proton cross sections decrease quickly with increasing \(Q^2\). For \(Q^2 < \Lambda^2\), the virtual photon–proton cross sections can be approximated by the respective ones for a real photon, and the pp cross section is logarithmically enhanced. The origin of \(\Lambda^2\) can be either kinematical (e.g. \(W^2\)) and/or dynamical (e.g. \(m^2\)). In either cases the upper \(Q^2\) limit in (13) should be taken as \(\Lambda^2\).

The validity of the WWA was investigated in Ref. [25] for the case of the total (hadronic) cross section at HERA. Modelling the \(Q^2\) dependence by generalized vector–meson dominance, it was shown how the size of the corrections to the WWA increase with \(Q^2_{	ext{max}}\). Since the corrections are negative, a naive use of the WWA would underestimate the true cross section. This feature is true in general. As another example, consider two-jet production. The cross section behaves like \(\sigma \sim (dQ^2/Q^2)(dm^2/m^2)\) at low \(Q^2\), i.e. for \(Q^2 \ll m^2\) where \(m^2\) is the jet-invariant mass. This dependence changes to \((dQ^2/Q^2)(dm^2/m^2)^2\) in the deep inelastic regime (\(Q^2 \gg m^2\)) because of a factor \(F \equiv m^2/(m^2 + Q^2)\) in the differential cross section. This factor, however, is missing in the WWA, because \(F = 1\) for \(Q^2 = 0\).

4.2 Photon structure functions

A real photon, and a time-like photon at low virtuality, behaves rather like a meson [36]. The total \(pp\) cross section and low-energy \(e^+e^-\) annihilation into hadrons are indeed two well known examples where the photon can be viewed as a hadron, as successfully described in the context of vector–meson dominance. At sufficiently high virtuality, the photon is essentially a point-like object, coupling directly to the quarks within the proton. The direct component to the reactions \(\gamma + N \rightarrow \gamma + X\) starts to show up in fixed-target photoproduction experiments [37] at high \(p_T\). At low virtuality one also expects contributions from the partonic constituents of the photons. Recently the AMY collaboration has shown that a non-zero gluon content of the photon is required by their data on jet production in two-photon collisions [38].

The hadronic component of the photon is described by the photon structure functions. These were introduced and tested experimentally in photon–proton collisions at high-energy \(e^+e^-\) colliders [39]. Particularly interesting is the kinematical region in which one of the photons with momentum \(q\) is very far off-shell, \(Q^2 = -q^2 \rightarrow \infty\), and the other photon with momentum \(p\) is close to its mass-shell, \(P^2 = -p^2 \rightarrow 0\). This kinematical region of the two-photon process corresponds to deep inelastic electron scattering off a photon target, \(e\gamma \rightarrow eX\), see Fig. 2. In the limit \(P^2 \ll Q^2 \ll Q^2\), the differential cross section for \(e\gamma \rightarrow eX\) is parametrized by two (real photon) structure functions \(F_2\) and \(F_3^1\) (or \(F_3^2\)), in complete analogy to the cross section of deep inelastic electron–proton scattering. Furthermore, the photon structure functions are functions only of \(Q^2\) and the Bjorken variable \(x = Q^2/(2p \cdot q)\).

In leading order (LO) QCD, the longitudinal-photon structure function vanishes, and \(F_3^3\) is the (charged weighted) sum of the quark and anti-quark distributions in the photon evolved in the leading-logarithmic \((Q^2)\) approximation (LLA).

\[
F_3(x, Q^2) = \sum_q x f_q(x, Q^2) + \sum_{\bar{q}} x f_{\bar{q}}(x, Q^2),
\]

(14)

The gluon distribution only enters through the QCD evolution of the singlet sector. New LO parametrizations of the parton distribution functions of the (real, i.e. \(P^2 = 0\)) photon were obtained in Ref. [10] based on all available two-photon data. For the first time,
the gluon distribution was also fitted from data. Three different parametrizations are provided in order to indicate the uncertainty of current determinations of the photon–parton distributions. In particular the gluon is poorly constrained from present data. Correspondingly, the parametrizations differ mainly by the ansatz for the input gluon distribution.

A consistent treatment of next-to-leading-order (NLO) calculations of photoproduction reactions requires the use of photon–parton distributions evolved beyond the LLA. A recent, an error in the NLO expression for the inhomogeneous gluonic term was pointed out [40]. Parametrizations of the parton distributions of the photon in NLO accuracy are expected in the near future [41, 42].

In principle, the photon structure functions $F_{2p}$ can be probed in $e^+e^-$ collisions when the tagged electron (the positron in Fig. 2) in the subprocess $e^+e^- \rightarrow e^+X$ is replaced by a quark and folded with the distribution of the quark in the proton. One obtains higher cross sections is $e\gamma$ collisions when replacing the probing (exchanged) photon by a gluon or quark. Then the incoming positron in Fig. 2 can also be a quark. In this way not only a different combination of the quark content of the photon is probed but also its gluon content, already in leading order. Expressing $F_{2p}$ in terms of the photon–quark distributions $q^a$, deep inelastic electron scattering off a photon target can be written as the product of $q^a$ times the elementary electron–quark scattering cross section

$$
\sigma(e^+e^- \rightarrow e^+X) = \sum_q q^a \otimes \sigma(eq \rightarrow eq).
$$

The analogous reactions in proton–photon scattering is jet production

$$
\sigma(p\gamma \rightarrow jX) = \sum_a \sum_b a^a \otimes b^b \otimes \sigma(ab \rightarrow cd),
$$

where $a, \ldots, d$ are either quarks or gluons, and $a^\gamma (a^g)$ is the distribution of parton $a$ in the photon (proton). A typical subprocess is shown in Fig. 3. The production of prompt photons (Fig. 3) is another example where the gluon content of the photon is probed. In these reactions where the photon is almost real, $e\gamma$ or $\gamma p$ scattering appears rather like hadronic collisions, cf. (16).

### 4.3 Direct and resolved photon contributions

In contrast to hadrons, photons also couple directly to quarks and not only through its partonic constituents. One-photon exchange cross sections at HERA thus contain two contributions in general, the direct contribution, and the resolved photon one:

$$
\sigma_{\gamma p} = \sigma_{\gamma q}[\text{direct}] + \gamma^* \sigma_{\gamma p}[\text{resolved}].
$$

Since some terms of the direct process are already included in the photon structure functions describing the resolved photon contributions, the sum (17) has to be performed with care. Consider the reaction $e\gamma \rightarrow X$, where $\gamma = \gamma^*$, $Z^0$, or $W^\pm$, see Fig. 4. The direct contribution involves an integration of $u$ with limits

$$
Q^2 \leq Q_{\text{lim}}^2 \leq W^2 - M^2 + Q^2 \sim M^2.
$$

Since $Q^2$ can become as small as $m^2 (W/2)^2$, the lower limit of the $u$ integration can become much smaller than the QCD scale parameter $\Lambda$. Then the quark–antiquark pair, coupling to the photon, rather behaves like a bound $q\bar{q}$ state. For a perturbative calculation, the $u$ integration has to be cut-off at a scale of order $\Lambda$. The leading $du/u$ integration then yields the large logarithm $\log(M^2/\Lambda^2)$. This logarithmic term arising at the zeroth order in $\alpha_s$ gets modified by terms of order one at higher orders: Multiple gluon radiation brings in a series of such logarithmic terms, compensating the additional powers of $\alpha_s$, because $\alpha_s(Q^2) \log(Q^2) = O(1)$. These logarithmically enhanced contributions are summed up in the photon structure functions (see e.g. [43] for a recent review). It is therefore only the remaining part of the direct cross section that is to be added to the resolved photon contribution [44, 43, 41]. The problem of matching the direct and resolved photon contributions of an almost real photon to hard $e\gamma$ scattering is in principle well defined, although technically quite involved in higher orders. So far, the matching has been taken into account in the calculation of gauge boson production, $e\gamma \rightarrow VX$ ($V = Z^0, W^\pm$) [44-46], and of prompt photon production, $\gamma p \rightarrow \gamma X$ [41].

When the virtuality of the exchanged photon exceeds the QCD scale $\Lambda$, then the use of real photon structure functions is no longer adequate. From (18) one can see how the logarithmic behaviour of the parton model result, $\sim \ln(M^2/\Lambda^2)$, changes to $\sim \ln(M^2/Q^2)$ if $Q^2 \gtrsim \Lambda^2$. A sizeable effect of the non-zero photon virtuality $Q^2$ on the cross section $e\gamma \rightarrow \gamma X$ was found in Ref. [47] based on the direct photon contribution $\gamma + q^* \rightarrow \gamma + q$.

1For heavy quarks, the $u$ integration of the direct contribution can be integrated down to the kinematical limit because the quark mass provides a physical cut-off.
and the resolved photon contributions \( q^2 + g^e \to \gamma + q \) and \( q^2 + g^e \to \gamma + g \). The resolved photon terms were calculated using parton-model expressions for the quark distributions in a real and virtual photon.

The parton-model result for a real photon can be analytically continued through the replacement \( A^2 \to A^2 + Q^2 \) yielding:

\[
q^e(x, Q^2, M^2)|_{\text{QPM}} = 3 \frac{\alpha}{2\pi} \gamma_1 \left[ x^2 + (1 - x)^2 \right] \log \left( \frac{M^2}{A^2 + Q^2} \right).
\]

(19)

Known results about real and virtual photon structure functions as well as about recent progress concerning the analytical properties of the moments \( F^n(n, Q^2, M^2) \) in \( Q^2 \) are reviewed in a contribution to these proceedings [43]. A consistent description of the photon structure function over all \( Q^2 \) is still missing. In Ref. [43] it is proposed to split the \( Q^2 \) range to at least achieve a partial matching. In the low-\( Q^2 \) region, direct and resolved photon contributions are matched as for a real photon, and in the high-\( Q^2 \) range, the total cross section is approximated by the direct contribution.

5 Calculations of reactions

The following reactions have been studied in the low-\( Q^2 \) study group during this workshop.

5.1 \( J/\psi \) production

The production of \( J/\psi \) is well studied in muo- and photoproduction experiments. The prospects at HERA were looked at in [48–52]. In [49] it is pointed out that the production of a \( J/\psi \) in association with a high-\( p_T \) photon is a unique probe of the photon-gluon distribution. Unfortunately, the rate is very small, e.g., \( \sigma(pp \to J/\psi \to l^+l^-\gamma X) \sim 0.3 \text{ pb for } p_T > 1.5 \text{ GeV} \). Of the same order of magnitude is the cross section for the production of open charm (with \( c \to l \)) associated with a high-\( p_T \) photon [53], which in principle is a way to measure the charm content of the proton and/or the photon.

Double charm production, i.e. the reaction \( pp \to J/\psi c \bar{c} X \), was calculated in Ref. [51]. Despite the additional factor of \( \alpha_s \), the cross section is a factor of about 10 smaller than inelastic \( J/\psi \) production via photon-gluon fusion. It will be interesting to see whether measurements at HERA will require a \( K \) factor similar to that for photon-gluon fusion into \( J/\psi \) plus gluon. This should help to clarify the origin of the large corrections found in inelastic \( J/\psi \) production at fixed-target experiments.

In Ref. [52], the authors report on a first investigation of how a modified evolution equation at small-\( x \) would affect the inelastic \( J/\psi \) production cross section. The weaker increase with the c.m. energy found in Ref. [52] can also result from a flatter "conventional" gluon distribution in the proton. The mass scale in \( J/\psi \) production is rather fixed, being of the order of the \( J/\psi \) mass. Differences between the conventional scenario and a new one based on transverse-momentum factorization have therefore to be looked for in the associated hadronic final state. Such studies will have to make use of the fact that in the latter approach the transverse momenta no longer have to be strictly ordered.

A thorough discussion of elastic and diffractive \( J/\psi \) production as well as of the various inelastic production processes can be found in Ref. [48]. There various models are proposed and compared with existing data. They predictions for HERA c.m. energies are given. The elastic and diffractive processes yield information about the Pomeron (trajectory, partonic content, etc.), the photon-\( J/\psi \) coupling, and the photon structure function \( F_2 \) at low \( x \) and \( Q^2 \). Particular emphasis was put on the question of how far inelastic \( J/\psi \) production can be described by photon-gluon fusion. Finally, a study to determine the photon-gluon distribution was presented, based on Monte Carlo data of complete events, including detector simulation.

5.2 Lepton-pair production

Lepton-pair production in hadronic collisions is dominated by the Drell-Yan process \( qq \to \gamma^* \to l^+l^- \) at low \( p_T \)'s. The Drell-Yan process also contributes to lepton-pair production in \( ep \) collisions: at low virtualities of the exchanged photon, the resolved photon contribution gives rise to \( qq \) annihilation processes, just as in the case of hadronic collisions. Two further lepton-pair-production mechanisms have to be considered in \( ep \) collisions: two-photon production (or Bethe-Heitler process) \( \gamma^*p \to l^+l^- , \) and production via virtual photon radiation from the electron line (Compton-like contribution), \( e^*p \to \gamma^* \to l^+l^- \). Of course, the above decomposition into three classes is justified in leading order only. First, a full \( O(\alpha^2) \) calculation has to include also the interference diagrams between the Bethe-Heitler and the Compton-like process. Second, one has to be careful when adding the Drell-Yan process, because its leading-order contribution is already contained in the \( O(\alpha^4) \) squared matrix elements.

It is well known (see e.g. Ref. [54] for a recent calculation) that in \( ep \) collisions, the Drell-Yan process is very small, and the Bethe-Heitler process typically dominates the Compton-like process. However, at least for electron-pair production, the full \( O(\alpha^4) \) calculation is needed [55]. Event generators for lepton-pair production at HERA are available and summarized in Ref. [3].

Lepton-pair production in elastic electron-proton scattering, \( ep \to e^*p l^+l^- \), is completely calculable in QED. The reaction is well suited to measure the \( ep \) luminosity, because it only depends on the well-known elastic proton form factors. Inelastic-lepton-pair production offers the possibility to measure the proton structure function \( F_2(x, Q^2) \) at very low \( x \) and very low \( Q^2 \) [46, 57]. First studies of the detection efficiency of lepton-pair events at HERA look promising [57].

5.3 Non-standard Higgs-boson production

The cross section for the Standard Model Higgs-boson at HERA is expected to be very small, given the bound from LEP [58]. At the same time, there is a large background, mainly coming from \( b \) quark production. In certain extensions of the Higgs-sector one may have enhanced couplings of a scalar Higgs-boson to down-like quarks. Because the Higgs-boson coupling to the fermions is proportional to the fermion masses, and because the \( b \) content of the proton is small, the leading contribution to the production of such a non-minimal Higgs-boson \( h \) should be the direct process \( gg \to b \bar{b}h \). This reaction was estimated in Ref. [59] and yields e.g. a cross section of about 1 fb at \( m_h = 50 \text{ GeV} \).
5.4 Production of direct photons

Contributions to the production of direct (or prompt) photons in ep collisions come from two sources: bremsstrahlung of the electron line and inelastic scattering on the proton. The full calculation can be done in Ref. [6]. Elastic scattering can be expressed in terms of the proton structure functions $F_2$ and inelastic collisions from photon bremsstrahlung from the electron line are also included. Even generators for photon bremsstrahlung in the sea state $p + p \rightarrow X$ is usually neglected when the proton is radiated. The hadronic final state is typically set in the proton. In turn, the three-momentum of the scattered photon is balanced in the region of interest. For the model-dependent part, the percent hadrons may be expressed in terms of the proton structure functions $F_2$ and $Q^2$.

In the low $Q^2$ region, the interest focuses on the deep inelastic Compton (DIS) process $p + q \rightarrow X$, where the photon has a high $z$, with respect to the beam axis. This can also be expressed in terms of the proton structure functions $F_2$ and $Q^2$. For $Q^2 > 2$ GeV$^2$, the hadronic final state is typically set in the proton. In turn, the three-momentum of the scattered photon is balanced in the region of interest. For the model-dependent part, the percent hadrons may be expressed in terms of the proton structure functions $F_2$ and $Q^2$. The full calculation can be done in Ref. [6]. Elastic scattering can be expressed in terms of the proton structure functions $F_2$ and inelastic collisions from photon bremsstrahlung from the electron line are also included. Even generators for photon bremsstrahlung in the sea state $p + p \rightarrow X$ is usually neglected when the proton is radiated. The hadronic final state is typically set in the proton. In turn, the three-momentum of the scattered photon is balanced in the region of interest. For the model-dependent part, the percent hadrons may be expressed in terms of the proton structure functions $F_2$ and $Q^2$. The full calculation can be done in Ref. [6]. Elastic scattering can be expressed in terms of the proton structure functions $F_2$ and inelastic collisions from photon bremsstrahlung from the electron line are also included. Even generators for photon bremsstrahlung in the sea state $p + p \rightarrow X$ is usually neglected when the proton is radiated. The hadronic final state is typically set in the proton. In turn, the three-momentum of the scattered photon is balanced in the region of interest. For the model-dependent part, the percent hadrons may be expressed in terms of the proton structure functions $F_2$ and $Q^2$.
calculation is needed, including the corrections to the contribution with a photon coming from the fragmentation of the final state. These corrections are obtained by convoluting the NLO distribution and fragmentation functions with partonic cross sections calculated at $O(a_s^2)$. Alternatively, one could try to experimentally suppress the contribution from bremsstrahlung photons by putting an isolation cone around the photon.

Concerning the phenomenology of the DIC process, it is noted [67, 68] that the tagged mode (i.e. $\gamma p \to X \gamma X$ at fixed $E_\gamma$) seems well suited to determine the photon-gluon distribution, see Fig. 3 of Ref. [68]. Similar observations were made [69] in the case of jet production (and to a lesser extent also in heavy-quark production). There it was found that the use of the luminosity monitor at HERA greatly improve the isolation of the contribution coming from gluons in the photon from the direct and quark contributions.

The prospects to measure the PDFs of the proton and the photon in the DIC process at HERA are summarized in Ref. [70].

5.5 Jet production

The production of high-$p_T$ jets at low $Q^2$ receives contributions from processes where the photon couples directly to the partons within the proton and from scatterings of its constituents off those of the proton. In leading order, $O(a_s)$, two jets are produced which balance in $p_T$. The scattered electron is hardly deflected and typically escapes undetected in the beam pipe. The direct contribution is described by the subprocesses $\gamma + q \to \gamma + g$ and $\gamma + g \to q + \bar{q}$. Two-to-two parton subprocesses constitute the resolved photon contributions, with one parton each coming from the photon and the proton.

In leading order, the direct and resolved photon contributions can simply (and have to be) added, and are available in Monte Carlo event generators [4]. They differ (i) in their angular distribution, and (ii) in their $p_T$ distribution. The difference in the angular distribution arises mainly from the presence of the diagrams with a gluon in the $t$ channel in the resolved photon contribution. The direct process dominates at high $p_T$, owing to the reduced energy of the partons in the photon. These differences, in addition to the presence of a photon spectator jet in the case of the resolved photon contribution, form the basis for the separation in experimental analyses at HERA [1]. In particular, the dominance of the gluon-gluon subprocesses at low $p_T$ makes jet production appear as the best way to extract the gluon distribution of the photon at HERA. Measurements of both the dijet cross section and the one-jet inclusive cross section seem to be suited.

The weak points of the leading order $O(a_s)$ calculation of jet production are well known. First, the predictions depend sizably on the mass and factorization scales. The dependence is expected to be reduced when higher-order corrections are included. Second, the QCD scale $\Lambda$ is first defined in NLO. Third, the theoretical jet cross section is independent of the jet definition. A comparison of experimental and theoretical jet cross sections as a function of the jet definition (e.g. cone size) is first possible when higher-order corrections are included in the calculation.

At NLO accuracy, $O(a_s^2)$, the separation into direct and resolved photon contributions is no longer unique. Consider the $O(a_s^2)$ diagram $\gamma g \to qg\bar{g}$. The kinematical configuration when the outgoing quark is collinear with the photon gives rise to a large logarithm, e.g. $\ln(p_T^2/\Lambda^2)$. The term associated with this logarithm is already contained in the leading-order expression for the quark distribution of the photon. It is thus only the remaining part of the process that contributes to the direct process. The precise expression depends on the scheme used to define the photon-parton distributions in NLO.

Complete NLO calculations to jet production are still missing. For the one-jet inclusive cross section, the calculation would be similar to the one for prompt photon production [41]. So far, the corrections to the one-jet inclusive cross section have been calculated, separately for the direct and resolved photon processes. In Ref. [70] the $O(a_s)$ corrections to the direct process were calculated. Terms already contained in the expression for the photon quark distributions were isolated and subtracted from the cross section. Higher-order corrections to the resolved photon process were calculated in Refs. [71, 42]. Their contribution is obtained by folding the $O(a_s^2)$ partonic cross section for $a + b 	o j + X$ with the respective (NLO) parton distributions in the proton and photon. Finite terms corresponding to the NLO definition of the photon-parton distributions can be obtained by considering the logarithmic enhanced diagrams of order $O(a_s a_s^2)$ (which then become $O(a_s^3)$). So far, results for the resolved process are calculated using photon structure functions evolved in leading order only. Even without a matching of the direct and resolved photon contributions, the existing calculations do improve the theoretical description of inclusive jet production when restricting to kinematical regions where either of the contributions dominates. This can clearly be seen in Fig. 1 of Ref. [70] for the direct process, and in Fig. 4 of Ref. [71] for the resolved process.

Instead of adding direct and resolved photon contributions, one may alternatively try to separate both contributions by suitable cuts. The individual contributions are then, of course, cut-dependent. Ideally, the experimental cuts should correspond to the ones used in the theoretical calculation (or vice versa). In the calculation of the $O(a_s^2)$ tree diagrams, one can avoid the collinear singularity associated with the photon quark distributions by demanding that there be at most a soft parton in the very forward direction. Such a calculation was performed in Ref. [72]. Experimentally, such a definition depends strongly on the ability to tag the photon remnant jet and/or on the possibility to estimate the fraction of the remnant jet that escapes undetected in the beam pipe.

6 Kinematical coverage of structure function measurements

Photoproduction processes at HERA receive contributions from reactions in which the photon couples directly to the partons within the proton, and from those mediated by the partonic content of the photon. Depending on which contribution dominates, a given reaction is better suited to extract the proton or the photon PDFs. Reactions dominated by direct photon contributions are

$$\gamma + p \to Q + \bar{Q} + X$$
$$\gamma + p \to J/\psi + X$$
$$\gamma + p \to \mu^+\mu^- + X$$

(20)

Also listed in (20) are the dominant subprocesses. Open heavy-quark production is studied in Ref. [73], $J/\psi$ production in [15, 50]. Both processes are well suited to determine the gluon distribution of the proton. In both cases detailed studies have been performed.
to actually reconstruct the gluon's momentum fraction $x_g$. The Drell-Yan process is dominated by two-photon production [51, 56, 57]. In turn it offers the prospect of measuring the proton structure function $F_2(x, Q^2)$ at low $Q^2$.

Reactions that receive large contributions from resolved photon processes are

$$\gamma + p \rightarrow \gamma + X \quad \begin{array}{c} q^+ + q^- \rightarrow \gamma + q \\ q^+ + g \rightarrow q + g + g \\ q^+ + q^- \rightarrow q + g + g \\ \end{array} \begin{array}{c} 4 \times 10^{-4} \\ 2 \times 10^{-4} \\ 2 \times 10^{-2} \\ \end{array}$$

Again, the dominant subprocesses are listed in (21). Prospects to extract PDFs from these reactions are discussed, for instance in Ref. [69] for the case of jet production and in Ref. [68] for the case of prompt photon production. Detailed Monte Carlo studies on how to separate the individual contributing processes, and ways to reconstruct the momentum fractions $x$, of the partons, have been performed in Refs. [74, 75] and in Refs. [74, 76], respectively. Results are summarized in Ref. [1].

In Table 3 a compilation of existing measurements of parton distributions of the proton is given. Also shown are reactions at HERA that offer the possibility to extract the PDFs of the proton and of the photon. The quark distribution functions of the proton are mainly determined from DIS data [77] and the Drell-Yan process (so far, mainly data from E665 [78] are used). In these reactions, the gluon distribution is not directly accessible, but can be constrained from a $Q^2$ analysis (together with $\Lambda$). The PDF parametrisation of Ref. [7] is based on these data. As pointed out in Ref. [8], prompt photon production in $p$ collisions is particularly suited to constrain the gluon due to the dominance of the gluon-initiated subprocess. The authors of Ref. [9] included the WA70 [79] prompt photon data in their parametrisation. Heavy-quark production does not yet have the status of providing structure-function measurements. With certain assumptions the New Muon Collaboration was able to extract the gluon density [10]; see [48] for a discussion of this point.

The experimentally measured distributions of the reactions listed in Table 3 can be translated into $x, Q^2$ regions, where $x$ and $Q^2$ are the momentum fraction and the scale at which the proton's PDFs are probed. These $(x, Q^2)$ ranges are given in Table 3 and also shown in Fig. 5, to illustrate the kinematical regions accessible to different high-energy reactions. Altogether an impressive region in $x$ and $Q^2$ is covered. At the same time, one can see the importance of the small-$x$ measurements at HERA. Also given in Fig. 5 and Table 3 are (preliminary) estimates of the $x$ ranges in which the proton distribution functions can be measured at HERA. Table 3 furthermore shows the reactions that might yield information about the photon distribution functions. One can clearly see that low-$Q^2$ processes have a big potential to determine PDFs, complementing those from DIS.

### 7 Summary

Measurements of low $Q^2$ reactions at HERA offer a wide spectrum of interesting physics, ranging from soft hadronic physics over hard QCD interactions and electroweak studies to explorations of extensions of the Standard Model. A central theme of low-$Q^2$ studies at HERA is the photon, whose nature can be probed in a new kinematical regime in high-energy photon-proton collisions, at both small and large momentum transfer.

Photoproduction reactions at HERA are expected to provide the first measurements of the parton distribution functions of the photon at small $x$ values (at still sufficiently
inelastic $Q^2$. The information will come mainly from jet production, but also from the production of heavy quarks, prompt photons, and pions. Moreover, deciphering the gluon content of the photon from such measurements looks promising.

Photoproduction reactions at HERA should also provide measurements of the proton-parton distributions. The gluon density, for example, can be extracted from heavy-quark production, both in open heavy quark and in $J/\psi$ production. Measurements of the parton distributions of the proton in low-$Q^2$ reactions will complement the ones in deep inelastic scattering at HERA.

In the course of this workshop, much progress has been achieved in the description, the simulation, and the phenomenology of low-$Q^2$ processes at HERA. Yet, more work is needed.

On the theoretical side, calculations of higher-order QCD corrections have to be continued. More studies are needed to investigate the photon structure function in the transition region from a virtual to a real photon.

Also studies on the phenomenology of low-$Q^2$ processes will continue. The abilities to measure the proton structure function $F_2$ in electron-photon and photon–photon reactions at HERA have to be quantified. Of course, all ways to measure the QCD scale $\Lambda$ in low-$Q^2$ processes at HERA will have to be investigated in great detail. Prospects are offered through QCD analyses of the photon structure function and through studies of the hadronic final state in photoproduction processes. An example of the latter kind is the ratio of photon–jet to jet–jet cross sections.

It is evident that low-$Q^2$ physics is going to be an exciting aspect of HERA physics, and the comparison with data is eagerly awaited.

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References


[37] For a recent review see e.g. G. Wormser, Oser preprint LAL 90–37, presented at the Workshop on Vector–Meson Dominance Phenomena, Göttingen, 1990.


[43] F. Borzumati and G.A. Schuler, “Real and virtual photon contributions to inelastic ep scattering, H* production at HERA, a case study”, these proceedings.


[50] R. Brognara, "$\pi^-$ determination using $J/\psi$ inelastic events", these proceedings.

[51] S.P. Baranov, "On the hadro- and leptoproduction of $J/\psi + c + \bar{c}$ states", these proceedings.

[52] V.A. Saleev and N.P. Zotov, "The $J/\psi$ electro- and photoproduction at high energies and the gluon distribution of the proton", preprint NPI MSU-91 (1991); and "$J/\psi$ photoproduction at low $x$", these proceedings.


[57] G. Leventis, "Muon pair production by two photon collisions at HERA", these proceedings.


[60] A.C. Bawa and M. Krawczyk, "Non-minimal neutral Higgs boson production in ep collisions due to the structure of the photon", these proceedings.

[61] H. Spiesberger, A. Akhundov et al., "Radiative corrections at HERA", these proceedings.


[68] A.C. Bawa and M. Krawczyk, "Probing the structure of the proton and the photon in the deep inelastic Compton process at HERA and LEP/LHC", these proceedings.