HIGH-LUMINOSITY INSERTION FOR A B-MESON FACTORY

B. Autin

ABSTRACT

The major characteristics of a very high-luminosity B-meson factory are presented. The machine would consist of two rings lying in the same horizontal plane and using a crab crossing scheme to restore the conditions of head-on collisions. Emphasis is put on very low-β values (≤1 cm) to relax the constraints imposed by the intensity. This results in a very strongly focusing lattice.
Abstract

The major characteristics of a very high-luminosity B-meson factory are presented. The machine would consist of two rings lying in the same horizontal plane and using a crab crossing scheme to restore the conditions of head-on collisions. Emphasis is put on very low-\(\beta\) values (\(\leq 1\) cm) to relax the constraints imposed by the intensity. This results in a very strongly focusing lattice.

INTRODUCTION

The B-factories [1,4] are characterized by very high luminosities combined with a low level of background. Two orders of magnitude in luminosity have to be gained over existing lepton rings. The solution consists of increasing the number of bunches and avoiding unwanted collisions by making electrons and positrons circulate in two different rings. The separation of the bunches remains a difficulty in the interaction region. If the collisions are head-on, one takes advantage of the different energies of the electron and positron beams to separate them magnetically but the bunch spacing can reach its minimum value only with a geometric separation which has the additional advantage of reducing the synchrotron background by eliminating any dipole field and being applicable to machines with symmetric energies. A design using a crossing angle suffers from several handicaps: it reduces the overlap of the bunches and therefore the luminosity, and it is a dangerous source of synchro-betatron oscillations which were a serious limitation in the first DORIS approach [5]. These problems can actually be solved with the crab crossing scheme [6,7] which restores the conditions of head-on collisions by using two rf deflectors located \(\pi/2\) apart from the interaction point. It is this approach which will be discussed in the context of a study of a B-factory in the CERN ISR tunnel where the two rings, like the old proton rings, would lie in the same horizontal plane.

DESIGN STRATEGY OF A VERY HIGH LUMINOSITY B-FACtORY

Central in the parameterization of a machine is the choice of the beam intensity and of the vertical \(\beta^*\) value at the crossing point. Here, we emphasize the arguments in favor of the smallest \(\beta^*\) value. A high beam intensity has adverse effects in at least two major areas: synchrotron power and higher order modes. The synchrotron power must be maintained within reasonable limits; it is an obvious waste of energy; for the energies and intensities of a B-factory, the synchrotron light beam impinging on the vacuum chamber imposes technical solutions which are costly; furthermore, the background in the experimental areas is directly related to the beam intensity. The higher order modes power per unit length generated by the interaction of a bunched beam with a cavity is given by

\[ P_{\text{norm}} = k \tau_b l^2 \]

where \(k\) is of the order of 0.15 V/pC/m at 350 MHz and increases with the square of the frequency, \(\tau_b\) is the time separation between consecutive bunches and \(l\) the beam current. This power exceeds 600 kW in the CERN-PSI design [8] and is clearly a major concern both for the development of couplers which have to evacuate this power and for the wake field which drives coupled bunches instabilities.

According to the expression of the luminosity, the intensity can only be reduced by diminishing \(\beta^*\) and the bunch length in the same proportion. A first limit is imposed by the rf voltage required to focus short bunches. Another difficulty lies in the correction of the high chromaticity associated with low-\(\beta^*\) values yet maintaining a sufficient dynamic aperture; this is the realm of non-linear optics where the experience accumulated with recent storage rings gives confidence in the finding of efficient correction schemes. Last, the bandwidth of the feedback systems which have to damp the coupled bunch instabilities has to be increased, but the developments, made especially in the domain of stochastic cooling, let hope that technical solutions exist [9].

Our discussion is limited to flat beams in the interaction area. In spite of the gain of a factor 2 in luminosity which could be theoretically accomplished with round beams, it turns out from more detailed studies on the beam-beam interaction [10] and optics schemes [11] that the gain increase with respect to flat beams may not exceed 40% and that the complications bound to such a scheme prevent the use of much more powerful methods of luminosity upgrading. In these conditions, the beam-beam tune shift, which is the measure of the beam-beam limit, is proportional to the beam density in the normalized horizontal phase-space \(N/\epsilon_x\) where \(N\) is the number of particles per bunch and \(\epsilon_x\) the horizontal emittance. For a given luminosity, the beam-beam tune shift decreases with the number of bunches but one may like nevertheless to work near the beam-beam limit by reducing the emittance. A low emittance has the substantial advantage of limiting the synchrotron radiation in the quadrupoles and is consistent with the short bunches necessary in very low-\(\beta\) scheme.

SCALING LAWS FOR SHORT BUNCHES AND LOW-\(\beta\) VALUES

A collider can work near the beam-beam limit (\(\approx 0.05\) for leptons) if the bunch length is roughly half the \(\beta^*\) value [12,13]. We shall discuss the implication of short bunches on rf voltage and single bunch longitudinal instability then how
to achieve low-\(\beta^*\) values. We assume that the regular part of the lattice is composed of FODO cells for which scaling rules are well known. In particular, the horizontal emittance is given by

\[
\varepsilon_x = \frac{2R}{Q_x} \sigma_x^2
\]

where \(R\) is the arc radius, \(Q_x\) the horizontal betatron tune and \(\sigma_x\) the energy spread. Another quantity which plays a central role is the momentum compaction which measures the rate of change of the orbit circumference with the momentum:

\[
\alpha_p = \frac{2\pi R}{C} \frac{1}{Q_x^2}
\]

where \(C\) is the machine circumference. Under these conditions, the voltage required to get a bunch of length \(2\sigma_x\) is

\[
V = \frac{cRE \sigma_x^2}{\epsilon f_{rf}} \frac{1}{(Q_x, \sigma_x)^2}
\]

where \(E\) is the bunch central energy and \(f_{rf}\) the rf frequency. In the same way, the maximum impedance permitted for the longitudinal stability of the bunch is proportional to the product \(\alpha_p \sigma_x^2/N\); replacing \(N\) by its expression as a function of the beam-beam tune shift and the emittance, it turns out that again this impedance depends on the product \(Q_x \sigma_x\). In brief, as far as the lattice structure is concerned, any reduction of the bunch length must be accompanied by a corresponding increase of the betatron tune.

For the low-\(\beta\) insertion, it is proposed that the two beams have independent focusing. Diminishing the constraints improves the simplicity and the performance of the structure. The insertion shown in Fig.1 is made of a single lens to achieve the shape of the beam at the crossing point followed by a quarter wavelength transformer to match the final focus to the regular FODO cell [14].

The betatron matching is performed between planes where the \(\beta\) functions are equal and the \(\alpha\) functions opposite so that the whole insertion is fully parameterized analytically within the thin lens model. The normalized focal length of the final lens is

\[
q = \frac{\beta_h + \beta_v - \sqrt{\beta_h \beta_v \left[ 4 + (\beta_h - \beta_v)^2 \right]}}{\beta_h - \beta_v}
\]

All the quantities in the above expression are normalized to the distance from the crossing point to the final lens. As expected, the method can only be applied if \(\beta_h\) and \(\beta_v\) are different. For flat beams, \(q\) is practically independent of \(\beta_v\) and roughly equal to 0.8. The measure of the flatness of the beams is given by the coupling coefficient \(c\), ratio of the vertical to horizontal \(\beta\) functions also equal to the ratio of the beam emittances. The scaling of the parameters is conveniently performed at constant \(c\) so that \(\beta_h\) is automatically related to \(\beta_v\). The distance \(d\) which separates the final lens from the transformer is then

\[
d = \frac{\sqrt{4 + (1-c)^2 \beta_h^2 \left(1 + c - \sqrt{\left[ 4 + (1-c)^2 \beta_h^2 \right]^2} \right)}}{2\sqrt{c \left[ 4 + (1-c)^2 \beta_v^2 \right] + (1-c)^2 (1+c) \beta_h^2}}
\]

The variations of \(d\) and \(q\) versus \(c\) are very similar except at the limit of very flat beams (\(c \approx 0\)) where \(d\) tends towards infinity whereas \(q\) cannot exceed 1.

The length of the insertion is not only determined by \(d\) but also by the length of the transformer which varies like \(\sqrt{\beta_h/\beta_v}\); the insertion can be maintained compact if the \(\beta\) function of the lattice \(\beta_0\) is small enough. This is one more aspect of the general rule: a small \(\beta_v\) at the crossing point implies a strong focusing lattice.

**CROSSING ANGLE**

The crossing angle is determined by the room available for the final quadrupole and the crab crossing cavity.

The final quadrupole in the type of insertion which has just been presented has a focal length of the order of 0.8 m and thus an integrated gradient at of 33 T at 8 GeV/c. It must have a width smaller than \(\alpha\) where \(\alpha\) is the total crossing angle and \(l\) the distance from the crossing point to the quadrupole. As a tentative set of parameters, we assume a coupling coefficient \(\beta_v/\beta_h\) or \(\epsilon_v/\epsilon_h\) equal to 0.01, a horizontal emittance of \(3 \times 10^{-8}\) m, a value of \(\beta_v^*\) of 1 cm and a distance \(l\) of 1 m so that the beam is round with a standard deviation of 0.25 mm.

A permanent quadrupole with an internal radius of 1 cm should therefore be well adapted to accommodate the vacuum chamber, a beam envelope of 10\(\sigma\), a closed orbit error of a few millimeters and to keep some flexibility for a further reduction of \(\beta_v^*\). With rare earth permanent magnets, a magnetization of 1 T and therefore a gradient of 100 T/m are typical characteristics; the magnetic length of the high-energy ring quadrupole would therefore be 33 cm.

![Fig. 1. Model of low-\(\beta\) insertion (\(\beta_v^* = 1\) cm, \(\beta_h^* = 1\) m, \(d^* = 1\) m).](image)
In order to have a magnet width compatible with a small horizontal crossing angle, slim geometries [15] which reproduce for permanent magnets the properties of figure of eight quadrupoles were investigated. The aperture is circular but the outer contour has an egg shape described by the polar equation

\[ r = r_0 \exp[k(1 - \cos 2\phi)] \]

When \( k \) is zero, the contour is simply a circle of radius \( r_0 \). When \( k \) increases, the contour is elongated in the vertical direction and, for small values of \( k \), takes a quasi-elliptical shape; a transition occurs at \( k = 0.25 \) value beyond which the contour has a waste in the horizontal medium plane of symmetry. The shape associated with \( k = 0.25 \) is called super-elliptical. The characteristics of two types of quadrupoles are summarized in Table 1.

<table>
<thead>
<tr>
<th>Outer shape</th>
<th>Gradient [T/m]</th>
<th>Field region with gradient error &lt;10^{-3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>1</td>
<td>63 mm</td>
</tr>
<tr>
<td>Super ellipse</td>
<td>1.18</td>
<td>52 mm</td>
</tr>
</tbody>
</table>

The higher gradient of the super ellipse type is at the cost of a reduced good field region. The circular shape is thus chosen as the reference design and a super elliptical shape may become an interesting alternative if one has to increase the inner radius without changing the overall width. In both cases, the residual field in the adjacent low energy ring quadrupole is negligible.

The rf deflector can be located near the second quadrupole of the transformer where \( \beta_h \) is maximum and the horizontal betatron phase close to \( \pi/2 \). At 500 MHz, the radius of the cavity is 24 cm and is compatible with a position 10 m far from the interaction point should the cryostat be the same for the cavities of the two rings.

A more detailed analysis is needed to determine the complete layout of the interaction region, it should address questions such as the choice of the exact frequency of the rf deflectors, the interest of a third cavity to compensate any phase imperfection between the deflectors, the exact quadrupole aperture for sub-centimeter \( \beta^* \) values and the vertical orbit distortion created by the detector. It seems, however, that 25 mrad would be the right choice for the half-crossing angle.

CONCLUSION

Scaling laws have been presented for the design of the interaction areas of B-meson factories. Luminosities as high as \( 10^{34} \text{ cm}^{-2}\text{s}^{-1} \) may be achieved with currents limited to 1.5 A for a 5 mm \( \beta \) value at the crossing point and a betatron tune of the order of 30. This seems to be possible if the beams cross horizontally with an angle such that the focusing is independent for the two beams. Technical solutions for the permanent quadrupoles and for the rf deflectors exist but require developments.

ACKNOWLEDGMENTS

V. Mokhov has studied the permanent quadrupoles. His calculations have been checked with the finite element program Flux2D by F. Rohrer.

REFERENCES