Inclusive Beauty Decays and the Spectator Model

G. Altarelli
CERN - Geneva
and
S. Petrarca
Dipartimento di Fisica. Universita' di Roma "La Sapienza"

Abstract

A complete updated analysis of the branching ratios of inclusive b decays in a pure parton picture with inert spectator light quark(s) is presented. In view of recent data that indicate a rather small value for the average semileptonic decay fraction, particular attention is devoted to evaluating the central value and the range of theoretical errors for this quantity. We find that, while the data are marginally consistent with the spectator model, there are perhaps indications for a moderate non-spectator contribution. We discuss possible terms of this kind and argue that the ratio $\tau^-/\tau^0$ should probably be around its present upper limit.

CERN-TH.6017/91
February 1991
In the present note we reconsider and update the theoretical predictions for the average semileptonic branching ratio $B_{SL}$ of $b$-flavoured particles and other interesting inclusive decay fractions. Our motivation mainly arises from recent experimental results that indicate a rather low value for $B_{SL} = B(B \rightarrow I^\pm X)$, $I = e$ or $\mu$. The most recent data from ARGUS [1], CLEO [2] and Crystal Ball [3], which refer to the average of $B^0$ and $B^-$ mesons (and their antiparticles), lead to the values:

$$B_{SL} = (10.2 \pm 0.5 \pm 0.2)\% \quad \text{ARGUS} \ e,\mu$$

$$= (10.5 \pm 0.3 \pm 0.4)\% \quad \text{CLEO} \ e,\mu$$

$$= (12.0 \pm 0.5 \pm 0.7)\% \quad \text{Crystal Ball} \ e$$

These results were obtained from the observed spectra by using a parton model approach corrected for non-spectator effects [4]. Other models lead to values well consistent with the above ones. When combined the results in eqs.1 give

$$B_{SL} = (10.6 \pm 0.4)\%$$

Finally new data on $B_{SL}$ are now being obtained by the LEP experiments. Assuming the standard model value for $\Gamma_b/\Gamma_h$ (the $Z$ partial widths into $b\bar{b}$ pairs and into hadrons) LEP results can be combined [5] into

$$B_{SL} = (10.7 \pm 0.5)\%$$

Note that at LEP the measured quantity is an average over all $b$-flavoured particles and not just over $B^0$ and $B^-$ mesons.

We see that there is a tendency by the most recent and precise experiments to favour smaller values of $B_{SL}$ than in the past [6].

Previous theoretical estimates [7],[8] of $B_{SL}$ in the parton model with spectator light quark(s) led to values around 14 %. One wonders whether the spectator model is not already disfavoured by the available data and whether some non-spectator contributions or even some new physics must be invoked to reproduce the experimental findings. Some prominent candidates for non-spectator effects, in a similar way as for charm decays, arise from interference [9] and/or W exchange [10]. It is indeed apriori conceivable that such power suppressed corrections to the parton approximation could still be appreciable for $b$ decays. However, as we shall discuss in the following, theoretical estimates of these contributions lead to rather small effects and available data on the ratio of $B^-$ and $B^0$ lifetimes already put stringent limits on the importance of these mechanisms for $B_{SL}$. In view of the fact that only a
moderate effect can be expected from these mechanisms, it is important to investigate how close the prediction of a pure spectator model can come to the data.

Even assuming that the b quark mass is large enough for a reasonably reliable application to b decays of the parton model with spectator(s), it is well known that the main uncertainty in the calculation of decay widths is introduced by the presence of the fifth power of $m_b$, the effective b quark mass, which is only defined with modest precision. This source of error drops away when ratios of widths are taken. As a consequence, the predictions for branching ratios are expected to be by far more accurate than for the individual widths. Yet the remaining uncertainties are still quite sizeable. They arise, for example, from the QCD sector (value of $\Lambda_{QCD}$, scale ambiguities) and from the uncertainties on the effective values of final state masses (e.g. current versus constituent quark masses). Our main purpose is to analyse the above sources of error in detail in order to derive not only central values, but also theoretical errors for $B_{SL}$ and other important branching ratios in the parton approximation with inert spectator(s).

For this we consider b decays in the parton model, neglecting at the moment all non-spectator effects. Note that Fermi motion and other bound state effects which were taken into account in ref.[4], while important for the detailed shape of the lepton spectrum near the end point and for relating the value of the effective mass $m_b$ to the properties of the observed spectrum, are essentially negligible in the calculation of inclusive branching ratios. We work in leading order in the weak Hamiltonian, i.e. we restrict ourselves to contributions of order $G_F^2$ (plus QCD corrections) to the various widths. Most of the time we do not explicitly mention $b\rightarrow u$ transitions, but we include them in the numerical calculations (we take $|V_{bu}/V_{bc}| = 0.1$ [6]).

In this approximation the semileptonic width is given by:

$$\Gamma_{SL} = \Gamma(b\rightarrow clv) = F \cdot \frac{I(m_c/m_b, m_l/m_b, 0)}{s^{2/3}} \cdot [1 - \frac{2\alpha_s}{3\pi} f(m_c/m_b)]$$

(4)

Here, the overall factor $F$ is given by

$$F = G_F^2 m_b^5 |V_{bc}|^2 / 192 \pi^3$$

(5)

The function $I(x_1, x_2, x_3)$, symmetric in $x_1, x_2$, normalized to $I(0, 0, 0) = 1$ and given explicitly in ref. [8], describes mass effects for $(V_{\pm A})\cdot(V_{\pm A})$
transitions (for the (V±A)-(V±A) cases, of interest later, one simply needs to exchange $x_2$ and $x_3$). The function $f(x_1)$, which specifies the QCD correction for the semileptonic width, was studied in refs.\cite{11} In the case of massless fermions it reduces to $f(0) = \pi^2 - \frac{25}{4}$. The lepton masses are always neglected in $f$, including for the $\tau$ lepton, (but not in $I$).

Most of the complications appear in the non-leptonic sector. Within the assumed set of approximations the dominant terms in the non-leptonic Hamiltonian before QCD corrections (i.e. at zeroth order in $\alpha_s$) are given by:

$$H_{NL}^{\text{free}} = \frac{G_F}{\sqrt{2}} V_{bc} [(\bar{b}c)_L (\bar{u}d')_L + (\bar{b}c)_L (\bar{c}s')_L]$$

where $(\bar{q}q')_{R,L} = \tilde{\gamma}_{\mu}(1+\gamma_5)q'$. For the $b\rightarrow c\bar{u}d'$ transition QCD effects do not introduce mixings with penguin-type operators while those mixings are present for the process $b\rightarrow c\bar{c}s'$. In the former case the corresponding width is given by:

$$\Gamma(b\rightarrow c\bar{u}d') = 3F \frac{2L_+^2+L_-^2}{3} \cdot J \cdot I(m_c/m_b, m_{d'}/m_b, m_u/m_b)$$

Strictly speaking $m_{d'}$ stands for the Cabibbo averaged mass $m_{d'} = \cos^2\theta_c m_d + \sin^2\theta_c m_s$. The factor $\frac{2L_+^2+L_-^2}{3} \cdot J$ contains the QCD correction \cite{12},\cite{13}. $L_+,- = (\frac{\alpha_s}{\alpha_s(m_W)})^{d_+,} -$ arise from resumming leading logs ($d_+, -$ = \frac{6}{23} \cdot \frac{12}{23}$), while:

$$J = 1 + \frac{2\alpha_s}{3\pi} (\frac{3}{4} - \pi^2) + \frac{2\alpha_s}{3\pi} (\frac{19}{4} + 3 \log f^2) \frac{2L_+^2 - L_-^2}{2L_+^2 + L_-^2} + 2 \frac{\alpha_s - \alpha_s(m_W)}{\pi} \frac{2L_+^2 \rho_+ + L_-^2 \rho_-}{2L_+^2 + L_-^2}$$

3
arises from the next-to-leading QCD correction. The expressions for $\rho_{+,-}$ can be found in ref.[13]. The quantity $\text{Log} \ f^2$ which appears in the third term of $J$ is not present in ref.[13]. For $f = 1$ the formula of ref. [13] is recovered which corresponds to choosing the scale $\mu = m_b$ for $\alpha_s = \alpha_s(\mu)$, i.e. the mass of the decaying heavy quark. For a different choice of scale $\mu' = f m_b$ the next-to-leading correction is modified as indicated. With this improvement the effect of a change of scale is displaced at order $\alpha_s^2$ in $\Gamma(b\rightarrow c\bar{u}d')$, as is the case also for $\Gamma_{SL}$ given in eq.(7), i.e. at the level of the neglected higher order terms. Actually $J$ as given in eq.(8) is strictly appropriate only for massless quarks (i.e. it is the analogue of $f(0)$ for the semileptonic width). We use it as an approximation for the exact two-loop correction which is not known.

The QCD corrections to the term $(\bar{b}c)_L (\bar{c}s')_L$ in the Hamiltonian of eq.(6) involve mixings with penguin diagrams [14]. It is convenient to rewrite this operator in the charge retention order, by using Fierz rearrangement for coloured quarks:

\[
(\bar{b}c)_L (\bar{c}s')_L = \frac{1}{3} (\bar{b}s')_L (\bar{c}c)_L + 2(\bar{b}tA s')_L (\bar{c}A c)_L =
= (\bar{b}_\alpha s'_\beta)_L (\bar{c}_\mu c_\alpha)_L
\]

(9)

where $t^A$ are the colour matrices, with the normalisation $\text{Tr}(t^A t^B) = \frac{1}{2} \delta^{AB}$ (summation over repeated colour indices, either $A = 1,...,8$ or $\alpha,\beta = 1,2,3$, is understood). In the second equality the notation of ref.[15] is adopted and its relation with our definitions is clarified. In the following we shall neglect the $d$ admixture in $s'$. Under QCD renormalisation the operator $(\bar{b}c)_L (\bar{c}s)_L$ is transformed into [15],[16]:

\[
(\bar{b}c)_L (\bar{c}s)_L \rightarrow A (\bar{b}s)_L (\bar{c}c)_L + A' (\bar{b}tA s)_L (\bar{c}A c)_L +
+ B (\bar{b}s)_L (\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c)_L +
+ C (\bar{b}tA s)_L (\bar{u}tA u + \bar{d}tA d + \bar{s}tA s + \bar{c}tA c)_L +
+ D (\bar{b}s)_L (\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c)_R +
+ E (\bar{b}tA s)_L (\bar{u}tA u + \bar{d}tA d + \bar{s}tA s + \bar{c}tA c)_R
\]

(10)

According to eq.(9), at zeroth order in $\alpha_s$ one has $A = \frac{1}{3}, A' = 2, B = C = D = E = 0$. In the past, the effective Hamiltonian at the scale $m_b$ was obtained by evolving from $m_W$ down to $m_t$ with 6 excited flavours, then from $m_t$ down to $m_b$ with 5 excited flavours. Now we know that $m_t >
\( m_W \) and that \( \log \left( \frac{m_t}{m_W} \right) \) is not large: \( 0 < \log \left( \frac{m_t}{m_W} \right) < 1 \). Thus there is only one single large logarithm that has to be resummed, i.e. \( \log \left( \frac{M}{m_b} \right) \) with \( M \approx m_W \approx m_t \). The corresponding results for \( A, A', B, ..., E \) can be obtained from ref. [15] in the limit \( \alpha_S(m_t) = \alpha_S(m_W) \). One has

\[
A = \frac{2}{3} L'_+ - \frac{1}{3} L'_-
\]

\[
A' = L'_+ + L'_-
\]

where \( L'_+ \), \( L'_- \), with \( Q = \frac{\alpha_S}{\alpha_S(M)} \), coincide with \( L_+, L_- \) for \( M = m_W \). The remaining coefficients are given by [15]:

\[
B = -0.0060 \ Q^{0.8994} + 0.1111 \ Q^{12/23} - 0.0999 \ Q^{0.42299} + \ 0.0063 \ Q^{-0.14564} - 0.0952 \ Q^{-6/23} + 0.0837 \ Q^{-0.40861}
\]

\[
C = 0.0313 \ Q^{0.8994} - 0.3333 \ Q^{12/23} + 0.2429 \ Q^{0.42299} + \ 0.0053 \ Q^{-0.14564} - 0.1429 \ Q^{-6/23} + 0.1968 \ Q^{-0.40861}
\]

\[
D = -0.0179 \ Q^{0.8994} + 0.0196 \ Q^{0.42299} + \ 0.0267 \ Q^{-0.14564} - 0.0285 \ Q^{-0.40861}
\]

\[
E = -0.0923 \ Q^{0.8994} + 0.0478 \ Q^{0.42299} + \ -0.0224 \ Q^{-0.14564} + 0.0670 \ Q^{-0.40861}
\]

By a first order expansion in \( \alpha_s \) one can check that the above resummation is consistent, given eq.(6), with the leading logarithm appearing in the penguin Hamiltonian obtained from the full one-loop calculation [14],[17],[18]:

\[
H_{penguin} = -\sqrt{2} \ G_F \frac{\alpha_s}{12\pi} \ V_{bc} \left[ \log \frac{m_t}{m_c} - \frac{2}{2} \prod(x_t) \right] \cdot (btA_s)_{L} \ (ctA_c)_{L+R}
\]

Note the important minus sign in front of \( H_{penguin} \): it derives from \( V_{bt}V_{st}^* \approx -V_{bc}V_{sc}^* \approx -V_{bc} \). It is encouraging for the validity of the parton approximation that the contribution of small virtual momenta below \( m_c \) cancels between the top and the charm loops. The function
\( \Pi (x_t) \) (which is finite at \( x_t = 1 \)) , with \( x_t = \frac{m_t^2}{m_W^2} \), describes mass corrections \([17],[18]\):

\[
\Pi (x) = \frac{x^2 \log x}{4(1-x)^4} \left( 15-16x+4x^2 \right) + \frac{x}{8(1-x)^3} \left( 18-11x-x^2 \right)
\]  

(17)

In the following we will replace \( C \) and \( E \) by

\[
C', E' = C, E \cdot \left( 1 - \frac{\Pi (x_t)}{\log \frac{m_t^2}{m_c^2}} \right)
\]  

(18)

in order to take the \( m_t \) dependence of the one-loop diagram into account. In this way for \( C \) and \( E \) (which turn out to be the dominant terms) the order \( \alpha_s \) is exact and the leading logarithms are resummed to all orders.

The penguin induced \( b \to s \) transitions lead to the following contribution to the widths:

\[
\Gamma (b \to su \bar{u}) \equiv \Gamma (b \to s d \bar{d}) = 3F \left[ |B|^2 + \frac{2}{9} |C|^2 + |D|^2 + \frac{2}{9} |E'|^2 \right] \cdot I(m_s/m_b, m_u/m_b, m_u/m_b)
\]  

(19)

\[
\Gamma (b \to ss \bar{s}) = 3F \left[ \frac{4}{3} |B|^2 + \frac{4}{27} |C|^2 + |D|^2 + \frac{2}{9} |E'|^2 \right] \cdot I(m_s/m_b, m_s/m_b, m_s/m_b)
\]  

(20)

\[
\Gamma (b \to sc \bar{c}) = 3F \cdot J \left[ |A+B|^2 + \frac{2}{9} |A'|^2 + |D|^2 + \frac{2}{9} |E'|^2 \right] \cdot I(m_s/m_b, m_c/m_b, m_c/m_b)
\]  

(21)

By comparison, in the same notation, the width \( \Gamma (b \to \bar{c} u d') \), given in eq.(7), can be written down in the form:

\[
\Gamma (b \to \bar{c} u d') = 3F \cdot J \left[ |A|^2 + \frac{2}{9} |A'|^2 \right] \cdot I(m_c/m_b, m_d/m_b, m_u/m_b)
\]  

(22)
Note that the interference between the two identical s quarks modifies the coefficients appearing in $\Gamma(b\rightarrow s\bar{s}s\bar{s})$. Also we included the factor of J in eq.(21) in order to quench the scale dependence of the leading A and A' terms.

As is well known, QCD effects also induce $b\rightarrow s+\text{gluons}$ transitions [19],[20]. The parton process with one real gluon in the final state is due to an effective vertex of the magnetic moment type. The corresponding form factor is numerically small in lowest order. The reason is that, while penguin diagrams are of order $\alpha_s/\pi \ \log(m_t^2/m_c^2)$, the leading logarithm is missing in the magnetic form factor (the GIM suppression is logarithmic in the former case and by a power in the latter). This is why the contributions to $b\rightarrow s\bar{q}q$ from the magnetic form factor have been neglected in eqs.(19-21). According to ref.[21] the QCD corrections are large but act as a further suppression of the width. It has been suggested in ref.[19] that the process $b\rightarrow sg\bar{g}$ could be important because it can proceed via a penguin diagram with the three-gluon vertex at the lower end. However this large logarithmic contribution is canceled by the diagrams with the external gluons emitted by the quarks in the loop. In fact, the leading penguin operator $\bar{q}_\mu t^A q D^V \gamma^A_y v_\mu$, by the equation of motion, is identical to a four-fermion operator. Thus, at leading order, it has vanishing matrix element for two on-shell gluons. As a consequence, we expect all $b\rightarrow s+$ gluons channels to be small and we neglect them in the following.

As expected the penguin contributions $B,...,E$ turn out to be small (see e.g. $\Gamma(b\rightarrow s+\text{no charm})$ in the table). In particular B and D, which are absent at order $\alpha_s$, are quite negligible. As a consequence the dominant effect of penguin terms on the individual widths is due to the linear term, proportional to $A' \ C$, which is present in eq.(21) for $\Gamma(b\rightarrow sc\bar{c})$ arising from the interference between penguin and ordinary four-fermion operators. It is to be noted that this interference term is negative: recall the minus sign in front of the one-loop penguin Hamiltonian in eq.(16). In the total hadronic width the negative shift of $\Gamma(b\rightarrow sc\bar{c})$ by itself nearly balances the sum of small positive terms added to all other channels. It follows that the net effect of including penguin operators in the hadronic width or, equivalently, in $B_{SL}$, is completely negligible. Typically $\Gamma(b\rightarrow sc\bar{c})$ decreases by about 3-5% while $\Gamma(b\rightarrow \text{hadrons})$ and $B_{SL}$ vary by a few per mille. Quite often $\Gamma(b\rightarrow \text{hadrons})$ is actually decreased by penguin terms so that $B_{SL}$ is correspondingly increased.
We now specify the value of $\alpha_s$, the choice of the scale $\mu$ and the sets of mass ratios that we use in the following calculations.

We need to specify $\alpha_s(M)$ and $\alpha_s(\mu)$ with $M \sim m_W \sim m_t$ and $\mu \sim m_b$. Before LEP data, the prediction from low energy experiments on $\alpha_s(m_Z)$ was given by [22] $\alpha_s(m_Z) = 0.11 \pm 0.01$. At present the average result for $\alpha_s(m_Z)$ from the LEP experiments is [23] $\alpha_s(m_Z) = 0.119 \pm 0.008$. In the following we take $\alpha_s(m_Z)$ in the range:

$$\alpha_s(m_Z) = 0.115 \pm 0.010 \quad (23)$$

Once $\alpha_s(m_Z)$ is fixed we compute $\alpha_s(M)$ and $\alpha_s(\mu)$ by letting $\alpha_s$ run with the two-loop expression for 5 flavours. In going down to $\mu < m_b$ at some point we should pass from 5 to 4 flavours and make a continuous connection in $\alpha_s$ between the two regimes. We have checked that the details of this transition at the $b$ threshold are practically irrelevant.

In order to estimate the error associated with the scale dependence we vary $\mu$ between $m_b/2$ and $m_b$. The inclusion of the next-to-leading correction $J$ in the dominant terms of the hadronic width turns out to be very effective in reducing the $\mu$ dependence. We use the same scale $\mu$ both in the leading terms $L_{+,-}$ and in the non-leading corrections $\alpha_s f, J, B, ... E$. The upper scale $M$ is usually fixed to $m_W$. Varying $M$ up to $m_t \leq 200$ GeV in penguin diagrams changes these small terms by less than 10%.

The quark masses are chosen in one of two sets made up of typical values for current and constituent masses, respectively. For the light (heavy) masses we take the values, in GeV:

$$m_{u,d} = 0 \quad (0.16), \quad m_s = 0.15 \quad (0.30) \quad (24)$$

$$m_c = 1.2 \quad (1.7), \quad m_b = 4.6 \quad (5.0) \quad (25)$$

We now present the results of our analysis of the spectator model. We start with a discussion of the semileptonic branching ratio $B_{SL}$ which is measured with good precision. The results for $B_{SL}$ are shown in fig.1, where the dependence on the various input parameters is also displayed in detail in order to derive an estimate of the theoretical errors. The conclusions from fig.1 can be summarised as follows. For $\alpha_s$ in the range specified in eq.(23), $m_b/2 < \mu < m_b$ and the masses fixed as in eq.(24,25) one obtains:
\[ B_{SL} = \begin{cases} 
12.2 \pm 0.45 \text{ (scale)} \pm 0.8 (\alpha_s) \% \text{ (light masses)} \\
14.4 \pm 0.45 \text{ (scale)} \pm 0.8 (\alpha_s) \% \text{ (heavy masses)} 
\end{cases} \] (26)

(The error from \( \alpha_s \) is increased up to \( \pm 1.2 \% \) if the range of \( \alpha_s \) is extended from 0.10 to 0.13).

We have checked that, for all values of the input parameters (in particular of \( m_b \)), the resulting values of \( \Gamma_{\text{Tot}}/(m_b^5 |V_{bc}|^2) \) are compatible within the quoted errors with the experimental values [6] of the \( b \) lifetime and of \( |V_{bc}|^2 \) (the latter value as obtained [6] from \( B \to D\ell v \) by using a set of theoretical models for the exclusive form factors so that the corresponding information is indeed to a large extent independent of the data on the inclusive widths). We find that relatively large values of \( |V_{bc}| \) (still within the stated error bars) are favoured by this argument. This is a way of taking advantage of the data on \( \Gamma_{\text{Tot}} \) of which we cannot make better use because of the uncertainty on \( m_b \).

From fig.1 one clearly realises that the choice of masses is a main source of error. The experimental results on \( B_{SL} \), given in eqs.1-3, appear to strongly favour the "light" set of masses in eq.(24,25), i.e. of current quark masses. Actually in this case the central value of \( B_{SL} \) is considerably lower than previously estimated.

A list of branching ratios of inclusive \( b \) decays, obtained from our analysis of the parton model with spectators, both for "light" and "heavy" masses, for different values of the input parameters, is presented in table 1. For \( b \to c \) and \( b \to u \) transitions the largest source of error is the mass ambiguity. In some cases it amounts to a factor of 2. This is the case for \( B(b \to c\bar{c}s) \) which varies between 11.2 - 11.6 \% for "heavy" masses and 22.6 - 23.8 \% for "light" masses. Such a large theoretical error on this inclusive width may be indicative of the uncertainty to be expected on \( B(B \to \psi K) \), a mode of great interest for CP violation studies in \( B \) decays. Similarly the fraction of \( b \to u \) decays (for \( |V_{bu}/V_{bc}| = 0.1 \)) varies in the range 1.7 - 2.5 \% depending on the choice of masses. On the contrary, for \( b \to s \) transitions, due to penguin diagrams, the major ambiguity arises from the value and the scale of \( \alpha_s \). The overall uncertainty is very large: the total \( b \to s + \text{no charm} \) fraction varies between 0.15 and 0.9 \%.

From eq.(26) we see that, although the experimental value of \( B_{SL} \) can still be considered as marginally consistent with a pure parton approach with inert spectator(s), certainly there are strong hints of sizeable non-spectator contributions in the hadronic width. In view of
this we now discuss possible corrections to the simplest parton picture. We concentrate on \( B^0 \) and \( B^- \) decays which are theoretically simple and the most studied by experiment. It is a-priori conceivable that moderate non-spectator effects could arise from interference with the spectator quark [9] and/or from W-exchange [10] mechanisms. However there are experimental bounds on the importance of these contributions which we now consider in the following.

Interference [9] can occur in \( B^- \) (and not in \( B^0 \)) decays: the spectator \( \bar{u} \) can interfere with the \( \bar{u} \) produced in the final state of the decay \( b \to c \bar{c} \bar{u}d \). This effect leads to a negative correction in \( \Gamma^- \) (due to the suppression implied by the Pauli principle): \( \Gamma^- = \Gamma_{sp} - \delta \Gamma_I \), where \( \Gamma_{sp} \) is the total \( B^- \) width in the spectator approximation and \( \delta \Gamma_I > 0 \) describes the interference effect. W exchange [10] (often referred to as "annihilation") can take place in \( B^0 \) decays (and not in \( B^- \) if \( V_{bu} \) is neglected) and it adds a positive contribution to the total \( B^0 \) width \( \Gamma^0 \):

\[
\Gamma^0 = \Gamma_{sp} + \delta \Gamma_W. \]

A stringent limit on the quantity \( \frac{\delta \Gamma_W + \delta \Gamma_I}{\Gamma_{sp}} \) is obtained from the experimental data on the ratio \( \tau^-/\tau^0 \) of the \( B^- \) and \( B^0 \) lifetimes. On one hand, by combining the ARGUS and CLEO results on \( \tau^-/\tau^0 \) , derived by several different methods [24], one obtains:

\[
\frac{\tau^-}{\tau^0} \approx 1 + \frac{\delta \Gamma_W + \delta \Gamma_I}{\Gamma_{sp}} = 1.03 \pm 0.17 \tag{27}
\]

On the other hand, for the average semileptonic branching ratio of \( B^0 \) and \( B^- \), given that \( \Gamma_{SL}^0 \approx \Gamma_{SL}^- \), one derives, at first order, the relation

\[
B_{\text{SL}} = B_{\text{SL}}^{sp} \left( 1 - \frac{\delta \Gamma_W - \delta \Gamma_I}{2\Gamma_{sp}} \right) \tag{28}
\]

By comparison with eq.(27) one obtains, at 1\( \sigma \):

\[
| \frac{B_{\text{SL}}}{B_{\text{SL}}^{sp}} - 1 | < \frac{\delta \Gamma_W + \delta \Gamma_I}{2\Gamma_{sp}} < 0.10 \tag{29}
\]

We see that for a negative contribution to \( B_{\text{SL}} \) the relation \( \delta \Gamma_W > \delta \Gamma_I \) must be satisfied and that its size is in any case tightly constrained by the present experimental bound on \( \tau^-/\tau^0 \) as shown in eq.(29).
Theoretical estimates indeed support the dominance of $\delta \Gamma_W$ over $\delta \Gamma_I$. In fact interference effects are predicted not to exceed the 1% level in $B$ decays [25]. For $W$ exchange without accompanying gluon emission, one obtains, for $m_{u,d} = 0$ [8]:

$$\Gamma(b\bar{d} \rightarrow c\bar{u}) = F (2L_L - L^0)^2 8\pi^2 \left(\frac{f_B}{m_b}\right)^2 \left(\frac{m_c}{m_b}\right)^2 (1 - \frac{m_c^2}{m_b^2})^2 \quad (30)$$

Here $\left(\frac{f_B}{m_b}\right)^2$ is the typical suppression factor of non-spectator effects ($f_B$ being the pseudoscalar decay constant for $B$ mesons), while $\left(\frac{m_c}{m_b}\right)^2$ is the helicity suppression factor. For $f_B = 0.1 - 0.2$ GeV one obtains a very small branching ratio of order $10^{-5}$. A much larger rate is instead possible if $W$ exchange is accompanied by gluon emission, although a perturbative estimate of the corresponding rate is not reliable. In this case [10] the amplitude for gluon emission from a light quark leg is proportional to $1/m_{\text{light}}$. As a consequence the non-spectator suppression factor $\left(\frac{f_B}{m_b}\right)^2$ is replaced by $\left(\frac{f_B}{m_{\text{light}}/m_b}\right)^2$ which is of order 1 (with the crude identification of $f_B$ for colour octet and singlet $B$ meson components). The order of magnitude of the width is expected to be [10]:

$$\Gamma(b\bar{d} \rightarrow c\bar{u}g) \approx F \pi \alpha_s \left(\frac{f_B}{m_b}\right)^2 \quad (31)$$

The presence of the infrared sensitive parameter $m_d$ is a warning against attempting a more precise perturbative evaluation. A branching ratio of order 10% could be quite realistic in this case.

In conclusion, we have studied $B_{SL}$ in the parton model with inert spectators. Values of $B_{SL}$ as low as 11.5-12% are perfectly compatible with the model. Penguin diagram terms are small and their contributions to the total hadronic width are further suppressed by strong cancellations. Recent precise determinations of $B_{SL}$ favour even smaller values, around 10-11%. Non-spectator effects could well account for the difference. The most plausible candidate is $W$ exchange with gluon emission. If this is the explanation then the value of $\tau/\tau^0$ should eventually settle at around 1.2 (corresponding to a reduction of $B_{SL}$ by 10%). It is fair to recall that a similar approach for charm decays has never worked satisfactorily. A semi-quantitative understanding of charm decays in terms of inclusive widths described in the spectator
model plus interference and annihilation has not so far been successful. For example if the large difference between the $D^+$ and $D^0$ lifetimes is dominantly attributed to $W$ exchange with gluon emission, then one would expect a substantial fraction of $D_s$ decays into final states without strange quarks. Such modes have up to now not been detected in a sufficient proportion. In view of this it is particularly interesting to see whether the picture can be established to work for $b$ decays where the spectator approach is a much better approximation.

**Acknowledgements.** We are grateful to our colleagues H.Fritzsch, M.Lusignoli, L.Maiani, V.Sharma, K.Schubert and N.G.Uraltsev for stimulating discussions and important exchanges of information.
References

24) The quoted value of $\tau^+ / \tau^0$ is obtained by a combination of six
    independent measurements: $1.32 \pm 0.50$ (S. Stone, CLEO Coll.,
    Proceedings of the Neutrino Conference, Ginosar, Israel, 1989);
    $0.89 \pm 0.23$ (CLEO Coll., R. Fulton et al., Phys. Rev. D43 (1991) 651);
    $1.04 \pm 0.15$ (CLEO Coll., A. Bean et al., Phys. Rev. Lett. 58 (1987) 183);
    $1.04 \pm 0.39$ (S. Schael, K. Schubert, ARGUS collab., private comm.,
    February 1991); $1 \pm 0.32$ (ARGUS Coll., H. Albrecht et al., Phys.
    Lett. B232 (1989) 554; $1.04 \pm 0.6$ (Y. Kubota, CLEO Coll., Proceedings
    of the AIP 196 (1989) 142). The above results assume equal
    production of $B^-$ and $B^0$ at the U(4S). We thank Prof. K. Schubert
    for this list of data and references.
27) M. B. Voloshin and M. A. Shifman, Sov. Phys. JETP 64 (1986) 698;
Figure Caption

The semileptonic branching ratio $B_{SL}$ as a function of $\alpha_s(m_Z)$ (taken as a physical measure of $\Lambda_{QCD}$). The upper and lower bands refer to the sets of "heavy" and "light" masses, respectively, as given in eq.(24,25). In each band the upper curves (labelled a and c) correspond to the choice of renormalization scale $\mu = m_b$ while the lower curves (b and d) refer to $\mu = m_b/2$.

Table Caption

Branching fractions (in %) of $b$ decays in the spectator model for $\alpha_s(m_Z) = 0.105, 0.115, 0.125$. For fixed $\alpha_s(m_Z)$ each column refers to "light" or "heavy" masses as given in eq.(24,25) and to the choices of the renormalization scale $\mu = m_b$ or $\mu = m_b/2$. 
<table>
<thead>
<tr>
<th></th>
<th>b→cev</th>
<th>cτν</th>
<th>cūd</th>
<th>cēs</th>
<th>TOT b→c</th>
<th>b→uev</th>
<th>uτν</th>
<th>uūd</th>
<th>uōs</th>
<th>TOT b→u</th>
<th>b→sūu</th>
<th>sōc</th>
<th>TOT b→s (no charm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>13.0</td>
<td>15.1</td>
<td>12.5</td>
<td>14.7</td>
<td>12.5</td>
<td>14.7</td>
<td>11.8</td>
<td>13.9</td>
<td>11.9</td>
<td>13.95</td>
<td>10.7</td>
<td>12.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.85</td>
<td>3.1</td>
<td>2.7</td>
<td>3.0</td>
<td>2.7</td>
<td>3.0</td>
<td>2.6</td>
<td>2.9</td>
<td>2.6</td>
<td>2.9</td>
<td>2.3</td>
<td>2.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>46.6</td>
<td>52.7</td>
<td>47.4</td>
<td>53.5</td>
<td>47.4</td>
<td>53.5</td>
<td>48.5</td>
<td>54.8</td>
<td>48.3</td>
<td>54.7</td>
<td>50.2</td>
<td>56.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>22.6</td>
<td>11.2</td>
<td>22.7</td>
<td>11.3</td>
<td>22.8</td>
<td>11.3</td>
<td>23.1</td>
<td>11.4</td>
<td>23.3</td>
<td>11.55</td>
<td>23.8</td>
<td>11.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>98.1</td>
<td>97.3</td>
<td>97.9</td>
<td>97.0</td>
<td>98.1</td>
<td>97.2</td>
<td>97.8</td>
<td>96.9</td>
<td>98.1</td>
<td>97.1</td>
<td>97.6</td>
<td>96.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Light</th>
<th>Heavy</th>
<th>Light</th>
<th>Heavy</th>
<th>Light</th>
<th>Heavy</th>
<th>Light</th>
<th>Heavy</th>
<th>Light</th>
<th>Heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>μ=mb</td>
<td>μ=mb</td>
<td>μ=(\frac{m_b}{2})</td>
<td>μ=(\frac{m_b}{2})</td>
<td>μ=mb</td>
<td>μ=mb</td>
<td>μ=(\frac{m_b}{2})</td>
<td>μ=(\frac{m_b}{2})</td>
<td>μ=mb</td>
<td>μ=mb</td>
</tr>
<tr>
<td>a_s(m_z)</td>
<td>0.105</td>
<td>0.115</td>
<td>0.125</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>