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From: Carmen Vasini <vasini@sif.it>
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Dear Jens Vigen,

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Kindest regards.

Carmen Vasini
On the Meaning of Fermi Coupling.

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(ricevuto il 16 Febbraio 1956)

Various authors have attempted to account for elementary particles, starting from a theory of Fermi-type coupling between some Fermion fields. However, no quantum attempt has been clearly successful so far. We shall here investigate the simplest example, that of a scalar self-coupling of a spinor $\psi$, and show that this theory is identical with a Yukawa-type theory of a scalar meson whose experimental mass $\mu$ and coupling constant $G$ are given functions of the Fermion constants $\gamma$. This identity will be used to express the unrenormalized quantities of the Fermi-theory in terms of the renormalized ones. We assume that both theories have a solution and that the Yukawa-theory still has a meaning when the renormalization constants tend to the critical values given below.

Let $L_F = L_4(\mu, \gamma) + (g_0/2)(\bar{\psi}\psi) + \frac{1}{\alpha} A_\alpha A_\alpha \bar{\psi}\psi$ be the Fermi-Lagrangian, as a function of the unrenormalized quantities, where $L_4$ is the free particle of the Dirac Lagrangian; $S_F$ will be the corresponding $S$-matrix. The Yukawa-Lagrangian $L_Y = L_Y(m_0, \gamma) + + L_Y(\mu, \nu) + \frac{1}{\alpha} A_\alpha A_\alpha \bar{\psi}\psi$, again in unrenormalized form ($L_Y$ being the free meson part), will lead to an $S$-matrix $S_Y$, of which the matrix elements in which no real mesons appear are denoted by $\langle S_Y \rangle_{\alpha\beta\alpha\beta}$. Comparing the matrix elements of $\langle S_Y \rangle_{\alpha\beta\alpha\beta}$ proportional to $G^2$, with those of $S_F$ proportional to $g^2$, one sees that

\begin{equation}
S_F = \langle S_Y \rangle_{\alpha\beta\alpha\beta}
\end{equation}

provided that

\begin{equation}
g_0 = \frac{G^2}{p^2 + \rho^2},
\end{equation}

(*) European Organization for Nuclear Research.

(\dagger) This demonstration has been carried out in more general and physical cases (but using regularization to treat the infinities) and various consequences explored by the author in a former work (\dagger). The model given here is easily generalized for different types of Fermi-coupling, and couplings between different spinor fields.

Both $G^2_0$ and $\mu_0^2$ must be infinite if this equality is to hold; thus the (contact) Fermi-interaction is equivalent to that resulting from the exchange of mesons of infinite mass $\mu_0$, with coupling constant $G_0$, also infinite. The point of interest is, however, that both these parameters are the unobservable unrenormalized ones. Because of the occurrence of $g_0$ in the divergent quantities, care must be taken to show that (2) actually leads to (1). By using a limiting process with a cut-off on the Fermion momentum, $p^2 < A$, one can give this proof providing that $G^2_0(A)$ and $\mu^2_0(A)$ satisfy

$$\left( \frac{\partial}{\partial A} \left( \frac{1}{G^2_0(A)} \right) \right)_{A \rightarrow \infty} = a = \text{const} \; ; \; \frac{\mu^2_0(A)}{g_0} = \frac{G^2_0(A)}{g_0}.$$  

Let us now deal with the renormalized quantities. If we assume that the usual renormalization constants of the Yukawa-theory are determined as functions of the observable parameters $(m, \mu, G)$, only the meson quantities may be renormalized at first

$$A_0 = Z_\tau \frac{1}{A}, \quad \mu^2_0 = \mu^2 + (\delta \mu^2)/Z_\tau, \quad G_0 = G_\tau Z^{-1}_\tau Z_\tau^{-1} G.$$  

Expressing $\langle S_Y \rangle_{\text{meson}}$ in terms of these quantities, the equality (1) holds when

$$Z_\tau = 0, \quad (\delta \mu^2) = G^2_\tau/g_0,$$  

and condition (3) leads to

$$\frac{\partial Z_\tau(A)}{\partial A} = aG^2_\tau/A.$$  

The unitarity of $S_\nu$ and $S_Y$ can be shown to imply that

$$S_Y|_{\text{meson}} = 0,$$  

which is the case if $\mu > 2m$ so that the meson has a finite life-time; $A_\nu$ then has a complex pole, and $\delta \mu^2$ is defined as the real part of the usual quantity $^{(*)}$. One may then renormalize the $\nu$ field in the usual way. Finally, one may renormalize the constant $g_\nu$, which is the value of the total Fermi-interaction between free particles when the exchanged momentum tends to zero, with

$$g_0 = g_\nu Z^2_\tau Z^{-2}_2; \quad g_\nu = G^2(\mu^2 + iG^2 R^Y_{\text{m}}(p^2 = 0))^{-1}.$$  

$K^Y_{\text{m}}(p^2)$ is the meson renormalized polarization operator. From this treatment one obtains two relations that we expect would determine the observable parameters

$$Z_\nu(m, \mu, G) = 0; \quad (\delta \mu^2(m, \mu, G))Z^2_\nu(m, \mu, G)g_\nu = G^2Z^2_\nu(m, \mu, G).$$  

$^{(*)}$ A similar case has been studied in detail in connection with the Lee model by V. Glaser (Colloquium, October 31st, 1955, Copenhagen).
Analogous equations should enable one to calculate, in principle, physical constants of elementary particles, such as electric charge, through the appropriate modifications of the old schematic model.

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