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DESIGN OF CERN SYNCHRO-CYCLOTRON MAGNET

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European Organization for Nuclear Research
CERN
GENEVA
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NOTE:

The present report contains the work for which — as a member of the CERN Synchro-cyclotron group — the author was mainly responsible. It covers the period July 1952 — October 1954. The work was discussed at several meetings of the CERN Synchro-cyclotron group during this period. The CERN Synchro-cyclotron group was composed of:

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SUMMARY. The procedure of design for the CERN 600 MeV cyclotron magnet is described with formulas, supported by model experiments, for calculation of fringing flux and shim shape, comparison with existing cyclotron magnets, analysis of cylindrical against tapered poles and economical study of designs with aluminium and copper in different sizes of coils. The resulting magnet has 5 m. diameter cylindrical poles and 45 cm. gap, a core weight of 2500 tons and a power demand with aluminium coils below 800 kW.

For a given particle energy in a cyclotron the product Br at exit radius is given by the relation

\[ E^2 - E_0^2 = (10^{-6} \cdot c \cdot Br \cdot q/e)^2 \]  

(1)

\[ E = E_0 + E_k \]  
Particle energy (MeV)

\[ E_0 \]  
Particle rest energy (MeV)

\[ q/e \]  
Number of unit charges of particle

\[ B \]  
Magnetic flux density (Wb/m^2)

\[ r \]  
Radius (m)

\[ c \]  
Velocity of light (299.8 \cdot 10^6 m/s)

The design value for the CERN synchro cyclotron is 600 MeV protons \( E_0 = 938 \) MeV), which gives \( Br = 4,066 \) Wb/m. The desired particle energy can be achieved either at a big radius and low flux density or at a smaller radius but higher density, the choice
being required to give a convenient magnet from the experimental point of view at a reasonable cost. As the cost of the magnet amounts to a third or more of the total cost of the machine, economy is an important consideration in design.

To keep the protons from the ion source focused on the median plane it is important that the field strength in the gap decreases continuously from the centre outwards. In the centre, where $dB/dr$ is zero, the number of protons that will be available for acceleration increases with the opening of the Dee, with the electric field strength near the Dee and with the second derivative $d^2B/dr^2$. To get a good beam current it is therefore desirable to have the largest possible gap, allowing a big opening in the Dee and high Dee voltage. In the centre big spikes from the poles improve the value of $d^2B/dr^2$.

From operating synchro cyclotrons it is found that most of the beam is lost in the vertical direction by a resonance coupling between vertical and radial oscillations in the region where $n = 0.2$

$$n = -\frac{r}{R} \cdot \frac{dB}{dr} \quad (2)$$

The value of $n$ should therefore be between zero in the centre and 0.2 at the largest possible radius. Particles accelerated beyond $n = 0.2$ (and other resonances at $n = 0.25$ and $n = 0.5$) will reach a theoretical maximum energy where $n = 1$. The design energy 600 MeV protons shall however correspond to $n = 0.2$.

Considerable information can be gained from existing cyclotron magnets. If the dimensions are scaled up the necessary number of ampere turns for the same field strength in the gap goes linearly with the scale factor. Flux densities in corresponding parts will be identical.

In Tables I and II performances and dimensions are given for some cyclotrons: Chicago (at two coil currents), Pittsburgh and Liverpool, as well as the same cyclotrons scaled up to 600 MeV. On figure 2 the latter are shown to the same scale. The pole base diameter $D$ is about 5 m for all. The gap is largest in the Chicago type, least in the Pittsburgh type, with the Liverpool type in between. To facilitate experiments and introduction of the Dee system the minimum gap $g$ should not be less than 360 mm. The maximum gap $G$ was chosen to be 450 mm. These values are about the same as in Chicago before scaling. In order to have the power demands for the 600 MeV magnets comparable a space factor of 0.5 and an average conductor temperature of 40°C have
been assumed. Coil designs resulting in lower power are possible.

The Pittsburgh type g has the lowest amount of iron and copper and demands least power, even with the reasonable space factor 0.5 instead of the exceptional value 0.787 attained in the original design. This has only a low number of turns (48) in the coils and demands a very high current, over 20 000 amps. It is felt that the supply for a more reasonable current, say below 2 000 amps at normal running, should be more reliable than the homopolar generator first used in Pittsburgh. The latter has been replaced by a transductor-regulated selenium rectifier, a system which also has some disadvantages, especially when a high accuracy in regulation is desired.

Cylindrical contra tapered poles. To see if tapered poles as in Pittsburgh design have any considerable advantages over cylindrical poles, model experiments have been performed. The dimensions of the model magnets, scaled up values of which are given in Table II j, k, are identical except that j has the part of the poles projecting out of the coils tapered to 90% of the pole base diameter, from 280 to 252 mm. For this experiment horizontal yokes, poles and coils from the Uppsala 200 MeV cyclotron model were used. The pole tips were shaped to give approximately 5% drop in fieldstrength from centre to exit radius at a current corresponding to 600 MeV. The values found are given in Table 1. The number of ampereturns needed for conical poles, \(1.3 \cdot 10^6\), is even higher than for cylindrical poles \(1.2 \cdot 10^6\). Neither corresponds to the best possible shape of the magnet pole tips, but the conclusion can be drawn that tapering of the poles alone gives no appreciable reduction in power demand for the same particle energy. The low consumption and low weight of the Pittsburgh magnet must therefore mainly be ascribed to the rather short gap, especially at the edge, and to the short poles and wide yoke.

Cylindrical poles have some advantages over conical pole tips (assuming the same pole base diameter, corresponding to about the same cost of magnet):
1. Easier and safer joint with the vacuum chamber.
2. Lower field strength in the gap allowing more effective magnetic channels for beam extraction.
3. More rapid reduction in fieldstrength outside the pole tips making some meson experiments simpler.

On these considerations it was decided to have cylindrical poles, and to design the magnet for the lowest cost consistent with good accessibility for experiments. The free height between coils (H) should not be less than 1.2 m and there
should be a 0.4 m wide walking space between the coils and the vertical yokes. To find a suitable design at low total cost different designs have been calculated along lines given below. These calculations have been supported by experiments where approximations are necessary.

**Required number of ampereturns.** The number of ampereturns inside a closed fluxline is given by

\[ NI = \int H_s \, ds = \sum H_n \cdot 1_n = \sum \frac{j_n}{\mu_r \cdot \mu_0} \cdot \frac{1_n}{a_n} \]  

(3)

The necessary number of ampereturns can therefore be calculated by subdividing the magnet core in a few parts, conveniently chosen as:

1. **Air gap**
2. **Poles**
3a. **Horizontal yokes**
3b. **Vertical yokes**

These sections are handled successively below:

1. **Air gap.** As the gap and field strength in the gap vary at different radii, and the iron has finite permeability, the number of ampereturns in air is not constant. The most representative value is at maximum gap \( G \) which usually is at about half exit radius. For 5% decrease in field strength from centre to exit radius the ampereturns for the gap is (allowing 0.5 % for the rounding off of fieldshape near the centre and the edge):

\[ NIA = 1.93 \cdot B_{0.2} \cdot G/\mu_0 \]  

(4)

2. **Poles.** The flux map for the upper pole of a cyclotron magnet is sketched on figure 3. The flux in the poles is composed of three components:

a. The flux inside \( n = 0.2 \). Assuming 5% drop from centre and adding 0.5 % for the curvature at the centre and at \( n = 0.2 \) one can put

\[ \phi_{0.2} = 1.022 \cdot \pi R_{0.2}^2 \cdot B_{0.2} \]  

(5)

b. The boundary flux entering the shim outside \( n = 0.2 \). The shim is: in
considered as the part of the pole tip with less than maximum gap.

The average value of \( B^p \) in this region is about 95% of \( B_{0.2} \cdot R_{0.2} \)
Exit radius \( R_{0.2} \) is about 0.5 G inside \( R \) (the pole radius measured
G/2 from median plane), decreasing with the ratio \( g/G \) and increasing
with the saturation at the pole edge. The outer limit of the boundary flux
will in the median plane be about 0.26 G outside \( R \). A reasonably accur-
ate value for this flux is

\[
\Phi_B = 0.95 \cdot (0.50 + 0.26)G \cdot B_{0.2} \cdot 2\pi R_{0.2} = 0.72 U \quad (6)
\]

where \( U = 2\pi R_{0.2} \cdot R_{0.2} G \) \( (7) \)

in the following will be called "Unit of fringing flux".

c. Flux entering the side of the pole.

The flux density at the side of the pole decreases with the distance \( z \)
from the median plane almost linearly.

Assuming the fluxlines were semicircles and the poles equipotentials with
a magnetomotive force \( B(\frac{R}{2}) \cdot G/\mu_0 \) one finds the flux to be

\[
2 R B G \int_{\frac{z}{2}}^{\frac{z}{2}} \frac{dz}{z} = 2 R B G \log_e \frac{2z}{G} \quad (8)
\]

Substituting for RB the approximately 12% lower value \( R_{0.2} \cdot B_{0.2} \)
and allowing a further 25% for the real, three dimensional shape of
the flux lines one finds, in good agreement with model experiments where
the flux at different heights of the pole has been measured :

\[
\Phi_s (z) = 0.45 \cdot 2\pi R_{0.2} \cdot B_{0.2} \cdot G \cdot \log_e \frac{2z}{G} = 0.45 U \log_e \frac{2z}{G} \quad (9)
\]

For tapered poles a correction factor considering the reduction in
length of flux lines must be applied.

Where the pole is surrounded by the coils the available m.m.f. decreases
gradually to zero at about 1/3 coil height from the yoke. In the vicinity of the yoke
some fluxlines will pass from the pole to the yoke through air. The driving m.m.f.
is here nearly proportional to the field strength in the iron near the pole base, as
only a few turns of the coil will be enclosed by the fluxlines. With an average field
strength \( \frac{\text{At}}{\text{m}} \) in the part of the pole considered, an approximate value of this flux is:

\[
d\phi_c = \mu_0 n_i \left( \frac{C}{2} - z \right) \cdot \frac{2\pi R \cdot 2}{w(C/2 - z)} \cdot dz \\
\phi_c = 4 \mu_0 n_i (z_2 - z) \quad (10)
\]

The constant \( z_2 = H/2 + \frac{2}{3} (C/2 - H/2) \)

As \( \phi_c \) is only a correction to \( \phi_p \), a rough value for \( n_i \) is sufficient, making calculation easier.

At the pole base the flux is

\[
\phi_{pb} = \phi_{0,2} + \phi_B + \phi_s \left( \frac{C}{2} \right) - \phi_c \left( \frac{C}{2} \right) \quad (12)
\]

If the coil has a soft iron steel flange of thickness \( J \) the values \( C \) should be replaced by \( C = 2J \).

Consequently the following formulas can be used to find the flux in different parts of the pole \( \phi(z) = \phi_{0,2} + \phi_B + \phi_s \)

For \( G/2 < z < z_1 = H/2 \):

\[
\phi_s = 0.45 \cdot U \cdot \log_{e} \frac{2z}{G} \quad (13)
\]

For \( z_1 < z < z_2 = \frac{1}{6} H + \frac{1}{3} (C - 2J) \):

\[
\phi_s = \phi_s(z_1) + 0.45 U \cdot \frac{z_2}{z_2 - z_1} \cdot \log_{e} \frac{z}{z_1} - 0.45 U \frac{z - z_1}{z_2 - z_1} \quad (14)
\]

For \( z_2 < z < z_3 = \frac{1}{2} (C - 2J) \):

\[
\phi_s = \phi_s(z_2) - 4 \mu_0 R n_i (z - z_2) \quad (15)
\]

The maximum value of the flux in the pole is
\[ \phi_p = \phi_{0.2} + \phi_B + \phi_s (z_2) \]  

(16)

3. **Yokes. External flux.** The flux in the pole, \( \phi_p \), will return mostly through the yokes, flux \( \phi_h \) in the horizontal yokes, \( \phi_v \) in the vertical yokes. A certain part of \( \phi_p \) will however return through air directly from upper to lower yokes, causing a stray field around the magnet. The flux \( \phi_p - \phi_v \) can be calculated by comparison with a homogeneously magnetized ellipsoid of the same volume and cross section, and with the same total magnetic potential difference.

**Equation of the ellipsoid:**

\[ \left( \frac{z}{b} \right)^2 + \left( \frac{\phi}{a} \right)^2 = 1 \]

\[ a^2 - b^2 = \phi_0^2 \]  

(17)

**Volume**

\[ V = \frac{4}{3} \pi a^2 b = A \cdot B \cdot (C + 2E) \text{ m}^3 \]  

(18)

**Horizontal cross section**

\[ \pi a^2 = AB \text{ m}^2 \]

**Magnetic moment**

\[ m = \frac{\phi V}{\mu_0 \pi a^2} \text{ At} \cdot \text{ m}^2 \]  

(19)

The magnetic potential is

\[ \varphi = \frac{m}{4\pi} \cdot \frac{3}{\phi_0^3} \left[ 1 - \frac{\sqrt{u - \phi_0^2}}{\phi_0} \arcsin \frac{\phi_0}{u} \right] \sqrt{\frac{\phi_0^2 - v^2}{\phi_0^2 - v^2}} \]  

(20)

or

\[ \varphi = \frac{m}{4\pi} \cdot \frac{z}{(z^2 + \phi^2)^{3/2}} + \ldots \]  

(21)

with only first term shown.

The elliptic coordinates \( u \) and \( v \) can be derived from

\[ u = \frac{uu}{\phi_0} ; \quad z = \frac{\sqrt{u^2 - \phi_0^2} \sqrt{2 - v^2}}{\phi_0} \]  

(22)
u = a gives the equation of the ellipsoid.

v = 0 is the z axis.

\( v = \frac{\rho}{c} \) is the median plane.

Now the magnetic potential difference shall be

\[
Nl_v + Nl_h = \mathcal{V}(+b) - \mathcal{V}(-b) = 2 \cdot \mathcal{V}(+b) =
\]

\[
= \frac{2m}{4\pi} \cdot \frac{3}{\rho^2} \left[ 1 - \frac{b}{\rho} \arcsin \frac{\rho}{a} \right] = \frac{\frac{m}{2}}{\frac{3\gamma}{\pi} \frac{a^2}{b^2}}
\]

where \( \gamma \) is the demagnetization factor

\[
\gamma = \frac{a^2}{\rho^2} \left[ 1 - \frac{b}{\rho} \arcsin \frac{\rho}{a} \right] \sim 1 + \frac{2}{15} \left[ 1 - \left( \frac{b}{a} \right)^2 \right] \text{if } b \approx a.
\]

For the magnet body the corresponding demagnetization factor is

\[
\gamma = \frac{1}{3} + \frac{2}{15} \left[ 1 - \frac{9\pi}{16} \left( \frac{C + 2E}{AB} \right)^2 \right]
\]

The flux through the median plane inside the ellipsoid = flux outside ellipsoid (or magnet) (eg 19, 21, 18):

\[
\phi_p - \phi_v = \phi_i = \frac{3 \mu m}{4b} = \frac{\mu}{2} \cdot \frac{\pi a^2}{b} \cdot (Nl_v + Nl_h) =
\]

\[
= \frac{2}{3\gamma} \cdot \frac{AB}{C + 2E} \cdot (Nl_v + Nl_h)
\]

The flux in the horizontal yokes is assumed to be the average of flux in the pole and in the vertical yokes

\[
\phi_h = \frac{1}{2} ( \phi_p + \phi_v )
\]

By trying a few values \((Nl_v + Nl_h)\) the correct value for given \( \phi_p \) can be found.

The vertical field strength component at large distances from the magnet is

\[
H_z = -\frac{d\phi}{dz} = -\frac{\gamma^2}{2 - 2\gamma^2} \frac{m}{4\pi} \left( \frac{m}{4\pi} \right)
\]
In the median plane:

\[ H = -\frac{m}{4\pi f^3} \quad (27) \]

Along the z-axis:

\[ H = \frac{3m}{4\pi z^3} \quad (28) \]

Here is

\[ m = \frac{2}{3y} \cdot A \cdot B \cdot (n_l + n_l_h) \quad (29) \]

Horizontal yokes. Length of flux lines and area:

\[ l_h = A - 0.9D - 2F + \pi E \quad (30) \]

\[ a_h = 2 \cdot B \cdot E \quad (31) \]

Vertical yokes. Length of flux lines and area:

\[ l_v = C \quad (32) \]

\[ a_v = 2BF \quad (33) \]

The distance \( A - 2F \) is given by the diameter of coil \( K \) and the space between coil and vertical yokes taken 0.45 m to allow easy access to the vacuum chamber from all sides. To reduce the length of the flux lines \( E \) should be small. \( B \) must then be large to give the wanted area. The minimum value of \( E \) must allow sufficient area in the boundary between pole and yoke:

\[ \pi D \cdot (E + J) > 0.25 \pi D^2 \]

or

\[ E + J > 0.25D \quad (34) \]
Design for minimum cost.

In order to find the most economical design of the magnet one must consider the costs of a) iron, b) coils and c) power dissipated. The sum of these costs during the period considered should be as low as possible, which is the aim of the following investigation.

Cost of Iron Core. The cost of iron core can be estimated to 1.8 Sw. Francs per kg. Special manufacturing problems related to the considerable size of the magnet may however cause the cost to rise rapidly with dimensions. The average magnetic quality of the steel should at least correspond to "Low Carbon Steel" in the magnetization curves Figure 4.

Cost of Coils and Power. The main coil dimension are according to Figure 1.

Outside diameter $K$

Inside diameter $D$

Height for two coils $C-H$

The volume of the coil is $W = \frac{\pi}{4} \left(K^2 - D^2\right) \left(C - H\right)$ (35)

The resistance is $R = \frac{\rho}{\alpha} \cdot \frac{\pi(K + D)}{(K - D)(C - H)} N^2$ (36)

where $\rho = \text{resistivity of winding (ohm m)}$.

$\alpha = \text{space factor}$

To supply $NI$ At the power consumption will be

$$P = \frac{(NI)^2}{N^2/R} = \frac{\rho}{\alpha q} (NI)^2 = \frac{\rho}{\alpha q} \left(NI_A + NI_p + NI_v + NI_h\right)^2$$ (37)

where $q = \frac{(K - D)(C - H)}{\pi(K + D)}$

The value $\frac{\alpha}{\rho}$ should be large to get a low power consumption but is limited by cost of coil and available window area of iron core.

The cost of power should consider:

a) primary power delivered as high voltage (18 kV) three phase current.

The primary power at the kWh-meter may be 10 to 15% higher than the coil power because of losses in b) and c).

b) motor-generator set

c) transformers to take down primary voltage to voltage suitable for MG-set (6 kV).
d) cost of cooling.

It is expected that the magnet will be running up to 4000 hours a year.

The number of years considered and the interest to be calculated is rather arbitrary. If the cost of primary power including all charges is 0.05 Sw. Francs/kWh it is reasonable to calculate with a total cost of power = 2 000 Sw. Francs/kW, which will cover running for at least 10 years.

For the coils two alternatives are considered: copper or aluminium winding, in both cases with conductor of square or rectangular cross section tubes with inside watercooling by purified water. An external heat exchanger transfers the heat to ordinary tap water. Aluminium tubes can be extruded in lengths of hundreds of metres whereas the lengths of copper tubes is limited by the size of the ingot, 50 to 100 kgs, that can be placed in the extrusion press. The lengths of copper, 5 to 10 m long, have to be silver soldered to each other, which increases the already high cost of the material. Aluminium is considerably cheaper so that most of the cost of the coils will be insulation material and winding work. A further advantage for aluminium is the lower weight, which will facilitate transport and erection. If the cost of the coils is put proportionate to the weight of the conductor:

For copper: Density = 8.92 \cdot 10^3 \text{ kg/m}^3

\text{Resistivity} = 0.0186 \cdot 10^{-6} \text{ ohm \cdot m (40^\circ C)}

Cost of coil : 10 Swiss Francs per kg of conductor

For aluminium: Density = 2.70 \cdot 10^3 \text{ kg/m}^3

\text{Resistivity} = 0.0294 \cdot 10^{-6} \text{ ohm \cdot m (40^\circ C)}

Cost of coil : 12 Swiss Francs per kg of conductor

For Power : 2000 Swiss Francs per kW

For iron core : 1.8 Swiss Francs per kg

Total costs for different designs. These costs and the earlier derived formulas have been applied on a dozen different designs of 600 MeV SC-magnets. All have the same gap, max. 0.45 m, min. 0.36 m, and the same distance between coils. The effect of different coil sizes is found by changing the height of the coils rather than their outside diameter, which is kept constant at 7.2 m, close to the limit that can be transported to the site. The results are found in Table III. Typical curves are drawn in figure 5.
The optimum should with the assumed relative costs give a total cost for
the magnet including power of about 6.6 M Sw. Frs. slightly lower for aluminium
and slightly higher for copper coils. The corresponding costs for the scaled mag-
nets in Table I and II are given below:

<table>
<thead>
<tr>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>l</th>
</tr>
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<tbody>
<tr>
<td>With copper coils</td>
<td>9.39</td>
<td>9.36</td>
<td>6.80</td>
<td>10.23</td>
</tr>
<tr>
<td>With aluminium coils</td>
<td>8.74</td>
<td>8.98</td>
<td>6.79</td>
<td>9.17</td>
</tr>
</tbody>
</table>

It is to be remembered that in these magnets the airgaps and the access-
ibilities of the experimental area are not equal.

**Final choice of main dimensions.** Under given assumptions there is a
wide choice of magnet dimensions giving nearly the minimum cost. The theo-
retical optimum should be a pole diameter of about 5.2 m and a pole height between
mild steel coil flanges (C - 2J) of about 2.8 m for aluminium coils and a little
lower for copper coils.

An increase of the size of the poles over 5 m diameter would however
be considerably more expensive than the presumed cost per kg, as they can be
forged in nothing but the biggest forging presses and even there only with diff-
culties. The more modest pole diameter 5.0 has therefore been chosen for the
CERN-SC magnet as seen from line 1 in Table II.

**Design of shimming.** The main dimensions of the magnet being fixed it
remains to shape the shimming to get an uniform decrease in field strength from
the centre outwards and to get the exit radius at largest possible radius. Because
of the saturation near the edges of the pole tips the field strength in the gap would
decrease much too rapidly if the shimming were not applied to make the gap max-
imum about half way out and reduced near the edge. The minimum gap should how-
ever be wide enough to give ample space for experiments and for magnetic channels
for extracting the beam. It has been possible by model experiments to attain 91 %
of pole radius with minimum gap only 20 % less than maximum gap (in Chicago 22%,
in Liverpool 26 % and in Pittsburgh even 37.5 % decrease in gap).

The procedure has been to machine approximately correct shimming and
to measure the resulting field shape. The necessary changes of shimming to com-
pensate the deviations from desired field shape can be estimated as described be-
low in conjunction with Fig. 6. Except at the central part of the pole tips the field
can be considered as two-dimensional.
If the shimming is increased in thickness by $t$, Fig. 6 a, the field strength in the gap will increase from $H_0$ (assumed almost constant over the width 2 $b$) to $H = H_0 + H_x$. The value of $H_x$ can be found by comparison with the field from a set of four straight wires, Fig. 6 b, with current.

$$ I = t \cdot H_0 \cdot \frac{\mu_r - 1}{\mu_r} = t \cdot \frac{B_0 - \mu_0}{\mu_0} \frac{H_{Fe}}{u_o} \rightarrow t \frac{B_s}{\mu_0} \text{ if } B_o \gg B_s \quad (38) $$

These currents alone would give a field

$$ H_x' = \frac{1}{\pi h^2} \left[ \frac{b + x}{1 + \left( \frac{b + x}{h} \right)^2} + \frac{b - x}{1 + \left( \frac{b - x}{h} \right)^2} \right] \quad (39) $$

which can be written:

$$ H_x' = \frac{2bt}{\pi \mu_0 h^2} \cdot \frac{H_0 (\mu_r - 1)}{F} \rightarrow 2bt \cdot \frac{B_s}{\pi \mu_0 h^2} \cdot \frac{F'}{(40)} $$

Here the influence of the proximity of iron in the pole tips has been neglected, which is fair at high saturation.

At low field strength the pole tip surfaces can be regarded as equipotentials and the resulting field correction can be calculated as caused by an indefinite set of mirror images, Fig. 6 c.

$$ H_x^{''} = \frac{2}{h^2} \left[ \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^2 + \left( \frac{b + x}{h} \right)^2} \right] + (b - x) \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^2 + \left( \frac{b - x}{h} \right)^2} $$

$$ = \frac{2}{\pi \mu_0 h^2} \cdot F $$

The sums can be written

$$ \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^2 + a^2} \simeq \frac{1}{1 + a^2} + \frac{1}{3^2 + a^2} \ldots + \frac{1}{(2m - 1)^2 + a^2} + \int_{m}^{\infty} \frac{dn}{(2n)^2 + a^2} \quad (42) $$

where

$$ \int_{m}^{\infty} \frac{dn}{(2n)^2 + a^2} = \frac{1}{2a} \arctg \frac{a}{2m} $$

With $m = 1$ the error is less than 1.5 %
For \( a = 0 \) the value of the sum is \( \Pi^2/8 = 1.2337 \ldots \)
so that for \( b = x = 0 \)
\[
F''(0) = \frac{\Pi^2}{4}
\]

In Fig. 6 d curves are given for \( F'' \) and \( F' \) for some values of \( b/h \). Generally the saturation of pole tips is neither complete nor negligible. A practical way to proceed is to use formula 40 where saturation is high and by experiment determine \( H_0 (\mu_r - 1)/\mu_r \) to fit at the corresponding radius. A value slightly higher than the saturation value of the iron may be found. Closer to the centre formula 41 with correspondingly chosen constant gives better agreement.

Within one airgap distance from the centre the twodimensional formulas are not applicable. To calculate the field correction from the iron spikes in the centre these can be regarded as magnetic dipoles, each of magnetic moment

\[
m = V \cdot H_0 \cdot (\mu_r - 1)/\mu_r
\]

The corresponding field correction in the median plane, neglecting the vicinity of the pole surfaces, is

\[
H'_{rr} = \frac{m}{\Pi h^3} \cdot \frac{1}{[1 + (r/h)^2]^{3/2}} \cdot \left[ 1 - \frac{1.5}{1 + (r/h)^2} \right] = \frac{VH_0 (\mu_r - 1)}{\Pi \mu_r h^3} \cdot G' \tag{44}
\]

With the pole surfaces considered as equipotentials the formula converts into:

\[
H''_{rr} = \frac{m}{\Pi h^3} \left[ \frac{1}{[1 + (r/h)^2]^{3/2}} + \frac{1}{[3^2 + (r/h)^2]^{3/2}} + \cdots \right. \\
- 1.5 \cdot \frac{1}{[(1 + (r/h)^2)^{5/2}] + \frac{1}{3^2 + (r/h)^2}^{5/2}} + \cdots \left. \right] \tag{45}
\]

\[
H''_{rr} = \frac{V H_0 (\mu_r - 1)}{\Pi \mu_r h^3} \cdot \mathcal{G}''
\]

Curves for \( G' \) and \( \mathcal{G}'' \) are given in Fig. 6 e.

With this method the shape of the shimming and field strength given in Fig. 7 has been found on the 1 : 10 magnet model with a steel quality of approximately 2.06 Wb/m² at 300 A/m. With different steel qualities small changes in total ampere
turns and in slope of the curves will follow. It is important that the machining tolerances are close and that the quality of the steel is constant in all parts facing the airgap to avoid irregularities in the field shape which could reduce the expected performance of the synchro cyclotron.
Literature


Appendix 1

Table I  Performance and power demand for Synchro-Cyclotron magnets

Table II  Main dimensions of Synchro-Cyclotron magnets

Table III  Calculated costs for different magnet designs

Table IV  CERN Synchro-Cyclotron magnet
Table I.

Performance and Power Demand for Synchro Cyclotron Magnets.

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Drafts for CERN-SC Magnets

- Tapered poles: 600 4.066 1.91 2.13 ~1.3 20.7 1.99 0.5 20.6 Al 1.63 1035
- Cylindrical poles: 600 4.066 1.80 2.26 ~1.2 20.7 1.99 0.5 20.6 Al 1.63 885
- Cylindrical poles: 600 4.066 1.79 2.27 1.17 19.2 1.76 0.5 16.8 Al 1.56 875

For Cu: $\gamma = 8.92 \cdot 10^3$ kg/m$^3$; $\rho_{40} = 0.0186 \cdot 10^{-6}$ Ω·m
For Al: $\gamma = 2.70 \cdot 10^3$ kg/m$^3$; $\rho_{40} = 0.0294 \cdot 10^{-6}$ Ω·m
Table II

Main Dimensions of Synchro-Cyclotron Magnets

Denotations from Figure 1.

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Drafts for CERN SC Magnet Models.

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non magnetic material
Table III

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Table IV
CERN-SC Magnet

Pole: diameter 5.00 m, area 19.6 m²
Horizontal yoke: area incl. coil flange 20.8 m²
  pathlength 11·0.9 · 5·2 · 1.5 +π·1.5 = 8.2 m
Vertical yokes: average area 18.4 m²
  pathlength 3 m
Gap: G = 0.45 m; g = 0.36 m
R₀.₂ = 2.27 m B₀.₂ = 1.79 Wb/m²
NI₀ = 1.03 · 1.79 · 0.45 · 10⁶/0.4π = 0.66 · 10⁶ At
B₀.₂ · R₀.₂ = 4.06 Wb/m
Unit of fringing flux: U = 2π · 4.06 · 0.45 = 11.45 Wb
ϕ₀.₂ = 1.022 · π · 2.27² · 1.79 = 29.8 Wb
ϕ₂ = 0.72 · 11.45 = 8.2 Wb
Flux entering pole tip 38.0 Wb

Assumed steel quality: 2.05 Wb/m² at 30 000 At/m
Flux density in tip 1.94 Wb/m² (17 000 At/m)
H = 1.2 m. . . . . . z₁ = 0.6 m
ϕₛ(z₁) = 0.45 · 11.45 · logₑ(2 · 0.6 / 0.45) = 5.1 Wb
ϕ(z₁) = 43.1 Wb
B(z₁) = 2.20 Wb/m² (80 000 At/m)
C - 2 J = 3.0 - 0.2 = 2.8 m
z₂ = 0.2 + 2.8/3 = 1.135 m

ϕₚ = ϕ(z₂) = 43.1 + 0.45 · 11.45 [1.135 / 0.535 logₑ(1.135 / 0.6) - 1] = 45.0 Wb
B(z₂) = 2.29 Wb/m² (145 000 At/m)
z₃ = 1.4 m
ϕ(z₃) = 45.0 - 4 · 0.4π · 10⁻⁶ · 2.5 · 0.145 · 10⁻⁶ · 0.265 = 44.6 Wb
B(z₃) = 2.27 Wb/m² (130 000 At/m)
Total ampereturns for poles: 0.23 · 10⁶ At
Equivalent ellipsoid: a = 4.75 m; b = 4.50 m.
πa² = 71 m²; Demagnetization factor 0.347.

ϕₚ - ϕᵥ / Nᵥ + N₉ = 0.4π · 10⁶ · 71 / 2 · 4.50 · 0.347 = 28 · 10⁻⁶ Wb/At
Table IV (Continued)

Assume $N_{V}l_{V} + N_{h}l_{h} = 0.27 \cdot 10^{6}$ At

$\phi_{V} = 45.0 - 7.7 = 37.3$ Wb; $\phi_{h} = 45.0 - 3.8 = 41.2$ Wb

$B_{V} = 2.03$ Wb/m$^2$ (28 000 At/m); $B_{h} = 1.98$ Wb/m$^2$ (22 000 At/m)

$N_{V}l_{V} + N_{h}l_{h} = (0.08 + 0.19) \cdot 10^{-6} = 0.27 \cdot 10^{6}$

Total ampereturns: Gap 0.66 + Poles 0.23 + Yoke 0.27 = 1.16 \cdot 10^{6}$ At
Appendix 2

Fig. 1  Denotation for dimensions

Fig. 2  600 MeV magnets

Fig. 3  Flux map around pole

Fig. 4  Magnetisation curves

Fig. 5  Cost size relations for cyclotron magnets

Fig. 6  Effects of small changes in shimming

Fig. 7  Field shape of model magnet
Fig. 3 Flux map around upper pole
Fig. 5
Cost-size relations for cyclotron magnets
Fig. 6

Effects of small changes in shimming