INCOHERENT BEAM-BEAM EFFECT - THE RELATIONSHIP BETWEEN TUNE-SHIFT, BUNCH LENGTH AND DYNAMIC APERTURE

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ABSTRACT

Simulation studies of the influence of long bunches of the beam-beam effect in particle colliders suggest that, despite the risk from synchro-betatron resonances, the attainable luminosity may be greater than that obtained for short bunches.

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INTRODUCTION

According to the thin-lens analysis of beam-beam effect in particle colliders, the bunch length, $\sigma_z$, the beta value at the interaction point, $\beta^*$, the linear tune-shift parameter, $\xi$, and the disruption parameter, $D$ are related as follows:

$$\frac{\sigma_z}{\beta^*} = \frac{D}{4\pi \xi}$$

Note that the bunch intensity and transverse dimensions are contained in the parameter $D$:

$$D_{xy} = \frac{2\pi N \sigma_y}{\gamma \sigma_x (\sigma_x + \sigma_y)}$$

where $r_e$ is the classical electron radius, $N$ the number of particles per bunch, $\sigma_x$, $\sigma_y$ are the transverse beam dimensions (Gaussian distribution) and $\gamma$ is the Lorentz factor.

Disruption is essentially the extent to which one bunch is focused or defocused within the length of the other bunch, with or without the mutual pinch. For a design tuneshift of, say, 0.05, it may be required that the ratio $\sigma_y/\beta^*$ should be less than 0.5 so that $D$ does not exceed 0.3, to avoid excessive phase-space dilution of the colliding beams. In this note we demonstrate that beam blow-up is not so simply related to longitudinal bunch length and that longer bunches could be beneficial.

THICK-LENS SIMULATIONS

To investigate the bunch crossing in the range of $\sigma_y/\beta^*$ from 0.05 to 1.0 we have used a program developed by one of us (L. Wood) for the CERN Compact Linear Collider (CLIC) study. This program simulates the beam crossing and includes the mutual pinch - the so-called strong/strong effect. To break down the overall effect, the mutual pinch can be turned off to simulate the weak/strong case, and the natural beta variation over the bunch length can be set to a constant value for the target bunch, leaving just the the thick-lens effect. Both bunches are sampled longitudinally during the crossing, however it is a single-pass event and so synchro-
betatron effects are not simulated. This has been done using a single-particle (weak/strong) tracking program that follows the particle in transverse and longitudinal phase space for typically 1000 revolutions. The particle is tracked with a variable longitudinal step size through the target bunch and a linear lattice transform is used between beam-beam crossings.

PARAMETERS

In this study we have computed the values of the linear tuneshift parameter that corresponded to a 10% increase in the beam divergence. This is equivalent to a value of $\xi = 0.04$ for short bunches - not far from the design limit for many e$^+$e$^-$ colliders. From an arbitrarily selected transverse bunch size and a chosen $\sigma_y/\beta^*$, the number of particles (and hence the tuneshift) is increased until the computed value of $X'/X'_0$ is equal to 1.1. Note that the values of $X'$ that are quoted are rms values. Both round and flat beams (aspect ratio 20) have been studied. Both bunches contain the same number of particles, although this only matters in the strong/strong cases.

PHENOMENOLOGY

In the weak/strong case and for a fixed number of particles, as the length of the target bunch is increased the focusing effect decreases simply because the incoming particles are pulled into a weakening field as they are drawn towards the axis of the target bunch (thick lens effect). Compared to the thin lens $X'/X'_0$ is reduced, but $X/X'_0$ is no longer unity. The variation of beta function over the length of the bunch further reduces the computed $X'/X'_0$ (and of course the luminosity). Both of these effects are compensated by the mutual pinch of the strong/strong interaction, but in our range of parameters the compensation is only partial and the same pattern of reduced $X'/X'_0$ with increasing bunch length is observed, although the pinch restores most of the luminosity lost due to beta variation in the weak/strong case.

ILLUSTRATIONS

Figures 1 to 4 illustrate the effects described in the previous section. The thick lens effect and the beta variation contribute approximately equally to the increased tolerance to the beam-beam effect as the ratio $\sigma_y/\beta^*$ increases from 0.1 to 1.0. The mutual pinch reduces this tolerance but, as the strong/strong simulation of Fig. 4 shows, for $\sigma_y/\beta^* = 1.0$ the thin-lens tuneshift of 0.04 can be increased to 0.08. Figures 5 and 6 show the equivalent cases for flat bunches. We find essentially identical results for round and flat bunches. The weak/strong and strong/strong simulations are indistinguishable for $\sigma_y/\beta^* \leq 0.2$. Longer bunches display differences between the two simulation techniques, these differences increasing with bunch length.

The tune shift (round bunch) is given by

$$\xi = \frac{\tau_e N}{4\pi\varepsilon}$$

This term contains no dependence on the transverse dimension or on energy. Weak/strong and strong/strong simulations returned the expected result that the behaviour of $X'/X'_0$ was independent of beam energy and beam widths.

Figure 7 illustrates the variation of $X'/X'_0$ with tuneshift for several values of $\sigma_y/\beta^*$. For short bunches the results can be superimposed. We find roughly an exponential increase in $X'/X'_0$ for short bunches ($\sigma_y/\beta^* \leq 0.2$) at $\xi$ limits of $0.2 \leq \xi \leq$
0.05. At larger values of $z$ the increase is linear. If $X'/X'_0 = 1.1$ represents the tuneshift limit then this exponential behaviour near $0 = 0.04$ can explain why even the addition of extra damping, e.g. via wigglers, has little effect on the tuneshift limit.

In Fig. 8 for the same range of tuneshift and bunch length we see that because of the mutual pinch the ratio of the luminosities calculated with and without pinch increases with bunch length and with tuneshift. This can be enough to offset the luminosity loss due to beta variation along the bunch.

SYNCHRO-BETATRON COUPLING

Of course, all of the advantages of long bunches may be wiped out by synchro-betatron effects. Figures 9 and 10 for the weak/strong and the strong/strong cases show the extent of variation of beam-beam effect (still expressed as $X'/X'_0$) over the length of the bunch for a single crossing. Since the weak and strong cases are not vastly different (mostly the variation over bunch length comes from the geometry of the interaction), we have applied a weak/strong particle tracking program to study the evolution over 1000 crossings in a linear machine with no transverse coupling.

Single particles of selected synchrotron amplitude are tracked through the target bunch repeatedly for previously chosen good values of the synchrotron and betatron tunes (i.e. away from low-order betatron and synchro-betatron resonances). The beam-beam effect is characterised by the increase in the transverse amplitude, $A$, of particles with starting amplitudes, $A_0$, equal to $2\sigma_x$. The target bunch is represented by a six-dimensional Gaussian distribution and in Fig. 11 the ratio $A/A_0$ is plotted against synchrotron amplitude (in units of $\sigma_x$) for short and long bunches ($\sigma_x/\beta^* = 0.1$ and $1.0$, respectively). To amplify the observed effects the example given is for a tuneshift parameter $\xi$ equal to 0.1. Values of $D$ are 0.13 and 1.3. At zero synchrotron amplitude particles passing through the long target bunch are blown up much less than by the short bunch. The difference becomes less marked at larger amplitudes, but the long bunch requires a lower dynamic aperture than the short bunch over the entire range of synchrotron amplitudes.

SUMMARY

The synchro-betatron study has just begun and we are examining ways of obtaining similar results for the strong/strong interaction. If, as we would expect, the long bunch is at least no worse than the short bunch (for the same overall luminosity), then it offers the advantage of economy of rf power. There could also be a net advantage in that larger values of the tuneshift parameter (i.e. higher luminosity) would be allowed. The future study must include a lattice transform containing nonlinear elements.
FIGURE CAPTIONS

1. Tuneshift parameter $\xi$ versus bunch length corresponding to $X'/X'_0 = 1.1$ weak/strong, beta off (see text) round beams.

2. Tuneshift parameter $\xi$ versus bunch length corresponding to $X'/X'_0 = 1.1$ weak/strong, beta on (see text) round beams.

3. As Fig. 1, but for strong/strong case.

4. As Fig. 2, but for strong/strong case.

5. As Fig. 2, with flat beams (aspect ratio 1:20).

6. As Fig. 2, but strong/strong case with flat beams (aspect ratio 1:20).

7. The ratio $X'/X'_0$ versus tuneshift parameter $\xi$ for various bunch lengths $\sigma_2/\beta^*$ from 0.1 to 0.9, strong/strong, round beams.

8. The computed luminosity divided by the theoretical luminosity without pinch versus the tuneshift, corresponding to $X'/X'_0$ of 1.1 for various bunch lengths as in Fig. 7, strong/strong, round beams.

9. Variation in $X'/X'_0$ along the bunch versus position along the bunch (in units of $\sigma_2$) for various bunch lengths, weak/strong, round beams.

10. As Fig. 9, but strong/strong.

11. Dynamic aperture versus synchrotron amplitude (in units of $\sigma_2$) for test particles in a weak/strong simulation after 1000 crossings with a tuneshift parameter $\xi = 0.1$, linear lattice, no transverse coupling, fractional part of transverse tune $q = 0.295$, synchrotron tune $Q_s = 0.0651$. Upper curve: short bunch, $\sigma_2/\beta^* = 0.1$, $D = 0.13$; lower curve: long bunch, $\sigma_2/\beta^* = 1.0$, $D = 1.3$. 
Figure 1

Figure 2
Figure 3

Strong-strong simulation of tune shift for $\epsilon / \epsilon' = 1.1$ (beta off)

Figure 4

Strong-strong simulation of tune shift value for $\epsilon / \epsilon' = 1.1$
Figure 5

Figure 6
Figure 7

Figure 8
Figure 9: Non-strong simulation of $X'/X_0$ against bunch position.

Figure 10: Strong-strong simulation of $X'/X_0$ against bunch position.
Dynamic aperture versus synchrotron amplitude

Dynamic aperture due to beam–beam force, A/Ao

Z-amplitude in units of sigma-Z

Short bunch

Long bunch

FIGURE 11: Variation of dynamic aperture with synchrotron amplitude