NEUTRINO COUNTING

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ABSTRACT

We examine various methods to determine the total number of neutrino species at LEP/SLC. The method of the width is considered and the influence of the physics beyond the Standard Model is taken into account. The radiative method is also studied together with its background and the effects of energy and angular cuts on the observed photons. The role played by statistical and systematic errors on the accuracy of ΔNν is briefly discussed.

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1 Introduction

The number of generations of fermions is among the important questions in the "Standard Model" [1]. The number of light neutrino types $N_{\nu}$ is related within the $SU(3) \times SU(2) \times U(1)$ model to the number of leptons and quark doublets.

There are several methods to determine the total number of light neutrino types. The abundance of primordial light chemical elements, like $^4$He, in the universe [2] supports a value of $N_{\nu}$ smaller than four. Collider experiments have recently placed upper limits on the total number of light neutrinos. The combined UA1 and UA2 data give a limit of $N_{\nu} \leq 5.9$ with the bound that $N_{\nu} \geq 3$ [3]. Limits from lower energy $e^+e^-$ collider experiments on $N_{\nu}$ have also been set [5]. Recently, the limit on $N_{\nu} = 3.0 \pm 0.9$ ($N_{\nu} < 4.4$ at 95% c.l.) has been given [4]. A review of the existing estimates on the number of generations including accelerator limits can be found in [6]. None of these measurements are precise enough to rule out a fourth generation of light neutrinos.

Light neutrinos − those with $m_{\nu} \leq \frac{1}{2} M_{W^*}$ − contribute to the $Z^0$ width via the decay $Z^0 \rightarrow \nu\bar{\nu}$. At LEP and SLC the number of neutrino types $N_{\nu}$ can be obtained by several different methods:

- The comparison of the measured value of the total $Z^0$ width with the width obtained in the Standard Model according to a given number of neutrino types [6].
- The measurement of the invisible width $\Gamma_{inv}=N_{\nu}\Gamma_{\nu\nu}$ from the total, the leptonic and the hadronic width and the comparison with $\Gamma_{\nu\nu}$.
- The direct method of radiative neutrino counting from the measurement of the cross section $e^+e^- \rightarrow \nu\bar{\nu} + \gamma$ and its comparison with the predicted dependence on the number of neutrino types $N_{\nu}$.

These methods use different theoretical inputs from the Standard Model and the accuracy on the number of neutrino types that can be obtained by the various methods can differ substantially depending on the various experimental conditions [7]. One important aspect is the complementarity between the different methods both on the theoretical and experimental side which results in producing independent determinations of $N_{\nu}$. Several studies are in the literature on both the width method [8] and the direct "radiative" method [9] [10] [11] [12] [13] [14].

2 The method of the width

High accuracy measurements of the $Z^0$ decays at the LEP and SLC colliders require very precise theoretical evaluations. This is especially true for the measurement of the total and partial widths of the $Z^0$.

Several corrections enter in the theoretical evaluation of the total and partial $Z^0$ widths [15]. The decay rate of the $Z^0$ into any fermion anti-fermion pair $f\bar{f}$ of the lepton and quark families can be calculated (see last two papers in Ref. [15]). The decay rate is given by:

$$\Gamma_{2\nu \rightarrow ff}^0 = \frac{N_{\nu} m_{Z^0}^3 G_F}{24\pi\sqrt{2}} \rho \left(1 - \frac{4m_f^2}{M_{Z^0}^2}\right) \left(1 - \frac{4m_f^2}{M_{Z^0}^2} + (2I_3^f - 4Q_f s_w^2)^2 \left(1 + \frac{2m_f^2}{M_{Z^0}^2}\right)\right)$$

with $N_{\nu}$ a color factor ($N_{\nu} = 1$ for leptons and $N_{\nu} = 3$ for quarks), $I_3^f$ is the third component of the weak isospin, $Q_f$ the charge and $s_w^2 = \sin^2 \theta_w$ with $\theta_w$ the weak mixing angle. The effect of weak corrections can be absorbed in the parameter $\rho$ and in the definition of an effective $s_w$ which reabsorbs a large part of the radiative corrections with the exception of those due to $b\bar{b}$ quarks [15]. In the comparison with the experimental data photoonic plus electroweak and QCD radiative corrections to the initial and final states have to be added [15]. By defining the various parameters in the above equation the various partial decay rates can be evaluated.

A measurement of the $Z^0$ width with the expected experimental and the attainable theoretical accuracies precisely determines the number of neutrino families. In the standard model the total width can be decomposed as

$$\Gamma_{tot} = N_{\nu} \Gamma_{\nu\nu} + \Gamma_{had} + \Gamma_{lept}$$

(2.1)

with $\Gamma_{lept} = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau}$. In the following we summarize several methods which provide experimental input to the above relation in order to extract $N_{\nu}$.

2.1 Comparison of the measured and expected total width

At LEP and SLC, the mass and the width of the $Z^0$ can be determined by measuring the $Z^0$ production cross section at various center-of-mass energies [4]. A review of the scanning strategies can be found in Ref. [16]. When comparing the measured value of $\Gamma_{tot}$ with the Standard Model expectation, one has to take into account strong and electroweak corrections. The result of the radiative corrections depends on the top and Higgs masses $m_t$ and $m_H$ and on the value of the strong coupling constant $\alpha_s$. Imposing an upper limit on $m_H$ of $100 \text{ GeV}$ and varying $m_t$ and $\alpha_s$ within their experimental limits [17], the theoretical uncertainty of $\Gamma_{tot}$ amounts to about $30 \text{ MeV}$. A measurement of $\Gamma_{tot}$ can be performed with a comparable accuracy [18] [19]. The number of additional neutrino species $\delta N_{\nu}$, with $N_{\nu} = 3 + \delta N_{\nu}$ can be obtained as

$$\delta N_{\nu} = \frac{\Gamma_{measured} - \Gamma_{SM}}{\Gamma_{tot}^M}$$

with

$$\Gamma_{SM}^{tot} = \Gamma_{\nu\nu} + \Gamma_{lept} + \Gamma_{had}$$

It is important to notice here that within this method any deviation of visible and invisible channels from the Standard Model (SM) enters in the difference $\Gamma_{measured} - \Gamma_{SM}^{tot}$ and therefore affects the determination of $N_{\nu}$.
2.2 Determination of the invisible width

Soon after data is taken near the $Z^0$ peak not only the total but also the partial $Z^0$ widths will become known. Assuming that all contributions from invisible channels originate from neutrinos one gets:

$$\Gamma_{inv} = \Gamma_{tot} - \Gamma_{had} - \Gamma_{elpt} \quad N_{\nu} = \frac{\Gamma_{inv}}{\Gamma_{\mu\nu}}$$

In the following we will discuss three different combinations of experimental and theoretical inputs in order to determine $\Gamma_{inv}$.

- **Method a**)

  This method uses as experimental inputs $\Gamma_{tot}$ and $M_{Z^0}$ determined in a scan around the peak together with the measured ratio between hadronic and leptonic width at the peak

  $$R = \frac{\Gamma_{had}}{\Gamma_{\mu\nu}} = \frac{\epsilon_{had} N_{had}}{\epsilon_{lad} N_{\mu\nu}}$$

  where $\epsilon_{had}$ and $\epsilon_{lad}$ are the hadronic and leptonic efficiencies and $N_{had}$ and $N_{\mu\nu}$ the numbers of hadronic and leptonic events respectively. The invisible width $\Gamma_{inv}$ can be determined using the measured $\Gamma_{tot}$ together with the measured ratio $R$ as

  $$\Gamma_{inv} = \Gamma_{tot} - \frac{R S^{M^2}(3 + R)}{C}$$

  In this case only $\Gamma_{had}$ is taken from the Standard Model and lepton universality is assumed. Contrary to the determination using the total width, the introduction of the measured ratio $R$ between hadronic and leptonic width will account for deviations of the hadronic width from the SM, if these are due to additional visible channels. This method, however, produces wrong results if $\Gamma_{\mu\nu}$ deviates from the SM and it does not account for virtual effects of new physics as these will almost cancel out in $R$.

- **Method b**)

  The assumption that $\Gamma_{\mu\nu}$ is taken from the Standard Model can be also replaced by a measurement of the hadronic peak cross section $\sigma_{had}^{peak}$. For a Breit-Wigner resonance a relation exists between the peak cross section, the total, the leptonic and the corresponding partial widths. Applied to $\sigma_{had}^{peak}$ this relation is

  $$\sigma_{had}^{peak} = C \Gamma_{tot} \frac{\Gamma_{had}}{\Gamma_{tot}}$$

  with $C = \frac{2\pi}{M_{Z^0}^2}$. Photonic corrections however cause substantial deviations of the $Z^0$ line shape from a Breit-Wigner. They have to be taken into account as an overall correction factor to the measured peak cross-section, i.e.

  $$\sigma_{had}^{peak} = \sigma_{had}^{measured} f'$$

  with $F (\frac{f}{M^2})$ and $\beta = \frac{2\pi ln(n_{	ext{fit}})}{n_{	ext{fit}}}$. $\sigma_{had}^{peak}$ can also be obtained directly as fitted parameter from a scan around the peak, by parametrizing the observed hadronic cross-section $\sigma_{had}$ as

  $$\sigma_{had}^{peak}(s) = \int_{min}^{max} \sigma_{had}(s') f(s,s') ds'$$

  with

  $$\sigma_{had}(s) = \sigma_{had}^{peak} \frac{s^{\frac{3}{2}}}{(s - M_{Z^0}^2)^{\frac{1}{2}} + s^{\frac{3}{2}}/M_{Z^0}^2}$$

  with $\sigma_{had}^{peak} = \sigma_{had}(M_{Z^0})$. The radiator function $f(s,s')$ takes into account initial state radiation (for a computation see Ref. [15]). Final state radiation, interference between initial and final state radiation and non-photic corrections have no or little $s$ dependence. They are taken into account as an overall factor in the fit result for $\sigma_{had}^{peak}$. The hadronic width can be calculated from (2.3) with $\Gamma_{\mu\nu} = \Gamma_{had}/R$ as

  $$\Gamma_{had} = \Gamma_{tot} \sqrt{\frac{\sigma_{had}R}{C}}$$

  The invisible width (2.2) then reads

  $$\Gamma_{inv} = \Gamma_{tot} (1 - 33/27/27) \sqrt{\frac{\sigma_{had}R}{C}} - \sqrt{\frac{\sigma_{had}R}{C}}$$

  (2.4)

  To calculate the error on $\Gamma_{inv}$ we have to discriminate whether $\sigma_{had}^{peak}$ is derived from a measurement at the peak or as fit parameter from a scan. If $\sigma_{had}^{peak}$ is derived from a measurement of the peak cross section (2.4) can be expressed in terms of basic experimental quantities

  $$\Gamma_{inv} = \Gamma_{tot} (1 - 33/27/27) \sqrt{\frac{\sigma_{had}R}{C}} - \sqrt{\frac{\sigma_{had}R}{C}}$$

  (2.5)

  where $L$ is the integrated luminosity and the error on $\Delta \Gamma_{inv}$ is

  $$\Delta \Gamma_{inv} = (0.5 \Gamma_{had} - 1.5 \Gamma_{\mu\nu}) (\Delta N_{\mu\nu} / N_{\mu\nu}) \oplus (\Delta \epsilon_{had} / \epsilon_{had}) \oplus (\Delta L / L) \oplus (\Delta \Gamma_{tot} / \Gamma_{tot})$$

  (2.6)

  where $\oplus$ refers to the addition of errors in quadrature. To treat the case where $\sigma_{had}^{peak}$ is derived as a fitting parameter from a scan we parametrize $\sigma_{had}^{peak} = \sigma_{had}^{peak} \epsilon_{had}$ where $\sigma_{had}^{peak}$ denotes the parameter obtained without acceptance correction. The purpose of this factorization is to account
for the likely correlation between the determination of $\sigma_{\text{had}}$ for the scan and $R'$. At an integrated luminosity $L \geq 1\, \text{pb}^{-1}$ the error on $\sigma_{\text{peak}}^\text{u}$ is completely dominated by the error on the luminosity measurement, i.e.

$$\frac{\Delta \sigma_{\text{peak}}^\text{u}}{\sigma_{\text{peak}}^\text{u}} = \frac{\Delta L}{L}. $$

The analogue of (2.5) and (2.6) then reads

$$\Gamma_{\text{inv}} = \Gamma_{\text{tot}}(1 - 3\sqrt{\frac{N_{\mu\mu}^\text{peak} \sigma_{\text{had}}}{C_{\mu\mu} N_{\mu\mu}}} - \sqrt{\frac{\varepsilon_{\mu\mu} N_{\mu\mu}^\text{peak} \sigma_{\text{had}}}{C N_{\mu\mu}^\text{peak} \sigma_{\text{had}}}})$$

(2.7)

and

$$\Delta \Gamma_{\text{inv}} = (0.5 \Gamma_{\text{had}} - 1.5 \Gamma_{\mu\mu})(\frac{\Delta N_{\mu\mu}}{N_{\mu\mu}} \oplus \frac{\Delta \varepsilon_{\mu\mu}}{\varepsilon_{\mu\mu}}) \oplus \Gamma_{\text{had}} \frac{\Delta \varepsilon_{\text{had}}}{\varepsilon_{\text{had}}}$$

$$\Theta(0.5 \Gamma_{\text{had}} - 1.5 \Gamma_{\mu\mu})(\frac{\Delta N_{\mu\mu}}{N_{\mu\mu}} \oplus (0.5 \Gamma_{\text{had}} + 1.5 \Gamma_{\mu\mu})(\frac{\Delta C}{C} \oplus \Gamma_{\text{inv}} \frac{\Delta \Gamma_{\text{tot}}}{\Gamma_{\text{tot}}})$$

(2.8)

Comparing (2.6) and (2.8) the only advantage using $\sigma_{\text{peak}}^\text{u}$ with respect to a direct measurement at the peak is related to the hadron statistics, which does not constitute a problem anyhow. Before discussing method b) in detail we want to introduce also another method.

- Method c)

Unlike the methods above, the method proposed by Feldman [8] requires no input from a scan only the $Z^0$ mass and not $\Gamma_{\text{tot}}$. All the other measurements are performed at the peak. As the cross-section varies only slowly in the vicinity of its maximum an accuracy of 200 MeV on $M_Z$ is sufficient for this method. The measurements needed at the peak are the ratio $R'$ of hadronic over muonic events and the absolute luminosity $L$. Using these quantities a value of the leptonic peak cross section $\sigma_{\mu\mu}^\text{peak}$ or of the hadronic peak cross section $\sigma_{\text{peak}}^\text{had}$ is obtained.

The photonic radiative corrections have to be applied to both quantities as described under method 2.2.b). Using as input $\sigma_{\text{peak}}^\text{had}$ and the SM expectation for the muonic width $\Gamma_{\mu\mu}$, together with lepton universality the total width can be expressed via

$$\sigma_{\mu\mu}^\text{peak} = \frac{C \Gamma_{\text{ee}} \Gamma_{\mu\mu}}{\Gamma_{\text{tot}}}$$

(2.9)

$$\Gamma_{\text{tot}} = \Gamma_{\mu\mu} \sqrt{\frac{C}{\sigma_{\mu\mu}^\text{peak}}}$$

(2.10)

Using $\Gamma_{\text{had}} = R' \Gamma_{\text{SM}}^\text{had}$ the invisible width is obtained as

$$\Gamma_{\text{inv}} = \Gamma_{\mu\mu} \left( \sqrt{\frac{C}{\sigma_{\mu\mu}^\text{peak}}} - 3 - R' \right)$$

(2.11)

or written in terms of basic experimental quantities

$$\Gamma_{\text{inv}} = \Gamma_{\mu\mu} \sqrt{\frac{C}{\sigma_{\mu\mu}^\text{peak}}} \Lambda_{\mu\mu} \frac{N_{\mu\mu}}{N_{\mu\mu}^\text{peak}} - \frac{\Delta \varepsilon_{\mu\mu}}{\varepsilon_{\mu\mu}} \frac{\Delta \varepsilon_{\text{had}}}{\varepsilon_{\text{had}} N_{\mu\mu} \varepsilon_{\text{had}}}$$

The total error $\Delta \Gamma_{\text{inv}}$ on $\Gamma_{\text{inv}}$ is then

$$\Delta \Gamma_{\text{inv}} = (-0.5 \Gamma_{\text{tot}} + \Gamma_{\text{had}})(\frac{\Delta N_{\mu\mu}}{N_{\mu\mu}} \oplus \frac{\Delta \varepsilon_{\mu\mu}}{\varepsilon_{\mu\mu}}) \oplus 0.5 \Gamma_{\text{tot}}(\frac{\Delta C}{C})$$

$$\Theta(0.5 \Gamma_{\text{had}} - 1.5 \Gamma_{\mu\mu})(\frac{\Delta N_{\mu\mu}}{N_{\mu\mu}} \oplus (0.5 \Gamma_{\text{had}} + 1.5 \Gamma_{\mu\mu})(\frac{\Delta C}{C} \oplus \Gamma_{\text{inv}} \frac{\Delta \Gamma_{\text{tot}}}{\Gamma_{\text{tot}}})$$

(2.12)

A suppression of the statistical errors is apparent in the $\mu\mu$ channel first term in the expression above. This weaker dependence on statistical errors is advantageous in determining $\Gamma_{\text{inv}}$ with a limited number of events [8] [16] [20]. Fig.1.a gives the luminosity dependence of $\Gamma_{\text{inv}}$ determined with the method 2.2.c) for various values of the luminosity scale error and various precisions of the acceptance determination. In Figures 1b and 1c, the dependance of $\Delta N_{\mu\mu}$ on the number of $Z^0$ events and on the error on luminosity with different values for the hadronic and leptonic acceptances are presented. From these figures, we conclude that the method is well suited to reach a precision of $\Delta N_{\mu\mu} \leq 0.5 \text{ neutrino generations in the early data with a modest integrated luminosity of } 100-200 \text{ nb}^{-1}$. The overall normalization has to be known however to 2-3% to reach this method $\Delta N_{\mu\mu} \leq 0.25$.

One may ask whether there is any physics advantage to rewrite eq.(2.11) deriving $\Gamma_{\text{tot}}$ from the hadronic section and the SM expectation for the hadronic width, i.e. writing (2.10) as

$$\Gamma_{\text{tot}} = \Gamma_{\mu\mu} \sqrt{\frac{C}{\sigma_{\mu\mu}^\text{peak}}}.$$  

Using $\Gamma_{\mu\mu} = R' \Gamma_{\text{SM}}^\text{had}$ the analogue of (2.11) then reads

$$\Gamma_{\text{inv}} = \Gamma_{\mu\mu} \sqrt{\frac{C \sigma_{\text{peak}}^\text{u}}{\sigma_{\mu\mu}^\text{peak}}} - 3 - R'$$

(2.13)

with an error
\[ \Delta \Gamma_{\text{inv}} = (0.5\Gamma_{\text{tot}} - 3\Gamma_{\mu\mu}) \left( \frac{\Delta N_{\mu\mu}}{N_{\mu\mu}} \oplus \frac{\Delta \epsilon_{\mu\mu}}{\epsilon_{\mu\mu}} \right) \oplus 0.5\Gamma_{\text{tot}} (\Delta \tilde{\Gamma} / \tilde{\Gamma}) \]

\[ \oplus (\Gamma_{\text{tot}} - 3\Gamma_{\mu\mu}) \left( \frac{\Delta N_{\text{had}}}{N_{\text{had}}} \oplus \frac{\Delta \epsilon_{\text{had}}}{\epsilon_{\text{had}}} \right) \oplus \Gamma_{\text{inv}} \frac{\Delta R^3(\mu^3)}{R^3(\mu^3)} \]  

(2.14)

Comparing (2.14) and (2.12) one observes that the cancellation accompanying \( \Delta N_{\mu\mu}/N_{\mu\mu} \) is less pronounced. In addition the term due to hadron statistics and systematics has become slightly bigger. It should be noticed here that the theoretical uncertainties for \( \Gamma_{\mu\mu} \) are slightly smaller than for \( \Gamma_{\text{had}} \) due to the top mass dependent vertex corrections. There is no new information in this quantity since one has that \( \Gamma_{\text{inv}} - \Gamma_{\text{tot}} = \Gamma_{\text{inv}} (1 - R^3(\mu^3)/R^3) \), i.e. the difference of both quantities can be calculated by comparing the measured \( R' \) with its SM prediction \( R'^{\text{SM}} \).

A variant of Feldman's method has been proposed in [21]. Instead of quoting \( \Gamma_{\text{inv}} \), which has been obtained by multiplying measured quantities with the calculated \( R'^{\text{SM}} \), it is argued that it is much more advantageous to give \( \Gamma_{\text{inv}}/\Gamma_{\mu\mu} \). This quantity can be experimentally defined without explicit choice of a particular model. In addition it has the advantage that loop corrections cancel in the ratio of partial widths and therefore it is not affected by the ignorance on \( m_{\text{top}} \).

Any deviation of this quantity from the SM expectation indicates the presence of additional neutrinos or of non-universal contributions of new physics, such as an additional boson \( Z' \).

### 2.3 Comparison of the Methods

In this section we compare the physics potential of the various methods and work out their deficiencies and their complementarity. A summary of this comparison is given in Table [1].

<table>
<thead>
<tr>
<th>Method</th>
<th>2.1</th>
<th>2.2a</th>
<th>2.2b</th>
<th>2.2c</th>
</tr>
</thead>
<tbody>
<tr>
<td>experimental input</td>
<td>( M_2, \Gamma_{\text{tot}} )</td>
<td>( M_2, \Gamma_{\text{inv}}, R' )</td>
<td>( M_2, \Gamma_{\text{inv}}, R', \tilde{\Gamma} )</td>
<td>( M_2, R', \tilde{\Gamma} )</td>
</tr>
<tr>
<td>theoretical input</td>
<td>( \Gamma^{\text{SM}}, R'^{\text{SM}} )</td>
<td>( \Gamma^{\text{SM}}, R'^{\text{SM}} )</td>
<td>( \Gamma^{\text{SM}}, R'^{\text{SM}} )</td>
<td>( \Gamma^{\text{SM}}, R'^{\text{SM}} )</td>
</tr>
<tr>
<td>deviation of ( \Gamma_{\text{inv}} ) from SM</td>
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<tr>
<td>deviation from SM</td>
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<tr>
<td>extra contrib. to the hadronic channel</td>
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<tr>
<td>misident. of hadronic sample</td>
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<tr>
<td>virtual effects of new physics</td>
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<tr>
<td>no precision on ( \Gamma_{\text{tot}} )</td>
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<tr>
<td>luminosity scale error</td>
<td>( \Delta_{\mu} )</td>
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</tr>
</tbody>
</table>

Table 1: Comparison of the methods to determine \( N_{\mu} \) based on the measurement of the total and partial widths of the \( Z^0 \). --: the result on \( N_{\mu} \) is affected, +: the result on \( N_{\mu} \) is not affected. For the numerical comparison a total integrated luminosity of \( 3pb^{-1} \) is assumed.
The total width or the invisible width is also sensitive to any new particle that escape detection. A possible example of this type is given by supersymmetric particles. If they exist with masses smaller than \( M_{\text{LSP}} \), then we can expect an increase in the nominal \( Z^0 \) width. The magnitude of the contribution to this width depends on the masses of the involved particles due to the phase space factor \( \beta = (1 - \frac{4m^2}{M_{\text{LSP}}^2})^{1.5} \). One has that [7] [22]:

- Sneutrinos contribute about \( \frac{1}{2} \beta \delta^0 \) of a neutrino generation.
- The lightest Neutralino that is a mixture of supersymmetric partners of gauge and Higgs bosons, gives a contribution equal to \( \beta^2 (N_{13}^2 - N_{14}^2) \), where \( N_{13}, N_{14} \) are the neutralino mixing matrix elements of \( \chi^0 \) corresponding to the two Higgsino components. Since these matrix elements are \( \lesssim 1 \), \( \chi^0 \) contributes at most to 1 neutrino generation, and usually much less for a typical choice of mixing angles [23].

Another example of model in which further contributions to the invisible width may appear is represented by the ones where a coupling of the \( Z^0 \) to Majorons [24] appear. The decay of the \( Z^0 \) into such states should be taken into account [25], and this, for the specific model of an additional triplet of Higgs with a neutral \( \delta^0 \) state, gives a decay width [26],

\[
\Gamma(Z \to \delta^0 \delta^0) = 2 \Gamma(Z^0 \to \nu_e \bar{\nu}_e)
\]

equivalent to two neutrino modes.

Other models with additional Higgs multiplets are possible giving different additional contributions to the invisible width [27].

None of the methods gives the correct answer if \( N_\nu \neq \Gamma_{\text{inv}}/\Gamma_{\nu} \) which suggests that in order to distinguish between \( N_\nu = 3 \) and \( N_\nu = 4 \) an independent search for new invisible particles is needed in the general strategy of determining \( N_\nu \). An independent measurement is also needed to test lepton universality, a common assumption of all methods.

For the following points all methods show subtle differences. The SM expectation for \( \Gamma_{\nu} \) enters into all methods except method 2.2b). A possible source of a substantial deviation of \( \Gamma_{\nu} \) from its SM expectation by up to 10% could originate from an additional \( Z^\prime \) [28]. \( E_8 \) models have the peculiar feature that the total and partial widths can be smaller than their SM expectation and that the relative impact on \( \Gamma_{\text{inv}} \) is stronger than on \( \Gamma_{\nu} \). It is quite interesting to compare the bias on \( N_\nu \) for the different methods using the maximal deviations derived in [28] within the present experimental limits on \( E_8 \) models:

\[
\begin{align*}
\Gamma_{\nu} &= 0.97 \Gamma_{\nu}^{SM} \\
\Gamma_{\nu}^{SM} &= 0.90 \Gamma_{\nu}^{SM} \\
\Gamma_{\nu}^{SM} &= 0.98 \Gamma_{\nu}^{SM} \\
\Gamma_{\nu}^{SM} &= 0.97 \Gamma_{\nu}^{SM} \\
\sigma_{\mu \mu}^{\text{peak}} &= 0.85 \sigma_{\mu \mu}^{\text{SM}} \\
\sigma_{\mu \mu}^{\text{SM}} &= 0.94 \sigma_{\mu \mu}^{\text{SM}} \\
R' &= 1.09 R^{SM}
\end{align*}
\]

Table 2: Illustration of a possible bias in the determination of the number of neutrinos induced by an \( E_8 \) predicted \( Z^\prime \). The partial widths have been modified according to [28]. \( N_\nu = 3 \) on input.

<table>
<thead>
<tr>
<th>method</th>
<th>2.1</th>
<th>2.2a)</th>
<th>2.2b)</th>
<th>2.2c</th>
</tr>
</thead>
</table>
| \text{\( N_\nu (\Gamma_{\nu} = \Gamma_{\nu}^{SM}) \)} | \begin{align*}
3.4 & \quad 1.6 \\
3.5 & \quad 1.6 \\
3.0 & \quad 1.6 \\
3.4 & \quad 1.6
\end{align*} |
| \text{\( N_\nu (\Gamma_{\nu} = \gamma_{\text{measured}}) \)} | \begin{align*}
3.4 & \quad 1.6 \\
3.5 & \quad 1.6 \\
3.0 & \quad 1.6 \\
3.4 & \quad 1.6
\end{align*} |

The above example shows that none of the methods except 2.2b) can account for the exchange of an additional gauge boson. The same is true for virtual corrections which only contribute to the self energy of the \( Z^0 \). The relative contribution of such corrections is about the same to all partial widths [15]. They therefore cancel in \( \sigma_{\text{peak}}^{\mu \mu} \) or \( R' \). In methods 2.2a) and 2.2c) \( \Gamma_{\nu}^{SM} \) is used to derive the hadronic or the total width from expressions where only \( \sigma_{\text{peak}}^{\mu \mu} \) or \( R' \) enter. This explains why they cannot accommodate unexpected contributions to the \( Z^0 \) self-energy.

The methods also behave differently with respect to visible channels. Only method 2.1 produces a wrong result if the contributions to the hadronic channel deviate from the prediction of the SM with three generations. As \( R' \) enters in the determination of the invisible width additional hadronic contributions are taken into account for the methods described in section 2.2. On the other hand \( R' \) is sensitive to a misidentification of both, the leptonic and the hadronic sample, whereas the measurement of the total width only needs a moderate understanding of the hadronic sample. The width is insensitive to contaminations which are proportional to the hadronic sample or an overall scale factor in the acceptance. A precision measurement of \( \Gamma_{\nu} \) however requires a high statistics scan, which is not needed for method 2.2c). As already mentioned there is no competitor for method 2.2c) at low statistics. An experiment which achieves \( \Delta \mathcal{L}/\mathcal{L} = 2\% \) within the first 100 pb$^{-1}$ can reach \( \Delta N_\nu = 0.3 \) with method 2.2c) whereas the other methods will give \( \Delta N_\nu = O(1) \) independently of systematics.

The numerical comparison of methods in the last row of Table [1] is based on an integrated luminosity of 3 pb$^{-1}$ for a scan and 3 pb$^{-1}$ for running on the peak. Once operating experience is gained with the detectors and the machine a realistic estimate of systematic errors may be:

\[
\begin{align*}
\frac{\Delta \mathcal{L}}{\mathcal{L}} &= 2\% \\
\frac{\Delta \mathcal{L}_{\text{had}}}{\mathcal{L}_{\text{had}}} &= \frac{\Delta \mathcal{L}_{\nu}}{\mathcal{L}_{\nu}} = 1\%
\end{align*}
\]
and for the high precision scan a point to point systematic error of 0.3%. At the luminosity quoted the methods are not very different concerning their accuracy on $N_{\nu}$. Table [1] and the specific example given in Table [2] show that it is important to cross check all methods for consistency before a high precision result on $N_{\nu}$ can be trusted.

3 The radiative method

A direct method for counting the number of neutrino types given by the measurement of the cross section for the process $e^+e^- \rightarrow \nu\bar{\nu} \gamma$ has been proposed by several authors [9] [10] [11] [12] [13] Only the neutrino partial width $\Gamma_{\nu}$ enters directly in this reaction and, contrary to the previous methods the experimental and the theoretical uncertainties only weakly affect the final answer. The main sources of error are the electromagnetic radiative corrections and the contamination of the signal given by the various background reactions.

The determination of the number of neutrino types can be performed by measuring the cross section for producing a single photon plus a neutrino antineutrino pair from the decay of the $Z^0$ in this channel. This cross section is proportional to the number of light neutrinos. The comparison of the measured photon distributions can be made with the theoretical prediction obtained with a given value of $N_{\nu}$. The deviations from the predicted distributions may be fitted having $N_{\nu}$ as a free parameter and give therefore a determination of the number of light neutrinos. The variation of the total cross section as a function of $N_{\nu}$ has an increase of about the 30% for each additional neutrino family. This sizeable change in $\sigma$ has to be however implemented with two potentially large effects due to:

- electroweak radiative corrections that are known [15] to influence both the shape and the position of the peak around the $Z^0$
- the background made up by all the possible reactions that can simulate the signal i.e. giving a single isolated photon collected within the detector acceptance with no other accompanying particle in the acceptance volume.

Both these sources of systematic errors have to be understood in order to carry the neutrino counting measurement.

3.1 Lowest order cross section

The diagrams within the standard model that contribute to the radiative neutrino production process are shown in Fig. 2. The cross section for producing a photon at an angle $\theta$, with respect to the beam axis and with a momentum $k$ is [29]:

$$\frac{d\sigma}{d\cos\theta, dk} = \frac{\alpha}{12\pi^2} G^2 M_W^2 \frac{s'k}{sk+k^2} \left[ \eta^2 F(\eta_+) + \eta^2 F(\eta_-) \right]$$

with

$$\eta_{\pm} = \frac{s - k_{\pm}}{M_W^2}$$

$$F(\eta_{\pm}) = N_{\nu} \frac{1}{2} \left( 4(g_\nu + g_\alpha)^2 + (g_\nu - g_\alpha)^2 \right) \frac{M_W^4}{|Z|^2}$$

$$+ 3(g_{\nu} + g_\alpha) \frac{M_W^4}{|Z|^2} \eta_{\pm} \left( 3 + \frac{2}{\eta_{\pm}} - 2(1 + 1/\eta_{\pm}) \ln(1 + \eta_{\pm}) \right)$$

$$+ \frac{6}{\eta_{\pm}} \left( 1 + \eta_{\pm}) (1 - \frac{2}{\eta_{\pm}} \ln(1 + \eta_{\pm})) + 1 \right)$$

$$s = (p_+ + p_-)^2 \quad s' = (q_+ + q_-)^2, k_{\pm} = 2p_{\pm} \cdot k, Z = s' - M_Z^2 + iM_Z \Gamma_Z$$

$$\eta_{\pm} = 1 - 2s \sin^2 \theta_W \quad g_\alpha = -1/2$$

where $p_{\pm}$ and $q_{\pm}$ are the momenta of $e^\pm$ and $\nu\bar{\nu}$ respectively.

In the limit $M_W \rightarrow \infty$, by neglecting the last graph in Fig. 2 with the double $W$ boson propagator and also discarding terms of order $\frac{s'}{M_W}$, the double differential cross section can be written in the form:

Fig. 2 Diagrams contributing in lowest order to the process $e^+e^- \rightarrow \nu\bar{\nu} \gamma$: a) $Z^0$ production b) $W$ exchange
\[
\frac{d^3 \sigma_0}{dx \, dy} = \frac{G^2_F \alpha_s \alpha(1-x) \left[ (1-\frac{1}{2}) + \frac{3}{4} \right]}{6\pi^2 x(1-y^2)} \left( 2 + \frac{N_w (g_1^2 + g_2^2) + 2 (g_0 + g_4) \left[ 1 - \frac{\alpha_s(x)}{\alpha_s(x_0)} \right]}{1 - \frac{\alpha_s(x)}{\alpha_s(x_0)} + \Gamma^2_3/\Gamma^2_3} \right). \tag{3.2}
\]

where \( G_F \) is the Fermi coupling constant, \( x = 2E_\gamma/\sqrt{s} \) is the fraction of energy carried away by the emitted photon, \( y = \cos \theta_\gamma \). \( N_w \) is the number of neutrinos and the couplings \( g_0 = -\frac{1}{2} + 2 \sin^2 \theta W \) and \( g_4 = -\frac{1}{2} \). In eq. (3.2) the term in the next factor containing \( N_w \) comes from the square of the \( Z^0 \) amplitude (Fig. 2a) the first "constant" term is the contribution from the square of the \( W \) exchange amplitude in the above limit and the last one originates from the \( Z - W \) interference. Clearly the diagrams containing the \( W \) exchange only contribute to electron neutrinos \( \nu_e \) production.

The accuracy of the \( M_W \to \infty \) approximation and the effect of the \( W \) exchanging diagrams with respect to those corresponding to the \( Z^0 \) direct production can be estimated [29][30][31][32]. The main conclusions are that:

- An exact expression for the lowest order cross section \( e^+e^- \to \nu\bar{\nu} \gamma \) in eq.(3.2) with a finite \( W \) mass can be derived [29][31]. The \( M_W \to \infty \) limit or Point Interaction Approximation (PIA) for the \( W \) boson propagator reproduces the exact calculation with an accuracy better than 1% within a sizeable range of energies around the \( Z^0 \) peak [31] and the error gradually increases for energies away from the \( Z^0 \) peak.

- The contribution due to the \( W \) pole diagrams Fig.2b with respect to the contribution due to the diagrams containing the direct \( Z^0 \) pole only, Fig.2a, is smaller than 1% only within a limited region around the \( Z^0 \) pole (see Fig.7 discussed in section 5.1). For instance for \( M_{Z^0} + 4 \mathrm{GeV} \leq \sqrt{s} \leq M_{Z^0} + 8 \mathrm{GeV} \) [29][31]. This observation means that for a theoretical accuracy below the 1% level in the determination of \( N_w \) at the \( Z^0 \) pole the annihilation channel approximation is therefore inadequate.

Let us now start considering the role played by the electroweak radiative corrections to the Born cross section in eq.(3.1) and (3.2).

### 3.2 Weak radiative corrections

The electroweak radiative corrections to the Born annihilation channel cross section \( e^+e^- \to \gamma Z^0 \) can be computed by evaluating diagrams containing both neutral and charged current one loop contributions [32]. These corrections have been found to be smaller than 1% if the Born cross section is calculated with the input of the Fermi constant \( G_F \) and therefore can be neglected around the \( Z^0 \) peak. They start to become larger at higher energies [32].

As discussed in the previous section a large part of the weak corrections can be absorbed in the parameter \( \rho \) and in the definition of an effective \( \alpha_s \), therefore the Born cross section (3.2) should be written as

\[
\frac{d^3 \sigma_0}{dx \, dy} = \frac{G^2_F \rho^2 \alpha_s \alpha(1-x) \left[ (1-\frac{1}{2}) + \frac{3}{4} \right]}{6\pi^2 x(1-y^2)} \left( 2 + \frac{N_w (g_1^2 + g_2^2) + 2 (g_0 + g_4) \left[ 1 - \frac{\alpha_s(x)}{\alpha_s(x_0)} \right]}{1 - \frac{\alpha_s(x)}{\alpha_s(x_0)} + \Gamma^2_3/\Gamma^2_3} \right). \tag{3.2}
\]

Another set of weak corrections is represented by the electroweak corrections to the \( Z^0 \) propagator where all the electroweak loop diagrams contribute. It has been stressed [33], that the \( O(\alpha) \) corrections to the \( \Gamma_Z \) should be taken into account. The inclusion of these electroweak corrections results in a change of the \( Z^0 \) propagator

\[
\frac{1}{s-M^2_Z+iM_Z\Gamma_Z} \to \frac{1}{s-M^2_Z+i(\Sigma (s)+i\frac{\mu}{6}M_Z)}.
\]

This replacement has the effect of introducing an energy dependence of the imaginary part that becomes the physical width of the \( Z^0 \) at \( s=M^2_Z \). Strong and weak corrections should be also included around the resonance. This modification has the effect of shifting the peak by 35 MeV with respect to the expression with constant \( \Gamma_Z \) [15].

### 3.3 Electromagnetic radiative corrections: \( O(\alpha) \) and higher orders

As has been noted for the case of the \( Z^0 \) line shape, the impact of the initial state electromagnetic radiative corrections dramatically changes the size and the position of the peak and therefore directly affects both the width and the mass determination of the \( Z^0 \) [15] [34] [35] [36] [37]. The same considerations also apply for the electromagnetic radiative corrections to the neutrino counting reaction since the same type of corrections as for the line shape apply to the initial electron and positron states for this reaction. The effect of the collinear and soft radiation should be also taken into account since the cross section itself depends on the angles and intensity of the radiation.

It should at this point be noticed that for the neutrino counting reaction the electromagnetic radiative corrections have a much simpler structure than for similar leptonic channels since, due to the absence of final charged states, they are really only initial state corrections. The comparison with theoretical evaluations is in this case possible also with the application of experimental type cuts analytically in a non-inclusive manner [30][31][38].
The attempt to include higher order radiative corrections within the Born cross section in eq.(3.2) has been considered in Ref. [30] [32] [39].

These corrections correspond to an effective $O(\alpha^2)$ contribution to the bare neutrino production process in eq.(3.2).

In references [30] [32] [39] the $O(\alpha)$ virtual corrections to the production channel diagram in Fig.2a,b together with the corresponding $O(\alpha)$ real contributions have been evaluated. The corresponding diagrams are represented in Fig. 3. The contributions to the $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ process have been evaluated in the cases where

a) The energy of the second photon associated with the observed one is smaller than the detection threshold. This configuration provides the cancellation between the infrared divergences from the virtual $O(\alpha)$ diagrams.

b) The second associated photon is hard and is emitted in the beam pipe thus also simulating a single photon final state.

c) The second associated photon is parallel to the observed one emitted within the minimum resolution angle in such a way that the two photons cannot be separately identified.

The matrix elements calculated in Ref. [30] [32] are in agreement with those evaluated in Ref. [39]. The $O(\alpha)$ corrections to the Born process in eq.(3.2) have been also computed in Ref. [29] by using the matrix elements given in Ref. [39]. The contributions coming from the hard additional photon and from the soft and virtual corrections as in the diagrams of Fig.3 have been implemented in an unweighted Monte Carlo generator of events [29].

\[
V = \frac{\alpha}{\pi} \left( \ln \frac{s}{m^2} - 1 \right) \left( \frac{1}{2} \ln \frac{s}{m^2} + \frac{1}{2} \ln \frac{m^2}{m^2} + \frac{1}{3} \pi^2 - 2 \right)
\]

\[
+ F(\kappa, \tau),
\]

\[
R = \frac{\alpha}{2} \left( \ln \frac{s}{m^2} - 1 \right) \ln \frac{2E_{\text{min}}}{\lambda} - \frac{1}{2} \ln \frac{s}{m^2} + \frac{1}{3} \pi^2 \right).
\]

$\lambda$ is a fictitious photon mass to regularize the infrared divergence and $\kappa$ and $\tau$ are kinematical factors given by the relations $\kappa = 1/2s(1 + y)$, $\tau = 1/2s(1 - y)$. The integral of $y$ of the function $F(\kappa, \tau)$ does not give logarithmic contributions, so that the leading and next to leading terms are contained in the $\kappa, \tau$-independent part of (3.4).

The radiative corrections to the $Z^0$ direct production process in Fig. 2a can be extended to the $W$ exchange diagrams of Fig. 2b in the $M_W = \infty$ point interaction limit [31] [38].

In Ref. [38] the formalism of the structure functions has been used to calculate the radiatively corrected neutrino production cross section. As is case for the radiative corrections to the initial states in $e^+e^- \rightarrow Z\gamma \rightarrow X$ the contributions given by the soft and collinear photons give large radiative corrections and a resummation of the corresponding large infrared and collinear logarithmic contributions has to be performed [34] [35] [39] [37] [38]. By using structure functions containing also transverse degrees of freedom [40] [41] one obtains the radiatively corrected cross section:

\[
\frac{d^2\sigma}{dz \ dy} = H^{(\alpha)}(z, y; s) \sigma_0 ((1 - z)s),
\]

where $\sigma_0$ is the "reduced" cross section for the process $e^+e^- \rightarrow Z, W \rightarrow \nu\bar{\nu}$ in eq.(3.2):

\[
\sigma_0 (s) = \frac{G_F^2 s}{12\pi} \left( 2 + \frac{N_v (s_0^2 + s_0^2) + 2(q_v + q_\nu) \left[ 1 - \frac{1}{M_W^2} \right]}{[1 - \frac{1}{M_Z^2}]^2 + \Gamma_Z^2 / M_Z^2} \right).
\]

and the $O(\alpha)$ radiator $H^{(\alpha)}(z, y, s)$ is given by the expression:

\[
H^{(\alpha)}(z, 0; s) = \frac{\alpha}{\pi} \frac{1 + (1 - z)^2}{(1 - \cos^2\theta) \left[ 1 - \frac{4m^2}{s} - |\cos \theta| \right].
\]

By using structure functions also higher orders can be evaluated due to soft and hard corrections.

Let us first consider the virtual and soft corrections. The total bare cross section for producing a real photon can be written by integrating eq.(3.3),
\[ \sigma^{(v)}(s) = \int_{z_{\text{min}}}^{1} dz \int_{-\cos \theta_{\text{min}}}^{\cos \theta_{\text{min}}} dy \, H^{(v)}(x, y; s) \sigma_{0}((1 - x)s). \]  

(3.8)

where, \( z_{\text{min}} = 2E_{\text{min}}, \) \( E_{\text{min}} \) is the minimum detectable energy, and \( \theta_{\text{min}} \) is the minimum detectable angle (for a schematic description of the experimental set-up see Fig. 4).

Fig. 4 Schematic description of a typical experimental set-up where \( \theta_{v} \) is the veto angle below which no particle can be detected. \( \theta_{v} \) defines the fiducial volume of the accepted photon.

In order to evaluate the \( O(\alpha) \) virtual and soft radiative corrections to the bare process, it is sufficient to correct the cross section \( \sigma^{(v)} \) by the \( O(\alpha) \) radiator \( H^{(s)}(x, s) \)

\[ H^{(s)}(x, s) = \frac{\beta}{z} - \frac{\beta}{2}(2 - z) \]

obtained by the following expression for the radiator [38]:

\[ H(x, s) = 2(2 - x) \left( \frac{\Delta(x) \beta x^{2 - \lambda} - \frac{1}{2} \beta(2 - x) + \frac{1}{8} \beta^{2} \left[ (2 - x) \left( 3 \ln(1 - x) - 4 \ln x \right) - \frac{4 \ln(1 - x)}{x} - 6 + x \right] \right). \]  

(3.2)

The fully corrected cross section \( \sigma^{(s)} \) is given by:

\[ \sigma^{(s)}(s) = \int_{z_{\text{min}}}^{1} dz \int_{-\cos \theta_{\text{min}}}^{\cos \theta_{\text{min}}} dy \, H(x, y; s) \sigma_{0}((1 - x)s). \]  

(3.10)

The differential spectrum becomes:

\[ \frac{d\sigma^{(s)}}{dz} = H^{(s)}(x, s) \int_{0}^{z_{\text{min}}} d\xi \, H(\xi, s) \sigma_{0}((1 - x)(1 - \xi)s). \]  

(3.11)

Eq. (3.11) describes the \( z \) spectrum of a real photon accompanied by soft and virtual radiation. The \( O(\alpha) \) corrected double differential spectrum can be written

\[ \frac{d^{2}\sigma}{dz dy} = \frac{d^{2}\sigma_{0}}{dz dy} \int_{0}^{z_{\text{min}}} d\xi \, H(\xi, s) \sigma_{0}((1 - x)(1 - \xi)s). \]  

(3.12)

It can be shown that the integral of the radiator in eq.(3.10) reproduces eq.(3.3) and gives a check that \( O(\alpha) \) corrections are included in eq. (3.10) [38].

Hard Corrections are those due to photons that are detectable but that are lost for various reasons. They may go undetected because they are emitted at angles outside the active region of the apparatus, for instance down the beam pipe.

These photons effectively contribute to the bare process and must be taken into account. The total differential spectrum for emitting two photons is given by:

\[ \frac{d\sigma}{dz_{1} dz_{2} dy_{1} dy_{2}} = H^{(s)}(x_{1}, y_{1}; s) H^{(s)}(x_{2}, y_{2}; 1 - x_{1})(1 - x_{2}) s_{0}((1 - x_{1})(1 - x_{2})). \]

- Photon lost in the beam pipe

These photons are lost being emitted at angles smaller than a veto angle (see Fig.4). The production spectrum must be integrated from the forward 0° angle direction to the veto angle. To the \( O(\alpha) \), the contribution coming from one photon lost in the pipe is given by:

\[ \frac{d\sigma^{(s)}}{dx} = H^{(s)}(x, s) \int_{\cos \theta_{v}}^{\cos \theta_{\text{min}}} dy_{1} H(x, y_{1}; (1 - x)s) \sigma_{0}((1 - x)(1 - x)s). \]  

(3.13)

where \( H(x, y_{1}; s) \) is the angle dependent radiator generalized in order to handle the collinear singularity resummation problem [38], \( \theta_{v} \) is the veto angle. The factor 2 takes into account the backward collinear singularity.

\[ c_{v} = \cos \theta_{v}. \]

- Photons parallel to the observed one. The contribution to the cross section given by photons parallel to the observed one can also be evaluated to be

\[ \frac{d\sigma^{(s)}}{dz} = 2 \int_{0}^{z_{\text{min}}} dy_{1} H(x, y_{1}; s) \int_{z_{v}}^{z_{\text{min}}} dz_{2} \int_{0}^{1 - z_{2}} dy_{2} H(x, y_{2}; s) \sigma_{0}((1 - x)(1 - x)s). \]  

(3.14)
where \( c_\gamma = \cos \theta_\gamma, s_\gamma = \sin \theta_\gamma, \theta_\gamma \) being half of the resolution angle. The spectrum of the observed photon \( \frac{dE_\gamma}{dz} \) is given by the following sum:

\[
\frac{d\sigma}{dz} = \frac{d\sigma_\text{Born}}{dz} + \frac{d\sigma_\text{res}}{dz} + \frac{d\sigma_\text{rad}}{dz},
\]

where the three contributions of the r.h.s. are respectively given by eqs. (3.11), (3.13) and (3.14).

We have performed a quantitative analysis with the choice of parameters:

\( G_F^2 = \frac{e^2}{\sqrt{2} \sin^2 \theta_W \sin^2 \theta_W}, \sin^2 \theta_W = 0.223 \)

\( M_W = 82 \text{ GeV}, M_Z = 93.2 \text{ GeV}, \Gamma_Z = 2.6 \text{ GeV}, N_e = 3 \)

\( E_{\gamma \min} = 1 \text{ GeV}, \theta_{\gamma \min} = 20^\circ, \theta_\gamma = 1.5^\circ \) and \( \theta_\gamma = 0.5^\circ \)

In Tables [3]-[10] are presented the results for various choices of cuts and center of mass energy.

The cross sections called \( \text{Born}, O(\alpha) \), and \( \text{resummed} \) correspond to the formulae (3.2), (3.3) and (3.15) for the photon energy distribution respectively.

<table>
<thead>
<tr>
<th>( E_\gamma )</th>
<th>( \text{Born} )</th>
<th>( O(\alpha) )</th>
<th>( \text{resummed} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.125</td>
<td>79.718</td>
<td>50.377</td>
<td>55.427</td>
</tr>
<tr>
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<td>4.885</td>
<td>5.280</td>
</tr>
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<td>5.143</td>
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<td>1.073</td>
<td>1.150</td>
</tr>
<tr>
<td>7.064</td>
<td>0.557</td>
<td>0.404</td>
<td>0.427</td>
</tr>
<tr>
<td>9.160</td>
<td>0.224</td>
<td>0.174</td>
<td>0.183</td>
</tr>
</tbody>
</table>

Table 3: \( d\sigma/dE_\gamma \) at \( \sqrt{s} = 93.2 \text{ GeV} \) with \( M_Z = 93.2 \text{ GeV}, E_{\gamma \min} = 1 \text{ GeV}, \theta_\gamma \geq 20^\circ, \theta_\gamma = 1.5^\circ \).

<table>
<thead>
<tr>
<th>( E_\gamma )</th>
<th>( \text{Born} )</th>
<th>( O(\alpha) )</th>
<th>( \text{resummed} )</th>
</tr>
</thead>
<tbody>
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<tr>
<td>3.047</td>
<td>3.704</td>
<td>2.480</td>
<td>2.881</td>
</tr>
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<td>4.095</td>
<td>1.563</td>
<td>1.076</td>
<td>1.154</td>
</tr>
<tr>
<td>5.143</td>
<td>0.778</td>
<td>0.548</td>
<td>0.584</td>
</tr>
</tbody>
</table>

Table 4: \( d\sigma/dE_\gamma \) at \( \sqrt{s} = 93.2 \text{ GeV} \) with \( M_Z = 93.2 \text{ GeV}, E_{\gamma \min} = 1 \text{ GeV}, \theta_\gamma \geq 45^\circ, \theta_\gamma = 1.5^\circ \).

The results for two particular cases are shown in Fig 5 and 6. The cross section is reduced by the radiative corrections to the 75% of the Born approx-

Fig. 5 Photon energy differential distribution at \( \sqrt{s} = 93.2 \text{ GeV} \) with \( E_\gamma \geq 1.0 \text{ GeV}, \theta_\gamma \geq 45^\circ, \theta_\gamma = 1.5^\circ \). Dashed line refer to the Born approximation, dash-dotted to the \( O(\alpha) \) radiative corrections and continuous line to the full resummed result.
Fig. 6 Photon energy differential distribution at $\sqrt{s} = 97.2$ GeV with $E_{\gamma} \geq 1.0$ GeV, $\beta_\gamma \geq 45^\circ$, $\theta_\gamma = 1.5^\circ$. Dashed line refer to the Born approximation, dash-dotted to the $O(\alpha)$ radiative corrections and continuous line to the full resummed result.

The resummation of the collinear and soft photons affects the distributions by a few percent. This behaviour is characteristic of the initial state radiatively corrected $Z^0$ width [15].

The use of a value for the mass of the $Z^0$ of 93.2 GeV will not affect the distributions compared to the 91.1 GeV value [5] since the cross-sections depend on the relative value of the photon energy with respect to the $Z^0$ mass.

4 The background

4.1 The radiative Bhabha background

Radiative Bhabha scattering is the dominant background to the radiative method of neutrino counting. The requirement that the final state electrons be below some minimum detector acceptance angle, limits the possible $p_1$ for photons from this source. Therefore, by imposing a minimum $p_1$ for the single photons, this background can be greatly reduced or eliminated. However, for center of mass energies near the $Z^0$ mass this also eliminates much of the signal. In this case the $p_1$ requirement must be relaxed, allowing a sizeable fraction of background events. Hence it is necessary to have a good understanding of this process, accurate at the few percent level, in the form of a Monte Carlo event generator to reproduce properly the effect of experimental resolution and detector gaps.

4.1.1 Lowest order

The lowest order calculation is complicated by the fact that the squared matrix element calculation cannot neglect all electron mass terms, since the region in which the momentum transfer squared is of the order of the mass of the electron squared is the one giving the bulk of the cross section. Further complication arises from the fact that the squared matrix element exhibits very sharp peaks which have to be handled very carefully to obtain a reliable integration and an efficient event generation. Nevertheless, the lowest order evaluation of the radiative Bhabha scattering background is well understood. Several Monte Carlo event generator programs for this process exist [56] [52] [57] and are in general agreement. Also, the process can be studied analytically using the method of quasi real photon exchange [60]. The work of Mana and Martinez and of Tominou and Shimizu show that the contribution from the weak sector is small and can be neglected.

Caffo, Gatto, and Remiddi have pointed out that [49] special care must be taken to ensure that the evaluation of this process is numerically stable. The various programs can be shown to be stable if their results do not strongly depend on the internal precision. This test was performed with the TEEGG [56] generator which, like the other programs, uses double precision variables. The total cross section was evaluated for center mass energy of 100 GeV, with the photons between 100 and 200 MeV and above 45$^\circ$ and the electrons below 10 mrad. A calculation with such low energy photons should be very sensitive to any instabilities. When all the internal variables were changed to single precision, the total cross section over the same phase space changed from 13.041 ± 0.016 to 13.379 ± 0.016. The results for quadruple and double precision are indistinguishable. Hence, it is expected that double precision is sufficient for the calculation to be numerically stable.

4.1.2 Radiative correction

Since radiative Bhabha scattering can be a large background, it is important to evaluate the contribution precisely and hence the radiative correction to the process should be considered. Due to the complexity of the problem, a method to deal with the correction for the general case has not yet been developed. However, for the case of low $Q^2$ scattering, the equivalent photon approximation (EPA) has been used [56] for both the one loop virtual and soft correction and the contribution from double radiative Bhabha scattering. An exact calculation of double radiative Bhabha scattering has been performed using the helicity amplitude method [59] and typically shows agreement within a few percent of the EPA calculation. The Table [11] shows the effect of the radiative correction on the total cross section of single photons above 45$^\circ$ at the center mass energy of 93.2 GeV.

In all cases, the radiative correction is small. The uncertainty in the radiative correction to the radiative Bhabha background is hence expected to be small; at the level of a few percent.

The total cross sections given in the Table [11] are for the case where both electrons are below the veto angle. A new topology must be considered for the four body final state, where an electron is above the veto angle but has too low an energy to be detected. The table [12] shows the additional cross section for events with an electron in the central (45$^\circ$ < $\theta$ < 55$^\circ$),
and forward ($\theta_{\text{veto}} < \theta < 10^\circ$) regions. It is assumed that the $p_{\text{t}}$ detection threshold in these three regions are 0.1, 0.2, and 0.5 GeV respectively.

Since the TEEGG program was not designed to deal with these configurations, very large weights can occur for certain events. The total cross section is best evaluated with weighted events in this case. In the near future, the phase space integration for this case will be modified to improve the behavior. This topology is seen to be a rather important correction to the total radiative Bhabha background. This is particularly important for measurements with a very low photon threshold.

### 4.2 The $e^+e^- \rightarrow \gamma \gamma \gamma$ background

This background can simulate a single photon configuration when only one photon is within the fiducial volume. To estimate the $e^+e^- \rightarrow \gamma \gamma \gamma$ contribution to the single photon channel the Monte Carlo code developed by Berends and Kleiss has been used [43]. With $k_0 = 0.01$, the photon energy cutoff parameter, we generate $10^8$ events both at $\sqrt{s} = 93.2$ GeV and at $\sqrt{s} = 97.2$ GeV. The background was determined from the events with a single photon satisfying the cuts as listed in Table [13] and with no photons with $E_{\gamma} \geq E_{\text{t}}/2$ and $\theta_{\gamma} \geq 1.8^\circ$.

From Table [15] the background is seen to be small in particular in the $\theta_{\gamma} \geq 45^\circ$ region and/or after the $E_{\gamma}$ or $p_{\text{t}}^2$ cut of 1.0 GeV. It should be remarked that the minimum forward veto angle of 1.5$^\circ$ is essential for the background suppression.

### Table 11: Radiative corrections on the total cross-section of a single $\gamma$ at $\sqrt{s} = 93.2$ GeV.

<table>
<thead>
<tr>
<th>$\theta_{\text{veto}}$ ($^\circ$)</th>
<th>$E_{\gamma\text{min}}$ (GeV)</th>
<th>$p_{\text{t}}^\text{min}$ (GeV)</th>
<th>$\sigma_{\text{cor}}$ ($\sigma^2$) (pb)</th>
<th>$\sigma_{\text{cor}}$ ($\sigma^4$) (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.5</td>
<td>-</td>
<td>20.25 ± 0.05</td>
<td>20.65 ± 0.05</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5</td>
<td>0.5</td>
<td>17.00 ± 0.05</td>
<td>17.30 ± 0.05</td>
</tr>
<tr>
<td>1.5</td>
<td>1.0</td>
<td>-</td>
<td>6.49 ± 0.02</td>
<td>6.33 ± 0.01</td>
</tr>
<tr>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
<td>3.53 ± 0.02</td>
<td>3.35 ± 0.01</td>
</tr>
<tr>
<td>3.0</td>
<td>1.0</td>
<td>-</td>
<td>19.50 ± 0.06</td>
<td>19.90 ± 0.05</td>
</tr>
<tr>
<td>3.0</td>
<td>1.0</td>
<td>1.0</td>
<td>16.27 ± 0.06</td>
<td>16.43 ± 0.05</td>
</tr>
</tbody>
</table>

Table 12: Total cross-sections for the events where an electron is above the veto angle, but it has not energy enough to be detected

<table>
<thead>
<tr>
<th>$\theta_{\text{veto}}$ ($^\circ$)</th>
<th>$p_{\text{t}}$ (GeV)</th>
<th>$\sigma_{\text{central}}$ (pb)</th>
<th>$\sigma_{\text{central}}$ (pb)</th>
<th>$\sigma_{\text{forward}}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.5</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1.5</td>
<td>1.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>3.0</td>
<td>1.0</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 13: Single photon background cross sections [pb] from the $e^+e^- \rightarrow \gamma \gamma \gamma$ reaction. The total cross section at $\sqrt{s} = 93.2$ GeV and at $\sqrt{s} = 97.2$ GeV is given between parenthesis at the top of the table.

<table>
<thead>
<tr>
<th>$\theta_{\gamma}$</th>
<th>$\sqrt{s} = 93.2$ GeV</th>
<th>$\sqrt{s} = 97.2$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\gamma} \geq 20^\circ$</td>
<td>4.6</td>
<td>4.3</td>
</tr>
<tr>
<td>$E_{\gamma} \geq 0.5$ GeV</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>$E_{\gamma} \geq 1.0$ GeV</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$p_{\text{t}}^2 \geq 1.0$ GeV</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

4.3 The $e^+e^- \rightarrow e^+e^- R_{\gamma X}$ background

Resonance production via the two-photon process, $e^+e^- \rightarrow e^+e^- R_{\gamma X}$ can appear as a single photon state when the scattered $e^\pm$ both remain in the beam pipe and when, in addition, either only the $R \rightarrow \gamma X$ decay photon is observed, or when two or more of the $R \rightarrow \gamma X$ decay photons cannot be separated experimentally.

The evaluation of the two-photon cross section is based on the cross section formulae as given in [44]. The details of its Monte Carlo implementation can be found in [45]. Here, our calculation of the cross-section is based on the complete lowest order two-photon diagram and does not rely on the often used equivalent photon approximation. Furthermore the coupling of the resonance to the $\gamma\gamma$ state as given in [46] is used. In agreement with the experimental measurements to date the $\rho$-pole form factor $F(\frac{q^2}{\mu^2}) = \frac{1}{1 - \frac{q^2}{\mu^2}}$ was used to extend the $q^2 = 0$ couplings to non-zero $q^2$. It is worthwhile to remark that the inclusion of this $\rho$-pole form factor gives a reduction of the cross section of at most a few %. For the decay of the produced resonances $R$ we used simple $n$-body phase space.

In this section, we present the background contributions from the most important two-photon resonance production channels i.e. the $\pi^0$, $\eta$, $\eta'$, $f$ and the $\omega$. Table [14] summarizes the experimentally measured partial width ($\Gamma_{\gamma\gamma}$)
Table 14: Resonance parameters. $M_R$, $\Gamma_{tot}$, and $\Gamma_{\gamma\gamma}$ represent the mass, total decay width, and two-photon partial decay width of the resonance $R$ respectively.

<table>
<thead>
<tr>
<th>$R$, $(\pi^0/\eta \to \gamma\gamma)$</th>
<th>$M_R$ (GeV)</th>
<th>$\Gamma_{tot}$ (MeV)</th>
<th>$\Gamma_{\gamma\gamma}$ (keV)</th>
<th>$Br(R \to \gamma\gamma)$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0 \to \gamma\gamma$</td>
<td>0.135</td>
<td>7.6x10^{-6}</td>
<td>7.6x10^{-3}</td>
<td>100 %</td>
</tr>
<tr>
<td>$\eta \to \gamma\gamma$</td>
<td>0.549</td>
<td>1.1x10^{-3}</td>
<td>0.43</td>
<td>36.0 %</td>
</tr>
<tr>
<td>$\eta' \to \eta\gamma$</td>
<td>0.988</td>
<td>0.2</td>
<td>4.6</td>
<td>22.5 %</td>
</tr>
<tr>
<td>$\eta' \to \eta\gamma\gamma^*$</td>
<td>0.988</td>
<td>0.2</td>
<td>4.6</td>
<td>22.5 %</td>
</tr>
<tr>
<td>$\eta' \to \pi^0\pi^0\gamma$</td>
<td>1.274</td>
<td>185.</td>
<td>2.8</td>
<td>209 %</td>
</tr>
<tr>
<td>$\eta' \to \pi^0\pi^0\gamma$</td>
<td>1.274</td>
<td>185.</td>
<td>2.8</td>
<td>209 %</td>
</tr>
<tr>
<td>$f \to \pi^0\pi^0\gamma$</td>
<td>3.138</td>
<td>110.</td>
<td>0.5</td>
<td>58.0 %</td>
</tr>
</tbody>
</table>

and branching ratio ($Br(R \to \gamma\gamma)$) used in our calculation. The behaviour of the cross-section as a function of these quantities is basically:

$$\sigma(e^+e^- \to e^+e^R_{\gamma\gamma})(J + 1) \frac{\Gamma_{\gamma\gamma}}{M_R} Br(R \to \gamma\gamma)$$

where $J$ and $M_R$ are respectively the spin and the mass of the resonance $R$.

Since the cross section of the two-photon process varies only slightly as a function of $\sqrt{s}$ ($\sigma \propto ln^2(s)$), we restricted the evaluation of the background to the single value of $\sqrt{s} = 93.2$ GeV; we generated 10$^6$ events for each of the decay modes listed in table [14]. The resulting background cross sections are very sensitive to the selection criteria, we used the following combination of cuts for the identification of the single photon state:

- A single photon satisfying the combination of cuts as given in table [15].
- No charged tracks with $p_T \geq 100$ MeV in the same fiducial region as used for the single photon (20° and 45° respectively).

Table 15: Single photon background cross sections [pb] from the $e^+e^- \to e^+e^-R_{\gamma\gamma}$ reaction. Cross sections between parentheses at the top of the table give the 'Low' estimate for the total cross section for each of the channels. «-» entries indicate that < 1 event was selected in that particular class.

<table>
<thead>
<tr>
<th>$E_R$, GeV</th>
<th>$\sigma(e^+e^- \to e^+e^-R_{\gamma\gamma})$ (20°)</th>
<th>$\sigma(e^+e^- \to e^+e^-R_{\gamma\gamma})$ (45°)</th>
<th>$\sigma(e^+e^- \to e^+e^-\eta\gamma)$ (20°)</th>
<th>$\sigma(e^+e^- \to e^+e^-\eta\gamma)$ (45°)</th>
<th>$\sigma(e^+e^- \to e^+e^-\eta'\gamma)$ (20°)</th>
<th>$\sigma(e^+e^- \to e^+e^-\eta'\gamma)$ (45°)</th>
<th>$\sigma(e^+e^- \to e^+e^-f\gamma)$ (20°)</th>
<th>$\sigma(e^+e^- \to e^+e^-f\gamma)$ (45°)</th>
<th>$\sigma(e^+e^- \to e^+e^-f\gamma)$ (20°)</th>
<th>$\sigma(e^+e^- \to e^+e^-f\gamma)$ (45°)</th>
<th>$\sigma(e^+e^- \to e^+e^-f\gamma)$ (20°)</th>
<th>$\sigma(e^+e^- \to e^+e^-f\gamma)$ (45°)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_R \geq 20$ GeV</td>
<td>7.8</td>
<td>9.2</td>
<td>18.6</td>
<td>17.8</td>
<td>108.0</td>
<td>6.3</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
</tr>
<tr>
<td>$E_R \geq 0.5$ GeV</td>
<td>2.3</td>
<td>3.3</td>
<td>6.7</td>
<td>6.7</td>
<td>43.2</td>
<td>2.3</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td></td>
</tr>
<tr>
<td>$E_R \geq 1.0$ GeV</td>
<td>2.3</td>
<td>3.3</td>
<td>6.7</td>
<td>6.7</td>
<td>43.2</td>
<td>2.3</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td></td>
</tr>
<tr>
<td>$p_T \geq 1.0$ GeV</td>
<td>2.3</td>
<td>3.3</td>
<td>6.7</td>
<td>6.7</td>
<td>43.2</td>
<td>2.3</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td>229</td>
<td></td>
</tr>
</tbody>
</table>

- No other photon with $E_\gamma \geq 250$ MeV separated by more than $10^\circ$ from the primary photon; again, the secondary photon must be within the fiducial volume.

We did not use the forward veto on the scattered (high energy) $e^\pm$; as a matter of fact, for the two-photon process, the forward veto turns out to be almost useless as a result of the relatively high $q^2$ corresponding to $\theta \geq 1.5^\circ$ (reduction of the background cross section < 1%). Table [15] summarizes our results for $\theta_R \geq 20^\circ$ and $\theta_\gamma \geq 45^\circ$, and $p_T^\gamma$ cuts of 0.5 and 1.0 GeV respectively. It is clear from this table that a $\theta_\gamma \geq 45^\circ$ cut is compulsory in order to keep the single photon background cross section from the two-photon reaction manageable. Moreover, it has to be underlined that the experimental input to the Monte Carlo calculation i.e. the two-photon decay widths $\Gamma_{\gamma\gamma}$, are only known to the 10% level. This sets a lower limit on the accuracy of the background cross section calculation of at least 10%. Hence, in order to have a good control on systematic errors, these contaminations have to be kept as small as possible.

The background can be further suppressed by the energy cut or, equivalently, by the $p_T^\gamma$ cut. A 1.0 GeV cut gives a background cross section of < 1 pb (largely coming from the $e^+e^- \to e^+e^-f\gamma$ reaction), to be compared with the signal cross section of about 30 pb.

4.4 The $e^+e^- \to e^+e^-\mu^+\mu^-\gamma$ and $e^+e^- \to e^+e^-e^+e^-\gamma$ backgrounds.

The role played by this reaction as a background to the single photon channel has been pointed out time ago [9]. Contrary to the two-photon resonance cross section, which suffered from the $\geq 10%$ uncertainty in the value of $\Gamma_{\gamma\gamma}$, the cross section of the $e^+e^- \to e^+e^-\mu^+\mu^-\gamma$ and $e^+e^- \to e^+e^-\mu^+\mu^-\gamma$ reactions are in principle entirely calculable within the QED framework. In spite of this our estimates are based on a combination of the exact cross section formula for the lowest order $e^+e^- \to e^+e^-\mu^+\mu^-\gamma$ and $e^+e^- \to e^+e^-e^+e^-\gamma$ processes [44] and a program described in Ref [47] to include photon radiation by the produced $\mu^+\mu^-$ or $e^+e^-$ pair.

The results are based on 20000 generated events from 0.2 GeV onwards for $e^+e^- \to e^+e^-\mu^+\mu^-\gamma$ and on 10000 generated events from threshold (0.21 GeV) onwards for $e^+e^- \to e^+e^-\mu^+\mu^-\gamma$ (threshold here refers to the invariant mass of the $\gamma\gamma$ produced $e^+e^-$ or $\mu^+\mu^-$ pair). With the same selection as the one of the previous subsection we got the results summarized in table [16]. The uncertainty in the listed background cross sections can be as large as 100% for the small cross sections since they are based on a single accepted event. As far as the case of two-photon resonance production it is again clear that the fiducial volume should be limited to the $\theta_R \geq 45^\circ$ region. A tracking device in front of the calorimeters is essential in controlling the $e^+e^- \to e^+e^-\mu^+\mu^-\gamma$ and $e^+e^- \to e^+e^-e^+e^-\gamma$ backgrounds. Finally, once more, $E_R$ or $p_T^\gamma \geq 1.0$ GeV cuts are necessary to keep the background cross sections well below the signal cross section.
<table>
<thead>
<tr>
<th>$\theta_\gamma \geq 20^\circ$</th>
<th>$e^+e^- \rightarrow e^+e^-e^-\gamma$</th>
<th>$e^+e^- \rightarrow e^+e^-\mu^+\mu^-\gamma$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_L \geq 0.5,\text{GeV}$</td>
<td>85.9</td>
<td>32.9</td>
<td>120</td>
</tr>
<tr>
<td>$E_L \geq 1.0,\text{GeV}$</td>
<td>4.1</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>$p_T &gt; 1.0,\text{GeV}$</td>
<td>2.1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$\theta_\gamma &gt; 45^\circ$</td>
<td>---</td>
<td>14.4</td>
<td>14</td>
</tr>
<tr>
<td>$E_L \geq 0.5,\text{GeV}$</td>
<td>---</td>
<td>2.1</td>
<td>2</td>
</tr>
<tr>
<td>$E_L \geq 1.0,\text{GeV}$</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Table 16: Single photon background cross sections [pb] from the $e^+e^- \rightarrow e^+e^-e^-\gamma$ and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-\gamma$ reactions. "--" entries indicate that no event was selected in that particular class.

4.5 Other sources of background

The list of the other possible sources of Background includes also the processes $e^+e^- \rightarrow q\bar{q}, e^+e^- \rightarrow \mu\bar{\mu}$. These backgrounds give however a negligible contribution to the "single photon like" cross section [48].

Two main sources of background which could give single photon events in the detector are synchrotron radiation and electron beam gas bremsstrahlung. The sources of synchrotron radiation are the dipole magnets and the straight section quadrupoles. The vacuum pipe size in the region of LEP detectors and the collimators have been optimized to reduce its flux [49]. According to these Monte Carlo simulations the photon spectrum goes from $10\,\text{KeV}$ to $1\,\text{MeV}$ with $E_\gamma > 180\,\text{MeV}$ giving no single photon type signal in the calorimeters.

Beam-gas background is by far the most dangerous one. The rates at LEP have been estimated via Monte Carlo [50] [51]. With a vacuum pressure of $2 \times 10^{-10}\,\text{torr}$, a beam current of $3\,\text{mA}$, a luminosity of $L = 10^{33}\,\text{cm}^{-2}\text{s}^{-1}$, the estimated rate on the electromagnetic barrel is $1.5\,\text{Hz}$. The maximum electromagnetic energy that can be seen in such conditions is of $0.150\,\text{GeV}$ [51]. In Ref. [51] we can see that by requiring $\theta_\gamma > 45^\circ$ this background is greatly reduced and no problems of contamination of the signal will be brought by beam-gas single photon at $\sqrt{s} = M_{Z^0}$.

5 Numerical estimates

In this section we compare the results obtained by using Monte Carlo generators for the signal and for the background. The programs used are NNGG03, KORALZ and MOE together with the ones mentioned before for the background.

The outcome from the various programs are compared and an evaluation of the statistic and systematic errors is presented.

5.1 Comparisons of Monte-Carlo Generators

5.1.1 Comparison between NNGG03 and KORALZ

NNGG03 is a Monte-Carlo generator including an exact calculation of the $O(a^2)$ matrix element, followed by an exponentiation of the soft photons [29] [52] [58]. It presently generates one or two photon final states. The electroweak corrections are those of the star scheme of B. Lynn [15].

KORALZ was originally a $\tau\tau$ and $\mu\mu$ generator suited for LEP energies. Initial state radiative corrections are treated through the YFS2 algorithm, which allows the generation of multiphoton final states. It has been recently upgraded to generate also neutrino pairs with multiphotons, without changes in the algorithm [53].

The physical parameters chosen for the comparison are the following: $M_{Z^0} = 92\,\text{GeV}$, $m_t = 60\,\text{GeV}$, $M_H = 100\,\text{GeV}$. The weak mixing angle, $\sin^2 \theta_W$, and the $Z^0$ width have been calculated from above parameters and found to be respectively $0.2203$ and $2.56\,\text{GeV}$. We define an idealized acceptance by $E_\gamma \geq 0.5\,\text{GeV}$ and $\theta_\gamma \geq 15^\circ$.

We focus here on the comparison of these two programs. It has been done [54] at three levels: first-order, $Z^0$ only, for which the comparison with analytical calculations is trivial, first-order including W exchange, and "all corrections included", which means the complete programs as described above. The contribution of the W exchange deserves some care. We first compare the effect of the $W$: $(1 + \frac{W}{Z^0}) - (\frac{Z^0}{Z^0})$. Fig. 7 shows an analytical calculation in the PIA approximation. We see that it agrees well with the corresponding KORALZ result in this approximation. NNGG has the W-exchange exactly implemented.

---

Fig. 7 $W$ contribution to the single $\gamma$ production $\sigma(1 + W/Z^0) - \sigma(Z^0/Z^0)$ in percent. The cuts described in the text are applied, as a function of center-of-mass energy.

---

27

---

28
The comparison of the NNGG points and the analytical calculation shows the level of this approximation. The W contribution is negative below the peak, due to the strong interference with the Z, and reaches 10% at 110 GeV c.m. energy (notice that this result is very sensitive to the cuts).

The comparison of the cross-sections was then carried out at the best level of accuracy of the two programs. In this case, the exact matrix element is used for the W exchange in both programs. The values of the cross-section are given at different center-of-mass energies in Table 17. Fig. 8 (ratio of KORALZ to NNGG03 cross-sections) shows the agreement between these two completely independent calculations, at the 0.5 - 1 % level.

<table>
<thead>
<tr>
<th>Program</th>
<th>90 GeV</th>
<th>92 GeV</th>
<th>94 GeV</th>
<th>99 GeV</th>
<th>110 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNGG03</td>
<td>38.4±0.1</td>
<td>144.9±0.2</td>
<td>245.4±0.5</td>
<td>117.8±0.1</td>
<td>46.5±0.1</td>
</tr>
<tr>
<td>KORALZ</td>
<td>38.4±0.2</td>
<td>143.7±1.0</td>
<td>245.1±1.2</td>
<td>117.9±0.6</td>
<td>46.3±0.3</td>
</tr>
</tbody>
</table>

Table 17: Cross-sections in pb.

Fig. 8 Ratio of the KORALZ to the NNGG03 cross-section, as a function of center-of-mass energy.

Differential cross-sections have also been compared. Starting from events with at least one photon in the acceptance, we call "first photon" the most energetic one, and "second" the second highest energetic one. The fraction of events with 1 and 2 photons in the acceptance are given in Table 18 at 92 GeV, 99 GeV and 110 GeV.

<table>
<thead>
<tr>
<th>Program</th>
<th>frac. 1 photon</th>
<th>frac. 2 photons</th>
<th>frac. &gt; 2 photons</th>
</tr>
</thead>
<tbody>
<tr>
<td>KORALZ</td>
<td>0.992</td>
<td>0.008</td>
<td>0.000</td>
</tr>
<tr>
<td>NNGG03</td>
<td>0.990</td>
<td>0.010</td>
<td>none</td>
</tr>
<tr>
<td>KORALZ</td>
<td>0.965</td>
<td>0.034</td>
<td>0.001</td>
</tr>
<tr>
<td>NNGG03</td>
<td>0.968</td>
<td>0.032</td>
<td>none</td>
</tr>
<tr>
<td>KORALZ</td>
<td>0.961</td>
<td>0.047</td>
<td>0.002</td>
</tr>
<tr>
<td>NNGG03</td>
<td>0.961</td>
<td>0.039</td>
<td>none</td>
</tr>
</tbody>
</table>

Table 18: Fraction of events with 1, 2 and more than 2 photons, at three center of mass energies for $M_{2\gamma} = 92$ GeV.

the fact that KORALZ takes into account the multibody kinematics in the final state.

Various distributions have been compared at three center-of-mass energies (92 GeV, 99 GeV, 110 GeV). At 99 GeV is shown in Fig. 9. At 110 GeV (Fig. 10), we see that the slight excess of events with a second photon is concentrated at low energy, which tends to confirm the interpretation of the difference as a soft photon effect.

Fig. 9 Comparison of the distributions relevant to the first and the second photon, from NNGG03 (continuous line histogram) and KORALZ (triangular points) at $\sqrt{s} = 99$ GeV. The angular distributions are weighted by $\sin \theta$ to render them flat.
Fig. 10 Comparison of energy distribution of the 2nd photon from NNGG03 and KORAL03, at $\sqrt{s} = 110$ GeV. The energy distribution of the 3rd photon (KORAL03 only) is also shown.

5.1.2 Comparison between MOE and NNGG03

MOE is a Monte Carlo generator based on the factorization properties of the structure function formalism [55] and it is described in the generator group report of this workshop. It generates, in an exclusive way, multiple photon configurations according to a dressed Born approximation. A comparison of MOE with NNGG03 [52] [58], with and without soft photon exponentiation, has been performed. We show the results obtained with the version of NNGG03 with exponentiation of soft photons and MOE that does naturally contain multiple soft photons configurations. By using the following parameters, $M_Z = 93.2$ GeV, sin$^2\theta_W = 0.223$, $\theta_0 = 2.5^\circ$, $E_\gamma > 1$ GeV, one has the results summarized in Table [19].

<table>
<thead>
<tr>
<th>Program</th>
<th>94 GeV</th>
<th>95 GeV</th>
<th>96 GeV</th>
<th>97 GeV</th>
<th>98 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNGG03</td>
<td>155.0</td>
<td>214.7</td>
<td>207.0</td>
<td>182.6</td>
<td>157.6</td>
</tr>
<tr>
<td>MOE</td>
<td>160.1</td>
<td>210.1</td>
<td>203.0</td>
<td>175.5</td>
<td>163.3</td>
</tr>
</tbody>
</table>

Table 19: Cross-sections in $p\bar{p}$

The results above differ by 1% to 3%. The choice of a small veto angle naturally constrains the comparison to be sensitive to the detailed structure of the electromagnetic radiation, particularly in the collinear region. If the $\theta_0$ constraint is relaxed to $10^\circ$ the agreement improves as shown in Table [20].

<table>
<thead>
<tr>
<th>Program</th>
<th>92.2 GeV</th>
<th>95.2 GeV</th>
<th>98.2 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNGG03</td>
<td>29.7</td>
<td>76.7</td>
<td>73.9</td>
</tr>
<tr>
<td>MOE</td>
<td>30.1</td>
<td>78.0</td>
<td>74.9</td>
</tr>
</tbody>
</table>

Table 20: Cross-sections in $p\bar{p}$ with $\theta_0 = 10^\circ$

This result is consistent with the one in the previous KORALZ and NNGG03 comparison where no minimum veto angle has been imposed.

5.1.3 Conclusion

In conclusion, a detailed comparison has been done between three of the $\nu\bar{\nu}$ generators, and they have been found to be in very good agreement. This gives confidence in the results of the programs and shows that they can be used for the neutrino counting experiments.

5.2 Signal versus background

The comparison of the signal with the background is presented in this subsection. For the various sources of background the results have been obtained from the programs described in the previous section. For the signal has been used the MOE Monte Carlo [55]. In Tables [21] and [22] the results at $\sqrt{s} = 93.2$ GeV and $\sqrt{s} = 97.2$ GeV are presented together with an evaluation of the pure statistical error on the number of the neutrino generations $\Delta N_\nu$ at $1\sigma$.

<table>
<thead>
<tr>
<th>$\theta_0 &gt; 20^\circ$</th>
<th>signal</th>
<th>Bhabha</th>
<th>$R \rightarrow \gamma X$</th>
<th>$\gamma\gamma$</th>
<th>$\text{cell} \gamma$</th>
<th>$\Delta N_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_\gamma \geq 0.5$ GeV</td>
<td>144</td>
<td>181</td>
<td>220</td>
<td>6.9</td>
<td>120</td>
<td>0.42</td>
</tr>
<tr>
<td>$E_\gamma \geq 1$ GeV</td>
<td>67</td>
<td>101</td>
<td>36</td>
<td>2.2</td>
<td>4</td>
<td>0.49</td>
</tr>
<tr>
<td>$P_T \geq 1$ GeV</td>
<td>41</td>
<td>17</td>
<td>0.5</td>
<td>0.8</td>
<td>2</td>
<td>0.38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta_0 &gt; 45^\circ$</th>
<th>signal</th>
<th>Bhabha</th>
<th>$R \rightarrow \gamma X$</th>
<th>$\gamma\gamma$</th>
<th>$\text{cell} \gamma$</th>
<th>$\Delta N_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_\gamma \geq 0.5$ GeV</td>
<td>74</td>
<td>20</td>
<td>66</td>
<td>1.3</td>
<td>14</td>
<td>0.39</td>
</tr>
<tr>
<td>$E_\gamma \geq 1$ GeV</td>
<td>34</td>
<td>6</td>
<td>1.1</td>
<td>0.6</td>
<td>2</td>
<td>0.37</td>
</tr>
<tr>
<td>$P_T \geq 1$ GeV</td>
<td>29</td>
<td>4</td>
<td>0.4</td>
<td>0.0</td>
<td>-</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 21: Signal (MOE Monte Carlo) versus background cross sections in picobarns at $\sqrt{s} = 93.2$ GeV with $M_Z = 93.2$ GeV, $\theta_{\text{veto}} = 1.5^\circ$ with a total integrated luminosity of $3$ pb$^{-1}$. $\text{cell} \gamma$ stands for $ee \rightarrow e\gamma\mu\nu$ and $ee \rightarrow ee\gamma\gamma$.

<table>
<thead>
<tr>
<th>$\theta_0 &gt; 20^\circ$</th>
<th>signal</th>
<th>Bhabha</th>
<th>$R \rightarrow \gamma X$</th>
<th>$\gamma\gamma$</th>
<th>$\text{cell} \gamma$</th>
<th>$\Delta N_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_\gamma \geq 0.5$ GeV</td>
<td>180</td>
<td>172</td>
<td>270</td>
<td>4.3</td>
<td>120</td>
<td>0.34</td>
</tr>
<tr>
<td>$E_\gamma \geq 1$ GeV</td>
<td>165</td>
<td>98</td>
<td>38</td>
<td>2.0</td>
<td>4</td>
<td>0.22</td>
</tr>
<tr>
<td>$P_T \geq 1$ GeV</td>
<td>144</td>
<td>20</td>
<td>0.5</td>
<td>0.3</td>
<td>2</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 22: Signal (MOE Monte Carlo) versus background cross sections in picobarns at $\sqrt{s} = 97.2$ GeV with $M_Z = 93.2$ GeV, $\theta_{\text{veto}} = 1.5^\circ$ at a total integrated luminosity of $3$ pb$^{-1}$.
results, for particular sets of cuts are also presented in Figs 11 and 12. The $ee \rightarrow e\gamma\gamma$ background results have been included only in the Tables.

Figures 11 a, b and 12 a, b show the effect of increasing the minimum photon angle from 20° to 45° on the photon energy distribution at $\sqrt{s} = 93.2$ GeV and $\sqrt{s} = 97.2$ GeV respectively. It is seen that both Bhabha and two photon resonance $E_\gamma \rightarrow \gamma\gamma\gamma$ background are strongly reduced. In Fig. 11 c, d and 12 c, d similarly a better signal versus background ratio is apparent when, with the same $\theta_\gamma$, a $p_\gamma$ instead of a $E_\gamma$ cut is applied.

These results based on the evaluation of the statistical errors on the signal versus background ratio show that a value of the center of mass energy of $\sqrt{s} = M_{\tau\tau} +4$ GeV is clearly favourable for the measurement of the number of neutrino families.

An accuracy of $\Delta N_\nu \sim 0.2$ can be reached if the minimum of the photon energy is set at $E_\gamma \geq 1$ GeV. At a lower value of the energy threshold $E_\gamma \geq 0.5$ GeV on the contrary this comparison shows a poorer signal to background ratio and therefore a lower accuracy on $\Delta N_\nu$. The cut on the transverse momentum of the photon $p_\gamma \geq 1$ GeV gives better results than the $E_\gamma \geq 1$ GeV cut.

The above results also show that on the peak a hard cut on $\theta_\gamma$ has a through-out positive effect on the accuracy of $\Delta N_\nu$. Off the peak the gain in the signal to background ratio is approximately compensated by the loss in statistics.

5.3 Errors on signal versus background

We have made some evaluation of various errors entering in the signal versus background ratio. The analysis we have carried can be described as follows:

We have looked at the signal at various ranges of energies around the peak or above the peak or at the peak itself.
We have chosen as a value of the mass of the $Z^0$ $M_{Z^0} = 93.2$ GeV and we have made for the case around the peak a scan at $\sqrt{s} = 93.2$, 94.2, and 95.2 GeV.

We have also carried a single energy signal evaluation at 97.2 GeV. The signal has been generated with the use of the NNG03 Monte Carlo [58]. For the background we have considered the channels: Bhabha [52], $e^+e^- \rightarrow \gamma\gamma\gamma$, $e^+e^- \rightarrow e^+\gamma$, $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-\gamma$.

The events for the signal and background have been generated as functions of the luminosity within the acceptance domain of $\theta_\gamma \geq 45^0$, $\theta_e = 1.5^0$ and demanding only one photon with an energy of $E_\gamma \geq 1$ GeV within this region. The results, where only the statistical errors are included, are shown for the scan at the three energies 93.2, 94.2, 95.2 in Fig. 13. The results of the application of the same procedure at the dedicated energy of $\sqrt{s} = 97.2$ GeV are shown by the continuous line in Fig. 14 for $E_\gamma \geq 1$ GeV. In these values a 95% confidence level is assumed.

![Fig. 13 Uncertainty on the determination of $N_\nu$ as function of the integrated luminosity in a scan around the $Z^0$ with three points.](image)

In order to get an estimate of the impact of the systematic errors on the determination of $\Delta N_\nu$ we have repeated the same procedure by assuming in addition to the statistical errors a 1% systematic shift in the luminosity for the single energy case. For the scanning combination there is a 1% systematic error included for every point. Furthermore we have assumed an energy dependence of the linear type of the luminosity with the energy with a 1% variation in a 10 GeV range.

A systematic error of the 1% has been assumed on the various backgrounds.

In all the figures the dashed line refers to this systematic error effect in the determination of $\Delta N_\nu$. An uncertainty of 150 MeV in the $M_{Z^0}$ mass affects the difference between the dashed and continuous line by a factor $\approx 1.5$ thus increasing the effect of the systematic uncertainties for the scanning case in Fig. 13.

Even if the systematic errors on the background due to the $2\gamma$ resonance production $e^+e^- \rightarrow e^+e^- R_{\gamma\gamma}$ can be larger than the 1% value assumed, the use of cuts in angles and energies is sufficient, as shown before, to considerably reduce the impact of this channel on the signal versus background ratio.

From the figures 13, 14 we can obtain a determination of $\Delta N_\nu$ consistent, at the same confidence level, with the estimates given in the tables above.

### 5.4 Fitting the single photon distribution

The interpretation of single photon data by the technique of subtracting the expected backgrounds minimize the statistical uncertainties. This method, however, can suffer from large systematic errors specific to each detector.

An alternative approach, less sensitive to systematic uncertainties, matches the observed single photon distribution to the expected signal and background distributions. The sensitivity of such an approach was studied with a maximum likelihood method. Monte Carlo data samples of 1 pb$^{-1}$ of signal and background (only radiative Bhabha scattering was considered) at various beam energies were generated. The combined signal and background energy and angular distribution was fit to analytic forms of the expected distributions by minimizing the negative log likelihood function,

$$\zeta(z) = -\sum_i \log \left( z_i \frac{d\sigma_i(E_{\gamma}, \theta_{\gamma})}{\sigma_{\gamma_{\gamma}}} + (1 - z_i) \frac{d\sigma_i(E_{\gamma}, \theta_{\gamma})}{\sigma_{\gamma}} \right)$$

where $i$ runs over the observed events. The analytical functions describing the expected energy and angular distributions were obtained from lowest order calculations. The number of neutrino generations is determined by,

$$N_\nu = 3 + \frac{2 E_{n_{\text{total}}} - n_{\text{exp}}}{4 n_{\text{exp}}}$$

where $n_{\text{total}}$ is the total number of single photon events observed (signal and background) and $n_{\text{exp}}$ is the number expected from the $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ process.


with 3 generations alone. The denominator in the above expression roughly represents the difference in the rate for 4 and 3 generations. The uncertainty in the number

$$\delta N_e = \frac{1}{n_{exp}} \left( \sum \delta_{\text{obs}}^2 + \sigma_{\text{obs}}^2 \right)^{1/2}.$$  

The uncertainty, $\delta N_e$, is determined in the minimization from the values which change the log likelihood by $1/2$.

Within 2 GeV of the $2\nu$ peak, the sensitivity of the measurement strongly depends on the minimum observed photon energy. For example on the peak, if only the region above 45° is considered, $\delta N_e = 1$ for $E_{\gamma}^{\text{min}} = 0.5$ GeV and $\delta N_e = 1.7$ for $E_{\gamma}^{\text{min}} = 2.0$ GeV.

The sensitivity improves for higher center of mass energies, until about 4 GeV above the peak where it becomes roughly constant. At this energy, $\delta N_e = 0.5$ and 0.6 for the two cases considered above. Far above the peak, the sensitivity decreases somewhat - $\delta N_e$ rises to about 0.7 for both cases 12 GeV above the peak. These errors are expected to scale roughly with the square root of the luminosity. Hence, for an integrated luminosity of 3 pb$^{-1}$, 4 GeV or more above the $2\nu$ mass, a sensitivity of 0.3 generations can be obtained.

The effect of various systematic errors was also considered. The method is most sensitive to systematic effects in the electromagnetic calorimeter at center of mass energies near the $2\nu$ mass. However, shifts of 100 MeV or 2% or excess noise of 5% cause errors of less than 0.2 generations for center of mass energies 4 GeV or more above the peak. Other effects such as uncertainty in the veto angle, $2\nu$ mass and luminosity can also be controlled below the 0.2 generation level.

This procedure can be modified to improve the statistical uncertainty (but at the same time allowing larger systematic errors) by adding to the likelihood fit information of the total number of expected background events. The optimal approach depends of the specific detector properties and the amount of available data.

6 Summary

In this report we have examined in detail various methods to determine the total number of light neutrino species. It appears that the width and radiative methods being independent and complementary, both should be used to decide if a new family and/or new physics is showing up beyond the Standard Model. A large amount of new theoretical information and results have been achieved in the last few years. For instance, concerning the width method, new calculations of electroweak corrections within the Standard Model and the effects of possible new physics have been included. Accurate evaluations of photonic corrections have put the radiative method on a firmer ground. Also on the side of the Monte-Carlo generators, the agreement among the different programs provides us with new tools to deal with the experimental results.

For the case of the single-photon approach, in order to measure the signal close to the $2\nu$ pole, a study of the background has been performed.

Our study of the systematic and backgrounds shows that both methods can achieve equal accuracy with a total integrated luminosity of a few pb$^{-1}$.

Concerning the width method, the Feldman approach gives the most accurate determination of the number of neutrinos within the Standard Model with low statistics. However, at high statistics, this approach has to be complemented with additional measurements to take into account new physics and the determination of the invisible width should include a measurement of the total width.

For the radiative method, it turns out that a measurement of low-energy photons (≤ 200 MeV) has to deal with the systematics coming from the two-photon resonance subtractions, which, therefore, requires the minimum photon energy to be above 1 GeV. From the systematics point of view, we also suggest to have a higher $\theta_v$ cut of, say, 45°, in order to overcome the problem of the two-photon background.

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References


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[51] P. Baratti, Nota interna 919/1988, Dipartimento di Fisica, Universita' di Roma "La Sapienza".
[53] see the "Monte Carlo Generators" group report at this Workshop.