Recent Studies on Transverse Beam Behaviour at the CERN PS

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Abstract The CERN Proton Synchrotron accelerates several types of particles with intensities up to 2E13 ppp. Field imperfections of the CPS magnet, space charge and impedance effects in the presence of high intensity beams perturb the betatron motion and limit the machine performances. Calculations of tune dependence on beam intensity have been performed and compared with measurements. Experimental observation of beam trajectories in the transverse phase plane have been carried out for dynamic aperture and resonance studies.

Incoherent Space Charge Tune Shift at 1 GeV

The incoherent detuning for a beam of elliptical cross section with arbitrary transverse particle distribution is

$$\Delta Q^J_{x,y} = -\frac{2 N r_p F_{ix,y}}{\pi \varepsilon_{x,y} \beta^2 \gamma^3 B} \sum_{m,n} \frac{\varepsilon_x^m \varepsilon_y^n}{2^{m+2n} (m)! (n)!} < v_{2m,2n} p_x^m p_y^n >$$

where \( N \) is the number of particles in the beam, \( r_p \) the classical proton radius, \( B \) the bunching factor, \( \varepsilon_x \) and \( \varepsilon_y \) the radial and vertical emittances, \( \beta_x \) and \( \beta_y \) the Twiss parameters. \( F_{ix,y} \) is a correction factor for image fields depending on the surroundings (close to unity for the CPS), \( v_{2m,2n} \) are the power series expansion terms of the beam potential, \( \beta \) and \( \gamma \) are the usual relativistic parameters. The integer \( p \) is equal to \( m \) or \( n \) in the radial or vertical plane respectively, and the brackets denotes the average over the machine circumference. For a beam of uniform density and for negligible image fields and momentum spread this equation reduces to the direct space charge Laslett tune shift

$$\Delta Q^J_{x,y} = -N r_p \left( \frac{\varepsilon_x + \sqrt{\varepsilon_x \varepsilon_y} Q_y / Q_x}{Q_x, y} \right)^{-1} (\pi \beta^2 \gamma^3 B)^{-1}$$

where \( Q_x \) and \( Q_y \) are the radial and vertical tunes.

The six-dimensional ACCSIM code\(^1\)\(^2\) has been used to calculate the incoherent tune shift. Elliptical particle distributions in transverse phase-space and a "binomial" distribution in longitudinal phase-plane...
have been assumed. Simulation takes into account image fields induced in the vacuum pipe. The calculated tune shifts for various amplitudes of oscillations of a 1.6E13 proton beam injected at 1 GeV are

\[
(0,0) \quad (r_x,0) \quad (0,r_y) \quad (r_x,r_y)
\]

\[
\Delta Q_x = -0.141 -0.103 -0.123 -0.087
\]

\[
\Delta Q_y = -0.185 -0.159 -0.141 -0.115
\]

where \(r_x\) and \(r_y\) are twice the rms x and y beam dimensions respectively.

**COHERENT TUNE SHIFT MEASUREMENTS AT 1 GeV AND 26 GeV/c**

At 1 GeV, the expression for coherent detuning is

\[
\Delta Q_{x,y}^c = -\frac{N R r_p}{\pi Q_{x,y} \beta^2 \gamma} F_{x,y}
\]

where \(F_{x,y}\) takes into account the image currents and the bunch length, \(R\) is the orbit radius. Analysing the response of a kicked beam with enhanced FFT techniques, the tunes can be measured with very high accuracy, up to 1E-4, in a short time (10 ms). Even higher frequency resolutions can be achieved by Wigner-Ville distribution analysis.

Figure 1a shows the coherent tune dependence on beam intensity.

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**FIGURE 1a.** Measured (dots) and calculated (solid line) coherent tune shift versus bunch intensity at 1 GeV.

**1b.** Measured (dots) and fitted (solid line) coherent tune shift versus \(Nb/4\sigma_t\) at 26 GeV/c.

At 26 GeV/c, the coherent tune shifts \(\Delta Q_{x,y}^c\) for proton bunches are dominated by the interaction between the beam and the inductive part of the wide band transverse impedance \(Z_{\perp}\) of the vacuum chamber

\[
\Delta Q_{x,y} = -\frac{e R N_b Z_{x,y}}{\pi^2 Q_{x,y} \beta (E/e) 4\sigma_t}
\]
where \( N_b \) is the number of particles in the bunch, \( \sigma_t \) the rms bunch length (in \( \mu \text{s} \)) and \( e \) the electron charge. From the measured data, plotted in figure 1b, the transverse impedances are evaluated as: \( Z_{\perp x} = 1.1 \ \text{MQ/m} \) and \( Z_{\perp y} = 2.8 \ \text{MQ/m} \). The corresponding longitudinal impedances may be written

\[
\left[ \frac{Z_n}{n} \right]_{x,y} = \frac{b_{x,y}^2 \beta}{2 R} Z_{\perp x,y}
\]

where \( b_{x,y} \) is the equivalent chamber radius defined as \( b_x = (h+w)/2 \) and \( b_y = h \), with \( w \) and \( h \) the half width and half height of the vacuum chamber respectively. The results are \( (Z_n/n)_x = 16 \ \Omega \) and \( (Z_n/n)_y = 17 \ \Omega \). These values are in very good agreement with measurements previously made with other methods (e.g. synchrotron frequency shift and potential well bunch lengthening)\(^7,\)\(^8\).

**DYNAMIC APERTURE MEASUREMENTS**

The dynamic aperture can be defined as the stable domain area of the six-dimensional particle phase space. The following studies have been performed to investigate the machine aperture at 1 GeV. Beam phase plane coherent trajectories were experimentally observed by sampling each turn a bunch position at two pickups \( 1/4 \) betatron wave length out of phase\(^4,\)\(^9\). Coherent motions of the beam were obtained by three methods: i) using the intrinsic beam instabilities (resistive wall), ii) applying the output of a beam position monitor to a kicker (self beam excitation), iii) kicking the beam with a pulse function (Q-meter). With nominal tunes away from resonances the trajectories trace out circles in the \((x,x')\) or \((y,y')\) planes (invariant surface). When the tunes lie close to a resonance the trajectories exhibit an island structure. Figure 2 demonstrates the existence of such islands for the resonance \( 6Q_x + 7Q_y = 81 \).

Observation of chains of islands in some cases showed the existence of resonances up to the 24th order. If the machine optics is nonlinear, the tunes change for particles of large amplitudes, involving distortions of the circular trajectory. As the amplitude increases the invariant surfaces break, producing chains of islands leading to unstable, i.e no longer closed trajectories. The maximum amplitude of the last stable trajectory gives the dynamic aperture.
FIGURE 2  Transverse phase space trajectories and fractional tunes measured with FFT on 128 turns at 1 GeV close to the $6Q_x+7Q_y=81$ resonance, excited for this purpose by the multipole components of the poleface windings.

The chromaticity of the CPS bare machine is equal to -1 in both planes and depends very little on the position of the orbit. Direct observation at 1 GeV shows that the tunes remain constant as the particle amplitude increases, confirming that the optics is almost perfectly linear. Finally the beam intensity starts to decrease without observing the breaking of the invariant surfaces. This shows that the dynamic aperture is larger than the physical aperture evaluated as: $A_x \approx 60 \text{ mm.mrad}$ and $A_y \approx 20 \text{ mm.mrad}$.

Processing the same data it was possible to view coupling resonances by plotting the action-angle variables $(J_x, \theta_x, \theta_y)$ or $(J_y, \theta_x, \theta_y)$ instead of the $(x,x')$ or $(y,y')^{10}$. Such 3 dimensional plots are shown in Figure 3 near the coupling resonances $4Q_y - Q_x = 19$ and $2Q_y + Q_x = 19$.

FIGURE 3 Trajectories in action-angle $J_x$, $\theta_x$, $\theta_y$ phase space showing the resonances: a. $4Q_y - Q_x = 19$, b. $2Q_y + Q_x = 19$

The evidence of nonlinear coupling resonances can be accentuated
by projecting the three-dimensional plot \((J_x, \theta_x, \theta_y)\) onto the angle variables \((\theta_x, \theta_y)\) plane, as in figure 4. The resonance \(mQ_x + nQ_y = p\) can be easily identified by the number of \(\theta_x\) axis crossings \(m\) and the number of \(\theta_y\) axis crossings \(n\). A positive slope means a difference resonance, a negative slope a sum resonance. By this method, coupling mode "0" or "\(\pi\)" may be recognized in the coupling resonance \(Q_x \cdot Q_y = 0\).

![Figure 4: Trajectories in \(\theta_x, \theta_y\) plane showing some types of resonances](image)

**FIGURE 4** Trajectories in \(\theta_x, \theta_y\) plane showing some types of resonances

a. \(2Q_x - Q_y = 6\) (difference)  
b. \(6Q_x + 7Q_y = 81\) (sum)  
c. \(7Q_y = 44\)  
d. \(6Q_x = 37\)  
e. \(Q_x = Q_y\) (mode 0: Skew Quad On)  
f. \(Q_x = Q_y\) (mode \(\pi\): No Skew Quad)

**EMITTANCE CONSERVATION AT 26 GeV/c**

Disadvantages of the initially used 26 GeV/c ejection optics were the large radial chromaticity \((Q'_x/Q_x = -2.5)\) and the closed orbit distortion (50 mm peak-to-peak). This introduces considerable dispersion variations across the large momentum spread of the short bunches. A recent upgrade of the poleface winding system and the introduction of two dipoles allow the correction of both the closed orbit and the chromaticity in the CPS ring\(^1\). These corrections linearize the orbit dispersion, thus improving the ejection efficiency and reducing the emittance blow-up to less than 0.2 \(\pi\).
CONCLUSIONS

Accurate betatron tune measurements using special FFT techniques allow evaluation of very small tune shifts induced by collective effects. They present good agreement with theory at low energy and lead to estimation of the machine impedances at high energy. Moreover, appropriate signal processing of sampled beam positions allows displays and clear identification of high order resonances. Conservation of beam quality up to the extraction channels is now ensured by adequate adjustments of the high energy machine optics.

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