EXPERIMENTAL RESULTS ON NEUTRINO-ELECTRON SCATTERING

The CHARM Collaboration

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(Submitted to Zeitschrift für Physik C)

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Abstract

A determination of $\sin^2 \theta_W$ based on measurements of elastic scattering of muon-neutrinos and muon-antineutrinos on atomic electrons is described. These purely leptonic processes were studied using the CHARM calorimeter exposed to neutrino and antineutrino wide-band beams at the CERN Super Proton Synchrotron. A total of $83 \pm 16$ neutrino-electron and $112 \pm 21$ antineutrino-electron events have been detected. From the measurement of the ratio of muon-neutrino and muon-antineutrino cross-sections a value of $\sin^2 \theta_W = 0.211 \pm 0.037$ was obtained.
1. Introduction

The mixing angle $\theta_w$ and the masses of the weak bosons $m_W$ and $m_Z$ are essential parameters of the Standard Model [1]. They are related by [2], [3]:

$$\cos \theta_w = m_W / m_Z,$$

(1)

where the minimal Higgs representation was assumed to be valid and, consequently, the ratio of the weak neutral-current (NC) and charged-current (CC) couplings, $\rho$, is equal to one [4].

Today, precise values of $m_Z$ and $m_W$ are available from the data obtained by the experiments UA1 [5] and UA2 [6] at the CERN pp Collider. Combining statistical and systematic errors in quadrature one finds the following mean values

$$m_W = (80.7 \pm 1.4) \text{ GeV}$$

(2)

$$m_Z = (92.0 \pm 1.8) \text{ GeV}.$$

An independent precise determination of $\theta_w$ from other processes, for example neutrino-electron scattering, allows the Standard Model to be tested at the loop-level. The values of $m_W$ and $m_Z$ are related to $\theta_w$, derived from low-energy experiments ($s \ll m_Z^2$, where $s$ is the centre-of-mass energy squared of the reaction) according to

$$m_W^2 = A^2 / [ (1 - \Delta r) \sin^2 \theta_w ]$$

(3)

$$m_Z^2 = A^2 / [ \rho (1 - \Delta r) \sin^2 \theta_w \cos^2 \theta_w ],$$

(4)

where $A^2 = \pi \alpha / \sqrt{2} G_F$. The experiments giving the most precise values of the fine structure constant $\alpha$ and of the Fermi constant $G_F$ are the measurements of the Josephson effect and of muon lifetime [7], respectively, yielding $A = 37.2810 \pm 0.0003 \text{ GeV}$. The quantity $\Delta r = 0.0713 \pm 0.0013$ [2], [8] accounts for the different one-loop radiative corrections entering into relations (3) and (4), compared with those of the low-energy process from which $\theta_w$ is derived.
The value of $\sin^2 \theta_w$ has been most accurately determined in recent experiments at CERN on semileptonic neutrino scattering [9], [10], giving the weighted average (for an assumed mass of the charm quark $m_c = 1.5 \pm 0.3$ GeV)

$$\sin^2 \theta_w = 0.230 \pm 0.004 \text{ (exp.)} \pm 0.005 \text{ (theor.)}.$$  (5)

The precision of $\sin^2 \theta_w$ derived from deep-inelastic semi-leptonic neutrino-nucleon scattering measurements is difficult to improve because the experimental errors have reached the limit of present theoretical uncertainties introduced by the quark model which is needed to derive $\theta_w$ from the semi-leptonic measurements.

The accuracy of $\sin^2 \theta_w$ obtained from the purely leptonic process of neutrino-electron scattering is not limited at this level by theoretical uncertainties. This paper describes the first statistically significant measurement of the cross-sections of neutrino and antineutrino scattering on electrons obtained by exposing the CHARM detector [11] to the CERN wide-band neutrino and antineutrino beams (WBB's).

2. Neutrino-electron scattering

In the Standard Model the cross-section of NC scattering of muon-neutrinos and muon-antineutrinos on electrons

$$\nu_\mu (\bar{\nu}_\mu) e \rightarrow \nu_\mu (\bar{\nu}_\mu) e$$  (6)

can be expressed in terms of three independent quantities, $G_F$, $\rho$, and $\theta_w$, or, alternatively, $\alpha$, $m_Z$, and $\theta_w$, according to the following expressions:

$$\sigma(\nu_\mu e) = (s/4\pi)G_F^2 \rho^2 [(-1 + 2 \sin^2 \theta_w)^2 + (4/3) \sin^4 \theta_w]$$  (7)

$$\sigma(\bar{\nu}_\mu e) = (s/4\pi)G_F^2 \rho^2 [4 \sin^4 \theta_w + (1 + 2 \sin^2 \theta_w)^2/3],$$  (8)

or

$$\sigma(\nu_\mu e) = (2\pi s a^2)/s [\sin^2 2\theta_w m_Z^2]((-1 + 2 \sin^2 \theta_w)^2 + (4/3)\sin^4 \theta_w]$$  (9)
\[ o(\bar{\nu}_e) = \frac{(2\pi s^2)}{(2\sin^2 2\theta_w m_Z^* Z^*)} \left[ 4 \sin^4 \theta_w + \left( -1 + 2 \sin^2 \theta_w \right)^2 / 3 \right] , \]  
\text{(10)}

where \( m_Z^* = m_Z \times \sqrt{(1 - \Delta r)} \) and \( m_Z^* \) is the measured physical mass of the Z boson. Figure 1 shows the dependence of (7)-(10) on \( \sin^2 \theta_w \), assuming in (a) \( \rho = 1 \) and in (b) \( m_Z^* = 93 \text{ GeV} \). As one can see in the figure, the cross-section (9), in particular, is strongly dependent on the value of \( \sin^2 \theta_w \) in the range around \( \sin^2 \theta_w = 1/4 \). The determination of the mixing angle from the absolute cross-section measurements has, however, two essential disadvantages: firstly, depending on the parametrization, the cross-sections are also a function of \( \rho \) or \( m_Z^* \); secondly, relatively important systematic errors can be expected on selection efficiency and neutrino flux determination.

The ratio \( R \) of the cross-sections of muon-neutrino to muon-antineutrino electron scattering \( R = o(\bar{\nu}_e)/o(\nu_e) \) depends on the value of \( \sin^2 \theta_w \) through the relation

\[ R = 3 \times \frac{1 - 4 \sin^2 \theta_w + (16/3) \sin^4 \theta_w}{1 - 4 \sin^2 \theta_w + 16 \sin^4 \theta_w} . \]  
\text{(11)}

The value of \( R \) is essentially free of theoretical uncertainties [3] and is almost independent of radiative corrections [12]. In the experimental determination of \( R \) the systematic uncertainties on the efficiency of the selection criteria cancel to a large extent and only a relative \( \nu/\bar{\nu} \) flux evaluation is necessary. Figure 2 shows \( R \) as a function of \( \sin^2 \theta_w \). For \( \sin^2 \theta_w \approx 1/4 \) one has \( \Delta \sin^2 \theta_w \approx 1/8 \Delta R/R \). When the mixing angle is known from the measurement of \( R \), the measured absolute cross-sections can be used to determine \( \rho \) following (7) and (8).

The main difficulty in determining the cross-sections of the processes (6) is due to the fact that they are about ten thousand times smaller than those of inclusive \( \nu N \) interactions. The CHARM detector consisted of a calorimeter of low-Z material, marble (CaCO₃), with a total mass of 156 tonnes. The fine granularity of the detector allowed very good identification of muons, and of electromagnetic and hadronic showers and a good determination of the electromagnetic shower direction. The electromagnetic shower events induced by reactions (6) could be identified with respect to the larger fraction of \( \nu N \) events having a hadronic shower or a hadronic shower and a muon in the final state on the basis of the different shower patterns.

The remaining background is due to semileptonic interactions with an electromagnetic shower in the final state, mainly coherent \( \nu^0 \) production and quasi-elastic scattering induced by the small admix-
ture of electron-neutrinos from $K_{e3}^-$ decays in the beam. The angle, with respect to the neutrino beam, of the shower induced by these reactions is large compared with that of the showers induced by reactions (6). In fact, the kinematics of the two-body elastic reactions (6) implies

$$2E_\nu E_\nu (1 - \cos \theta) = 2(E_\nu - E)m_e,$$

where $E_\nu$ is the incoming $\nu_\mu$ energy, and $m_e$ and $E$ are the mass and the recoil energy in the laboratory system of the electron. Introducing the variable $y = E/E_\nu$ one obtains for $E_\nu >> m_e$

$$E\theta^2 = 2(1 - y) m_e < 2m_e.$$

In this experiment $<E> \approx 12$ GeV and thus $<\theta> \approx 10$ mrad. The background reactions, on the other hand, are characterized by a relatively wider angular distribution owing to the large value of the nucleon mass as compared to the electron mass. An accurate measurement of the direction of the electromagnetic shower therefore allowed a statistical separation of the signal from the background.

The measurement of the elastic scattering of muon-neutrinos (muon-antineutrinos) on electrons can give information on phenomena not described in the context of the Standard Model. Deviations of the cross-sections of reactions (6) from the prediction of the Standard Model for a fixed value of $\sin^2 \theta_{w'}$ derived from the experimentally known values of the masses of the gauge bosons can be interpreted [13] as due to i) a modification of the electron propagator owing to the existence of an excited electron $e^*$ and/or to ii) the exchange of an additional heavier neutral gauge boson $Z'$ between the muon-neutrino and the electron. If the deviations of $\sigma(\nu_\mu, e)$ and $\sigma(\bar{\nu}_\mu, e)$ are parametrized in terms of the departure of the axial-vector $g_A$ ($\Delta g_A$) and vector coupling constants $g_V$ ($\Delta g_V$) from the Standard Model predictions, upper limits on the mass of an excited electron $m_{e^*}$ and on the mass of an additional $Z$ boson, $Z'$, of mass $m_{Z'}$, can be obtained using the following relations [14, 15]:

$$m_{e^*}^2 = (1.8 \times 10^{-2} \lambda^2 \Delta A)/\Delta g_V = (1.8 \times 10^{-2} \lambda^2 \Delta A)/\Delta g_A$$

and

$$m_{Z'}^2 = (g'/g)^2 m_{Z'}^2 5/3 \Delta g_V = (g'/g)^2 m_{Z'}^2 10/3 \Delta g_A.$$
In (14) \( \lambda \) and \( \Lambda \) are respectively the coupling and the scale of the \( Z^0 \)-\( e^\ast \) interaction. It was assumed that \( m_{e^\ast} = m_{\mu^\ast} \), that there are no other excited leptons, and that \( \sin^2 \theta_w = 0.215 \) consistent with the result of this experiment. In (15) \( g' \) and \( g \) are the \( Z' \) and \( Z \) coupling strengths in a model that is based on the extended gauge group \( SU(2) \times U(1) \times U(1)' \) and in which \( Z \) and \( Z' \) do not mix. The values of \( g_V \) and \( g_A \) are related to \( \sin^2 \theta_w \) according to \( g_V = (-1/2 + 2 \sin^2 \theta_w) \) and \( g_A = -1/2 \).

Deviations of \( \sigma(\nu_\mu e) \) and \( \sigma(\bar{\nu}_\mu e) \) from the Standard Model predictions can also indicate a non-vanishing magnetic moment of neutrinos (\( \mu_\nu \)) or an anomalous electric charge radius of neutrinos (\( r_\nu \)). If \( E_t \) is the lower electron energy threshold the total cross-section is modified according to the following expression [16]:

\[
\sigma(\nu_\mu e) = \sigma_{SM}(\nu_\mu e) + \mu_\nu^2 \alpha \left[ \frac{E_t}{E_\nu} - 1 - \ln \left( \frac{E_t}{E_\nu} \right) \right],
\]

where \( \sigma_{SM} \) is the NC scattering cross-section mediated by the \( Z^0 \) exchange in the frame work of the Standard Model and \( \alpha \) the fine structure constant.

Charged-current loops give rise to an electric charge radius of neutrinos [17]. In the frame work of the Standard Model it can be estimated to \( <r^2_\nu>_{SM} \approx 2 \times 10^{-33} \) cm\(^2\) [18]. In \( \nu_\mu e \) scattering its contribution is almost completely cancelled by other loop terms [19]. The value of \( \sin^2 \theta_w \) derived from measurements of the ratio \( R \) of \( \sigma(\nu_\mu e) \) and \( \sigma(\bar{\nu}_\mu e) \) (11) is affected by \( <r^2_\nu>_{SM} \) at the level of 1% only. This cancellation allows to search directly for any anomalous contribution to the electric neutrino charge radius, \( <r^2_\nu>_{\text{anom}} \) owing e.g. to anomalous properties of the mediating W boson or to compositeness of neutrinos [20]. It will modify the cross-section according to the following expression [21]:

\[
\sigma(\nu_\mu e) = \sigma_{SM}(\nu_\mu e) + \frac{(4/3\pi)}{G_\mu^2 E_\nu m_{\nu} \sin^2 \theta_w m_w^2} \times <r^2_\nu>_{\text{anom}} \times \\
\left[ \frac{(8/3)\sin^2 \theta_w - 1 + (8/9)\sin^2 \theta_w m_w^2}{m_{\nu}^2} <r^2_\nu>_{\text{anom}} \right],
\]

for the \( \nu_\mu e \) scattering the expression in the bracket is replaced by

\[
\left[ \frac{(8\sin^2 \theta_w - 1)/3 + (8/9)\sin^2 \theta_w m_w^2}{m_{\nu}^2} <r^2_\nu>_{\text{anom}} \right].
\]

The ratio \( R \) will be modified by a factor \( (1 - 2\Delta \sigma/\sigma_{SM}) \). Comparing the experimental value of \( R \) with the value expected for \( \sin^2 \theta_w \) as determined e.g. from the boson masses, a limit on anomalous contributions to the neutrino electric charge radius can be determined [22].
In this paper we describe the data-taking procedures and the analysis of the $\nu_\mu(\bar{\nu}_\mu)e$ events obtained in two different exposures of the CHARM detector in 1979-1981 [23] and 1983 [24]. In the following we will refer to them as the first and the second exposure. More emphasis will be placed on the description of the analysis of the data collected in the second exposure. In Section 3 of the paper the neutrino beam is described. The main features of the CHARM detector related to the measurement of the neutrino-electron processes are recalled in Section 4. The determination of the neutrino fluxes is described in Section 5. Section 6 is devoted to a description of the data analysis. The results obtained in the analysis of the data collected in the two periods are combined in Section 7. In this section a comparison with other experimental results is also reported. Section 8 is dedicated to determining limits on the possible existence of excited electrons $e^\ast$ or of an additional neutral boson $Z'$ and on the electromagnetic properties of muon-neutrinos.

Muon-neutrinos can interact with atomic electrons also through the CC process

$$\nu_\mu e^- \rightarrow \mu^- e^-, \quad (18)$$

the so-called inverse muon decay. Events induced by this reaction appear in the detector as a single muon track at a small angle ($\leq 5$ mrad) with respect to the neutrino direction. The main background is due to quasi-elastic scattering of $\nu_\mu(\bar{\nu}_\mu)$ on nucleons. From the ratio of the cross-sections of $\nu_\mu e^-$ elastic scattering and of inverse muon decay a value of $\rho$ can be determined using leptonic reactions only (Section 9).

3. The wide-band neutrino beam

Because of the smallness of the cross-section of neutrino-electron scattering the use of intense neutrino beams is required. In this experiment the WBB facility of the Super Proton Synchrotron (SPS) at CERN has been used to obtain the highest possible flux of neutrinos and antineutrinos.

A schematic view of the target and focusing system of the WBB is shown in Fig. 3. The target for the neutrino (antineutrino) beam consisted of 5 (11) beryllium rods of 10 cm length and 3 mm diameter, equally spaced over a distance of 200 cm. The total target length corresponded to 1.6 (3.5) interac-
tion lengths. This configuration optimized the flux of positive (negative) pions and kaons, taking into account the reabsorption of the secondary particles in the target.

The positive (negative) particles that decayed to $\nu_\mu$ ($\bar{\nu}_\mu$) were achromatically focused by a magnetic horn [25] and two reflectors. The horn consisted of two conductors that were arranged symmetrically around the beam. A sinusoidal pulse of 6 ms half period and a current of 100 kA at the maximum was applied along the inner conductor and returned along the outer conductor. The shape of the conductors was designed to focus secondary particles of the desired momentum and angle range into a nearly parallel beam. The reflectors operated on the same principle; positioned 85 m downstream from the target and 40 m in front of the decay tunnel, they reflected the particles which were too far off-axis back towards the decay tunnel.

Particles which are produced in the very forward direction pass through the centre of the inner conductor without being affected by the magnetic field of the horn. Since in the antineutrino beam configuration the majority of the forward-going particles was positive, a substantial fraction of the wrong-sign particles was not defocused. To reduce this source of background, the hole in the horn was shielded by a 118 cm long beryllium stopper ($\approx 3.6$ interaction lengths). This 8 mm thick rod absorbed the hadrons leaving the target at angles $\leq 2$ mrad. The stopper also absorbed the remaining protons which, by interaction in the air or in the entrance window of the decay tunnel, would also contribute to the creation of wrong-helicity neutrinos.

The secondary particles were allowed to decay in a region of 410 m length. In the first 120 m the target, horn, and reflectors were installed. It was followed by a 290 m long decay tunnel which was evacuated to a residual pressure of 0.15 Torr. The entrance window to this region consisted of a 2 mm thick titanium foil. The decay tunnel was followed by a shield consisting of 180 m of iron and 230 m of earth to absorb hadrons and to range out muons by energy loss.

Silicon detectors were installed at various depths in the iron shield to measure the muon flux and thereby monitor the neutrino flux [26]. For the analysis presented in this paper these measurements were used only as a consistency check. The neutrino energy spectra and the absolute fluxes were determined experimentally, as described in Section 5, from quasi-elastic and deep-inelastic neutrino interactions recorded in the CHARM neutrino detector.
4. Description of the CHARM experiment

4.1 The detector

Because of the smallness of the cross-sections of reactions (6) a detector of large target mass is required for the study of neutrino interactions on electrons. In order to distinguish between different processes by accurate measurements of the kinematic variables, the detector was required to provide good spatial and energy resolutions.

The CHARM detector [11] was designed to combine the advantages of a calorimetric detector with those of a fine-grain tracking device. It consisted of a target calorimeter of high mass and low density, in which the neutrino interactions took place, and of a muon spectrometer (see Fig. 4). In material of low atomic number \( Z \) the ratio of the width over the length of electromagnetic showers is small and a good determination of the shower direction can therefore be made.

The fine transverse and longitudinal subdivision of the target calorimeter made it possible to determine accurately the energy and the direction of the particle showers produced in neutrino interactions, and to distinguish between hadronic and electromagnetic showers and muons. The muon spectrometer, consisting of magnetized iron disks, proportional drift tubes, and scintillators, determined the sign of the charge of muons and their momentum.

The target calorimeter consisted of 78 equal units, each 20 cm thick and with a cross-section of 4 \( \times \) 4 m\(^2\). Each unit comprised a marble plate followed by layers of proportional drift tubes, scintillation counters and streamer tubes. Each marble plate, having a surface area of 300 \( \times \) 300 cm\(^2\) and a thickness of 8 cm, was surrounded by a 45 cm wide and 8 cm thick magnetized iron frame at a distance of 5 cm. Two coils produced a toroidal magnetic field of 1.5 T on average in the iron. Muons leaving the detector sideways could thus be identified and their charge and momentum measured. The scintillators served to measure the energy loss of single-particle tracks and of particle showers. This information was also used to reconstruct the shower profile. Each scintillator plane consisted of 20 scintillation counters 15 cm wide, 300 cm long, and 3 cm thick. The proportional drift tubes [27] in the calorimeter were used to determine the shower profile and the shower direction. They were also used to measure muon tracks in order to determine their direction and deflection in the magnetized iron. Each plane of proportional drift tubes consisted of 128 aluminium tubes, 3 cm wide, 400 cm long, and 3 cm thick. The drift-time and the charge collected on the wire were measured. From the drift-time measurement
the track position was determined with an error of 1 mm. The streamer tubes [28] were operated in the limited streamer mode. Their main function was to measure the vertex and the angle of electromagnetic showers. They were also used to measure the spatial density of the energy flow of the showers. Every plane was equipped with 256 aluminium tubes, each having a cross-section of 1 cm² and a length of 265 cm. They were added for the second exposure. The active detector elements were mounted alternately in horizontal and vertical direction. The orientation of streamer tubes and proportional drift tubes between two absorber plates was orthogonal thus providing a space point information. The scintillators had the same orientation as the streamer tubes.

A 'veto' plane detected charged particles entering the detector from the front. It was installed in front of the first marble plate of the calorimeter and consisted of a hodoscope of 29 scintillators, 3 cm thick, 15 cm wide and 4 m long, arranged in such a way that they mutually overlapped each other, covering a total surface area of $4 \times 4 \text{ m}^2$.

The end system served both as a muon spectrometer and as a coarse calorimeter for measuring the energy of showers which were not fully contained in the target calorimeter. It consisted of 30 magnetized iron disks, interleaved with 18 planes of 128 proportional drift tubes each and 6 planes of 18 scintillators each. The first 15 iron disks had a thickness of 5 cm; the remaining disks were 15 cm thick. They were magnetized toroidally. The scintillation counters were 44 cm wide and covered the magnetized region. They measured the energy of photons radiated by high-energy muons. The reconstruction of multiple muon tracks was made possible by three additional proportional drift-tube planes with wire the direction rotated by $\pm 12.5^\circ$. The left-right drift-path ambiguity could be solved because each second plane with the same orientation was staggered by half a tube width.

4.2 The selection of electromagnetic showers

In this subsection we discuss the methods used to distinguish between showers induced by electrons and hadrons in the calorimeter. To study the features of these showers a part of the detector was exposed to a test beam [29], [30] of electrons and pions with momenta in the range 5-30 GeV. Figure 5 shows typical electron- and pion-induced showers of 15 GeV energy in the detector. The electromagnetic showers have a more regular development and a narrower transverse profile while the average length of the two kinds of showers is similar. According to the theory of shower development in the
low-Z calorimeter material one expects that 90% of the total energy of electromagnetic showers is contained in a cylinder with radius \( r = 9 \) cm. The corresponding radius for hadronic showers is \( r = 35 \) cm. The length of electromagnetic (hadronic) showers containing 90% of the deposited energy turns out to be similar, \( L = 210 \) cm (220 cm).

The start of the shower was defined by hits in two consecutive scintillator planes with a pulse height corresponding to at least 2 MeV each. The properties of the shower were studied in the following 20 planes. The shower axis was determined first and different estimators of the shower pattern were constructed in such a way as to be independent of the shower angle \( \theta \). The lateral profile of a shower corresponds approximately to a Cauchy distribution:

\[
f(L) = \frac{1}{\pi \Gamma} \left( \frac{1}{1 + \left( \frac{L}{\Gamma} \right)^2} \right),
\]

(19)

where \( L \) is the distance from the shower axis and \( \Gamma \) is the half-width. For electromagnetic showers \( \Gamma \approx 1 \) cm. A Cauchy distribution was fitted to the profile determined from each scintillator plane, using 7 or 8 scintillators. A mean value of the half-width in each projection was determined and weighted with the energy depositon in each plane. The quadratic average of the values measured in the two projections (\( \Gamma^2 \)) was determined and multiplied by a correction factor which ensured that the efficiency of cuts on \( \Gamma \) was independent of the shower energy

\[
\Gamma = \Gamma^2 \times \left[ 1 + 0.3 \times (\sqrt{\frac{E}{15 \text{ GeV}}} - 1) \right].
\]

(20)

The \( \Gamma \) distribution measured for 10 GeV electron and pion events in the test beam is shown in Fig. 6a. As can be seen, the mean values for pions and electrons, \( <\Gamma_\pi> \) and \( <\Gamma_e> \), differ by a factor 10.

The distributions of the streamer tube hits and of the hits of proportional drift tubes relative to the shower axis were also determined. For the streamer tubes the hits were weighted using the pulse height of the nearest scintillator in the same unit. In the case of the proportional drift tubes the pulse height of the tubes was used as a weight. The standard deviation \( \sigma \) of these distributions is a measure of the shower width. Figure 6b shows the distribution of the standard deviation measured with the streamer tubes \( \sigma_{\Gamma} \) for 10 GeV electron and pion showers.

Although the average longitudinal behaviour of electromagnetic and hadronic showers in the target material with \( <Z> \approx 12 \) is rather similar, the fluctuations of the individual shower lengths are
much larger for hadronic showers than for electromagnetic ones. To exploit this fact the following quantity was calculated for each shower using the scintillator information:

\[ L_{\text{shower}} = \sum E_i \chi_i/E - X_0 \ln (E/20 \text{ GeV}), \]  

(21)

where \( E_i \) is the energy deposited in the scintillator plane \( i \), \( E = \sum E_i \), \( \chi_i \) is the plane position relative to the vertex of the shower, \( X_0 \) is the average radiation length of the calorimeter, and \( E \) the shower energy. The first term in (21) is the distance of the barycentre of the shower from the vertex. The energy dependence was parametrized normalizing to the length of 20 GeV showers. The distribution of \( L_{\text{shower}} \) for electron- and pion-induced showers of 20 GeV is shown in Fig. 7.

The regularity of the electromagnetic shower development as measured by the streamer tubes was also used to separate electromagnetic and hadronic showers. For each streamer tube plane \( i \), the two outer tubes with a hit were determined. The number of tubes without a hit between the two outer hits, i.e. the number of gaps \( G_i \), was then determined. Only those hits for which the preceding scintillator shows a pulse height of at least one minimum ionizing particle were taken into account. In order to reject hadronic showers, an upper limit on the number of gaps in the planes 2 to 8 of the shower and on their sum was fixed as a function of the shower energy using the test beam data. This separator, called 'helicity', did not need any calibration by cosmic-ray events and was used to filter the events.

The shower pattern near the vertex also displays characteristic differences for electromagnetic and hadronic showers. In the neutrino-electron scattering process the shower is induced by one single electron and there will be an odd number of minimum ionizing particles in the vertex region, the original electron and, if any, an electron-positron pair from a converted bremsstrahlung photon. In processes where the electromagnetic cascade is induced by single \( \gamma \)'s or \( \pi^0 \)'s an even number of particles will be observed at the shower origin. In the case of \( vN \) interactions pions are produced and the nucleus can break up or evaporate nucleons and the fragments scatter around the interaction point.

The variables used to quantify these differences were: \( M_i^d \) (the hit multiplicity in the first proportional drift-tube plane), \( M_i^s, G_i^s \) (the hit multiplicity and the gap multiplicity in the first streamer tube plane), and \( E_i \) (the energy deposited in the first scintillator plane of the shower). The distributions of these quantities were measured for electron and pions in the test beam. To simulate the uniform vertex distribution of neutrino-electron interactions in the detector plates using incident electrons, an aluminium wedge was used in front of the first scintillator plane. Figure 8 shows the distribution of \( M_i^d \)
measured in the test beam for 20 GeV incident electrons. In about 75% of the cases, the electromagnetic cascade in the first tube plane is limited to the width of one tube.

The pulse height resolution of scintillators allowed to distinguish between one, two, or more than two particles crossing the counter. Figure 9a shows the distribution of the energy deposited in the first scintillator plane of the shower for electrons as measured in the test beam. In neutrino-electron scattering the recoiling electron is accompanied by radiated photons. A simplified treatment of radiative corrections of the process (6) has been used to compute the probability of photon emission from the emerging electrons in the scattering process [31]. The radiated photon energy corresponds to the bremsstrahlung energy lost by an electron crossing a material of 3% of a radiation length. The $E_T$ distribution of the events induced by the scattering of neutrinos on electrons is expected to be very similar to that shown in Fig. 9a. The figure also shows the $E_T$ distribution (b) of neutrino-induced events belonging to a kinematic region where the production of $\pi^0$ dominates. The sample shown has been obtained by applying the cuts $E^2\theta^2 > 0.54 \text{ GeV}^2$ and $7.5 < E < 17.5 \text{ GeV}$, where $E$ is the energy and $\theta$ the angle of the shower. A cut of $E_T < 8 \text{ MeV}$, allows to select a class of events that started with a single particle and rejects electromagnetic showers induced by $\gamma$'s and $\pi^0$'s.

4.3 The measurement of the angle of electromagnetic showers

The shower direction in each projection was determined as the line that starts from the shower vertex and devides the shower energy into equal halves. The longitudinal vertex coordinate $x$ was defined by the central point of the marble plate in which the shower started. The transverse vertex coordinates $y$ and $z$ were defined by the proportional drift tubes and by the streamer tubes. For the latter the mean value of the wire coordinates of the hit streamer tubes was computed. Only tubes with a corresponding scintillator hit were considered. For the proportional drift tubes the vertex was determined from the centroid of the wire coordinates weighted with the pulse height. In the detector configuration without streamer tubes one vertex projection was determined in the first sensitive layer of the shower and the other in the following one, 1.5 radiation lengths away from the shower vertex. After the installation of the streamer tubes the vertex could be determined in the first unit for both projections, thereby improving the precision of the vertex reconstruction. If only one proportional drift tube was hit in the first plane the drift-time information was used to improve the vertex determination.
The left-right ambiguity could be solved in the case of drift distances larger than 4 mm. To do that the data collected without streamer tubes were subjected to a visual scan and the ambiguity was solved by extrapolating the shower direction obtained from the proportional drift tubes in the shower core region backwards to the vertex. With the installation of the streamer tubes the ambiguity was solved by using the position of the streamer tube hit in the subsequent unit.

The shower direction $\theta$, determined from the proportional drift-tube information, was defined by the solution of the equation

$$\Sigma P_i \times \arctan\left(\frac{L_i - \tan(\theta) \times x_i}{\sigma_{PT}}\right) = 0 ,$$  \hspace{1cm} (22)

where $P_i$ is the pulse height in the proportional drift tube $i$, $L_i$ the lateral position of the tube $i$, and $x_i$ the longitudinal position relative to the shower vertex. The arctangent function was chosen to give the best angular resolution, $\sigma_{PT}$ is the r.m.s. width of the lateral shower profile measured by the proportional drift tubes. Equation (22) has also been used to determine the shower angle, making use of the streamer tube information. In this case $L_i$ and $x_i$ were the transverse and the longitudinal wire coordinates, and the weights $P_i$ were obtained from the energy measured in the scintillator nearest to the streamer tube. In the weight function $\sigma_{PT}$ was replaced by $\sigma_{ST}$. In the second exposure the average of the two angle determinations obtained from the streamer tubes and from the proportional tubes was used. In Fig. 10 the average resolution of the projected angles is shown for the two configurations of the detector as measured with the events taken in the test beam exposure.

4.4 Measurement of the energy of electromagnetic showers

The energy response of the calorimeter was measured in the test beam. The total visible energy $E_{\text{vis}}$ was obtained by summing the deposited energy in the scintillation counters. All pulse heights were calibrated by recording the known energy loss of cosmic-rays in the scintillators. Corrections were applied for light attenuation in the counters. The energy calibration factor, for electrons with energy $E$ in the range 5-30 GeV, was found to be energy independent with a value [30]

$$\frac{<E_{\text{vis}}>}{E} = (11.3 \pm 0.3)\% ,$$  \hspace{1cm} (23)

where $<E_{\text{vis}}>$ is the mean visible energy. This value was slightly lower compared with that obtained for the detector configuration without streamer tubes [29] because of the additional material of the
streamer tubes. In the data analysis the appropriate factor was used for the two configurations of the detector in the first and in the second exposure.

For a given electron energy $E_{\text{vis}}$ had a Gaussian distribution. The fractional energy resolution, defined as the ratio of the standard deviation and the mean value of the distribution, was found to be

$$\Delta E/E = (18 \pm 1)\% / \sqrt{(E/\text{GeV})}. \quad (24)$$

5. Determination of neutrino fluxes

The value of the cross-section ratio $R$ as defined in (11) was obtained from the ratio of $\nu_{\mu}^e$ and $\bar{\nu}_{\mu}^e$ events (see Section 6) and the energy-weighted antineutrino and neutrino flux ratio $F$,

$$R = (N_{\nu_{\mu}^e}/N_{\bar{\nu}_{\mu}^e}) \times F.$$ The flux ratio was determined from the number of events recorded in the neutrino and antineutrino beams induced by inclusive CC and NC processes on nucleons

$$\nu_{\mu}(\bar{\nu}_{\mu})N \rightarrow \bar{\nu}_{\mu}(\nu_{\mu})X, \quad \nu_{\mu}(\bar{\nu}_{\mu})N \rightarrow \mu^-(\mu^+)X \quad (25)$$

and the known cross-section ratios, and, independently, from the recorded number of quasi-elastic CC processes on nucleons

$$\nu_{\mu}(\bar{\nu}_{\mu})N \rightarrow \mu^-(\mu^+)N^\prime. \quad (26)$$

The inclusive processes (25) have a cross-section rising linearly with energy, and the ratio of the energy-weighted fluxes is given by

$$F_1 = [N_1(\nu)/\sigma_0(\nu)]/[N_1(\bar{\nu})/\sigma_0(\bar{\nu})], \quad (27)$$

where $N_1(\nu)$ [$N_1(\bar{\nu})$] is the observed number of events and $\sigma_0(\nu)$ [$\sigma_0(\bar{\nu})$] the cross-section slope for deep inelastic scattering [32] of neutrinos and antineutrinos.

Since the cross-sections of neutrino and antineutrino quasi-elastic scattering for low $Q^2$ and high neutrino energy are nearly equal and independent of the neutrino energy (a correction for the small difference was applied), the ratio of the energy-weighted fluxes determined from quasi-elastic events is given by

14
\[ F_Q = [N_Q(\nu) <E_\nu>] [N_Q(\bar{\nu}) <E_{\bar{\nu}}>] , \]  
(28)

where \( N_Q(\nu) \) [\( N_Q(\bar{\nu}) \)] is the observed number of events and \( <E_\nu> \) \( [<E_{\bar{\nu}}>] \) the mean value of the energy of the muon-neutrino [muon-antineutrino] beam obtained by correcting the measured energy distributions for the experimental resolutions using an unfolding procedure.

To derive neutrino-electron and antineutrino-electron cross-sections, absolute measurements of the neutrino fluxes are necessary. The normalization factor \( G \) was obtained using the recorded number of inclusive events:

\[ G(\nu[\nu]) = \sigma_0(\nu[\nu])/N_Q(\nu[\nu]) . \]  
(29)

The inclusive and quasi-elastic reactions were also used to study the relative fractions and the spectra of the different components, \( \nu_\mu \), \( \bar{\nu}_\mu \), \( \nu_e \), and \( \bar{\nu}_e \) of the beams.

The WBB parameters for the first and second data periods were nearly equal. In the following subsections we describe in detail how the flux information was extracted for the second data period. A similar method was used for the first data period [23]. All neutrino flux quantities were determined as mean values averaged over the fiducial volume of the detector.

5.1 Measurement of inclusive semileptonic neutrino interactions

The analysis is based on events recorded with a 'minimum-bias' trigger which were classified as NC or CC interactions by the CHARM reconstruction program. The minimum-bias trigger required a hit in at least 4 scintillators. Events were accepted if the total energy deposited in the calorimeter was larger than 1.5 GeV. To reduce the trigger rate only one event out of fifty was recorded. The total numbers of the selected events were:

\[ \nu \text{ beam}: 20179 \pm 142 \quad \text{and} \quad \bar{\nu} \text{ beam}: 11903 \pm 109 . \]  
(30)

The fractions of \( \nu_e \)- and \( \bar{\nu}_e \)-induced events were determined from a comparison of the shower-energy distributions of NC and CC events, assuming equal y distributions for CC and NC \( \nu_\mu \) \( (\bar{\nu}_\mu) \) events. Normalizing the NC events to the CC shower energy distribution in the region between 10 and 30 GeV an excess of 282 \( \pm \) 121 NC events in the neutrino beam and 190 \( \pm \) 95 NC events in the antineutrino beam were observed at shower energies above 30 GeV. The large systematic error is due to un-
certainties in the classification of events. The excess is due to CC events induced by electron-(anti)neutrinos. The NC events induced by electron-neutrinos contribute to both the low-energy and the high-energy parts of the shower-energy spectrum and therefore cannot be discriminated from \( \nu_\mu \) NC interactions in the approximation of equal energy distributions for \( \nu_\mu \) and \( \nu_e \). Taking into account the contamination of wrong-sign muon-neutrinos (see subsection 5.2), the following energy-weighted flux ratios were found in the neutrino and antineutrino beams respectively:

\[

\begin{align*}
\nu \text{ beam: } & \quad \Phi(\nu_e)/\Phi(\nu_\mu) = 0.017 \pm 0.009, \quad \Phi(\bar{\nu}_e)/\Phi(\nu_\mu) = 0.004 \pm 0.002 \\
\bar{\nu} \text{ beam: } & \quad \Phi(\bar{\nu}_e)/\Phi(\bar{\nu}_\mu) = 0.012 \pm 0.006, \quad \Phi(\nu_e)/\Phi(\bar{\nu}_\mu) = 0.006 \pm 0.003.
\end{align*}
\]

(31)

The relative contributions of electron-neutrinos and electron-antineutrinos were calculated by Monte Carlo methods assuming that the cross-sections of \( \nu_e(\bar{\nu}_e) \) are equal to the \( \nu_\mu(\bar{\nu}_\mu) \) ones [34]. The simulation was constrained by the experimental data on \( \nu_\mu \) and \( \bar{\nu}_\mu \) interactions described below.

The normalization factor \( F_1 \) was finally obtained by applying corrections to the raw numbers of events (30) measured in the two beams. The most important corrections had to be applied for i) the trigger efficiency, \(( +6 \pm 1)\% \) in the neutrino and \(( +22 \pm 4)\% \) in the antineutrino beam respectively, ii) the \( \nu_e(\bar{\nu}_e) \) contamination \(( -2 \pm 1)\% \) in both beams, and iii) the contamination of wrong-sign muon-neutrinos, \(( -3 \pm 0.5)\% \) in the neutrino and \(( -19 \pm 2)\% \) in the antineutrino beam, respectively. After these corrections we found 19885 \( \pm 329 \) events in the neutrino beam and 11619 \( \pm 495 \) events in the antineutrino beam. Using for the \( \nu \) and \( \bar{\nu} \) deep-inelastic cross-section ratio the value measured with the same detector [32], the following value of \( F_1 \) was derived

\[
F_1 = 1.12 \pm 0.01 \text{ (stat.)} \pm 0.08 \text{ (syst.)}.
\]

(32)

To obtain the absolute energy-weighted neutrino fluxes the data were further corrected taking into account that the longitudinal fiducial volume of the \( \nu_\mu e \) analysis contained 57 absorber plates and that of the normalization analysis only 49 and the down-scaling factor of the minimum-bias trigger (a factor 50). These corrections cancel in the determination of \( F_1 \). Using the cross-sections of Ref. [32] we found\(^1\) for the normalization factors (29)

\(^1\) The value of the total neutrino and antineutrino cross-sections have been corrected by a factor 1.13 as compared to earlier measurements due to a better estimation of the neutrino and antineutrino fluxes in the narrow-band beam used to measure the total cross-sections [33].
\[ G(\nu) = (1.56 \pm 0.06) \times 10^{-44} \text{ cm}^2/\text{GeV}, \]
\[ G(\bar{\nu}) = (1.39 \pm 0.05) \times 10^{-44} \text{ cm}^2/\text{GeV}. \]

(33)

Here the ratio of two between the numbers of nucleons and electrons in marble was taken into account.

5.2 Measurements of quasi-elastic neutrino interactions

Characteristic features of quasi-elastic muon-neutrino nucleus interactions (25) are the small visible hadronic energy and the small four-momentum transfer \( Q^2 \). Candidates were therefore selected by applying the following criteria: i) presence of a muon having generated hits in at least two scintillator planes of the end magnet, ii) longitudinal vertex position in units 9-63 and lateral vertex coordinates \( y \) and \( z \) in the region \( |y|, |z| \leq 1.15 \text{ m} \), iii) the distance \( r \) between the detector axis and the muon position at the entrance to the end system is \( 0.20 \text{ m} \leq r \leq 1.80 \text{ m} \), iv) only one reconstructed muon in the detector with the angle of the track fitted in both projections, and a momentum between 1 and 200 GeV/c with an estimated relative measurement error of \( \Delta p/p \leq 25\% \), v) \( p_T^2 = p_\mu^2 - 1 < Q^2 \leq 1 \text{ GeV}^2 \), and vi) the scintillator energy in the first six planes starting from the vertex \( E_6^{\text{scint}} \leq 92 \text{ MeV} \).

The third condition eliminates those muons which would pass through the hole in the centre of the end system or escape laterally from the muon spectrometer. The fourth condition makes the probability of a wrong identification of the sign of the muon charge negligible. The limit on the sum of the scintillator energies, \( E_6^{\text{scint}} \), corresponds to the energy deposited by \( \approx 2.5 \) minimum ionizing particles. About 90\% of the quasi-elastic events fulfil condition (vi). Applying the described criteria to the \( 2.2 \times 10^6 \) neutrino and \( 1.6 \times 10^6 \) antineutrino interactions recorded, the following event numbers were obtained:

\[ \nu \text{ beam: } 26293 \ (\mu^-) \quad 1976 \ (\mu^+) \]
\[ \bar{\nu} \text{ beam: } 39849 \ (\mu^+) \quad 3184 \ (\mu^-). \]

(34)

The sign of the muon charge was determined from the curvature of the track in the magnetic spectrometer. A very small background of cosmic muons which entered backwards through the end system.
and stopped in the target calorimeter was determined in a separate run without beam, and subtracted [35].

The samples (34) contain events induced by the quasi-elastic reactions (26) and by neutrino-nucleon interactions producing nucleon resonances. Since neutrino- and antineutrino-induced reactions have different hadronic final states a Monte Carlo simulation was used to determine the efficiency of the selection criteria for the two sets of data.

The processes simulated were: i) quasi-elastic scattering (26), ii) production of the $\Delta (1232)$ resonance with $\Gamma = 120$ MeV, and iii) production of the $N^\ast (1500)$ resonance with $\Gamma = 150$ MeV. The excited state $N^\ast$ at 1500 MeV simulates the three isospin 1/2 resonances that are close together at 1440, 1520, and 1535 MeV. The fractions of the events for each reaction in the simulation sample were 40%, 45%, and 15% respectively. The Monte Carlo simulation was divided into three stages: i) neutrino-nucleon scattering, ii) intranuclear interactions, and iii) detector simulation. The basic assumption was that neutrinos interact with 'quasi-free' nucleons inside the nucleus. The neutrino (antineutrino) energy spectra were calculated using a Monte Carlo simulation [36] of the neutrino beam. For $I = 1/2$ resonances the recoiling nucleon is a neutron; in the case of the resonance $\Delta$ the relative neutron and proton production probabilities are given by isospin symmetry. The resonance mass was defined on the basis of a relativistic Breit-Wigner distribution. Fermi motion of the nucleons and Pauli suppression of quasi-elastic events with low $Q^2$ was taken into account and final-state radiative corrections were applied. Interactions inside the nucleus and decays of hadronic particles generated in the neutrino-nucleon interaction were simulated using the program ISOBAR [37]. In addition to two-body decays of $\Delta$ resonances the processes of nucleon-nucleon scattering, pion-nucleon interactions, and $\Delta$-nucleon reactions were simulated. For the $I = 1/2$ resonances ($N^\ast$) the same cross-sections as for the $\Delta$ resonance were used. A simple thermodynamical model was used to simulate the transformation of the excitation energy of the nucleus into evaporation nucleons. Finally, all particles were tracked through the CHARM target calorimeter. Decays, electromagnetic processes, and hadronic interactions were simulated in detail [38] and inefficiencies and random hits in the proportional and streamer tubes, as well as the fluctuations of the number of photons reaching the photomultipliers from the scintillators, were taken into account. Figures 11 and 12 compare the measured and calculated distributions of the important variables $p_{T^2}$ and $E_{\text{scint}}$. The Monte Carlo simulation reproduces rather well the experimental results; the small differences between the simulated and the experimental data are about the
same for \( \nu \) and \( \bar{\nu} \) events and therefore cancel in the ratio. The neutrino data contain 352 \( \pm 61 \) events with \( p_{T}^{2} < 0.02 \text{ GeV}^{2} \) due to the reaction \( \nu_{\mu} e \rightarrow \mu^{-}\nu_{e} \) (see Section 9) for which a correction was applied. Applying the selection criteria defined above to the Monte Carlo simulation yielded an efficiency of 62\% for the neutrino beam and 66\% for the antineutrino beam.

The ratio of the energy-weighted fluxes of muon-neutrinos and muon-antineutrinos was calculated from a subsample of events (34) having a value of \( p_{T}^{2} \) in the range \( 0.1 < p_{T}^{2} < 0.3 \text{ GeV}^{2} \) where there is a good agreement between measurement and simulation [35]. The following additional cuts were applied: \( \sum_{2}^{6} N_{i}^{\text{scint}} \leq 60 \text{ MeV}, \sum_{2}^{6} N_{i}^{\text{prop}} \leq 9 \), and \( \sum_{2}^{6} N_{i}^{\text{str}} \leq 9 \). The sum \( \sum_{2}^{6} \) extends over planes two to six starting from the vertex. The first plane was excluded because of the differences between \( \nu_{\mu} \) and \( \bar{\nu}_{\mu} \) which were larger than in the following planes. Applying this additional selection we found

\[
N_{Q}(\nu) = 8882 \pm 94 \quad \text{and} \quad N_{Q}(\bar{\nu}) = 13571 \pm 116 .
\]  

The ratio of the efficiencies of the selection criteria for the neutrino and antineutrino samples (35), obtained using the Monte Carlo simulation, was 0.94 \( \pm 0.035 \). The systematic error was computed varying the selection cuts and the assumption on the Monte Carlo simulation. The wrong-sign muon-neutrino contamination has been determined from the measurement of the wrong-sign muons, taking into account the efficiency in the defocussing magnet configuration.

The measured distribution of the muon momentum \( f_{\mu}(p) \) allowed a determination of the average values of the muon-neutrino and muon-antineutrino energies from the convolution integral

\[
f_{\mu}(p) = \int A(p,E) \times f_{\nu}(E) \, dE ,
\]

where \( f_{\nu}(E) \) is the neutrino energy spectrum to be determined and \( A \) the energy resolution function, which gives the probability of finding an event induced by a neutrino of energy \( E \) with a measured muon momentum \( p \). The quantity \( A \) is defined by the momentum-dependent efficiency of the selection criteria, the momentum transfer of the neutrino to the muon, and the momentum resolution of the muon spectrometer (about 15\%) whose contribution is the most essential. The unfolding is performed by a standard mathematical procedure [39]. The resulting spectra from the first period are shown in Fig. 13; they are similar to those obtained in the second data period. The ratio of the mean
muon-antineutrino energy in the antineutrino beam and of the mean muon-neutrino energy in the neutrino beam, obtained from this analysis, was [35]:

\[
\frac{\langle E_{\nu} \rangle}{\langle E_{\mu} \rangle} = 0.803 \pm 0.009 \text{ (stat.)} \pm 0.010 \text{ (syst.)} . \tag{37}
\]

The energy spectra were used also to correct for the energy dependence of the selection criteria and for the acceptance of the energy cuts.

Taking into account the difference of the cross-sections of neutrino and antineutrino quasi-elastic scattering averaged over the neutrino energy spectra \((\sigma_\nu/\sigma_\mu) = 1.065 \pm 0.025 [35] \) and the small excess of protons over neutrons in the target calorimeter \((N_p/N_n = 1.009 \pm 0.004)\), we found

\[
FQ = 1.22 \pm 0.02 \text{ (stat.)} \pm 0.06 \text{ (syst.)} , \tag{38}
\]

in good agreement with the flux ratio determined using inelastic neutrino-nucleon events. The systematic error was estimated by varying the selection criteria and the fraction of resonance events. The weighted mean value of the normalization factors \(F\) as calculated from the deep-inelastic and quasi-elastic events is

\[
F = 1.18 \pm 0.05 . \tag{39}
\]

In order to make the total normalization factors \(G (33)\) consistent with this ratio \(F\) they were corrected by 3\% and the following numbers were used to determine the neutrino and antineutrino electron cross-sections:

\[
G(\nu) = (1.60 \pm 0.06) \times 10^{-44} \text{ cm}^2/\text{GeV} , \tag{40}
\]

\[
G(\bar{\nu}) = (1.35 \pm 0.05) \times 10^{-44} \text{ cm}^2/\text{GeV} .
\]

The relative energy-weighted fluxes of the different neutrinos,

\[
\begin{align*}
\nu \text{ beam: } & \nu_\mu : \bar{\nu}_\mu : \nu_e : \bar{\nu}_e = 1 : 0.06 : 0.017 : 0.004 \\
\bar{\nu} \text{ beam: } & \bar{\nu}_\mu : \nu_\mu : \bar{\nu}_e : \nu_e = 1 : 0.09 : 0.012 : 0.006
\end{align*} \tag{41}
\]

for the second period, and
\[ \nu_{\mu} : \bar{e}_{\mu} : \nu_{\bar{e}} : \bar{e}_{\bar{e}} = 1 : 0.06 : 0.009 : 0.007 \]
\[ \bar{\nu}_{\mu} : e_{\mu} : \bar{\nu}_{\bar{e}} : e_{\bar{e}} = 1 : 0.10 : 0.010 : 0.005 \]

for the first period, were used to attribute the neutrino electron scattering events (see Section 6) to the various reactions of $\nu_{\mu}$, $\bar{\nu}_{\mu}$, $\nu_{\bar{e}}$, and $\bar{\nu}_{\bar{e}}$ on electrons. The relative errors in (41) and (42) are 5% for the muon-neutrino background and 50% for the electron-neutrino flux.

6. Measurement of neutrino-electron reactions

This Section describes the analysis of neutrino-electron scattering events. These events have a single electron in the final state and have to be separated from the large number of hadronic events produced by the scattering of neutrinos on nucleons by making use of the different event patterns of electromagnetic and hadronic showers. These events were then further separated from the residual background due to neutrino-nucleon reactions with an electromagnetic shower in the final state by applying statistical methods that exploit the differences in the distributions of shower angle $\theta$, shower energy $E$, and the energy $E_q$ deposited in the first scintillator plane of the shower. A value of $\sin^2 \theta_w$ was determined from the neutrino and antineutrino cross-section ratio $R$. Also the measurement of the individual cross-sections allowed a determination of the mixing angle, although with larger systematic errors.

6.1 Analysis of the data from the second exposure

A total of $2.2 \times 10^8$ neutrino and $1.6 \times 10^8$ antineutrino interactions were recorded in the second exposure of the CHARM detector to the WBB. To select events a rough filter was first applied to the data. i) The visible energy $E_{\text{vis}}$ in the scintillators of the calorimeter was required to be at least 200 MeV. ii) The ratio $T$ of $E_{\text{vis}}$ over the number of hits in the streamer tubes was required to be smaller than 35 MeV/tube; this cut eliminates events that are mainly outside the detector volume covered by the streamer tubes; for electron showers of 5 GeV a mean value of $<T> = 13$ MeV/tube with a standard deviation $\sigma_T = 2$ MeV/tube was measured; for 50 GeV, $<T> = 25$ MeV/tube and $\sigma_T = 2$ MeV/tube. iii) Events with the vertex in the units 71-78 were rejected. iv) At least 4 hits detected in the bremsstrahlung counters as a signature of a muon from $CC\nu_{\mu}$ events were used to reject these events. v) The separator 'helicity' described in subsection 4.2, was applied. Events with a total number of gaps
in the streamer tube hit pattern of less than three are due to muon tracks and were rejected. For electromagnetic showers the efficiency of this separator was measured in a test beam and was found to be $(96 \pm 1)\%$. For energies in the range $4 < E < 50 \text{ GeV}$ and $0 < \theta_{\text{proj}} < 100 \text{ mrad}$ the efficiency was independent of shower energy and angle. A total of 11000 events recorded in the neutrino beam and 6700 events in the antineutrino beam survived the described selection criteria.

These events were further analysed by the reconstruction program. About 50% of the events had a visible muon track and were rejected. For the remaining events, important quantities, such as energy $E$, width of the lateral shower profile, shower vertex and angle $\theta$, and the value of $E_f$ were calculated using the algorithms described in Section 4. The following selection criteria were then applied. i) Fiducial volume: $7 \leq \text{unit} \leq 63 \text{ (longitudinal)}$ and $|y|, |z| \leq 1.15 \text{ m (transversal)}$, events with a vertex outside this range were not completely recorded by all detector elements and were therefore rejected. ii) Shower profile: lateral $\Gamma < 1.8 \text{ cm}$, $\sigma_{\text{ST}} \leq 10-13 \text{ cm}$, and longitudinal $92 \text{ cm} \leq L_{\text{shower}} \leq 165 \text{ cm}$. In Section 4 the selection of electromagnetic cascades, according to width and length of the shower, is discussed in detail. The cut in $\sigma_{\text{ST}}$ varied from 10 to 13 cm for shower energies varying from 4 to 30 GeV. For elastic neutrino-electron interactions the efficiency of the cuts (ii) was $(85 \pm 3)\%$, nearly independent of angle and energy. iii) Vertex topology: $3 \text{ MeV} \leq E_f \leq 50 \text{ MeV}$ (1 minimum ionizing particle corresponds to 6 MeV), and $M^d_4 = 1$, and $0 < M^s_4 < 7$, and $G^5_4 < 3$. The multiplicities in the proportional and streamer tubes of the vertex plane are denoted by $M^d_4$ and $M^s_4$, respectively. The number of holes in the streamer tube pattern is denoted by $G^5_4$. The efficiency of neutrino-electron events fulfilling the vertex criteria is $(73 \pm 4)\%$, nearly independent of the shower energy. The vertex cuts improved the angular resolution, and reduced the background due to showers induced by the scattering of neutrinos on nucleons that have on average higher particle multiplicity at the shower vertex. iv) Kinematic range: $4 \text{ GeV} \leq E \leq 30 \text{ GeV}$ and $E^2 \theta^2 \leq 0.54 \text{ GeV}^2$. The choice of the energy window was based on the consideration that below 4 GeV the trigger efficiency is small (less than 50%) and above 30 GeV the background of quasi-elastic $\nu_eN$ interactions is very high. On average, electron-neutrinos produced in kaon decays have a higher energy than muon-neutrinos and almost the total energy of the neutrino is transferred to the electron in quasi-elastic scattering on nucleons, while for neutrino-electron scattering events only 40% of the neutrino energy is transferred on average to the electron.
For $\sin^2 \theta_w = 0.215$ and for the neutrino energy spectra determined from quasi-elastic events (Section 5) the efficiency of the cuts in energy is 51% for $\nu_\mu e$ and 54% for $\bar{\nu}_\mu e$ scattering. The upper limit of $E^2 \theta^2 < 0.54 \text{ GeV}^2$ was chosen in order to make sure that this sample contains a sufficient number of background events in a kinematic region to which neutrino-electron scattering does not contribute. The background can be determined from the measured distribution in this region and extrapolated to the region $E^2 \theta^2 < 0.06 \text{ GeV}^2$ where, according to a Monte Carlo simulation, 85% of the neutrino-electron interaction events are expected.

These selection criteria reduced the numbers of candidate events to

$$N^\nu_{\text{tot}} = 339 \quad \text{and} \quad N^{\bar{\nu}}_{\text{tot}} = 376 \quad .$$

(43)

The following reactions can contribute to these samples of events: i) neutrino-electron scattering, ii) coherent NC scattering of muon-neutrinos on nuclei with the production of a $\pi^0$ [40], iii) inclusive neutrino-nucleon NC interactions with a dominantly electromagnetic final state, and iv) quasi-elastic electron-neutrino CC scattering on nucleons. The distribution of the energy, angle, and $E_f$ variables are different for the neutrino-electron scattering events and for the background reactions (ii)-(iv), as shown in Figs. 14-17. The distributions shown refer to neutrino events; those of the antineutrino-induced events are similar. For neutrino-electron events the angular distribution is narrow and peaks in the forward direction; it is considerably broader for all the background processes. The $E_f$ distributions show clear differences between $e^-$- and $\pi^0$-induced showers. Also the energy distributions are different for the various reactions. The distributions shown in Figs. 14-17 refer to those events fulfilling the selection criteria. They were obtained using a Monte Carlo simulation of the beam and of the detector response for the different reactions. Experimental resolution functions, energy dependence of the trigger efficiency, and the event selection criteria were taken into account.

To determine the number of neutrino-electron events in the candidate samples (43) the measured distributions $N^\nu(E, \theta, E_f)$ and $N^{\bar{\nu}}(E, \theta, E_f)$ were compared with the expected distributions $X^\nu$ and $X^{\bar{\nu}}$ [35]:

$$X^\nu(a_1, ..., a_4; E, \theta, E_f) = N^\nu_{\text{tot}} \times \sum_i a_i f_i(E, \theta, E_f) \quad$$

$$X^{\bar{\nu}}(\bar{a}_1, ..., \bar{a}_4; E, \theta, E_f) = N^{\bar{\nu}}_{\text{tot}} \times \sum_i \bar{a}_i \bar{f}_i(E, \theta, E_f) \quad .$$

(44)

The functions $X$ are linear combinations of the distributions $f_i$ and $\bar{f}_i$ ($i = 1, 2, 3, 4$) of the two times
four reactions contributing to the candidate samples. The distributions were calculated by a Monte Carlo simulation and normalized according to

\[ \int f_i(E, \theta, E_f) \, d\theta \, dE_f = 1 \quad \text{and} \quad \int f_i(E, \theta, E_f) \, dE \, dE_f = 1. \]

The integral covers the regions of the variables \(E, \theta,\) and \(E_f\) according to the selection criteria applied in the analysis. The space set up by \(E, \theta,\) and \(E_f\) was divided into \(K\) small cells \(C_j\) for which the corresponding measured and expected event numbers \(N_j\) and \(X_j\) were determined:

\[ N_j = \int_{C_j} N(E, \theta, E_f) \, d\theta \, dE_f, \quad X_j = \int_{C_j} X(E, \theta, E_f) \, d\theta \, dE_f. \]

The value of \(K\) must be large compared to \(N^v_{\text{tot}} \) and \(N^\nu_{\text{tot}}\) and the cell size not larger than the experimental resolutions. The eight coefficients \(a_i\) and \(\overline{a}_i\) (\(i = 1, \ldots, 4\)) were determined by minimizing the negative logarithm of the likelihood function \(L\):

\[ L(a_1, \ldots, a_4, \overline{a}_1, \ldots, \overline{a}_4) = -2 \log \left( \prod_j \text{prob} \left[ N^\nu_{j}, \; X^\nu_{j}(a_1, \ldots, a_4) \right] \right) \times \prod_j \text{prob} \left[ N^\nu_{j}, \; X^\nu_{j}(\overline{a}_1, \ldots, \overline{a}_4) \right]. \]

The Poisson distribution \(\text{prob} (N,X)\) gives the probability of finding \(N\) events if \(X\) are expected.

The number of independent parameters \(a_i\) and \(\overline{a}_i\) was reduced from eight to four by making additional assumptions. The first assumption, \(X^\nu_{\text{tot}} \left( X^\nu_{\text{tot}} \right) = N^\nu_{\text{tot}} \left( N^\nu_{\text{tot}} \right),\) is not entirely correct because the number of events is not fixed by the experiment; however, since \(N^\nu_{e} \ll N_{\text{tot}}\) the assumption introduces a negligible bias in the determination of the value and the error of \(N^\nu_{e}\). It follows that \(\Sigma a_i = \Sigma \overline{a}_i = 1.\) As far as the two muon-(anti)neutrino-induced backgrounds (\(i = 2, 3\)) are concerned, both the relative normalization of the fluxes in the antineutrino and the neutrino beam and the ratios of antineutrino and neutrino cross-sections are known: \(\overline{a}_2/a_2 = 0.85 \pm 0.15\) and \(a_3/a_4 = 0.50 \pm 0.10.\)

The uncertainties on these ratios contribute to the systematic errors. In the case of electron-neutrino (electron-antineutrino) interactions the flux ratio was not measured with high precision (Section 5), and, hence, no restriction was made on \(\overline{a}_4/a_4.\) With these assumptions only four of the coefficients \(a_i\) and \(\overline{a}_i\) are independent and \(a_1, a_2, a_3,\) and \(\overline{a}_1\) have been chosen.

The \(K\) cells were defined by dividing the \(E\) and \(\theta\) variables into 50 equal intervals \((\Delta E = 0.5\) GeV, \(\Delta \theta = 4\) mrad\) and the \(E_f\) variable into 25 intervals \((\Delta E_f = 2\) MeV\). With the restriction \(E^2 \theta^2 \leq\)
0.54 GeV² the total number of cells is \( K \approx 25000 \). To simplify the Monte Carlo calculations the functions \( f_1 \) and \( \tilde{f}_1 \) were factorized in the following way:

\[
f(E, \theta, E_d) = f'(E, \theta) \times f''(E_d). \tag{48}
\]

In the considered intervals of \( E \) and \( \theta \) this is a good approximation. The values of \( f''(E_d) \) were calculated using the EGS simulation program [41].

The parameters \( a_1 \) and \( \tilde{a}_1 \), corresponding to the minimum of the likelihood function \( L \) (47), were determined using the program MINUIT [42]. In Table 1 the number of events attributed to the different processes with their statistical and systematic errors are reported. The errors are discussed below. A detailed study of the likelihood \( L \) near the minimum revealed that the number of neutrino-electron and antineutrino-electron events is almost uncorrelated with the number of the different background events. However, the number of coherent \( \pi^0 \) events and NC reactions with dominant electromagnetic background are strongly (negatively) correlated. This is to be expected because the distributions \( f \) and \( f'' \) for these two processes are very similar. The stability of the solution for \( a_1, a_2, a_3, \) and \( \tilde{a}_1 \) was checked and the systematic errors were estimated by modifying the underlying assumptions and using slightly different theoretical distributions \( f(E, \theta, E_d) \) [35]. It was verified that the assumption \( \sin^2 \theta_w = 0.215 \) used to calculate \( f_1 \) and \( \tilde{f}_1 \) does not bias the results obtained. With the numbers for \( a_1 \) and \( \tilde{a}_1 \) obtained from the fit several important distributions were calculated and compared with the measured ones. Good agreement was found as is shown by the comparison of the \( E^2 \theta^2 \) distributions in Fig. 18.

Multiplying the ratio of the number of the neutrino-electron and antineutrino-electron events with the ratio of the energy-weighted neutrino fluxes, as determined in Section 5, we obtained

\[
R' = 1.21 \pm 0.10 \text{ (syst.)} \tag{49}
\]

The systematic error takes into account the uncertainty of \( F \) and the (positively) correlated errors of \( N_{\nu e} \) and \( N_{\bar{\nu}e} \). The quantity \( R' \) is not identical to \( R = \sigma(\nu_{\mu} e)/\sigma(\bar{\nu}_{\mu} e) \) defined in (11) since the neutrino beam contained different neutrino flavours and the energy cut \( 4 < E < 30 \text{ GeV} \) has been applied in the analysis. It is given by the expression

\[
R' = \frac{p_\nu(\nu_{\mu}, e)\sigma(\nu_{\mu} e) + p_\nu(\bar{\nu}_{\mu} e)\sigma(\bar{\nu}_{\mu} e) + p_\mu(\nu_{\mu} e)\sigma(\nu_{\mu} e) + p_\mu(\bar{\nu}_{\mu} e)\sigma(\bar{\nu}_{\mu} e)}{p_\nu(\bar{\nu}_{\mu}, e)\sigma(\bar{\nu}_{\mu} e) + p_\nu(\nu_{\mu} e)\sigma(\nu_{\mu} e) + p_\mu(\nu_{\mu} e)\sigma(\nu_{\mu} e) + p_\mu(\bar{\nu}_{\mu} e)\sigma(\bar{\nu}_{\mu} e)}, \tag{50}
\]

25
where \( p \) is denoting the energy-weighted fractions of the different neutrino types in \( \nu \) and \( \bar{\nu} \) beams as determined in Section 5 and \( \rho \) the neutrino-electron scattering cross-sections. Since NC and CC interactions contribute both to the scattering of electron-neutrinos and electron-antineutrinos on electrons, \( \Gamma' \) depends on \( \sin^2 \theta_w \) and \( \rho \). However, since the measured \( \bar{\nu}_e \) cross-section [43] is compatible with \( \rho = 1 \) and the fraction of electron-(anti)neutrinos in the beams is small, a value of \( \rho = 1 \) was assumed to derive \( \sin^2 \theta_w \) from (50). The quantities \( \epsilon \) also depend on \( \sin^2 \theta_w \); they denote the efficiencies of the kinematic cuts and take into account the trigger efficiency as a function of the electron energy. Figure 2 shows the expected dependence of \( \Gamma' \) on \( \sin^2 \theta_w \) (solid line). For comparison \( \Gamma(\sin^2 \theta_w) \) is also shown (dashed line). From the measured value of \( \Gamma' \) (49) the value

\[
\sin^2 \theta_w = 0.215 \pm 0.050 \text{ (stat.)} \pm 0.010 \text{ (syst.)}
\]  

(51)

was determined. This determination of the mixing angle is independent of \( \rho \) and \( m_Z \).

In order to calculate the total \( \nu_\mu e \) and \( \bar{\nu}_\mu e \) cross-sections the contributions of other neutrino types were subtracted from the numbers \( N_{\nu e} \) and \( N_{\bar{\nu}e} \) assuming \( \sin^2 \theta_w = 0.215 \). After corrections for efficiencies the following cross-sections were found:

\[
\begin{align*}
\sigma(\nu_\mu e)/E_\nu &= [1.9 \pm 0.4 \text{ (stat.)} \pm 0.3 \text{ (syst.)}] \times 10^{-42} \text{ cm}^2/\text{GeV} \\
\sigma(\bar{\nu}_\mu e)/E_\nu &= [1.5 \pm 0.4 \text{ (stat.)} \pm 0.3 \text{ (syst.)}] \times 10^{-42} \text{ cm}^2/\text{GeV}.
\end{align*}
\]  

(52)

The absolute neutrino-flux determinations (40) were used.

6.2 Analysis of the data from the first exposure

In the first exposure of the CHARM detector in the WBB \( 1.3 \times 10^6 \) neutrino and \( 1.4 \times 10^6 \) antineutrino interactions were recorded. The selection criteria used to identify events with electromagnetic showers [23], [44] were very similar to those used in the analysis of the second period data described in the previous subsection, except for the streamer tubes which were not installed.

The neutrino-electron scattering events were statistically separated from the residual background on the basis of their distribution as a function of \( E^2 \theta^2 \). The distributions of the 267 neutrino and 665 antineutrino candidate events with \( E^2 \theta^2 < 0.54 \text{ GeV}^2 \) and \( 7.5 \text{ GeV} < E < 30 \text{ GeV} \) are shown in Fig. 19. The lower energy cut, made at 7.5 GeV, was higher than in the analysis of the second period data because of the higher energy threshold of the trigger in this exposure [44]. The measured electron en-
ergy and angular resolutions imply that 90% of the $\nu_\mu(\bar{\nu}_\mu)$ events have $E^2\theta^2$ less than 0.12 GeV$^2$, corresponding to the first two bins in Fig. 19.

The number of neutrino-electron scattering events in the region $E^2\theta^2 < 0.12$ GeV$^2$ (forward region) was obtained by extrapolating the background from the region $0.12 < E^2\theta^2 < 0.54$ GeV$^2$ (reference region). In this analysis we assumed that the background was due only to: i) the elastic and quasi-elastic CC events induced by the $\nu_e$ and $\bar{\nu}_e$ contamination of the beams and ii) coherent NC scattering on nuclei with the production of $\pi^0$. Inclusive neutrino-nucleon NC interactions with a predominantly electromagnetic final state (incoherent $\pi^0$ production) were not separately described because they have very similar distributions to those of background ii).

The normalization of the two backgrounds was obtained by a study of $E_\perp$. This analysis was based on the observation that electromagnetic showers initiated by one or more photons deposit in the first scintillator plane an energy larger than one minimum ionizing particle (6 MeV), whilst a large fraction of the showers is due to single electrons and gives an energy deposition corresponding to one particle.

The number of events attributed to background (i) was obtained from the number of events with $E_\perp < 8$ MeV in the region $0.12 < E^2\theta^2 < 0.54$ GeV$^2$. The efficiency was determined using the events with visible energy between 30 and 50 GeV and $0.12 < E^2\theta^2 < 0.54$ GeV$^2$ which are dominantly due to the $\nu_e(\bar{\nu}_e)$ quasi-elastic scattering. The result is $(26 \pm 6)\%$, for neutrino and antineutrino events. This number agrees, within the errors, with the measured efficiency for single electrons $(32 \pm 5)\%$.

The efficiency for these quasi-elastic events is expected to be somewhat smaller than that measured for an isolated electron in a test beam, because of the energy detected for recoiling nucleons. The remaining events were attributed to background (ii).

The $E^2\theta^2$ distribution of background (i), known to be approximately energy-independent, has been determined by folding the measured $E^2\theta^2$ distributions of quasi-elastic CC reactions induced by muon-neutrinos and muon-antineutrinos with the measured electron energy and angular resolution. For this process, the ratio of events (forward region)/(reference region) is 0.49. The $E^2\theta^2$ distribution of background (ii) was calculated using the predictions for coherent $\pi^0$ production by neutrinos on nuclei [40]. The ratio of events (forward region)/(reference region) is 0.39. Since the two ratios are not very different, a possible error in the decomposition of the total background makes only a small difference on the final result.
In Fig. 19 the shapes of the two backgrounds are shown. The amount of background (i) found in the neutrino and antineutrino exposures agrees with the background expected from the calculated $\nu_e$ and $\bar{\nu}_e$ contamination of these beams. The number of neutrino and antineutrino events attributed to background (ii) corresponds to a cross-section ratio compatible with one. This supports the hypothesis that these events are due to coherent scattering of muon-neutrinos on nuclei with $\pi^0$ production.

From the cross-section ratio a value of

$$\sin^2\theta_W = 0.206 \pm 0.048 \text{ (stat.)} \pm 0.013 \text{ (syst.)}$$

(53)

was determined. Using the absolute neutrino fluxes (Section 5) the following cross-section values were derived:

$$\sigma(\nu, e)/E_{\nu} = [2.45 \pm 0.6 \text{ (stat.)} \pm 0.6 \text{ (syst.)}] \times 10^{-42} \text{ cm}^2/\text{GeV}$$

(54)

$$\sigma(\bar{\nu}, e)/E_{\bar{\nu}} = [1.75 \pm 0.4 \text{ (stat.)} \pm 0.4 \text{ (syst.)}] \times 10^{-42} \text{ cm}^2/\text{GeV}.$$  

This analysis method was also applied to the data from the second period [24]. The results are in good agreement with those obtained in the previous section by applying the maximum-likelihood method. Taking into account the improved angular resolution and the differences of the acceptance criteria, the signal-to-background ratio and the background composition of the first and the second period agree within the errors.

7. Combined results

In this section the results obtained from the analysis of the first and second period data will be combined. The $E^2\theta^2$ distributions of all $\nu_\mu e$ and $\bar{\nu}_\mu e$ candidate events collected in the first and in the second exposure of the CHARM detector are shown in Figs. 20a and 20b taking into account the different angular resolutions. Fig. 20c and 20d show the analogous distributions for events satisfying the additional condition $E_T < 8$ MeV applied to select unambiguously events with a single electron at the shower vertex. After the background subtraction, the number of events found [23], [24] in the region $E^2\theta^2 < 0.12$ GeV$^2$ was 83 \pm 16 (neutrino) and 112 \pm 21 (antineutrino). From the $E^2\theta^2$ distributions of the events with $E_T < 8$ MeV, signals of 24 \pm 6 and 35 \pm 9 events, respectively, were obtained. The
ratios of the signals found with $E_T < 8$ MeV and with $E_T < 50$ MeV is $0.30 \pm 0.08$, in good agreement with the relative selection efficiency of $(32 \pm 5)$%, as measured in the electron test beam. This agreement supports the hypothesis that the signals are due to events with a single recoil electron.

The weighted mean of the $\sin^2 \theta_W$ values obtained in the analysis of the first and second exposure is

$$\sin^2 \theta_W = 0.211 \pm 0.035 \text{ (stat.)} \pm 0.011 \text{ (syst.)}. \quad (55)$$

For the cross-sections the values

$$\sigma(\nu_{\mu},e)/E_\nu = [2.2 \pm 0.4 \text{ (stat.)} \pm 0.4 \text{ (syst.)}] \times 10^{-42} \text{ cm}^2/\text{GeV}$$
$$\sigma(\bar{\nu}_{\mu},e)/E_{\bar{\nu}} = [1.6 \pm 0.3 \text{ (stat.)} \pm 0.3 \text{ (syst.)}] \times 10^{-42} \text{ cm}^2/\text{GeV} \quad (56)$$

were obtained. The statistical errors were averaged quadratically and the systematic errors were averaged linearly because of their correlations.

Using the parametrization (9) for the $\nu_{\mu},e$ cross-section and as input from other experiments $m_Z = (92 \pm 1.8)$ GeV (see Section 1) a value of

$$\sin^2 \theta_W = 0.199 \pm 0.015 \text{ (stat.)} \pm 0.020 \text{ (syst.)} \quad (57)$$

was obtained.

The cross-sections and the mixing angle are in good agreement with the results of other recent $\nu_{\mu},(\bar{\nu}_{\mu}),e$ measurements [45] - [52], which are summarized in Table 2. From the cross-sections vector and axial-vector coupling constants of the electron, $g_{eV}^e$ and $g_{eA}^e$, can be calculated up to a fourfold ambiguity. The ambiguity was eliminated (see Fig. 21) by using measurements of other leptonic processes, $\bar{\nu}_\tau e \rightarrow \bar{\nu}_\tau e$ [43] and $e^+e^- \rightarrow \mu^+\mu^-$ [53], leaving a unique solution:

$$g_{eV}^e = -0.06 \pm 0.07 \text{ (stat.)} \pm 0.02 \text{ (syst.)}$$
$$g_{eA}^e = -0.57 \pm 0.04 \text{ (stat.)} \pm 0.06 \text{ (syst.)}, \quad (58)$$

in good agreement with the prediction of the Standard Model $g_{eA}^e = -1/2$ based on the assumption that the left-handed electron transforms as a doublet under weak isospin rotation.

Comparing the sum of $\sigma(\nu_{\mu},e)$ and $\sigma(\bar{\nu}_{\mu},e)$ with the cross-section values expected in the Standard Model for the value of $\sin^2 \theta_W$ given in (55) we derived the value of the ratio of NC and CC coupling constants:
\[ \rho = 1.14 \pm 0.07 \text{ (stat.)} \pm 0.12 \text{ (syst.)} \]  

This result is compatible with our recent determination in semileptonic reactions, \( \rho = 0.99 \pm 0.01 \) [10], and the value \( \rho = 1.02 \pm 0.03 \) as determined from W and Z measurements at the \( pp \bar{p} \) Collider [53]. In the Standard Model with a minimal Higgs sector \( \rho \) is predicted to be equal to one [4].

8. Limits derived from the measurements of \( \sigma(\nu_\mu e) \) and \( \sigma(\nu_\mu e) \)

8.1 Limits on the mass of an excited electron and of an additional neutral boson

From the ratio of the Z and W masses [5], [6] a value of

\[ \sin^2 \theta_w = 0.22 \pm 0.02 \]  

was derived. Comparing the measured values of \( g^e_V \) and \( g^e_A \) in (58) to those obtained using this independent determination of \( \sin^2 \theta_w \) the upper limit

\[ \lambda^2 \Lambda^2 / m_{e^*} \leq 2.6 \quad (95\% \text{ CL}) \]  

was determined making use of (14). With assumptions on the scale factor \( \Lambda > 1000 \text{ GeV} \) and on the coupling constant \( \lambda = 1 \) a lower limit for the mass of the excited electron is derived

\[ m_{e^*} > 620 \text{ GeV} \quad (95\% \text{ CL}) \].  

Using the same assumptions, a limit \( m_{e^*} > 72 \text{ GeV} \) (95% CL) was derived previously from a measurement of the cross-section of the process \( e^+e^- \rightarrow \gamma\gamma \) [54] (see also [55] - [57]). Figure 22 shows the limit on \( \lambda \) as a function of \( m_{e^*} \), together with the previous limits [56].

Using (15) the upper limit

\[ (g'/g)^2 \times (m_Z/m_{Z'})^2 < 0.11 \quad (95\% \text{ CL}) \]  

was derived; \( g \) (\( g' \)) and \( m_Z \) (\( m_{Z'} \)) are the coupling constants and the masses of the Z boson and of an additional \( Z' \) boson, respectively. With the assumption \( g' = g \) a lower limit of

\[ m_{Z'} > 280 \text{ GeV} \quad (95\% \text{ CL}) \].
was obtained. Analysing semileptonic processes at small \( Q^2 \) another limit has been derived which is a factor of 2-3 smaller [15]. For different models with an additional neutral boson, lower bounds between 100 GeV and 300 GeV were obtained [58]. From a direct search the limit \( m_{Z'} > 188 \) GeV (90% CL) has been determined [59].

### 8.2 Limit on the magnetic moment of the muon-neutrino

From the value of \( \sin^2 \theta_W \) (60), deduced from the measurements of the masses of the W and Z bosons, the number of neutrino (antineutrino)-electron events expected in this experiment can be predicted in the Standard Model. Using the data collected in the second exposure the limits

\[
\Delta N(\nu_\mu e) < 26 \quad \text{and} \quad \Delta N(\bar{\nu}_\mu e) < 32 \quad (95\% \ CL)
\]  

(65)

were obtained for an excess of events over the Standard Model prediction. From this result we derived an upper limit on the magnetic moment of muon-neutrinos using (16) and assuming \( \mu(\nu_\mu) = \mu(\bar{\nu}_\mu) \)

\[
\mu(\nu_\mu, \bar{\nu}_\mu) < 1.0 \times 10^{-9} \mu_B \quad (95\% \ CL).
\]  

(66)

The following 90% confidence limits have been derived previously from measurements of \( \bar{\nu}_e e \) and \( \nu_\mu e \) cross-sections in other experiments

\[
\mu(\nu_e) < 1.5 \times 10^{-10} \mu_B \quad [60],
\]

\[
\mu(\bar{\nu}_e) < 9.5 \times 10^{-10} \mu_B \quad [61],
\]

\[
\mu(\nu_\tau) < 4.0 \times 10^{-6} \mu_B \quad [62].
\]  

(67)

From astrophysical observations the following 90% confidence limit was obtained [63]:

\[
\mu(\nu) < 0.8 \times 10^{-11} \mu_B.
\]  

(68)

### 8.3 Limit on anomalous neutrino electric charge radius

Comparing the measured cross-section ratio \( R = \sigma(\nu_\mu e)/\sigma(\bar{\nu}_e e) \) with the value expected for \( \sin^2 \theta_W \) from (60) we derived from (17) a limit on anomalous contributions to the neutrino electric charge radius:
\[ \langle r^2 \rangle_{\text{anom}} < 10^{-32} \text{ cm}^2 \ (90\% \ CL) . \] (69)

A similar limit \( (0.8 \times 10^{-32} \ \text{cm}^2) \) has been derived in [61]. It is interesting to note that this limit is similar in magnitude to the limit on a possible internal structure of the electron and the muon derived from \( e^+e^- \) annihilation at high energy [20]. As the contribution arising in the Standard Model are almost completely cancelled in \( \nu_\mu \) scattering the sensitivity can be further increased in future high statistics experiments [64].

9. Charged-current \( \nu_\mu \) e interactions: inverse muon decay

The cross-section of the inverse muon decay reaction

\[ \nu_\mu e^- \rightarrow \mu^- \nu_e \] (70)

has also been measured in the same exposures; a discussion of the analysis can be found in [65], [66], [35]. Recoil-less single-muon events were selected in the transverse momentum range \( p_T^2 \leq 0.1 \ \text{GeV}^2 \). The energy deposited in the vertex region was required to be compatible with that of a single minimum ionizing particle. The neutrino and antineutrino samples thus selected contained quasi-elastic \( \nu_\mu (\bar{\nu}_\mu)N \) events, events in which nucleon resonances are excited, and, in the neutrino case, inverse muon decay events. The latter are characterized by a small value of \( p_T \). In the range \( 0 \leq p_T^2 \leq 0.02 \ \text{GeV}^2 \) an excess of

\[ N(\nu_\mu \rightarrow \mu^- \nu_e) = 648 \pm 48 \ (\text{stat.}) \pm 20 \ (\text{syst.}) \] (71)

events was found in the neutrino data (see figure 23). The background was subtracted using the anti-neutrino \( p_T^2 \) distribution normalized to the number of neutrino events in the range \( 0.02 \leq p_T^2 \leq 0.10 \ \text{GeV}^2 \).

Normalizing to the number of inelastic neutrino-nucleon events and using our recent CC cross-section measurements [33] a cross-section of

\[ \sigma(\nu_\mu e^- \rightarrow \mu^- \nu_e)/E_\nu = [1.81 \pm 0.14 \ (\text{stat.}) \pm 0.16 \ (\text{syst.})] \times 10^{-41} \text{cm}^2/\text{GeV} \] (72)
was determined for \( E_\nu \gg m_\mu^2/m_e \). This number is in good agreement with the theoretical prediction of \( 1.72 \times 10^{-41} \text{ cm}^2/\text{GeV} \).

The ratio of the sum of the NC neutrino- and antineutrino-electron cross-sections and the inverse muon decay cross-section is proportional to \( \rho^2 \), the ratio of the NC and CC couplings

\[
\frac{\sigma_{\text{NC}}^{\nu} + \sigma_{\text{NC}}^{\bar{\nu}}}{\sigma_{\text{CC}}^{\nu}} = \frac{2}{3} \left( g_{\nu}^{eV} + g_{A}^{eA} \right) \times \rho^2.
\]

(73)

From the measured cross-sections, assuming \( g_{A}^{eA} = -1/2 \) and using the measured value of \( \sin^2 \theta_W \) (55) to determining \( g_{\nu}^{eV} \) we derived

\[
\rho = 1.10 \pm 0.08 \text{ (stat.)} \pm 0.13 \text{ (syst.)}.
\]

(74)

This result is compatible with that obtained by comparing the measured \( \nu_\mu \text{e} \) and \( \bar{\nu}_\mu \text{e} \) cross-sections (56) and the calculated cross-sections using the value of \( \sin^2 \theta_W \) derived from the measurement of their ratio (55) [see (59)].

10. Conclusions

The high-mass fine-grain calorimeter of the CHARM collaboration in conjunction with the intense neutrino beams produced at the 400 GeV CERN Proton Synchrotron, has been successfully used for a determination of \( \sigma(\nu_\mu \text{e}) \) and \( \sigma(\bar{\nu}_\mu \text{e}) \). The value of \( \sin^2 \theta_W \) that has been derived from the ratio \( R = \sigma(\nu_\mu \text{e})/\sigma(\bar{\nu}_\mu \text{e}) \) is one of the most accurate experimental values of this fundamental quantity obtained from the measurement of reactions involving leptons only. The present result, based on the detection of \( 83 \pm 16 \) neutrino-electron and \( 112 \pm 21 \) antineutrino-electron events is limited by statistical accuracy. A new experiment (CHARM II) with a detector of improved performance [64] is now taking data at CERN with the aim to determine \( \sin^2 \theta_W \) with an error of \( \Delta(\sin^2 \theta_W) = 0.005 \). This determination, together with the improved measurements of the boson masses \( m_Z \) and \( m_W \), will allow a verification of the Standard Model at the loop-level.

The experimental values of \( \sigma(\nu_\mu \text{e}) \) and \( \sigma(\bar{\nu}_\mu \text{e}) \) have been used to derive new limits on masses and coupling constants of a hypothetical excited electron \( e^* \) and a new neutral boson \( Z' \) and on the magnetic properties and on the anomalous electric charge radius of muon-neutrinos.
We would like to express our gratitude and appreciation to our numerous technical collaborators. The successful realization of the detector was only possible thanks to their skill and dedication. In particular, we wish to thank W. Albrecht and his group for technical help; G. Basti, P. Cesaroni, R. Donnet, M. Ferrat, B. Friend, V. Gemanov, S. Guerra, E. Gygi, M. Jimenez, A. King, G. Lunadei, Y. Perrin, G. Petrucci, G. Pozzo, F. Schneider, J. Schütt, L. Sokolov, J.C. Tarlé, A. Tusi, P. Veneroni and H. Verweij; the SPS staff for the operation of the accelerator; and P. Lazeyras and his group for operating the horn-focused beams. J. Audier and M. Busi had helped with the data handling. We wish to express our gratitude to J. Aspiazu, R. Biancastelli, V. Blobel, A.N. Diddens, W. Kozanecki, E. Lilletun, K. H. Mess, E. Metz, J. Meyer, R.S. Orr, P. Pistilli, K. H. Ranitzsch, P. Schütt, and A.M. Wetherell for their contributions to this experiment.
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Table 1
Composition of the sample of $\nu e$ candidates for the data of the second period

<table>
<thead>
<tr>
<th>Process</th>
<th>Neutrino</th>
<th>Antineutrino</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu e \to \nu e$</td>
<td>$44 \pm 9 \pm 4$</td>
<td>$4 \pm 10 \pm 4$</td>
</tr>
<tr>
<td>$\nu_\mu A \to \nu_\mu A\pi^0$</td>
<td>$198 \pm 26 \pm 30$</td>
<td>$191 \pm 25 \pm 30$</td>
</tr>
<tr>
<td>$\nu_\mu N \to X_{\text{elm}}$</td>
<td>$47 \pm 35 \pm 20$</td>
<td>$26 \pm 19 \pm 20$</td>
</tr>
<tr>
<td>$\nu_e N \to cN'(\pi)$</td>
<td>$51 \pm 16 \pm 20$</td>
<td>$116 \pm 14 \pm 30$</td>
</tr>
</tbody>
</table>

Table 2
Measurements of $\sigma(\nu_\mu e)$, $\sigma(\bar{\nu}_\mu e)$, and $\sin^2 \theta_W$ derived from R

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\sigma(\nu_\mu e)$</th>
<th>$\sigma(\bar{\nu}_\mu e)$</th>
<th>$\sin^2 \theta_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GGM-PS [45]</td>
<td>$&lt; 3.0$</td>
<td>+1.3</td>
<td></td>
</tr>
<tr>
<td>AC-PD [46]</td>
<td>$1.1 \pm 0.6$</td>
<td>1.0</td>
<td>$0.35 \pm 0.08$</td>
</tr>
<tr>
<td></td>
<td>+1.2</td>
<td>-0.6</td>
<td></td>
</tr>
<tr>
<td>GGM-SPS [47]</td>
<td>2.4</td>
<td>$&lt; 2.7$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BNL-COL [48]</td>
<td>1.6 \pm 0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FMMS [49]</td>
<td>$&lt; 2.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BEBC-TST [50]</td>
<td>$&lt; 3.4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VMWOP [51]</td>
<td>1.4 \pm 0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBKOPS [52]</td>
<td>1.85 \pm 0.4</td>
<td>1.16 \pm 0.3</td>
<td>$0.209 \pm 0.03$</td>
</tr>
<tr>
<td>This experiment</td>
<td>2.2 \pm 0.6</td>
<td>1.6 \pm 0.5</td>
<td>$0.211 \pm 0.04$</td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1 Dependence of $\sigma(\nu_{\mu}e)$ and $\sigma(\bar{\nu}_{\mu}e)$ on $\sin^2 \theta_W$.

Fig. 2 Ratio of neutrino-electron to antineutrino-electron scattering cross-sections as a function of $\sin^2 \theta_W$. The dashed curve represents the expectation in the case of full energy acceptance and pure muon-neutrino and muon-antineutrino beams. The full curve represents the expectation for events in the electron energy range 4-30 GeV and takes into account the small contaminations of wrong-sign muon-neutrinos, electron-neutrinos, and electron-antineutrinos in the beams. The value of $R'$ measured in the second exposure and its statistical error are shown together with the corresponding values of $\sin^2 \theta_W$.

Fig. 3 Layout of the wide band neutrino beam.

Fig. 4 Layout of the detector.

Fig. 5 Typical displays of an $e$- and a $\mu$-induced shower. The dots represent the scintillator and the proportional tube hits.

Fig. 6 Distributions of the width of 20 GeV electron and pion showers: a) as measured by the scintillators and b) as measured by the streamer tubes. The width parameters $\Gamma$ and $\sigma_{ST}$ are discussed in the text.

Fig. 7 Normalized shower-length distributions for electrons and pions of 20 GeV. The parameter $L_{\text{shower}}$ is discussed in the text.

Fig. 8 Hit distribution in the first proportional drift-tube plane ($M_1^d$) of showers induced by 20 GeV electrons.

Fig. 9 Measured distributions of the energy deposition in the first scintillator plane following the shower vertex $E_l$: a) shower induced by 15 GeV electrons traversing, on the average, half a marble slab, b) showers produced by neutrino and antineutrino beams in an energy-angle range where photon-induced showers due to coherent processes dominate ($7.5 < E < 17.5$ GeV, $E^2 \theta^2 > 0.54 \text{ GeV}^2$). The contamination due to electron-induced showers is estimated to be 15%.
Fig. 10  Average resolution of the projected angles of electron showers as a function of the electron energy. The streamer tubes were installed for the second exposure.

Fig. 11  Distribution of transverse momentum squared of quasi-elastic events in (a) the neutrino and (b) antineutrino beam. The Monte Carlo expectations are shown.

Fig. 12  Distribution of energy deposited in the first six scintillator planes of quasi-elastic events in (a) the neutrino and in (b) the antineutrino beam. The Monte Carlo expectations are shown.

Fig. 13  Neutrino energy spectra.

Fig. 14  Characteristic distributions of events induced by the reaction $\nu_\mu e \rightarrow \nu_\mu e$. Fig. (a) shows the shower angle ($\theta$) versus the shower energy (E). The event distributions are shown in (b) as a function of E, in (c) as a function of $\theta$, in (d) as a function of $E^2\theta^2$, and in (e) as a function of $E_T$.

Fig. 15  Characteristic distributions of events induced by the reaction $\nu_\mu A \rightarrow \nu_\mu A\pi^0$. Fig. (a) shows the shower angle ($\theta$) versus the shower energy (E). The event distributions are shown in (b) as a function of E, in (c) as a function of $\theta$, in (d) as a function of $E^2\theta^2$, and in (e) as a function of $E_T$.

Fig. 16  Characteristic distributions of events induced by the reaction $\nu_\mu N \rightarrow \nu_\mu X$ with a large fraction of electromagnetic energy in the shower. Fig. (a) shows the shower angle ($\theta$) versus the shower energy (E). The event distributions are shown in (b) as a function of E, in (c) as a function of $\theta$, in (d) as a function of $E^2\theta^2$, and in (e) as a function of $E_T$.

Fig. 17  Characteristic distributions of events induced by the reaction $\nu_e N \rightarrow eN'(n)$. Fig. (a) shows the shower angle ($\theta$) versus the shower energy (E). The event distributions are shown in (b) as a function of E, in (c) as a function of $\theta$, in (d) as a function of $E^2\theta^2$, and in (e) as a function of $E_T$.

Fig. 18  Distribution of the candidate events from the second exposure as a function of $E^2\theta^2$, a) in the neutrino, b) in the antineutrino beam. The data are compared with the Monte Carlo expectations normalized to the total number of events.
Fig. 19 Distribution of the candidate events from the first exposure as a function of \( E^2 \eta^2 \), a) in the neutrino, b) in the antineutrino beam. The backgrounds of quasi-elastic (CC) \( \nu_e \) (\( \bar{\nu}_e \)) events and neutral-current (NC) \( \nu_\mu \) (\( \bar{\nu}_\mu \)) events are shown in the figures and discussed in the text.

Fig. 20 Distribution of (a) neutrino and (b) antineutrino events collected in the first and in the second exposure. The data from the second exposure are shown using a horizontal scale expanded by a factor of 2 in order to take into account the different angular resolution. The same scale correction has been applied in the computation of the two backgrounds. Figures (c) and (d) show the analogous distributions for the events satisfying the additional condition that the energy deposited by the shower in the first scintillator plane be \( E_f < 8 \text{ MeV} \). In this case the background is due only to \( \nu_e \) (\( \bar{\nu}_e \)) quasi-elastic scattering.

Fig. 21 Neutral-current coupling constants \( g^e_A \) and \( g^e_V \) of electrons as determined by this experiment and by \( \bar{\nu}_e e \) scattering [43] and \( e^+ e^- \rightarrow \mu^+ \mu^- \) [53].

Fig. 22 Lower limits (95% CL) on the coupling constant \( \lambda \) for excited electrons as a function of \( m_e^* \). A value of \( \Lambda = 1000 \text{ GeV} \) was assumed. This experiment gives a limit of \( m_e^* > 620 \text{ GeV} \) for \( \lambda = 1 \).

Fig. 23 Distribution of recoil-less events with a single muon as a function of \( p_T^2 \). Muon-neutrino and antineutrino distributions have been normalized in the range \( 0.02 < p_T^2 < 0.10 \text{ GeV}^2 \). The excess of muon-neutrino induced events for \( p_T^2 < 0.02 \text{ GeV}^2 \) is due to inverse muon decay.
Fig. 1
Fig. 2
Fig. 3
Fig. 5

Fig. 6
Fig. 9
Fig. 10

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This is a graph showing the relationship between $\sigma(\theta)$ (in mrad) and $E$ (in GeV). The graph compares the effects of using or not using streamer tubes; circles represent data without streamer tubes, and diamonds represent data with streamer tubes. The data points show a general decrease in $\sigma(\theta)$ with increasing energy $E$. The y-axis represents $\sigma(\theta)$ in milliradians (mrad), while the x-axis represents energy $E$ inGeV.
Fig. 11

- Data
- MC
- MC quasi-elastic
Fig. 12
Fig. 13
Fig. 14
$\nu_\mu A \rightarrow \nu_\mu A \pi^0$

Fig. 15
\( \nu_\mu N \rightarrow \nu_\mu X \)

(a)

(b)

(c)

(d)

(e)

Fig. 16
\[ \nu_e N \rightarrow eN'(\pi) \]

(a)

\[ \theta \text{ (rad)} \]

\[ E \text{ (GeV)} \]

(b)

\[ \text{NUMBER OF EVENTS} \]

\[ E \text{ (GeV)} \]

\[ 4 \rightarrow 30 \]

(c)

\[ \theta \text{ (rad)} \]

\[ 0 \rightarrow 0.2 \]

(d)

\[ E^2 \theta^2 \text{ GeV}^2 \]

\[ 0 \rightarrow 0.54 \]

(e)

\[ E_f \text{ (MeV)} \]

\[ 0 \rightarrow 48 \]

Fig. 17
Fig. 20
Fig. 23