REVIEW OF NEW EXPERIMENTAL RESULTS ON
SOFT HADRONIC PHYSICS

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ABSTRACT

Recent data from a few reports on soft-hadronic (and some leptonic) interactions are presented. The topics discussed are: multiplicity distributions and fits to the negative binomial distribution; transverse momentum; particle correlations in rapidity and the transverse-momentum plane; as well as Bose–Einstein correlations. The data are compared with models, mainly FRITIOF, PYTHIA and DPM. Finally, the UA1 and UA2 minimum-bias data on transverse energy prompts the question, Where is the limit of ‘soft physics’?

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1. Introduction

This review of recent low-\(p_T\) experimental data was inspired by the excellent dinners the author (most correctly) anticipated having at the conference. Composing a review talk is like composing a dinner menu. Thus the menu put together for this talk consists of
- Consommé de Multiplicité avec œuf Negbin
- Momentum Salade Transverse
- Steak à la Corrélation de la \(p_T\) et y
- Mousse Bose et Einstein
- Fromage de UA1 \(E_T\) UA2
- Café avec biscuit Summary.

The intention is to give a pleasant mixture of some of the subjects in this large field of low-\(p_T\) physics, subjects where new data have recently appeared. Furthermore, the aim is to put the data in a larger context and compare it with models. The models that are used are FRITIOF [1], PYTHIA [2], and the Dual Parton Model (DPM) [3-8]. These are currently the models that give the best general explanations of the hadron-hadron physics discussed here, although it should be noted that PYTHIA is intended only for collision energies from about 100 GeV and above. To some extent a phenomenological cluster model based on a longitudinal phase space, GENCL [9], is also used.

2. Multiplicity distributions

Charged-particle multiplicity distributions have received a lot of attention during the last few years, in particular in connection with the negative binomial distribution (NBD),

\[
P_n(n, k) = \binom{n + k - 1}{k - 1} \left( \frac{n}{n+k} \right)^n \frac{1}{(1 + \frac{n}{k})^k}.
\]

In 1985 it was found by the UA5 Collaboration that multiplicity distributions in full phase space in pp collisions at 546 GeV and in pp collisions at c.m. energies between 11.5 and 62 GeV [10], as well as multiplicity distributions in limited symmetric intervals in pseudorapidity [11], could all be very well described by the NBD. Since then, many groups have performed similar analyses in different reactions and using various cuts [12-25]. An overview is given in Table 1.

The vast majority of these fits are good (at the available statistics!), except for a few (denoted with a 'x' in the table) that are mostly in a limited region of \(\eta\) or \(y\) away from the central region. Several explanations for the NBD have been proposed [26-32]. The most consistent seems to be the 'clan picture', developed mainly by Giovannini and Van Hove [27, 33]. However, this picture is not completely free from problems, e.g. it breaks down when the parameter \(1/k\) is less than 0 (which happens in some cases [14, 15, 21, 22], and when studying the growth of the clan size with rapidity, one finds inconsistencies [34].

The UA5 Collaboration has now finished the analysis of its pp 200 and 900 GeV multiplicity data [25]. The numbers of events (4156 and 6839, resp.) are five to six times larger than the ones used in an early publication on full phase-space distributions [18], where the

*) All three models exist in Monte Carlo forms. However, whereas the first two are well specified, there are several different DPM Monte Carlo programs. Thus, in this review we will refer to comparisons with the DPM from three different experiments (NA22, NA23, UA5), each of which uses a different program.
Table 1

Multiplicity data to which the NBD has been fitted [10-25]. The analysis for which all fits were successful are marked with a \( \checkmark \); for those marked \( \times \), some of the fits were not acceptable. In addition, the NBD has been successfully fitted to published, full phase-space data at various energies between 11.5 and 62.6 GeV [10].

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<th>Collaboration Reaction</th>
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NBD was found to fit the data well. In the new paper [25], distributions are presented for the true number of charged particles produced. To obtain these, a new method was used to correct for acceptance limitations and inefficiencies [25, 35, 36]. It was realized that it is not possible to correct the observed multiplicity samples in order to obtain the sample of the true distribution. Instead, the new method uses the maximum entropy principle with linear constraints [37] to find a smooth distribution which represents the data well. The constraints used are that some of the observed (algebraic) moments shall be exactly reproduced by the true distribution. In this way, UA5 eliminates the unphysical waves which could be seen in the 'corrected' 546 GeV distributions [11, 19, 38]. The true distributions obtained at 200 and 900 GeV for full phase space are shown in Fig. 1 [25], plotted in the 'KNO form', with \( \langle n \rangle \)P(n) as a function of \( z = n/\langle n \rangle \). It can be seen that not only does the tail increase with energy, showing KNO scaling [39] violation (already noted at 546 GeV [38]), but also that at 900 GeV the peak looks different than at 200 GeV. At 900 GeV the peak is higher, more narrow, and at a lower z-value.

This 'peakier' distribution at 900 GeV cannot be well described with the NBD (Fig. 2). A \( \chi^2/\text{DF} \) value of 160/73 is obtained when fitting the NBD to the 900 GeV data, corresponding

![Fig. 1: The distributions of charged particles in full phase space for inelastic, non-single diffractive \( pp \) events at 200 and 900 GeV, plotted in 'KNO' form. Note that the errors are strongly correlated [25].](image1)

![Fig. 2: The same distributions as in Fig. 1, compared with the best fits of the negative binomial distribution [25].](image2)
to a probability of about $10^{-8}$ [25]. The fit to the observed data is done through the acceptance matrix and trigger efficiencies. It is the regions of the observed multiplicity of 12–15 and 26–36, corresponding to true multiplicities around 20 and 45, respectively, which give the main contribution to $\chi^2$. The tail of the distribution, on the other hand, is well described by the NBD. At 200 GeV, the distribution is very well fitted by the NBD, giving $\chi^2/DF = 50/43$, corresponding to a probability of 21%.

The fits to the NBD at 900 GeV get better when limiting the phase space with a symmetric cut in pseudorapidity: $|\eta| < \eta_c$. For $\eta_c \approx 2.5$, the fits are acceptable (with the convention of UA5 [25] that accepts probabilities larger than 0.1%).

There is a more general formula than that of the NBD, called the partially coherent laser distribution (PCLD) (or general Glauber-Lachs, or Laguerre), which, it has been suggested [40], is the appropriate distribution for multiplicities, motivated by quantum statistical arguments:

$$ P_{ul}(N,S,k) = \frac{(N/k)^p}{(1 + N/k)^{p+k}} \exp\left(-\frac{S}{1 + N/k}\right) L_n^{k-1} \left(\frac{kS/N}{1 + N/k}\right). \quad (2) $$

Here $N$ and $S$ are the average number of particles from a chaotic and a coherent part, respectively, contributing to the particle production, and $L_n^k$ denotes the $n^k$ Laguerre polynomial of order $k$. In the limits $S \to 0$ and $N/k \to 0$, the PCLD tends towards an NBD and a Poisson distribution, respectively. The PCLD has also been fitted to the UA5 data [25, 41], but cannot describe the 900 GeV data better than the NBD can. In fact, in general one obtains an $S$-value which is consistent with 0, i.e. the NBD limit.

The multiplicity distributions at 200 and 900 GeV have been compared [25] with FRITIOF [1] and PYTHIA [2], and with a tuned ABR version [7, 42] of the DPM [3–8] (Fig. 3). However, although the overall features are not too bad, a $\chi^2$ test shows that in general the descriptions of the distributions are far worse than the NBD one. FRITIOF and another version [43] of the DPM have also been compared with NA22 data from 22 GeV $\pi^0 \cdot p$ collisions (Fig. 4) [15]. At 22 GeV, both models are below the data in the tail, in contrast to the

**Fig. 3:** Predictions of 'observed' distributions from three models compared with the data at 900 GeV in limited intervals of $|\eta|$. The comparison at 200 GeV looks very similar [25].

**Fig. 4:** Multiplicity distributions for $\pi^0 \cdot p$ data for full phase space and $|\eta| < 1$. The lines are predictions from the one-string Lund model (solid), FRITIOF (dashed), and DPM (dash-dotted) [15].
Collider energies where they tend to be above the data. It may be that the two models have too strong an increase of the multiplicity tail with energy.

One can speculate about the reason for the 'peaky' appearance of the 900 GeV distribution. It could, for example, be due to a two-component production mechanism. Several two-component models have been proposed, e.g., based on quark- and gluon-initiated QCD bremsstrahlung processes [44], or on valence quarks and gluon-gluon interactions [45], or on pionization and fragmentation [46], or on a soft component plus a component of a convoluted soft and hard interaction [47]. However, in two-component models it is often assumed that one or both of the components KNO-scale, and it is very doubtful if the change of shape between 200 and 900 GeV can be explained with such assumptions [44, 45, 47]. It seems that one would need one component around \( z = 0.5-1 \) that gets narrower with energy, and one high-multiplicity component that gets broader with energy. In fact, the model of Ref. [46] has such features. It also predicts a small bump with a change of slope around \( z = 1.8 \) at 1 TeV, qualitatively similar to the 900 GeV data.

It has been pointed out during this conference [48] that it was predicted [6, 49], within the DPM, that the multiplicity distribution should exhibit a pronounced peak at high energies (Fig. 5). This peak comes from the one-Pomeron exchange, giving two quark-gluon chains, each with a multiplicity distribution which is close to a Poisson distribution. A shoulder is then expected for the two-Pomeron exchange, and the tail is due to multi-Pomeron exchanges, i.e. one actually has a kind of multicomponent model.

3. Transverse momentum

The first results from the Fermilab Tevarron have now been published [50], showing the average \( p_T \) as a function of the multiplicity for proton-antiproton collisions at \( \sqrt{s} = 1.8 \) TeV.
Fig. 6: $\langle p_T \rangle$ versus $N_c$ for uncorrected positive and negative tracks separately, and for corrected negative tracks, at 1.8 TeV [50]. The UA1 data are from 546 GeV [51].

(Fig. 6). The E735 Collaboration measured the transverse momentum for charged particles in the region $-0.36 < \eta < 1.0$. A cut was made for $p_T < 0.15$ GeV/c, and then corrected for by extrapolating to $p_T = 0$ by fitting $dN/dp_T \propto \exp(-b p_T)$ in $0.15 < p_T < 0.50$ GeV/c. Figure 6 shows both the uncorrected and the corrected distributions. The variable $N_c$ is the true (i.e., corrected) charged-particle multiplicity in $|\eta| < 3.25$, with a quoted uncertainty of $\pm 10\%$. The $\langle p_T \rangle$ increases strongly with $N_c$ up to $N_c = 50$, where it flattens out. For $N_c$ larger than about 180, there is a weak hint of a new strong increase, which could signal a quark-gluon plasma. Indeed, Experiment E735 is designed to search for this plasma, but claims nothing with the present statistics. For comparison, the UA1 data at 546 GeV [51] are also shown in Fig. 6, but note that these are for the region $|y| < 2.5$. (The energy dependence of $\langle p_T \rangle$ versus multiplicity is discussed in Ref. [52].)

The total $\langle p_T \rangle$ found by the E735 Collaboration at 1.8 TeV is $\langle p_T \rangle = 0.46 \pm 0.01$ GeV/c, where the error is dominated by systematics. Note also the large particle density that was measured: up to $\langle dN_c/dp_T \rangle = 32$.

A multiplicity dependence of $\langle p_T \rangle$ for kaons has also been seen in $p\bar{p}$ collisions at 900 GeV [53]. Figure 7 shows $\langle p_T \rangle$ (in $|y| < 2.5$) versus the number of observed charged particles in the UA5 detector at collision energies of 200 and 900 GeV. The statistics is rather poor, but a clear rising trend is seen at 900 GeV, whereas not much can be seen at 200 GeV. A similar but weaker trend, as the 900 GeV kaon data show, was previously seen at 546 GeV [54]. The full line in Fig. 7 represents the UA1 data for all charged particles [51].

In Fig. 8 [53] the $p_T$ distributions for kaons in $p\bar{p}$ collisions at 200, 546, and 900 GeV are compared with the same models as in Fig. 3, i.e., FRITIOF [1], PYTHIA [2], and a DPM [7, 42]. All three models agree fairly well with the data, although PYTHIA is a bit low in the tail at 900 GeV. This also shows up in the $\langle p_T \rangle$, where FRITIOF and the DPM do well, but PYTHIA is somewhat low at 546 and 900 GeV [53]. However, in FRITIOF, $\langle p_T \rangle$ increases too quickly with energy.

The energy dependence for $\langle p_T \rangle$ for various particle species is shown in Fig. 9. One can see the new points at 1.8 TeV from E735 [50] and CDF (prelim.) [55], and the kaon points at 200 and 900 GeV [53]. The $\Lambda$ and $\Xi$ points at 200 and 900 GeV are preliminary results from UA5 [56]. The lines are drawn to guide the eye. It seems that $\langle p_T \rangle$ is increasing faster with $\sqrt{s}$
above the ISR energies than at ISR and below. (The ISR data and below are from the compilation of [57].)

4. Correlations in rapidity and transverse momentum

The NA23 Collaboration has studied two-particle correlations in pp reactions at $\sqrt{s} = 26$ GeV [58]. They looked at correlations, both in rapidity and in the transverse momentum plane, and compared the data with models. In the $p_T$ plane they define the asymmetry $B$ of the distribution $W(\phi)$ of the angle $\phi$ between the $p_T$ of the two particles:

$$B = \frac{\int_0^{\pi/2} W(\phi) \, d\phi - \int_{\pi/2}^{\pi} W(\phi) \, d\phi}{\int_0^{\pi/2} W(\phi) \, d\phi + \int_{\pi/2}^{\pi} W(\phi) \, d\phi}.$$

Conservation of $p_T$ leads to a value $B = 0.06$, independent of the rapidity gap $\Delta y$ between the particles. Figure 10 shows data of $B$ as a function of $\Delta y$, together with several models. Note, first, that the simple one-string Lund model [59] and the two-chain DPM strongly overestimate the correlation. This overestimate is considerably reduced — but not enough — when the two-string FRITIOF model (which also includes QCD effects) or the multichain DPM is used. In contrast, the cluster Monte Carlo GENCL*) [9] reproduces the data better than the

*) GENCL is tuned to low-$p_T$ data in the energy region 200 to 900 GeV, and, in order to describe correlation data well, it was necessary to introduce clusters with an average number of charged particles of about two, roughly Poissonian-distributed.
other models do. It is a general conclusion of NA23 that the one-string Lund and two-chain DPM overestimate correlations in $p_T$ and underestimate them in rapidity. The discrepancies are reduced with FRITIOF and with the multichain DPM, but not sufficiently (Fig. 11), whilst GENCL shows agreement with the data.

It is somewhat strange to see that, at 26 GeV, FRITIOF gives two-particle rapidity correlations that are too small, whereas at 900 GeV the corresponding pseudorapidity correlations are well described (Fig. 12) [41, 60]. Is this due to the difference in energy? or is it
due to a change of version in FRITIOF? — at 26 GeV, version 3.1 was used, whereas version 2 was used at 900 GeV — or is the explanation something else [61]?

Preliminary pp data at 31, 44, and 62 GeV from the ABCDHW Collaboration [17] show that there are genuine three-particle correlations in rapidity (the observed two-particle correlations are subtracted), if the three particles do not have the same charge. When the three particles have the same charge, no correlation signal is seen. A possible explanation for this could be that clusters exist with three charged particles, but few (if any) where the three charges are the same.

5. Bose–Einstein correlations

5.1 Introduction

When studying the normalized two-particle distribution $R(q)$ of identical bosons, as a function of their momentum difference $q$, one can find an increase at small $q$. This increase is commonly interpreted as interference phenomena by nearby boson emitters, and hence called Bose–Einstein correlations (or the GGLP effect, from the first paper [62] suggesting this explanation, after its original discovery in 1959 [63]). If certain assumptions are made regarding the boson source, it is possible to deduce the size of it from fits to the measured $R(q)$, because $R(q) - 1$ is proportional to the Fourier transform of the source distribution. Two assumptions that are common in the literature are: i) particles are emitted from independent sources with lifetime $\tau$ distributed on a spherical surface of radius $r_K$ [64], and ii) the source has a Gaussian shape, in the c.m.s. of the pair, of size $r_G$ [62, 63]. In the first case, $R$ gets the (Kopylov–Podgoretskii) form [64]

$$R(q_T, q_0) = 1 + \lambda_K \left[ \frac{2J_1(q_T r_K)}{q_T r_K} \right]^2 \frac{1}{1 + (q_0 \tau)^2},$$

(4)

where $q_0 = |E_1 - E_2|$; $q_T$ is the component of $p_1 - p_2$ transverse to $p_1 + p_2$ [$p_i = (E_i, p_i)$ is the four-momentum of particle $i$]; and $J_1$ is the first-order Bessel function. The parameters to fit are $r_K$, $\tau$, and $\lambda_K$ (see below). In the second case, $R$ has the (Goldhaber) form [62, 63]

$$R(Q^2) = 1 + \lambda_G \exp \left(-r_G Q^2\right),$$

(5)

where $Q^2 = -(p_1 - p_2)^2 = M^2 - 4m^2$ ($M$ is the invariant mass of the pair), and one fits $r_G$ and $\lambda_G$. This second form is Lorentz-invariant. The strength parameter (or chaoticity) $\lambda_K$ or $\lambda_G$ should, for a completely chaotic source, have its maximum value of 1, but several effects can cause it to decrease, e.g. resonance production, local charge conservation, Coulomb repulsion, and final-state rescattering. In practice, one usually fits to some approximation of Eq. (4) or Eq. (5), e.g. by including an overall normalization constant, or by allowing for a slow variation of $R$ with $q$ at large $q$, or by decomposing $Q^2$ in Eq. (5). Fits to forms other than those of Eqs. (4) and (5) have also been done [65]. In general, charged pions have been studied, but also kaons have been looked at [66, 67]. For a recent compilation of references to BE experiments, see for example, Ref. [68].

In this section I will try to summarize the ‘trends’ in BE correlation data. However, one must be careful when comparing results from different experiments, and bear in mind that the methods of analyses often differ. Different experiments correct for different things; these can be due to detector inefficiencies (e.g. misidentification of particles [69–71]) or to physics (e.g. resonances [72]). (See Ref. [69] for discussions on corrections.) Various methods of obtaining unbiased reference samples are used (see Ref. [68] for details), and there are several ways of parametrizing $R(q)$ and making the fits.
5.2 Energy and reaction dependence

No significant dependence on $r$ or $\lambda$ on the collision energy has been seen. In Fig. 13 a compilation of data on $r_K$ for hadron–hadron collisions is shown [68, 73]. The $\lambda_K$ values found are in the range 0.3–0.5. In Fig. 14 the dependence on different reactions is sketched (error bars are drawn for the $K^0$ data points only). It can be seen that $r_G \approx 0.6$–1.0 fm, with no clear dependence on the reaction. (In heavy-ion reactions $A + A$, however, it has been found that $r_G \approx 1.0 A^{1/3}$ fm [74].) A comparison of Figs. 13 and 14a shows that $r_K \approx 2r_G$, which is expected from the exponential approximation of Eq. (4). From Fig. 14b it seems that $\lambda_G$ is somewhat smaller for hadron–hadron data than for the lepton data (apart from the $K^0$ point). Note also the large difference in the CLEO data ($e^+ e^-$ around $\sqrt{s} = 10.5$ GeV) between the uncorrected $\lambda_G$ value and the value when corrected for long-lived resonances [72].

A recent study from MARK II [69] supports the conclusion of CLEO [72]: that when correcting for long-lived resonances in $e^+ e^-$ collisions, $\lambda$ is consistent with being 1. MARK II has studied four different samples—the $J/\psi$ region at 3.1 GeV; two-photon interactions giving a hadronic energy of about 5 GeV; and annihilation in the regions $\sqrt{s} = 4–7$ GeV and $\sqrt{s} = 29$ GeV—and has corrected for Coulomb effects and pion misidentifications, obtaining the $\lambda_G$ values shown in Fig. 15 ($r_G = 0.8$ fm in all cases). Furthermore, with a Monte Carlo

Fig. 13: Momentum dependence of $r_K$ ($= R$ here) in hadron–hadron interactions [68, 73].

Fig. 14: Reaction dependence of $r_G$ and $\lambda_G$ [65, 68–72, 77, 79–81]. Note that the $e^+ e^-$ data are from various types of data sets.

Fig. 15: The strength $\lambda_G$ for the four sets of data of MARK II, arranged roughly in the order of increasing energy available for hadron production. The two sets of values show results with different reference samples. The errors indicated are statistical (inner bars) and systematic [69].
study they show that the decrease of \( \lambda_0 \) from 1 in the two annihilation samples could well be due to the presence of charmed and bottom mesons, the latter being present in the higher-energy sample only. The radius \( r_0 \) is essentially unaffected by these mesons. Finally, MARK I1 has studied \( \lambda_0 \) and \( r_0 \) for three bins of \( p_{T\text{max}} \) in the two-photon sample (where \( p_{T\text{max}} \) is the largest pion \( p_T \) in the event). For low \( p_{T\text{max}} \), \( \lambda_0 = 1 \), and it decreases to about 0.8 for the highest bin (\( \approx 0.8 \text{ GeV}/c \)) \( r_0 \approx 0.8 \text{ fm} \) in all three bins). This can also be explained with charmed production, which is expected when \( p_{T\text{max}} \) is large.

5.3 Multiplicity dependence

In \( pp \), \( pp \), and \( \alpha \alpha \) collisions at the ISR, an increase of \( r_0 \) with multiplicity has been seen [66, 75–77]. Such a dependence has not been observed in hadron-hadron collisions at lower energies [68, 78] or in \( e^+e^- \) collisions [70, 72]. The increase with multiplicity becomes stronger between 31 GeV and 62 GeV [77]. Preliminary data from UA1 \( (\sqrt{s} = 200–900 \text{ GeV}) \) [79] are similar to those at 62 GeV [77] and do not seem to show any energy dependence. Whereas \( r_0 \) increases with multiplicity above \( \approx 30 \text{ GeV} \), \( \lambda_0 \) decreases slowly [66, 75–77, 79]. The WA25 Collaboration reported that charged-current neutrinos (from 400 GeV protons) on deuterium events may show a small increase of \( r_0 \) with multiplicity [80].

5.4 Directional dependence of the source

In general the shape of the source is found to be compatible with spherical sources [65, 68–71, 80, 81], although there are also indications of elongated ones [69, 70]. The AFS Collaboration has found evidence of an elongation along the beam axis in minimum-bias pp and \( pp \) collisions at \( \approx 60 \text{ GeV} \) [76]. Figure 16 shows \( r_K \) as a function of cosine for the angle \( \theta \) (in c.m.s.) between the beam axis and the momentum difference between the pions (\( \Delta p \)). It is the dimension parallel to \( \Delta p \) that is probed by the correlation [82], and a clear increase of \( r_K \) is seen for directions close to the beam axis in Fig. 16. The AFS data have recently been reanalysed [83] with a method [84] that does not require a particular model of the radial source distribution. It was found that the ratio of the sources sizes transverse and parallel to the beam axis is about 0.8 [83]. The authors note, however, that care must be taken, because the result will in general depend on where, in phase space, the pions are selected, but an extrapolation to full phase space still indicates an elongation of the pion source.

![Fig. 16: The size \( r_K \) as a function of \( |\cos \theta(\Delta p)| \) [76].](image-url)
5.5 The form of $R(q)$

Experiments that have tried both the Kopylov-Podgoretskii (KP) form [Eq. (4)] and the Lorentz-invariant Goldhaber form [Eq. (5)] have usually found both forms to be compatible with data [68, 70, 81], but CLEO [72] found that the KP form does not describe their $e^+e^-$ data adequately at about 10 GeV. Recently, the AFS Collaboration showed that neither KP nor the Goldhaber form fits their data [65]. They find that a double Gaussian describes their data well (Fig. 17), with radii of 0.66 ± 0.08 fm and 2.1 ± 0.5 fm. One possible interpretation for this is short-lived resonances (such as the $\varphi$), which could enlarge the source of pion production to a few fermis, assuming that the primary source is about 0.7 fm. Alternative forms that fit the AFS data are an exponential or $R = 1 + \lambda/[1 + (\mu Q)^2]$ [65]. The interesting part of the AFS data is the increase of $R(Q)$ at small $Q$, which has been seen because of large statistics (> 100,000 events) and good momentum resolution.

5.6 Three-pion correlations

Three experiments [65, 69, 81] have measured BE correlations between three identical pions. With the present statistics, however, they can all be explained by the BE correlations for pion pairs (cf. the preliminary rapidity correlation results [17] mentioned in Section 4).

5.7 BE correlations between kaons

Cooper et al. [67] have studied correlations between $K^0$ pairs in $p\bar{p}$ annihilations at $p_{lab} = 0.76$ GeV/c. They found $r_K = 0.9 ± 0.2$ fm (with $\lambda_K$ fixed at 1), which is slightly less than the value of around 1.4 fm usually measured for pions. The AFS Collaboration compared BE correlations between charged kaons and charged pions produced at the ISR and got similar results, i.e. $r_K(KK) \approx r_K(\pi\pi)$ [66].

5.8 BE correlations in string models

Instead of assuming a spherical or a Gaussian shape of the boson source, it can be assumed that particles are produced according to the string model [85], as used in the Lund model [86]. The expected BE correlations can then be computed either by introducing amplitudes for the string break-up points (instead of probabilities) [87], or by Fourier-transforming the pion-source intensity as given by the string [88]. It has been argued that there is a close correspondence between the two methods [89], but this statement has been queried [90]. However, the two methods seem to give the same conclusion: that BE correlations in $e^+e^-$ can be explained with the string model, but only if the $\eta'/\eta$ production ratio is much less than 1 (as assumed in the Lund Monte Carlo) (Fig. 18)—otherwise the string model may have
a problem with the BE correlations. In a recent, related paper [91], the string is quantized, yielding propagation amplitudes and supporting the method of Ref. [87].

5.9 Conclusions

The experimental situation is not completely clear, although some features seem to be present: the strength parameter $\lambda$ is larger in $e^+e^-$ than in hadron–hadron collisions, and the size $r$ of the source increases, whilst in hadron–hadron collisions at energies $> 30$ GeV, $\lambda$ decreases with multiplicity. However, it is not excluded that these apparent trends could be due to experimental limitations. For example, in $e^+e^-$ it has been shown that correcting for resonances can give $\lambda$ values that are consistent with 1 [69, 72]. If the hadron–hadron data are corrected for resonances (and for other effects, e.g. Coulomb interaction), the values obtained may also be consistent with 1. Furthermore, there is an indication that none of the ‘classical’ forms [Eqs. (4) and (5)] for parametrizing $R(q)$ works at small $q$ [65], and that the source could be elongated [76], but these points need confirmation.

Whilst the experimental situation is somewhat unclear, there is also a lack of theoretical predictions. Some work has been done on $e^+e^-$ using the string model [87–89, 91], but little on hadron–hadron collisions [92]. Recently, the effect of $2\pi I = 2$ S-wave rescattering has been analysed [93]. It would be very nice to have BE correlations explicitly built into a Monte Carlo program (e.g. the Lund fragmentation) using amplitudes, although it would probably be very time-consuming. However, with such a tool and with improved experiments measuring accurately at small $q$, a better understanding of this interesting quantum mechanical phenomenon might be achieved.

6. The limit of ‘soft physics’; or rather, How far can we apply QCD?

Since this is a review on soft physics—or low-$p_T$ physics—it could be appropriate to ask the above question. I am afraid, though, that it is not possible to give a complete answer, but two recent studies by UA1 [94] and UA2 [95] give some interesting insight. Both collaborations analyse events as a function of transverse energy $E_T$: UA2 uses $E_T$ in $|\eta| < 1$, full azimuth, whereas UA1 searches for clusters with $E_T > 5$ GeV.

6.1 Global event features versus transverse energy à la UA2

The UA2 Collaboration [95] investigated the collective shape variables in $p\bar{p}$ events at 630 GeV, as a function of $E_T$ (the total transverse energy in $|\eta| < 1$), and compared observables such as sphericity, thrust, and aplanarity with predictions from a QCD jet model [96] and a low-$p_T$ model (GENCL) [9]. Figure 19 [95] gives an example where the sphericity...
distribution for four intervals in $E_T$ is compared with the models. Here, as in the other cases, one sees a region ($E_T < 25$ GeV) where soft physics dominates, and the data are well described by the low-$p_T$ model. When $E_T$ is larger than about 100 GeV, hard processes dominate, and the jet model based on perturbative QCD works very well. In between there is a transition region where it is not possible to reproduce data with any linear combination of the two models. Events are neither of the ‘low-$p_T$ type’ nor ‘QCD-type’, but exhibit intermediate features.

6.2 Local energy fluctuations à la UA1

The UA1 Collaboration has now completed a paper [94] on ‘minijets’; some of the results have been presented at various conferences [97–99]. The ordinary jet-finding algorithm [100] of UA1 is applied to minimum-bias $p\bar{p}$ events at energies between 200 and 900 GeV. The algorithm searches for clusters of calorimeter cells having a total $E_T^c > 5$ GeV in a circle $\sqrt{\Delta \eta^2 + \Delta \phi^2} = 1$, and an axis in $|\eta| < 1.5$ which has its azimuth $\phi \approx 30^\circ$ from the vertical (where the two calorimeter halves join). UA1 studies the shape of the clusters in $\eta$ and the angular distribution of jet axes for events with two clusters, and compares the production rates of observed $E_T$ clusters with predictions from MC models. For example, at 900 GeV, 17% of all events have one or more high-$E_T$ cluster [97], about one third containing a second cluster. This last fraction is in very good agreement with the QCD Monte Carlo [101] used by UA1. However, the strongest evidence for $E_T$ clusters signalling hard scattering, is that the
polar angle distribution of the two highest-E_T clusters in their c.m.s. show Rutherford scattering (Fig. 20 [94]). This is clear if the invariant mass of the two clusters is larger than about 20 GeV/c^2, which corresponds to E_T^1 \approx 7.5 GeV. But one should be aware that large cuts are imposed [99] to produce the points in Fig. 20, so the implication is not necessarily that a majority of all E_T clusters are due to hard scattering. Nevertheless, the conclusion of UA1 is that E_T^1 = 5 GeV is a reasonable limit where experimental results can be compared with those of QCD models**.

Thus, UA1 connects the E_T^1 of the clusters with the p_T of the jets. After several corrections, reasonable agreement with QCD (within a factor of about 2) for the jet transverse-momentum distribution is obtained. But, as noticed, the trend in the data indicates that d^2\sigma/dp_Td\eta_{\perp}=0, at fixed p_T, increases more quickly with energy than is predicted by leading-order QCD. The corrections are large, however, and rely greatly on Monte Carlo simulation, both low-p_T MC and QCD MC. The indicated discrepancy with QCD could, for example, be due to underestimating the low-p_T background at high energy or overestimating it at low energy. For subtracting the 'low-p_T noise', i.e. the number of E_T clusters not originating from hard scatterings, UA1 uses a Monte Carlo [103] that might be called a 'minimal minimum-bias' MC. In this MC, \langle p_T \rangle is independent of v and equal to the \langle p_T \rangle of low-multiplicity events at the Collider. The number of E_T clusters in this MC is 18% of the number in the data. This is much less than what the low-p_T MC GENCL gives**, which is about 50% of the observed value at 900 GeV [9]. Furthermore, this fraction increases between 200 and 900 GeV. Thus, if GENCL was used to correct for the low-p_T background, one would perhaps get an increase of d^2\sigma/dp_Td\eta_{\perp}=0 with energy, which agrees better with QCD. With GENCL to estimate the background of the E_T clusters that are not due to hard scattering, the cross-section for jets would also be smaller. UA1 defines \sigma_{jet} as the cross-section for producing at least one cluster with E_T > 5 GeV (as defined by the UA1 algorithm) at any rapidity (Fig. 21 [94]). The \pm 20% error shown in this figure accounts for uncertainties in the background, efficiency, and acceptance corrections [94]. However if one were to assume that

Fig. 20: Two E_T-cluster polar angular distributions for different mass thresholds [94]: m > 4.5 GeV/c^2 (a), m > 15 GeV/c^2 (b), m > 22.5 GeV/c^2 (c), and m > 30 GeV/c^2 (d). The curves are QCD predictions.

*) UA8, in single diffractive-like events at 630 GeV, has lately obtained similar results [102].

**) This is with an exponential p_T distribution for the (particle-) clusters, tuned to \langle p_T \rangle for all charged particles. The default version of GENCL contains a power-law tail of p_T for the clusters as well, and gives about 50% more E_T cluster events [9].
GENCL [with exponential $p_T$ distribution for (particle-) clusters] gives the correct amount of background, it would seem that $\sigma_{\text{jet}}$ could be even lower, roughly as indicated by the lower dashed line in Fig. 21.

6.3 Conclusions

The 'ordinary' low-$p_T$ phenomenology (as typically used in the Monte Carlo GENCL [9]) works well up to $E_T = 25$ GeV in $|\eta| < 1$ [95], or as long as there is no localized cluster in the $\eta-\phi$ space (as defined by UA1) with $E_T^\eta$ larger than about 10 GeV. UA1 argues that QCD is applicable down to $E_T^\eta = 5$ GeV, but one can question whether the region in $E_T^\eta$ of 5–10 GeV is not too much dominated by low-$p_T$ events. A cluster of 10 GeV in $\sqrt{\Delta \eta^2 + \Delta \phi^2} = 1$, or $E_T = 25$ GeV in $|\eta| < 1$, agrees well, since a cluster in one direction should be balanced with roughly the same $E_T$ in the other direction. Then, adding up in the rest of azimuth, one could well expect about 25 GeV in the full azimuth. It should also be mentioned that the models FRITIOF [1], PYTHIA [2], and DPM [8, 104] all include some hard-scattering features to fully describe data. Finally, it is clear that the results concerning hard scattering in this 'limit region' are rather sensitive to what is assumed or believed to be the correct underlying mechanisms of the low-$p_T$ phenomenology.

7. Summary

Recent data from a few topics in the rich field of soft-hadronic (and some leptonic) interactions have been presented. The main points are summarized below.

- Multiplicity distributions of many kinds are, in general, well described by the negative binomial distribution, but not at 900 GeV for regions $|\eta| < \eta_c$, when $\eta_c \geq 3.0$. These distributions show a relatively narrower peak at 900 GeV than at lower energies, but also a higher tail [25].
- Average $p_T$ increases with multiplicity at 1.8 TeV, in roughly the same way as at 546 GeV, but there is an indication of the onset of a stronger increase at very high multiplicities. For all charged particles, $\langle p_T \rangle = 0.46 \pm 0.01$ GeV/c at 1.8 TeV [50]. A multiplicity dependence of $\langle p_T \rangle$ for kaons has been observed at 900 GeV [53]. It seems that $\langle p_T \rangle$, for various particle species, increases more quickly with $\sqrt{s}$ above about 100 GeV than below it.

- Three-particle correlations in rapidity have been observed, but only when the particles do not have the same charge [17].

- Bose-Einstein correlations have been studied by many experiments in several different reactions. The size of the source is independent of the reaction, but the strength is greater in $e^+e^-$ than in hadronic reactions. The size increases with multiplicity in hadronic collisions when $\sqrt{s}$ is larger than about 30 GeV. One experiment (AFS) has found an elongation of the source [74], and that neither of the two traditional forms [Eqs. (4) and (5)] can fit the AFS data [65].

- The ‘limit’ of low-$p_T$ phenomenology in terms of $E_T$ was discussed in the light of recent UA1 [94] and UA2 [95] data. It is not clear how far down in $E_T$ QCD is applicable, but either $E_T > 25$ GeV in $|\eta| < 1$ or UA1 $E_T$ clusters having $E_T > 10$ GeV are rough limits where ‘soft physics’ is not enough to explain the data (in the energy range of the CERN pp Collider).

- The models FRITIOF [1], PYTHIA [2], and the Dual Parton Model (DPM) [3-8] are in rough agreement with multiplicity data and kaon $p_T$ data at Collider energies. FRITIOF and DPM also agree with multiplicity data at 22 GeV. However, both these models show too strong two-particle correlations in $p_T$ and too weak correlations in rapidity at 26 GeV, whilst at 900 GeV FRITIOF is in agreement with the correlation data for pseudorapidity.

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Discussion

A. CAPELLA: It turns out to be very bad to use two-jet Lund fragmentation in DPM when you
are looking at things like two-particle correlations (NA23) or multiplicity distributions at
relatively low energies (NA22). This is due to the fact that the ratio of $D^2/\langle n \rangle$ (which is a
measure of the 'clustering') in the two-jet Lund is much smaller than the experimental one
(from e$^+e^-$ or deep-inelastic $p$ scattering). In some cases it is even smaller than 1 for central
rapidity intervals, indicating clustering. When one uses, in the chain, the experimental values
of $D^2/\langle n \rangle$, one gets good agreement with data.