Polarization at LEP

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ABSTRACT

This report contains a collection of papers covering the most important part of studies carried out by five study groups in view of a programme of experiments with polarized beams at LEP, the Large Electron–Positron collider under construction at CERN. The emphasis is on precision measurements at the Z peak. Such measurements are shown to be of considerable theoretical interest as well as very clean from the point of view of theoretical and experimental uncertainties. The measurement of the beam polarization can certainly be performed with sufficient accuracy, thanks to the availability of both $e^+$ and $e^-$ beam polarization. The normalization of the data taken with different beam helicities poses certain constraints that are described. Substantial progress has been made in understanding the possibility of providing longitudinally polarized beams in the LEP machine: the design of new wigglers and spin rotators, the study of correction procedures and results of numerical simulations are presented.
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Measurements of Polarization in LEP

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Abstract: This report summarizes the work of the Polarimeter Working Group. The aim was to study the feasibility of measuring the degree of polarization to a precision where its contribution to the uncertainty on the Left–Right asymmetry, $\Lambda_{L,R}$, is small compared to the required uncertainty for this parameter. Including the systematic effects considered in this report, we conclude that the electron longitudinal polarization can be measured with a precision of $\approx 0.3\%$ in less than 20 minutes of LEP operation.

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1. Introduction

The polarimeter working group has studied the feasibility of measuring the degree of polarization in LEP to a precision required for an accurate measurement of the Left–Right asymmetry (\( \Delta A_{LR} = 0.0025 \)). The chosen method is the Compton scattering of laser light by the LEP beams, which, in the case of transverse (vertical) polarization, gives rise to an up–down asymmetry. For longitudinally polarized beams, the energy spectrum of the recoil photons changes drastically when the circular polarization state of the laser light is reversed, forming the basis of our proposal for a longitudinal polarimeter.

Chapter 2 describes the proposed scheme for measuring \( A_{LR} \) and establishes the precision with which the polarization has to be measured. The principles behind the measurement are outlined in chapter 3, followed by a chapter on the laser system. Transverse and longitudinal polarimetry are dealt with in chapters 5 and 6. Finally our conclusions are given in chapter 7.

2. The Required Precision of the Polarization Measurements

We assume that the LEP physics program with polarized electrons will use the so called "Blondel Scheme" [1], where the 4 electron and positron bunches have the following longitudinal polarization states:

<table>
<thead>
<tr>
<th>electron bunches</th>
<th>1— 2 3 4—</th>
</tr>
</thead>
<tbody>
<tr>
<td>positron bunches</td>
<td>1 2— 3 4—</td>
</tr>
<tr>
<td>cross sections</td>
<td>( \sigma_1 ) ( \sigma_2 ) ( \sigma_3 ) ( \sigma_4 )</td>
</tr>
<tr>
<td>event numbers</td>
<td>( N_1 ) ( N_2 ) ( N_3 ) ( N_4 )</td>
</tr>
</tbody>
</table>

Electron bunches 2 and 3 and positron bunches 1 and 3 are unpolarized and the other bunches are either left or righthanded longitudinally polarized as indicated by the arrows.

The four measured cross sections can be expressed via the polarization of the electron bunches (\( P_{-e} \)) and the positron bunches (\( P_{+e} \)), namely
\[
\sigma_1 = \sigma_u (1 - P^- \Lambda_{LR}) \\
\sigma_2 = \sigma_u (1 + P^+ \Lambda_{LR}) \\
\sigma_3 = \sigma_u \\
\sigma_4 = \sigma_u [1 - P^+ P^- + (P^+ - P^-) \Lambda_{LR}]
\]

This procedure yields a measurement of the Left–Right asymmetry \( \Lambda_{LR} \) at the \( Z^0 \) peak and an absolute calibration of the polarimeters. The importance of a very precise measurement of \( \Lambda_{LR} \) for the physics programme at LEP has been underlined elsewhere [2]. To achieve an accuracy of \( \Delta \Lambda_{LR} = 0.0025 \) with about \( 10^9 \) \( Z^0 \) events, the polarization measurements should reach the following precision [2,3]:

a) \( \delta P_e \approx (1 - 2) \times 10^{-2} \) in several minutes to average cross-sections taken with different polarization levels;

b) \( \delta (\Delta P_e) \approx 3 \times 10^{-3} \) where \( \Delta P_e \) is the difference in polarization between two polarized like-sign bunches;

c) \( \delta P_e \approx 5 \times 10^{-3} \) for the “unpolarized” bunches.

3. The Principles of Compton Polarimetry

3.1 Kinematics

Although the theoretical treatment of Compton scattering refers to the electron centre of mass system, from the experimental point of view the laboratory system is the relevant one. We summarise below some required transformation formulae. We define \( k_0 \) and \( k \) as the momenta of the incoming and outgoing photons in the laboratory system, \( \delta \) as the laboratory incident angle of the laser photon with respect to the electron beam direction and \( \theta_{\text{lab}} \) as the scattered photon polar angle. The quantity \( \gamma \) will as usual be defined as \( \gamma = E_e / m_e c^2 \). In the electron CM system, \( q_0 \) and \( q \) are the incoming and outgoing photon energies and \( \theta \) and \( \phi \) the scattering angles. All energies and momenta are in units of electron rest mass. With these definitions (see also Fig. 1) we have the following relations:
\[ \frac{1}{q} - \frac{1}{q_0} = 1 - \cos \theta \]
\[ y \tan \theta_{\text{LAB}} = \sin \theta / (1 - \cos \theta) \]
\[ q_0 / y k_0 \approx 1 + \cos \theta_{\text{lab}} \approx 2 \]
\[ k \approx y q (\cos \theta - 1) \]

To apply these formulae to LEP we will use throughout this paper the value of \( F_c \approx M_Z^2/2 \) (46 to 47 GeV). If not otherwise stated, we will discuss polarimetry with a circularly polarized pulsed laser, Nd:YAG (frequency doubled) with \( \lambda = 532 \text{ nm} \), equivalent to a photon energy of 2.33 eV.

The backward laboratory angle of the photon as a function of the scattered angle in the electron CMS is given in Fig. 2. The same figure also shows the corresponding backward scattered photon energy. This one to one correspondence between the angle \( \theta \) and the photon energy is further illustrated in Fig. 3.

Note that:

a) The very small backward angle range of the recoiling photons which typically is of the order of 10 to 20 \( \mu \) rads.

b) Even though no direct measurement of the scattering angle is possible, it can be measured indirectly through the scattered photon energy \( E_\gamma \) which is spread between 0 and about 29.5 GeV. For a fixed LEP energy the \( E_\gamma \) limit will increase as the laser energy increases.

c) The energy of the recoil electron in the Compton scattering is also well defined and may be measured instead (as proposed by SLC) or even in coincidence with the final state photon.

### 3.2 Cross Sections

The total Compton scattering cross section for unpolarized particles is given in the electron center of mass system by the Klein–Nishina formula:

\[ \sigma_c = 2\pi r_e^2 \left\{ \frac{(2(1 + \frac{q_o}{q_0})^2)}{(q_o^2 + 2q^2)} - \frac{(1 + \frac{q_o}{q_0})}{q_0^2} \ln(1 + 2q_o) \right\} + 0.5(\frac{1}{q_o}) \ln(1 + 2q_o) - (1 + 3q_o) / (1 + 2q_o)^2 \} \]
where $q_0$ is the incoming photon energy (in the electron centre of mass system) and $r_e$ is the classical electron radius. The dependence of this cross section on the laser energy is shown in Fig. 4, from which a cross section value of about 300 mb results for a laser of $\lambda = 530$ nm and an electron beam having an energy of $E_e = 47$ GeV.

The differential Compton scattering cross section is given by [4]:

$$
\frac{d\sigma}{d\Omega} = 0.5 \left( r_e \frac{q}{q_0} \right)^2 (\Phi_0 + \Phi_1 + \Phi_2)
$$

with

$$
\begin{align*}
\Phi_0 &= (1 + \cos^2 \theta) + (q_0 - q)(1 - \cos \theta) \\
\Phi_1 &= \xi_1 \sin^2 \theta \\
\Phi_2 &= -\xi_3 (1 - \cos \theta) (\vec{\mathbf{z}} \cdot (\vec{\mathbf{q}}_0 \cos \theta + \vec{\mathbf{q}}))
\end{align*}
$$

where $q$, $\theta$ and $\phi$ are the scattered photon momentum and angles in the electron CM system, with

$$
\zeta = (\xi_1, \xi_2, \xi_3) = \text{Electron Polarization Vector (unit normalization)}
$$

and

$$
\xi = (\xi_1, \xi_2, \xi_3) = \text{Photon Polarization Vector}
$$

where the $x$, $y$, and $z$ axes are respectively oriented horizontal, longitudinal and vertical.

Here we will consider a fully circular polarised laser beam i.e. $\xi = (0, 0, \mp 1)$.

**Case I. Transverse polarized electron beam**, i.e. $\zeta = (0, 0, \pm P)$. In this case one has for the $\Phi_i$ the following expressions:

$$
\begin{align*}
\Phi_0 &= (1 + \cos^2 \theta) + (q_0 - q)(1 - \cos \theta) \\
\Phi_1 &= 0 \\
\Phi_2 &= \mp P (1 - \cos \theta) q \cos \phi \sin \theta
\end{align*}
$$

which implies a $\phi$ dependence, the magnitude of which is proportional to the electron polarization $P$.

From this it follows that for a given angle $\theta$ the maximum up–down asymmetry $\Lambda(\theta)$ is:

$$
\Lambda(\theta) = \frac{d\sigma / d\Omega (\phi = 0^\circ) - d\sigma / d\Omega (\phi = 180^\circ)}{d\sigma / d\Omega (\phi = 0^\circ) + d\sigma / d\Omega (\phi = 180^\circ)}
$$

which for transverse polarized electrons case reduces to:

$$
\Lambda(\theta) = \frac{\Phi_2 (\phi = 0^\circ)}{\Phi_0}
$$

This asymmetry property has been used in the past and will also be utilized in IEP to measure the degree of transverse polarization of the electron and positron bunches.
Case II. longitudinal polarized electron beam, i.e., \( \zeta = (0, \mp P, 0) \). In this case one obtains for the \( \Phi_1 \)

\[
\Phi_0 = (1 + \cos^2 \theta) + (q_0 - q)(1 - \cos \theta)
\]

\( \Phi_1 = 0 \)

\( \Phi_2 = \mp P (1 - \cos \theta) (q_0 + q) \cos \theta \)

which means that no \( \phi \) asymmetry is present and \( \Lambda(\theta) \equiv 0 \). However, a difference in the scattering cross section exists between left-hand and right-hand circularly polarized photon beams, which can be expressed either as a function of \( \theta \) or as a function of \( P_y (= k) \) as shown in Figs. 5 and 6. The maximum difference between the two cross sections occurs at the CMS backward direction corresponding to the highest energy value, \( P_y \), of the scattered photons. One can therefore define for the longitudinal polarization case, an asymmetry quantity \( \Lambda_{lo} \) which is equal to

\[
\Lambda_{lo} (\theta) = \frac{(d\sigma/d\Omega (P_y = -1) - d\sigma/d\Omega (P_y = +1))}{(d\sigma/d\Omega (P_y = -1) + d\sigma/d\Omega (P_y = +1))}
\]

which is shown in Fig. 7 as a function of \( \cos \theta \) where \( P_y = +1 \) and \( P_y = -1 \) corresponding respectively to a righthand and lefthand circularly polarized photon beams.

4. The Laser System

The choice of the multi–photon technique [5], to be discussed in chapter 5 and section 6.3.2, implies the use of high peak power, low repetition rate lasers.

4.1 Choosing the Laser

For a given machine energy \( E_c \), the analysing power of a polarimeter depends on the laser wavelength. For longitudinal polarimetry shorter wavelength lasers have better analysing power, as shown in Fig. 8 where \( \Lambda_{lo} (\cos \theta = -1) \) is plotted against \( \lambda_{\text{laser}} \). Furthermore, a shorter wavelength laser will also increase the maximum energy available to the recoil photon, which may be advantageous in avoiding some background sources. At the same time, due to energy conservation, the recoil electrons will have lower energy which could facilitate their extraction from the collider ring for analysis.

For transverse polarimetry for \( \approx 50 \) GeV beams, lasers operating in the visible region are optimal. A Nd–laser operating on the fundamental frequency (\( \lambda_{\text{laser}} = 1060 \) nm) or an Excimer laser (XeCl,
308 nm) would produce a slightly reduced transverse asymmetry while a CO₂ laser (10600 nm) would give a much smaller one.

Excimer lasers with 20 MW peak power at \( \lambda_{\text{laser}} = 308 \) nm up to 250 Hz repetition rate are at present commercially available. The higher repetition rate would certainly constitute an advantage provided the associated average power can be transmitted through the optical system. Drawbacks are cost, a more complex structure including toxic gas manipulation (XeCl, KrCl or KrF) and the need of quartz lenses due to the shorter wavelength.

Nd:YAG lasers with 50 \( \pm \) 100 MW peak power at 532 and 1060 nm are available; their repetition rate is in the range of 10 \( \pm \) 50 Hz due to the solid state structure.

Both devices can produce pulses durations in the range of 5 to 8 ns. More reasonable costs and wider experience at DESY and SLAC with Nd:YAG lasers have substantial advantages and this type of laser has indeed been chosen.

4.2 The Laser pulse duration \( \tau_\gamma \)

The luminosity from two Gaussian distributions intersecting at an angle \( 2\delta_\gamma \) depends on the longitudinal rms sizes of the interacting bunches. In our case the dependence of the luminosity on the laser pulse duration has been investigated for the interaction with an electron bunch of the appropriate rms dimensions. The results are illustrated in Fig. 9, where the luminosity for crossing angles \( 2\delta_\gamma = 2, 4 \) and 6 mrad is shown as a function of the laser pulse duration. It can be seen that a \( \approx 20\% \) reduction in the relative luminosity occurs for an rms pulse duration of 3 ns when colliding at an angle of 2 mrad and it reaches \( \approx 38\% \) for \( \tau_\gamma = 5 \) ns.

The laser pulse duration should be chosen to be \( \leq 3 \) ns rms (7 ns FWHM) to keep the reduction in the luminosity within reasonable limits.

4.3 Present Laser performance

In order to gain experience prior to installation of the LEP transverse polarimeter, a Nd:YAG laser with the following characteristics has been purchased. The 7 mm diameter by 115 mm long rod is placed in a cavity defined by a convex mirror associated to a converging lens on one side and a flat
semitransparent mirror on the other side. The fundamental wave length is 1064 nm, in the infrared. A frequency doubler gives a final 532 nm wave length in the green, that is 2.33 eV or 3.7 \( 10^{-19} \) Joules per photon. The laser is pulsed at 30 Hz, providing 250 mJ at 1064 nm and 90 mJ at 532 nm, i.e. 7.5 Watts and 2.7 Watts respectively. The pumping light is provided by two flash lamps dissipating 30 Joules per pulse from the discharge of a 30 \( \mu \)F capacitor bank. The full width half height duration of the pumping light signal is .25 msec, leading to a .28 msec fluorescence time delayed by .2 msec relative to the flash signal. When the fluorescence reaches a maximum, a 3.6 kV voltage is applied to a Pockels Cell inserted in the optical cavity. Its starts a laser signal of 7 nsec at FWHM. The jitter is less than .5 nsec and the amplitude variation is small as shown in Fig. 10.

At the laser output the light density has an oval form. The vertical extension (z axis) is 7 mm, the horizontal one is 4 mm. The emittance is however 5 to 8 times greater than expected from pure diffraction. The structure of the beam light is not Gaussian and the optics will have to be a compromise between the smallest spot and the best homogeneity at the interaction point. The laser parameters are collected in Table 1.

Before installation an amplifier will boost the existing laser output to 190 mJ at 532 nm. Furthermore new products are already available and it is possible that a new higher performance Nd:YAG laser be provided.

**Table 1: Present Laser performance.**

Laser type: Nd:YAG Quantel – longitudinal monomode

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>532 nm</td>
</tr>
<tr>
<td>Photon energy</td>
<td>2.33 eV</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>30 Hz</td>
</tr>
<tr>
<td>Peak power</td>
<td>14 MW</td>
</tr>
<tr>
<td>Pulse length (FWHM)</td>
<td>7 ns</td>
</tr>
<tr>
<td>Pulse energy</td>
<td>90 (190) mJ/pulse</td>
</tr>
<tr>
<td>Peak intensity</td>
<td>( \approx 0.24 ) ( 10^{18} ) ph/pulse</td>
</tr>
<tr>
<td>CW power</td>
<td>2.7 W</td>
</tr>
<tr>
<td>Time jitter (rms)</td>
<td>(&lt; 0.5 ) ns</td>
</tr>
<tr>
<td>Output emittance</td>
<td>4.1 ( \pm 1 ) mm-mrad</td>
</tr>
</tbody>
</table>

5. The LEP Transverse Polarimeter

A condense version of the detailed account on the measurement of the electron transverse polarization in LEP, given in reference 5a, is presented where we concentrate on relative bunch to bunch effects.
5.1 Introduction

The transverse laser polarimeter has become a part of the standard equipment in e± storage rings above 1 GeV. The laser polarimeter is based on spin dependent Compton scattering of circularly polarized photons from a high energy electron/positron beam, already discussed in chapter 3. The Compton rate is given by

\[ n_\gamma = \mathcal{L}_{\gamma e} \sigma_{C}(P_e \cdot P_\gamma) \]

where \[ \mathcal{L}_{\gamma e} = \frac{N_e N_\gamma}{\Sigma} \]

is the luminosity of the electron beam – laser interaction, \( f \) the laser repetition rate, \( N_e \) the number of electrons bunch, per bunch, \( N_\gamma \) the laser pulse intensity and \( \Sigma \) the interaction area.

From these formulae one can calculate a value of \( n_\gamma \) of \( 7 \times 10^4 \) for the ideal intersection situation using the nominal values of the LEP I electron bunches and the laser specifications as given in Table 1. In practice, however, it is expected that several factors will reduce significantly the backward Compton scattered photon flux. Among them are the laser light losses along the optical path, the laser pulse duration and incomplete overlap of the laser spot with the electron bunches. We therefore assumed conservatively throughout this work that at each laser beam – electron bunch collision, the effective number of Compton photons useful for polarization measurements is about 1000 photons.

In Table 2 we present some parameters which are relevant for the LEP polarization physics programme.

<table>
<thead>
<tr>
<th>Table 2: Some relevant LEP parameters [6]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
</tr>
<tr>
<td>No. of bunches</td>
</tr>
<tr>
<td>Particles per bunch</td>
</tr>
<tr>
<td>Circulating current</td>
</tr>
</tbody>
</table>

The backscattered high energy \( \gamma \)-rays travel towards a detector which records their vertical angular distribution. If the electron beam is transversally (vertical) polarized, an up–down asymmetry in the
\( \gamma \) - rate is present and two different vertical distributions are obtained depending on the left – right polarization \( P_\gamma \) of the photons.

The electron polarization level can be deduced, by comparing the backscattered \( \gamma \) - rates for the two helicity configurations of the laser beam through the up – down asymmetry

\[
\Lambda(z) = \frac{n_{\gamma R}(z) - n_{\gamma L}(z)}{n_{\gamma R}(z) + n_{\gamma L}(z)}
\]

where \( n_{\gamma R}(z) \) and \( n_{\gamma L}(z) \) are the counts of the backscattered \( \gamma \)'s at a vertical position \( z \), for the two helicities ( R, L ) of the laser beam.

The multi - photon technique of illumination is used, requiring a high peak power laser to produce about \( 10^5 \) photons per interaction.

### 5.2 Monitoring the Laser Polarization

The circularly polarized light is generated by inserting a \( \lambda/4 \) - wave plate on the original linearly polarized laser beam. Left – and right–hand polarization light states are obtained by rotating by \( \pm 90^\circ \) the \( \lambda/4 \) plate or by an electro – optical device (Pockels cell). The photon helicity has to be changed rather often during data taking to average out false asymmetries coming from drifts in the closed orbit slope at the interaction region.

The photon circular polarization level might be affected by strains in the window separating the machine vacuum from the atmosphere due to birefringence of the material. This effect cannot be numerically quantified and has to be studied in the laboratory prior to the installation of the apparatus. Cure of possible effects could impose larger window diameters. Optical elements in the light path can also induce some depolarization in the photon beam.

The effective circular polarization \( P_\gamma \) of the laser beam at the Laser Interaction Region (LIR\(\gamma \), chosen for the transverse polarimeter, has to be properly measured if the absolute polarization of the electron beams is aimed at. A possibility, illustrated in Fig. 11 , envisages a “remote polarimetry” on the light deflected after the interaction into the U14 junction where some instrumentation can be installed.

It is estimated that the laser polarization can be known in absolute terms to about 1\%, and that long
term drifts can occur. It is clear however that the laser polarization will be identical for the different bunches at a given time to a much better accuracy.

5.3 Background Considerations

The background will consist of two components: beam-gas bremsstrahlung and synchrotron radiation. In the following the background conditions in the straight section 1.5S1 are investigated.

5.3.1 Beam-Gas bremsstrahlung.

The total cross section \( \sigma_{gb} \) [mb] for the beam-gas bremsstrahlung can be found in Ref. 7 and is given by

\[
\sigma_{gb} (\epsilon_s) = 57.3 \left[ 6.37\epsilon_s - 6.37 \ln(\epsilon_s) - 2.34\epsilon_s^2 - 4.03 \right]
\]

where \( \epsilon_s = k/E \) is the ratio between the photon and the electron energies. Photons below \( \epsilon_s = 0.2 \) will not reach the detector. Computing the rate of gas bremsstrahlung photons with the following parameters

- residual gas \( <Z> = 5 \)
- average pressure = 5.10^{-9} \text{ Torr}
- path length = 500 m

leads to an upper limit of 2 photons per interaction at 1 mA / bunch.

5.3.2 Synchrotron radiation.

A formula giving the number \( N_{sr} \) of photons/s/mA/m emitted above a certain energy \( u(\text{keV}) \) as a function of the energy \( E \) and the bending radius \( \rho \) is given in [8]:

\[
N_{sr} = 4.6 \times 10^{16} \exp(-0.45 u_{p}/E^{1.5}) \sqrt{(E^{1.5}/u_{p}^{3})}
\]

The layout, described later in section 5.4, has been chosen in order to eliminate all background from the main dipoles. The remaining backgrounds to which the detector is sensitive \([E_{p} > 0.5 \text{ MeV}]\), are quantified in Table 3, where arc trajectories of 10 m (10% dipole), 2 m (both miniwigglers), 6.5 m (16 c.o. correctors) and 24 m (24 quadrupoles) are considered as contributing \( N_{d} \) photons to the s.r. background.
Table 3: Flux of synchrotron radiation photons with energy > 0.5 Mev.

<table>
<thead>
<tr>
<th>Source</th>
<th>ρ</th>
<th>εc</th>
<th>N_d</th>
<th>P_d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(km)</td>
<td>(keV)</td>
<td>(10^{17} /mA/s)</td>
<td>(GeV/crossing)</td>
</tr>
<tr>
<td>10% dipole</td>
<td>30.96</td>
<td>12</td>
<td>5.6·10^{-17}</td>
<td>2·10^{-12}</td>
</tr>
<tr>
<td>quadrupoles</td>
<td>≈10</td>
<td>≤40</td>
<td>≤1.5·10^{-3}</td>
<td>≈ 50</td>
</tr>
<tr>
<td>corr. dipoles:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100%</td>
<td>2</td>
<td>184</td>
<td>220</td>
<td>7.5·10^{6}</td>
</tr>
<tr>
<td>50%</td>
<td>4</td>
<td>92</td>
<td>5.2</td>
<td>2·10^{5}</td>
</tr>
<tr>
<td>30%</td>
<td>6.7</td>
<td>55</td>
<td>.07</td>
<td>2·10^{3}</td>
</tr>
<tr>
<td>mini-wigglers</td>
<td>0.877</td>
<td>420</td>
<td>10^{3}</td>
<td>3.6·10^{7}</td>
</tr>
</tbody>
</table>

It can be seen that the dominating effects come from the mini-wigglers and the correcting dipoles for which the deposited power \( P_d = u I N_d \) for \( I = 3 \text{ mA/beam} \) is

\[
3.4 \cdot 10^{11} \text{ GeV/s} \rightarrow 7.5 \cdot 10^6 \text{ GeV/crossing} \quad \text{(corr: dipoles at 100%)}
\]

\[
P_d = 1.6 \cdot 10^{12} \text{ GeV/s} \rightarrow 3.6 \cdot 10^7 \text{ GeV/crossing} \quad \text{(miniwigglers)}
\]

These figures have to be compared with the energy deposited by the backscattered \( \gamma \)-beam. If \( \approx 10^3 \) \( \gamma \)'s per interaction are produced with an average energy of about 25 GeV with the multiphoton technique, the deposited energy is \( \approx 2.5 \cdot 10^4 \text{ GeV/crossing} \).

Two conclusions can be drawn from Table 3:

(i) The operation of the polarimeters is incompatible with the miniwigglers.

(ii) The 16 correcting dipoles in LSS1 should not be powered (if all used at the same time) to more than \( \approx 30\% \) of their maximum. Orbit correction procedures will have to take this recommendation into account.
5.4 Layout

According to the previous considerations a layout for the polarimeter has been adopted which prevents the synchrotron radiation from the main dipoles from falling into the detector.\(^1\)

As shown in Fig. 11 the photon beam produced by a laser installed in the Optical Laboratory in the building US15 in front of IP1 is directed towards the LEP tunnel through a \(\approx 16\) m long channel drilled in the rock. The light is then deflected towards the LIR\(_T\) located between the quadrupoles QL4 and QL5 at both sides of IP1, \(\approx 66\) m downstream.

A more detailed layout of the polarimeter in LSS1 is shown in Fig. 12. The Compton–backscattered \(\gamma\)’s will travel together with the LEP beams along the LSS1 straight section and will be separated from the electrons (positrons) in the 10\% dipole B4WL when entering the arcs. As shown in Fig. 13 the \(\gamma\)–rays will leave the LEP vacuum chamber at the end of the first dipole B4/1 and after passing between the two external coils of the quadrupole QL13 will reach the \(\gamma\)–detector through a \(\approx 40\) m long evacuated path.

The detector will be installed between the dipole B4/3 and the tunnel wall at about 275 m from the LIR\(_T\) (\(\approx 341\) m downstream IP1).

The following modifications are required to provide the necessary channel for the extraction of the photons.

1) The dipole B4/2 must be reversed.
2) The two external coils of the quadrupole QL13 need to be modified to provide a 20 mm vertical aperture to the \(\gamma\)–beam.
3) The vacuum chamber in dipole B2R of B4/1 has to be enlarged.
4) The orbit correcting dipole MCHA next to QL13 has to be reversed.

---

\(^1\) The synchrotron radiation emitted in the first normal bending magnet B4/1 does not hit the detector because of the 0.754 mrad deflection in the B4WL 10\% dipole.
5.5 The Laser Beam and the Interaction Region (LIR T)

5.5.1 The illumination point.

For a given value of the degree of polarization of the circulating beams the asymmetry depends on the optical parameters at the LIR T and on its distance from the detector. For the foreseen LEP optics an illumination point 1 meter outside the D− quadrupole Q14 has been chosen. The LEP beam characteristics in the Vertical (V) and Horizontal (H) directions are given in Table 4. Vertical beam profiles for both laser helicities are shown in Fig. 14.

Table 4: Twiss parameters for the electron beam at the LIR T

(E_e = 55 GeV, \( \varepsilon_h = 55.6 \) nm, \( \varepsilon_v = 2.2 \) nm)

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>( \beta )</th>
<th>( \sigma_e )</th>
<th>( \sigma'_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.105</td>
<td>23.599</td>
<td>1.096</td>
</tr>
<tr>
<td>V</td>
<td>2.779</td>
<td>123.438</td>
<td>0.525</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>( \beta )</th>
<th>( \sigma_e )</th>
<th>( \sigma'_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>-0.069</td>
<td>31.533</td>
<td>1.325</td>
</tr>
<tr>
<td>V</td>
<td>1.896</td>
<td>65.00</td>
<td>0.38</td>
</tr>
</tbody>
</table>

5.5.2 The special vacuum insertion.

Experience at PETRA has shown that total internal reflection prisms inside the vacuum chamber were seriously damaged. Metallic mirrors will be used in LIR T for the final steering of the laser onto the electron beam.

The laser beam from the optical lab will reach a special vacuum insertion and enter the vacuum chamber through a quartz window to be deflected by the retractable metallic mirror M6 (Fig. 15). This will be introduced at the end of the LEP acceleration cycle and after the beam adjustments at the flat top so that the beam−to−mirror clearance can be reduced to about 10 mm (i.e. \( \approx 26 \sigma_e \)). A second mirror M6' will recuperate the laser light after the interaction.
5.5.3 Collision angle and laser spot size.

The collision angle should be chosen to maximize the luminosity of the interaction for a given laser pulse length. The necessity of having a small crossing angle to produce a long interaction region has to be balanced with the importance of reducing the sensitivity to the vertical orbit misalignment. Interactions at very small angles are moreover limited by the position of the last mirror relative to the electron beam, which is in turn defined by the mirror dimensions and hence by the laser spot size at the interaction, for a given emittance of the laser beam.

The influence of the laser spot size on the luminosity has been studied. The relative luminosity is shown in Fig. 16 as a function of the laser spot size at the interaction for collision angles $2\delta_0 = 0, 2, 4$ and $6$ mrad. Requiring a $10$ mm clearance between the beam and the wedge of the mirror $M_e$, and taking into account the laser beam emittance, a $2$ mrad collision angle can be obtained with a nominal laser spot size

$$\sigma_y^* = 0.6 \, \text{mm}$$

A smaller laser size at LIR-1 would require to increase the interaction angle to accommodate the larger mirror dimensions within the $10$ mm clearance, with no net advantage in the luminosity.

5.6 The Recoil $\gamma - \text{beam} - \text{Apertures and Diaphragms}$

For LEP I ($E_c = 45 \pm 60$ GeV), the backscattered $\gamma - \text{beam}$ energy lies in the range 15 to 40 GeV. The transmission of the Compton backscattered $\gamma$'s to the detector is limited by the available apertures on the line of flight (Fig.15). The diaphragms are essentially located in the modified vacuum chamber inside the B4/1 dipole ($\text{Horizontal diaphragm } D_H = 50 \, \text{mm}$) and between the external coils of the quadrupole QL.14 ($\text{Vertical diaphragm } D_V = 20 \, \text{mm}$).

The $\gamma - \text{beam}$ dimensions at the diaphragms and at the detector location are collected in Table 5 for $E_c = 45$ GeV.
Table 5: \( \gamma \) – beam dimensions at diaphragms and detector for \( E_c = 45 \text{ GeV} \)

<table>
<thead>
<tr>
<th>position</th>
<th>dist. from LIR</th>
<th>( \sigma \gamma \text{H} ) (mm)</th>
<th>( \sigma \gamma \text{V} ) (mm)</th>
<th>diaphragm (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mid B4/1</td>
<td>220.6</td>
<td>9.5</td>
<td>2.3</td>
<td>50 (H)</td>
</tr>
<tr>
<td>out QL13</td>
<td>235.6</td>
<td>10.2</td>
<td>2.5</td>
<td>20 (V)</td>
</tr>
<tr>
<td>detector</td>
<td>( \approx 275 )</td>
<td>11.8</td>
<td>2.9</td>
<td></td>
</tr>
</tbody>
</table>

Note that at 45 GeV:

(i) the horizontal aperture in B4/1 is 5.3 \( \sigma \gamma \text{H} \)

(ii) the vertical aperture in QL13 is 8.0 \( \sigma \gamma \text{V} \)

(iii) the minimum required sensitive area of the detector is

\[ S_{\text{det}} \geq 63 \text{ mm} \times 24 \text{ mm (H x V)} \]

5.7 Considerations on the \( \gamma \) – detector

A very compact calorimeter will monitor the rate of the backscattered \( \gamma \)'s. The sandwich-like structure, derived from the solution adopted for the Bhabha relative luminosity monitors \([9]\), will be composed of two vertical Silicon double half-planes separated by 10 r.l. Tungsten converters and followed by a 20 r.l. Tungsten shielding (Fig. 17). Two Silicon strip planes\(^2\) will allow one to check the profiles of the incoming \( \gamma \) – beam in both planes during the calibration phase.

A Silicon/Tungsten calorimeter \([9]\) with only two sensitive planes has a resolution of \( \approx 125% / \sqrt{E} \). At an average \( \gamma \) energy of about 25 GeV this resolution amounts to \( \approx 25\% \) and will contribute by only 6% to the statistical fluctuations when a bunch of \( \gamma \)'s are recorded.

The detector has an horizontal plane of symmetry and allows for simultaneous recording of the backscattered \( \gamma \)'s above and below the center of gravity of the distribution. The total energies \( E_{\gamma \text{Ru}} \)

---

\(^2\) 32 mm / 16 channels covering \( \pm 6 \sigma \) of the \( \gamma \)'s distributions at the detector.
+ \( E_{\gamma Ld} \) and \( E_{\gamma Rd} + E_{\gamma Lu} \) recorded in the upper (u) and the lower (d) halves of the calorimeter for both photon helicities can be combined to give the asymmetry

\[
\Lambda = \frac{E_{\gamma Ru} - E_{\gamma Rd} + E_{\gamma Ld} - E_{\gamma Lu}}{E_{\gamma Ru} + E_{\gamma Rd} + E_{\gamma Ld} + E_{\gamma Lu}}
\]

This asymmetry is insensitive to systematic errors originating from drifts in the closed orbit at LIR-T. An analogous algorithm can be set up to monitor a possible offset at the detector, and this information will be used as a feedback to control its vertical position for a proper compensation. The simultaneous acquisition would also make the measurements insensitive to fluctuations in the nominal electron beam rms dimensions at LIR-T.

The evaluation of the analyzing power of the polarimeter for the nominal beam gives

\[
\Pi_T (P_{\gamma} = 1) = \frac{\partial \Delta}{\partial P_{e}} \approx 11.33\%
\]

One can also determine the beam polarization by a measurement of the centroid displacement, \( \Delta < V > \), of the recoiling beam in the Silicon strip detector. For \( P_{e} = 1 \) the distribution width is 3 mm rms and the displacement is 0.812 mm when the laser polarization is reversed (see Fig. 14).

### 5.8 Rates, Accuracies and Measuring Time

The adoption of the multi-photon technique, as in the case of the longitudinal polarimetry (see sec. 6.3), provides a backscattered \( \gamma \) - rate

\[
r_{\gamma} \approx (1 + 3) \times 10^4 \text{ Hz}
\]

The relative statistical accuracy \( \delta \Lambda / \Lambda \) for the asymmetry is a function of \( \Lambda \):

\[
\frac{\delta \Lambda}{\Lambda} \approx \frac{1}{\Lambda \sqrt{2 \langle n_{\gamma L,R} \rangle}}
\]

where \( \langle n_{\gamma L,R} \rangle \) is average number of counts at each photon helicity required for a given accuracy.

Introducing the analyzing power the measuring time in terms of the polarization level \( P_{e} \) and the accuracy on \( \Lambda \) is
\[ T_{\text{meas}} = \frac{\langle n_{yL,R} \rangle}{r_\gamma} \approx 10^{-4} \quad \text{[s]} \]
\[ 2 \left( \frac{\Pi_T}{P_c} (\Delta \Lambda/\Lambda) \right)^2 \]

The measuring time \( T_{\text{meas}} \) is shown in Fig. 18 as a function of \( P_c \) for different accuracies \( \delta \Lambda/\Lambda \).

For the centroid measurement method a precision of \( \delta P_c = 1\% \) is reached for \( \approx 10^6 \) photons corresponding to about 1 to 2 minutes data collection time.

### 5.9 Systematic Effects.

In this section we address the question of how accurately the transverse polarimeter can measure the absolute and the relative bunch to bunch polarization. Systematic errors coming from offsets in the electron beam position or divergence are large. However they can easily be eliminated by by centering the \( \gamma \) detector on the average of the recoil beam. Other systematic errors which arise from the effects of uncertainties in the electron beam size and divergence, due to the finite measuring resolution and knowledge of the \( \beta \) functions, have been investigated. This has been done, using the layout shown in Fig. 12 and the polarimeter detector of Fig. 17 with a slit of 2 mm, by generating about \( 3 \times 10^5 \) Compton scattering events with a simulation program. It was found that the asymmetry method is sensitive to systematic effects of the beam vertical spot size. This uncertainty in the polarization measurement can be eliminated by a normalization of the measured asymmetry to the vertical rms dimensions of the recoil photon beam or by using the centroid method.

Finally we note that if an accurate spatial detector were to be added to the front of the \( \gamma \) detector, it might be possible to use it to measure more accurately the beam divergence — however there are no plans at present to implement such an improvement.

### 6. Longitudinal Polarimetry

#### 6.1 Introduction

As discussed in chapter 3, the longitudinal polarimeter proposed for IEP is based on Compton scattering of a circularly polarized laser beam on the IEP \( e^\pm \) beams, where the degree of the electron
(positron) polarization is derived from the different energy spectra of the recoil photons when the laser beam polarization is reversed.

Fig. 19 illustrates the detection end of the system. Scattered photons exit from the LEP straight section through special vacuum tubes and windows (not shown in Fig. 19) where they are converted into $e^+ e^-$ pairs in a thin lead or crystal converter. A sweeping magnet then provides a vertical momentum dependent separation and calorimeters 1–4 detect the converted $e^\pm$ in appropriate energy ranges. This scheme will also separate the wanted $e^+ e^-$ pairs from the backgrounds. Arguments will be presented for the choice of the multi–photon mode of operation.

Many of the considerations of the chapters on the laser system (chapter 4) and transverse polarimeter (chapter 5) are also relevant for the longitudinal polarimeter. In particular we assume for the time being that the same type of laser will be used, although we are aware of new products in this field which have better performance than the device described in chapter 4. As for monitoring the laser beam and apertures, their problems and solutions are similar to those outlined for the transverse polarimetry (sections 5.2 and 5.6) and thus will be adopted also here.

### 6.2 Detector Design

The detector for the longitudinal polarimeter might well resemble that used successfully by the UA7 collaboration [10], namely a 20 radiation length Silicon/Tungsten calorimeter.

The longitudinal development of the shower is measured at 11 sampling points. A 4" diameter Silicon wafer is placed every two radiation lengths. Four of these wafers are segmented into 5 mm wide strips, for the horizontal and vertical coordinates, and placed at 6 and 10 radiation lengths. The strip width is 11 mm for the front vertical plane and for the 45° oriented U plane placed at 8 radiation lengths.

The shower signal was amplified by a linear preamplifier and shaping amplifiers and transmitted to a Lecroy 2281 ADC system. The calibration of the preamplifier and ADC system was made by injecting a test pulse in front of the preamplifier.

The energy deposit in the Silicon had been calibrated using 976 keV electrons from decay of isotope Bi$^{207}$. The thickness uniformity of each Silicon wafer was better than 25 $\mu$m by construction. The absolute thickness varies from 8 to 1.2 mm.
The longitudinal development of the showers generated by electrons at different energies is shown in Fig. 20. Superimposed is a fit to the data using current formulas, such as the one given by the Particle Data Group [11].

Fig. 21a represents the energy resolution of the calorimeter as a function of the incident beam energy. The data points agree well with the Monte Carlo prediction \(25\%/\sqrt{E} + 1.1\%\). The constant term comes from the limited number of Tungsten plates. After correction for the losses (1% at 10 GeV, 3.2% at 100 GeV) due to the limited number of radiation lengths, the linearity of the calorimeter was better than 1%.

The position resolution is shown in Fig. 21b as a function of the energy and can be expressed by \(3.8(\text{mm})/\sqrt{E(\text{GeV})}\). Lower points are for the horizontal direction (5 mm strip width) and upper points correspond to the U plane (11 mm strip width).

Fig. 22 (from reference 12) shows a typical example of the lateral distribution of the shower at incident electron energy of 4.5 GeV. The response does not change much when the incident beam is close to the edge of the detector [13].

This detector looks perfectly suited to our needs for the measurement of longitudinal polarization.

6.3 Counting Rates

Two modes of operation are in principle possible:

6.3.1 The Single Photon Mode

In this mode one analyses single Compton scattering events. This requires the average number of recoil photons entering the polarimeter detector per laser shot to be less than one, \(N_y \approx 0.1\) being reasonable. The photon detector of the polarimeter can then be adjusted to have a low photon energy threshold, \(E_c\). We define \(E_L\) and \(E_R\) as the average recoil photon energy (above the cutoff) respectively for left and right circular polarized photons which can be expressed in the following way:

\[
E_L = E_0 (1 + \Pi L P_c)
\]
\[
E_R = E_0 (1 - \Pi L P_c)
\]

where \(E_0\) is the average energy for \(P_c = 0\) and \(\Pi L\) is the analyzing power which depends on \(E_c\). From these one can then calculate \(P_c\) through the relation.
\[ P_e = (1/\Pi_L) \left( (F_L - F_R)/(F_L + F_R) \right) \]

The obvious advantages of this single photon detection mode are

a) The measurement is independent of the photon-electron luminosity \( \mathcal{L}_{ey} \) which is hard to control and monitor; and

b) One is able to cope with the background of low energy photons.

However this method has two severe drawbacks.

a) The first is its high sensitivity even to a very low background of high energy photons with \( E > E_c \),
   e.g. those from radiative Bhabha scattering,
   \[ e^+ e^- \rightarrow e^+ e^- \gamma \]
   or from beam-gas bremsstrahlung in the straight section.

b) The second concerns the sampling time. The relative error in measuring the polarization is approximately given by

\[ \delta P_e = (1/\Pi_L) \left( N_s \mathcal{L}_{ey} F \right)^{-0.5} \]

where \( N_s \) is the number of laser pulses and \( F \) is the fraction of Compton photons with \( E_{\gamma} > E_c \).

For \( \Pi_L = 0.15 \) and \( F = 0.3 \) the precision of \( \delta P_e = 0.003 \) will be reached when \( N_s \approx 1.5 \times 10^7 \) which is a long time if one uses a 10 Hz laser. The sampling time may be somewhat reduced with the choice of a higher frequency laser.

The two drawbacks mentioned above make the single photon mode a rather unpractical proposition.

6.3.2 The Multi-photon Mode

We next consider the multi-photon mode where a high power laser is used and a high \( \mathcal{L}_{ey} \) results. The polarization is given by the same expression as for the case of the single photon mode. The total energy of the beam of recoiling photons is measured, many such recoil photons being recorded for each laser shot. This total energy is sensitive to the beam polarization, but the technique has at first sight a reduced analysing power \( \Pi_L \) and is sensitive to the relative fluctuations of \( \mathcal{L}_{ey} \), which may be large, between the different electron and positron bunches. In this mode the relative polarization error is given by

\[ \delta P_e = (1/\Pi_L) \delta (\Delta \mathcal{L}_{ey}) (N_s)^{-0.5} \]
If we take for example $\delta (\Delta \mathcal{L}_{\text{e}\gamma} ) = 0.05$ (just a guess until we have some operational experience) and $\Pi_L = 0.15$ one needs to have $N_s = 1.2 \times 10^4$ shots to obtain $\delta P_e \approx 0.003$. This gives an acceptable sampling time of about 20 min for a 10 Hz laser.

A variation on the straightforward multi-photon mode overcomes the problems mentioned above and increases significantly the analyzing power. To measure the polarization free of uncertainty in $\mathcal{L}_{\text{e}\gamma}$ one has to be able to retrieve the spectral information of the individual Compton photons. This can be achieved by converting the photons into electron pairs the momenta of which are then magnetically analysed. A schematic outline of such a system is shown in Fig. 19. The Compton photons hit a thin convertor of about 0.1 radiation length. The thickness of the convertor is determined by two factors. The first is the need to minimize the radiation corrections and the second to have a sufficient yield of photons conversion to electron pairs in order to keep the sampling time within an acceptable limit. The electron pairs are then passing through a sweeping magnet of $\approx 0.2$ Tm and are detected by four calorimeters situated symmetrically to have equal efficiency detection for electrons and positrons. A sandwich of Tungsten and Silicon detectors having some 20 radiation lengths should be an adequate solution for the proposed calorimeters. A considerable reduction of synchrotron radiation background in the calorimeter can be achieved by deflecting the electrons and positrons away from the LEP ring plane. Finally additional information could be obtained from the unconverted photons which are captured by an additional counter placed in the Compton straight flight line (not shown in the the figure). This counter may also be utilized as supporting luminosity counter since it will be sensitive to the radiative Bhabha scattering, $e^+ e^- \rightarrow e^+ e^- \gamma$, (see also ref. 14).

The beam polarization $P_e$ in this setup is given by:

$$P_e = \frac{1}{\Pi_L} \left( \frac{Q_L}{Q_R} \right) \frac{Q_L}{Q_R}$$

with $Q_L = (E_{23} / E_{14})_L$ and $Q_R = (E_{23} / E_{14})_R$

where $E_{ij}$ is the energy deposited in the calorimeters $i$ and $j$. For a setup where calorimeters 2 and 3 detect electrons between 20 and 29.5 GeV and the calorimeters 1 and 4 between 5 and 10 GeV, the analysing power $\Pi_L$ is equal $\approx 52\%$. With this system about 3% of the recoil photons will be recorded in the electron calorimeters and at least $\approx 10^3$ Compton photons/laser shot are expected for the type of laser under consideration. About 34000 laser pulses will be needed to achieve an accuracy of $\delta P \approx 0.003$.
6.4 Analyzing Power and Systematic Errors

Two independent programmes have been developed and used to evaluate the analyzing power \( \Pi_L \) of the proposed longitudinal polarimeter and its systematic errors. They arrive at the same conclusions, which are described below.

6.4.1 Analyzing Power and Resolution.

Referring to Fig. 19, the Compton photon distributions were generated using the equations of chapter 3 and the beam parameters of section 6.6. The electron energy spectrum of the converted photons is shown in Fig. 23. Using a 0.15 Tm sweeping magnet following a 10\% radiation length converter the energy limits of the low energy calorimeters (1 & 4) and the high energy calorimeters (2 & 3) were varied in order to optimize the resolution.

Results for the analyzing power [15] are shown in Table 6, where the horizontal variable is the upper energy limit of calorimeters 1 & 4, and the vertical variable is the lower energy limit of calorimeters 2 & 3. Table 7 shows the precision of the polarization measurement obtained with 17,000 laser shots for each laser helicity, corresponding to about 20 mins with a 10 Hz rep. rate. Note that the optimum resolution does not correspond to maximum analyzing power – the optimum of the latter occurs at points where the counting rate would be very low.

Fixing the upper limit of calorimeters 2 & 3 at or above the maximum energy of the recoil photons ( \( \approx 29.5 \text{ GeV} \) ), Fig. 24 shows how the resolution varies with the lower energy limit. The different curves correspond to various upper limits of the calorimeters 1 & 4 fixing their lower limit to 5 GeV. As a result of these studies we have chosen the following ranges:

- Calorimeters 1 & 4: \( 5 < E < 10 \text{ GeV} \)
- Calorimeters 2 & 3: \( 20 < E < 29.5 \text{ GeV} \)

which yield for 17,000 double laser shots the following analyzing power and resolution values for the nominal beam:

\[ \Pi_L = 0.522 \quad \text{and} \quad \frac{d\theta_e}{\theta_e} = 0.314 \% \]

Some redundancy could be built into the system if a set of position measuring devices (MWPC's or Silicon strips) were positioned in front of the calorimeters which will provide independent momentum values of the converted electrons. Furthermore they would not be subject to "edge" effects and so might turn out to be essential in analyzing the calorimeter data to the required high precision.
### Table 6: Analyzing powers $\Pi_L (%)$ for different energy ranges.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{14}$ range [GeV]</td>
<td>73.1</td>
<td>71.0</td>
<td>68.7</td>
<td>66.3</td>
<td>63.7</td>
<td>61.2</td>
<td>58.5</td>
<td>55.9</td>
<td>53.3</td>
<td>50.7</td>
<td>48.2</td>
<td>45.7</td>
<td>43.3</td>
<td>41.0</td>
</tr>
<tr>
<td>5 – 6</td>
<td>72.9</td>
<td>70.9</td>
<td>68.6</td>
<td>66.1</td>
<td>63.6</td>
<td>61.0</td>
<td>58.3</td>
<td>55.7</td>
<td>53.1</td>
<td>50.5</td>
<td>47.9</td>
<td>45.5</td>
<td>43.0</td>
<td>40.7</td>
</tr>
<tr>
<td>5 – 7</td>
<td>72.8</td>
<td>70.7</td>
<td>68.4</td>
<td>65.9</td>
<td>63.4</td>
<td>60.8</td>
<td>58.1</td>
<td>55.5</td>
<td>52.8</td>
<td>50.2</td>
<td>47.7</td>
<td>45.2</td>
<td>42.8</td>
<td>40.4</td>
</tr>
<tr>
<td>5 – 8</td>
<td>72.6</td>
<td>70.5</td>
<td>68.2</td>
<td>65.7</td>
<td>63.1</td>
<td>60.5</td>
<td>57.8</td>
<td>55.2</td>
<td>52.5</td>
<td>50.2</td>
<td>47.3</td>
<td>44.8</td>
<td>42.8</td>
<td>40.4</td>
</tr>
<tr>
<td>5 – 9</td>
<td>72.3</td>
<td>70.3</td>
<td>67.9</td>
<td>65.4</td>
<td>62.8</td>
<td>60.2</td>
<td>57.5</td>
<td>54.8</td>
<td>52.2</td>
<td>49.9</td>
<td>46.9</td>
<td>44.4</td>
<td>42.4</td>
<td>40.1</td>
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<tr>
<td>5 – 10</td>
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<td>70.0</td>
<td>67.6</td>
<td>65.1</td>
<td>62.4</td>
<td>60.2</td>
<td>57.1</td>
<td>54.4</td>
<td>51.7</td>
<td>49.5</td>
<td>46.5</td>
<td>44.0</td>
<td>42.0</td>
<td>39.7</td>
</tr>
<tr>
<td>5 – 11</td>
<td>71.3</td>
<td>69.6</td>
<td>67.2</td>
<td>64.6</td>
<td>62.0</td>
<td>59.3</td>
<td>56.6</td>
<td>53.9</td>
<td>51.2</td>
<td>49.1</td>
<td>45.9</td>
<td>43.4</td>
<td>41.5</td>
<td>38.5</td>
</tr>
<tr>
<td>5 – 12</td>
<td>70.8</td>
<td>69.2</td>
<td>66.7</td>
<td>64.2</td>
<td>61.5</td>
<td>58.8</td>
<td>56.1</td>
<td>53.3</td>
<td>50.6</td>
<td>48.5</td>
<td>45.3</td>
<td>42.7</td>
<td>40.2</td>
<td>37.9</td>
</tr>
<tr>
<td>5 – 13</td>
<td>70.3</td>
<td>68.7</td>
<td>66.2</td>
<td>63.6</td>
<td>60.9</td>
<td>58.2</td>
<td>55.4</td>
<td>52.7</td>
<td>50.6</td>
<td>49.9</td>
<td>47.2</td>
<td>44.5</td>
<td>42.0</td>
<td>39.5</td>
</tr>
<tr>
<td>5 – 14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 – 15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### Table 7: The error $\delta P_e (%)$ for different energy ranges.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{14}$ range [GeV]</td>
<td>.842</td>
<td>.567</td>
<td>.458</td>
<td>.407</td>
<td>.383</td>
<td>.374</td>
<td>.375</td>
<td>.382</td>
<td>.395</td>
<td>.413</td>
<td>.433</td>
<td>.457</td>
<td>.484</td>
<td>.513</td>
</tr>
<tr>
<td>5 – 6</td>
<td>.841</td>
<td>.561</td>
<td>.446</td>
<td>.389</td>
<td>.359</td>
<td>.343</td>
<td>.337</td>
<td>.337</td>
<td>.343</td>
<td>.352</td>
<td>.364</td>
<td>.373</td>
<td>.397</td>
<td>.417</td>
</tr>
<tr>
<td>5 – 8</td>
<td>.852</td>
<td>.565</td>
<td>.446</td>
<td>.385</td>
<td>.351</td>
<td>.334</td>
<td>.332</td>
<td>.337</td>
<td>.343</td>
<td>.350</td>
<td>.362</td>
<td>.373</td>
<td>.396</td>
<td>.417</td>
</tr>
<tr>
<td>5 – 15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.4.2 Systematic Errors.

As for the transverse polarimeter, we have investigated the effect of off-centring the beam at the converter, both in position and angle, and of adjusting the LEP beam properties (spot size, divergence and beam energy) by amounts typical of the relative precision with which they are known. Offsets in position or divergence could be eliminated experimentally by exploiting the possibility to switch off the laser circular polarization or by measuring the recoil beam centroid with an on-axis calorimeter. We found that the systematic errors introduced in the knowledge of the analyzing power due to the above uncertainties in the LEP beam properties, are small compared to the required precision. In Table 8 we present the dependence of the analysing power on the bunch to bunch relative beam properties. We conclude that the bunch to bunch systematic is of the order of 1%. The systematic for absolute measurement of the longitudinal polarization is estimated to be larger by an order of magnitude.

Table 8: Variations in analyzing power ($\Pi_{L}$).

<table>
<thead>
<tr>
<th>Relative bunch to bunch change</th>
<th>Change relative to nominal beam (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>nominal beam $+ \delta V = 50 \mu m$</td>
<td>$+0.026$</td>
</tr>
<tr>
<td>nominal beam $+ \delta \sigma Y = 50 \mu m$</td>
<td>$+0.051$</td>
</tr>
<tr>
<td>nominal beam $+ \Delta V' = 1.0 \mu rad$</td>
<td>$+0.004$</td>
</tr>
<tr>
<td>nominal beam $+ \delta \sigma Y' = 0.5 \mu rad$</td>
<td>$-0.002$</td>
</tr>
<tr>
<td>nominal beam with $\Delta F / F = \pm 0.02%$</td>
<td>$\mp 0.01$</td>
</tr>
</tbody>
</table>

6.5 Background Estimates

The background estimates are very different for the two types of spin rotator under study. Arc rotators have no magnets in the experimental straight sections and are designed so that the weak (10%) bends at the extremities of the straight sections are preserved. Their background situation is therefore analogous to that described in section 5.3 for the transverse polarimeter. Straight section rotators, however, change the layout significantly in the straight sections surrounding the experiment by introducing a strong bend near ($\approx 30 - 50 \text{ m}$) to the experiments. The two cases are dealt with separately.
6.5.1 Background Estimates in the Case of Arc Rotators.

The main sources of background in the longitudinal polarimetry are due to Synchrotron radiation, beam–gas bremsstrahlung, radiative Bhabha Scattering and 2–photon annihilation processes, where the last two are proportional to luminosity. These backgrounds are identical to those encountered in the LEP transverse polarimetry and apart from the last, are described in some detail in section 5.3. The introduction of arc rotators in the accelerator ring should not affect the background level. In any case due to the fact that the electron calorimeters (1, 2, 3 and 4; Fig. 19) may have a lower energy cutoff, \( E_c \) and are situated outside the LEP ring plane, these background sources are by far less harmful than they are for transverse polarimetry.

I) Synchrotron Radiation

The sources of synchrotron radiation that may affect LEP longitudinal polarimetry are the weak (10%) dipole magnets, the orbit correctors and the straight section quadrupoles. Note that the straight sections where the LEP experiments have their detectors do not contain miniwigglers. It is assumed that the orbit correctors are all running at 30% of their maximum strength and that the beam is off axis in the quadrupoles as in section 5.3.2. The effects of these accelerator ring elements can be estimated with the formula given in section 5.3.2 and are quantified in Tables 3 and 10.

II) Electron Beam – Gas Bremsstrahlung

As in the case of the transverse polarimeter (see section 5.3.1), the estimate of this background is based on formulae given in Ref. 7. The background from beam–gas Bremsstrahlung depends on the gas residues in the LEP straight section of about 600 m seen by the converter and on the number of electrons in a bunch. One can safely assume that the residual gas in the ring behaves like an ideal gas and that it consists mainly of Carbon and Oxygen. Hence, for bunches with \( 4.16 \times 10^{11} \) electrons and a 600 m length, the probability \( P_r \) for an electron with an energy \( E_c \) to radiate a photon with an energy \( k \) is:

\[
P_r (E_c, k) \, \mathrm{d}k \approx 0.09 \, \tau \, \mathrm{d}k / k
\]

where \( \tau \) is the pressure in the ring in units of \( 10^{-10} \) Torr. If the detector system has a lower energy cutoff of 5 GeV, the beam–gas background will be less than 1 photon/shot (Table 10) for the estimated average pressure of \( \tau = 4.7 \) in the LEP straight sections equipped with r.f. cavities. In the
straight sections without r.f. cavities the pressure is expected to be $\approx 2 \times 10^{-10}$ Torr and the background scales down accordingly.

III) Radiative Bhabha Scattering

The cross section for radiative Bhabha scattering, $e^+ e^- \rightarrow e^+ e^- \gamma$ has been studied in reference 16 for the case of unpolarized electron beams. Here we will use that study to estimate the expected background from that radiative process. Since some of the colliding bunches are longitudinally polarized, the background estimate given below will be somewhat overestimated. The basic formula for this process is:

$$d\sigma / dy = \left\{ (2\pi r^2_{e^+} ) / (1 - y) \right\} \left\{ 2(1 + y^2) - (4/3) y \right\} \left\{ \ln \left( y^2 / 4y/(1-y) - 0.5 \right) \right\}$$

where $\gamma = E_e / m_e$ and $E_e - E'_e = k$ is the energy of the radiated photon and $y$ is defined to be $y = E'_e / E_e$.

Using this expression it is estimated that for a luminosity of $\rho_{ee} = 10^{31}$ cm$^{-2}$ s$^{-1}$, a total of about 500 GeV is deposited per bunch crossing. As mentioned before, this background essentially rules out the single photon polarimetry. With the multi-photons detection system proposed here about 10% of these photons will be converted and due to the energy cut only 10% of these will be recorded.

IV) $e^+ e^- \rightarrow \gamma \gamma$ Annihilation into 2 Photons

The cross section for this $e^+ e^- \rightarrow \gamma \gamma$ process produces a pair of photons having the same energy as the beam electrons. The differential cross section, which has maxima in the forward and backward directions, is given by:

$$d\sigma / d(\cos \theta) = \pi \alpha^2 / (2E_{e^-}^2) \left\{ A^2 + B^2 \cos^2 \theta \right\} / \left\{ A^2 - B^2 \cos^2 \theta \right\}$$

where $A = 0.5 s - m^2_e$ and $B = 0.5 s (1 - 4m^2_e / s)^{0.5}$ and $s = (2E_e)^2$.

Without radiative corrections the calculated background is negligible. The effect of radiative corrections on this cross section has been studied by F. Berends and R. Kleiss [17] – the cross section increases by about 23%.

V) Summary of Background Estimates in the case of Arc Rotators.
Table 9: Signal and background estimates with arc rotators.

<table>
<thead>
<tr>
<th>Source</th>
<th>At Converter</th>
<th>At Calorimeters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E &gt; .1</td>
<td>E &gt; 5</td>
</tr>
<tr>
<td></td>
<td>1000 γ</td>
<td>850 γ</td>
</tr>
<tr>
<td>Compton signal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sync. Radiation</td>
<td>&gt; 6 x signal</td>
<td></td>
</tr>
<tr>
<td>Beam – Gas</td>
<td>3γ</td>
<td>1 γ</td>
</tr>
<tr>
<td>e⁺ e⁻ → e⁺ e⁻ γ</td>
<td>78γ</td>
<td>23 γ</td>
</tr>
<tr>
<td>e⁺ e⁻ → γ γ</td>
<td>&lt; 1γ</td>
<td></td>
</tr>
</tbody>
</table>

In Table 9 we present rough estimates for the background effects at the photon converter and at the calorimeters which are compared with the expected Compton signal for $L_{\text{cc}} = 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ and $4.16 \times 10^{11}$ particles/bunch. The energies (E) in the Table are in GeV.

Finally one should note that the remaining background which enters the calorimeters can be experimentally monitored by observing the counting rate with laser on and off and partially understood by having circulating beams in the accelerator without producing luminosity.

6.5.2 Backgrounds in the Case of Straight Section Rotators

The sources of background are the same as for arc rotators, namely synchrotron radiation, electron beam–gas bremsstrahlung, radiative Bhabha scattering and annihilation into 2 photons. Clearly the last two will be the same as for the arc rotators, since they depend only on luminosity.

A description of a straight section rotator and polarimeter is given in section 6.7 and shown in Fig. 25 (where electron detection is also considered). At the converter the synchrotron radiation background will be much higher than reported in the last subsection due to the proximity of "bend 2" of the spin rotator. However it should not be troublesome since it cannot produce electron–positron pairs which would be detected in the calorimeters 1–4. The beam–gas bremsstrahlung background will be smaller than that reported in Table 9, due both to the shorter length of straight section "seen" by the detectors and the better vacuum near the experiments.
6.6 Layout, Installation and Cost (Arc Rotator)

Many of the arguments which determined the layout, laser interaction region (to be denoted by LIRL, when referring to the longitudinal polarimeter), crossing angle and apertures (sections 5.4, 5.5 and 5.6) of the transverse polarimeter are also valid for the longitudinal polarimeter and will therefore not be dealt with in detail here. The requirements of having a good overlap of the round laser beam with the LEP electron/positron beam, of achieving a high luminosity and of being able to extract most of the recoil photons for analysis leave us with a clear choice for the laser interaction region, which is chosen to be at the exit of the quadrupole QS10 (see Fig. 12). The Twiss parameters and electron beam sizes, together with the recoil photon beam parameters are collected in Table 10.

Note that the electron beam sizes are comparable with those of the transverse polarimeter. The photon beam dimensions include the spread of the scattering process (10.9 μrad).

The case for having an optical laboratory for monitoring purposes has already been made. Since the longitudinal polarimeter will be installed in one of the experimental straight sections, it is assumed that the laser will be housed near to the middle of the straight section (near an experiment) in an accessible place. The need for "remote polarimetry" will be reviewed in the light of experience with the LEP transverse polarimeter.

The extraction channel of the scattered γ rays, similar to that shown in Fig. 13 for the transverse polarimeter, requires the following to the standard LEP layout.

<table>
<thead>
<tr>
<th>Table 10: Twiss parameters and beam sizes at the LIRL and converter.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E = 55 GeV, ( \eta_h = 55.6 \text{ nm} ), ( \eta_v = 2.2 \text{ nm} ))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( e^{\pm} ) at LIRL</th>
<th>( \gamma ) at Converter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-0.469  2.782</td>
</tr>
<tr>
<td>( \beta )</td>
<td>11.745  84.115  937.7  424.9  m</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>0.808   0.430  7.29   1.38   mm</td>
</tr>
<tr>
<td>( \sigma_e' )</td>
<td>76.00   15.12  76.8   18.6   μrad</td>
</tr>
</tbody>
</table>

32
1) The horizontal component of B4/2 must be reversed. Probably the straight section side of B4/2 will be the first vertical bend of the arc rotator.

2) The dipoles B4/3 have to be reversed.

3) The two external coils of the quadrupole QL13 need to be modified to provide a 20 mm vertical aperture to the γ-beam.

4) The vacuum chamber in dipole B2R of B4/1 has to be enlarged.

5) The orbit correcting dipole MCHA next to QL13 has to be reversed.

Note that the above modifications are identical to those for the transverse polarimeter, except for the turning of bending magnets B4/3 and the vertical bend of the rotator.

The costs of the above modifications are given in Table 11, for the case of a polarimeter on one side only of the interaction point i.e. for either e+ or e−, not both.

A major cost is the laser itself, the optical transport elements, the monitoring devices, the sweeping magnet and of course the detector. Finally it should be noted that this first cost estimate should be considered as a lower limit and that costs of the magnet power supply, cabling etc. are not included.

<table>
<thead>
<tr>
<th>Table 11: Cost estimates of the LEP changes and polarimeter components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum chambers in B4/1, QS13 and B4/2.</td>
</tr>
<tr>
<td>Coils of QS13.</td>
</tr>
<tr>
<td>Reversed B4/2,3 magnets. Coil connections.</td>
</tr>
<tr>
<td>Laser.</td>
</tr>
<tr>
<td>Optical path elements</td>
</tr>
<tr>
<td>Laser monitoring devices</td>
</tr>
<tr>
<td>Sweeping Magnet</td>
</tr>
<tr>
<td>γ calorimeters.</td>
</tr>
<tr>
<td>TOTAL (1 side only)</td>
</tr>
</tbody>
</table>

6.7 Longitudinal Polarimetry by Electron Detection

In the previous chapters we have discussed polarization measurements utilizing only the backward scattered Compton photon. An additional information on the beam polarization can clearly also be
extracted from the outgoing Compton electron energy which may be measured alone or in conjunction with its associated Compton photon.

For a given Compton scattering event, the relation between the energy of the final state electron and photon is simply given by \( E_e = E_{\text{beam}} - E_\gamma \) so that the differential cross section \( d\sigma/dE_e \) is readily inferred from the expression for \( d\sigma/dE_\gamma \) (see also Fig. 6). For \( E_{\text{beam}} = 47 \) GeV the Compton electron energy spectrum ranges from about 17 GeV to 47 GeV. This means that Compton electrons well below the beam energy, say less than 35 GeV, will be deflected out of the LEP.

To achieve a successful setup for a polarimeter which uses both the Comptons photon and electrons, the following obvious requirements are needed:

1) compatibility with the LEP and rotators design
2) acceptable low background of photons and off–momentum electrons
3) sufficient space for the detection equipment

Here we outline a scheme for such a longitudinal polarimeter keeping in mind the fact that both the final state electrons and photons do emerge in a very narrow cone around the accelerator beam line. This is illustrated in Fig. 25 where a preliminary setup of the equipment is shown. The Laser beam illumination point and the rotator are placed in the space of about 25 m between the quadrupoles QT1 and QT2 of the LEP straight section. The rotator, schematically drawn in the insert of the figure, is of the Richter–Schwitters type (more details can be found in the machine physics section of this Yellow Report). The vertical bend can be realized by 3 C–shaped magnets placed on their “back” so that the Compton electrons can be extracted out of the LEP ring and be detected and momentum analysed by a pair calorimeters (5 and 6 in Fig. 25) which define two momentum ranges.

The Compton photon analyzing method is similar to that described previously in Chapter 6. The Compton photon hits a thin converter and the emerging electron–pair are further momentum analyzed by a flat magnet so that it will not interfere with the circulating LEP beams which pass right above it. The magnetic field, of about 1.5 Tm, sweeps horizontally the electron pairs into a set of four calorimeters (1, 2, 3 and 4 in Fig. 25). As in previous chapters we will also here consider the operation of this polarimeter setup in the multi–photon mode. With this arrangement and a similar Laser electron bunches illumination system as discussed in the previous chapters, the estimated off–momentum electron and photon backgrounds can be controlled and do not seem to pose any serious problem.

The analyzing power of this polarimeter system can be defined as
\[ \Pi^{\gamma L} = (T_L + Q_L - T_R - Q_R) / (T_L + Q_L + T_R + Q_R) \]

with \( T_L = (E_s / E_0)_L \) and \( T_R = (E_s / E_0)_R \)

measured by the electron calorimeters 5 and 6 and \( Q_L \) and \( Q_R \) are energy sum ratios obtained from the photon detection system as defined in chapter 6.3.2.

In Table 12 we give the values of the combined analyzing power \( \Pi^{\gamma L} \), for several operating energy range values of the electron counters 5 and 6 taking a pointlike laser beam Compton scattering. The analyzing power values were obtained for the optimal setup of the Compton photon detection system alone (with an analyzing power of \( \approx 52\% \)), namely:

\[ 5 < E_{14} < 10 \text{ GeV} \quad \text{and} \quad 20 < E_{23} < 29.5 \text{ GeV}. \]

These table values are higher than that for a Compton photon polarimeter and do as expected depend on the energy gap between the counters 5 and 6. A rough estimate indicates that for a given statistics of Compton scattering collisions the addition of information on the scattered electron improves the polarization precision measurement by about a factor 2.

**Table 12: Combined Electron - Photon Analysing Power (\( \Pi^{\gamma L} \))**

<table>
<thead>
<tr>
<th>( E_5 ) Range (GeV)</th>
<th>( E_6 ) Range (GeV)</th>
<th>Gap (GeV)</th>
<th>( \Pi^{\gamma L} ) %×10</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 – 26</td>
<td>27 – 35</td>
<td>1.0</td>
<td>558</td>
</tr>
<tr>
<td>15 – 24</td>
<td>28 – 35</td>
<td>4.0</td>
<td>595</td>
</tr>
<tr>
<td>15 – 23</td>
<td>29 – 35</td>
<td>6.0</td>
<td>615</td>
</tr>
<tr>
<td>15 – 22</td>
<td>30 – 35</td>
<td>8.0</td>
<td>633</td>
</tr>
</tbody>
</table>

**6.8 New Ideas on Longitudinal Polarimetry using Crystals**

Several novel and interesting ideas concerning the use of single crystals for the polarization measurement of electron bunches have been proposed. Since these are given with some details in two separate papers [18,19] following this one, we will here briefly summarize their main ideas.

The use of crystals can be applied in several ways, either as an improvement on the Compton scattering analysis scheme outlined here, or as an independent polarization measurement tool.

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One considerable improvement may be obtained by replacing the "amorphous" metal converter of the longitudinal polarization detector (see chapter 6) by a crystal. With an optimum choice of crystal and orientation angles, the ratio of the photon absorption coefficient to the inverse radiation length is then at least 2.2 times better than for a conventional converter. This will allow a higher electron-pair conversion rate without increasing the subsequent unwanted radiation effects.

The possibility of measuring the polarization of electron bunches using the radiative Bhabha scattering

\[ e^+ e^- \rightarrow e^+ e^- \gamma \]

with a crystal detector is described in ref 19. In this scattering process, the single bremsstrahlung photons are circularly polarized whenever the electrons (positrons) are longitudinally polarized. The circular polarization of the photons can be measured in two different ways, either by a thin crystal converter or even better, by a thick crystal used as a photon absorber. An estimate of the statistical error on a polarization measurement using a copper crystal gives an error of \( \approx 0.38\% \) for a luminosity of \( 10^{31} \text{ cm}^{-2} \text{ sec}^{-1} \), a polarization level of 50% and a 5 hour run. A further analysis is required to evaluate the systematic errors.

7. Conclusions

7.1 Transverse Polarimeter

A detailed study of the LEP transverse polarimeter showing attractive performance has been presented. A layout optimised on the basis of background estimates has been defined and the laser-electron beam interaction region designed to achieve the best conditions for the Compton interaction. The laser has been chosen to optimize the performances of the polarimeters in the 40 to 100 GeV per beam LEP energy range.

The proposed structure for the \( \gamma \)-detector makes use of compact Silicon calorimeters similar to those adopted for the relative luminosity monitors and provides a measurement of the asymmetry which is insensitive to systematic errors from closed orbit drifts within the acceptance of the \( \gamma \) transport channel.
A study of systematic effects indicates that the error in the absolute transverse polarization measurement will be about 2%, mainly due to the laser degree of polarization uncertainty, when the centroid method is used. This precision is more than adequate for the LEP energy calibration and transverse polarization studies. The systematic effects coming from the beam parameters and which affect the relative transverse polarization measurements are much smaller and in the order of a few %.

7.2 Longitudinal Polarimeter

The detailed design of the overall system depends on the design of the spin rotators. Nevertheless, we have shown that even a longitudinal polarimeter based on Compton photons alone, can be built with the following properties:

1) separation of the Compton photons from the electron beams.
2) insensitivity to fluctuations in $\mathcal{L}_y$.
3) very low level of background.
4) adequate sampling rate.
5) insensitivity to bunch differences.
6) systematic effects are very small.

We have shown that the bunch to bunch relative level of longitudinal polarization of electrons and positrons beams in LEP, can be measured in less than 20 minutes, to the precision of 0.3% needed for conclusive Left – Right asymmetry measurements in $e^+ e^-$ reactions.

The error on the absolute longitudinal polarization measurement however, will be comparable to the one to be achieved with the transverse polarimeter.

For the case of a straight section rotator, an improvement of the analysing power and resolution can be obtained by adding an electron detection system.

Acknowledgements

Our thanks are due to P. Hobson for his detailed technical remarks on the laser system and to R. Rossmanith for his many helpful comments concerning the transverse polarimetry.
References


   b) J. Badier et al. (The ALEPH Polarization Working Group), ALEPH 87–17 NOTE 87–5.


Fig. 1 Diagram for Compton scattering in the laboratory frame of reference and in the electron centre of mass system.

Fig. 2 The Compton scattering angle, $\theta_{\text{LAB}}$, in the laboratory system as a function of the electron center of mass scattering angle $\theta$. The corresponding recoil photon laboratory energies are also indicated.
Fig. 3  The relation in Compton scattering between the recoil photon laboratory energy and the \(\cos \theta\) in the electron center of mass system.

Fig. 4  The total Compton scattering cross section as a function of photon energy in the electron center of mass system. The cross section value for a beam energy of 47 GeV and a laser wavelength 530 nm is also shown.

Fig. 5  The Compton differential cross section as a function of \(\cos \theta\) is shown for longitudinally polarized electrons with \(P_x = +1\) scattered by lefthanded \((P_y = -1)\) and righthanded \((P_y = +1)\) laser beams.

Fig. 6  The Compton differential cross section as a function of the recoil photon energy in the laboratory system for longitudinally polarized electrons with \(P_x = +1\) scattered by lefthanded \((P_y = -1)\) and righthanded \((P_y = +1)\) laser beams. The corresponding \(\cos \theta\) values are also indicated.
Fig. 7 The asymmetry $A_{10}$ $(\cos \theta) = \frac{d\sigma/d\Omega(P_\gamma = -1) - d\sigma/d\Omega(P_\gamma = +1)}{d\sigma/d\Omega(P_\gamma = -1) + d\sigma/d\Omega(P_\gamma = +1)}$ as a function of $\cos(\theta)$.

Fig. 8 The laser analysing power, defined as $A_{10} (\cos \theta = -1)$, for longitudinal polarimetry is given as a function of the laser wavelength.

Fig. 9 Effect of the laser pulse duration on luminosity.
Fig. 10  The measured Laser output pulse (Oscilloscope trace).

Fig. 11  The laser beam optical path and the 'remote light polarimetry' in the UJ14 junction for the LEP transverse polarimeter.
Fig. 12  A sketch of the LEP half straight section containing the transverse polarimeter, shown from the e⁺e⁻ interaction region to the arc. Also given is a schematic drawing of the laser optical path from the laser laboratory to the colliding point with the LEP positron beam (taken from reference 5). The exit place of the recoil photons and the location of a Compton photon detector are also shown.

Fig. 13  Detail of arc layout showing e⁺ and γ beam paths for the transverse polarimeter.
Fig. 14  Vertical beam profiles at the transverse polarimeter $\gamma$ detector

Fig. 15  Detailed layout of the special vacuum insertion at the transverse polarimeter LIR$_T$. 

LIR Layout

Interaction angle $\delta = 2$ mrad
Fig. 16 Effect of transverse rms round laser beam dimensions on the luminosity for different values of the crossing angle $\delta$.

Fig. 17 Schematic of the $\gamma$-detector layout of the transverse polarimeter.
Fig. 18  Measuring time versus the $e^*$ polarization level $P_e$ for the transverse polarimeter.

Fig. 19  A schematic view of a proposed Compton photon detector for longitudinal polarimetry of electrons.
Fig. 20  Longitudinal electron shower development.

Fig. 21  Energy resolution (a) and position resolution (b) of the UA7 calorimeter.
Fig. 22  Lateral electron shower distribution at 4.5 GeV incident electron energy in the UA7 calorimeter.

Fig. 23  Normalized positron energy spectrum of converted photons in the longitudinal polarimeter. The two curves are for opposite laser circular polarizations.
Fig. 24 The resolution on the measurement of longitudinal polarization versus the lower energy limit of calorimeters 2 & 3 keeping their upper energy limit at 29.5 GeV. The different curves are for the upper energy limit of calorimeters 1 & 4 keeping their lower limit at 5 GeV. Upper energies of 9, 10, 11, and 12 GeV of calorimeters 1 & 4 correspond to solid, dashed, dashed-triple dotted and dashed-dotted lines respectively.

Fig. 25 A scheme for longitudinal polarimetry in a straight section (Richter–Schwitters) rotator, where both recoil photons and electrons are detected. The upper left insert shows a side view of the rotator—the other insert shows a top view of the photon detection system. The main figure is a side view relative to the incident electron beam direction.
USE OF CRYSTALS IN $\gamma$-RAY POLARIMETRY

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ABSTRACT.

We describe some methods of $\gamma$-ray polarimetry which make use of coherent $e^\pm$ pair production in crystals and can be applied to the measurement of LEP $e^\pm$ longitudinal polarization, via the measurement of the circular polarization of secondary photons. At LEP, high energy circularly polarized photon beams are produced by Compton back scattering of Laser circularly polarized photons against longitudinally polarized $e^\pm$ (see ref. 1), or by longitudinally polarized $e^\pm$ single bremsstrahlung (see ref. 2). In the former case the asymmetry in $e^\pm$ pair production from right- and left-circularly polarized photons is detected. In this context we discuss the optimization of the photon conversion process by the use of a crystal. For the latter case the first step is to convert circular to linear polarization via a quarter period crystal plate. In this context we propose a method of trimming the conversion process by two wedge-shaped crystals. The second step consists in the measurement of the photon linear polarization with a thick crystal analyzer. We propose a calibration method using a second crystal which can be either thin or thick. In addition we show that a thin crystal can be used stand-alone, as the method is self-calibrating.

1. INTRODUCTION.

The following notes have been written with the purpose to complement the report prepared by the Polarimetry Working Group$^1$, concerning the measurement of polarization at LEP. We show that a crystal can be conveniently used in different instances as an alternative to the conventional polarization measurement. The level of detail is not as deep as in parts of ref. 1, the aim being principally to introduce new ideas. We limit our consideration to the measurement of longitudinal polarization, as for transverse polarization a well established method exists$^1$.

The principle of the proposed $e^\pm$ longitudinal polarization measurement presented in ref. 1 rests on the detection of the asymmetry effect in Compton back scattered right- and left-circularly polarized laser photons against longitudinally polarized $e^\pm$. The essential instrument to this purpose is a magnetic spectrometer, in which photons are converted to $e^\pm$ pairs.

An improvement in performances can in effect be obtained: here is where a first conventional use of a crystal in place of the "amorphous" converter could be of help. The cross sections for $e^\pm$ pair production and bremsstrahlung in a crystal depend on the orientation of the crystal with respect to the primary particle. The orientation angle can luckily be chosen so as to
relatively increase pair production with respect to bremsstrahlung. With an optimum choice of
crystal and angles, the product of the photon absorption coefficient by the radiation length is at
least 2.2 times that for a normal converter. The results of some detailed calculations are
illustrated in Sect. 5.

A different proposal for an integrated luminometer-polarimeter project has also been
presented\(^2\). It makes use of the single photon bremsstrahlung process of \(\text{e}^\pm\). For longitudinally
polarized \(\text{e}^\pm\), the highest energy emitted photons are completely circularly polarized. The
measurement of the circular polarization provides informations on the longitudinal \(\text{e}^\pm
\) polarization. To determine the photon circular polarization, the measurement of the asymmetry
in the angular distribution of \(\text{e}^\pm\) produced pairs can be performed. Here again a crystal
converter can be useful in making the angular selection easier.

Still another proposal\(^3\) of photon circular polarization measurement has appeared. The
method\(^4\) uses a crystal as the analogous of the quarter wave plate known in optics. It consists
of a slab of crystal material which introduces a change of a quarter period between the
components of the photon polarization vector \(\text{e}\). This acquires thus a linear polarization which
can be easily measured by other well established crystal methods.

To understand this possibility, details concerning the description of the polarization state
of a photon beam are given in Sect. 2. The basic principles of coherent pair production in
crystals which allow the determination of the photon polarization are given in Sect. 3. It is
shown that two different type of analyzer can be used, employing a thin\(^4\) and a thick\(^5\) crystal,
respectively. In Sect. 4 we propose some methods for practically obtaining a quarter period
plate and for calibrating analyzers.

2. THE PARAMETERS DESCRIBING A POLARIZED PHOTON BEAM.

We adopt an orthogonal system of axes, with the x- and y-axis in a plane perpendicular
to the photon momentum \(\text{k}\) (the x-axis is chosen arbitrarily in this plane) and the z-axis along \(\text{k}\).

In terms of the density matrix elements, the 3 Stokes parameters describing the
polarization state of a photon beam are given by

\[
P_1 = \rho_{11} - \rho_{22}; \quad P_2 = \rho_{12} + \rho_{21} = 2 \Re(\rho_{12}); \quad P_3 = i(\rho_{21} - \rho_{12}) = 2 \Im(\rho_{12})\;
\]

they are all real, due to the hermiticity of the density matrix \(\rho\), and possess a direct experimental
definition (see below). \(P_1\) represents the *degree of linear polarization along the x- or y-axis*; \(P_2\)
represents the *degree of polarization along directions at 45\(^\circ\) or 135\(^\circ\) to the x-axis*; \(P_3\) represents
the *degree of circular polarization*.
In terms of Stokes parameters, the degree of polarization \( P \) is given by

\[
P = \sqrt{P_1^2 + P_2^2 + P_3^2} = \sqrt{(\rho_{11} - \rho_{22})^2 + 4|\rho_{12}|^2} = \rho' - \rho'',
\]

where \( \rho' \geq \rho'' \geq 0 \) are the eigenvalues of \( \rho \) (normalized in such a way that \( \rho_{11} + \rho_{22} = \rho' + \rho'' = 1 \)). \( P \) is invariant under rotations around the \( z \)-axis. We will use a vector notation \( \mathbf{P} = (P_1, P_2, P_3) \).

To characterize completely a beam containing many photons, an additional Stokes parameter \( I \), representing the beam intensity, is considered.

The determination of the density matrix of a beam (or equivalently, of its Stokes parameters) is accomplished by means of a detector characterized by the Stokes parameters \( \mathbf{P}^d \) of the photons to which it responds. Then the probability of detecting a photon characterized by Stokes parameters \((1, \mathbf{P}^d)\) in a beam characterized by Stokes parameters \((I, \mathbf{P})\) is \( w = (1 + \mathbf{P} \cdot \mathbf{P}^d)/2 \).

We want now calculate \( w \) for an idealized analyzer experiment. If the photon beam is passed through an absorbing medium, there will be in general changes both in the amplitudes and in the phases of the beam components. This is treated by introducing a complex refractive index. For the time being let us examine only the case of negligible absorption. Then the medium introduces only a phase difference \( \delta \) between the \( x \)- and \( y \)-component of the polarization directions; for this reason this device is called phase compensator. The beam emerging from the compensator is then sent to the analyzer, which is a projector device capable of detecting only the component of the photon polarization vector \( \mathbf{e} \) at an angle \( \phi \) to the \( x \)-axis. This analyzer also is supposed to have a negligible absorption. By letting \( I_0 \) be the incident beam intensity and \( I(\phi, \delta) \) the intensity detected for the settings \( \phi, \delta \), we have

\[
w(\phi, \delta) = \frac{I(\phi, \delta)}{I_0} = \rho_{11} \cos^2 \phi + \rho_{22} \sin^2 \phi + \rho_{12} \exp(-i\delta)(\sin \phi \cos \phi + \rho_{21} \exp(i\delta) \sin \phi \cos \phi)
\]

\[
= \frac{1}{2} \left[ 1 + P_1 \cos^2 \phi + P_2 \sin^2 \phi \cos \delta + P_3 \sin^2 \phi \sin \delta \right] = \frac{1}{2} \left( 1 + \mathbf{P} \cdot \mathbf{P}^* \right), \quad (1)
\]

where \( \mathbf{P}^* \equiv (\cos 2\phi, \sin 2\phi \cos \delta, \sin 2\phi \sin \delta) \) is vector describing an ideal analyzer \( (P^* = 1) \). Thus eq. (1) represents the detection probability of an ideal analyzer which is the combination of a phase compensator and a projector.

From eq. (1) we obtain the operational definition of Stokes parameters:

\[
I_0 = I(0^\circ, 0^\circ) + I(90^\circ, 0^\circ) \quad P_1 = \frac{I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ)}{I_0} ;
\]

\[
P_2 = \frac{I(45^\circ, 0^\circ) - I(135^\circ, 0^\circ)}{I_0} ; \quad P_3 = \frac{I(45^\circ, 90^\circ) - I(135^\circ, 90^\circ)}{I_0}.
\]
\( P_1 \) (\( P_2 \)) is measured by the relative excess in intensity detected by a perfect analyzer accepting linear polarization at the azimuth \( \phi=0^\circ \) (\( \phi=45^\circ \)) over the intensity detected by the same analyzer at \( \phi=90^\circ \) (\( \phi=135^\circ \)). To determine the circular polarization \( P_3 \), we first introduce a phase difference of a quarter period (e.g., by means of the so-called quarter period compensator - see Sect. 4 for a practical realization of the device) between the x- and y-components of the polarization. Then the analyzer is again used as in the case of the \( P_2 \) measurement. We see that the action of the quarter period compensator is to convert circular to linear polarization, in such a way that a linear polarization analyzer can be used.

A real detector can be characterized by \( P^d = A P^* \), \( A = (M-m)/(M+m) \); \( M \) is the maximum signal for polarization \( P \), \( m \) the minimum signal for a polarization orthogonal to the previous one, and \( A \) is the analyzing power. If \( m=0 \), then \( A=1 \) and \( P^d = P^* \).

A convenient way of determining the degree of polarization of a partially polarized beam consists in finding the absolute maximum \( N_{\text{max}(\phi,\delta)} \), and the absolute minimum \( N_{\text{min}(\phi,\delta)} \), with respect to both \( \phi \) and \( \delta \), of the counting rate (in a given energy range) corresponding to a fixed incident number of photons; we then obtain

\[
P = \frac{1}{A} \frac{N_{\text{max}(\phi,\delta)} - N_{\text{min}(\phi,\delta)}}{N_{\text{max}(\phi,\delta)} + N_{\text{min}(\phi,\delta)}}.
\]

In conclusion, we have shown that the combination of a quarter wave compensator and a linear polarization analyzer is capable of analyzing the general elliptical polarization state of radiation. However, for a real detector the analyzing power \( A \) is less than 1, and the central problem of a polarization measurement is that of optimizing and carefully determining it. In Sect. 4 we will show how \( A \) can be experimentally determined.

### 3. COHERENT INTERACTIONS AND POLARIZATION IN CRYSTALS.

When a high energy photon or electron interacts with the nuclei of a crystal, the primary particles feels a periodical field over a macroscopic length. The pair production and bremsstrahlung show coherence peaks for specific orientation angles. These are specified by the polar angle \( \Theta \), i.e., the angle between the primary momentum and a major crystal axis (say, \( <110> \) for a cubic crystal), and by the azimuthal angle \( \alpha \), i.e., the angle between a major crystal plane (say, (001) for a cubic crystal) and the plane determined by the major crystal axis and the primary momentum. The fundamental low governing the phenomenon is the same as for elastic scattering of X-rays (Laue-Bragg law): a coherent recoil of the target crystal as a whole takes place when the momentum transfer coincides (in suitable units) with a reciprocal lattice vector of the crystal. The kinematics is very different with respect to X-ray scattering, however. A full account of these type of phenomena is given in ref. 6.
For both pair production and bremsstrahlung the component of the minimum momentum transfer along the direction of motion (which, on the basis of the uncertainty principle, determines the maximum length over which the interaction is coherent) is inversely proportional to the primary energy. The same is true for the $\theta$ angle of orientation at which the coherent peaks are produced. Typical values of $\theta$ for $\alpha=0$ are a few mrad at some tenths GeV. At the same time the coherent cross section increases linearly with primary energy (this behaviour, typical of Born approximation, should obviously break down at some high energy). A systematic search among all existing crystals\textsuperscript{7} led to the conclusion that, as regards the coherency properties, the best crystal is diamond; it has a coherent cross section for pair production at about 40 GeV which is one order of magnitude larger than the Bethe-Heitler one. In general all low-$Z$ crystals, like beryllium, beryllium carbide, etc., are best suited for coherency, but diamond has in addition the highest Debye temperature. So the smearing effect of thermal vibration on the field of the nuclei is negligible even without cooling. Silicon crystals are also used, due to the advanced technology of growing. However its coherent cross sections are roughly a factor of 3 smaller than diamond ones.

In addition, thermal vibrations produce an incoherent background which has a cross section some % smaller than the Bethe-Heitler one. The best crystal under this respect is again diamond.

Recently a series of experiments performed at CERN in the energy range 40-150 GeV with Ge crystals cooled at 100 K, both for pair production\textsuperscript{8} and bremsstrahlung\textsuperscript{9}, have shown the failure of the Born approximation and the spectacular rise of the so called constant field effect: the electrons (in bremsstrahlung as well as in pair production), captured in the so called axial or planar channeling regime, behave as classical particles in a macroscopic transverse field. For example, the radiation process is synchrotron-like. It is interesting to remark that in the pair production experiment\textsuperscript{8} the departure from Born theory begins to show up at an angle $\theta \approx 1$ mrad, roughly independent of photon energy. This should be of interest to us in the following.

As a crystal is an anisotropic medium, we have to expect also polarization effects with respect to some privileged crystal axes. Thus, for instance, the coherent bremsstrahlung cross section from unpolarized $e^{\pm}$, differential in photon energy, is sensitive to the linear polarization direction, while the coherent pair production cross section from linearly polarized photons, differential in electron energy, displays an asymmetry which depends on the angles $\theta$ and $\alpha$ defined above. It is interesting to note that, differently from the case of the incoherent process, these properties persist even if integrated over the entire emitted cone of particles.

The use of a thin crystal (i.e.: a crystal having a negligible absorption) for measuring the linear polarization of photons has been proposed\textsuperscript{10} and experimentally\textsuperscript{11} verified. Its
exploitation requires a magnetic spectrometer, but is rewarding in the sense that the asymmetry ratio

\[ A = \frac{d\sigma(0^\circ) - d\sigma(90^\circ)}{d\sigma(0^\circ) + d\sigma(90^\circ)}, \]  

(1)

(where the symbols on the right side represent the pair production cross section, differential in electron energy, for \(\phi=0^\circ\) and \(90^\circ\) respectively) is quite large: asymptotically (\(k \geq 70\) GeV/c) \(A \approx 0.7\) in a diamond crystal, for equal \(e^\pm\) energies. (However, the problem of Born approximation failure at these large energies should be taken in mind - see ref. 8). The device can well be used as the projector part in the combination discussed in Sect. 2 and so eq. 2 (2) should be applicable. We will discuss the problem of experimentally determining \(A\) in Sect. 4.

In fig. 1a we show, in arbitrary units, the pair production cross section, differential in electron energy, while in fig. 1b we show the asymmetry ratio (1), for equal energy \(e^\pm\), versus \(\theta\), for 50 GeV photons and a diamond crystal with main axis <110> perpendicular to the crystal plate; the angle between the plane (110) and the plane determined by the photon momentum \(k\) and the axis <110> is \(\alpha=5^\circ\); the \(x\)-axis, from which the azimuth \(\phi\) is reckoned, is chosen in the latter plane. In the calculation we used the Born approximation and so its validity is restricted to angles \(\theta \geq 1\) mrad. In fig. 1a the value at \(\theta=0\) represents the incoherent cross section, which is some \% lower than the Bethe-Heitler one.

The breaks in both figures represent a real effect: by increasing \(\theta\), each time the projection of a reciprocal lattice vector along the photon direction reaches (in suitable units) the minimum allowed momentum transfer, a constructive interference of amplitudes is produced. The angle \(\alpha=5^\circ\) has been so chosen that the most important coherent contribution (i.e.: the largest break in the figures), due to the reciprocal lattice vector 220, falls at \(\theta=12\) mrad. It is seen that this break is well isolated from the others. Different values of \(\alpha\) (for example \(0^\circ\) and \(90^\circ\)) can cause the collapsing of several breaks into one, and \(A\) could be smeared out as a consequence.

The same interference structure is shown of course in pair energy distribution. The upper curve in fig. 2 represents, in arbitrary units, the differential cross section for 30 GeV photons, versus the electron fractional energy, for \(\theta=7\) mrad and \(\alpha=90^\circ\)(same crystal and axes as above). Here many breaks have collapsed, and the asymmetry ratio (not reported) is smaller as a consequence. The lower curve refers to the Bethe-Heitler cross section.

Another method\(^{12}\) for measuring (and producing) the photon linear polarization exploits the dependence of the photon absorption coefficients on the linear polarization direction. Photons of different polarization are differently absorbed in a thick crystal and even if the incident beam is unpolarized (i.e.: it is the superposition of two equal intensity perpendicularly
Fig. 1. a: Pair production differential cross section (arbitrary units); b: Asymmetry ratio, versus θ. Diamond, axis <110>, reference plane (110), α = 5°. Photon energy kc=50 GeV. Equal energy e².

Fig. 2. Pair production differential cross section (arbitrary units) versus electron fractional energy. Photon energy: kc=30 GeV. Upper curve: Diamond, axis <110>, θ = 7 mrad, reference plane (110), α = 90°. Lower curve: Bethe-Heitler cross section.
polarized beams), the transmitted radiation becomes partially linearly polarized. The device is
analogous to the dichroic filter for visible light. Here also the experimental verification has been
performed\textsuperscript{13}.

Our interest in the method resides in that it can be used as polarimeter for linear
polarization. As we have now an absorbing medium, in place of eq. 2 (1) we use, for a linearly
polarized beam,

\[ T(\phi) = T_0 \left( 1 + P P_0 \cos 2\phi \right), \]

where \( T(\phi) \) represents the \textit{transmission}, i.e., the ratio of the beam intensity at the exit to the
beam intensity at the entrance of the crystal, for photons with polarization vector \( e \) at the angle \( \phi \)
to the \textit{x}-axis (this being bound to a suitable direction in the crystal); \( P \) is the degree of
polarization after transmission; \( T_0 \) and \( P_0 \) are similar quantities referring to an incident beam
which is unpolarized or linearly polarized at 45\textdegree{} or 135\textdegree{} degrees to the \textit{x}-axis (\( P_0 \) is called the
\textit{polarizing power of the crystal}). It results

\[ T_0 = \frac{\exp\left\{-L[\Sigma(0^\circ)+\Sigma(90^\circ)]/2\right\}}{\sqrt{1 - P_0^2}}, \]

\[ P_0 = \tanh\left\{-L[\Sigma(0^\circ)-\Sigma(90^\circ)]/2\right\}; \]

\( L \) is the crystal thickness and \( \Sigma(\phi) \) the absorption coefficient for photons with polarization
vector \( e \) at the angle \( \phi \) to the \textit{x}-axis. If \( P_0 \) is experimentally known, the unknown linear
polarization of a beam can be obtained from eq. (2):

\[ P = \frac{1}{P_0} \frac{T(0^\circ) - T(90^\circ)}{T(0^\circ) + T(90^\circ)}. \]

The transmission coefficients are obtained as the counting rates (corresponding to a
fixed luminometer read-out) accepted in a suitable energy window by a calorimeter placed
behind the crystal. These coefficients must be determined first for a specified \( \theta, \alpha \) setting, with
\( \phi = 0 \), and then after a rotation about the photon direction by \( \phi = 90^\circ \), leaving \( \theta \) and \( \alpha \) unaltered.
In Sect. 4 we will learn how to determine \( P_0 \) by a calibration with another crystal.

The polarizing power of a Cu crystal cooled at 77 K, for 40 GeV photons moving
parallel to the (010) plane at a best angle \( \theta = 0.46 \text{ mrad} \) with respect to the <101> axis, and for a
transmission of 0.1 (corresponding to a crystal thickness of 2.4 cm), is\textsuperscript{12} \( P_0 = 0.37 \).

Last, but very important for us, it has been shown\textsuperscript{14} that as a consequence of the
previous effect, crystals are also birefringent, i.e., complex refractive indexes \( n_x \) and \( n_y \) can be
introduced for directions parallel and perpendicular to a suitable crystal axis, respectively. This
could be useful for handling circular polarization. In particular, it is possible to construct a
quarter period compensator (see Sect. 2), analogous to the quarter wave plate used for instance in the LEP laser system (see ref. 1), to convert linear to circular polarization and vice versa. It is sufficient to set

$$\Re \left( n_x - n_y \right) \frac{kL}{\hbar} = \frac{\pi}{2}$$

(k photon momentum), from which the quarter period plate thickness $L$ can be obtained. The real part of the refractive index is obtained by dispersion relations from the absorption coefficients, calculated by means of the coherent pair production theory. The thickness of a Cu crystal quarter period compensator, cooled at 77 K, for a photon energy of 40 GeV, with the axis $<110>$ set at the optimum angle of $\theta = 0.46$ mrad with respect to the photon beam (this being in the (001) plane and $n_x$ being referred to the $<1\bar{1}0>$ axis), is $14$ $2.7$ mm. No experimental test has been made yet. In effect this $\theta$ value seems to be already in the range of invalidity of Born approximation ($Z=29$ for Cu, very similar to $Z=32$ for the experiments of ref. 8). However estimates have been made even in the constant field approximation for the planar channeling case. Luckily enough, this theoretical incertitude can be experimentally settled, by finely trimming the phase compensation, as we will now show.

4. PROPOSAL FOR COMPENSATOR AND CALIBRATION.

In order to determine the quarter period compensator thickness appropriate for each energy, we propose the following arrangement of crystals. The compensator consists of two wedges $W'$, $W''$, cut from the same crystal. Wedge $W'$ can slide over $W''$, the upper face of $W'$ being parallel to the lower face of $W''$. We obtain in this way a parallel plate whose variable thickness can be adjusted to give the required quarter period phase difference. The compensator can in some cases be made integral part of the analyzer.

Two different types of polarization analyzer can now be built up, according to how the compensator is combined with the linear polarization detector:
1) Association with the projector, i.e., with the thin crystal.
2) Association with the dichroic analyzer, i.e., with the thick crystal.

In instance 1), eq. 2 (2) can be directly used (provided $A$ is known). Thus a practical procedure to measure the circular polarization $P$ should be to set approximately the thickness of the compensator to the correct value for a specified energy, then look for a relative maximum and minimum of intensity by changing the azimuth $\phi$ of the projector, and finally adjust the compensator thickness to obtain the absolute maximum and minimum.
In instance 2) the procedure should be similar, with the only difference that now transmission coefficients must be determined. Then the wanted polarization should be given by

$$P = \frac{1}{P_0} \frac{T_{\text{max}(\phi, \delta)} - T_{\text{min}(\phi, \delta)}}{T_{\text{max}(\phi, \delta)} + T_{\text{min}(\phi, \delta)}}$$

(1)

where the meaning of the symbols is the same as in Sects. 2 - 3. For a statistical error analysis, see ref. 3.

The two methods are in principle equivalent. Method 1) is interesting in that the asymmetry ratio, at least for diamond, is large (see fig. 1b). Diamonds could be available up to a size of 0.8 - 1 cm² and a thickness 4 - 5 mm (= 0.03 - 0.04 amorphous radiation length). Thus the possibility to use them depends on the distance of the pair spectrometer to the interaction point in LEP and on the tolerated loss in intensity after collimation. Probably the best use of this method is for calibration purposes (see below), for which the measurement time is not critical. Up to the size mentioned, there should not be a cost problem. For what we need is not a perfect stone from the optical point of view. In the past yellowish stones have been used with success in producing coherent beams; yet the commercial value was negligible.

We want to show how linear polarization detectors can be calibrated. As a practical example, let us consider the calibration of a dichroic analyzer D, oriented at the appropriate angles, by another crystal C which can be either a thin crystal or another dichroic analyzer not necessarily identical to D (if two identical dichroic analyzers can ever be obtained, a very simple cross-calibration can be performed - see ref. 3). With C we experimentally determine a quantity of the form PR (see eqs. 2 (2) and 3 (4)), where R is of the type given either by eq. 3 (1)) or by eq. 3 (3). Consider then the following successive measurements with a linearly polarized incident photon beam (after compensation), $P_1$ being its degree of polarization:

1) D off beam; C on beam - Measured quantity: $c_1 = P_1 R$

2) D on beam, with azimuth $\phi$ set for maximum signal; C behind D - Final polarization after D: $P_f$ - Measured quantity: $c_2 = P_f R$.

3) D on beam, with azimuth set at 45° degrees larger than in 2); C behind D as in 2) - Measured quantity: $c_0 = P_0 R$ ($P_0$: polarizing power of D). The same result is obtained with an unpolarized incident beam and any azimuth.

The ratio of any two quantities out of $c_0, c_1, c_2$ is independent of R. Now, it is easy to show that, if $\phi$ is set as in 2), then

$$P_f = \frac{P_0 + P_1}{1 + P_0 P_1};$$

then

$$P_0 P_1 = \frac{c_0 c_2 - 1}{c_2} \frac{c_1}{c_2} = C_1,$$

$$P_1 = P_0 \frac{c_1}{c_0}.$$
and finally

\[ P_0 = \sqrt{\frac{c_0}{c_2} (1 + \frac{c_0}{c_1})} - \frac{c_0}{c_1}. \]

In conclusion, by means of 3 measurements we were able to determine the two needed ratios. All polarizations were determined. In particular, we have calibrated the dichroic analyzer, by determining its polarizing power \( P_0 \), without any knowledge of the parameters for crystal C. In order that this calibration can successively be used, D crystal scans in \( \theta \) and \( \alpha \) must be performed and the best angle coherence maximum retrieved with the calorimeter.

Finally we illustrate now a possible self-calibration procedure for a thin crystal, which can lead to the determination of the asymmetry ratio, eq. 3 (1). To understand the principle, we have to discuss the structure of the coherent cross section. We assume that Born approximation is still valid. The differential cross sections\(^{10} \) \( d\sigma(0^\circ) + d\sigma(90^\circ) \) and \( d\sigma(0^\circ) - d\sigma(90^\circ) \) display a structure in which the kinematical part is well separated from the crystal structure part, the latter being common to both cross sections (this determines the amplitudes of the breaks in figs.1 and 2). The idea is then to extract this common part from the measured \( d\sigma(0^\circ) + d\sigma(90^\circ) \) and apply it to \( d\sigma(0^\circ) - d\sigma(90^\circ) \). A magnetic pair spectrometer is needed for this purpose; the experimental data should look like fig. 1a or 2, depending on wether an angular scan or a pair energy distribution is measured. This same idea can be exploited to determine the influence of experimental resolutions such as photon beam divergence, mosaic spread of crystal axes, etc. A nice feature of this procedure is that additive errors cancel out to the first order in the needed ratio of cross sections. The accuracy that is attainable is limited by statistical fluctuations.

To illustrate the method, we borrow an example from the cross channel (coherent bremsstrahlung), in which exactly the same structure functions are contained. Fig. 1a in ref. 11 show the spectrum and the linear polarization of a coherent bremsstrahlung beam. The continuous curve is derived from the theoretical cross section (dashed curve) by introducing the effects of measurable experimental resolutions. The agreement with the data points is remarkable. The polarization given by the continuous curve in fig.1b of ref. 11 was obtained from the parameters determined in the spectrum. Its accuracy is better than that obtained by the direct measurement of the asymmetry (open circles).

5. COMPTON POLARIMETRY IMPROVEMENT.

We will now investigate wether a crystal, used as converter in the Compton Polarimeter described in ref. 1, could improve the performances of the instrument. The instrument is essentially a magnetic \( e^n \) pair spectrometer. The pair converter cannot be made thick, otherwise the development of an electromagnetic shower will modify the expected distribution.
The best converter is that with highest $\gamma$-ray absorption and lowest $e^\pm$ absorption. A global performance parameter should be

$$\frac{\alpha_\gamma}{1/X_R} = \alpha_\gamma X_R,$$

where $\alpha_\gamma$ is the $\gamma$-ray absorption coefficient and $X_R$ is the radiation length of the converter. For a normal converter and ultrarelativistic $e^\pm$ we have $\alpha_\gamma X_R = 7/9 = 0.8$, independent of material and energy. For a crystal this product depends on crystal orientation and even on primary energy. (As $X_R$ is energy dependent, it does not have the meaning of radiation length in the usual sense). Now, for kinematical reasons, the coherent behaviour of the two parameters is completely different: while $1/X_R$ is a regularly decreasing function, $\alpha_\gamma$ displays a maximum. To be precise, we have

$$\alpha_\gamma = \frac{N_A \rho}{M_A} \frac{Z^2 r_0^2}{137} \left[ \frac{k}{m c} \phi_R \left( \frac{k}{m c} \theta \right) + \phi_0 \right],$$

where $N_A$ is the Avogadro number, $\rho$ is the density, $M_A$ is the atomic mass, $Z$ is the atomic number, $r_0$ is the classical electron radius, $k$ is the photon momentum, $m$ is the electron rest mass, $c$ the velocity of light in vacuum, $\phi_0 = 11.3$ is an incoherent background for diamond, ($\approx 10\%$ lower than Bethe-Heitler contribution) and $\phi_R$ is a universal function plotted in fig. 3 for diamond (choice of axes as in Sect. 3), with $\alpha = 90^\circ$ (this gives the highest coherent cross section). Written in this way, $\alpha_\gamma$ displays a coherent part proportional to the photon energy, typical of Born approximation. $1/X_R$ can be written in a similar way, and is monotonically decreasing. A compromise exists for a $\theta$ value which maximize $\alpha_\gamma X_R$. From our calculation it turns out that a good compromise for 30 GeV photons (which is the highest expected energy) is $\theta = 5$ mrad, for which $\alpha_{\gamma,\text{crystal}}/\alpha_{\gamma,\text{amorphous}} = 5.2$ and $(\alpha_\gamma X_R)_{\text{crystal}}/(\alpha_\gamma X_R)_{\text{amorphous}}$ levels around 2.2 for $e^\pm$ between 15 and 30 GeV and increases to 6.4 for low energy $e^\pm$. (Remember that now the “radiation length” is energy dependent). Born approximation is valid at these energies and angles. Calculations for silicon do not give interesting results. For the availability of diamond, see Sect. 4.

Then a detailed calculation of the detector response has been made. Starting from the Compton back scattering cross section of 2.33 eV circularly polarized photons against 48 GeV longitudinally polarized electrons with $P_e = +1$ (maximum back-scattered photon energy of 30 GeV), and considering a pair spectrometer field of 1.5 Tesla, a field length of 2.5 m and a drift length of 2 m, then folding with the coherent- as well as the Bethe-Heitler pair production cross sections given in fig. 2, the results of fig. 4 have been obtained. The figure represents, in arbitrary units, the energy deposition per unit detector width, versus the spectrometer particle energy, for a normal (label n) and a diamond converter (label d) of the same thickness in amorphous radiation length units. The upper and lower curves refer to the two possible circular
Fig. 3. Universal function $\phi_p(k\theta/m_c)$ for the coherent photon absorption coefficient, versus $\theta$ (units of $m_c/k$). Diamond, axis $<110>$, reference plane $(110)$, $\alpha = 90^\circ$.

Fig. 4. Energy deposition per unit detector width (arbitrary units), versus pair spectrometer electron energy (GeV) for a normal (label n) and a diamond converter (label d). Upper curves (label left): left-circularly polarized photons. Lower curves (label right): right-circularly polarized photons. LEP longitudinal polarization $P_e=+1$. 

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polarization state. The curves for diamond display ripples which reflect the discontinuous nature of the cross section (see fig. 2). The shape of the curves is about the same for the two cases, whereas the numerical values of the maxima for diamond are larger by a factor = 3. At the same time the concurrent bremsstrahlung process has not increased so much its contribution.

It is stressed that this proposal by no means represents an entirely new and major piece of work. It is only an addition for an already foreseen instrument, which can nevertheless be rewarding if the maximum tolerated amorphous converter thickness is \( \equiv 0.05 \) radiation length and if counting rate becomes critical.

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USE OF SINGLE BREMSSTRAHLUNG WITH CRYSTALS
FOR MEASURING LONGITUDINAL POLARIZATION AT LEP

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1.0) Introduction

In the previous sections of this report dedicated to the longitudinal polarimetry it has been considered in some details a well established method of measuring the longitudinal electron polarization. It is based on the detection of asymmetry effect on the Compton backscattering of circularly polarized laser photons against the longitudinally polarized \(e^+\) or \(e^-\) beams. The photon detector considered there was essentially a magnetic pair spectrometer.

Another method on the same subject, is proposed in this section.

This method [1] is related to the fast monitor for measuring luminosity and beam angular divergence at the interaction point (IP), which is discussed in another section of this proceedings [2]. It is essentially based on the detection of single bremsstrahlung photons emitted at IP due to the \(e^+e^-\) interaction, a well known physical process called also radiative Bhabha scattering. In fact for longitudinally polarized \(e^+e^-\), the highest energy single bremsstrahlung photons are fully circularly polarized [3]. Therefore if the photon polarization is measured, the electron polarization is also determined. The advantages of this method are, essentially, an high and stable (as far as the luminosity is also stable) photon rate and the measurement of electron polarization value \textit{inside} the interaction region. The experimental apparatus is mainly based on an electromagnetic calorimeter, of the same type as the one used for luminosity measurements [2].

We propose [4,5], as it will be explained in the section 1.2), to exploit single crystal properties in order to measure the single bremsstrahlung (SB) photon polarization.

Both single bremsstrahlung photons and backscattered laser photons need to cross a special low absorption "window" placed at the end of the LEP straight sections. For the SB photons a window of size \(7 \times 12\) cm\(^2\) has been proposed [2] for the SS-2, while for the laser photons a window of size \(2 \times 5\) cm\(^2\) has already been built for both SS-1 and SS-4. For the reader convenience we summarize the polarimetry proposals by the following list.

**Polarimetry proposals**

**SS-1 - Window** \(2 \times 5\) cm\(^2\)
AIM: measurement of \(e^+e^-\) transverse polarization
METHOD: angular distribution asymmetry of backscattered laser photons circularly polarized.
SS-2 - Window $7 \times 12 \text{ cm}^2$
AIM: measurement of luminosity, beam divergence and $e^+e^-$ longitudinal polarization
METHOD (for longitudinal polarization): absorption in thick crystals of circularly polarized photons from single bremsstrahlung. Circular polarization converted to linear.
Measurement of linear photon polarization: a) by absorption in a second thick crystal and measuring photon energies with a calorimeter; or b) by using a thin crystal as $e^+e^-$ converter and a magnetic pair spectrometer.

SS-4 - Window $2 \times 5 \text{ cm}^2$
AIM: measurement of $e^+e^-$ longitudinal polarization
METHOD: asymmetry in energy distribution of backscattered laser photons circularly polarized.
Or the same method as in SS-2.

1.1) Polarimetry by single bremsstrahlung

A very fast monitor for measuring luminosity and beam angular divergence at the interaction point (I.P.) of the experimental regions, has been proposed in another section of this report [2]. It is essentially based on the detection of single bremsstrahlung photons emitted at the I.P. due to the reaction $e^+e^- \rightarrow e^+e^- \gamma$. It is known since a long time [6] that the bremsstrahlung emitted in the interaction with nuclei of ultrarelativistic electrons longitudinally polarized of energy $E_0$, has a certain degree of circular polarization $P_{\gamma}^c$.
According the Ref [6]:

$$P_{\gamma}^c(X) = P_{\gamma}^1(X) \cdot P_{e^-}$$  \hspace{1cm} (1)

where $P_{\gamma}^c(X)$ is the circular polarization of the photons with fractional energy $X = \frac{K}{E_0}$, $P_{\gamma}^1(X)$ is the previous function in the case of a fully polarized electron beam, i.e. $P_{e^-} = 1$.

While the unpolarized cross section of bremsstrahlung from collisions in the $e^+e^-$ centre of mass, has been calculated by several authors (see Ref [7]), the polarized cross section has been evaluated only recently by Caffo, Gatto and Remiddi [3]. These authors are able to obtain the cross sections when both the incoming electron and the outgoing photon are polarized. They results are summarized by the expression:

$$\frac{d\sigma}{dX} = \frac{1}{2} \left[ \frac{d\sigma_0}{dX} + h_\gamma h_e \frac{d\sigma_p}{dX} \right]$$  \hspace{1cm} (2)

where $\frac{d\sigma}{dX}$ is the bremsstrahlung cross section for the helicities of the incoming electron and the outgoing photon given by $h_e$, $h_\gamma$ respectively;
$\frac{d\sigma_0}{dX}$ and $\frac{d\sigma_p}{dX}$ are the unpolarized and polarized part of the cross section respectively, given by:

$$\frac{d\sigma_0}{dX} = 4\alpha r_0^2 \frac{1}{X} \left[ \frac{4}{3} (1 - X) + X^2 \right] \left( L - \frac{1}{2} \right)$$  \hspace{1cm} (3)

$$\frac{d\sigma_p}{dX} = 4\alpha r_0^2 \frac{1}{3} (4 - X) \left( L - \frac{1}{2} \right)$$

where $L = \ln[(4E_0^2/m^2)(1 - X)/X]$. 

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By defining the photon polarization \( P_\gamma^1(X) \), \( h_\gamma = +1 \), as:

\[
P_\gamma^1(X) = \frac{\frac{d\sigma}{dX}(h_\gamma = +1) - \frac{d\sigma}{dX}(h_\gamma = -1)}{\frac{d\sigma}{dX}(h_\gamma = +1) + \frac{d\sigma}{dX}(h_\gamma = -1)}
\]  

(4)

By using (1),(2),(3) and (4) the following expression is obtained:

\[
P_\gamma^c(X) = P_e^- P_\gamma^1(X) = P_e^- \frac{X(1 - \frac{X}{4})}{1 - X + \frac{3}{4}X^2}
\]  

(5)

According to the above expression \( P_\gamma^1(X) \) is going to one for \( X \to 1 \) as shown in Fig.1. By concluding, in order to determine \( P_e^- \) we need to measure the degree of polarization \( P_\gamma^c(X) \) of the emitted photons.

1.2) Crystal analyzers for measuring high energy photon polarization

1.2a) Thin crystal method

The measurement of \( P_\gamma^c(X) \) is possible only by converting the \( \gamma \) rays in electron pairs using a thin or a thick crystal target. These two different methods require a completely different experimental approach. Now we start to explain in detail the first method.

The differential cross section for pair production by circularly polarized high energy photons on nuclei has been derived in Ref [6,8] and, after summing up over positron and electron spins, it has the form:

\[
d^3\sigma = (a + b\vec{P}_\gamma \cdot \vec{n})dE_+d\Omega_+d\Omega_-
\]  

(6)

where:
- \( \vec{n} = \vec{u} \times \vec{v} \);
- \( \vec{u} \) and \( \vec{v} \) are, respectively, the components of the momenta \( \vec{P}_+ \), \( \vec{P}_- \) of the positron and the electron perpendicular to the photon momentum \( \vec{K} \);
- \( \vec{P}_\gamma = \pm P_\gamma^c \frac{\vec{K}}{|\vec{K}|} \) where \( \pm P_\gamma^c \) is the photon polarization and the double sign is referred to right or left circular polarization;
- \( a \) and \( b \) are dependent on \( \vec{P}_+ \), \( \vec{P}_- \), \( \vec{K} \), on the energies of the positron and electron \( E_+ \) and \( E_- \) and on the atomic number \( Z \) of nucleus.

The quantity \( b \) is zero in the first order Born approximation and the contribution to it comes from higher order approximation. If the second term of (6) is different from 0, then it is possible to define an asymmetry ratio \( A \) as follows.

Let us define for the positron polar and azimuthal angles \( \theta_+ \) and \( \phi_+ \) respect to the photon momentum direction \( \vec{K} \) (see fig. 2) so that \( \vec{u} = \vec{P}_+ \sin \theta_+ \). Similarly the corresponding quantities \( \theta_- \), \( \phi_- \) and \( \vec{v} \) are defined for the electron. For a right handed incoming photon, let us define the cross section \( d\sigma_R \), \( d\sigma_L \) for emission of the electron to the right or to the left of the positron emission plane respectively, with respect to an observer looking at the direction of \( \vec{K} \) while \( \vec{u} \) is pointing upward.
Then $d\sigma_R$ is obtained by integrating eq.(6) over all $\theta_+$ and $\theta_-$ and over $(\phi_+ - \phi_-)$ from 0 to $+\pi$, while $d\sigma_L$ is obtained by integrating eq.(6) also over all $\theta_+$ and $\theta_-$, but over $(\phi_+ - \phi_-)$ from 0 to $-\pi$.

Finally the asymmetry ratio $A$ turns out to be linearly dependent on $P_c^z$; in fact we get:

$$A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = P_c^z \cdot F(K, E_+, Z)$$

(7)

where $F(K, E_+, Z)$ is the result of integration explained above.

The analytic integration of eq.(6) is quite difficult. For a particular choice of values $(\theta_+ \simeq 1/E_+ , \theta_- \simeq 1/E_- ; E_+ = E_- = K/2 ;$ recoil momentum of nucleus $q = |\vec{q}| = \sqrt{2}$; - all in $mc^2$ and $mc$ units - ) we calculated numerically the asymmetry $A$ versus $Z$ and the result is compared with Kolbenstvedt and Olsen calculations [9] for identical choices (see Fig.3). Note a great discrepancy at high $Z$ between the two curves due to some approximation in Kolbenstvedt and Olsen calculation, while the exact calculation † is always less than one, as it must be.

The integration over all the angles of eq.(6), for finding $A$ has been performed with a Monte Carlo multidimensional integration method. This method is based on randomly extraction of the five independent kinematic parameters that characterize the kinematic of the pair production process. Then all the other parameters are evaluated consistently with the allowed phase-space region. Finally the cross section is integrated with usual Monte Carlo method.

This integration gives an asymmetry ratio, in the case of an amorphous germanium converter ($Z = 32$), for fully right circularly polarized photons and for symmetric energies of the $e^+e^-$ pair ($E_+ = E_- = K/2$), $A = 0.080$.

This asymmetry ratio $A$ is difficult to measure because the cross section of pair production decreases quickly at high transverse recoil momentum of nucleus, therefore the production planes of $e^+$ and $e^-$ are almost coplanar. To avoid problems in the determination of the left or right position of the electrons, due to multiple scattering, the use of a thin crystal as converter, has been suggested [1].

Coherent high energy electromagnetic interactions in crystals have been studied extensively both experimentally and theoretically [10].

In a crystal we can distinguish a coherent part $\sigma_c$ of pair production cross section from an incoherent one $\sigma_i$. The coherent part is not zero only when the recoil momentum $\vec{q}$ of nucleus coincides with a point of the reciprocal lattice of the crystal, i.e. when $\vec{q} = \vec{g}$ where $\vec{g}$ is a reciprocal lattice vector.

Since $\frac{\sigma_c}{\sigma_i} \propto \frac{1}{2}$ and $A$ is an increasing function of $Z$ a good compromise seems to be a germanium crystal. The ratio $\frac{\sigma_c}{\sigma_i}$ can be increased by cooling the crystal.

The features of the coherent cross section are determined by the allowed recoil momentum space. In fact since the component of $\vec{q}$ along $\vec{K}$, i.e. $q_\parallel$, cannot be less than a $q_{\parallel\min}$ value due to the quadri-momenta conservation and the cross section is going to zero for $q_\parallel > 2q_{\parallel\min}$, the allowed recoil momentum space is a thin disk shaped region as shown in Fig.4. This feature combined with the coherent effects in crystals permits to "fix" the value and/or direction of the recoil momentum $\vec{q}$, orienting the crystal so that only a single reciprocal lattice point or a special row of points are inside the disk shaped region.

† For "exact" calculation we intend our "numerical" evaluation of $\frac{d^3\sigma}{dE_+d\Omega_+d\Omega_-}$ in [8] at an order higher than the Born approximation
Since a thin crystal "fix" the recoil momentum $\vec{q}$, we can obtain a well defined semi-plane $[\vec{q}, \vec{K}]$ in which $\vec{q}$ lies. Let us call $\vec{q}_\perp$ the component of $\vec{q}$ perpendicular to $\vec{K}$, while $\vec{q}_\perp + \vec{u} + \vec{v} = 0$. Assume that we have fixed $\vec{q}$ in the left semi-plane respect to $\vec{K}$. If the electron travels to the right (left) of positron emission plane, the positron is up (down) with respect to the plane $(\vec{q}, \vec{K})$ (see Fig.5a).

By this way the experimental measurement of the asymmetry ratio $A$ is simplified. The collimated and polarized bremsstrahlung photons crosses a thin crystal where $e^+e^-$ pairs are coherently produced and analyzed by a magnetic spectrometer. The crystal orientation is chosen so that the $(\vec{q}, \vec{K})$ plane is coincident with the medium magnetic plane of the spectrometer. Assuming the incoming photons are right circularly polarized we can obtain $A$, simply by counting the number of electrons (or positrons) up and down the medium plane of spectrometer (see Fig.5b) and subtracting the incoherent pair production contribution. The experimental value of asymmetry, for a selected momentum range, is given by:

$$A_{exp} = \frac{N_{e^+}^{up} - N_{e^-}^{down}}{N_{e^+}^{up} + N_{e^-}^{down}} = \frac{N_{e^+}^{up} - N_{e^-}^{down}}{N_{e^+}^{up} + N_{e^-}^{down}}$$

(8)

Possible sources of systematic errors can be checked comparing second and third member of eq.8, that must be equal inside the statistic, or changing crystal orientation so that $\vec{q}_\perp$ is inverted, up-down distribution are exchanged and the same $A_{exp}$ should be obtained.

A Monte Carlo simulation of this experimental situation has been done. We have chosen a germanium ($Z = 32$) monocystal which has a diamond lattice with a constant $a = 5.66\text{Å}$. The orientation of the crystal can be chosen so that only one row of the first reciprocal lattice point is involved. The resulting asymmetry ratio for fully right circularly polarized photons and for symmetric positron and electron energies ($E_+ = E_- = K/2$) is given in Fig.6 as a function of the recoil momentum $\vec{q}$, while after summation over allowed $\vec{q}$, the value $A = 0.052$ is obtained. For a tungsten crystal ($Z = 74$) the expected value is $A \approx 0.11$.

It has been shown as the use of a thin crystal, instead of an amorphous converter, makes the measurement of the $e^+e^-$ pair asymmetry ratio possible. The electron pairs from the crystal converter are energy selected using a pair spectrometer, while a detector such as a microstrip device of 20 $\mu$m pitch can be used in order to measure the electron and positron angular vertical distribution.

A draw back of this method consists on the necessity to use crystals having a thickness $< 10^{-3}$ radiation length, in order to make the multiple scattering rms angle smaller than the natural $e^+e^-$ emission angle $\theta = \frac{m}{E}$, which is $\approx 10^{-5}$ rad for LEP-I.

1.2b) Thick crystal method

For the reason stated above another method has been proposed [4,5], based on the property that the absorption cross sections for polarized high energy photons crossing thick crystals depend on the direction of the unit polarization vector $\vec{e}$ with respect to a crystal axis in the form [11]:

$$\Sigma(\vec{K}, \vec{e}) = A + B(\vec{e} \cdot \vec{\varepsilon})^2$$

(9)
where $\vec{t}$ is a unit vector orthogonal to the momentum $\vec{K}$ of the photons and related to the orientation of the lattice.

Let us consider, for example, a cubic crystal and a photon whose momentum $\vec{K}$ lies in the (010) plane of the crystal making a small angle $\alpha$ with the [100] axis (see Fig.7).

The polar plot of the absorption cross section $\Sigma(\vec{K}, \vec{e})$ as a function of the angle between $\vec{e}$ and the crystal plane (010) is an ellipse. Since the (010) plane is, in our example, a symmetry plane, an axis of the ellipse lies in this plane and we can choose the vector $\vec{t}$ along the same axis.

If $\vec{y}$ is a unit vector perpendicular to the plane $(\vec{K}, \vec{t})$, i.e. to the (010) plane, the polarization vector will be in general of the form:

$$\vec{e} = \epsilon_\parallel \vec{t} + \epsilon_\perp \vec{y}$$

The components $\epsilon_\parallel, \epsilon_\perp$ have a different degree of absorption in the crystal so that their intensities after having penetrated a thickness $x$ of the crystal are given by:

$$I^\parallel(x) = I^\parallel(0) \cdot \exp[-\Sigma^\parallel x] \quad (11a)$$

$$I^\perp(x) = I^\perp(0) \cdot \exp[-\Sigma^\perp x] \quad (11b)$$

where

$$\Sigma^\parallel = \Sigma(\vec{K}, \vec{e} = \vec{t}) \quad (12a)$$

$$\Sigma^\perp = \Sigma(\vec{K}, \vec{e} = \vec{y}) \quad (12b)$$

We can define a degree of polarization $P(x)$ for originally unpolarized beam ($I^\parallel(0) = I^\perp(0)$), by:

$$P(x) = \frac{I^\parallel(x) - I^\perp(x)}{I^\parallel(x) + I^\perp(x)} = \tanh[\frac{1}{2}x(\Sigma^\perp - \Sigma^\parallel)] \quad (13)$$

Moreover, the phase of the components will change progressively, differently for each one, while the photon beam propagates along the crystal thickness. The components before and after having penetrated a thickness $x$ of the crystal are related by a $2 \times 2$ matrix [12]:

$$\begin{pmatrix} \epsilon^\parallel(x) \\ \epsilon^\perp(x) \end{pmatrix} = \begin{pmatrix} \exp[\imath n^\parallel \omega x] & 0 \\ 0 & \exp[\imath n^\perp \omega x] \end{pmatrix} \cdot \begin{pmatrix} \epsilon^\parallel(0) \\ \epsilon^\perp(0) \end{pmatrix} \quad (14)$$

where $\omega = |\vec{K}|$ † and $n^\parallel$ and $n^\perp$ are complex quantities analogous to the refraction index in optics. The imaginary parts of $n^\parallel$ and $n^\perp$ are related to the absorption cross sections $\Sigma^\parallel$ and $\Sigma^\perp$:

$$\text{Im}(n^\parallel) = \frac{\Sigma^\parallel}{2\omega} \quad (15a)$$

$$\text{Im}(n^\perp) = \frac{\Sigma^\perp}{2\omega} \quad (15b)$$

The real parts can be derived from them by using the dispersion relations [12].

The crystal acts as a $\frac{\lambda}{4}$ plate if the relative phase of the two components is changed by $\frac{\pi}{2}$ so that:

$$\text{Re}(n^\perp - n^\parallel) \omega x = \frac{\pi}{2} \quad (16)$$

† in the usual unit $\hbar = c = 1$
For measuring the circular polarization $P^c_\gamma$ of the incoming photon we can convert it in a linearly polarized photon by a first crystal acting as a $\frac{\lambda}{4}$ plate and then analyzing the linear polarization of the outgoing photon with a second crystal (see Fig.8).

In order to give an example of the method proposed and an indicative idea of the quantities involved, let us consider two cubic Cu crystals cooled at $77^\circ K$ and a monochromatic circularly polarized photon beam of 40 GeV (following Ref.[12]).

The best result at this energy is obtained, for both crystals, with the direction of the momentum $\vec{K}$ lying in the (010) plane and making an angle $\alpha = 0.456$ mrad with the [101] axis. The first crystal will act as a $\frac{\lambda}{4}$ plate if we choose its thickness to be:

$$
\chi_0 = \frac{\pi}{2} \frac{1}{\omega \text{Re}(n^\perp - n^\parallel)} = 0.273 \text{ cm}
$$

(17)

The outgoing photons from the first crystal are now polarized with a degree of linear polarization $P_l = P^c_\gamma$, where $P^c_\gamma$ is the initial circular polarization of the photon beam. The linear polarization $P_l$ is modified to elliptical one by changing either the crystal thickness or the photon energy.

The linear polarization of the outgoing photons from the first crystal can be analyzed with the second one. In fact the transmission coefficient $T(x)$ for linearly polarized photons depends on the angle $\phi$ between the polarization vector $\vec{P}_1 = P_1 \hat{e}$ and the plane in which $\hat{e}$ lies (i.e. (010) plane), as follows:

$$
T(x) = \begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 = 0} = \begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 = 0} \cdot [1 + |\vec{P}_1| \cdot P(x) \cos 2\phi]
$$

(18)

It is possible to measure $P_l$ - and then $P^c_\gamma$ - rotating the second crystal and measuring the transmission coefficient at different angles $\phi$, being known $\begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 = 0}$ and $P(x)$.

The last quantities are given by theory:

$$
\begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 = 0} = \exp \left[ - \frac{\tanh^{-1} P(x)}{E} \right] \sqrt{1 - P(x)^2}
$$

(19)

where $E = \frac{\Sigma^\perp - \Sigma^\parallel}{\Sigma^\perp + \Sigma^\parallel}$ and $P(x)$ is given in eq.(13).

It can be shown that in order to minimize the error $\Delta P_l / P_l$, a crystal thickness $x \approx 2$ cm should be chosen. Moreover, by taking from Ref.[11] the values for $\Sigma^\parallel$, $\Sigma^\perp$, we obtain $\begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 = 0} = 0.133$ and $P(x) = 0.319$.

The behaviour of the transmission coefficient defined in eq.(18) as a function of the angle $\phi$, assuming as initial circular polarization $P^c_\gamma = 50\%$, is shown in Fig.9.

It would be very convenient to avoid the knowledge of the theoretical values of $\begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 = 0}$ and $P(x)$, in order to obtain the value of $P_l$.

This is indeed possible with the following experimental procedure.

First let us determine the following asymmetry ratio:

$$
C = \frac{\begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 (\phi = 0)} - \begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 (\phi = \pi / 2)}}{\begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 (\phi = 0)} + \begin{bmatrix} I(x) \\ I(0) \end{bmatrix}_{\vec{P}_1 (\phi = \pi / 2)}} = P_l P(x)
$$

(20)
where \( \frac{I(x)}{I(0)} \hat{P}_1 \) for \( \phi = 0 \) and \( \phi = \frac{\pi}{2} \) are given by the maximum and the minimum respectively of the normalized photon counting rate, measured by the calorimeter behind the second crystal (see Fig.8) in a narrow energy range up to 40 GeV.

Second, we need to calibrate the second crystal of Fig.8, in order to determine its analyzing power \( P_a = P(x) \). For this aim, let us take another crystal equal to the previous one, and put both crystals equally oriented on an unpolarized photon beam of 40 GeV. Then the linear polarization \( P_0 \) of the photons emerging from the upstream crystal turns out to be equal to the analyzing power \( P_a = P_0 \) of the downstream crystal. Therefore finally we have \( C_0 = P_0 P_a = P(x)^2 \) and \( P(x) = \sqrt{C_0} \).

Now we have obtained a calibrated analyzer to be used, for the polarization measurement, as the second crystal shown in Fig.8. Therefore finally from eq.(20) we get:

\[
P_t = \frac{C}{P(x)} = \frac{C}{\sqrt{C_0}}
\]

(21)

A more general treatment of the calibration method using thick or thin crystal is discussed in Ref.[13].

We estimate now the statistical error which affects the measurement of \( P_t \).

In fact, calling

\[
T_{0^0} = \frac{I(x)}{I(0)} \hat{P}_1 (\phi = 0^0)
\]

\[
T_{90^0} = \frac{I(x)}{I(0)} \hat{P}_1 (\phi = 90^0)
\]

the eq.(20) becomes

\[
C = \frac{T_{90^0} - T_{0^0}}{T_{0^0} + T_{90^0}} = P_t P(x)
\]

(22)

and we have:

\[
\frac{\Delta P_t}{P_t} = \frac{\Delta C}{C} + \frac{\Delta P(x)}{P(x)}
\]

(23)

From the eq.(22) we obtain:

\[
\frac{\Delta C}{C} = \left| \frac{1}{T_{0^0} - T_{90^0}} - \frac{1}{T_{0^0} + T_{90^0}} \right| \Delta T_{0^0} + \left| \frac{1}{T_{0^0} - T_{90^0}} + \frac{1}{T_{0^0} + T_{90^0}} \right| \Delta T_{90^0}
\]

(24)

The transmission coefficients are given by:

\[
T_{0^0} = \frac{N_1}{N_0}
\]

\[
T_{90^0} = \frac{N_2}{N_0}
\]
where $N_0$ is the photons rate incident on the second crystal and $N_1$, $N_2$ are the photons rates behind the second crystal.

The errors on the transmission coefficients $T_{0^0}$ and $T_{90^0}$ are given by:

$$\frac{\Delta T_{0^0}}{T_{0^0}} = \frac{1}{\sqrt{N_0}} + \frac{1}{\sqrt{N_1}}$$

$$\frac{\Delta T_{90^0}}{T_{90^0}} = \frac{1}{\sqrt{N_0}} + \frac{1}{\sqrt{N_2}}$$

which allow the evaluation of $\Delta T_{0^0}$ and $\Delta T_{90^0}$ in eq.(24).

The statistical error $\frac{\Delta P(x)}{P(x)}$ is obtained from the relation $P(x) = \sqrt{C_0}$ as follows:

$$\frac{\Delta P(x)}{P(x)} = \frac{1}{2} \frac{\Delta C_0}{C_0}$$

(25)

where $C_0$ refers to the calibration procedure of the second crystal explained above. Such a calibration procedure can be performed in a test photon beam. Assuming to accumulate an high enough statistics, the term $\frac{\Delta P(x)}{P(x)}$ will be negligible respect to the other term $\frac{\Delta C}{C}$ so that, concerning the statistical error, we have $\frac{\Delta P}{P_1} \approx \frac{\Delta C}{C}$.

In fig.10a and 10b we plot $\Delta P_1/P_1$ vs the thickness $x$ of the second crystal. The polarization of the beam is assumed $P_0^\gamma = P_1 = 50\%$. Two case of photon flux emitted from LEP interaction point are taken into account at a luminosity of $10^{31}$ cm$^{-2}$ sec$^{-1}$: $10^5$ $\gamma$/sec $(10^4$ $\gamma$/sec), corresponding approximatively to an energy range of $\approx 10$ GeV ($\approx 1$ GeV) of the upper part of bremsstrahlung spectrum.

Then we estimate $N_0$ by taking into account (see also Ref.[2])

- acceptance of photon window (0.185 for $7 \times 5$ cm$^2$ window),
- beryllium absorber
- the transmission coefficient of the $\frac{1}{4}$ plate (the first crystal)
- a running time of 2.5 hours

We obtain the photon flux $N_0$ on the second crystal for the two cases: $N_0 \approx 4.5 \cdot 10^7$ ($N_0 \approx 4.5 \cdot 10^8$). Assuming a total measuring time of 5 hours ($2.5 \times 2$) the minimum errors $\Delta P_1/P_1$ obtained would be 0.38% (1.2%).

In Fig.11a and 11b we plot - taking into account the two cases - $\Delta P_1/P_1$ vs the polarization $P_1$ with a thickness of the second crystal fixed at $x \approx 2$ cm.

We did not take into account the systematic errors. Those due to the misalignment of the crystals can be experimentally estimated by performing appropriate "rocking" curves.

Conclusions

We suggested to measure the circular polarization of the high energy SB photons in order to determine the longitudinal electron polarization inside the LEP interaction region.
The circular polarization could be measured with two different method involving (a) the use of a thin crystal photon converter, or (b) photon absorption in thick crystals. The method (b) seems to be experimentally the most convenient.

An estimation of statistical error in polarization measurement has been performed in case of using a copper crystal. It is shown that an error $\Delta P_i / P_i \approx 0.38\%$ can be achieved at a luminosity of $10^{31}$ cm$^{-2}$ sec$^{-1}$ in a total of 5 hours, assuming a constant polarization value of 50%.

A further analysis needs in order to evaluate the systematic errors. If they will be kept under 0.3% then this method could be applied also to the so called "Blondel scheme" [14].

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contributed paper to these proceedings.

Figure captions

Fig.1
Circular polarization of the bremsstrahlung photons emitted with fractional energy $X = \frac{K}{E_0}$ for fully longitudinal polarized $e^+, e^-$ beam with energy $E_0$.

Fig.2
Kinematics of $e^+ e^-$ pair production.

Fig.3
Comparison between the Kolbenstvedt and Olsen asymmetry ratio calculation and our numerical evaluation obtained from Ref [8]

Fig.4
The diamond reciprocal lattice in the incidence plane and the intersection of the recoil momentum space.

Fig.5
a) Kinematics of $e^+ e^-$ pair production in the reciprocal lattice plane normal to the photon momentum $\vec{K}$.

b) Sketch of the angular distribution for positrons and electrons up and down the medium magnetic plane.

Fig.6
Montecarlo evaluation of the asymmetry ratio vs the recoil momentum $\vec{q}$ in a germanium crystal. Simmetric pair case.

Fig.7
The intersection between the reciprocal lattice with the allowed recoil momentum space for copper crystal case (see text).

Fig.8
Sketch of the experimental set-up for measuring $P_t$. 

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Fig. 9
Transmission coefficient of the crystal vs the angle $\phi$ between the polarization vector and the (010) plane. $P^c_\gamma = 50\%$, crystal thickness $x = 2$ cm (Copper, $T = 77^\circ K$).

Fig. 10
$\frac{\Delta P_t}{P_t}$ vs the thickness $x$ of the second crystal shown in Fig. 8 in the case of a photon flux (a) $N_0 \approx 4.5 \cdot 10^7$ and (b) $N_0 \approx 4.5 \cdot 10^6$

Fig. 11
$\frac{\Delta P_t}{P_t}$ vs the polarization $P_t$ in the same cases of Fig. 10.
Fig. 1

Fig. 2
Fig. 5a

Median magnetic plane

Fig. 5b
total asymmetry ratio = 5.2%

recoil momentum $q$ (mc units)
(germanium crystal - $Z=32$ - $E^+/K=0.5$)

Fig. 6

[Diagram showing crystal lattice with labels and axes]

Fig. 7
\[ \frac{\lambda}{4} \text{ plate equivalent for high energy circularly polarized } \gamma \]

**Fig. 8**

**Fig. 9**

**TRANSMISSION COEFFICIENT**

![Graph showing transmission coefficient as a function of \( \phi \) (radians)](image)
NORMALIZATION

Normalization, G. Coignet
Monitoring of the LEP luminous region, G. von Holtey
Normalization in ALEPH, H. Burkhardt et al.
Normalization in DELPHI, L. Bugge et al.
Normalization in L3, M. Athanas et al.
Precision attainable on relative luminosity measurements with the OPAL detector, G. Alexander et al.
A single bremsstrahlung monitor for measuring luminosity and beam divergence at LEP, M. Chen et al.
NORMALIZATION

G. Coignet
Convener of the Working Group

The luminosity monitors of the four LEP experiments have been initially designed to determine the luminosity with an absolute error \((\Delta L/L)_{\text{abs}} \leq 2\%\). For this purpose each experiment has built a "standard" Small Angle Tagger (SAT) to detect Bhabha events with a minimum scattering angle \(\theta_{\text{min}} \geq 30-50\) mrad depending on the experiment. Working at the Z energy with the nominal luminosity \(L \approx 10^{31}\) cm\(^{-2}\) s\(^{-1}\), each SAT is expected to reach a statistical precision of about 2% on a run (fill) to run (fill) basis. The uncertainty in the absolute luminosity will finally be dominated by other run independent errors: theoretical uncertainty on the Bhabha cross section, manufacturing tolerances and positioning of the detectors, etc. Run-dependent errors may arise from uncertainties in estimating the effects of machine backgrounds on detection efficiencies.

To reduce the statistical error to a negligible level, most experiments have designed a very small angle luminosity monitor (called VSAT, SALM or FFD) to measure Bhabha events at an angle of 5 to 7 mrad. A very high rate detector to measure the photons emitted at zero degrees in the reaction \(e^+e^- \rightarrow e^+e^-\gamma\) has also been proposed. This detector would also provide information on beam divergence over a reduced angular acceptance region.

With polarized beams, in the four bunch scheme proposed for LEP by A. Blondel, the left-right asymmetry \(A_{\text{LR}}\) could be measured with a precision \(\Delta A_{\text{LR}}\) which can be expressed as:

\[
\Delta A_{\text{LR}} = \sqrt{\frac{(1 + 1/\gamma)(1 + \rho)^2}{p^2 N}} + \frac{1 + \rho^2}{4p^2} \left( \frac{\Delta L_i}{L_j}\right)_{\text{syst}}^2 + A_{\text{LR}}^2 \left( \frac{\Delta F}{F}\right)_{\text{syst}}^2
\]

with

\(P\) : beam polarization
\(N\) : total number of Z events
\[ \gamma = \frac{\text{Bi}}{N_i} \quad \mid \quad i = 1, 4 \quad : \quad \text{ratio of Bhabha events to Z events} \]

\[ \rho = \frac{A_{LR} \left( \frac{1}{F} - P \right)}{1 - A_{LR}^2 P^2} = \frac{N_3 + N_4}{N_1 + N_2} \]

\[ \Delta \left( \frac{L_i}{L_j} \right)_{\text{syst}} : \text{systematic error on the relative bunch } i \text{ to bunch } j \]

\[ \left( \frac{\Delta P}{P} \right)_{\text{syst}} : \text{systematic error on polarization measurement} \]

Table 1 shows an estimate of the total number of Z events, as a function of \( \gamma \) and for different values of \( \Delta \left( \frac{L_i}{L_j} \right)_{\text{syst}} \), needed to measure \( A_{LR} = 0.15 \) with an error \( \Delta A_{LR} = 3^\circ/_{\circ\circ} \). Two values of beam polarization \( P \), assumed to be determined with \( \left( \frac{\Delta P}{P} \right)_{\text{syst}} = 3^\circ/_{\circ\circ} \), are considered.

<table>
<thead>
<tr>
<th>( P ) (%)</th>
<th>( \Delta \left( \frac{L_i}{L_j} \right)<em>{\text{syst}} ) ( (\circ/</em>{\circ\circ}) )</th>
<th>( \gamma = 1 )</th>
<th>( \gamma = 4 )</th>
<th>( \gamma = 15 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.0</td>
<td>1.6</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.9</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>2.7</td>
<td>1.7</td>
<td>1.5</td>
</tr>
<tr>
<td>30</td>
<td>0.5</td>
<td>6.2</td>
<td>3.9</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>9.3</td>
<td>5.8</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>48.0</td>
<td>30.0</td>
<td>25.5</td>
</tr>
</tbody>
</table>

Since run-independent errors will cancel out, the systematic errors affecting the relative luminosity measurement \( \Delta \left( \frac{L_i}{L_j} \right)_{\text{syst}} \) will be due to systematic differences between various bunches circulating in the machine. The following sources of systematic errors have been considered:
i) shape and position of the luminosity region,
ii) beam divergence and tilt at the interaction region,
iii) machine backgrounds,
iv) physics backgrounds.

The sensitivity of the SAT and of the VSAT measurements to the effects i) and ii) have been investigated by Monte Carlo simulation. Typical values of beam position and angle, which will be monitored by the LEP instrumentation and independently by the central detector and the SAT detector of each experiment, have been assumed. Two types of machine background have been taken into account: the photons from synchrotron radiation and the off-momentum electrons (positrons) due to beam gas bremsstrahlung. It is hoped that these parasitic fluxes will be sufficiently reduced by a set of collimators implemented in the LEP main ring. It is planned to measure the off-momentum electrons due to beam gas interactions and then to subtract them statistically. The synchrotron radiation will add energy rather homogeneously and hence only change the effective trigger thresholds. Finally the physics background, mainly two photon physics event, has been estimated to be negligible.

From this study, the following conclusions can be drawn:

- The relative luminosity measurement, with the accuracy required to carry out polarized beam experiments, relies on the monitoring of the beam position and divergence and of machine background. It appears that the beam parameters can be measured with the necessary precision by LEP instrumentation and also by the experiments themselves. The effects due to off-momentum electrons and to synchrotron radiation are expected to be negligible on the relative bunch to bunch luminosity measurement.

- The SAT detector measurements are stable against variations of the beam at the interaction point. Under the assumptions considered, the required systematic precision accuracy on the relative bunch to bunch luminosity measurements should be achievable with these luminosity detectors. Their main limitation comes from their counting rate.

- The VSAT detector measurements can provide high statistical accuracy but are more sensitive to variations of the interaction parameters, and especially to horizontal divergence and position changes. These variations
cause systematic errors which render the use of these detectors more difficult; a good position measurement would certainly improve the situation. At present, there is no general agreement that these detectors can be used to measure the relative luminosity with the accuracy needed in the case of polarized beam experiments.

- The proposal based on the measurement of the single bremsstrahlung photons emitted around zero degrees offers the possibility to monitor certain parameter changes in the luminous region very quickly. Its main limitation, for the purpose discussed here, comes from its limited angular acceptance. However, it could be considered as a first part of a device to measure the longitudinal polarization.

- After some running time at LEP (without polarization) has occurred, information on monitor behavior and LEP instrumentation performances will be obtained. Realistic conditions will then be available to reassess the present conclusions. It appears anyway from this study that a reduction of the beam pipe radius (to 6 cm or less) would allow for coverage of the $\simeq 25$ to $\simeq 60$ mrad angular region, which is the best region for normalization as far as systematic and statistical errors are concerned. Except for the L3 experiment which already has a luminosity monitor covering this angular region, the other experiments would need to construct new luminosity detectors.

Contributors to the Normalization Working Group:

MONITORING OF THE LEP LUMINOUS REGION

G. von Holtey

1. Introduction

A very high accuracy is required for the determination of the left-right asymmetry $A_{LR}$ in order to make physics measurements with longitudinal polarized beams in LEP fruitful. The aimed precision is $\Delta A_{LR} \leq 0.003$ [1].

The error $\Delta A_{LR}$ depends on the precision in determining the ratio of annihilation cross-sections $\Delta(\sigma_1/\sigma_2)$ and therefore, besides statistical fluctuations in the counting rates, on the error in the measurement of the luminosity ratio $\Delta(L_1/L_2)$. The indices 1 and 2 refer to two different bunch pairs with different polarization configurations, crossing at an interaction point [2].

As is shown in the following papers of this chapter [3], the above goal in the measuring precision of $A_{LR}$ asks for a determination of relative luminosity changes between bunch pairs (with different polarisation configuration) to less than 0.1%.

All four LEP experiments incorporate luminosity monitors, that rely on the detection of Bhabha events at very small scattering angles.

$$\hat{N}_{\text{Bhabha}} = 4r^2 \left(\frac{\mu c^2}{E}\right)^2 \int_{\Delta \Omega_i} d\theta d\phi$$

(1)

where the geometric acceptance $\Delta \Omega_i$ of the luminosity detectors depends on the position and extend of the luminous region (LR) created by the different bunch pairs i around the interaction point. Systematic bunch-to-bunch changes of the geometric acceptance can therefore be induced by bunch-to-bunch variations of luminous region parameters.

This sets stringent requirements on the monitoring of the luminous region. Some of the LR-parameters can be measured directly by the experimental detectors, others will have to be induced from...
measurements of beam- and optics parameters outside the interaction region by the LEP instrumentation.

Since the luminosities of interest are always measured over a large number of beam revolutions in LEP, the bunch-to-bunch correction can be derived from average values of LR-parameters, provided their relationship can be linearized.

2. Definition of the luminous region

The luminous region*) spanned by two crossing bunches around the interaction point is characterized by the position and extend of the overlapping bunches (see Fig. 1, $x =$ horizontal, $y =$ vertical, $z =$ longitudinal):

- the bunch centroid position $<x^*>; <y^*>; <z^*>$
- and tilts $<x'^*>; <y'^*>$
- the bunch r.m.s. widths $\sigma^*_x; \sigma^*_y; \sigma^*_z$
- and angular spreads $\sigma^*_x; \sigma^*_y; \sigma^*_z$
- the vertical beam separation $\Delta h^*$

![Fig. 1 - Luminous Region](image)

Nominal bunch parameters for LEP beams at 45 GeV and 3 mA current are taken from [4] and listed in Table 2. Bunch size and angular spread will increase by about 30 % if beam currents of 5 mA can be

*) Note that for perfectly overlapping bunches with $\sigma^*_x, \sigma^*_y, \sigma^*_z$, the luminous region is an ellipsoid with r.m.s. values $\sigma^*_x/\sqrt{2}, \sigma^*_y/\sqrt{2}, \sigma^*_z/\sqrt{2}$.
reached, they will decrease (width) and increase (angular spread) by 23% if the beta-values at the interaction points can be squeezed down to 60% of their present nominal values of $\beta_x^* = 1.75 \text{ m}$ and $\beta_y^* = 0.07 \text{ m}$.

The range for the LR position and tilt is estimated from a closed orbit amplitude of 1 mm r.m.s. measured at the nearby pick-up electrodes which we believe can be obtained with a careful correction of the orbit around the interaction region.

The nominal bunch length $\sigma_z^* = 12.8 \text{ mm}$ increases to $\sigma_z^* = 33 \text{ mm}$ if dedicated polarisation wigglers are added [5].

In optimum luminosity conditions beams are colliding head-on, but a small jitter in the beam separation of $\Delta h = \pm 1 \mu\text{m}$ remains due to the voltage stability of the electrostatic separator generators.

3. Monitoring of luminous region parameters

None of the LR-parameters defined in para. 2 can be measured directly by the LEP machine instrumentation. Their values must be inferred from measurements outside the interaction region, using appropriate optics transformations to the interaction point.

The bunch average position and direction at the crossing point are derived from measurements of the bunch position at two neighbouring pick-up electrodes located at positions $\pm s$ upstream and downstream of the crossing.

$$
\begin{pmatrix}
\langle x \rangle^* \\
\langle x' \rangle^*
\end{pmatrix}
= M \begin{pmatrix}
\langle x \rangle^* \\
\langle x' \rangle^*
\end{pmatrix} 
= M \begin{pmatrix}
\langle x \rangle^* \\
\langle x' \rangle^*
\end{pmatrix} $$

(2)

The transformation $M$ depends on the betatron-amplitude functions at the interaction point and at the two pick-up positions as well as on the betatron-phase advance between them. The bunch width and angular spread at the crossing point are deduced from a measurement of the bunch transverse profile $\sigma_\theta$ in one of the bending arcs

$$
\sigma^* = \sigma_\theta \sqrt{\beta^*/\beta_0}, \quad \sigma'^* = \sigma_\theta \sqrt{\beta^*/\beta_0}
$$

(3)
where $\beta_0$ is the measured betatron-amplitude at the position of the profile-meter.

Twiss parameters and bunch transverse profiles are determined with absolute and relative errors as given in Table 1.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Measurement absolute value</th>
<th>Accuracies relative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse bunch positions</td>
<td>300 $\mu$m</td>
<td>25 $\mu$m</td>
</tr>
<tr>
<td>Twiss parameters</td>
<td>5 %</td>
<td>2 %</td>
</tr>
<tr>
<td>Transverse bunch profiles</td>
<td>5 %</td>
<td>1 %</td>
</tr>
<tr>
<td>Bunch intensities</td>
<td>$&lt; 1%$</td>
<td>$2 \times 10^{-4}$ in 1 sec</td>
</tr>
</tbody>
</table>

**Table 1**

Measurement uncertainties of LEP beam parameters (outside the interaction regions)

A sophisticated procedure can be used to establish the twiss-parameters at a collision point with an ultimate accuracy of about 1 % [6]. But these parameters correspond to a machine without beam-beam effect (separated beams). When beams with nominal intensity and emittance are collided during a physics run the betatron amplitude functions $\beta^*$ change by about 30 %. This fact can be introduced as a correction on the previously determined values, but the uncertainty on the correction reduces the accuracy to which the twiss parameters at the interaction point are known in absolute terms.

Bunch-to-bunch differences of the LR-parameters, however, can be monitored by measuring individual bunch positions, profiles and intensities outside the interaction region and thereby assessing the equality of the beam-beam forces. This equality is valid for nominal LEP optics and optimum luminosity conditions to within two percent, as long as bunch-to-bunch variations of intensity and emittance stay below about five percent [7].
The longitudinal position \( \langle z \rangle^* \) of the interaction point and the bunch length \( \sigma_z^* \) will be measured directly by the experimental detectors with a statistical uncertainty of \( \pm 0.7 \) mm in \( \langle z \rangle^* \) and \( \pm 0.5 \) mm in \( \sigma_z^* \) for a collected sample of 2500 clean two-track events [8]. \( \langle z \rangle^* \) is completely defined by the longitudinal alignment of the RF-cavities with respect to the crossing point, and should not differ by more than \( \pm 1 \) mm from its theoretical position.

Resulting uncertainties in the relative bunch-to-bunch measurements of LR-parameters are summarized in Table 2, column 3. It has been assumed that they will be measured in time intervals of typically several minutes. In column 2 of this table nominal values of the luminous region parameters at the Z\(^0\)-peak energy are given together with their likely range of variation (r.m.s.).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal value</th>
<th>Uncertainty of bunch-to-bunch variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle x \rangle^* )</td>
<td>(0 ( \pm ) 250) ( \mu m )</td>
<td>15 ( \mu m )</td>
</tr>
<tr>
<td>( \langle y \rangle^* )</td>
<td>(0 ( \pm ) 72) ( \mu m )</td>
<td>5 ( \mu m )</td>
</tr>
<tr>
<td>( \langle z \rangle^* )</td>
<td>(0 ( \pm ) 1) mm</td>
<td>0.7 mm</td>
</tr>
<tr>
<td>( \langle x' \rangle^* )</td>
<td>(0 ( \pm ) 33) ( \mu \text{rad} )</td>
<td>2 ( \mu \text{rad} )</td>
</tr>
<tr>
<td>( \langle y' \rangle^* )</td>
<td>(0 ( \pm ) 175) ( \mu \text{rad} )</td>
<td>10 ( \mu \text{rad} )</td>
</tr>
<tr>
<td>( \Delta h^* )</td>
<td>(0 ( \pm ) 1) ( \mu m )</td>
<td>---</td>
</tr>
<tr>
<td>( \sigma_x^* )</td>
<td>(300 ( \pm ) 90) ( \mu m )</td>
<td>10 ( \mu m )</td>
</tr>
<tr>
<td>( \sigma_y^* )</td>
<td>(12 ( \pm ) 4) ( \mu m )</td>
<td>1 ( \mu m )</td>
</tr>
<tr>
<td>( \sigma_z^* )</td>
<td>(12.8 ( \pm ) 3) mm</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>( \sigma_{x'}^* )</td>
<td>(175 ( \pm ) 50) ( \mu \text{rad} )</td>
<td>5 ( \mu \text{rad} )</td>
</tr>
<tr>
<td>( \sigma_{y'}^* )</td>
<td>(175 ( \pm ) 50) ( \mu \text{rad} )</td>
<td>5 ( \mu \text{rad} )</td>
</tr>
</tbody>
</table>

Table 2
Luminous region parameters
REFERENCES

[3] See e.g.: G.M. Dallevalle et al., Achievable Precision on the Relative Luminosity Measurement with the OPAL Detector, this proceedings.
NORMALIZATION IN ALEPH

H.Burkhardt\textsuperscript{1)}, E.Fernandez\textsuperscript{2)}, J.D.Hansen\textsuperscript{3)}, R.Møllerud\textsuperscript{3)} and J.A.Perlas\textsuperscript{2)}

Geneva, 4 July 1988

Abstract: The design of the Standard Aleph Luminosity detector and of the Very Small Angle Luminosity Monitor (SALM) and its expected performances in the bunch to bunch normalization for measurements with polarized beams will be described.

1. Short description of the design of the Standard Monitor

The Standard Aleph Luminosity monitor consists of two parts, a drift chamber called Small Angle Tracking chamber (SATR) and a shower counter called Luminosity Calorimeter (LCAI). These detectors were designed to measure the absolute Luminosity with a systematic error of less than 2 \%. The track detector is built by the University of Siegen and the Luminosity Calorimeter by the Niels Bohr Institute Copenhagen. One aim in the design of the Luminosity Calorimeter has been to make it similar to the Aleph barrel and endcap calorimeters.

As shown in figure 1, the counters are placed close to the beam pipe at about 2.6 meters from the interaction region. The tracking part consists of 9 identical layers of drift tubes. They are stacked one behind the other in \( z \). Compared to the first layer, the second layer is rotated in \( \phi \) by 15 degrees and the third layer by another 15 degrees. This is repeated such that the layers 4,5,6 and 7,8,9 have the same orientation as 1,2,3. Inefficiencies due to overlap of dead zones from tube walls and gas channels are avoided this way and a high energy particle traverses at least 6 sensitive layers. The spatial resolution has been measured in a test beam. It was found to be better than 300 \( \mu \)m per plane. For a particle

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coming from the center of the interaction region the angular resolution is 50 µrad in θ and 4.5 mrad in φ. The tracking chamber covers a solid angle of 2π in φ and 41 to 91 mrad in θ. There are in total 2016 wires, connected to 1152 read-out channels. More details about the SATR can be found in [1], [2].

The Luminosity calorimeter is a stack of 38 lead converter sheets and chambers adding up to 24 radiation length. The expected energy resolution is \( \sigma_E / E = 20 \% / \sqrt{E} \) (more details [1], [3]). The
spatial resolution for high energy electrons is expected to be similar in $\phi$ and $\theta$ and to vary between 1 and 2 mm depending on the position within the calorimeter. The LCAL covers about 95 % of $2\pi$ in $\phi$ and angles of about 53 to 165 mrad in $\theta$.

The combined acceptance of the tracking device and calorimeter is then 95 % in $\phi$ and 53 to 91 mrad in $\theta$, corresponding to a Bhabha cross section of 27 nb at the energy of the Z. Including losses in cuts and inefficiencies we usually quoted an expected cross section of 25 nb at Z energies corresponding to 0.25 Hz event rate at the design luminosity of $10^{31}$ cm$^{-2}$ sec$^{-1}$. A typical run with 100 nb$^{-1}$ Luminosity would contain 2500 Bhabha events in the Luminosity detectors leading to a statistical uncertainty of 2 %. For the bunch to bunch normalization one could consider to use the full $\theta$ range of the calorimeter up to 165 mrad in $\theta$ which would increase the normalization cross section to 37 nb.

Fast energy sums of the calorimeter will be used in the trigger. The subdivision in $\phi$ is such, that an acoplanarity cut of about $\pm$ 45 degrees can already be applied at the first trigger level. Several trigger schemes will be used simultaneously. A two side coincidence on a high threshold (probably around 40 % of the beam energy) will be used to select the signal. Low threshold and one side downscaled triggers will allow a monitoring of the trigger efficiency. The background from off momentum electrons is expected to be well below the 1 % level. The background will be measured accurately using a delayed coincidence. The delay time will be made equal to the time an electron bunch needs for a full revolution in the LEP ring, in order to compare signal and background for the same bunch configuration.

2. Study of Bunch to Bunch Systematics for the Standard Monitor

Experience at PETRA and PEP has shown that it might be difficult to obtain the absolute luminosity with 2 % precision. Most of the work in design, building and Monte Carlo simulation of the Standard Aleph Luminosity detector has been concentrated in reducing the systematics for the absolute measurement ([4], [5], [6]). The largest error contributions are expected to come from uncertainties in the knowledge of the acceptance, including geometry folded with trigger and reconstruction efficiencies. This and many other uncertainties such as from radiative corrections and positioning errors do not depend on the properties of differently polarized bunch types and will therefore cancel in the relative bunch to bunch normalization. The only remaining errors should be linked to the bunch properties.
The relativ bunch to bunch normalization needed to measure $A_{LR}$ with longitudinally polarized beams can therefore be obtained with potentially higher systematic accuracy than any absolute cross section measurement.

We do not expect problems from background sources. The coincidence rate from off-momentum particles will be small and accurately measured. Moreover the different bunch types will have very similar currents and geometry so that remaining uncertainties will largely cancel in the relative measurement. The total flux of energy from synchrotron radiation is small. Its effect would be to add rather homogeneously energies and hence to change the effective trigger thresholds. The differential systematic effect on different bunch types is estimated to be negligible.

We have done Monte Carlo studies to check the uncertainty in the knowledge on the difference between the bunch to bunch geometry on the relative luminosity measurement. The results are summarized in table 1. We studied separately uncertainties in the average bunch position (systematic differences in the mean $<x>, <y>, <z>$ position between bunches of different polarization) in their size ($\sigma_x, \sigma_y, \sigma_z$) and mean and sigma of small tilt angles in the interaction region ($<x', <y'>, \sigma_x', \sigma_y'>$).

Except for $z$ and $\sigma_z$ the parameters have been varied within their measurement uncertainty as expected to be known from the LEP machine monitors [7] in time intervals of typically 10 minutes. $z$ and $\sigma_z$ instead will be measured directly using the Aleph central tracking chambers and their errors are purely statistical ($z/\sqrt{N}$ for $<z>$ and about $\sigma_z/\sqrt{2N}$ for $\sigma_z$). The number used in table 1 is derived using a bunch length of 33 mm (using dedicated wigglers, 12.8 mm otherwise) and a measurement of the longitudinal bunch size and position using 2500 clean two track events which we should collect in a typical run in the central detector.

The numbers in table 1 have been obtained using a symmetric $\theta$ acceptance from 50 to 90 mrad at $\pm$ 2.5 meters from the interaction region. At a small cost of cross section and therefore slightly increased statistical error the dependance on the bunch geometry could be further reduced in making tighter cuts on one side only, as proposed to do for the ALEPH SALM detector. Radiative corrections have been included in the Monte Carlo simulations using the event generator of [8]. They were found to decrease the normalization uncertainty slightly. We studied also how the different error contribu-
tions add up and scale with the $\theta$ range. It was found that the different uncertainties add up quadratically in very good approximation. Angular uncertainties were found to increase inversely proportional to $\theta_{\text{min}}$. The sensitivity on the bunch length, the largest single contribution to the error, decreases with increasing the $z$ distance to the interaction region. For this reason, the ALEPH SALM detector depends less on the bunch length and more on the angular uncertainties of the bunch geometry.

To summarize the result of this chapter we believe that the bunch to bunch normalization can be done using the Standard ALEPH Luminosity detector with high systematic accuracy. Systematic errors from angular and transverse shifts between bunches can be excluded using the information from LEP monitors to well below 1%. The largest contribution is expected to come from uncertainties in the longitudinal ($z$) bunch size and position which is measured to better than 1 mm statistical accuracy per run using the ALEPH central detector. The total error of 0.8% in table 1 should be obtainable per run and could be further reduced averaging over longer periods.

<table>
<thead>
<tr>
<th>Parameter at I.R.</th>
<th>typical value</th>
<th>known to</th>
<th>absolute loss in % for typ. val.</th>
<th>systematic uncert. in %$<em>{\text{sys}}$ $\Delta L</em>{\text{i}}/L_{\text{i}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle x \rangle$</td>
<td>100 $\mu$m</td>
<td>15 $\mu$m</td>
<td>0.9</td>
<td>0.14</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>300 $\mu$m</td>
<td>10 $\mu$m</td>
<td>2.4</td>
<td>0.08</td>
</tr>
<tr>
<td>$\langle y \rangle$</td>
<td>100 $\mu$m</td>
<td>5 $\mu$m</td>
<td>0.9</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>12 $\mu$m</td>
<td>1 $\mu$m</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>$\langle z \rangle$</td>
<td>1 mm</td>
<td>0.7 mm</td>
<td>0.8</td>
<td>0.59</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>33 mm</td>
<td>0.5 mm</td>
<td>28.0</td>
<td>0.38</td>
</tr>
<tr>
<td>$\langle x' \rangle$</td>
<td>0</td>
<td>2 $\mu$rad</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma_{x'}$</td>
<td>175 $\mu$rad</td>
<td>5 $\mu$rad</td>
<td>2.6</td>
<td>0.08</td>
</tr>
<tr>
<td>$\langle y' \rangle$</td>
<td>0</td>
<td>10 $\mu$rad</td>
<td>0</td>
<td>0.24</td>
</tr>
<tr>
<td>$\sigma_{y'}$</td>
<td>175 $\mu$rad</td>
<td>5 $\mu$rad</td>
<td>2.6</td>
<td>0.08</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
<td>O (3%)</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

Table 1: Uncertainties in the bunch to bunch normalization from systematic changes in the bunch geometry
3. Possible changes in the design to optimize for polarization

As discussed in the previous chapter, we believe that it is possible to obtain a systematic precision at the 1 % level in the bunch to bunch normalization using the Standard ALEPH Luminosity detectors. For the present design the accepted Bhabha cross section in the Luminosity detector equals about the peak Z resonance cross section (both about 25 nb). The detector is limited by the geometry of the LCAL. The detector starts to be efficient at a radius of in average 14 cm or about twice the beam pipe radius. Given the present design and timescale, it will be impossible to make the LCAL efficient down to lower angles approaching the beam pipe radius. A possible upgrade, possibly coinciding with the installation of a smaller beam pipe in the interaction region, would more likely involve a replacement of the track detector by a very compact shower detector with at the same time very good spatial resolution. It has been proposed to study in future a design using tungsten as converter and silicon strips or scintillating fibers as active material. It has been varyied that in an extended acceptance down to 25 mrad the errors shown in table 1 scale as expected: The errors connected with transverse quantities (x,y) all increase by about 40 % while the errors for the longitudinal quantities (z,σz) decrease by about 40 %. Since errors from z,σz still dominate, the total systematic error would even decrease and errors below 1 % should not be a problem even on the run per run basis.

4. SALM location, design and acceptance

The Small Angle Luminosity Monitor for the ALEPH detector [9] (SALM) consists of four identical calorimeters located symmetrically on each side of the beam pipe with respect to the horizontal (bending) plane, and on each side of the interaction point along the beam (Figure 3). The SALM is presently being built by the group of the Universidad Autonoma de Barcelona.

The main purpose of this detector is to count Bhabha events at the smallest possible angles available in the ALEPH region. In its present location the SALM will be hit by Bhabhas at a rate approximately 20 times higher than the rate hitting the ALEPH main luminosity monitor. The counting rate will be made available on-line, so that the relative LEP luminosity can be constantly monitored.

The four monitors which make up the SALM are located on each side of the beam pipe in the (x−y) plane and on each side of the interaction point. (We use a coordinate system in which the z axis
is along the beam and the x axis is in the horizontal plane and perpendicular to the beam). The beam pipe is elliptical in the region from 7.66 m to 7.91 m in z, in order to locate the monitors as close as possible to the beam line. The active area of the counters starts at 6.5 cm from the beam.

The superconducting (mini-beta) quadrupole located in the region 3.7 m < z < 5.7 m will defocus Bhabha electrons and positrons going towards the monitors from the interaction point so that the effective minimum angle seen by the monitors is $\theta_{\text{min-eff}} = 5.1$ mrad. On the other hand beam-pipe elements before the region of the monitor define a window such that the maximum acceptance angle in the x-z plane is $\theta_{\text{max-eff}} \simeq 6.7$ mrad. In the x direction this translates into an acceptance which is less than 2 cm wide.

Each of the four counters consists of a sampling calorimeter made with tungsten converter sheets interspersed with sampling layers made of plastic scintillator and a plane of vertical silicon strips as described below. The overall shape of the calorimeter is that of a rectangular box of 2 cm × 5 cm × 12 cm as shown in Figure 4.

The first tungsten layer has four radiation lengths, which are needed to protect the sampling layers from the high flux of synchrotron radiation photons. The next 4 tungsten layers are each 2 radiation lengths thick. The first sampling plane has both a plane of silicon strips and a scintillator layer, while the next four sampling planes have only one layer of scintillator. Finally a thick plate of tungsten 2.1
cm (6 radiation lengths) protects the sampling planes from synchrotron radiation photons entering the back of the calorimeter.

The energy resolution with the scintillator read out is dominated by the lateral leakage, since most of the electrons enter the counters very near the edge. Since a Bhabha event is defined as a back to back shower with an energy above a certain threshold the effect of the leakage is to reduce the Bhabha detection efficiency. A detailed simulation using the GEANT program [10] shows that with a threshold energy cut of 60 % the average energy of a totally contained shower, one can obtain an efficiency of 75 %. This efficiency will be improved using shower position information obtained with the sampling plane of vertical silicon strips.

The acceptance of the monitor has been calculated by Monte Carlo taking into account complete Q.E.D. radiative corrections to third order and Z self-energy diagrams [11] as well as the effects of the quadrupole fields. This calculation gives a total acceptance, with the typical running conditions described below, of 0.67 μbarns. With an efficiency for detecting Bhabhas of 75 % this gives a counting rate of 2.5 Hz at a luminosity of $5 \times 10^{30}$ cm$^{-2}$ sec$^{-1}$. The statistical precision that can be obtained is thus 5 % in about 2.5 minutes of monitoring time.
5. Bunch to bunch systematics with the SALM

To measure $A_{LR}$ with high precision the aim is to measure the relative luminosity between the various bunch configurations to about $10^{-3}$. Statistically this precision can be obtained with the SALM, but it is less clear if systematic differences between bunches with different polarization can be kept at this level.

Systematic errors in the relative luminosity can be induced by possible bunch to bunch changes in the beam geometry, namely in the beam position and divergence, as well as to possible changes in bunch to bunch background rates.

The geometrical effects have been computed with the help of a Monte Carlo program. Beam divergences are simulated by generating $e^+e^-$ beams with directions gaussianly distributed with respect to the $+z$ and $-z$ axis. The resulting Bhabha scattering between the acollinear $e^+e^-$ produces an also acollinear $e^+e^-$ pair in the final state. Since Bhabha events are only counted by the simultaneous observation of $e^+e^-$ showers in a pair of counters located back-to-back, an increase on the beam divergence produces a decrease on the counting rate. Likewise, a systematic displacement of the collision point from $x=y=z=0$ will decrease the amount of back to back coincidences. A proper simulation of these effects has to include the fact that radiative corrections produce a natural smearing in the angle at which the particles collide, and beam widths smear the exact position of the collision point.

Shown in table 2 below are the nominal values of the beam position ($<x>$, $<y>$, $<z>$), beam widths ($\sigma_x$, $\sigma_y$, $\sigma_z$) beam tilts ($<x'>$, $<y'>$, $<z'>$) and beam divergences ($\sigma_{x'}$, $\sigma_{y'}$, $\sigma_{z'}$) as well as the errors with which these quantities can be measured. The third column in the table shows the "typical" values for these parameters used in the computations explained below. Columns 4 and 6 show the expected maximum bunch to bunch changes and the errors in the measurements of these changes. In the 5th column we show the decrease in the counting rate when the corresponding variable changes by its maximum bunch to bunch change (column 4) from the typical value (column 3) and all the other quantities stay at their typical values. Column 7th is the decrease in counting rate when the change in the corresponding variable is equal to the error in the bunch to bunch difference (column 6) and all the other variables stay at their typical values. In all the calculations the beam energy was 46.1 GeV.
Table 2: Uncertainties in $\Delta L_i/L_i$ to the bunch to bunch normalization for the SALM

<table>
<thead>
<tr>
<th>quantity</th>
<th>nom. err.</th>
<th>typ. val.</th>
<th>bunch change</th>
<th>bunch change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>val.</td>
<td></td>
<td>diff. %</td>
<td>error %</td>
</tr>
<tr>
<td>$&lt;x&gt;$</td>
<td>$\mu m$</td>
<td>0 300</td>
<td>100</td>
<td>0 0</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>$\mu m$</td>
<td>300 35</td>
<td>300</td>
<td>30 4</td>
</tr>
<tr>
<td>$&lt;x'&gt;$</td>
<td>$\mu rad$</td>
<td>0 45</td>
<td>0</td>
<td>0 0</td>
</tr>
<tr>
<td>$\sigma_{x'}$</td>
<td>$\mu rad$</td>
<td>175 20</td>
<td>175</td>
<td>18 7</td>
</tr>
<tr>
<td>$&lt;y&gt;$</td>
<td>$\mu m$</td>
<td>0 300</td>
<td>0</td>
<td>0 0</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>$\mu m$</td>
<td>12 1.5</td>
<td>12</td>
<td>4 &lt;1</td>
</tr>
<tr>
<td>$&lt;y'&gt;$</td>
<td>$\mu rad$</td>
<td>0 45</td>
<td>0</td>
<td>0 0</td>
</tr>
<tr>
<td>$\sigma_{y'}$</td>
<td>$\mu rad$</td>
<td>175 20</td>
<td>175</td>
<td>50 1</td>
</tr>
<tr>
<td>$&lt;z&gt;$</td>
<td>$mm$</td>
<td>0 1</td>
<td>0</td>
<td>0 0</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>$mm$</td>
<td>33 33</td>
<td>33</td>
<td>0.4 &lt;1</td>
</tr>
<tr>
<td>column</td>
<td></td>
<td>1 2 3</td>
<td>4 5 6</td>
<td>7</td>
</tr>
</tbody>
</table>

The most important effect comes from the beam divergence and beam tilt in the $x$ direction. The dependence of the acceptance on the beam divergence is shown in more detail in the top curve of Figure 5. All the other variables were kept at their “typical” values to calculate this curve.

The dependence of the acceptance on beam divergence and beam displacements would be reduced if we were able to define a restricted acceptance region characterized by a smaller area in the front face of the calorimeters. A Bhabha event could be defined by a coincidence of a shower on the inner acceptance region of one counter with a shower anywhere on the opposite counter. For a hit in a given inner region of one counter we can guarantee that the other particle hits the opposite side counter provided the acollinearity angle and the displacement of the beam from the nominal interaction points are smaller than certain values. This effect can be seen in Figure 5 where the dependance of the acceptance on $\sigma_x$ is also shown for two asymmetric acceptances characterized by inner regions of 1.8 and 1.6 cm. The dependance is softer than for the symmetric acceptance, but the gain in precision is not significant since the acceptance is also reduced.
6. Background in the SALM

Two kinds of background will hit the monitors. One is the synchrotron radiation photons emitted from the quadrupole fields in the straight section around the interaction points. The other is the off-momentum electrons produced by beam gas bremsstrahlung in the straight sections and in the arcs.

The synchrotron radiation backgrounds depends critically on the position of the collimators just behind our calorimeters. [12] The energy spectrum of the photons ranges from 10 keV up to about 5 MeV. Most of them will be absorbed by the thick tungsten layers on each side of the active layers of the calorimeters but a tail remains that can reach these active layers. There are in addition the photons entering the sides of the monitor. The effect of this background will be a small signal present with every beam crossing which will be almost completely suppressed by the threshold cut on the Bhabha triggers.

Off-momentum electrons and positrons reaching the area of the interaction region are produced by beam gas bremsstrahlung in the straight section of the beam pipe around the interaction point as well as in the arcs. New calculations by G. von Holtey [13] led to the design of new collimators.
It should be noted that the background rate can be inferred from rates of showers in a single counter, the rate of simultaneous showers in back to back counters (the signature of Bhabha events) and the rate of back to back showers where one of the showers comes from a given beam crossing and the opposite shower comes from the next crossing of the same bunches. We intend to monitor these three rates in order to measure the background.

References

Normalization in DELPHI

L. Bugge¹, M. Dam¹, G. Jarlskog², A. L. Read¹, G. Rinaudo³

Oslo, July 15, 1988

1 Introduction

We report on the results of a study of the performance of the DELPHI small angle detectors as luminosity and beam monitors. The detectors considered are the Small Angle Tagger (SAT) and the Very Small Angle Tagger (VSAT).

The SAT consists of two modules each containing a track detector (three Si detector planes) and an electromagnetic calorimeter (lead/scintillating optical fibres). It covers the polar angular range of 53–116 mrad. When a smaller diameter second phase beam pipe is installed, both the calorimeter and the track detector can be extended inwards, allowing the angular coverage to start at a lower angle. For further specifications, see Ref. [1] and the references quoted therein.

The VSAT consists of two sets of two tungsten calorimeters with Si planes as active elements. Three of the planes have strip read out to allow precise determination of the electron impact point. One calorimeter is placed at each side of the beam tube at $\pm 7.7$ m from the interaction point. The calorimeters have rectangular cross sections, covering between 6 and 9 cm from the beam axis in the horizontal plane and $\pm 2.5$ cm in the vertical direction. The VSAT covers the polar angular region of 4.5–9 mrad. For further specifications, see Ref. [2].

The luminosity monitoring will be performed by the detection of Bhabha scattering. The two detectors are complementary in the sense that:

1. SAT is a "low rate" monitor, but stable against variations in the interaction region parameters.

2. VSAT is a "high rate" monitor, but very sensitive to some variations in the interaction region parameters.

The studies have been performed by using the Bhabha event Monte Carlo of Berends, Hollik, and Kleiss [3], and for the VSAT additionally a beam transport program of von Holtey [4]. The beam energy was 47 GeV and $k_0$ — the cut off between soft and hard bremsstrahlung — was 0.001.

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In the present study we concentrate on contributions to the uncertainty on the relative bunch to bunch luminosity.

2 The Relative Luminosity

As discussed elsewhere in this report, the longitudinal polarization asymmetry $A_{LR}$ is derived from the experimental asymmetry $A_{\text{exp}}$, by the relation:

$$ A_{LR} \approx \frac{1}{P} \left[ A_{\text{exp}} \left(1 + \frac{A_{\text{exp}}}{P} \frac{P^+ - P^-}{2}\right) + \frac{L_1 - L_2}{L_1 + L_2} \right]. \quad (1) $$

Here $P^-$ and $P^+$ are the longitudinal polarization of the electrons and the positrons, respectively, whereas $P$ is the average of the two.

The relation between the precision on $A_{LR}$ and the precision on the relative luminosity is discussed in the general introduction to the Normalization section of this report. It is shown that the systematic uncertainty on $L_1/L_2$ — the ratio of the integrated luminosities corresponding to bunch collisions of polarization state 1 and 2 — has to be below 0.1–0.2% in order to allow a measurement of $A_{LR}$ to the desired precision, 0.3%.

From Eq. (1) we see that the directly relevant quantity to study is $(L_1 - L_2)/(L_1 + L_2)$. Since $\delta(L_1/L_2) \approx 2\delta((L_1 - L_2)/(L_1 + L_2))$, we prefer to discuss the uncertainty in terms of the relative luminosity:

$$ D_L = \frac{\Delta L}{L} = 2 \frac{L_1 - L_2}{L_1 + L_2}, \quad (2) $$

Introducing the cross sections $\sigma_{1,2}$ and the observed numbers of events $N_{1,2}$ into Eq. (2) we get

$$ D_L = 2 \frac{N_1 \sigma_2 - N_2 \sigma_1}{N_1 \sigma_2 + N_2 \sigma_1}. \quad (3) $$

Using that $N_1 \approx N_2 = N$ (this assumption is correct within 10% [5]) and $\sigma_1 \approx \sigma_2$, and observing that the event samples from the two kinds of collisions are statistically independent, we get for the statistical uncertainty:

$$ \delta D_L^{\text{stat}} = \sqrt{\frac{2}{N}}. \quad (4) $$

To study the systematic uncertainty of $D_L$, we write the cross sections as

$$ \sigma_1 = \sigma(\xi) = \sigma_0 \cdot f(\xi) \quad (5) $$

$$ \sigma_2 = \sigma(\xi + \Delta \xi) = \sigma_1 + \frac{\partial \sigma}{\partial \xi} \Delta \xi = \sigma_0 \cdot \left( f(\xi) + \frac{\partial f}{\partial \xi} \Delta \xi \right). \quad (6) $$

Here $\xi$ denotes all parameters which might vary from one type of bunch collision to another (frequently denoted "bunch dependent" parameters). All bunch independent effects are contained in $\sigma_0$ and cancel in $D_L$. The quantities $\Delta \xi$ are the differences in parameters from one bunch collision to another, $\Delta \xi = \xi_1 - \xi_2$. The variables $\xi$ and $\Delta \xi$ are independent. Therefore the systematic uncertainty in $D_L$ from one of the parameters $\xi_i$ can be evaluated as:
\[ \delta D'_L = \sqrt{\left( \frac{\partial D'_L}{\partial \xi_i} \cdot \delta \xi_i \right)^2 + \left( \frac{\partial D'_L}{\partial (\Delta \xi_i)} \cdot \delta (\Delta \xi_i) \right)^2}, \]  

with \( \delta \xi_i \) and \( \delta (\Delta \xi_i) \) being the errors on \( \xi_i \) and \( \Delta \xi_i \), respectively. With the assumption of independent uncertainty contributions from the different \( \xi_i \), we obtain

\[ \delta D'^{\text{yst}}_L = \sqrt{\sum_i (\delta D'_L)^2}. \]

From Eqs. (3), (5), (6), and (7), and assuming \( N_1 \approx N_2 \) and \( \frac{\partial L}{\partial \xi}, \Delta \xi \ll f \) we get

\[ \frac{\partial D'_L}{\partial \xi_i} = \frac{(\partial f/\partial \xi_i)^2 - \partial^2 f/\partial \xi_i^2 \cdot f}{2f^2} \cdot \Delta \xi_i \]

\[ \frac{\partial D'_L}{\partial (\Delta \xi_i)} = \frac{\delta f/\delta \xi_i}{2f} \]

The cross section dependences on the parameters we have studied are well described by parabolas

\[ f(\xi_i) = 1 + b_i \xi_i + a_i \xi_i^2, \]

as seen from Figs. 4 and 5.

3 Bhabha rates, statistical uncertainties

The statistical uncertainty on the relative luminosity is found from Eq. (4). The effective rate of coincident Bhabha events depends on the acceptance definition (see Refs. [1,6]). For the VSAT it depends in addition on the interaction region parameter values, such as the horizontal orbit position and the beam divergence.

Realistic values of the effective Bhabha cross sections are:

**SAT:** 30 nb, or a rate of 0.3 Hz at the design luminosity, \( L = 10^{31}\text{cm}^{-2}\text{s}^{-1} \). With four different bunch collisions this means 0.075 Hz per bunch. A statistical uncertainty of 1% on the relative luminosity would thus require 74 hours running, whereas reaching 0.1% would require 7400 hours (67 pb\(^{-1}\) per bunch). A factor 2.6 (5.4) can be gained by starting the lower angle of the acceptance at 35 (25) mrad. which is possible after the installation of a 120 (80) mm diameter beam pipe.

**VSAT:** 500 nb, or a rate of 1.25 Hz per bunch. A statistical uncertainty of 1% would thus require 4.4 hours running, whereas reaching 0.1% would require 440 hours (4 pb\(^{-1}\) per bunch).

4 Contributions to the systematic uncertainty

The uncertainty contributions to the absolute luminosity measurements include:

- Theoretical uncertainties (radiative corrections, weak effects),
- Physics backgrounds,
- Machine backgrounds,
- Production tolerances on luminometer,
- Alignment/surveying inaccuracies,
- Trigger and reconstruction efficiencies,
- Beam geometry.

These are discussed elsewhere, see for example Refs. [1,2,6,7]. Most of these contributions cancel during the measurement of the relative luminosity. The different treatment of the differently polarised bunches may, however, introduce bunch to bunch differences in the machine background and the beam geometry.

No information is available on possible bunch to bunch differences in machine backgrounds. In order to make a precise background subtraction possible, efforts will be made during runtime to measure the background through use of, for example, delayed coincidences. The possible bunch to bunch difference in the beam geometry makes a precise surveying of the beam and interaction region geometry necessary.

5 Beam monitoring

Some surveying of the interaction region parameters can be performed both by the SAT and the VSAT by using the traditional method, where asymmetries in counting rates between different parts of the monitors are utilised (see for example Refs. [1,8]). The precision tends, however, to be poor.

The presence of the SAT tracker allows us to obtain rather precise measurements of the interaction point position [1,6]. This is especially important for the determination of the longitudinal position, where the information from the LEP beam monitoring will be rather poor.

Reconstruction of \(Z^0\) events by the DELPHI central tracking detectors allows us to measure directly the mean \(z\) and \(\sigma_z\) of the luminous region. The measurement will be dominated by statistical uncertainties. With dedicated wigglers for polarization installed the nominal \(\sigma_z\) is 33 mm (12.8 mm otherwise). In a run where 2500 \(Z^0\) events are collected a precision of \(\delta z = 0.7\) mm and \(\delta \sigma_z = 0.5\) mm can be obtained.

The good spatial resolution of about 0.3 mm and the high rate in the VSAT makes very precise measurements of the bunch to bunch differences of some beam parameters feasible. Since the systematic errors on the measured beam parameters tend to cancel in the difference, the uncertainties on the bunch to bunch differences of the beam parameters are expected to be dominated by statistics. For our investigation of these measurements we have used Bhabha
Figure 1: The distribution $Dx = x_1 - x_2$ for, a) different values of the horizontal orbit position and, b) different values of the horizontal divergence.

events with both electrons having more than 90% of the beam energy, resulting in an event sample with a very narrow acollinearity peak. To avoid border effects in the calorimeter a 2 mm restricted acceptance has been used.

To determine the beam parameters we measure the difference between the $x$ coordinate of the electron in arm 1 and arm 2, $Dx = x_1 - x_2$. The same is done for the $y$ coordinate. We thus obtain four sets of Gaussian like distributions, two for each pair of detector modules. The central value of $Dx$ reflects two contributions: detector mis-alignment and beam displacement. The first effect will remain constant from run to run, and thus can be corrected for. The remaining shift now reflects a displacement of the beams. In a similar way the widths $\sigma_{Dx}$ and $\sigma_{Dy}$ of the $Dx$ and $Dy$ distributions will integrate three effects: measurement errors, beam width, and beam divergence.

Figures 1 a) and b) show the $Dx$ distribution for different values of the horizontal displacement and the beam divergence, respectively. The correlations between the $Dx$ and $Dy$
Figure 2: a) The correlation between the $Dx$ central value and $\langle x \rangle$ and the width of $Dx$ and the beam divergence. b) The same for $y$.

central positions and $\langle x \rangle$ and $\langle y \rangle$ are shown in Fig. 2, together with the correlations between the widths $\sigma_{Dx}$ and $\sigma_{Dy}$ and the beam divergences $\sigma_x'$ and $\sigma_y'$. As one would naively expect, linear relationships between the displacements and the central positions are observed. The combination of the nominal beam width, the defocusing effect of the quadrupole, and the measurement error (0.5 mm) gives a 2 mm width of the $Dx$ distribution without beam divergence. Within the studied interval, a linear relationship between the beam divergence and the width of the $Dx$ distribution is a good approximation. At the nominal divergence the width is 3.3 mm. In a typical run $2 \times 10^4$ useful events per bunch will be accumulated and the uncertainty on the average shift will therefore be about 0.025 mm, whereas the uncertainty on the width will be 0.017 mm. These numbers correspond to a 0.008 mm statistical uncertainty on the average beam displacement, and a 0.0017 mrad statistical uncertainty on the divergence. There are additional significant systematic errors, but these cancel in the determination of the bunch to bunch differences. The uncertainties on these are therefore found from the numbers above by multiplication by a factor $\sqrt{2}$, so that finally $\delta(\Delta(x)) = 0.011$ mm and $\delta(\Delta \sigma_x') = 0.0024$ mrad. By similar considerations it is found that $\delta(\Delta(y)) = 0.041$ mm and $\delta(\Delta \sigma_y') = 0.0019$ mrad.

As a check Fig. 3 shows the width of the $Dx$ distribution as a function of the transversal displacement and width of the interaction region. A weak correlation is observed implying that information about these parameters has to be taken into account when the divergence is calculated.
Figure 3: The width of $Dx$ as a function of the transversal position and size of the luminous region.

Table 1: Uncertainties on the monitoring of beam parameters. The SAT and VSAT numbers correspond to one run, whereas the LEP numbers are available after much shorter time intervals.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Absolute</th>
<th>Rel. bunch to bunch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SAT tracker inf.</td>
<td>VSAT asym.</td>
</tr>
<tr>
<td>$(x)$ mm</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma_x$ mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(x')$ mrad</td>
<td>0.8</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_{x'}$ mrad</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(y)$ mm</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_y$ mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(y')$ mrad</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{y'}$ mrad</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(z)$ mm</td>
<td>5.</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_z$ mm</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

6 Systematic uncertainties from the beam geometry

We have performed a precise study of the systematic uncertainties due to the possible bunch to bunch variations of the following ten interaction region and beam parameters:

- $(x)$, the horizontal orbit position at the interaction point
- $\sigma_x$, the horizontal size of the luminous region
- $(x')$, the average beam direction at the interaction point in the horizontal plane
- $\sigma_{x'}$, the beam divergence at the interaction point in the horizontal plane
- $(y)$, the vertical orbit position at the interaction point
- $\sigma_y$, the vertical size of the luminous region
- $(y')$, the average beam direction at the interaction point in the vertical plane
Figure 4: Bhabha cross section for SAT as function of interaction point parameters. Three different acceptance definitions used. Solid circles: full acceptance; open circles: asymmetric acceptance, 3 mrad; solid boxes: asymmetric acceptance, 6 mrad.

- $\sigma_y$, the beam divergence at the interaction point in the vertical plane
- $\langle z \rangle$, the average longitudinal position of the interaction region
- $\sigma_z$, the longitudinal size of the interaction region

Figures 4 and 5 show the dependence of the accepted Bhabha cross section on some of the parameters for the SAT and the VSAT, respectively. The three curves on each plot correspond to acceptance limitations in the asymmetric acceptance definitions [1,6] of 0.0, 3.0, and 6.0 mrad for the SAT, and 0.0, 2.5, and 5.0 mm for the VSAT. The dotted lines represent the parabolic fits.

From these fits we obtain the results for the systematic uncertainties on the relative luminosity. The numbers used for the uncertainties on the bunch-averaged values, $\delta \xi$, and on the bunch to bunch differences, $\delta(\Delta \xi)$, are given in Tab. 2. The systematic error on $D_{\xi}$...
Figure 5: Bhabha cross section for VSAT as function of interaction point parameters. Three different acceptance definitions used. Solid circles: full acceptance; open circles: asymmetric acceptance, 2.5 mm; solid boxes: asymmetric acceptance, 5 mm.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Monit. uncert.</th>
<th>Bunch to bunch</th>
<th>Nominal</th>
<th>Pessimistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta \xi$</td>
<td>$\Delta \xi$</td>
<td>$\delta (\Delta \xi)$</td>
<td>$\xi$ SAT</td>
</tr>
<tr>
<td>$\langle x \rangle$</td>
<td>0.3</td>
<td>0.015</td>
<td>0.011</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.035</td>
<td>0.030</td>
<td>0.010</td>
<td>0.3</td>
</tr>
<tr>
<td>$\langle x' \rangle$</td>
<td>0.045</td>
<td>0.002</td>
<td>0.002</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_{x'}$</td>
<td>0.020</td>
<td>0.018</td>
<td>0.0024</td>
<td>0.175</td>
</tr>
<tr>
<td>$\langle y \rangle$</td>
<td>0.3</td>
<td>0.005</td>
<td>0.005</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>0.0015</td>
<td>0.004</td>
<td>0.001</td>
<td>0.012</td>
</tr>
<tr>
<td>$\langle y' \rangle$</td>
<td>0.045</td>
<td>0.010</td>
<td>0.010</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_{y'}$</td>
<td>0.020</td>
<td>0.050</td>
<td>0.0019</td>
<td>0.175</td>
</tr>
<tr>
<td>$\langle z \rangle$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.5</td>
<td>0.4</td>
<td>0.5</td>
<td>33.</td>
</tr>
<tr>
<td>Total</td>
<td>0.2</td>
<td>2.0</td>
<td></td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 2: Systematic uncertainties on the relative luminosity.
is calculated substituting Eq. (11) into Eqs. (9), (10), and (7), making the assumptions that $a_i \xi_i^2 \ll 1$, $b_i \xi_i \ll 1$, $a_i \xi_i \Delta \xi_i \ll 1$, and $b_i \Delta \xi_i \ll 1$. It is calculated for the nominal and for a “pessimistic” value of $\xi_i$ as given in the table. During the study of one quantity all other quantities have been kept at their nominal values. The results for the SAT were obtained using the 3 mrad restricted asymmetric acceptance. By going to the full acceptance the uncertainty on the relative luminosity was doubled. For the VSAT the results were obtained by using the full acceptance. The small size of this detector allows only small acceptance restrictions. In combination with the strong angular dependence of the Bhabha cross section in the VSAT region, and the resulting large loss of rate, these acceptance restrictions do not give significant improvements. We estimate a systematic error in the $\delta D_C$ values of less than 15%, mainly due to the approximations discussed above.

For the uncertainties $\delta (\Delta \xi_i)$ we have exploited the VSAT method as discussed in Sec. 5, whereas for the absolute uncertainties $\delta \xi_i$ we use the LEP group numbers [9].

From the figures we expect the maximum effects for the SAT to come from the longitudinal degree of freedom, $\langle z \rangle$ and $\sigma_z$, and for the VSAT from $\langle x \rangle$ and $\sigma_x$. These expectations are confirmed in the $\delta D_C$ columns of Tab. 2. The different characteristics of the two monitors are clearly exposed:

**SAT** Stable against variations in the interaction region parameters. The systematic uncertainty can realistically be kept at the 0.03% level. A possible extension of the SAT to 35 or 25 mrad has no dramatic effect on the systematic uncertainty.

**VSAT** Unstable against variations in some of the interaction region parameters, particularly horizontal divergence, position, and size. It seems realistic, however, that the systematic uncertainty on the relative luminosity for one run will be below 0.3% (we get 0.2% when all parameters are at their nominal values). These numbers may be reduced, however, as discussed in the following section.

### 7 A possible improvement to the VSAT analysis

It has been observed in our Monte Carlo studies that the correction to the Bhabha cross section depends mainly on the two parameters of the $x$ difference distribution independent of the physical origin of the values of these parameters. Figure 6 shows the cross section as a function of the width of $Dx$. The open circles correspond to a variation of the beam divergence in the interval 0.125–0.225 mrad, the solid ones to a variation of the horizontal beam width in the interval 0.0–2.0 mm. All other parameters were kept at their nominal values. All the points seem to fall on the same curve. Assuming that the correction to the effective Bhabha cross section depends only on the mean value and width of the $Dx$ distribution, the relative luminosity can be determined directly by measuring these quantities, even without a detailed knowledge of the individual luminosity region parameters. With the precision quoted
Figure 6: The Bhabha cross section for the VSAT as a function of the with of the $Dx$ distribution. The open circles are obtained by varying the beam divergence, the solid ones by varying the horizontal beam width. All points fit on the same curve.

In Sec. 5, the uncertainty on the relative luminosity from the determination of the bunch to bunch difference between the $Dx$ widths will be about 0.13%, whereas the uncertainty arising from the determination of the difference between the central values will be about 0.08%. This gives a total uncertainty of about 0.15%.

Being independent of the detailed knowledge of the beam parameter values, this method results in an uncertainty on the relative luminosity which is dominated by statistical contributions: the usual contribution of Eq. (4), and the contribution coming from the $Dx$ parameter determination. Furthermore, assuming that the variations in the $Dx$ distribution parameters are small, so that the approximation of a linear relation between the cross section and the parameters can be made, statistics integrated over several runs can be used. This strategy could reduce the uncertainty significantly. However, further studies are required to confirm the validity of this approach.

8 Conclusion

In order to determine $A_{LR}$ with an accuracy of 0.3% the luminosity asymmetry $D_L$ should be measured to an accuracy of the order 0.1–0.2%.

With the SAT the relative luminosity can be monitored with negligible systematic errors. A very long run-time corresponding to an integrated luminosity of 67 pb$^{-1}$ per bunch is
needed, however, to reach a 0.1% statistical uncertainty. When a reduced diameter beam pipe is installed, allowing the acceptance to start at 35 (25) mrad, the same accuracy can be obtained after 26 (13) pb⁻¹ per bunch. The systematic uncertainty will still be negligible.

With the VSAT an integrated luminosity of 4 pb⁻¹ per bunch is needed to reach 0.1% in the statistical uncertainty on the relative luminosity. The systematic uncertainty obtained will be less than 0.3%, using the direct measurement of the beam parameters in the single runs. Under ideal conditions, with all the beam parameters at their nominal values, 0.2% can be reached. The good spatial resolution of the VSAT seems to allow a determination of the relative luminosity, even without a detailed knowledge of all the beam parameters, cf. Sec. 7. This method would result in systematic errors about 25% smaller than the ones cited above. Furthermore, integrating over many runs could reduce the uncertainty significantly.

References


[9] G. von Holtey, Transparencies from Workshop on Polarization in LEP, p. 179, CERN 1987. See also the contribution by G. von Holtey to this report.
NORMALIZATION IN L3

L3 Luminosity Monitor Group
(M. Athanas et al.)

Abstract

The design of the L3 luminosity monitor is presented. Its expected performance for the absolute luminosity measurement is given. The influence on the relative luminosity measurement of the uncertainty in the variation of the bunch parameters from bunch to bunch is estimated.

1. DESIGN DESCRIPTION

The L3 luminosity monitor device was designed specifically for reliable luminosity measurements in the Z energy range at LEP [1]. It is located in an angular region forward enough to become independent of the Z exchange and yet not too far forward so as to allow easy Bhabha event selection unaffected by systematics. It consists of a charge tracking device to achieve good position resolution, followed by a highly segmented BGO array to measure precisely the showering energy as well as to assure good radiation hardness properties.

With this system shown in Fig. 1, one will be able to study in great detail the Bhabha process including the radiative tail. A carefully designed trigger [2] will permit the measurement and removal of background events like beam-gas. Furthermore, one can apply a sophisticated analysis to remove any Bhabha events that develop only a fraction of their energy in the BGO detector but otherwise pass the trigger condition. Thus, by comparing the tracking information with the energy profile deposited in the crystal array, one can define a very precise geometrical acceptance region. Lastly, an off-line asymmetric trigger can be easily implemented to further reduce systematics.

For the tracking device, it has been decided to use Proportional Inclined Chambers (PIC) [3], [4]. They will have a tilt against the vertical of 30° and a wire spacing of 2 mm. Each chamber (which in turn consists of two half
Hadron Calorimeter Endcap

Figure 1

The L3 Luminosity Monitor Region

chambers) will be composed of 8 sectors covering 45° each in azimuth. The wires extend over four sectors at a time, stretched over combs to guide them across sectors. In addition, cathode strips are provided with a width of about 3° in order to measure the azimuth. The tracking device consists of two PIC chambers per arm located less than 10 cm away from the BGO array and rotated by 22.5° relative to each other to guard against possible inefficiencies near the combs.

The BGO array is cylindrically symmetric. The crystals are arranged in eight rings, as shown in Fig. 2, each covering 15 mm radially, parallel to the beampipe. Azimuthally, they are arranged in sixteen sectors of 22.5° each. Each sector consists of 19 crystals of different sizes. To ensure optimum
shower containment, a software trigger condition is defined that limits the acceptance to the inner six of the eight rings. This also matches the full efficiency range of the PIC chambers. The BGO array is split into two halves that separate during each filling of the LEP ring. A hydraulic device with a measured positioning accuracy of 10 microns will close the array again for a run. A lead shield between BGO and beampipe provides for further radiation protection. In addition, any possible radiation damage and recovery will be monitored by LED pulsing. The main characteristics of the L3 Luminosity Monitoring System are summarized in Table 1.

The trigger consists of two sub-triggers. Apart from an energy trigger responsible for large amounts of energy deposited in total in the two BGO arrays (e.g. for tags in two-gamma physics), there will be a geometrical trigger that requires the observation of a minimum amount of energy (e.g. half the beam energy) in each of the two BGO arrays, in coincidence. The azimuthal width of the overlap region for the coincidence is defined as two BGO sectors, i.e. 45°. This trigger scheme allows the observation of radiative (and non
Table 1
Main characteristics of the L3 Monitoring System

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from the Interaction Point (m)</td>
<td>2.65 - 2.9</td>
</tr>
<tr>
<td>$Z_{\text{min}} - Z_{\text{max}}$</td>
<td></td>
</tr>
<tr>
<td>Amount of material in front ($X_0$)</td>
<td>0.10 - 0.15</td>
</tr>
<tr>
<td>Beam pipe radius (cm)</td>
<td>6.0</td>
</tr>
<tr>
<td>Radial extent of physical BGO array (cm)</td>
<td>6.8 - 19.0</td>
</tr>
<tr>
<td>$R_{\text{min}} - R_{\text{max}}$</td>
<td></td>
</tr>
<tr>
<td>Radial extent of acceptance area (cm)</td>
<td>8.5 - 17.5</td>
</tr>
<tr>
<td>$R_{\text{min}} - R_{\text{max}}$</td>
<td></td>
</tr>
<tr>
<td>Effective polar angle coverage ($\theta$) (mrad)</td>
<td>30 - 62</td>
</tr>
<tr>
<td>$\theta_{\text{min}} - \theta_{\text{max}}$</td>
<td></td>
</tr>
<tr>
<td>Effective Bhabha cross-section ($\sigma$) (nb)</td>
<td>100</td>
</tr>
<tr>
<td>Length of BGO crystal (cm)</td>
<td>26.0</td>
</tr>
<tr>
<td>Length of BGO crystal ($X_0$)</td>
<td>24</td>
</tr>
<tr>
<td>Tracking chamber resolutions (entire track)</td>
<td></td>
</tr>
<tr>
<td>* $\Delta R$ (um)</td>
<td>&lt; 300</td>
</tr>
<tr>
<td>$\Delta \theta$ (mrad)</td>
<td>&lt; 0.12</td>
</tr>
<tr>
<td>* $\Delta \phi$ (degrees)</td>
<td>0.8</td>
</tr>
<tr>
<td>Calorimetry</td>
<td></td>
</tr>
<tr>
<td>* $\Delta E/E(%)$</td>
<td>0.5 - 1.0</td>
</tr>
<tr>
<td>* $\Delta R$ (um)</td>
<td>&lt; 800</td>
</tr>
<tr>
<td>$\Delta \theta$ (mrad)</td>
<td>&lt; 0.3</td>
</tr>
<tr>
<td>* $\Delta \phi$ (degrees)</td>
<td>&lt; 0.6</td>
</tr>
<tr>
<td>* has been measured in a 50 GeV electron test beam</td>
<td></td>
</tr>
</tbody>
</table>

Radiative Bhabha events as well as background interactions. Software event studies will then control the amount of background to be admitted into the luminosity event sample. The tracking chambers are not included in the trigger scheme.

2. PRECISION GOALS FOR THE L3 LUMINOSITY MONITOR

The Luminosity Monitor will accept an angular region of about 30 to 62 milliradians with full efficiency. This corresponds to an effective Bhabha cross-section $\sigma \approx 100$ nb. Assuming an average luminosity of $10^{31}$ cm$^{-2}$s$^{-1}$, a trigger rate of about 1 Hz will result. It has to be compared to the
0.25 - 0.3 Hz expected rate from Z events. Thus, a statistical error of about 1\% on the luminosity will be achieved in a 2.5 - 3 hour run.

We aim at a precision of better than 2\% on the absolute luminosity on a run to run basis. The main limitation will then come from systematic errors which can be separated into four major sections as follows:

2.1 Theoretical uncertainties.
These include uncertainties in the Bhabha cross-section, e.g. weak interaction effects, vacuum polarization, or multiple photon emission beyond order $\alpha^3$. These errors are being studied and are at present estimated to contribute about 1\% to the systematic errors.

2.2 Detector performance
The basic features of the luminosity detector have been listed above. The inherent limitations are expected to contribute to the systematics well below the percent level. In particular, the loose total energy trigger and the requirement of good lateral shower containment will produce no significant loss of Bhabha events. Longitudinal shower containment is almost complete in 24 radiation lengths and can be ignored against the more incomplete lateral containment. The tracking chamber resolution of better than 300 microns, taken at the critical inner radius $R = 8.5$ cm, produces a luminosity error of less than 0.6\% per event and this will be reduced to zero by the statistics of a three hour run. Chamber production tolerances, as well as final alignment and survey, should be below the 100 micron level and thus contribute 0.2\% to the systematics. The chambers will be mounted in a fixed position on the beampipe, and they will be surveyed with respect to the LEP quadrupoles. Chamber efficiencies have probably the most critical effect in the systematics. They are required to be known to better than 1\% per wire, and also stable at that level. When requiring 8 out of 10 wires per track, a drop by 1\% in all wire efficiencies would translate into a systematic effect of 0.1\%, whereas a requirement of 3 out of 4 wires (near the comb regions where the chambers are inefficient) would produce 0.3\% in systematics.

Thus, by carefully monitoring any wire inefficiencies, especially near the onset of full efficiency at $R = 8.5$ cm, one should be able to control their effect to well below the percent level.
2.3 LEP beam backgrounds

Minor contributions to the systematics come from non Bhabha interactions mixed into the event sample. Synchrotron radiation induced events are easily removed by a suitable energy threshold in the trigger. Off-momentum electrons resulting in beam gas or beam wall interactions will be studied as part of the event sample. They are identifiable as highly acoplanar events which violate the geometric trigger but which manage to be accepted by the energy trigger. A study of these events will permit a reliable background subtraction accurate to better than the percent level.

2.4 LEP beam parameters

One of the most difficult obstacles to keep luminosity systematics below the percent level is the dependence of the observed Bhabha rates on variations of the beam parameters at the interaction point. In addition, the parameters are predictions by the LEP staff for values at the I.P. derived from single separated beam measurements, and some parameters are thus difficult to quote. Hence, a major effort was undertaken to largely eliminate the dependence of the Bhabha calibration on the precise values of the beam parameters. After consultation with the LEP instrumentation group, the following parameters were picked as the most essential to control [5]:

I.P. position \( (\langle x \rangle, \langle y \rangle, \langle z \rangle) \)
I.P. width \( (\langle \sigma_x \rangle, \langle \sigma_y \rangle, \langle \sigma_z \rangle) \)
Beam dispersion \( (\langle \sigma_{x'} \rangle, \langle \sigma_{y'} \rangle) \)
Angular beam offset \( (\langle x' \rangle, \langle y' \rangle) \) called "Beam Tilt".

A Monte-Carlo study was undertaken to investigate the effects of beam parameter values, in combination with the trigger design and event selection [2].

From the start, the L3 luminosity monitor has foreseen the use of a (software) asymmetric trigger [6]: only one of the two detector arms requires the nominal geometrical trigger between 30 to 62 milliradians, whereas the other arm has to satisfy the geometrical trigger within a loosened angular range from 30-d to 62+d milliradians, with "d" a parameter to be suitably chosen. As an example, the event rates drop almost linearly with \( \langle x \rangle \) when \( d = 0 \). With increasing \( d \), the dependency becomes parabolic until for a particular value of \( d \), there is no more dependency for a wide range of \( \langle x \rangle \) values. With still larger \( d \) values, one obtains even a rise in rate with \( \langle x \rangle \) for small
a) $d = 0$, $\sigma_x$, $\sigma_y$, $\sigma_z$ as for (c).

b) $d = 0.6$ mrad, $\sigma_x = \sigma_y = \sigma_z = 0$

c) $d = 0.6$ mrad, $\sigma_x = 0.3$ mm, $\sigma_y = 0.012$ mm, $\sigma_z = 33$ mm

Figure 3
Effect of asymmetric trigger

Calibration change (%)

values of $\langle x \rangle$. This behaviour is illustrated in Fig. 3. In order to guarantee an absolute luminosity systematic error below 1% for a wide range of beam parameter values, a value of $d = 0.6$ milliradians has been chosen which is in this sense overcompensating.

During the study, a value of 400 microns for the chamber resolution has been used. The energy showers were not simulated, but lateral non-fluctuating shower spreading has been allowed for. The trigger as explained in section (1) has been simulated with an energy trigger threshold of 78% of beam energy per arm. The events themselves were produced by a standard generator [7] including first order radiative events.

It is important to realize that the results from the study on the systematic error apply only in case that a particular beam parameter setting cannot
be measured (or predicted). If a parameter can be measured (e.g. by the central detector), it can clearly be corrected for and its effects on systematics removed. A study will be undertaken to investigate the feasibility of measuring beam parameters (beam spot offset, beam width, beam tilt, ..) with the luminosity monitor itself. Hence, the stated results are worst case only. Table 2 shows "typical values" for the parameters as they have been estimated by the LEP instrumentation group [5]. The column labelled "absolute change" shows the corresponding change in the luminosity calibration, in per mille.

In conclusion, it can be stated that the systematic error, due to ignorance of beam parameter settings, on the absolute luminosity can be kept below the percent level for values several times larger than the column labelled "Typical Value" in Table 2 hereafter, using a value of \( \delta = 0.6 \) milliradians.

<table>
<thead>
<tr>
<th>Parameter at I.P.</th>
<th>Typical Value</th>
<th>Known to</th>
<th>Absolute Change in ( % ) for Typical Value</th>
<th>Systematic Uncert. ( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle x \rangle )</td>
<td>100 ( \mu m )</td>
<td>15 ( \mu m )</td>
<td>0.1</td>
<td>0.10</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>300 ( \mu m )</td>
<td>10 ( \mu m )</td>
<td>1.5</td>
<td>0.05</td>
</tr>
<tr>
<td>( \langle y \rangle )</td>
<td>100 ( \mu m )</td>
<td>5 ( \mu m )</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>12 ( \mu m )</td>
<td>1 ( \mu m )</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>( \langle z \rangle )</td>
<td>1 ( mm )</td>
<td>0.7 ( mm )</td>
<td>0.1</td>
<td>0.11</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>33 ( mm )</td>
<td>0.5 ( mm )</td>
<td>1.5</td>
<td>0.08</td>
</tr>
<tr>
<td>( \langle x' \rangle )</td>
<td>0</td>
<td>2 ( \mu rad )</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>( \sigma_{x'} )</td>
<td>175 ( \mu rad )</td>
<td>5 ( \mu rad )</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>( \langle y' \rangle )</td>
<td>0</td>
<td>10 ( \mu rad )</td>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td>( \sigma_{y'} )</td>
<td>175 ( \mu rad )</td>
<td>5 ( \mu rad )</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>0.45</td>
</tr>
</tbody>
</table>

* these errors \( \Delta(L)/L \) are obtained under the assumption that all distributions are gaussian.
3. BUNCH TO BUNCH NORMALIZATION FOR L3

For the purpose of this report, it is not of vital importance to ensure a systematic error below 1% in the absolute luminosity. As demonstrated in the introduction to chapter "Normalization" in these proceedings [8], it is relevant instead to control the relative luminosity systematics between different bunch collisions to the per mille level. All of the systematic errors listed in sections 2.1, 2.2 and 2.3 will cancel in a relative luminosity measurement. Thus, one has only to reevaluate the Monte Carlo study of 2.4 for bunch-to-bunch parameter variations. The results are listed in Table 2.

The last column lists the per mille change in the Bhabha rate upon a variation of a parameter by the "Known to .." amount around the "Typical Value" setting [5], all remaining parameters being fixed at their respective nominal values. In particular, this means that the study lists results when the beam spot widths were at their nominal values. If one were to set the beam spot width artificially to zero, the systematics for the beam spot position, beam tilt, etc., would be much more severe. However, \( \sigma_x \) and \( \sigma_y \) were set to zero when not taken as parameter under study.

Under these conditions, the final figure of \( \Delta(L)/L \) was obtained by adding linearly the various contributions:

\[
\frac{\Delta(L)}{L} \approx 0.4^\circ/_{oo}
\]

is the error in luminosity calibration due to unknown and uncontrollable fluctuations in the geometric parameters as listed in Table 2. In order to arrive at the "bunch to bunch" luminosity systematics as used in the introduction, one has to set:

\[
\Delta(L_{Li}/L_j) = \sqrt{2} \times \frac{\Delta(L)}{L} = 0.6^\circ/_{oo}
\]

to obtain the relative error in calibration between two different bunch collision classes.

In conclusion, the L3 luminosity monitor will collect Bhabha statistics at a rate fourfold higher than the Z events. Given a polarization
programme at LEP, it appears that, under the conditions specified in the
text, it could control the relative luminosity measurement from bunch to
bunch with the required accuracy.

* * * * * *

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Precision Attainable on Relative Luminosity Measurements with the OPAL Detector

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Abstract: A successful exploitation of a physics program with polarized beams at LEP depends on measuring changes in luminosity from bunch to bunch with a precision better than 0.1\%. This precision can be obtained in a run of 80 pb\textsuperscript{-1} with the foreseen LEP parameters and the present OPAL detector. The main sources of uncertainty are reviewed.

1. Introduction

The availability of longitudinally polarized electron and positron beams in LEP at the $Z^0$ peak can enlarge the experimental capability to measure the fundamental parameters of the Standard Model. Longitudinally polarized beams will offer, among others, the following opportunities:

i) the measurement of the “Left-Right” asymmetry $A_{LR}$ between the $Z^0$ production cross sections from left-handed and from right-handed polarized electrons;

ii) “$e$ counting” at the $Z^0$ peak, because the background from radiative Bhabhas is “left-right” symmetric and cancels out when comparing the cross sections from bunches with opposite longitudinal polarization;

iii) an increased sensitivity to the asymmetries of the final state (the forward-backward asymmetry and the polarization asymmetry).

The main reason for having longitudinal polarization has certainly to be seen in the measurement of $A_{LR}$, which is only possible with longitudinally polarized beams. However, the determination of $A_{LR}$ is of value only when performed with very high accuracy. The precision aimed for is $\Delta A_{LR} \leq 0.003$ [1]. This sets stringent requirements on luminosity monitoring, in particular on how well changes in luminosity between bunches with different polarizations can be monitored.

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\textsuperscript{4)} University of Maryland, USA
2. Requirements from $A_{LR}$

At LEP, using the strategy proposed by A.Blondel [2], four electron bunches will hit four positron bunches with the following polarization scheme:

\[
\begin{align*}
e^- & \quad \Rightarrow \quad \varnothing \quad \varnothing \quad \varnothing \quad \varnothing \\
\varnothing & \quad \varnothing \quad \varnothing \quad \varnothing \quad \Rightarrow \quad e^+ \\
1 & \quad 2 \quad 3 \quad 4
\end{align*}
\] (1)

If the electron bunches 1 and 4 have a same level of longitudinal polarization $P^-$, and similarly the positron bunches 2 and 4 have an identical $P^+$, from the measured annihilation cross sections $\sigma_1$, $\sigma_2$, $\sigma_3$, $\sigma_4$ in (1) at $\sqrt{s}=M_{Z^0}$ one can extract values for $P^+$, $P^-$, and for the asymmetry $A_{LR}$. When $P^+=P^-=P$,

\[
A_{LR} = \frac{1}{P} \left( 1 - \frac{\sigma_2}{\sigma_1} \right) / \left( 1 + \frac{\sigma_2}{\sigma_1} \right),
\]

and the error $\Delta A_{LR}$ depends on the precision in determining the cross sections ratio $\Delta(\sigma_2/\sigma_1)$ and on the relative error on the level of longitudinal polarization $\Delta P/P$. The typical value $A_{LR}=0.16$ is assumed in the following.

In Fig. 1 the dependence of $\Delta A_{LR}$ on $\Delta(\sigma_2/\sigma_1)$ is sketched for $\Delta P/P=0.5\%$ and $2\%$, and for $P=30\%$ and $50\%$ average longitudinal polarization. It is observed that asking for $\Delta A_{LR}<0.003$ sets stringent requirements on the above parameters. With $\Delta P/P=2\%$, the measurement is unfeasible. With $P=30\%$, even when $\Delta P/P=0.5\%$, $\sigma_2/\sigma_1$ has to be measured to $\Delta(\sigma_2/\sigma_1)<0.0015$, which is lower than the foreseen statistical error (an integrated luminosity of 40 pb$^{-1}$ [1] yields 10$^6$ $Z^0$ events).

We are lead to assume a high average longitudinal polarization in LEP (we optimistically take $50\%$) and that it is measured to $\Delta P/P=0.5\%$. From Fig. 1, $\Delta A_{LR}\leq0.003$ requires $\Delta(\sigma_2/\sigma_1)<0.003$. The

![Fig. 1](image)

A sketch of $\Delta A_{LR}$ ($A_{LR}=(1/P)\left( 1 - \frac{\sigma_2}{\sigma_1} \right) / \left( 1 + \frac{\sigma_2}{\sigma_1} \right)$) as a function of the error in determining the annihilation cross sections ratio $\sigma_2/\sigma_1$, for an average longitudinal polarization $P_L=30\%$ and $50\%$ with a relative error of (a) $\Delta P/P=0.5\%$ and (b) $\Delta P/P=2\%$. It is assumed $A_{LR}=0.16$. 

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main contributions to $\Delta(\sigma_2/\sigma_1)$ come from the statistical error on the numbers of collected $Z^2$ events $N_1$ and $N_2$, and from the error on the luminosities ratio $\Delta(L_1/L_2)$, that is

$$\Delta(\sigma_2/\sigma_1) \approx \sqrt{(1/N_1 + 1/N_2 + (\Delta(L_1/L_2))^2)} < 0.003.$$  \hspace{1cm} (3)

With 40 pb$^{-1}$, $N_1 \sim N_2 \sim 250,000$ events in eq. (3), which implies that the error on the relative luminosity measurement must satisfy

$$\Delta(L_1/L_2) < 0.001,$$  \hspace{1cm} (4)

including both the statistical error on the collected Bhabha events and the systematics influencing the measurement. If the integrated luminosity is doubled (to 80 pb$^{-1}$) then, for the same precision on $\Delta_{LR}$, the required precision on the luminosity measurement becomes

$$\Delta(L_1/L_2) < 0.002.$$  \hspace{1cm} (5)

In the following the main uncertainties so far envisaged in the relative luminosity measurement are discussed for the particular case of the OPAL luminosity monitors.

3. The OPAL Luminosity Monitors

The OPAL detector has two luminosity monitors: the Forward Detector (FD) and the Far Forward Detector (FFD). Both of them rely on the detection of Bhabha events at very small angle. A short description is given below; for more detailed descriptions the OPAL Technical Proposal and its updates should be consulted. At present, the Forward Detector is ready for final testing; the Far Forward Detector is in an advanced stage of design. Their characteristics are summarized in Tab. 1. The quoted polar angle coverage of the FFD is with the low-\(\beta\) quadrupoles switched on.

<table>
<thead>
<tr>
<th>Tab. 1</th>
<th>Relevant characteristics of the OPAL luminosity monitors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>\begin{itemize} \item effective polar angle coverage \item $\theta_{\text{min}} - \theta_{\text{max}}$ [mrad] \item location \item $z_{\text{min}} - z_{\text{max}}$ [m] \item effective Bhabha $\sigma$ [nb] at $\sqrt{s} = 93$ GeV \item Tracking: \item $\Delta R$ [$\mu$m] \item $\Delta \theta$ [mrad] \item $\Delta \phi$ [mrad] \item Calorimetry: \item total $X_0$ \item $\Delta E/E$ ((\sigma) at $E = 45$ GeV) \item $\Delta R$ [$\mu$m] \item $\Delta \theta$ [mrad] \item $\Delta \phi$ [mrad] \end{itemize}</td>
<td>\begin{itemize} \item Forward Detector \item 47 - 120 \item 2.1 - 2.9 \item 24 \item drift chambers \item 140 \item 0.06 \item 2.5 \item Pb/Scint \item 26 \item 0.20/$\sqrt{E}$ (3%) \item 2000 \item 1 \item 3.5 - 20 \end{itemize}</td>
</tr>
</tbody>
</table>
3.1 The Forward Detector

The FD will provide a precise measurement of the absolute luminosity at the level of 2 - 3 %. Fig. 2 shows a side view of the detector. The lead-scintillator sandwich of the pre-sampler and main calorimeter measures electron energies to $\Delta E/E \approx 20\% / \sqrt{E}$. Three layers of tube chambers at 4X$_0$ depth give shower position to a few millimetres. TC1 and TC2 are drift chambers which measure single tracks to $\pm 0.5$ mm radially and a few millimetres in azimuth. The fine luminosity monitor has four pairs of precision scintillators of different areas (asymmetric acceptance) which will be used to make the absolute measurement of luminosity. Light from the calorimeter scintillators is wavelength shifted and detected by vacuum phototetrodes. The fine monitor scintillators use 12-stage mesh-dynode photomultipliers.

3.2 The Far Forward Detector

The FFD is intended for a fast monitoring of the absolute luminosity. The calorimeter has 20X$_0$ of lead-scintillator sandwich, with some segmentation of the BBQ readout to give coarse position resolution on showers. Two BaF$_2$ counters of different area in front of each calorimeter define a forward-backward asymmetric acceptance.

4. Error on the Relative Luminosity

The contributions to $\Delta(L_1/L_2)$ from the various effects hereafter discussed are summarized in Tab. 2; the total uncertainties for the FD and the FFD have to be compared with the required precision given in (4) and (5).

4.1 Statistical Uncertainty

With a luminosity of 40 pb$^{-1}$ integrated over the four polarization configurations of (1) ( 10 pb$^{-1}$ per configuration ) as suggested in [1], the relative luminosity measurement has a statistical error of 0.0029 for the FD and of 0.0007 for the FFD. The statistical uncertainty on the relative luminosity is given by

$$\Delta(L_1/L_2) \approx \sqrt{\left( 1/Nb_1 + 1/Nb_2 \right)},$$

(6)

where Nb$_1$ and Nb$_2$ are Bhabha events in two different polarization configurations. For the FD the Bhabha cross section is comparable to the $Z^0$ cross section, and relation (6) together with (3) implies that the required statistical accuracy can only be reached with $\sim 80$ pb$^{-1}$ (20 pb$^{-1}$ per polarization configuration), i.e. about two years of polarized beams using the assumptions in [1].

4.2 Systematic Uncertainties

The systematic errors which degrade the absolute luminosity measurement are reduced in the relative luminosity measurement. Errors due to geometric misalignment, showering before counters, geometrical uncertainties (size of counters, etc.) will cancel to a great extent in the luminosities ratio. However, whether this will be negligible in comparison to 0.001 is still matter of study.

In the following, some plausible sources of uncertainty in the relative luminosity measurement are discussed and the results are reported in Tab. 2. Emphasis is given to bunch-to-bunch systematic differences in the beam parameters, too small to be detected, but big enough to generate changes of a $0.00\%$ in the luminosity. A typical example is the displacement of the bunch crossing region. A tiny displacement (of the order of 10 $\mu$m) can cause a loss of the electrons on the lower edge of the detector acceptance and hence decreases the geometrical acceptance for Bhabha events. If such a systematic difference exists between two consecutive bunch crossings (corresponding to different polarization configurations), it remains undetected because it is smaller than the machine resolution. A simulation study has been performed, where the electron and positron beams are generated with the beam parameters at
Tab. 2 A list of the main uncertainties so far envisaged which contribute to the error on the relative luminosity measurement with the OPAL luminosity monitors. The values for the systematic uncertainties are calculated by means of a simulation program, as described in the text, and should be regarded as lower limits. For the FFD a forward-backward asymmetric acceptance is used.

<table>
<thead>
<tr>
<th>Statistics:</th>
<th>( \Delta(L_1/L_2) ) in FD</th>
<th>( \Delta(L_1/L_2) ) in FFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated Luminosity 40 pb(^{-1})</td>
<td>0.0029</td>
<td>0.0007</td>
</tr>
<tr>
<td>Integrated Luminosity 80 pb(^{-1})</td>
<td>0.0020</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

**Systematics:**

- **Bunch-to-bunch uncertainty in:**
  - **Beam Position:**
    - \( \Delta < x > \) (horiz.) = 15\( \mu m \)
    - \( \Delta < y > \) (vert.) = 5\( \mu m \)
    - \( \Delta < z > \) = 0.5mm
    - Beam Width:
      - \( \Delta \sigma_x = 10\mu m \)
      - \( \Delta \sigma_y = 1\mu m \)
      - \( \Delta \sigma_z = 0.3mm \)
    - Beam Tilt:
      - \( \Delta < x' > = 2\mu rad \)
      - \( \Delta < y' > = 10\mu rad \)
    - Beam Divergence:
      - \( \Delta \sigma_{x'} = 5\mu rad \)
      - \( \Delta \sigma_{y'} = 5\mu rad \)

**Total Uncertainty (40 pb\(^{-1}\))**

- Statistical: 0.0030
- Systematic: >0.0009
- Required Uncertainty: <0.001

**Total Uncertainty (80 pb\(^{-1}\))**

- Statistical: 0.0022
- Systematic: >0.0009
- Required Uncertainty: <0.002

their nominal LEP values and collide producing Bhabha events. Input for this study (beam parameters and machine resolutions) has been kindly provided by G. Von Holtey [3] and the LEP machine group. The changes in the geometrical acceptance of the luminosity monitors FD and FFD induced by variations in the beam parameters are studied. No shower simulation is used; real detector efficiency and shower development will reduce the acceptance and tend to increase the uncertainty, in particular for the FFD which has poorer shower containment because of its small lateral dimensions. In this respect the values for the systematics in tab.2 are lower limits.

Concerning the FFD, all numbers hereafter quoted refer to operation with the low-\( \beta \) quadrupoles switched on, whose effect has been taken into account in the simulation. Notice that for the FFD a
forward-backward asymmetric acceptance is used: a Bhabha event is defined by the coincidence of the small counter with the large counter opposite to it. The acceptance region for the small and the large counters differs in x by \( \Delta x = 2.5 \text{ mm} \), the small counter defining an inner acceptance region of the large counter. This allows an acollinearity between the electron and the positron, and smooths out the losses due to tilt and divergence of the beams.

It has to be stressed that alignment errors have not been taken into account. Their influence is stronger for the FFD, because it is very far from the bunch crossing region and is located behind the low-\( \beta \) quadrupole. Furthermore, its stability with respect to variations in tilt and divergence of the beams depends on the forward-backward asymmetric acceptance, which is distorted in case of alignment errors. Thus, the values shown for the systematics in Tab.2 are clearly lower limits.

4.2.1 Displacement of the Bunch Crossing Region

With the LEP instrumentation the position of the bunch centroid at the interaction point is likely to be determined to \( \Delta <x> (\text{hor}) = 100 \mu \text{m}, \Delta <y> (\text{vert}) = 30 \mu \text{m}, \Delta <z> = 1 \text{ mm} \). However, the uncertainty on the relative position of two consecutive bunches is \( \pm 15 \mu \text{m} \) in \( x \), \( \pm 5 \mu \text{m} \) in \( y \), \( \pm 1 \text{ mm} \) in \( z \). The average position in \( z \) is statistically better determined. In a run with an integrated luminosity of 100 \( \text{nb}^{-1} \), about 700 \( Z^0 \) events will be collected per polarization configuration in scheme (1), yielding a statistical uncertainty of \( \pm 0.5 \text{ mm} \) on \( <z> \). It is the displacement along \( z \) that induces the largest uncertainty on \( \Delta(L_1/L_2) \) in the FD.

4.2.2 Beam Size

Assumed values for the bunch width are \( \sigma_x = 300 \pm 35 \mu \text{m}, \sigma_y = 12 \pm 4 \mu \text{m}, \sigma_z = 12.8 \pm 1.3 \text{ mm} \), with a bunch-to-bunch uncertainty limited to \( \pm 10 \mu \text{m} \) in \( x \), \( \pm 1 \mu \text{m} \) in \( y \), \( \pm 0.5 \text{ mm} \) in \( z \). Notice that \( \sigma_x \) strongly increases in presence of wigglers into the LEP ring. With 700 \( Z^0 \) events \( \Delta \sigma_x \) for a polarization configuration can be determined to \( \pm 0.3 \text{ mm} \), which yields in the FD the dominant uncertainty on the relative luminosity measurement.

4.2.3 Beam Tilt

The assumed beam direction is \( <x'> (\text{hor}) = 0 \pm 15 \mu \text{rad}, <y'> (\text{vert}) = 0 \pm 60 \mu \text{rad} \), with a bunch-to-bunch uncertainty limited to \( \pm 2 \mu \text{rad} \) in \( x \), \( \pm 10 \mu \text{rad} \) in \( y \). The strongest effect is in the FD and is due to the relative large value of the uncertainty in \( y' \).

4.2.4 Beam Divergence

The angular spread of a beam bunch in LEP is likely to be \( \sigma_{x'} = 175 \pm 20 \mu \text{rad}, \sigma_{y'} = 175 \pm 20 \mu \text{rad} \), with a bunch-to-bunch uncertainty limited to \( \pm 5 \mu \text{rad} \) in \( x \), \( \pm 5 \mu \text{rad} \) in \( y \).

The FFD measurement is strongly influenced by the bunch-to-bunch uncertainty in the beam divergence along \( x \), as shown in Fig.3 where \( \Delta(L_1/L_2) \) is plotted as function of \( \Delta \sigma_{x'} \), uncertainty in the

![Graph](image)

**Fig. 3**
A sketch of the dependence of the uncertainty of the relative luminosity measurement \( \Delta(L_1/L_2) \) on the bunch-to-bunch uncertainty in the beam divergence \( \Delta \sigma_x \) for the Forward Detector (FD) and for the Far Forward Detector (FFD).
bunch-to-bunch difference about the nominal value of $\sigma_{X'} = 175 \mu\text{rad}$. An uncertainty of $30 \mu\text{rad}$ increases the error for the relative luminosity in the FFD up to the 1.0% level. On the contrary, the FD is observed to be quite stable against a bunch-to-bunch uncertainty in $\sigma_{X'}$.

4.2.5 Off-momentum Electrons

Off-momentum electrons are generated in the bremsstrahlung collision of beam electrons with the residual gas inside the beam pipe. The rate of off-momentum electrons coming out of the beam pipe inside the acceptance region of each LEP detector has been calculated in [4], and it is large (of the order of 500 Hz per beam). Most of these off-momentum electrons are at small polar angles and produce fake Bhabha events. The rate of off-momentum electrons is proportional to the bunch intensity, which can vary from bunch to bunch by as much as 10%. In the FD the energy distribution of the off-momentum electrons has a sharp peak at $\sim 30$ GeV, much below the beam energy; a rejection based on the energy measurement reduces this background to a negligible amount. Furthermore, the tracking devices and the azimuthal segmentation of the calorimeter allow to measure the alignment of Bhabha showers. For the FFD, the rejection of fake Bhabhas is less effective, because the off-momentum electrons hitting the FFD have energy near the beam energy, and also the FFD layout does not allow a good acollinearity measurement. However, it is possible to measure this background with high accuracy by monitoring, for each polarization configuration in (1), the shower rate on one side only, and the rate of delayed coincidences of one shower with a shower on the opposite side on the next crossing of the same bunches. The background is subtracted from the Bhabha coincidences for each polarization configuration. The uncertainty induced on the relative luminosity can be kept below 0.1%.

5. Conclusions

Assuming the essential condition of an average longitudinal polarization $P_L$ of at least 50% measured to $\Delta P/P = 0.5\%$, the success of an experimental programme with polarized beams still depends on monitoring to a very high accuracy the variations in luminosity between different polarization configurations, that is

$$\Delta(L_1/L_2) < 0.002,$$  \hspace{1cm} (7)

including both the statistical and the systematic errors, for a run of 80 pb$^{-1}$ with polarized beams.

The OPAL luminosity monitors are the Forward Detector (FD) (47 to 120 mrad) and the Far Forward Detector (FFD) (5.6 to 10 mrad). With an integrated luminosity of 80 pb$^{-1}$ (20 pb$^{-1}$ per polarization configuration), the FD relative luminosity measurement is limited to $\Delta(L_1/L_2) = 0.002$ by Bhabha statistics. The FFD can achieve a statistical error on $L_1/L_2$ of 0.0005. On the other hand, the simulation studies so far performed on the systematic errors show that the FFD is much poorer than the FD in controlling the systematics, and can not match the above requirement on the relative luminosity monitoring.

We are studying at least two possible upgrades which would improve on the precision of the luminosity measurement in OPAL.

i) Fine position sensitive detectors could be placed inside the FFD calorimeter to measure the precise alignment of Bhabha showers, and hence to reduce the systematic error due to $\Delta\sigma_{X'}$ [5]. Provision will be made in the FFD design to allow for such extra detectors.

ii) With a reduced beampipe diameter (and redesign of the pressure-bell window in front of the FD) the inner acceptance angle of the FD could be reduced to $\sim 30$ mrad, giving a comfortable excess of Bhabha statistics over the $Z^0$ statistics. This would be a major modification of the present OPAL layout.
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A SINGLE BREMSSTRAHLUNG MONITOR FOR MEASURING LUMINOSITY AND BEAM DIVERGENCE AT LEP.

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3) Università di Roma "La Sapienza", and INFN Sezione di Roma, Rome, Italy
4) In partial fulfilment of doctoral thesis

ABSTRACT
The luminosity and the beam divergence at an interaction point of LEP can be measured by detecting the energy and the angular distribution of the single bremsstrahlung photons emitted at very forward angle. This method is much faster than the conventional method of detecting small angle Bhabha scattering. Both luminosity and divergence can be measured with an absolute accuracy of 1%.

1. INTRODUCTION
Luminosity and beam divergence are very important parameters in colliding beam experiments. Furthermore luminosity monitoring is essential in experiments with $e^+e^-$ colliding beams in order to determine the cross sections from experimental rates.

Measurement and monitoring of luminosity is based on detection of QED processes of which the cross section can be evaluated with acceptable accuracy [1].

Two requirements in a monitoring system are essential:
a) speed: a short measurement time is needed in order to check and to optimize continuously the machine operating conditions;
b) precision: a high precision in absolute luminosity determination is required for experiments that measure cross sections.

We have compared the following processes to be used for luminosity determination at storage rings [2]; namely the Bhabha scattering (BS), the single bremsstrahlung (SB) (which is also called radiative Bhabha scattering), and the double bremsstrahlung (DB):

\[ e^+e^- \rightarrow e^+e^- \quad \text{(BS)} \]
\[ e^+e^- \rightarrow e^+e^- \gamma \quad \text{(SB)} \]
\[ e^+e^- \rightarrow e^+e^- \gamma\gamma \quad \text{(DB).} \]
BS events can be detected both in the experimental apparatus and/or by special
dedicated monitors. SB and DB events require dedicated monitors, as the photons are
mostly emitted at very small angle along the e± beam directions.

The BS reaction is by far the most often utilized. The main features of BS can be
summarized as follows.

BS cross section decreases with the total c.m. energy squared s as s−1, and it depends
on the e± scattering angle θ as θ−4. At LEP energy the scattered electrons have to be
detected at small angle (<102 mrad) in order to get a reasonable counting rate which in any
practical case hardly exceeds 1+10 s−1, at a luminosity of 1031 cm−2 s−1.

The comparison of the BS production rate at small angle for storage rings of different
energy and luminosity is shown in Fig.1 and in Table 1. In contrast to the sharp decreasing
of the BS rate with increasing energy, the rate of SB is almost constant. Therefore SB
becomes more favourable at LEP than at low energies machines.

<table>
<thead>
<tr>
<th></th>
<th>SPEAR</th>
<th>PETRA</th>
<th>LEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>luminosity (cm−2s−1)</td>
<td>1031</td>
<td>1030</td>
<td>1031</td>
</tr>
<tr>
<td>beam energy (GeV)</td>
<td>2.6</td>
<td>8.5</td>
<td>55</td>
</tr>
</tbody>
</table>
| BS rate at interaction point (Hz)
  (10<θ<20 mrad)        | 3.5 103 | 30 | 6.5 |

The BS angular and energy dependences make also the measurement sensitive to
many machine parameters as energy spread, displacement of the interaction region, shape
and size of the beams, beam tilt at the interaction point, etc... The BS absolute
measurement is also sensitive to detector position and alignment. It should be noted that
very small angle BS e± cannot be detected in the main experimental apparatus, so that a
problem of normalization could be there as well as in case of SB and DB.

The main advantage of this method is a comparatively low background, as the emitted
electron and positron are detected in coincidence.

However at high energy the SB shows important advantages over BS and in this note
we will discuss the SB method for LEP [3].

The expression of the differential cross section of e+e− → e+e−γ reaction as given by
Y.S.Tsai [4] is:

\[
\frac{d\sigma}{d\Omega dK} = \frac{2\alpha^2}{\pi K m^4} \left[ \frac{2y-2}{(1+\ell)^2} + \frac{12\ell(1-y)}{(1+\ell)^4} + \left[ \frac{2-2y+y^2}{(1+\ell)^2} - \frac{4\ell(1-y)}{(1+\ell)^4} \right] \left[ \frac{2\ell n}{(m^2-1)} \right] \right],
\]
where: \( E = \text{beam energy} \)
\( m = \text{electron mass} \)
\( K = \text{photon energy} \)
\( y = K/E \)
\( \theta = \text{angle of photon} \)
\( \zeta = \theta^2 E^2 / m^2 \)
\( \delta = m^2 y / 2E(1-y) \)

Assuming a LEP luminosity of \( 10^{31} \text{ cm}^{-2}\text{s}^{-1} \), the rate of SB photons emitted only on one beam side from interaction point (IP) above a given cutoff energy \( e \) is shown in Fig.2.

The most important SB feature at 50 GeV beam energy is the very high rate of events that is \( \sim 10^2 / \text{crossing} \), or \( \sim 4 \times 10^6 / \text{sec} \). This value has to be compared with 0.25+ 1 Hz expected for the standard luminosity monitors normally used in LEP experiments.

Furthermore almost all SB photons are emitted at very small angle (< 10 \( \mu \text{rad} \)) for the whole photon energy spectrum. As a consequence the beam angular divergence, which is expected [5] to be around 175 \( \mu \text{rad} \) at LEP (in low \( \beta \) operation) is much larger than the emitted SB photons angular spread, and can be measured and continuously monitored (see Fig.3). Of course this requires that the photons are not shadowed by any obstacle so that the complete angular distribution is "seen" by the photon detector (this important point is discussed in Sect.2).

In order to measure the absolute luminosity we need to install a calorimeter and to perform the following measurements:
i) count the number of photons with an energy larger than a certain photon threshold energy (single counting regime, see below); or measure the total energy deposited by SB photons (multi-photon counting regime);
ii) determine the beam divergence by measuring the angular distribution of SB photons.

The SB monitor works in two different regimes of photon counting, depending on the luminosity value:
i) the single photon counting, i.e. no more than one photon/crossing, as expected at \( L = 10^{29} + 10^{30} \text{ cm}^{-2}\text{s}^{-1} \); ii) the multi-photon counting, as expected at \( 10^{31} \text{ cm}^{-2}\text{s}^{-1} \).

In the first case a calorimeter can be used to measure both the energy and the direction (i.e. the axis of the e.m. shower). In the second case the calorimeter measures the total energy deposited/crossing, but cannot determine the photon direction, as many e.m. showers overlap into the calorimeter fiducial volume at each crossing. To measure the SB photon angular distribution it is then necessary to detect the photon conversion vertex by a special thin detector placed before the calorimeter.

Moreover later on it could be possible to extend the role of this detector to measure the longitudinal polarization by detecting the circular polarization of the SB photons [3].

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2. **SINGLE BREMSSTRAHLUNG DETECTION AT LEP**

The SB photons emitted at the interaction point (IP) must go out of the machine (vacuum pipe, magnets, etc) and reach the photon-detector. In Fig.4 a sketch of a LEP half straight section from the IP to the arc is shown; the detector is located at $\ell \approx 350$ m from the IP.

In order to set up a thin window to minimize the absorbing material (essentially aluminium and iron) traversed by SB photons, some modification of the machine is needed.

The situation has been already investigated by other authors with the aim to measure the electron transverse polarization at LEP by detecting back scattered laser photons [6]. In this last case in order to set up a window of 2 cm (vertical) $\times$ 5 cm (horizontal), the magnet B4/2 must be reversed while the coils of the QL13 quadrupole and the vacuum chamber inside the QL12 quadrupole have to be modified.

In the case discussed here the largest possible window, which is centred relative to the theoretical beam line, cannot exceed $\pm 60$ mm (horizontal) due to vacuum pipe radius and $\pm 35$ mm (vertical) due to magnet aperture [7]. In Fig. 3 this "window" is compared to the angular distribution of SB photons due to the beam divergence. In first approximation the SB photon angular distribution is assumed to be the same for the electron and positron angular distribution at IP, i.e. a bivariate normal distribution with equal variances. (The last assumption, however, is inessential). With the expected r.m.s. spread $\sigma_x = \sigma_y = \sigma_D = 175$ $\mu$rad, it turns out that only a fraction of about 38% of SB photons emitted at IP can impinge the detector, through a "window" of $\pm 1.15 \sigma$ (horizontal) and $\pm 0.67 \sigma$ (vertical) where $\sigma = \sigma_D \ell$.

The sketch of the experimental lay-out to detect SB photons emitted in IP is shown in Fig.5. The main components of a possible set-up are:

(a) a beryllium thick absorber (A) of the low energy photons produced by synchrotron radiation; 60 cm of Be reduce SR photons in an energy range 0.1 to 0.5 MeV by a factor of $10^6 \div 10^4$ respectively, while the SB photons in the range 0.1 to 50 GeV are only reduced by a factor three (see Sect.4 for detailed discussion on SR background);

(b) a sweeping magnet (M) sweeps out electrons and positrons coming out of A; a 3 T m bending power is enough to sweep out electrons up to 5 GeV/c;

(c) a thin converter (T) placed in front to a tracking device (V) to convert a small fraction (e.g. 10%) of SB photons and to detect the conversion vertex; in this way the photon angular distribution is measured and monitored; a spatial resolution of 2+5 mm is sufficient as discussed in Sect.3;

(d) finally an electromagnetic calorimeter (C) measures the energy deposited by SB photons and is then used to monitor continuously the machine luminosity; a detector with high tolerance to radiation damage is imperative. Therefore we propose to use a calorimeter made of scintillating fibres (polystyrene) and lead as described in Ref.[8].
An energy resolution of 10%/√E(GeV) and a spatial resolution of 2+5 mm are acceptable for our purpose. A full containment of e.m. shower both transverse and longitudinal is needed. The main features of the calorimeter are summarized in Table 2.

**Table 2**

Main features of the e.m. calorimeter proposed to measure energy and position of SB photons.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>scintillator</td>
<td>polystyrene</td>
</tr>
<tr>
<td>fibre diameter</td>
<td>1 mm</td>
</tr>
<tr>
<td>absorber</td>
<td>lead</td>
</tr>
<tr>
<td>filling factor (scint./total)</td>
<td>30%</td>
</tr>
<tr>
<td>global density</td>
<td>6.5 g cm⁻³</td>
</tr>
<tr>
<td>global rad. length</td>
<td>1.4 cm</td>
</tr>
<tr>
<td>sensitive volume</td>
<td>30x25x20 cm³</td>
</tr>
<tr>
<td>(length x width x height)</td>
<td>~20000</td>
</tr>
<tr>
<td>fibre total number</td>
<td>100 PMT (14 mm diameter)</td>
</tr>
</tbody>
</table>

3. **STATISTICAL ERROR ON LUMINOSITY AND DIVERGENCE**

As the angular accepted fraction of photons is limited the luminosity measurement depends on the assumption that the electron divergence distribution is of gaussian shape.

The resolutions on luminosity $(\Delta L/L)_{stat}$ and on divergence $(\Delta \sigma_D/\sigma_D)_{stat}$ have been investigated by a Montecarlo simulation. The total photon flux $N_T$ emitted at IP is proportional to the machine luminosity $L$.

If $N$ is the detected SB photon rate, then we have

$$N = \frac{N_T}{2\pi \sigma^2} \int_{-a}^{b} \int_{-\tilde{x}}^{\tilde{x}} \exp \left[ -\frac{(x - \tilde{x})^2 + (y - \tilde{y})^2}{2\sigma^2} \right] \ dx \ dy \tag{1}$$

where $\sigma, \tilde{x}, \tilde{y}$ are the variance and the means respectively of a bidimensional gaussian distribution. This distribution of photons emitted at IP is limited by a rectangular aperture $(2a, 2b)$ in the plane $(x, y)$; the $z$-axis represents the photon line of flight.

The statistical error in the determination of $L$ and $\sigma_D$ is evaluated by a minimum $\chi^2$ method (program MINUIT). The theoretical spectrum of SB photons (1) is compared to a simulated spectrum; $\sigma, \tilde{x}$ and $\tilde{y}$ are the free parameters.

The results on $\Delta L/L$ and $\Delta \sigma_D/\sigma_D$ could depend in principle on: (i) the total number of photons passing through the window; (ii) the chosen binning for points $(x, y)$ of the
window which is related to the spatial resolution of the detector; (iii) the displacement of \((\tilde{x}, \tilde{y})\) out of the centre of the rectangle (which can be produced by a tilt of the beams or by a transverse displacement of IP).

In the computation it has been assumed that the beam divergence varies by \(\Delta \sigma_D = 30 \mu\text{rad}\) around a central value of \(\sigma_D = 175 \mu\text{rad}\). We recall that the beam divergence should be monitored with a precision of \(\pm 20 \mu\text{rad}\) with the LEP instrumentation [5].

The Montecarlo results are summarized in Table 3. This result is practically independent of the binning chosen for the collected data, ranging from 1 to 10 mm\(^2\).

In Table 3 results obtained for two windows smaller than the proposed one (A:7x12 cm\(^2\)) are also presented. These conditions refer to: (i) the insertion of horizontal collimators (B:7x5 cm\(^2\)) required by experiments in order to reduce background on apparatus [9]; (ii) a small window (C:2x5 cm\(^2\)) originally designed for longitudinal polarimetry monitor [6]. The determination of luminosity and divergence will actually be made in conditions corresponding to case B.

The rate of detected SB photons was estimated by taking into account an overall efficiency which includes absorption of photons (30%), efficiency in e\(^+\)e\(^-\) pair conversion (10%), threshold of minimum energy in \(\gamma\) detection (0.3 GeV), etc.

At a luminosity of \(10^{29}+10^{30} \text{ cm}^{-2}\text{s}^{-1}\) the SB photon flux allows to operate the \(\gamma\)-detector in single-photon regime, so that the running time scales by a factor ten only respect to \(10^{31} \text{ cm}^{-2}\text{s}^{-1}\), as shown in Table 3.

The following points should also be noted: (a) the running time needed is very short especially in comparison to proposed BS luminosity monitor rate; (b) for machine operation purposes L can be monitored with an accuracy of 10% instantaneously (0.05+0.7 s at \(10^{31} \text{ cm}^{-2}\text{s}^{-1}\) with window A and B respectively).

Finally, Fig. 6 shows the statistical error achieved on the luminosity determination as a function of the photon statistics.

4. BACKGROUNDS AND SYSTEMATIC ERRORS

As the off-momentum electron background is negligible in our case, we consider in the following the two main sources of background:

(i) photons due to beam-gas interactions (BG) inside the vacuum chamber;

(ii) photons due to synchrotron radiation (SR) emitted essentially inside correcting dipoles.

The rate of BG at LEP has been estimated by a Montecarlo simulation [10]. With a vacuum pressure of \(2 \times 10^{-10} \text{ torr}\), a beam current of 3 mA (\(L = 10^{31} \text{ cm}^{-2}\text{s}^{-1}\)) and setting a minimum photon energy to 0.3 GeV, the estimated rate is \(1.3 \times 10^4 \text{ photons s}^{-1}\) (0.3 photons/crossing). This rate corresponds to a 0.4% contamination of the SB rate independent of the luminosity and of the threshold energy.
The flux of photons by SR at LEP has been already estimated in Ref.[6] where a transverse polarization monitor, with a geometry similar to the present proposed layout, is proposed.

According to the result of Ref.[6] we made the following assumptions: (a) beam energy 50 GeV; (b) luminosity $10^{31}$ cm$^{-2}$ s$^{-1}$; (c) correcting dipoles powered up to 25% of their maximum value; (d) finally a safety factor of ten was also taken. The result is then that the contribution of SR to the total energy/crossing deposited in the calorimeter, by taking a threshold energy of 0.5 MeV, is less than 5%. By using a 60 cm thick beryllium absorber (see also Sect.2) the energy due to SR deposited in the calorimeter, is estimated to be less than 0.1%. In these conditions, if the gaussian width of the photon distribution is supposed to be constant during the monitoring time, a simple measurement of the total energy would be convenient for a very fast relative monitoring of L.

On the other hand, for an accurate absolute measurement of L, we need to measure both the number of photons and their angular distribution.

An evaluation of SR photon rate detected by the $\gamma$-vertex detector V (see Fig.5) strongly depends on the detector threshold energy. For example by moving the threshold from 0.5 MeV to 100 MeV the contamination of SR to the photon rate, with Be absorber, decreases from 20% down to 1%. Nevertheless the SR photon rate can be measured with spatially separated beams and thus can be subtracted to the total photon rate on $\gamma$ detector.

It should be noted that in case of single photon counting (i.e. no more than one photon/crossing) a threshold energy can be fixed in the e.m. calorimeter. Therefore in this case the SR contamination could become negligible(*).

Main systematic errors come from fluctuations of machine parameters and from incertitudes introduced by the experimental set-up.

We have considered the following sources: (a) IP displacement; (b) beam "tilt"; (c) beam energy variation; (d) beam divergence fluctuation; (e) non-gaussian tail fluctuations of SB photon angular distribution.

The points (a) and (b) have been investigated by the Montecarlo method quoted in Sect.3. The simulation shows that a 10 mm displacement of the axis of the photon angular distribution affects luminosity and divergence relative error by less than 0.1%. This displacement corresponds to a 33 $\mu$rad "tilt" of one beam.

Due to the logarithmic dependence with the beam energy of the SB cross section, the relative error on luminosity is of the order of 0.1% for a fluctuation of 0.5% in maximum beam energy.

(*) If the SR flux is higher than that considered here, also by several orders of magnitude, the situation can be restored simply by increasing the absorber thickness at a price of some reduction in SB flux. Nevertheless this reduction can be compensated by the transition to the single photon counting regime, when appropriate.
As stated above during absolute measurement of L the beam divergence at IP is also monitored by the proposed detector. For the statistical errors and measuring times involved see Table 3.

**Table 3**

Results of a Monte Carlo calculation on absolute luminosity and divergence determination assuming a gaussian angular distribution of SB photons.

<table>
<thead>
<tr>
<th>luminosity (cm(^{-2}) s(^{-1}))</th>
<th>10(^{31})</th>
<th>10(^{29})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma) window (cm(^{2}))</td>
<td>A 7x12</td>
<td>B 7x5</td>
</tr>
<tr>
<td>(\gamma) acceptance(%)</td>
<td>37.5</td>
<td>18.5</td>
</tr>
<tr>
<td>SB photons rate on detector (Hz)</td>
<td>3.8 (10^4)</td>
<td>1.9 (10^4)</td>
</tr>
<tr>
<td>Photon number for a statistical error (\Delta\sigma_D/\sigma_D \approx 0.5%)</td>
<td>(10^5)</td>
<td>1.5 (10^6)</td>
</tr>
<tr>
<td>(\Delta L/L \approx 1%)</td>
<td>(10^7)</td>
<td>(1.5 \times 10^8)</td>
</tr>
<tr>
<td>corresponding running time</td>
<td>2.6 s</td>
<td>80 s</td>
</tr>
<tr>
<td></td>
<td>26 s</td>
<td>13.3 min</td>
</tr>
<tr>
<td>Photon number for a statistical error (\Delta\sigma_D/\sigma_D \approx 0.5%)</td>
<td>(10^7)</td>
<td>1.5 (10^8)</td>
</tr>
<tr>
<td>(\Delta L/L \approx 1%)</td>
<td>(10^7)</td>
<td>(1.5 \times 10^8)</td>
</tr>
<tr>
<td>corresponding running time</td>
<td>4.3 min</td>
<td>2.2 h</td>
</tr>
<tr>
<td></td>
<td>43 min</td>
<td>22 h</td>
</tr>
</tbody>
</table>

This result has been estimated under the assumption that SB photons have a gaussian angular distribution. Therefore in absolute measurement of luminosity a systematic error may be present due to possible fluctuations in the non-gaussian tail of the angular distribution. In fact the tail is not "seen" by the proposed experimental set-up, due to the limited acceptance of the window.
Nevertheless the experimental measurements of the (vertical) beam profile performed at CESR have shown that the particle distribution inside the bunch core remains gaussian, as it falls by about 7 orders of magnitudes, or out to about $5\sigma$ [12].

For a fixed measurement time (see Table 3) the luminosity error is almost independent from the fluctuations of the beam parameters listed above ((a), (b), (c) and (d)). The proper time needed to reach statistical errors $\Delta L/L \approx 1\%$ and $0.1\%$ are 80 sec and 2.2 hours respectively at $10^{31}\text{cm}^{-2}\text{s}^{-1}$ using the window $(7\times5\text{ cm}^{2})$ with collimator in place.

Uncertainty in knowledge of photon absorption coefficient in the Be absorber will affect the absolute luminosity measurement at the $1\%$ level.

As quoted in Sect. 3, by changing the spatial resolution of the SB photon conversion vertex (in detector V and/or in calorimeter C) from 2 to 5 mm, divergence (and luminosity) measurement is not affected.

Systematic errors on absolute luminosity measurement may be introduced by systematic uncertainties in SB photon energy determination.

In case of single-photon counting, $(\Delta L/L)_{\text{syst}}$ has been investigated by a Montecarlo calculation, taking into account for instrumental effects (calorimeter): (a) variations of threshold energy; (b) calibration errors. In this computation a simulated experimental energy spectrum is compared to the energy theoretical distribution of SB photons; a normalization factor and an energy-scale factor are the free parameters. Variations of the detection threshold energy have essentially no effect on the systematic error $(\Delta L/L)_{\text{syst}}$. Uncertainties in energy calibration, or instabilities in calibration constants ranging from 1% to 0.2% contribute to $(\Delta L/L)_{\text{syst}}$, as much as 1% and 0.1% respectively.

5. **CONCLUSIONS**

We have shown that a single bremsstrahlung monitor at LEP should be very useful. If at least one window of the dimensions specified above $(7\times12\text{ cm}^{2})$ is installed, it should be possible to measure luminosity and beam divergence at the level of 1% in a very short time (few seconds at $L = 10^{31}\text{ cm}^{-2}\text{s}^{-1}$), which are most useful both for the machine tuning and for the luminosity monitoring in data taking.

**ACKNOWLEDGEMENTS**

We are indebted to Massimo Placidi for his contribution to this paper, especially for the design of the machine modifications requested for the proposed experimental set-up.

We are grateful also to Claude Bovet and Georg von Holtey for their interest and for many useful discussions.
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Fig. 1  Comparison of the BS (left scale) and SB (right scale) cross section ratios vs the centre of mass energy for $e^+e^-$ colliders.

Fig. 2  Rate of SB photons emitted at IP in LEP for a luminosity of $10^{31} \text{ cm}^{-2} \text{s}^{-1}$ and a beam energy of 50 GeV as a function of a photon threshold energy. This rate has been computed by QED theory.
Fig. 3  Angular distribution of SB photon emitted at 0 degree due to a beam divergence ($\sigma_D = 0.175$ mrad). The SB angular spread at some energy is also shown for a comparison.

Fig. 4  A sketch of LEP half straight section from IP to SB photon detector. Notice that photons go out of LEP vacuum pipe at about 300 m from IP.
Fig. 5 The experimental lay-out for SB photon detection system: W = window; A = Be absorber; M = sweeping magnet; T = thin γ converter; V = vertex detector; C = e.m. calorimeter.

Fig. 6 Statistical error on luminosity as a function of the total SB photon counting rate. The corresponding running time is estimated at $10^{31}$ cm$^{-2}$ s$^{-1}$, for a (7x5) cm$^2$ window.
BEAM POLARIZATION IN e+ ACCELERATORS

Polarization in LEP-Scenario-Feasibility-Cost, E. Keil
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Polarization simulation studies for LEP, J.P. Koutchouk and T. Limberg
Wigglers for polarization, A. Blondel and J.M. Jowett
Calculation of higher order spin resonances, S.R. Mane
Depolarization enhancement due to the energy spread, J. Buon
A matched spin rotator for LEP, A. Blondel and E. Keil
Photon background at the LEP detectors from Richter-Schwitters spin rotators, D. Treille and G. von Holtey
1. INTRODUCTION

This paper describes the steps needed to obtain longitudinally polarized beams in LEP, their interference with the LEP commissioning programme and energy upgrade, and their cost. The remainder is organized as follows: Chapter 2 describes the present status of theoretical studies and simulations aiming at predictions of the degree of polarization in LEP. Chapter 3 describes further studies, both theoretical and experimental, which need to be done between now and the time when LEP is fully commissioned with unpolarized beams, in order to put the decision whether or not to install spin rotators on a firm basis. The conceptual design of a Richter-Schwitters spin rotator [1], its installation in LEP, and its cost are discussed in Chapter 4. The conclusions are in Chapter 5.

It helps to distinguish between the transverse and longitudinal spin-physics programmes:

i) The transverse spin-physics programme is directed towards a prediction of the degree of polarization which will be achieved in LEP and towards its experimental verification once LEP is in operation.

ii) The longitudinal spin-physics programme is directed towards the design of the spin rotators for turning the spin into the beam direction, towards a verification of their proper functioning by the same analytical and computer techniques that were used for the transverse spin physics, and their installation and commissioning in LEP.

When discussing degrees of polarization, it is important to distinguish between the following quantities:

i) The average equilibrium polarization $\langle P \rangle$, often called polarization level for short, which is reached in the limit of infinite time in an ensemble of machines. Its upper limit, $\langle P \rangle = 92\%$, given by the Sokolov-Ternov effect, can only be reached in an ideal machine without reversed-field dipoles and without distortions of the closed orbit. Wiggler magnets with reversed-field dipoles and errors in the alignment and excitation of LEP components reduce $\langle P \rangle$. Estimates for $\langle P \rangle$ can be obtained by simulation with computer programs, e.g. SLIM [2].
ii) The average effective polarization \( \langle \langle P \rangle \rangle \) during a physics run, discussed by Blondel and Jowett [3] and defined by:

\[
\langle \langle P \rangle \rangle = \left[ \frac{\int_0^T P(t)^2 L(t) dt}{\int_0^T L(t) dt} \right]^{1/2}
\]

It takes into account the buildup of the polarization \( P(t) \) and the decay of the luminosity \( L(t) \) during a physics run of duration \( T \). The average effective polarization \( \langle \langle P \rangle \rangle \) is always smaller than the equilibrium polarization \( \langle P \rangle \).

2. **PRESENT STATUS**

Preliminary computer simulations, using the linear spin-dynamics program SLIM [2] and realistic assumptions about errors in the alignment and excitation of LEP components and the resulting errors of the closed orbit after correction were presented at the workshop in November 1987 by Kouchch [4]. He showed that a more realistic treatment of the remaining orbit errors than in an earlier simulation [5] led to a reduction of \( \langle P \rangle \) from 70% to about 50% even after a correction of several orbit harmonics responsible for exciting the nearest spin resonances. These simulations did not take into account the finite amplitudes of betatron and synchrotron oscillations.

Blondel and Jowett [3] proposed additional wigglers for LEP, dedicated to improving the beam polarization. Since these polarization wigglers are much more asymmetric than the existing ones, they lead in an ideal LEP without orbit distortions to a higher equilibrium polarization of 88% instead of 74% with the existing wigglers at the \( Z_0 \) energy. The polarization time can be reduced from 90 minutes with the existing wigglers to 36 minutes. This makes empirical correction of depolarizing effects feasible in a reasonable time and largely improves the effective polarization degree \( \langle \langle P \rangle \rangle \) in a physics run. Since the polarization time is smaller the depolarizing effects are weaker and an empirical correction, applying several vertical orbit bumps and using SLIM to obtain \( \langle P \rangle \), increased the average equilibrium polarization from 50% to 80% in about 20 steps.

Further simulation work was done by Kouchch and Limberg [6], using SLIM [2] and SITF [7], as well as the spin-tracking program SITROS [8], which takes into account the finite amplitudes of betatron and synchrotron oscillations, and hence the effects of the energy spread in the beam, and which shows several synchrotron satellites of the integer spin resonances instead of the single ones shown by linear programs, e.g. SLIM and SITF.
With the dedicated wigglers [3] excited, they find that a degree of polarization above 50% is possible in LEP at 46 GeV, after precise cancellation of the integer spin resonances and their first synchrotron satellites.

An analytical estimate of the effects of the finite energy spread $\sigma_e$ was obtained by Buon [9] who calculated an enhancement factor $C$ by which depolarizing effects computed in first order theory have to be multiplied. The estimate includes two effects, the modulation of the spin tune due to synchrotron oscillations, and the spin diffusion due to the synchrotron radiation. Applying the enhancement factor $C$ thus obtained to the results of linear computations [3] gives much lower degrees of polarization than spin tracking [6]. The reason might be that the estimate assumes synchrotron radiation to be emitted uniformly around the circumference while spin tracking concentrates the synchrotron radiation in the dedicated wigglers which is true to a very good approximation.

Blondel and Keil [10] designed a Richter-Schwitters spin rotator [1] which satisfies all spin-matching conditions which Buon [11] and Blondel [12] asked for. As expected, the spin matching resulted in a very small depolarization at the horizontal betatron resonances with $\nu = \pm Q_X$, where $\nu$ is the spin tune and $\{ \}$ denotes the fractional part, while the depolarization at the vertical betatron resonances with $\nu = \pm Q_Y$ is also very small although they are not included in the spin matching.

3. FUTURE DEVELOPMENTS

3.1 Development up to LEP commissioning

The approach of Koutchouk and Limberg [6] can be pursued to include a complete and simultaneous compensation of the 104th and 105th harmonics of the vertical closed orbit and of the vertical dispersion. This should further improve the degree of polarization beyond the 50% achieved already.

Further computational and analytical work is needed to clarify the apparent contradiction between Buon's estimates [9] and Koutchouk's and Limberg's spin-tracking results [6] about the consequences of the energy spread, in particular concerning the increased energy spread when the dedicated polarization wigglers [3] are excited.

A polarimeter based on the backscattering of a circularly polarized laser beam is being built for the e- beam [13]. The transverse feedback kickers are being equipped for their use as depolarizers. This equipment can be commissioned in parallel to the LEP commissioning proper, starting in the summer of 1989.
The design and fabrication of dedicated polarization wigglers should be launched because they would at least speed up the transverse polarization programme once LEP is in operation.

The design of the spin rotators should be actively pursued. Their fabrication may have to be launched, depending on the sequence of events adopted below.

3.2 Developments after LEP commissioning

If the theoretical studies continue to predict a useful degree of polarization, the dedicated polarization wigglers, the polarimeter and transverse feedback kickers will be used to verify the degree of polarization and to calibrate the LEP energy with spin resonances to an accuracy of $10^{-4}$, a factor of 5 better than can be achieved by magnetic measurements alone. The experiments on transverse polarization will initially require dedicated shifts of LEP operation with the experimental solenoids switched off. It remains to be seen whether a scheme as proposed by Rossmanith [14] for the compensation of the transverse spin rotation by the solenoids or variants of it are practical. It is important to determine experimentally the relation between luminosity and degree of polarization $\langle P \rangle$. A drop in $\langle P \rangle$ was observed in PETRA when the vertical beam-beam tune shift reached a value $\Delta Q_y = 0.025$ [15].

Turning the spin into the beam direction requires the design, installation and commissioning of the spin rotators, and a verification of their proper functioning in terms of $\langle P \rangle$. A sizeable fraction of LEP has to be modified and new equipment installed. In order to avoid interfering with the regular operation of LEP, this work must be synchronized with LEP shutdown(s) for other reasons. Basically, two choices can be made with the fabrication of the spin rotators either before or after the experimental verification of a useful degree of transverse polarization in LEP. In this context, it should be remembered that HERA will come into operation before LEP, and that experiments on transverse and longitudinal spin are foreseen there [15]. The electron beam energy in HERA is much closer to the LEP beam energy than in other machines, and the same simulation programs are used for both machines. Therefore, experimental evidence from HERA should be very valuable for LEP.

i) If one waits for experimental data from LEP on transverse polarization before launching the fabrication of the spin rotators, the sequence of events is as follows: it is unlikely that a useful degree of transverse polarization is established within less than a year from initial LEP
operation, i.e. before the summer of 1990. The fabrication and installation of the spin rotator will take about 13 months. Hence, commissioning LEP with longitudinal spin will not start before 1992, when the LEP energy upgrade will permit operation up to about 67 GeV. This choice does not require a financial commitment into spin rotators before transverse polarization is established experimentally.

ii) If the fabrication of the spin rotators is decided on the basis of theoretical evidence on the achievable degree of polarization in LEP, and on experimental data from HERA, it can be launched much sooner. The installation of the spin rotators could take place when the theoretical evidence is confirmed by experiments in LEP, i.e. during a shutdown after the summer of 1990. In this case, the commissioning of LEP with longitudinal spin could take place in 1991, i.e. about one year sooner than in the previous scenario. The funds for the spin rotators, estimated below, would have to be committed before a useful degree of polarization is found experimentally.

4. **SPIN ROTATORS**

Spin rotators rotate the spin from the vertical direction into the beam direction, by horizontal-field bending magnets. A spin rotation by 90° is obtained in a dipole of integrated field 2.3 Tm, independent of the particle energy. At the workshop in November 1987, spin rotators were discussed by Plane [16] and by Keil [17]. Since then, work was only done on a Richter-Schwitters rotator [1] which is installed in a straight section of the LEP lattice and consists only of horizontal-field bending magnets.

4.1 Spin rotator layout

A schematic sketch is shown in Ref.[10]. The slope of the orbit at the interaction point, 14.9 mrad, is determined by the spin rotation. The maximum vertical offset is about ±0.6 m. Because of symmetry, this rotator re-establishes the original spin direction at all energies even if the spin is not directed parallel to the beam at the interaction point. The horizontal-field dipoles close to the interaction point are about 23 m long each. Their field is 2.46 times higher than in the normal LEP dipoles. The synchrotron radiation from these dipoles irradiates a vertical strip at the interaction point. At 46.5 GeV the critical energy is 177 keV, and at 3 mA in each beam 251 W of synchrotron radiation power remain inside a beam pipe of 65 mm radius. The collimation of the synchrotron radiation was studied by G. von Holtey and D. Treille [18].

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Fig. 1 - Cross section of the spin rotator dipoles, one half of the weak (strong) dipole is shown on the left (right).

Fig. 2 - Cross section of the vacuum chamber for the spin rotator dipoles.
For the following discussion of geometrical interferences between the spin rotators and the rest of LEP it is assumed that the spin rotators are installed for operation near the $Z^0$ energy, but are not removed later on when the LEP energy is increased, even though they are not suitable for rotating the spin into the longitudinal direction. Consequently, the magnets have been designed for operation up to 100 GeV. The vertical offset of the LEP orbit due to the spin rotators requires that components be installed above and below the present beam level which is 0.8 m above the floor. Components above the present beam level interfere with the large hadron collider LHC [19] in the LEP tunnel. Components below the present beam level may have to be installed in holes or slots in the tunnel floor. The implications of the vertical offset and of the vertical dispersion on the present Cu RF system near Pits 2 and 6, and on the future extensions of the RF system with superconducting cavities are being studied. Clearly it is desirable to have a spin rotator layout which works in all experimental pits.

4.2 Spin rotator magnets

The quadrupoles within the spin rotator have to be moved up or down following the design orbit. Their excitations are different from their standard settings. However, the maximum gradient of the existing quadrupoles is adequate for operation up to 78 GeV [10].

The spin rotator horizontal-field dipoles were studied by J.P. Gourber. Because of the aspect ratio of the LEP vacuum chamber they must have a larger gap height than the standard vertical-field LEP dipoles. Therefore the standard LEP yokes cannot be used, and new magnet designs are needed.

Two types of horizontal-field dipoles are needed, strong ones close to the interaction points, and weaker ones further away. One half of each of these magnets is shown in Fig. 1. The yokes are the same but the coils are different. The vacuum chamber which is shown in Fig. 2 has the pump channel on the top of the beam channel in order to limit the gap height of the magnets. The strong and weak magnets are powered in series; since there are only a few magnets in the circuit, it is advantageous to have many turns in the coils thus reducing the current and increasing the voltage. A trim coil located on the weak magnets permits the fine adjustment of the field.

4.3 Spin rotator installation and commissioning

The installation of spin rotators around an experimental interaction point in LEP was studied by G. Bachy. It involves the following tasks:
i) Remove about 0.45 km of LEP straight section.

ii) Modify the LEP tunnel and infrastructure over this distance, i.e. cut holes or slots into the floor, modify cabling, piping, etc.

iii) Install about 0.45 km of LEP straight section with spin rotators.

To complete these tasks in a LEP shutdown of about six months requires careful preparation and planning, and parallel installation at different interaction points. It may be possible to do some of the work in a shutdown before the modifications of the LEP machine proper are made. Once LEP is complete again it must be brought back into operation with the spin rotators, at first only dealing with the optics, and afterwards also dealing with the spin.

4.4 Spin rotator costs

The fabrication costs of the new dipoles for the spin rotators were estimated. Their costs per interaction region are summarized in Table 1. Not included are the costs for cabling, power convertors, modifications of the vacuum and cooling systems, installation, survey and civil engineering.

Table 1 - Fabrication costs for rotator magnets

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost (kSFr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 dipole cores and tooling</td>
<td>900</td>
</tr>
<tr>
<td>8 coils for weak dipoles and tooling</td>
<td>90</td>
</tr>
<tr>
<td>16 coils for strong dipoles and tooling</td>
<td>340</td>
</tr>
<tr>
<td></td>
<td><strong>1,330</strong></td>
</tr>
</tbody>
</table>

Detailed cost estimates for spin rotator components other than the dipoles, for their installation and for the civil engineering have not been made. Multiplying the estimate for the dipoles by a factor 1.5 gives 2 MSF in round figures for each interaction region.

5. CONCLUSIONS

Computer simulations have shown that it is possible to achieve a high average equilibrium polarization in LEP when twelve dedicated polarization wigglers are installed which reduce the polarization time to 36 minutes, and permit an empirical optimization within a fews hours. A spin rotator which fits into the straight sections surrounding the experimental interaction points and satisfies the important spin-matching conditions has been
designed and shown to have very small depolarizing effects. Further computer simulations and analytical work are needed to refine the estimates of the average polarization. The engineering design of the dedicated wigglers and of the spin rotators should be pursued. Once LEP is in operation, the dedicated wigglers should be installed and experiments on transverse polarization should be rapidly launched and vigorously carried out. The installation of the spin rotators in order to obtain longitudinally polarized beam in LEP at the $Z^0$ energy must be carefully synchronized with normal LEP shutdown(s). If the spin rotator components are ready for the first LEP shutdown after the positive outcome of transverse polarization experiments, LEP could be commissioned with longitudinal polarization starting in 1991 or 1992. The dedicated wigglers would cost about 2 MSF, the spin rotators about 2 MSF for each interaction region.

ACKNOWLEDGMENTS

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Polarization at SLC

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Abstract

The SLAC linear Collider has been designed to readily accomodate polarized electron beams. Considerable effort has been made to implement a polarized source, a spin rotation system, and a system to monitor the beam polarization. Nearly all major components have been fabricated. At the current time, several source and polarimeter components have been installed. The installation and commissioning of the entire system will take place during available machine shutdown periods as the commissioning of SLC progresses. It is expected that a beam polarization of 45% will be achieved with no loss in luminosity.

1. Introduction

The utility of polarized electron (and positron) beams for the production and study of the $Z^0$ boson has been discussed in in many publications [1] and will be discussed in detail at this workshop. It is sufficient to say that the use of polarized beams provides an effective increase in luminosity of one to two orders of magnitude for some standard model measurements and improves the systematic uncertainties associated with those measurements to a level unattainable without polarization. (As an example, the precision of the electroweak mixing parameter $\sin^2\theta_w$ as obtained from a measurement of the left-right polarization asymmetry is compared with that obtained from the forward-backward muon asymmetry in Figure 1.)

In order to exploit these advantages, the SLAC Linear Collider (SLC) has been designed to readily accomodate polarized electron beams. At the current time, nearly all of the additional components that are necessary to produce, accelerate, and monitor the polarized electron beam have been fabricated. It is hoped that the installation and testing of various parts will take place during the coming year. The implementation of the polarized electron beam is not expected to affect the luminosity of the machine in any way. The cost of the entire project is approximately $2 million.
2. The Polarized SLC

A layout of the polarized SLC is shown in Figure 2. The orientation of an electron spin vector is shown as the electron is transported from the electron gun to the interaction point.

A gallium arsenide based photon emission source produces pulses of up to $10^{11}$ longitudinally polarized electrons at repetition rates of up to 180 Hz. The electrons are then accelerated in the first sector of the linac. The beam pulse achieves an energy of 1.21 GeV as it arrives at the entrance of the LTR (Linac To Ring) transfer line.

The electrons must be stored in the North Damping Ring for one machine cycle (the cycle time is $\geq$ 5.5 ms). A system consisting of the LTR bend magnets and a superconducting solenoid is used to rotate the spins into the vertical direction that is necessary for storage in the damping ring. After one machine cycle, the bunch is extracted and passed through another spin rotation system consisting of the bend magnets of the RTL (Ring-To-Linac) transfer line and two superconducting solenoids. The system is sufficiently flexible to provide essentially any spin orientation as the bunch reenters the linac at the beginning of sector 2.

The beam pulse is then accelerated to nearly 50 GeV in the linac. To insure that the spin gymnastics in the damping ring have worked properly and to study many of the potential sources of depolarization, a Møller polarimeter is located at the end of the linac near the PEP injection line. This polarimeter is used primarily for diagnostic purposes.

The beam pulse is then transported through the north machine arc and the final focus section to the interaction point. At full energy, the spin vectors precess roughly 26 times. Vertical precession also occurs in the nonplanar arcs. Since longitudinal polarization is required at the interaction point, the total precession angle must be calculated for the exact machine energy and the polarization at the arc entrance must be adjusted appropriately.

After colliding with the unpolarized positron bunch, the electron beam is transported through the south final focus system where a Compton polarimeter is located. The beam continues to the south extraction line where a second Møller polarimeter is located. The bending magnets of the final focus and extraction line cause an additional spin precession of roughly 540° between the interaction point and the Møller target. Both polarimeters continuously monitor the beam polarization.
3. The Polarized Source

In 1976, Pierce and Meier [2] observed the photoemission of polarized electrons from negative electron affinity gallium arsenide (NEA GaAs). Since then, nearly all polarized electron sources that have been used with accelerators have been based on this technique [3]. These sources have the advantages of relative simplicity, easy reversibility, and good beam characteristics, but are limited to a maximum polarization of fifty percent. The SLC source is an improved version of a GaAs photoemission source that was developed for a previous SLAC experiment [4]. The first part of this chapter describes the physical principles of the source operation and the second part describes the current status of the SLC source.

3.1. Gallium Arsenide Sources

Gallium arsenide is a well-known semiconductor with two very important properties that make it useful as a polarized electron source:

1. Its band structure permits a given spin state to be preferentially pumped into the conduction band.

2. Its surface can be treated to develop a negative work function (hence the term, \textit{negative electron affinity}).

The band structure of GaAs at the energy maximum of the valence band and energy minimum of the conduction band is shown in Figure 3. The band energy versus momentum is shown on the left-hand side and the energy level structure is shown on the right-hand side of the figure. The band gap of the material is $E_g = 1.52 \text{ eV}$. At the minimum of the conduction band and the maximum of the valence band, the electron wavefunctions have $S$ and $P$ symmetry, respectively. Spin-orbit splitting causes the $P_{3/2}$ states to reside in energy above the $P_{1/2}$ states by an amount $\Delta = 0.34 \text{ eV}$. The absorption of single photons proceeds via an electric dipole transition. The selection rules for the absorption of right- and left-handed circularly polarized photons are $\Delta m_j = +1$ and $\Delta m_j = -1$, respectively. They are indicated by the solid and dashed arrows in Figure 3. Since the electric dipole operator changes the orbital angular momentum of the initial state by one unit, the spin of the electron remains unchanged in the process.

Let's consider what happens when a right-circularly polarized photon is incident upon a GaAs crystal. The photon direction is the only vector in the system. All angular momentum projections refer to the incident photon direction. If the photon energy $E_\gamma$ is in the range $E_g \leq E_\gamma \leq E_g + \Delta$, then transitions can only occur from the
$P_{3/2}$ states to the $S_{1/2}$ states. Specifically, the $P$ state with $m_j = -3/2$ can make a transition to the $S$ state with $m_j = -1/2$ and the $P$ state with $m_j = -1/2$ can make a transition to the $S$ state with $m_j = +1/2$. In the former case, the emitted electron has spin antiparallel to the incident photon direction (or parallel to its ejected direction). In the latter case, the spin of the emitted electron is parallel to the incident photon direction (antiparallel to its ejected direction). Due to Clebsch-Gordon coefficients (the $P$ state with $m_j = -3/2$ is a pure spin state whereas state with $m_j = -1/2$ is not), the former transition is three times more likely than the latter. The relative transition rates are indicated by circled numbers in Figure 3. This implies that the absorption of a right circularly polarized photon produces a right-handed electron with a polarization

$$P = \frac{3 - 1}{3 + 1} = 50\%.$$  

Actually, all that's been shown so far is that polarized electrons can be pumped into the conduction band with a beam of circularly polarized photons. In order to make a polarized source, the electrons must leave the material. In normal GaAs, the energy gap from the bottom of the conduction band to the free electron state is approximately 2.5 electron volts. Even with a large applied electric field, pure GaAs is a poor photoemitter. The magic that is necessary to make it an efficient photoemitter is shown in Figure 4. The energy of the various bands is shown as a function of depth near the surface for several materials: pure GaAs, GaAs with a cesiated surface, and GaAs with a surface layer of Cs$_2$O. The energy of the free electron state is shown as $E_\infty$. The addition of cesium to the surface causes the energy gap between the conduction band and the free electron state to decrease to zero. The addition of Cs$_2$O to the surface causes the gap to become negative! Quantum efficiencies as large as 5\% have been been observed for GaAs photocathodes that have been treated with Cs$_2$O (actually CsF is currently used instead). At photon energies that are appropriate for polarized electron production, quantum efficiencies in the range 0.1\% → 0.5\% are typical.

In practice, the photoexcited electrons can become depolarized by spin flip scattering processes that occur before emission from the photocathode. Attempts are currently being made to minimize this effect by making very thin photocathodes. The measured electron polarization from several thin photocathodes is presented in Figure 5 for several photon wavelengths. Note that the polarization increases to a value in the range 45\% → 50\% as the photon energy is decreased to a value near the bandgap of the material. Although the systematic errors of these measurements are typically 10\%, electron polarizations near the theoretical maximum are attained.
3.2. THE SLC POLARIZED SOURCE

A drawing of the SLC polarized source is shown in Fig. 3.2. The source is installed on a Y section to facilitate switching from one source to the other. At the current time, the source is installed on the accelerator. Several tests were conducted during the winter 1987-1988 shutdown. A high vacuum bakeout of the Y section was performed. The photocathode was then activated with a CsF treatment. A quantum efficiency of approximately 1% was measured. The vacuum valve that isolates the photoemission gun from the accelerator vacuum was then opened. The lifetime* of the photocathode when exposed to the accelerator environment was measured. In the laboratory tests, the cathode lifetime has been measured to be approximately 1000 hours. Unfortunately, the lifetime of the cathode when exposed to the accelerator environment was only 100 hours. The solution to this problem is to improve the accelerator vacuum near the source.

The second major component of the polarized electron source (also shown in Figure 6) is the laser light source. It is a model TFDL-10 flashlamp pumped dye laser built by the Candela Corporation and extensively modified at SLAC. The laser has been operated with two dyes: oxazine 720 and rhodamine 700. The laser power is shown in Figure 7 as a function of wavelength (which depends upon the dye concentration). The power is the average power at 60 Hz operation. The actual pulse width is 500 ns (full width). Also shown in Figure 7 is the expected electron polarization for each photon wavelength. It is clear that the rhodamine dye produces more power in the long wavelength region that is desirable for high polarization.

Unfortunately, this is only part of the story. The dyes are gradually destroyed by exposure to the high intensity illumination of the flash lamps. The lifetime of each dye determines the frequency of laser maintenance that is necessary in actual operations. At 60 Hz operation, the lifetimes of the oxazine and rhodamine dyes have been measured to be 216 hours and 41 hours, respectively. The operational maintenance interval is determined by a number of parameters. If one assumes that the cathode lifetime is 100 hours and that the initial quantum efficiency is 1%, then the production of $10^{11}$ electrons at 120 Hz would require source maintenance each 8.3 days with the oxazine dye and each 2.9 days with the rhodamine dye. The rhodamine dye is capable of producing higher polarization but requires more frequent maintenance. In either case, the situation is

* The photocathode lifetime refers to the decrease in quantum efficiency with time. When the quantum efficiency becomes too low, it can be restored by a CsF activation. The physical photocathode surface is capable of many such reactivation cycles.
tolerable but not entirely satisfactory. There are plans to investigate more dyes in the near future.

The time structure of the laser pulse differs substantially from the required time structure of the electron pulses. The each machine cycle, the SLC source must produce two two-nanosecond electron pulses that are separated in time by 60 ns. The solution to the problem is to modulate the the 500 ns laser pulse with the apparatus that is shown in Figure 8. The photon beam emerges from the laser and passes through a double prism to insure 100% linear polarization. It is then passed through a Pockels cell, a second double prism, and a second Pockels cell. The fast and slow axes of the Pockels cells are rotated by 45° with respect to the polarization direction of the beam. The polarizing axis of the second prism is rotated by 90° with respect to the first prism. In the absence of a voltage applied to the first Pockels cell, no light is transmitted through the system. The laser pulse is modulated by applying a high voltage signal to the first Pockels cell that has the correct time structure (two 2-ns pulses that are separated by 60 ns). The amplitude of the voltage is adjusted to produce a phase shift of 180° between the axes of the cell. This causes the polarization vector of the beam to rotate by 90°. The beam is thus fully transmitted through the second prism for the duration of the high voltage pulse and the correct time structure is established. The voltage applied to the second Pockels cell is held constant for the duration of the machine cycle and adjusted to produce a 90° phase shift between the fast and slow axes. The cell therefore transmits either right- or left-circularly polarized photons (depending upon the sign of the voltage).

4. The Spin Rotation System

The second major element of the polarized SLC is the spin rotation system. As was mentioned in chapter 1, the spin rotation system has two functions:

1. To rotate the (initially longitudinal) polarization vector of the electron bunch into the vertical direction for storage in the North Damping Ring.

2. To allow the orientation of the electron polarization vector to be controlled as the bunch reenters sector 2 of the linac. This is necessary to compensate for precession in the machine arcs.

A detailed representation of the north damping ring, the north LTR transfer line, and the north RTL transfer line is shown in Figure 9. The orientation of the polarization vector at various places is shown by the double arrow. The electron bunch arrives at

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the entrance to the LTR transfer line with an energy of 1.21 GeV. At this energy, the spins precess by 90° for each 32.8° that the electron trajectories are bent by a transverse magnetic field. The initial bend angle of the LTR has been chosen to be 5 × 32.8°. The longitudinal polarization of the beam emerging from sector 1 of the linac is therefore rotated into the horizontal direction. A superconducting solenoid of strength 6.34 T·m is introduced into the LTR optics after the first bend. The solenoid has only a small effect on the optics of the transport system but causes a rotation of the spin vector about the beam axis by 90°. The spins are therefore rotated into the vertical (downward) direction. After one machine cycle (≥ 5.5 ms), the electron bunch is extracted from the damping ring with a horizontal kicker magnet and passed through a second superconducting solenoid magnet. The horizontal bend magnets of the RTL transfer line then deflect the beam by an angle of 3 × 32.8° before it reenters the linac at the beginning of sector 2. A third superconducting solenoid is introduced into the linac lattice just downstream of the reentry point. If the second (RTL) solenoid is adjusted to have the same strength as the first (LTR) solenoid has, the system will restore the longitudinal beam polarization. If it is not energized, the beam polarization will be vertical upon reentry into the linac. The third (linac) solenoid can then rotate the polarization vector to any transverse orientation. The combination of the two solenoids and the RTL bending magnets permits the selection of any orientation of the polarization vector.

Status of the Spin Rotation System

At the current time, all three spin rotation solenoids have been fabricated and delivered to SLAC. The magnets and their cryogenic and control systems have been assembled in a large test facility at SLAC. The entire system is undergoing extensive testing. The installation of the spin rotator solenoids into the LTR and RTL transport lines requires relatively minor changes to the optics and instrumentation of the beam lines. The installation of the solenoid into sector 2 of the linac is somewhat more involved. A single accelerator section must be removed (which results in a loss of 50 MeV of energy to the electron beam). Three quadrupole magnets must be moved and two others must be installed.

5. Polarimetry at SLC

The polarization of the SLC electron beam will be monitored by three polarimeters. Two of the polarimeters are based upon polarized Møller scattering. They provide three-axis polarimetry and have moderate precision (δP/P ≤ 5%). The third polarimeter is
based upon polarized Compton scattering and may provide high precision \( \delta P/P \approx 1\% \) measurements of the longitudinal beam polarization. The Møller polarimeters are located at the end of the linac and in the south extraction line. The Compton polarimeter is located at the last bend magnet of the south final focus.

5.1. THE MØLLER POLARIMETERS

The Møller polarimeters make use of the polarized asymmetries of the cross section for electron-electron elastic scattering. At tree level, the differential cross section for this process in the center-of-mass frame (in the \( m_e \to 0 \) limit) is given by the following expression

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 (3 + \cos^2 \theta)^2}{s \sin^4 \theta} \left\{ 1 - P_z^1 P_z^2 A_z(\theta) - P_t^1 P_t^2 A_t(\theta) \cos(2\phi - \phi_1 - \phi_2) \right\}
\]

where: \( s \) is the square of the total energy in the cm frame; \( \theta \) is the cm frame scattering angle; \( \phi \) is the azimuth of the scattered electron (the definition of \( \phi = 0 \) is arbitrary); \( P_z^1, P_z^2 \) are the longitudinal polarizations of the beam and target, respectively; \( P_t^1, P_t^2 \) are the transverse polarizations of the beam and target, respectively; \( \phi_1, \phi_2 \) are the azimuths of the transverse polarization vectors; and \( A_z(\theta) \) and \( A_t(\theta) \) are the longitudinal and transverse asymmetry functions which are defined as

\[
A_z(\theta) = \frac{(7 + \cos^2 \theta) \sin^2 \theta}{(3 + \cos^2 \theta)^2}
\]

\[
A_t(\theta) = \frac{\sin^4 \theta}{(3 + \cos^2 \theta)^2}.
\]

The differential cross section is the product of the unpolarized cross section and the sum of one and two polarization dependent terms. The first is the product of the longitudinal polarizations of the beam and target particles and the function \( A_z(\theta) \). The second is the product of the transverse polarizations of the two electrons, an azimuthal factor, and the function \( A_t(\theta) \). The unpolarized cross section, \( A_z(\theta) \), and \( A_t(\theta) \) are plotted as functions of \( \cos \theta \) in Figure 10. Both asymmetry functions are maximal for 90° scattering. The longitudinal asymmetry function becomes quite large \( A_z(90°) = 7/9 \) whereas the transverse asymmetry function never exceeds 1/9. The analyzing power of any polarimeter scales as the product of the unpolarized cross section and the square of the asymmetry. This combination is also largest at \( \theta = 90° \) but has a rather broad maximum.

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The laboratory momentum and angle of the scattered electron, $P'$ and $\theta_{lab}$ are related to the center-of-mass frame scattering angle by the following expressions,

$$P' = \frac{P}{2} [1 + \cos \theta]$$
$$\theta_{lab} = 2m_e \left[ \frac{1}{P'} - \frac{1}{P} \right]$$

where $P$ is the momentum of the incident electron and where $m_e$ is mass of the electron. The extraction line polarimeter has been designed to accept an interval of momentum $\Delta P'/P' = \pm 5\%$ about $P' = P/2$. Its acceptance is therefore centered about $\theta = 90^\circ$. The linac polarimeter makes use of the first bend magnet of the PEP injection line as an analyzing element. The PEP injection line cannot transport electron momenta larger than 15 GeV/c. At a beam energy of 46 GeV, this corresponds to a minimum center-of-mass angle of $110^\circ$. The corresponding laboratory angles, unpolarized cross sections, and polarization asymmetries are summarized in Table I below:

**Table I**

The accepted cm and laboratory angles, cross section, longitudinal asymmetry and transverse asymmetry for the linac and extraction line Møller polarimeters. The beam energy is assumed to be 46 GeV. The cross sections are given in units of $\alpha^2/s$.

<table>
<thead>
<tr>
<th>Polarimeter</th>
<th>$\theta$</th>
<th>$\theta_{lab}$ (mRad)</th>
<th>$d\sigma/d\Omega$</th>
<th>$A_x$</th>
<th>$A_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linac</td>
<td>$110^\circ$</td>
<td>6.8</td>
<td>12.5</td>
<td>0.64</td>
<td>0.08</td>
</tr>
<tr>
<td>Extraction Line</td>
<td>$90^\circ$</td>
<td>4.7</td>
<td>9.0</td>
<td>0.78</td>
<td>0.11</td>
</tr>
</tbody>
</table>

The larger scattering angle regime accepted by the linac polarimeter reduces the analyzing power only slightly (by 6%) as compared with the extraction line device.

**Polarimeter Design**

It is clear that the electron beam polarization can be monitored by providing a target containing polarized electrons and by measuring the asymmetry of the cross section caused by reversing either the beam or target polarizations. The beam polarization $P_b$ is then related to the measured asymmetry $A_{exp}$ by the following simple expression

$$P_b = \frac{A_{exp}}{P_{tgt}A_{theor}}$$
where $R_{tg}$ is the target polarization and $A_{theor}$ is the theoretical longitudinal or transverse asymmetry given above. A diagram of the extraction line polarimeter is shown in Figure 12. Note that the horizontal bending magnets have no function for the polarimeter (they make synchrotron radiation for the SLC energy spectrometer and background for the polarimeter). The linac polarimeter differs only in the choice analyzing magnet and detector. The main elements of both polarimeters are:

1. A magnetized foil target.

2. A collimator to define a scattering plane (defines the azimuth of the scattered particles).

3. A magnet and aperture that select the momentum range to be accepted (which is equivalent to selecting the cm scattering angle). Note that the bending plane of the magnet is perpendicular to the scattering plane defined by the collimator. This is done to decouple $P'$ from $\theta_{lab}$.

4. A detector that is capable of measuring the electron rate as a function of position. A shower counter sampled with very small (2 mm diameter) proportional tubes is used in the extraction line. The linac polarimeter uses a silicon strip detector.

The counting rates in both polarimeters are in the range $50 \rightarrow 100$ electrons per pulse. It is therefore expected that longitudinal polarization measurements will require $1 \rightarrow 2$ minutes to achieve a statistical precision of 5%. Transverse polarization measurements will require roughly one hour to achieve the same precision.

**Møller Target**

A single polarized target design has been used for both polarimeter targets. A *beam's eye view* of the target assembly is shown in Figure 11. A holder containing four iron foils can be moved through the beam. Two foils are transverse to the beam axis and two are tilted at an angle of 20° with respect to the beam axis. A three sets of interleaved Hemholz can produce a 100 gauss magnetic field along any of the three axes. This field is sufficient to magnetically saturate a foil along its axis. At saturation, the spins of two of the 26 valence electrons are (anti)aligned with the external field (the maximum polarization is therefore 8%). The actual target polarization is measured by the following procedure:

1. A small pickup coil is placed around the target foil.

2. The direction of the driving field is reversed and the induced coil voltage is measured.
3. The target is removed from within the pickup coil and step 2 is repeated.

Step 2 measures the total $B$ field induced within the foil. Step 3 measures just the driving $H$ field. The difference between the measurements determines the magnetization density of the foil. The magnetization density is closely related to the target polarization (it must be corrected for orbital effects) [6].

Backgrounds

The Møller scattering signal that was measured in the polarimeter of the SLAC polarized electron deuteron scattering experiment [4] is shown in Figure 13. The number of detected electrons is shown as a a function of scattering angle (the upper plot). The signal consists of a smoothly varying distribution and a peak. The Møller scattering peak occurs at the angle which corresponds to the accepted momentum (according to the electron-electron two body kinematics). The continuous distribution provides a 15 → 20% background to the Møller signal. Both the shape and the magnitude of the background are well described by the process $e^- + N \rightarrow e^- + N + \gamma$ (also known as the Bethe-Heitler process) [7]. A similar background is expected to be present in the extraction line polarimeter. The larger angle regime accepted by the linac polarimeter is expected to be less contaminated by background (by a factor of 2-3).

Systematic Errors

The ability to operate the polarimeters in four polarization modes (two beam polarization directions × two target polarization directions) will help to study many systematic problems. Nevertheless, it is expected that the precision that can be achieved will be limited by the following systematic effects:

1. The target polarization can be measured with a precision of two or three percent. Better precision probably requires a good understanding of the shape of the magnetization density distribution within the target.

2. The uncertainty in the subtraction of the intrinsic background will probably be less two percent.

3. A complete set of radiative corrections for polarized Møller scattering has not been calculated. The effect on the size of the cross section is quite large (of order 20%). The effect on the asymmetry is estimated to be quite small (less than 1%).

4. The measurement of an asymmetry is sensitive to the linearity of the detector. The non-linearities can probably be corrected and controlled to less than one percent.
5. The passage of the beam through the target can disrupt the spin alignment of the electrons. At SLC, the effect is estimated to be less than a one percent uncertainty on the target polarization.

The combined effect of the above uncertainties is less than 5%. It appears that with a bit of care and good luck, the systematic error could be controlled to $2 \rightarrow 3\%$.

Status of the Møller Polarimeters

At the current time, all components of both polarimeters have been fabricated. The linac polarimeter target assembly has been calibrated and installed. A number of background studies have been conducted with a prototype silicon detector. The extraction line polarimeter target assembly and collimator will be installed during the next extended shutdown of the machine. The lead-proportional tube detector will be installed in the near future and background studies will be conducted.

5.2. THE COMPTON POLARIMETER

The Compton polarimeter makes use of the large polarization asymmetry in the cross section for elastic electron-photon scattering. The light source initially will be a frequency doubled Nd:YAG laser which produces 2.23 eV photons (a later upgrade to an excimer laser is being considered). The kinematics of the scattering of optical photons with high energy electrons in the laboratory frame are discussed first. After the variables have been defined, the polarized cross section is presented.

Compton Scattering Kinematics

The kinematical properties of the scattering of a high energy electron with an optical photon seem quite strange to those accustomed to working in reference frames that are nearer the center-of-mass frame. The energy of the electron is typically 10 orders of magnitude larger than that of the photon. It is clear that all final state particles are swept into the forward direction (along the incident electron direction). It is therefore convenient to define all angles with respect to the incident electron direction. The direction of the outgoing photon, $\theta_K$, differs from the normal definition of the scattering angle by $180^\circ$ (if the colliding $e-\gamma$ are collinear). Let $E$, $E'$, $K$, and $K'$ be the incident electron energy, scattered electron energy, incident photon energy, and scattered photon energy, respectively. The maximum energy of the scattered photon $K'_{\max}$ and the minimum energy of the scattered electron $E'_{\min}$ can then be written as
\[ K'_{\text{max}} = E(1 - y) \]
\[ E'_{\text{min}} = Ey \]

where the parameter \( y \) is defined as

\[ y \equiv \left( 1 + \frac{4EK}{m^2} \right)^{-1}. \]

The emission angle of the scattered photon \( \theta_K \) is related to the scattered photon energy by the following expression,

\[ K' = K'_{\text{max}} \left[ 1 + y \left( \frac{E\theta_K}{m} \right)^2 \right]^{-1} = K'_{\text{max}} \cdot x \]

where the definition of \( x \) is obvious. The parameter \( x \) varies from unity at zero emission angle to zero at larger angles. The scale of the angular range is set by the angle for which the energy has been reduced by a factor of two. This occurs when \( E\theta_K \sqrt{y}/m = 1 \) or at the angle \( \theta_K = m/E\sqrt{y} \). For the SLC Compton polarimeter operating with a 46 GeV beam, the value of the parameter \( y \) is 0.389. Therefore, the maximum photon energy is 28.1 GeV and the minimum electron energy is 17.9 GeV. The angle at which the photon energy has been decreased by a factor of two is \( 1.8 \times 10^{-5} \) radians. The scattered electron and photon both remain along the beam direction.

The Polarized Cross Section in the Laboratory Frame

The polarized cross section can be expressed in terms of the laboratory variables \( x, y, \) and the azimuth of the photon with respect to the electron transverse polarization \( \phi \) as follows [8],

\[
\left( \frac{d^2 \sigma}{dx d\phi} \right)_{\text{Compton}} = \left( \frac{d^2 \sigma}{dx d\phi} \right)_{\text{unpol}} \left\{ 1 - P^\gamma \left[ P_z \cos^2 A_2^\gamma(x) + P_t \sin \phi A_t^\gamma(x) \right] \right\}
\]

where: the unpolarized cross section is defined as

\[
\left( \frac{d^2 \sigma}{dx d\phi} \right)_{\text{unpol}} = r^2 y \left\{ \frac{x^2(1-y)^2}{1-x(1-y)} + 1 + \left[ \frac{1-x(1+y)}{1-x(1-y)} \right]^2 \right\};
\]

\( P_z, P_t \) are the longitudinal and transverse polarizations of the electron; \( P^\gamma \) is the circular polarization of the photon; and where the longitudinal and transverse asymmetries are defined as
\[ A_z^{x\gamma}(x) = r^2 y \left[ 1 - x(1 + y) \right] \left\{ 1 - \frac{1}{1 - x(1 - y)^2} \right\} \left( \frac{d^2\sigma}{dx d\phi} \right)_{\text{unpol}}^{-1} \]

\[ A_t^{x\gamma}(x) = r^2 y x(1 - y) \left[ 4xy(1 - x) \right]^{1/2} \frac{1}{1 - x(1 - y)} \left( \frac{d^2\sigma}{dx d\phi} \right)_{\text{unpol}}^{-1} \]

These equations are difficult to visualize and interpret without a bit of assistance. The unpolarized cross section and the longitudinal and transverse asymmetries are plotted as functions of \( x \) in Figure 14 for \( y = 0.389 \). The unpolarized cross section is very large (several hundred millibarns) and peaked at \( x = 1 \). The longitudinal asymmetry has a maximum of 75% also at \( x = 1 \). Note, however, that as \( x \) is decreased, \( A_z^{x\gamma} \) decreases rapidly and becomes negative near \( x = 0.72 \). It reaches a minimum of -25% near \( x = 0.47 \) and returns to zero at \( x = 0 \). The transverse asymmetry is zero at both endpoints and reaches a maximum of 33% near \( x = 0.75 \).

Polarimeter Design

The location SLC Compton polarimeter is shown in Figure 15. The laser is brought into collision with the electron beam after it has passed through and immediately before it enters the first bend magnets of the final focus region. Since no precession occurs between the interaction point and the Compton collision point, it is sufficient to measure only the longitudinal component of the electron polarization. As is shown in Figure 14, the largest cross section and largest longitudinal asymmetry occur near \( x = 1 \). Since both quantities are very sensitive to the precise value of \( x \) (or alternatively, energy), there is a clear advantage in measuring the scattered electrons. The electrons can be momentum analyzed rather precisely in large numbers. The main elements of the polarimeter are as follows:

1. The pulsed laser. As was stated previously, the polarimeter will initially use a frequency doubled Nd:YAG laser operating at 4 millijoules per pulse. The pulse length is 8 nanoseconds and the repetition rate is 15 Hz.

2. The laser optics and light path. The beam is circularly polarized with a Pockels cell and transported to a collision point that is approximately 12 m downstream of the SLC interaction point. The beam size at the collision point is 1.2 mm and the crossing angle is 20 milliradians.

3. The analyzing magnet is the first bending magnet of the final focus region. The endpoint (17.9 GeV) electrons are deflected by approximately 15 milliradians with respect to the unscattered beam.

4. The detector is a multicell gas Cerenkov counter that has a threshold energy of 20 MeV. It is located approximately 3 m downstream of the magnet center.
The counting rate is expected to be approximately 190 events per machine pulse. Because the laser target has essentially 100% polarization (as compared with the Møller target polarization of 8%), many fewer counts are required to achieve a given statistical precision than are required for the Møller polarimeters. It is expected that a 1% statistical precision can be achieved in a 70 second run.

Backgrounds

While Compton polarimeters have do not have the intrinsic backgrounds of the Møller technique, they can have serious background problems. The electron-positron collisions produce low energy electrons that are collinear with the beam axis via the process $e^+e^- \rightarrow e^+e^-\gamma$. This background decreases as the energy of the accepted electrons is decreased. For electron energies below 25 GeV, it is expected to be less than 15% of the signal. A second and potentially more serious background is the off-energy beam halo that is produced by the scattering of the beam tails from various apertures. Linear collider experiments are affected much more by this type of problem than storage ring experiments are. It is difficult even to estimate the size of this background since it depends upon machine parameters that are not measured well and upon details of the precise machine tune.

Both types of background are directly measureable by turning off the laser for machine pulses. They can be therefore be tolerated to fairly high levels.

Systematic Errors

The list of possible systematic errors that apply to the Compton polarimeter is very similar to that given the Møller section:

1. As with Møller scattering, it is essential to understand the degree of polarization of the target. Although the laser beam must be passed through windows and reflected from mirrors, it appears that the uncertainty on the beam polarization can be controlled to $\Delta P/\gamma \leq 1\%$.

2. Unless the machine related backgrounds are very severe, it appears that they and the radiative Bhabha background can be measured and subtracted from the total signal. The effect on the measured asymmetry can be kept to a few tenths of one percent.

3. A complete set of first order radiative corrections to polarized Compton scattering has been calculated by Gongora and Stuart [9]. The theoretical uncertainty on the
longitudinal asymmetry is certainly small as compared with the other systematic errors discussed in this section.

4. As with the Møller case, it is essential to monitor non-linearities of the detection system. Most detection systems are not linear to better than a percent or several percent. It appears that these can be corrected to the level of a few tenths of one percent.

5. Since the longitudinal asymmetry is a strong function of the scattered particle momentum, it is very important to understand the energy scale of the detector. At $y = 0.389$, an energy shift of 100 MeV causes a fractional change in the asymmetry of 1.1%. A good energy calibration can be derived from endpoint of the scattered electron energy distribution. Since the scattering cross section is largest at the endpoint, it should be relatively straightforward to observe. This does require good spatial resolution of the detector (less than a few hundred microns).

With some care, it appears that the total systematic uncertainty can be controlled to the one percent level.

**Status of the Compton Polarimeter**

At the current time, the laser light path has been installed. The laser has been operated successfully for many years as part of the SLAC backscattered photon beam. A prototype gas Čerenkov cell has been installed downstream of the analyzing magnet. Background studies are being conducted in an attempt to measure the machine luminosity via the radiative Bhabha process.

6. **Depolarization Effects**

There are numerous possible sources of electron beam depolarization. None of them are expected to be serious. The following is a summary of the most important.

6.1. **DEPOLARIZATION IN THE LINAC**

The depolarization of a longitudinally polarized electron beam by the SLAC linac has been calculated to be very small [10]. This has been verified by several experiments [11, 4]. The polarized SLC does differ in two respects from the old SLAC linac:

1. The electron bunches are much smaller than they were for the fixed target experiments. It was pointed out by W.K.H. Panofsky [12] that the intra-bunch fields
could depolarize the bunch via an effect that is analogous to Thomas precession. More detailed calculations indicate that this effect causes less than a one percent depolarization of the beam.

2. The SLC must accelerate beams with transverse components of the polarization vector. This is not expected to be a problem, however, detailed calculations and experimental verification are still needed.

6.2. Depolarization in the Damping Ring

Since the electrons are stored in the north damping ring for only one machine cycle \(\approx 8ms\) or several damping times), only resonant depolarization is a serious possibility. The resonance condition is

\[
\nu = N + I\nu_z + J\nu_y + K\nu_s
\]

where: \(\nu\) is the spin tune of the damping ring (the number of spin precessions per orbit); \(N, I, J, K\) are integers; \(\nu_z\) and \(\nu_y\) are the horizontal and vertical betatron tunes, respectively; and \(\nu_s\) is the synchrotron tune of the damping ring. The SLC damping ring is designed to operate at an energy \(E = 1.21\) GeV. The spin tune at this energy is given by the expression

\[
\nu = \frac{g - 2}{2} \cdot \frac{E}{m_e} = \frac{E}{440.65 MeV} = 2.746
\]

(where \((g - 2)/2\) is anomalous magnetic moment of the electron). The horizontal and vertical betatron tunes are \(\nu_z = 7.20\) and \(\nu_y = 3.20\), respectively (the \(\nu_z - \nu_y = 4\) coupling resonance is used to produce round beams). The synchrotron tune is very small \((\nu_s \approx 0.04)\). Therefore, the nearest spin depolarizing resonance occurs when \(N, I, J = 6, 0, -1\) (the synchrotron tune is ignored since only relatively weak resonances are associated with it). The right hand side of the resonance equation is equal to 2.80 in this case. Since the natural width of this sideband resonance is expected to be less than 0.001, no serious resonant depolarization is expected.

6.3. Depolarization in the Arches

The SLC arcs are fairly achromatic transport systems (they can transport a momentum interval \(\Delta P/P = 5\%\)). Since the total precession angle is a sensitive function of the beam energy, the finite energy spread of the beam \((\Delta P/P = 0.2\%)\) causes a spread
in the final spin directions of the electrons. The average longitudinal polarization at the interaction point is reduced by 2.2%.

6.4. Depolarization from Beam-Beam Interactions

Because the SLC beams are very small at the interaction point, each beam is subjected to very strong electromagnetic fields during the collision. These fields cause some depolarization of the electron bunch. The size of the effect is given by the expression

$$\Delta \theta_s = \frac{g-2}{2} \cdot \frac{E}{m_e} \cdot \theta_d$$

where: $\Delta \theta_s$ is the average precession angle of beam particles; $E$ is the beam energy; and $\theta_d$ is the disruption angle of the beam. Since the disruption angle at SLC is roughly one milliradian, the average depolarization is less than one percent.

6.5. Systematic Effects

It is possible that the average beam polarization as measured by the two downstream polarimeters be different from the luminosity weighted average polarization. There are two possible causes for this effect.

1. The beam-beam interaction obviously changes the polarization before it arrives at the Compton and extraction line Møller polarimeters. The size of this effect is estimated to be less than 0.5%.

2. If the electron beam at the interaction point has a non-zero dispersion function, it is possible that a beam-beam targeting error could cause the luminosity weighted beam energy and polarization to differ from the average beam energy and polarization. The beam-beam deflection process allows the beam to be targeted to within a small fraction of the beam sizes. Therefore, even if the dispersion function at the interaction point were as large as 3 mm (which is quite large), the fractional deviation of the monitored polarization from the average one is less than two percent. If the dispersion function is the more normal 1 mm, this effect is a few tenths of one percent.

7. Summary

Polarized beams can rather naturally be incorporated into a linear collider like SLC. Although some care is required, there are no technical reasons that make the machine
difficult to polarize. Since the polarized electron source is capable of performing as well as the normal unpolarized thermionic source, no degradation in machine performance is expected to result from the implementation of polarized beams. The hardware for producing, accelerating, and monitoring a polarized electron beam in SLC is in a reasonably advanced state of preparation. The installation schedule is being determined almost entirely by the rate of progress in commissioning the machine.

REFERENCES

1. A complete list of every publication that has discussed this topic would be huge. The following is just a sample of those that discuss the most basic asymmetries: *Proceedings of the LEP Summer Study at Les Houches, 1978* CERN 79-01 (1979); C.Y. Prescott, SLAC PUB 3120 (1983); *Physics at LEP*, edited by J. Ellis and R. Peccei, CERN 86-02 (February 1986); B.W. Lynn and C. Verzegnassi, *Physical Review* D35, 3326 (1987); A. Blondel, B.W. Lynn, F.M. Renard, and C. Verzegnassi, Montpellier preprint PM/87-14 (March 1987).


FIGURE CAPTIONS

1) The expected uncertainty of a measurement of the left-right asymmetry $A_{LR}$ as a function of the number of events used. The beam polarization is taken to be 45%. The $Z^0$ mass is assumed to be 92.5 GeV. The corresponding uncertainty on $\sin^2\theta_w$ and on $M_Z$ is shown on the right-hand scales. The three branches of the $A_{LR}$ curve refer to the precision of the polarization monitoring. From top to bottom, $\Delta A_{LR}$ is shown for $\Delta P/P = 5\%$, 3\%, and 1\%, respectively. A sample of $10^5$ to $10^6$ events is sufficient to saturate the asymptotic limit, depending upon the precision of the polarization monitoring. The expected uncertainty on $\sin^2\theta_w$ from a measurement of the leptonic forward-backward asymmetry is also shown. The beams are assumed to be unpolarized and the number of muonic decays is assumed to be given correctly by the Standard Model.

2) A layout of the SLAC Linear Collider. The orientation of an electron spin vector is shown as the electron is transported from the electron gun to the interaction point.

3) The band structure of GaAs near the bandgap minimum (from references 2 and 3). The energy levels of the states are shown on the right. Allowed transitions for the absorption of right (left) circularly polarized photons are shown as solid (dashed) arrows. The circled numbers indicate the relative transition rates.

4) The band structure of Gallium Arsenide near its surface \cite{18} for: (a) pure GaAs, (b) GaAs with a cesiated surface, and (c) GaAs with a layer of Cs$_2$O on its surface.

5) The polarization of electrons emitted from several GaAs photocathodes as functions of photon wavelength. The Illinois and Stanford samples were measured at SLAC by T. Maruyama. The data from Alvarado et al are taken from reference 5.
6) A drawing of the SLC polarized source. The photoemission source is installed on a Y vacuum section with the thermionic source to facilitate switching from one source to the other. The laser polarization and chopping optics are not shown.

7) The power of the SLC laser is shown as a function of wavelength (which depends upon dye concentration) for two different dyes. The laser power is the time averaged at 60 Hz operation.

8) The modulation and circular polarization optics for the SLC polarized source laser.

9) The spin rotation system as incorporated into the north damping ring complex. The orientation of the polarization vector at several points is shown by the double arrow.

10) The unpolarized differential cross sections for Møller and Bhabha scattering are presented as a function of the center-of-mass scattering angle. The longitudinal and transverse asymmetry functions for both processes are also shown.

11) The beam’s eye view of a Møller scattering target to be used at SLC. A holder containing four iron foils can be moved through the beam. Two foils are transverse to the beam and two are tilted at angle of 20° with respect to the beam axis. A set of three Hemholz coils can produce a 100 gauss field along any of the three axes.

12) A schematical representation of a Møller polarimeter for the SLC extraction line.

13) The electron signal as measured in a Møller polarimeter. The top part of the figure shows the number of scattered electrons as a function of scattering angle. The signal appears at the angle that corresponds to the scattered momentum. The background is well described by the Bethe-Heitler process.

14) The unpolarized cross section and the longitudinal and transverse polarization asymmetries are shown as a function of $x = K'/K_{max}$ for the scattering of a 2.23 eV photon by a 46 GeV electron ($y = 0.389$).

15) The location of the SLC Compton polarimeter is shown. A pulse from the laser located on the surface is directed into collision with the electron beam after it has passed through the interaction point.
Fig. 1
Polarization in the Overall SLC Layout

Fig. 2
Fig. 7

Fig. 8
Fig. 9

Fig. 10
PLANS FOR POLARIZATION AT HERA

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ABSTRACT
A report on the status of the HERA project is presented. The machine, the
proposed experiments and the plans for longitudinal polarization are described.

1. THE MACHINE
HERA is a high energy electron (or positron)-proton colliding beam facility that
is being constructed at the Deutsches Elektronen-Synchrotron (DESY) laboratory in
Hamburg, Federal Republic of Germany. The name HERA is short for Hadron Electron
Ring Anlage.

The electrons and protons are stored in separate vacuum pipes 6336 m in circumfer-
ence and will have energies of (up to) 35 GeV and 820 GeV respectively. The luminosity
at each interaction point will be about 1.5 \cdot 10^{31} \text{cm}^{-2} \text{sec}^{-1}. Fig. 1 shows a plan view
representation of the machine. The ring tunnel is 10-20 m underground. Construction
began in May 1984 and the boring of the tunnel was completed in August 1987. We
hope to have colliding beams in 1990. The countries participating in the construction
are Germany, Canada, France, Holland, Israel, Italy, Poland, the People's Republic of
China, Switzerland, the United Kingdom and the United States of America.

Electrons and protons will be supplied with the aid of a modified version of the existing
storage ring PETRA and this in turn will receive them from a series of purpose-built
lower energy machines. More details may be found in the HERA proposal[1]. Electrons
and positrons have already been successfully transferred from PETRA to HERA.

HERA incorporates a number of special features.
The high proton energy requires that the proton ring use superconducting magnets.
Among these, the dipoles are 9 m long and run at 4.5 T.

HERA is being designed with the aim of storing polarized electrons and achieving
longitudinal polarization at the interaction points. Experience has already been gained
at PETRA and DORIS where vertical polarization of over 70 % and of over 80 % at
beam energies of 16.5 and 5.01 GeV respectively have been observed[2].

Tracking simulations of the beam dynamics have shown[3] that the electrons and
protons must collide "head on" in order to avoid synchro-betatron orbit resonances
which could destroy the proton beam.

This "zero angle" geometry requires that the electrons be deflected into the path of
the protons and then out again with the aid of combined function quadrupoles po-
ositioned close to the interaction points[4]. Naturally, such an arrangement leads to
engineering constraints and in particular, in the West area where the electron and pro-
ton beams will be injected, it has been decided that during the starting up phase no
electron-proton collision point will be provided.

2. PHYSICS AIMS
Naturally, an important aim of a machine like HERA is to study deep inelastic e-p
scattering as a means of probing the structure of quarks. The c.m. energy extends up to
314 GeV and the energy transfer, $\nu$, to the protons extends up to 52 TeV. The accessible kinematic region in the $Q^2 - \nu$ plot is the triangle in Fig. 2. The kinematically accessible region for $10^3$ GeV electrons scattering on a fixed target is the tiny shaded area in the lower left corner. Clearly, HERA provides for a vast expansion of the accessible region and it is expected that in two years of data collection enough data should be available to explore distances down to $3 \cdot 10^{-18}$ cm. In addition, these structures can be explored not only by exchange of virtual photons but also with $Z^0$, $W^\pm$. In particular, $Z^0$ and $\gamma$ will interfere and the helicity dependence of the weak couplings will lead to differences in the positive and negative helicity electron and positron cross sections (Fig. 3) and provide a further test of the standard model[5]. Polarization will also be used to search for right handed weak currents and for supersymmetric interactions. Studies of the $q^2$ dependence of structure functions will enable quark substructure to be investigated. Detailed measurements on weak interaction effects will be sensitive to the existence of additional $Z^0$ and $W^\pm$. These and other aspects of the physics programme are discussed in more detail in[6].

3. THE EXPERIMENTS

So far, two experiments have been approved for HERA, namely "H1" and "Zeus". They will be installed in the North and the South interaction regions respectively and both are designed to do the physics described above. H1 is a collaboration of groups from Fed. Rep. Germany, France, German Dem. Rep., Italy, Switzerland, U.K., USA and USSR. Zeus is a collaboration from Fed. Rep. Germany, Canada, Holland, Israel, Italy, Poland, Spain, U.K. and USA.

Since the proton fragmentation products will be scattered mainly in the direction of the 820 GeV protons, both detectors are asymmetric in internal arrangement. Both utilize calorimeters for measuring the shower energies. H1 relies on liquid argon calorimetry and Zeus on uranium-scintillator calorimeters. In both detectors it is important for unambiguous identification of the kinds of events expected, that all particle tracks are registered. Thus, an essential feature of both detectors is that they provide complete coverage. Both use transition radiation detectors in the proton direction to help discriminate electrons from hadrons. Further details can be found in the proposal documents[7].

4. POLARIZATION

The notion that electron beams in storage rings could become polarized originated with the work of Loskutov, Korovina, Sokolov and Ternov[8,9], who pointed out that the emission of synchrotron radiation in a simple storage ring can cause electron spins to flip from up to down and vice versa and that there is a difference in the rates. Only a very small fraction of the synchrotron radiation power causes spin flip but nevertheless this asymmetry in the rates should lead to a build up of polarization along the vertical guide field. The equilibrium polarization should be $8/5\sqrt{3} = 92.4\%$ and the time constant for the exponential build up should be[9,10]

$$\tau_p^{-1} = \frac{5\sqrt{3} \gamma^5 \hbar \varepsilon^2}{8p^3 m^2 c^2}$$

(1)

where $\gamma$ is the Lorentz factor and $p$ is the radius of curvature in the magnets. At HERA the time constant is 40 mins and 11 mins at 27 and 35 GeV respectively.
As is well known, a classical spin magnetic moment precesses around a static magnetic field. The polarization vector of a relativistic electron in a storage ring also precesses, but this time according to the BMT equation\textsuperscript{[11]}

\[ \frac{d\vec{P}}{ds} = \frac{e\vec{P}}{mc\gamma} \wedge ((1 + a)\vec{B}_\parallel + (1 + a\gamma)\vec{B}_\perp) \quad , \quad a = \left(\frac{9 - 2}{2}\right) \]

(2)

where \(\vec{B}_\parallel, \vec{B}_\perp\) are the magnetic fields parallel and perpendicular to the trajectory and \(s\) is the longitudinal coordinate. In real storage rings there are quadrupole focussing magnets and possibly vertical bend magnets. The electrons oscillate in the transverse fields of the quadrupoles and one expects in the most naive classical picture that the spin vectors will precess in a complicated way in the fields along the (oscillating) orbits\textsuperscript{[12]}.

A correct quantum statistical treatment must account for this in the appropriate way and the theory will need to be more subtle than that given in the original treatments \textsuperscript{[8,9,13]}.

The resulting expression for the equilibrium polarization, \(|\vec{P}|\), originally derived by Derbenev and Kondratenko (D.K)\textsuperscript{[10]}

\[ |\vec{P}| = \frac{8}{5\sqrt{3}} \frac{<|\rho|^{-3}\hat{B} \cdot (\hat{n} - \hat{d})>}{<|\rho|^{-3}(1 - \frac{2}{9}(\hat{n} \cdot \hat{d})^2 + \frac{11}{18}d^2)>} \]

(3)

where \(\hat{n}\) is a classical unit spin vector defined at each point in the (position, momentum) phase space of the particle ensemble and \(d(s) = \gamma \frac{\partial s}{\partial \rho}\) is a partial differential describing how \(\hat{n}\) varies with particle energy. \(\hat{d}\) depends on the azimuthal position, \(s\), in the ring. The brackets, \(<\cdot\>\), denote an ensemble and azimuthal average. This formula describes the polarizing mechanism in the presence of particle oscillations (which are also excited by synchrotron radiation). Another quantum mechanical derivation and elucidation of the D.K. formula has recently been given by Mane\textsuperscript{[14]} who derives it from a point of view somewhat different from Derbenev and Kondratenko. Mane has also written a program, SMILE, for calculating \(\hat{n}\) and \(\hat{d}\) analytically and hence the polarization under the assumption that the orbital motion is linear. If \(d^2\) is large, then \(|\vec{P}|\) is small. \(d\) is particularly large when the natural spin precession frequency, \(\nu\), (the "spin tune") is near to the resonant condition

\[ \nu = k + k_zQ_z + k_zQ_z + k_zQ_z \]

(4)

where the \(k\)'s are integers and the \(Q\)'s are the orbital tunes\textsuperscript{[10,12]}. \(d\) is called the spin-orbit coupling vector. For a flat machine, \(\nu = a\gamma\). The D.K. formula only takes into account the excitation of the orbital motion caused by the change of energy accompanying photon emission. If the effect of immediate (very small) transverse momentum recoil is included\textsuperscript{[15]}, a modified D.K. formula predicts that for perfectly aligned weak focussing machines, polarizations as high as 99.2 % could be obtained over a narrow energy range. However, in practical high energy storage rings the relevant terms are submerged by the conventional \(d(s)\) terms and so this enhancement is not expected to be of practical importance.

The direction of the polarization at each point in the ring is given by \(<\hat{n}>\). Away from resonances, \(\hat{n}\) is closely parallel to \(\hat{n}_a\) which is the periodic solution to the BMT equation on the closed orbit. Thus
\[ \bar{P} \approx |\bar{P}| \bar{n}_o \]  \hspace{2cm} (5)

Therefore, from the numerator of the D.K. formula one sees that to obtain high polarization it is essential for \( \bar{n}_o \) to be parallel to the guide field in most of the ring. This is anyway its natural orientation in simple flat rings where the dipole fields are all vertical.

At HERA, longitudinal polarization will be obtained by allowing \( \bar{n}_o \) to remain vertical in the arcs and by rotating \( \bar{P} \) (i.e. \( \bar{n}_o \)) into the horizontal plane just before an interaction point (I.P.) and by returning it to vertical just after the I.P. The horizontal bend magnets in the interaction region then rotate \( \bar{P} \) into and out of the longitudinal direction at the I.P. According to the BMT equation the spin precession angle in a transverse field is \( a \gamma \) times larger than the orbit deflection angle. At 30 GeV, \( a \gamma \sim 68 \). Thus, small orbit deflections lead to large precession angles and since finite rotations do not commute, the required rotation can be achieved with the aid of a series of interleaved vertical and horizontal bends. This is the basis of the "Mini-rotator" scheme of Buon and Steffen[16] which has been adopted for HERA. The layout of the ring near the I.P. is sketched in Fig. 4 where the horizontal (H) and vertical (V) rotator bends are marked. The bend angles depend on the electron beam energy. Helicity is reversed by reversing the vertical bend polarity. Thus, the beam pipe must be flexible and all rotator magnets mounted on adjustable jacks. Since \( \bar{n}_o \) is not parallel to the bending field except in the arcs, the maximum polarization (corresponding to \( d = 0 \)) is several percent less than 92.4%. In the start up phase, only the East area will be equipped with a rotator pair so that machine physics investigations can be carried out in relative isolation from the high energy physics experiments.

To obtain high polarization it is still necessary to minimize \( \langle d^2 \rangle \) at those points in the ring where \( 1/\rho \) is non-zero. In a perfectly aligned flat ring \( d \) is zero (we neglect the transverse recoil effects). However, in a machine like HERA, which has rotators and regions where \( \bar{n}_o \) is horizontal, \( d(s) \) is normally non-zero even in a perfectly aligned ring. If both spin motion (as well as orbital motion) are calculated in lowest order (linear approximation)[14] one finds that \( d(s) \) can be written as a linear combination of one turn integrals whose integrands are products of orbital and spin motion factors[14,17]. Each integral is multiplied by a resonance factor leading to one of the first order resonances:

\[ \nu = k \pm Q_i; \quad i = x, z, s \]  \hspace{2cm} (6)

\( \langle d^2 \rangle \) and the polarization can be calculated in linearized theory using SMILE set to run in lowest order mode or by using the program SLIM of A. Chao[18]. The results are algebraically and numerically identical. For a normal "untreated" HERA electron optics the curve of calculated polarization versus energy often looks something like the lower curve in Fig. 5. In this example two pairs of rotators are switched on in North and South, the closed orbit is perfectly aligned and the rotators have been adjusted to give longitudinal polarization at each energy. Under these conditions and for \( d = 0 \), the maximum attainable polarization would be about 87%.

We see, however, that this value is not reached; the polarization is limited by strong first order resonance effects.

One can then adjust the optics to minimize \( \langle d^2 \rangle \). We call this procedure "linear spin matching" and at DESY this is carried out using the program SPINOR[19] of L. Hand. Although, in linear approximation \( d \) can be represented in terms of one turn integrals and spin matching has often been discussed in terms of those integrals[17], at
DESY I prefer to carry out the spin matching using the transfer matrices and spin-orbit eigenvectors contained in the SLIM formalism[18]. This approach, which has been discussed by H. Mais and G. Ripken[20] has several advantages.

The upper curve of Fig. 5 shows the polarization predicted after spin matching this optics at 29.23 GeV. The polarization now reaches the full value of 87 % and, although optimization (which is energy dependent) was carried out only at one energy, this seems to be sufficient to ensure high calculated polarization over a broad range of energies. The dips are due to remaining first order resonance effects.

We cannot conclude from this, however, that we will achieve the full 87 % electron polarization at HERA. This calculation was for a perfectly aligned HERA and only first order resonances were included. Fig. 6 shows the result of a preliminary recalculation of almost\(^1\) the same spin matched optic using SMILE set to include the resonances

\[
\nu = k \pm Q_x \pm Q_z \pm k_s Q_s, \quad |k_s| \leq 5
\]

\[
\nu = k \pm k_s Q_s, \quad |k_s| \leq 6
\]

(7)

No resonances above 6th order are included in this example so we can only get a partial impression. But it is clear already that the result of Fig. 5 is too optimistic and that the ability to calculate beyond lowest order is very significant advance.

This is not the end of the story. Apart from including resonances above 6th order, one should also consider the effect of machine misalignments, mistuning and beam-beam effects[12,21]. Even at lowest order these can lead to a reduction of polarization below that predicted in Fig. 5. Techniques have already been developed for dealing with the effects of misalignments and mistuning[17,22]. Simulations show that these methods are very successful in softening resonances.

In calculating \(\hat{n}\) and \(\hat{d}\) the SMILE algorithm assumes that the orbital motion is linear. But in reality orbit motion is slightly non-linear. In electron rings, an important contribution to non-linearity comes from the sextupoles which are installed to correct chromatic effects. Such a non-linearity considerably complicates the calculation of \(\hat{n}\) and \(\hat{d}\). However, Yokoya[23] has proposed an algorithm which will enable polarization to be calculated analytically even in the presence of non-linearities. In addition to the analytical studies described above, T. Limberg[24] has been carrying out numerical studies using the program SITROS of J. Kewisch[25]. This program undertakes Monte-Carlo spin-orbit tracking of classical spins and includes non-linear orbit motion and beam-beam effects. Strong resonance effects are also seen in these calculations.

5. CONCLUSION

The HERA project is running according to schedule and two large high energy physics experiments are being prepared. The machine is being designed with the aim of achieving longitudinal electron polarization. This facility will be the first of its kind and high energy physics measurements will commence in 1990.

\(^1\)The calculations of Figs. 5 and 6 are for a perfectly aligned orbit. In Fig. 5 thick lens optics is used but in Fig. 6, with SMILE, the same optics is treated in thin lens approximation. Thus, the tunes and resonance positions are slightly shifted.
ACKNOWLEDGMENTS

The author wishes to acknowledge the continued encouragement of Prof. G.-A. Voss and helpful discussions with colleagues, in particular R. Brinkmann, J. Buon, Yu. Eidelman, J. Jowett, T. Limberg, J. Kewisch, R. Kose, J.-P. Koutchouk, H. Mais, V. Muratov, G. Ripken, R. Rossmanith, Yu. Shatunov, K. Steffen, A.A. Zhdents and K. Yokoya. The author is particularly indebted to S. Mane for making the program SMILE available for use at DESY.

* * *

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Fig. 1 Schematic view from above of the HERA ring and preaccelerators.

Fig. 2 The kinematical region available at HERA.
**Fig. 3** Interference between photon and $Z^0$ exchange in the reaction $e + p \rightarrow e + X$ for electrons and positrons of positive and negative helicity. The cross section is normalized to the photon exchange cross section and plotted against the scaled energy transfer $y$ for $x = 0.25$ (see ref. 6).

**Fig. 4** Schematic plan and side views of the electron and proton rings at the normal straight sections. The rotator bend magnets ($H, V$) are shown, as are the separating magnets (combined function quadrupole triplet and dipoles).
**Fig. 5** Calculations with the program SLIM for a typical electron optics before (lower curve) and after (upper curve) spin matching.

**Fig. 6** Calculations with the program SMILE for the optics of Fig. 5.
POLARIZATION SIMULATION STUDIES FOR LEP

Jean-Pierre Koutchouk and Torsten Limberg

1 INTRODUCTION

In a flat storage ring, the natural polarization direction is vertical. The radiative polarization (Sokolov-Ternov effect) is in competition with resonant depolarization, further enhanced by the quantum nature of synchrotron radiation; it induces a stochastic component of the orbital motion which couples to the spin precession.

Physics with polarized beams requires the spin to be rotated longitudinally and the interaction products to be analyzed with detector solenoids. Both schemes involve magnetic fields which are further potential sources of depolarization. It is thus necessary to demonstrate the feasibility of a high degree of transverse polarization.

An exact solution for the depolarization would require to solve a system of stochastic equations for the orbital motion and the Thomas-BMT equation for the spin. Although progress is being made in various institutes, the only experienced theory requires the assumption of vanishing emittances; it allowed to reproduce correctly the polarization pattern in SPEAR and, to a reasonable extent, in PETRA. It is equally the basis for polarization optimization and spin matching of insertions.

However the LEP beam rms energy spread (37 MeV) is not small compared to the major spin resonance spacing (440 MeV); it is further enhanced to 78 MeV by the wiggler magnets necessary to reduce the polarization rise time. The spin precession modulation due to the energy oscillation results in presumably strong satellite resonances not accounted for in the simplified theory.

The aims of this study are essentially twofold:

- to calculate and optimize the polarization in a realistic imperfect machine using the linear theory.
- to attempt evaluating the higher-order effects with a tracking code and possibly carry out further optimization.

2 THE POLARIZATION MODELS

2.1 The linearized theory and programs

The linear theory was developed by Chao [1] and aims at calculating successively the orbital motion, the spin precession and the consequences of quantum excitation.

Basically, the orbital motion is described by a linear 3-oscillation (horizontal, vertical, longitudinal) about a generalized 3-closed orbit which itself may take into account optics imperfections and non-linear elements. Once the closed orbit is identified, the knowledge of the magnetic fields allows to compute the spin rotations $\Omega$ of the BMT equation.

\[
\frac{d\vec{S}}{ds} = \Omega \times \vec{S}
\]
where \( s \) is the machine azimuth. The real eigenvector of the one turn spin rotation is the periodic spin axis \( \vec{n} \). An arbitrary spin of an oscillating particle precesses around \( \vec{n} \); the number of precessions per machine turn is the spin tune \( \nu \). The corresponding oscillation dependant spin transformation is then found for each element by integrating the Thomas-BMT equation with some simplifications: the spin motion is assumed to be plane instead of spherical:

\[
\vec{S} \approx \vec{n} + \alpha \vec{m} + \beta \vec{l}
\]

with \((\vec{n}, \vec{m}, \vec{l})\) spin precession frame, \((\alpha, \beta)\) spin coordinates. Only first-order terms in small quantities are retained. It becomes thus possible to find a linear transformation of the 6 orbital and the 2 spin coordinates which expresses the spin-orbit coupling.

The depolarization due to a single photon emission is calculated by expressing the relation between the initial state (a pure energy loss) and the final state after damping (a pure spin tilt). Subsequent averaging over all photon emissions yields the total depolarization.

The limitations of the method arise from the linearization which discards the consequences of finite amplitude oscillations. Only linear resonances of the type

\[
\nu = k, \quad \nu = k \pm Q_x, \quad \nu = k_x \pm Q_x, \quad \nu = k_y \pm Q_y
\]

may appear, where \( Q_x, Q_y \) are the synchrotron and betatron tunes of the orbital motion and \( k, k_x, k_y \in Q \).

Two programs have been used: SLIM [1], using thin-lens approximation and STTF [2], newly available, using a thick-lens model both for the optics and the spin motion. Whilst the essentials of a perturbed optics may be represented in SLIM, the optical part of STTF uses a generalization of the PETROS program [3]. Therefore all the possibilities of that program to simulate displacement and strength errors of the elements as well as closed orbit correction can be used also for polarization calculations. The thick-lens model suppresses the need to cut the accelerator elements into pieces; This enhances the speed of the code and its accuracy.

### 2.2 The tracking method

The principle of the program STTROS [4] is to trace the orbital and spin coordinates of an ensemble of 50 particles for several damping times. The polarization degree of the particles as a function of time is measured by taking the vector sum of the spins of the ensemble. The depolarization time constant \( \tau_d \) is calculated by fitting the function

\[
P(t) = P_0 \exp \frac{-t}{\tau_d}
\]

to the measured values. \( P_0 \) is the polarization degree without depolarizing effects, the so-called Sokolov-Ternov level. It is calculated from the knowledge of the \( \vec{n} \)-axis and so is the polarization time constant \( \tau_p \). The asymptotic polarization level is found from

\[
P_\infty = \frac{P_0}{1 - \frac{\tau_p}{\tau_d}}
\]

The calculation of the spin rotation in every magnet of the storage ring (LEP has more than 2000 magnets) would be far too time consuming on available computers.
Therefore the ring is divided into sections. The emission of synchrotron radiation and the beam-beam effect can only occur between sections.

The quantum fluctuations are simulated by a random energy loss occurring at six places around the storage ring. The amplitude of the stochastic fluctuation of the energy is adjusted so as to reproduce the beam distribution in all three dimensions within 10%. First-order polarization calculations where all the radiation takes place in the points chosen for the tracking procedure show no significant difference compared to the calculations with radiation in every bending magnet. Furthermore different azimuths and number of positions (2, 3 and 6) for the radiation have been tried to check that its simulation is acceptable.

The transformation of a particle through a section is done up to second order in the coordinates, using 6x27 matrices which allow to take into account the quadratic terms. The spins of the particles are represented by three dimensional vectors. Their rotations in a section are described by spinors, the components of which are computed up to second order in orbital coordinates using 4x27 matrices. The spinors are transformed into 3x3 rotation matrices which are then used to perform the transformation of the spins. So axis and angle of the rotations are described taking into account quadratic terms whilst the spins undergo a purely spherical motion.

3 POLARIZATION DEGREE BEFORE OPTIMIZATION

3.1 Energy scanning around the Z & the simulation of imperfections

In the linearized theory, the depolarization calculated from the Derbenev-Kondratenko formula can be shown to increase with the square of the beam energy [5]. However more indirect energy dependences may be expected from the energy dependance of the spin resonances [6] [7].

In order to identify a favourable energy range about the energy of the Z (46 GeV), a large energy scan was carried using SLIM [8], between 40 and 52 GeV. Instead of the expected polarization decrease like \((1 + E^2)^{-1}\), a large amplitude periodic oscillation is observed to dominate, partially represented on figure 1. The complete analysis [9] shows that the oscillation is intimately related to the Fourier harmonics of the closed orbit observed in the spin precession frame; random optics imperfections are shown to only drive a subset of the spin integer resonances ('systematic' resonances) fulfilling:

\[ \nu = \lfloor Q_s \rfloor + B \cdot k \]

with \(\lfloor Q_s \rfloor\) integer part of the tune (78), \(B\) superperiod of the bending (8), \(k \in \mathbb{Q}\)

The other integer spin resonances are only weakly excited. This pattern is smoothed by closed orbit correction.

The consequences are:

- the spin tune corresponding to the Z energy, including its uncertainty, is in a favourable situation with respect to systematic integer resonances.

- a realistic simulation must incorporate a realistic spectrum of the corrected closed orbit, otherwise the spin resonances around the Z (\(\nu = 104\)) are artificially weak,

- any additional random perturbation to a corrected optics will mainly change the polarization in the vicinity of the systematic resonances, i.e. \(\nu = 102, 110\) for LEP, away from the Z energy.
Following these findings, the closed orbit was simulated by allowing all the elements to be mispowered and misaligned and by correcting the resulting orbit with the MICADO algorithm of the CERN version of PETROS [3].

![Graph showing polarization around the Z](image)

Figure 1: linear polarization around the Z

### 3.2 Polarization on the standard LEP optics

Ten random optics were generated and the horizontal and vertical closed orbits corrected to reach:

\[
\langle x \rangle = 0.93 \pm 0.22 \text{mm} \quad \langle y \rangle = 0.91 \pm 0.13 \text{mm}
\]

which is a reasonable target in practice.

The polarization degree was then computed around the Z energy, without wigglers to simplify the situation. Figure 1 shows the average over the 10 machines of

- the peak polarization observed between two integer spin tunes
- the peak polarization degree as measured using SLIM (refer to the previous chapter)

The polarization degree is found significantly less than predicted by SLIM and the strong spin tune dependence has almost disappeared. There is still a depression at the "systematic" integer resonances which should thus be avoided. Table 1 summarizes the results obtained at the Z energy.

The same calculation with the nominal or the new dedicated wigglers [10] shows essentially the same results. The dominant depolarizing mechanism is related to the integer resonances and their synchrotron satellites. Figure 2 shows indeed a very good correlation between the resulting tilt of the \( \vec{n} \) axis and the polarization level. The
<table>
<thead>
<tr>
<th>Polarization</th>
<th>peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average over 10</td>
<td>19.3%</td>
</tr>
<tr>
<td>Peak over 10</td>
<td>32.9%</td>
</tr>
</tbody>
</table>

Table 1: Statistics of polarization

Figure 2: linear polarization versus tilt of spin axis

betaresonances are weaker. In a flat electron ring this result may be expected because the vertical closed orbit deviations are large compared to the vertical beam size and because the tilt of the $\vec{n}$ axis is small.

In first-order approximation, the wiggler magnets positioned at zero dispersion can be shown to further damp the already small depolarization due to the betatron resonances but not to change the main depolarizing terms related to the integer resonances and their synchrotron satellites [10].

One of the best optics (40 % polarization) with dedicated wigglers on was tested with the tracking code SITROS. It shows weak polarization peaks (5 to 20 %) separated by wide satellite resonances. Bootstrapping the polarization optimization is thus likely to require special measures.

4 OPTIMIZATION OF THE POLARIZATION

4.1 Harmonic spin matching

4.1.1 Principle

The imperfect vertical closed orbit being at the root of the depolarization and the depolarization being essentially resonant, it seems natural to attempt changing the
Figure 3: Linear and non-linear polarization on one of the best optics before polarization optimization

spectral properties of the closed orbit. This method was developed and successfully used in PETRA [11]. It is shown in [12] that the tilt of the \( \vec{n} \) axis can be expressed as a function of the orbit harmonics:

\[
|\vec{d}n| \propto \sum_k \frac{1}{(k - \nu)^2} (a_k^2 + b_k^2)
\]

where \( a_k, b_k \) are the Fourier components of the vertical closed orbit calculated in the spin precession frame. The principle is to filter out the harmonics of the orbit closest to the spin tune.

4.1.2 Implementation

Although in practice the \( k = 104, 105, ... \) closed orbit harmonics are not measurable, they are of course provided by the simulation program. The correction method aimed thus at a direct cancellation of the offending closed orbit harmonics, in an attempt to simulate the best achievable correction. It was found later that a further empirical optimization provides another improvement not consistent with the above-mentioned principles [13].

The most straightforward implementation uses four vertical orbit correctors to correct the two closest harmonics. The more sophisticated method developed for PETRA [11] needed an extension to eight families of eight symmetric correctors to avoid too large orbit deviations; it allows to correct the four closest harmonics.

Finally, advantage was taken of the peculiarities of the LEP cell: over three cells, the betatron phase advance is equal to \( \pi \), whilst the spin precession is close to \( 2\pi + \frac{\pi}{3} \) around the \( Z \) energy. It follows that a three cell orbit bump is almost as efficient as an
equivalent orbit distortion all around the machine. Four of these bumps were thus used to correct the two closest harmonics, with the advantage of a reduced perturbation to the machine.

4.1.3 Results

Figure 4 shows the selectivity and degree of efficiency of the technique.

![Graph showing harmonic spin matching](image)

Figure 4: harmonic spin matching

Without wigglers, the compensation of two and four integer resonances yields:

<table>
<thead>
<tr>
<th>Method</th>
<th>4 correctors</th>
<th>Petra/64 correctors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak polarization</td>
<td>46% (was 27%)</td>
<td>58% (was 27% )</td>
</tr>
</tbody>
</table>

With the new dedicated wigglers on, an initially "bad" optics (high vertical dispersion) can reach 72% polarization; further empirical adjustments of the bumps yields 85% polarization (figure 5). The gain due to the wigglers is to be attributed to the additional damping of the spin betatron resonances. However this is only a truncated view which does not take into account the enhancement of the energy spread caused by the same source.

4.2 Optimization of the vertical dispersion

A first SITROB run showed that the 85% polarization optics selected would hardly provide any polarization in practice due to the strong resonance satellites. The high emittance coupling (27%) being pathologic, the first attempt was to reduce it by a minimization of the vertical dispersion in the machine and especially in the wigglers. The
vertical dispersion indeed enhances the vertical spin resonances through the dispersion invariant $H_y$. Their driving term is given by [5]:

$$\Delta_{\pm y} = \frac{(\nu + 1)e^{\mp i \phi_y}}{e^{2i\alpha(n + Q_y)} - 1} \sqrt{\beta_y} \int (\tilde{m} + i \tilde{l}) e^{\mp i \phi_y} ds$$

The dispersion $D_y$ enters in a less straightforward way in the driving term of the synchrotron resonances:

$$\Delta_{\pm s} = \frac{(\nu + 1)e^{\pm i \phi_s}}{e^{2i\alpha(n + Q_s)} - 1} \int (\tilde{m} + i \tilde{l})(\tilde{c}_y D_x + \tilde{c}_x D_y) K ds$$

where $\nu$ is the spin tune, $(\tilde{n}, \tilde{m}, \tilde{l})$ the spin precession frame, $(\tilde{c}_x, \tilde{c}_y, \tilde{c}_s)$ the geometrical frame, all the other parameters being the standard accelerator quantities.

### 4.2.1 Minimization of the rms vertical dispersion

The rms vertical dispersion was reduced both by suppressing the dominating effect of orbit offsets in the strong low-$\beta$ quadrupoles and by minimizing the fundamental harmonic of the remaining residual dispersion. For that purpose four-corrector asymmetric orbit bumps were used to straighten the orbit in the short insertions, whilst another of these bumps placed in a long insertion, essentially in quadrature with the others, allowed a minimization process. The rms vertical dispersion and the emittance coupling became:

**Before** \quad $< D_y > = 198 \text{ mm} \quad \kappa = 27.2\%$

**After** \quad $< D_y > = 30 \text{ mm} \quad \kappa = 0.5\%$
As may be seen from the emittance coupling $\kappa$, the dispersion at the wiggler magnets was significantly reduced.

Linear polarization calculations show no improvement after this drastic reduction until a reoptimization of the harmonic spin matching is done. It reveals then that, as could be expected, the vertical betatron resonances have almost disappeared, and that the spin matching has brought a further small improvement (figure 5). The non-linear simulation using SITROS does not show any significant improvement following the dispersion minimization and still shows the dominance of resonance satellites (figure 6).

Figure 6: Non-linear polarization after spin matching and $D_y$ minimization

These observations confirm that the vertical spin betatron resonances are not critical, even at very high emittance ratio.

4.2.2 The harmonic dispersion correction

As already mentioned, changing the dispersion required a reoptimization of the orbit harmonics. However the antisymmetric orbit bumps used to create the dispersion are positioned in the straight sections; they are thus spin transparent, i.e. they do not contribute to the orbit harmonics observed in the spin precession frame.

In order to explain the need for a reoptimization, it is necessary to reconsider the driving term of the synchrotron resonance $\Delta_{\perp s}$. Neglecting the multiplying factor, the integral part $J_s$ may be reexpressed in the unperturbed spin base, to allow the quantity $\tilde{d}n$ to appear:

$$J_s = \oint K[\psi D_y - D_x(\tilde{d}n)] ds$$

where $\psi$ is the spin phase advance.
The first term is the Fourier transform of the function $K D_y$ whilst the second is dominated by the harmonic of the closed orbit closest to the spin tune. The harmonic spin matching using orbit correctors or orbit bumps changes both the orbit and dispersion spectra. The final empirical tuning already mentioned likely finds the best compensation between the two terms.

In order to disentangle them, a spectral analysis of the dispersion was carried in the spin precession frame and the sine component of the dispersion harmonic 104 was cancelled using the 4 asymmetric orbit bumps in the short insertions.

Once again, no improvement in the linear polarization was observed until the harmonic spin matching was empirically adjusted. A drastic improvement was obtained by this technique, although only one of the four significant terms was cancelled. The non-linear calculation with SITROS shows a large weakening of all the synchrotron satellites in the vicinity of the partially corrected synchrotron spin resonance (figure 7). Spectral

![Graph showing polarization degree vs spin tune](image)

**Figure 7:** Effect of a partial harmonic correction of $D_y$ (nh=104/sine)

analysis of the final closed orbit and dispersion shows that the empirical harmonic spin matching resulted in the weakening of the sine component of the closed orbit harmonic 104, demonstrating the interplay between the dispersion and closed orbit harmonics.

The next step that will be performed is a complete and simultaneous compensation of the harmonics 104 and 105 of the vertical orbit and dispersion.

5 CONSEQUENCES OF THE DISCONTINUOUS REPLACEMENT OF RADIATED ENERGY

The closed orbit of electrons and positrons in a storage ring is different due to radiation effects [14]. The energy loss which occurs all around the machine is compensated locally at the azimuth of the RF cavities. In an ideal plane machine, the closed orbits would
describe sawtooth like curves in the horizontal plane around the design orbit. Besides this systematic effect which is symmetric for the two kinds of particles the sextupole fields and the rotated quadrupole fields in a real storage ring causes differences in the vertical closed orbits.

This difference (0.04 mm rms at 46 GeV [15]) is of the same order of magnitude as the orbit bumps required by the harmonic spin matching technique. It was thus necessary to calculate the polarization difference between electron and positron taking into account the radiation.

Linear polarization calculations shows that the difference between the polarization degrees of electrons and positrons might be some per cent in cases when the harmonic spin match is not perfect. For the fully optimized machine, the difference is below one per cent.

This result may be understood in the light of chapter 3 : the additional optics imperfections which result in different vertical orbit for electrons and positrons are essentially random. They can thus be expected to produce a weak effect between systematic integer resonances.

6 CONCLUSION

Within the limitations of the models, and taking into account that the accelerator description does not incorporate spin rotators and the beam-beam effect, it appears that a high degree of polarization seems possible in LEP at 46 GeV. The key to high asymptotic polarization has been shown to lie in the precise and independent cancellation of the integer spin resonances and their first synchrotron satellites, resulting in a very significant weakening of the higher-order satellites. Given the only partial harmonic correction carried out, even better suppression of the higher order satellites may be expected.

However, a direct correction would require the knowledge of the closed orbit and dispersion to very high absolute accuracy (better than 0.001 mm and 0.1 mm for their harmonics 104). This is far beyond what can be achieved in practice. The polarization optimization in the control room will rely on an empirical search in, at least, a 8 parameter space. Its implementation will require dedicated studies.

References


WIGGLERS FOR POLARIZATION

A. Blondel and J.M. Jowett
CERN

Abstract
LEP should be equipped with new wigglers which would reduce the polarization
time to 36 minutes, the minimum consistent with the aperture constraints. This
makes empirical correction of depolarizing effects feasible in a reasonable time and
improves the effective polarization degree during a physics run. The new wigglers
can be made more "asymmetric" than the existing ones, giving an ideal asymptotic
polarization degree of 88 % instead of 74 % at the $Z$ energy. If installed in the
low-$\beta$ straight sections, they would also reduce the depolarizing effects of horizontal
betatron oscillations. We have found that these powerful wigglers dominate the rest of
the machine as far as both polarizing and depolarizing effects are concerned, bringing
substantial simplification to their analysis and correction. Beneficial effects on the
asymptotic polarization degree have been found in first-order simulations. The cost
of installing the complete wiggler system is estimated at less than 2 MSFR.

1 Introduction

The first obstacle encountered in the attempt to produce an acceptable degree of polar-
ization for experiments at the $Z$ resonance is the very long polarization time in LEP—
some 300 minutes in the "bare" machine (i.e. no wigglers). This exceeds the luminosity
decay time which is expected to be about 200 minutes. Wiggler magnets—the only
known, practicable means of accelerating the Sokolov-Ternov polarization mechanism—
are therefore indispensable.

This paper is based on a recent LEP Note [1] in which we proposed new wigglers to
produce the shortest possible polarization time. These magnets would greatly facilitate
the machine physics studies of transverse polarization that have to be performed in an
early phase of LEP operation. These studies are needed for the beam energy calibration
and should help us to resolve some of the uncertainties related to our chances of event-
ually obtaining longitudinal polarization. In any case, these wigglers are a prerequisite
for the exploitation of spin rotators.

Besides these fairly obvious—but nonetheless important—benefits, we have found
that a wiggler design which achieves the maximum possible polarization rate also reduces the effect of many sources of depolarization, giving us hope that a much higher level of
polarization can be achieved.

1.1 The existing wigglers and the new wigglers

LEP is already destined to be equipped with 8 wiggler magnets [2] which affect the
beam in various ways (emittance increase, bunch-lengthening at injection, increase of
damping rates, ...) calculated to obtain the highest possible luminosity. A wiggler
configuration which fulfils these purposes is simply not powerful enough to produce the
shortest possible $\tau_p$ at the $Z$ energy; it was always expected (see [3] and p. 11 of [4]) that
further wigglers would be required (along with spin-rotator magnets etc.) if longitudinal polarization were ever to be implemented. Nevertheless the existing wigglers bring the polarization time, $\tau_p$, down from 310 min to 90 min at the energy of the Z-pole and, with polarization in mind, they were made "asymmetric" (at little extra cost) so as to minimise the reduction in $P^I_\infty$.\footnote{Symbols and notations not defined in the text are listed in Appendix A.} However, since polarization was not a high-priority item at the time they were specified, the extra cost of a configuration fully effective for polarization was not incurred. Moreover, since that time, our understanding of the effects of powerful wigglers on polarization has changed significantly.

Half of the existing wigglers must be installed in a part of the ring where the dispersion, $\eta_z$, is non-zero (so that they can be used to increase emittance) [5]. We shall see that this is far from being ideal for polarization purposes. The new wigglers which we now propose should be located in straight sections around two of the non-experimental crossing points (P3 and P7, say), where the nominal $\eta_z = 0$. Their parameters are chosen to produce the maximum possible acceleration of the radiative polarization process that can be had at the Z energy with a very small reduction of $P^I_\infty$. In addition we shall show that, at least to first-order in depolarizing effects, they considerably improve the asymptotic polarization level $P_\infty$.

2 Effect of wigglers on beam parameters

Let us summarise those effects of wigglers on the beam which limit our ability to decrease $\tau_p$. Further details will be found in [1] and related discussions were previously given in [6,7,8] among others.

Apart from the wigglers, LEP has an isomagnetic bending strength, $1/\rho_0$ to a very good approximation.

In essence, we have to fix four free parameters:

1. The total length of the central poles of the wigglers, $N_W L_+$.

2. The field in the central poles $B_+$.

3. The wiggler asymmetry parameter $r = -B_+/B_- = L_-/L_+ > 1$ (see Figure 1).

4. The design value of the horizontal dispersion at the wigglers.

Our choice of these parameters should increase the polarization rate $\alpha_p = 1/\tau_p$ but respect the constraints which arise through the effects on the polarization level, the energy spread and beam size.

2.1 Polarization rate, emittance and energy spread

The wigglers change the emittance by a factor [1]

$$
\mathcal{F} \{ \varepsilon_z \} \equiv \frac{\varepsilon_z (\text{wigglers})}{\varepsilon_z (\text{no wigglers})} = \frac{1 + \frac{N_W L_+ |B_+|^2}{2\pi \rho_0 B_0^2} \left(1 + \frac{1}{r^2}\right)}{1 + \frac{N_W L_+ |B_+|^2}{2\pi \rho_0 B_0^2} \left(1 + \frac{1}{r}\right)}.
$$

(1)
In order not to increase the emittance, it is best to locate the wigglers in straight sections. Then the value of \( \langle H \rangle_w \), which is ideally zero when the wigglers are switched off, will be very small and its contribution can be neglected. A stronger incentive to do so is the fact that this also reduces the depolarizing effects of betatron oscillations (see Section 3). This fixes one of our four free parameters. In this case the emittance is actually reduced:

\[
\mathcal{F} \{ \epsilon_x \} \simeq \frac{1}{1 + \frac{N_W L_B |B_+|^2}{2\pi \rho_0 B_0^2} \left( 1 + \frac{1}{r^2} \right)} < 1. \tag{2}
\]

In contrast, the effect of the wigglers is always to increase the r.m.s. energy spread:

\[
\mathcal{F} \{ \sigma_e \} \overset{\text{def}}{=} \frac{\sigma_e(\text{wigglers})}{\sigma_e(\text{no wigglers})} = \frac{\sqrt{1 + \frac{N_W L_B |B_+|^3}{2\pi \rho_0 B_0^3} \left( 1 + \frac{1}{r^2} \right)}}{\sqrt{1 + \frac{N_W L_B |B_+|^2}{2\pi \rho_0 B_0^2} \left( 1 + \frac{1}{r^2} \right)}} > 1, \tag{3}
\]

The rate at which the polarization builds up by the Sokolov-Ternov mechanism is given by:

\[
\alpha_p = \frac{1}{\tau_p} = \frac{5\sqrt{3}}{8} \frac{h \rho c}{m_e c^2} \left( \frac{E}{m_e c^2} \right)^5 \left( \frac{e}{p_0 c} \right)^3 \int |B|^3 \, ds. \tag{4}
\]

where the integral is taken over the machine circumference. The wigglers increase this by a factor

\[
\mathcal{F} \{ \alpha_p \} \overset{\text{def}}{=} \frac{\alpha_p(\text{wigglers})}{\alpha_p(\text{no wigglers})} = \frac{\tau_p(\text{no wigglers})}{\tau_p(\text{wigglers})} = 1 + \frac{N_W L_B |B_+|^3}{2\pi \rho_0 B_0^3} \left( 1 + \frac{1}{r^2} \right) > 1, \tag{5}
\]

i.e. any wiggler increases the polarization rate. The existing wigglers give a factor 300 min/90 min \( \simeq 3.3 \) at 46 GeV and we require the new wigglers to do substantially better. It follows that the \textit{contribution of the wigglers to} \( \int B^3 \, ds \) \textit{must dominate the contribution from the normal bending magnets}:

\[
\frac{N_W L_B}{2\pi \rho_0} \left( \frac{B_+}{B_0} \right)^3 \gg 1. \tag{6}
\]

In an \textit{ideal, absolutely planar} machine the asymptotic polarization level is given by

\[
P_\infty = \frac{8}{5\sqrt{3}} \frac{\int B^3 \, ds}{\int |B|^3 \, ds} \tag{7}
\]

which is just a special case of (19) with all depolarizing effects neglected. Using the wigglers to increase the rate of polarization reduces \( P_\infty \) by a factor

\[
\mathcal{F} \left\{ P_\infty \right\} \overset{\text{def}}{=} \frac{P_\infty(\text{wigglers})}{P_\infty(\text{no wigglers})} = \frac{1 + \frac{N_W L_B |B_+|^3}{2\pi \rho_0 B_0^3} \left( 1 - \frac{1}{r^2} \right)}{1 + \frac{N_W L_B |B_+|^3}{2\pi \rho_0 B_0^3} \left( 1 + \frac{1}{r^2} \right)}. \tag{8}
\]

In the limit of a wiggler-dominated machine, (6), this depends only on \( r \):
Figure 1: Schematic design of an asymmetric wiggler

Figure 2: Polarization level in an ideal wiggler-dominated machine; the dashed line shows $P_\infty (\text{no wiggler}) = 92.4\%$.

The existing wigglers have $r = 2.5$. We see that, if the new wigglers have $r \gtrsim 6$, there will be no significant loss in equilibrium polarization level.

Now two of the four free parameters (nos. 3 and 4) are fixed.

For a given improvement in polarization rate we can derive a relationship [1] giving the price paid in increased energy spread. Since the wigglers occupy only a small fraction of the circumference of LEP, (3) may be approximated as

$$P_\infty (\text{wigglers}) = \frac{8}{5\sqrt{3}} \frac{(r^2 - 1)}{(r^2 + 1)},$$

and is plotted in Figure 2.

The first term on the right-hand side is several times larger than the second. Moreover the weak dependence of the second term on the wiggler parameters shows that, for a given increase in polarization rate, $F\{\alpha_p\}$, there is very little which can be done to avoid the corresponding increase in $\sigma_x$.

Doubling the total length of wigglers (and reducing their field by a factor $1/\sqrt{2} = 0.8$ to keep the same $\tau_p$) only decreases $\sigma_x$ by some 4% for $F\{\alpha_p\} \approx 10$. But this would double their cost(!) while such a reduction in $\sigma_x$ can probably be obtained otherwise.

So the choice of $NWL_+$ is not crucial and can be determined by considerations of cost and practicability.

At a given azimuth, $s$, the r.m.s. horizontal beam size is

$$\sigma_x = \sqrt{\epsilon_x \beta_x(s) + \eta_x(s)^2 \sigma_\beta^2}.$$  

Although $\epsilon_x$ is decreased by the wigglers, we must be wary of increasing the beam size too much at the maxima of the dispersion, $\eta_x$, occurring at the horizontally focussing quadrupoles in the arcs.
The formula analogous to (10) which gives the price paid in increased beam size at
azimuth $s$ (optical functions $\eta_x$ and $\beta_x$) for a given $\mathcal{F}\{\alpha_p\}$ is [1]:

$$\mathcal{F}\{\sigma_x\} \simeq \sqrt{1 + \frac{\mathcal{F}\{\alpha_p\} - 1}{1 + \beta_x \langle \Pi \rangle_B / \eta_x^2} \left(1 - \frac{N_W L_+}{16 \pi \rho_0} \right)^{1/3} \left(\mathcal{F}\{\alpha_p\} - 1\right)^{2/3} \left(1 + \frac{1}{r}\right) + \ldots}\right).$$

(12)

Similar arguments apply: we can do essentially nothing to influence the overall factor
and the dependence of the small second term inside the curly brackets on the wiggler
length is very weak.

To find the upper limit on the polarization rate, we take the usual aperture criterion
[9], that $10\sigma_x$ and $10\sigma_z$ should still be contained within the physical aperture.

2.2 Practical design, cost and location of the wigglers

Although both (10) and (12) favour long wigglers, the benefits come very slowly with
rapidly increasing length. However it is clear that there is nothing to be gained by
resorting to high field superconducting wigglers. To keep their length (and cost) rea-
sonable we propose a design [10] using strong conventional magnets which satisfy the
space constraints in the LEP tunnel:

$$\max B_+ \simeq 1.3 \text{T}, \quad L_+ = 0.65 \text{ m}, \quad L_- = 4.0 \text{ m}, \quad L_g = 0.25 \text{ m}. \quad (13)$$

The asymmetry parameter $r = L_- / L_+ = 6.15$.

We find that, at $E_0 = 46.5$ GeV, the aperture is filled with 12 such units, $B_+ = 1.3$ T
and $\tau_p = 36$ min. The total installed length of wigglers is then a manageable 65 m.
Accordingly, all the numerical calculations in this report have been made with $N_W = 12$
and the parameters in (13).

It is estimated [10] that this new system would cost less than 2 MSFR, including the
cost of power converters and cabling.

The possibility of using a pair of the mass-produced LEP dipole cores for the weakly
bending poles of the wigglers may turn out to be a little cheaper [10] and has also been
considered. Parameters for such a system could be:

$$\max B_+ \simeq 1.3 \text{T}, \quad L_+ = 1 \text{ m}, \quad L_- = 11.6 \text{ m}, \quad L_g = 0.5 \text{ m}. \quad (14)$$

With 8 such units we procure the same reduction of polarization time obtained with
the wigglers described above. The 5.75 m long dipole cores cause a larger closed orbit
displacement $x_c \simeq 1.5$ cm at the centre of the wiggler but otherwise our results are
essentially unchanged.

At 46 GeV, some 3 kW of synchrotron radiation power is generated in the wigglers
per mA of beam current so some special cooling of the vacuum chamber may become
necessary if high beam currents are reached.

The wigglers must be installed in straight sections where the dispersion is zero. The
potential radiation problems for the detectors and the space requirements of the RF
systems preclude putting them in the even-numbered straight sections. Therefore they
must be installed in the long straight sections around the odd-numbered pits.

Since the radiative energy losses in these wigglers are a significant fraction of the
total energy loss per turn, $U_0$, they must also be positioned with a certain amount of
symmetry to keep the centre-of-mass energies of $e^+e^-$ collisions equal in all 4 detectors.

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With an eye to possible economies in installation, power supplies and cabling, it seems desirable to install them beside the existing damping wigglers where possible.

In [1] we proposed positioning 3 wigglers on either side of points 3 and 7. A slightly less symmetric arrangement, e.g., 3 on either side of points 5 and 7 would also be acceptable. Wigglers can be installed in the spaces between quadrupoles in the “RF” part of the long straight sections (where these are not already occupied by damping wigglers).

2.3 Beam parameters and performance

Figures 3 and 4 show the actual values of the parameters discussed in Section 2. All the computations have been made over the energy range of 40–50 GeV with constant field \( B_+ = 1.3 \, \text{T} \) in the positively bending central poles of the wigglers. The damping partition numbers have their natural values, \( J_z = 2, J_x = J_y = 1 \).

Figure 5 shows the minimum RF voltage needed to maintain an adequate longitudinal quantum lifetime in the face of the increased radiative energy loss per turn, \( U_0 \), and the enhancement of quantum diffusion, \( \sigma_z \), brought about by the wigglers. The bunch length turns out to be rather constant in this energy range.

Figure 6 was prepared using the standard predictive model for the LEP luminosity due to the new wigglers increases the beam-beam tune-shift for a given stored current. Therefore the stored current at the beam-beam limit is somewhat smaller\(^2\) than the nominal 3 mA. Just above \( E = 46 \, \text{GeV} \), the aperture constraint is violated, not because of a large emittance (as more commonly occurs in this range of beam energies) but

\(^2\)At this point it is worth noting the contrast with the emittance wigglers: these are used to increase the emittance and permit a larger current to be stored.
because of the contribution of the large energy spread. At the maxima of the dispersion function occurring in the arc cells, the notional $10\sigma_a$ exceeds the available physical aperture. To indicate this, the luminosity graph is terminated by the program. To operate at higher energy, the field in the wigglers would have to be reduced.

It goes without saying that these conclusions are only as valid as the details of the luminosity model. If the available aperture turns out to be less in practice then it may be necessary to reduce the wiggler excitation and accept a longer polarization time and smaller luminosity. Conversely, if it turns out to be possible to run with bigger beams, then we could reduce $\tau_p$ still further or have higher luminosity.

### 2.4 Comments on machine performance

In both the performance estimates and polarization simulations we have not availed ourselves of the extra degree of freedom afforded by the variation of damping partition numbers (exchange of total amount of radiation damping between betatron and synchrotron motion, see, e.g., [9]). This involves changing the RF frequency slightly in order to displace the equilibrium orbit and it is not yet clear whether such shifts will be compatible with the stringent requirements of correction schemes for the polarization.

If polarization could be maintained with this technique, then there is good hope that the luminosity can be increased and the depolarization due to the energy spread decreased. In any case, with powerful wigglers, the beam-beam depolarizing effect is reduced in the same proportion as the polarization time (see Section 3.1).

Other likely benefits of these wigglers, applicable even when polarization is not required, are:
• Increased radiation damping between beam-beam interactions; this may help to raise the beam-beam limit on luminosity.

• Increased bunch lengths and radiation damping at injection should help to raise the limit on the number of particles per bunch imposed by the transverse mode-coupling instability. This is believed to be the main performance limitation of LEP.

2.5 Polarization figure of merit

Reducing \( \tau_p \) while increasing \( P_\infty \) has two immediate advantages: First, it reduces the time it takes to maximise the polarization degree in the presence of depolarizing effects which have to be compensated empirically (see Section 3.3). Second, it will substantially improve the polarization figure of merit of a physics run [11,12]:

\[
F = \int_0^{T_{\text{fill}}} P^2(t)L(t) \, dt.
\]

(15)

This improvement will now be substantiated numerically.

Because of depolarizing effects, the asymptotic polarization will diminish and the polarization time will be reduced to an effective value:

\[
P_\infty = \frac{P_{\text{I}}} {1 + \frac{\tau_p}{\tau_d}}, \quad \tau_{\text{p eff}} = \frac{\tau_p}{1 + \frac{\tau_p}{\tau_d}},
\]

(16)

where \( \tau_d \) is the depolarization time.

The quantity \( \tau_p/\tau_d \) measures the strength of depolarizing effects compared to polarizing effects. The effective polarization during a physics run is defined by

\[
\langle \langle P \rangle \rangle = \sqrt{\frac{\int_0^{T_{\text{fill}}} P^2(t)L(t) \, dt}{\int_0^T L(t) \, dt}}.
\]

(17)

To maximise the polarization figure of merit, we need to make some assumptions about the initial luminosity, \( L_{\text{peak}} \), the time it takes to dump, re-inject and accelerate the beams, \( T_{\text{-disable}} \), the luminosity decay time, \( \tau_L \), and the polarization rise-time, \( \tau_{\text{p eff}} \). Given values of these quantities, the polarization figure of merit can be maximised by varying the duration of each fill, \( T_{\text{fill}} \). Naturally, this maximisation is different from the usual maximisation of integrated luminosity.

<table>
<thead>
<tr>
<th>Luminosity maximised</th>
<th>( \tau_p )</th>
<th>( T_{\text{fill}} )</th>
<th>Luminosity loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine without wigglers</td>
<td>310 min</td>
<td>450 min</td>
<td>33 %</td>
</tr>
<tr>
<td>Machine with existing wigglers</td>
<td>90 min</td>
<td>300 min</td>
<td>17 %</td>
</tr>
<tr>
<td>Machine with new wigglers</td>
<td>36 min</td>
<td>220 min</td>
<td>7 %</td>
</tr>
</tbody>
</table>

Table 1: Fill duration and luminosity loss in maximising the polarization figure of merit
Figure 7: Polarization figure of merit, \( F \), and effective polarization as a function of the relative strength of depolarizing effects \( \tau_p/\tau_d \) for different wiggler configurations.

Figure 8: Effective polarization as a function of the relative strength of depolarizing effects \( \tau_p/\tau_d \) for different wiggler configurations.

We have taken the same values:

\[
L_{\text{peak}} = 10^{31} \text{ cm}^{-2} \text{ sec}^{-1}, \quad \tau_{\text{inj}} = 60 \text{ min}, \quad \tau_L = 180 \text{ min},
\]

of these parameters for each of the three cases given in Table 1. This table gives the resulting optimum fill time and the corresponding loss in integrated luminosity. We have not attempted to include the beam-beam depolarizing effect here but we shall argue in Subsection 3.2 that strong wiggles will help to reduce it too.

The shorter the polarization time, the closer the optima for integrated luminosity and polarization figure of merit will be.

The values obtained for the polarization figure of merit of a 100 day long running period and the effective polarization are shown in Figures 7 and 8.

It is clear that reaching an effective polarization \( \langle P \rangle = 50\% \), or a polarization figure of merit of 10 pb\(^{-1}\) in 100 days, is impossible not only for the machine without wiggles but also for the machine equipped with the existing wiggles, no matter how well depolarizing effects are cancelled out.

With the new wiggles, on the contrary, such values are attainable for a strength of depolarizing effects of \( \tau_p/\tau_d \lesssim 0.6 \).

3 Depolarizing effects

Let us now discuss the depolarizing effects in a wiggler-dominated machine. We shall show, rather generally, that photon emission in the wiggles dominates depolarizing effects in the same way that it dominates polarizing effects. Accordingly, the two principal strategies in the battle against depolarizing effects due to machine imperfections—design
choices and empirical correction procedures—are considerably simplified by the localisation of photon emission and the faster response of the machine to corrections. These assertions have been substantiated by first-order simulations.

3.1 The wiggler-dominated machine

The asymptotic polarization degree in the presence of depolarizing effects is [13]

\[ P_\infty = -\frac{8}{5\sqrt{3}} \frac{\langle |\rho^{-3}| \mathbf{\hat{b}} \cdot (\mathbf{n} - \mathbf{\Gamma}) \rangle}{\langle |\rho^{-3}| (1 - \frac{2}{3} (\mathbf{n} \cdot \mathbf{\hat{v}})^2 + \frac{11}{18} |\mathbf{\Gamma}|^2) \rangle}, \]  

where the angle brackets represent integrals around the ring and averages over the phase space of the beam, \( \mathbf{n} \) is the periodic solution of the spin precession equation, \( \mathbf{\hat{b}} \) and \( \mathbf{\hat{v}} \) the unit vectors along the magnetic field and the particle velocity. The spin-orbit coupling vector, \( \mathbf{\Gamma} \), is the derivative of \( \mathbf{n} \) with respect to a fractional change of particle energy due, e.g., to photon emission. If there are no spin-rotators, we can rewrite this formula taking into account the fact that, even in the presence of imperfections, \( \mathbf{n} \) is nearly aligned with \( \mathbf{\hat{b}} \) all around the machine so that \( \mathbf{\hat{b}} \cdot \mathbf{\Gamma} \approx \mathbf{n} \cdot \mathbf{\hat{v}} \approx 0 \). Integrals around the ring can be expressed as a sum with an index \( j \) running over all magnets:

\[ P = -\frac{8}{5\sqrt{3}} \frac{\sum_j |\mathbf{B}_j|^3 L_j}{\sum_j [|\mathbf{B}_j|^3 L_j (1 + \frac{11}{18} |\mathbf{\Gamma}_j|^2)]} = P_0 \frac{\frac{1}{\tau_p}}{\frac{1}{\tau_p} + \frac{1}{\tau_d}}, \]  

where \( P_0 = -8/5\sqrt{3} \) and we have identified the polarization and depolarization rates:

\[ \frac{1}{\tau_p} \propto \sum_j |\mathbf{B}_j|^3 L_j, \quad \frac{1}{\tau_d} \propto \sum_j \frac{11}{18} |\mathbf{B}_j|^3 L_j |\mathbf{\Gamma}_j|^2. \]  

Numerically the wigglers described in (2.2) contribute 18.1 T\(^3\)m to \( \sum_j |\mathbf{B}_j|^3 L \) whereas the whole ring with 18 km of 0.05 T normal bending magnets only contributes 2.37 T\(^3\)m! As we have seen, this reduces the polarization time considerably.

The consequence for depolarizing effects is less obvious but: no less striking: the wigglers are designed to produce negligible perturbation to the orbit, thus no change in the vectors \( \mathbf{n} \) and \( \mathbf{\Gamma} \). Photon emission in an individual magnet \( k \) makes the same contribution to the total depolarization rate but its effect on the polarization level will be:

\[ \frac{\Delta P}{P} \text{ (from magnet } k \text{)} = -\frac{\frac{11}{18} |\mathbf{B}_k|^3 L_k |\mathbf{\Gamma}_k|^2}{\sum_j |\mathbf{B}_j|^3 L}. \]  

**Powerful wigglers actually reduce the loss in polarization due to the \( \mathbf{\Gamma} \)-vector in the same proportion as they reduce the polarization time, i.e., by a factor 8.6 in our case.**

Very similar arguments apply to other depolarizing effects such as the beam-beam interaction and the Sokolov-Ternov effect in spin rotators (via the \( \mathbf{\hat{b}} \cdot \mathbf{n} \) term in (19)).

Depolarizing effects coming from photon emission in the wigglers themselves dominate with a relative weight

\[ \frac{\sum_{\text{wigglers}} |\mathbf{B}|^3}{\sum_{\text{wigglers}} |\mathbf{B}|^3 + \sum_j |\mathbf{B}_j|^3 L(\text{ring})} = 88\%. \]  

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To eliminate these, we must make \( \Gamma \) vanish at the location of the wigglers. Since \( \Gamma \) is a derivative of the unit vector \( n \), this represents 2 conditions per wiggler magnet. Since the wigglers are clustered in essentially two locations we can expect that 4 tunings will be largely sufficient. The magnitude of this task is considerably less than that of making \( \Gamma \) vanish at every magnet. It may even be somewhat simpler than harmonic resonance compensation.

3.2 Spin-matching of the wigglers

Making the spin-orbit coupling \( \Gamma \) vanish at its location is called spin-matching a magnet. Sufficient conditions for spin-matching at azimuth \( s \) are obtained by decomposing \( \Gamma \) into contributions arising from the 3 normal modes of orbital motion:

\[
\Gamma(s) = 0 \iff \begin{cases} 
J_{\pm x}(s) = 0 \quad \text{or} \quad \eta_{x}(s) = \eta'_{x}(s) = 0, \\
J_{\pm y}(s) = 0 \quad \text{or} \quad \eta_{y}(s) = \eta'_{y}(s) = 0, \\
J_{z} = 0,
\end{cases}
\] (24)

where the spin-orbit coupling integrals are defined by

\[
J_{\pm x}(s) = \int_{s}^{s+C} (m(s') + il(s')) \cdot e_{x}K(s')/\sqrt{\beta_{x}(s')} \exp(\pm i\mu_{x}(s')) \, ds',
\] (25)

\[
J_{\pm y}(s) = \int_{s}^{s+C} (m(s') + il(s')) \cdot e_{y}K(s')/\sqrt{\beta_{y}(s')} \exp(\pm i\mu_{y}(s')) \, ds',
\] (26)

\[
J_{z}(s) = \int_{s}^{s+C} (m(s') + il(s')) \cdot (\eta_{x}e_{y} + \eta_{y}e_{x})K(s') \, ds',
\] (27)

where \( K \) is the local focussing strength, \( e_{x}, e_{y}, e_{z} \) and \( m \) and \( l \), the spin reference vectors, perpendicular to \( n \) and precessing around \( n \) with the spin precession frequency.

Since the wigglers have \( K = 0 \) they do not contribute to the spin orbit coupling integrals for the other magnets of the ring. Most of the depolarization coming from photon emission in the latter will be suppressed.

In a perfect, flat machine, \( n \equiv e_{y} \) and both \( \eta_{y} \) and \( \eta'_{y} \) vanish so that \( J_{x} = J_{z} = 0 \). Although the integrals \( J_{y} \) are not small their contributions to the depolarization are multiplied by \( \eta_{y} \) or \( \eta'_{y} \) which are zero.

In an imperfect machine, depolarization occurs because

(i) the vector \( n \) is not parallel to \( e_{y} \), but makes a typical angle \( \delta n \) with it;

(ii) parasitic vertical and horizontal dispersions of typical size \( \delta \eta \) are generated by imperfections.

Realistic values for LEP are \( \delta n \simeq 10 \text{ mrad} \) and \( \delta \eta \simeq 5 \text{ cm} \).

The depolarizing effects due to excitation of the three normal modes by radiation in the wigglers will then scale as:

- horizontal betatron: \( |\Gamma_{x}|^2 \propto (\eta_{x}^W + \delta \eta)^2 \delta n^2 \),
- vertical betatron: \( |\Gamma_{y}|^2 \propto \delta \eta^2 \),
- synchrotron: \( |\Gamma_{z}|^2 \propto A\delta \eta^2 + B\delta n^2 \).

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Here $\eta_z^W$ denotes the design horizontal dispersion in the wigglers. Having made $\eta_z^W = 0$ by design (by installing the wiggler in the straight sections), we reduce the effect of horizontal betatron oscillations to $(\delta\eta)^2(\delta n)^2$ which is fourth order in the imperfections and totally negligible in practice. We still have to correct empirically the parasitic vertical dispersion and the depolarization arising from synchrotron oscillations.

### 3.3 Empirical spin-matching

We have simulated the LEP lattice in first-order with the program SLIM [14]. The machine simulated was flat with no experimental solenoid, skew-quadrupole or spin rotator. Defects were generated and corrected according to the method described in [15]. At first, no spin correction procedure was attempted, and results with and without the wiggler were compared. The components of depolarization corresponding to horizontal and vertical betatron oscillations and synchrotron oscillations were isolated and their dependence on the beam energy is shown in Figures 9–11; the net polarization is shown in Figure 12.
Figure 11: Contribution to depolarization from synchrotron oscillations with and without the wigglers (evaluated to 1st order with SLIM, before empirical spin-matching).

Figure 12: Polarization degree with and without the wigglers (evaluated to 1st order with SLIM, before empirical spin-matching).

Figure 13: Layout of the vertical bumps separated by 90° spin phase-advance at one side of the machine.

Figure 14: The four bumps used to maximise the polarization level.
The following conclusions can be drawn:

(i) As expected from our discussion above, the relative effect of horizontal betatron oscillations is reduced by the same amount as the polarization time. The depolarization due to horizontal excitations generated in the wigglers themselves is typically \( < 10^{-5} \) and totally negligible.

(ii) The energy-dependence of the depolarization from vertical oscillations is dominated by the systematic resonances, \( \nu = 4k \pm Q_y \) but, for reasons which remain to be understood, the contributions with and without wiggles behave quite differently. By a happy coincidence, the wiggles suppress this depolarizing effect at the energy corresponding to the expected mass of the \( Z \) peak, \( E = 46 \text{ GeV} \) or \( \nu = 104.5 \pm 1.5 \).

(iii) Synchrotron oscillations are therefore the dominant source of depolarization, at least in the energy range around 46 GeV. This remains true close to half-integer spin tunes where the depolarization is least. Energy oscillations generate synchrotron sidebands of the integer resonances but the relative strength of this source is not substantially modified by the wiggles. This indicates that the component \( \Gamma_z \) is of similar magnitude in the wiggles and in the rest of the ring. It is the dominant source of depolarization to be cancelled empirically.

Empirical spin matching consists in maximising the polarization degree with orbit correctors. This process has been simulated with four vertical \( \pi \)-bumps situated as shown in Figures 13 and 14. By another fortunate coincidence, these correctors turn out to be uncorrelated in the sense that the best setting for any one of them is found to be independent of the settings of the others.

The results of the simulations of empirical spin-matching are described in more detail in [16]. A correction of this kind, performed at \( \nu = 104.5 \), yields a polarization degree of 80–85%. The depolarization by photon emission in the wiggles is almost perfectly cancelled; all residual depolarizing effects come from photon emission in the arcs.

The correction process terminates in 20 steps and, with \( \tau_p = 36 \text{ min} \), should take less than 500 minutes on the real machine. This time could probably be reduced by more astute optimum-searching techniques and further corrections can be devised to improve the polarization degree even more.

Carrying out the same procedure on the machine without wiggles yields a polarization level of only 50% for settings of the correctors very close to those attained with the wiggles on. This is compatible with previous studies [17]. But, with \( \tau_p = 300 \text{ min} \), the correction process would take three days—utterly impractical on the real LEP.

Our interpretation in terms of dominance of the integer resonances is confirmed by the fact that the corrector settings are close to those obtained when we tried to perform harmonic spin-matching [18] by cancelling out the two closest integer resonances.

Finally, then, we can summarise our understanding of the improvement brought about by the wiggles:

*Most (88 %) of the depolarizing effects are concentrated in the wiggles. Because the wiggles are positioned in straight sections, the depolarization coming from horizontal and vertical betatron oscillations is extremely small by design and the longitudinal component can be made to vanish with very few (4) correctors. Moreover, the quantum excitation of synchrotron oscillations is also the dominant component of depolarization in the arcs of the machine.*
We conclude that wigglers installed in straight sections allow a high degree of polarization to be reached with knobs that not only are fewer but also respond much more quickly.

3.4 Comments on the effect of energy spread

The simulations described in the previous section were all performed in the well understood first order approximation. Higher order resonances are likely to appear, especially synchrotron satellites of the integer resonances. The evaluation of such effects, particularly for high energy machines like LEP and HERA is notoriously difficult. Despite many efforts, neither their theoretical understanding nor their computation has been brought into a form which would permit practical predictions of the polarization level to be made. However some recent estimates of their importance are given in other contributions to this report [18,19,20].

4 Conclusions

- If they were available in time, these new wigglers would greatly reduce the time required for the early transverse polarization studies and may help to produce a higher level of transverse polarization. Indeed, these wigglers could well make the difference between the practicability and impracticability of the transverse polarization programme. So the timing of the sequence of events leading up to their installation would have a significant bearing on the evolution of the programme of development of polarized beams.

- The new wigglers are essential for experimentation with longitudinally polarized beams.

- From present knowledge, it appears that no significantly better set of dedicated polarization wigglers can be envisaged.

- The wigglers inevitably cause a large increase in energy spread whose effect on the final polarization level remains difficult to quantify but, if an overall $\tau_p/\tau_d \simeq 0.6$, i.e. $P_\infty \simeq 50-60\%$ can be achieved, then the shorter polarization time leads to a much higher polarization figure of merit.

Acknowledgements: We thank T.M. Taylor for advice concerning possible designs and cost of wiggler magnets. We are also indebted to D.P. Barber, J. Buon, E. Keil, J.-P. Koutchouk and other colleagues in the Polarization Working Group, for numerous helpful discussions.
References


[18] T. Limberg and J.-P. Koutchouk, contribution to this report.


## A  List of symbols

\[ E_0 = mc^2 \gamma = \sqrt{p_0^2c^2 + m_e^2c^4} \]  

beam energy

\[ C = 2\pi R \]  
circumference of machine

\[ s \]  
machine azimuth \((0 \leq s < C)\)

\[ e, m_e, r_e \]  
electron charge, mass, classical radius

\[ P(t) \]  
polarization degree at time \(t\)

\[ P_\infty \]  
asymptotic polarization degree

\[ P_\infty^I \]  
asymptotic polarization degree in ideal machine

\[ \tau_p = 1/\alpha_p \]  
Sokolov-Ternov polarization time

\[ \tau_d \]  
de polarization time

\[ \tau_{p\text{eff}} \]  
effective polarization time, see (16)

\[ B_0 \]  
bending field in dipoles of LEP arcs

\[ 1/\rho_0 = eB_0/p_0c \]  
bending strength in normal LEP dipole

\[ K(s) \]  
quadrapole focussing strength

\[ N_W \]  
number of wiggler units

\[ L_+ \]  
length of + block of one wiggler unit

\[ L_- \]  
combined length of 2 – blocks of one wiggler unit

\[ L_g \]  
length of gaps between wiggler poles

\[ B_\perp \]  
field in centre pole of wiggler

\[ r = L_-/L_+ = B_+/B_- \]  
asymmetry parameter of wiggler

\[ \langle H \rangle_B, \langle H \rangle_W \]  
usual quadratic form related to dispersion

\[ J_z, J_y, J_e \]  
radial, vertical, longitudinal damping partition numbers

\[ \epsilon_z \]  
total horizontal emittance (no coupling)

\[ \sigma_z, \sigma_e \]  
r.m.s. fractional energy deviation

\[ \sigma_\perp \]  
r.m.s. radial beam size

\[ \eta_z, \eta_y, \eta_e \]  
radial, vertical dispersion function

\[ \eta_z^W \]  
design dispersion at wiggler

\[ \nu = \gamma a, a = (g - 2)/2 \]  
spin tune, gyromagnetic anomaly of the electron

\[ \sigma_z \equiv z_{\text{rms}} \]  
r.m.s. bunch length

\[ U_0 \]  
total radiation energy loss per turn

\[ V_{\text{RF}} \]  
total peak RF voltage

\[ I \]  
total beam current

\[ L(t) \]  
luminosity at time \(t\)

\[ L_{\text{Peak}} \]  
peak luminosity at start of fill

\[ T_{\text{ini}} \]  
time to refill LEP

\[ \tau_L \]  
luminosity decay time

\[ T_{\text{fil}} \]  
duration of a fill

\[ F \]  
polarization figure of merit

\[ Q_z, Q_y, Q_s \]  
betatron and synchrotron tunes

\[ \tau_p \]  
Sokolov-Ternov polarization time

\[ \tau_d \]  
de polarization time

\[ F\{A\} \]  
factor by which wigglers increase the beam parameter \(A\)

\[ \Gamma = \partial n/\partial \epsilon \]  
spin-orbit coupling vector

\[ J_{\pm x, \pm y, z} \]  
(complex) spin-matching integrals
CALCULATION OF HIGHER ORDER SPIN RESONANCES

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ABSTRACT
We calculate higher order resonances for various high-energy
electron storage ring models, using a computer program called
SMILE. The results are used to elucidate aspects of the behav-
ior of the polarization, such as the effects of lattice super-
periodicity and beam energy spread, and we present some
results for LEP. We also discuss how to use SMILE in conjunc-
tion with other programs to obtain practical results for LEP. We
describe some analytical constraints on the behavior of spin
resonances, and show that SMILE satisfies these constraints.
Reasonable agreement is also obtained with some experimental
data from measurements at SPEAR.

1. INTRODUCTION
The chief goal of polarization calculations is to calculate the equilibrium polar-
ization, including spin resonances of arbitrary order, at least in principle. A formula
for the equilibrium polarization in an electron storage ring was given in 1973 by Der-
benev and Kondratenko[1], and rederived in Ref. [2]. However, no practical algorithm
was given in Ref. [1] for evaluating the polarization formula for a given storage ring
model. Such an algorithm was developed in Ref. [3], and a computer program called
SMILE [3] has been written to to calculate spin resonances to arbitrary orders, in the
approximation of treating only linear orbital dynamics.

Here we present results of polarization calculations for storage ring models, to
elucidate various aspects of the behavior of the polarization. We consider such effects
as the lattice superperiodicity and the beam energy spread, and present some results
for LEP. We also describe some analytical constraints on resonance behavior, and
show that SMILE satisfies these constraints. We fit some experimental data from
measurements at SPEAR [4], and obtain reasonable agreement. Finally, we discuss
how SMILE may be used to obtain useful practical results for LEP in conjunction with
other computer programs.

Various topics are beyond the scope of this report. In particular, SMILE is not,
in its present form, a fitting program. Thus, it does not perform closed orbit correc-
tions or spin matching. SMILE also assumes the beam is in equilibrium. It does not
calculate the time evolution of the orbital and/or spin distribution, e.g. by tracking of
a set of electrons.

2. GENERAL REMARKS

Before discussing the equilibrium spin distribution, let us briefly review the cal-
culation of the orbital equilibrium, i.e. the orbital emittances. The procedure is well
known: one first diagonalizes the Hamiltonian, i.e. one finds the betatron and syn-
chrotron amplitudes and phases. One then considers the effect of a single photon
emission on the action-angle variables, averages over photon emissions and particle
trajectories, and derives the phenomena of radiation damping and stochastic excita-
tion. The emittances are then obtained by demanding balance between these effects.

The calculation of the equilibrium polarization, which is the emittance of the
spin distribution of the beam, proceeds along similar lines. We first determine the spin
action-angle variables (in classical language), or the spin eigenstates of the Hamiltonian,
in quantum language. This is accomplished by finding the quantization axis, say \( \hat{n} \),
such that the operator \( \vec{\mathcal{J}} \cdot \hat{n} \) commutes with the Hamiltonian, just as for the orbital
motion we want the Poisson Bracket \( \{ I, \mathcal{H} \} \) to vanish. The technical details can be
found in Refs. [1,2]. The formal solution for \( \hat{n} \) was given by Derbenev and Kondratenko
[1,5], and a practical algorithm to evaluate it for a given storage ring model was given
in Ref. [3]. The axis \( \hat{n} \) depends on the particle orbit, \( \hat{n} = \hat{n}(I, \psi, \theta) \), and satisfies the
Thomas-BMT equation

\[
\frac{d\hat{n}}{d\theta} = \vec{\Omega}(I, \psi, \theta) \times \hat{n} ,
\]

and the periodicity conditions [5,6]

\[
\hat{n}(I, \psi, \theta) = \hat{n}(I, \psi + 2\pi, \theta) = \hat{n}(I, \psi, \theta + 2\pi) ,
\]

where \( \{ I, \psi \} \) denote the orbital action-angle variables, and \( \vec{\Omega} \) is the spin precession
vector. Because there is a distribution of vectors \( \hat{n} \), the full equilibrium polarization
vector \( \vec{P}_{eq} \) is given by (see Ref. [2] for details)

\[
\vec{P}_{eq} = \langle \vec{\mathcal{J}} \cdot \hat{n} \rangle (\hat{n}) .
\]

The factor \( \langle \vec{\mathcal{J}} \cdot \hat{n} \rangle \) is determined by balance between spin-flip transition rates. It contains
contributions not only from the recoil of an electron due to energy loss during photon
emission, but also from transverse momentum recoils. The latter effects have been
studied in Refs. [7–9], and it has been shown that their numerical contribution for real
storage rings is negligible [10], including HERA [11]. They will be ignored below. It
has also been shown that the contribution of the \( \langle \hat{n} \rangle \) term is negligible [12], and so it
will also be ignored below. This leads us to the Derbenev-Kondratenko formula [1],
which gives the value of \( \langle \hat{s}.\hat{n} \rangle \) by considering electron recoils due to energy loss only. It is adequate for practical purposes. The formula is

\[
P_{eq} = \frac{8}{5\sqrt{3}} \left\langle \left| \rho \right|^{-3} \hat{b} \left[ \hat{n} - \gamma \frac{\partial \hat{n}}{\partial \gamma} \right] \right\rangle .
\]

Here \( \hat{v} \) is the particle velocity, \( \gamma \) is the particle energy in units of rest mass energy, \( \hat{b} \equiv \hat{v} \times \hat{v}'/|\hat{v} \times \hat{v}'| \), \( \hat{n} \) is the spin quantization axis and \( \rho \) is the local radius of curvature of the particle trajectory. The angular brackets denote an equilibrium ensemble average over the distribution of particle orbits and the ring azimuth. The corresponding polarization build-up time, \( \tau_{pol} \), is given by

\[
\tau_{pol}^{-1} = \frac{5\sqrt{3} e^2 \hbar \gamma}{8 m^2 c^2} \left\langle \frac{1}{|\rho|^3} \left[ 1 - \frac{2}{9} (\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \gamma} \right|^2 \right] \right\rangle,
\]

where \( m \) and \( e \) are the particle mass and charge, respectively.

The computer program SMILE calculates \( \hat{n} \) and \( \gamma (\partial \hat{n} / \partial \gamma) \) in the approximation of linear orbital dynamics [3]. The program uses a recursive algorithm, and is therefore able to calculate the polarization up to arbitrary orders of spin resonances without requiring new code to be written for each new order. The formalism uses perturbation theory, and the perturbation expansion parameters are the orbital amplitudes, which are all treated on the same footing. The user can instruct the program to calculate to different orders in the various modes; thus one can, for example, calculate to higher order in the synchrotron oscillations than in the betatron oscillations, if desired. The program can also accept storage ring lattices of arbitrary geometry, including magnet misalignments and spin rotators (or other non-planar lattices). It uses a fully symplectic six-dimensional formalism to calculate the orbital motion. Overlapping spin resonances can be calculated without requiring any special treatment. The beam energy spread, as well as the transverse beam sizes, are taken into account when calculating the polarization. In the results presented below, it is assumed that the orbital distribution is Gaussian in all planes. If desired, a more general orbital distribution can be implemented into the code.

3. PERFECTLY ALIGNED RING

In all the results that appear below, the symbol \( \nu \) denotes the spin tune, \( a = (g - 2)/2 \), while the orbital tunes are \( Q_x, Q_y \) and \( Q_z \), in standard notation. For a perfectly aligned planar ring, \( \nu = a \gamma = E(\text{GeV})/440652 \). In this section we consider a perfectly aligned storage ring. Figure 1 shows the polarization for a two-fold symmetric machine with only horizontal bends, vertical bends and quadrupoles in the lattice.
The orbital tunes are $Q_z = 2.552$, $Q_x = 0.715$ and $Q_y = 0.0617$. The calculation is to second order in all orbital modes. The first order resonances $\nu = Q_z$, $4 - Q_z$, $\nu = 2 + Q_z$, $4 - Q_z$ and $\nu = 2 + Q_z$, $4 - Q_z$ are visible, but because the lattice superperiodicity equals 2, the odd harmonics $\nu = 3 \pm Q_z$, $\nu = 3 \pm Q_z$, etc. are absent. Similarly, one can verify that only even harmonics of the higher order resonances appear.

![Graph](image)

**Fig. 1** Polarization vs. $\alpha\gamma$ for a perfectly aligned ring with superperiodicity= 2. All resonances with odd harmonics vanish.

4. **ANALYTICAL FORMULAS FOR RESONANCE WIDTHS**

One of the principal difficulties in obtaining quantitative results pertaining to the polarization is that the resonances vanish for a perfectly aligned planar ring, and almost all ring designs are planar, including LEP. Hence to obtain nonzero spin resonances, say in a model of LEP, one must introduce imperfections in the lattice, and this requires the use of random numbers, hence a statistical analysis. In this section we describe analytical results which do not depend on the the imperfections, and so can serve as the basis of quantitatively precise tests of the accuracy of numerical calculations.

Although the absolute resonance widths depend on imperfections, the ratios of resonance widths are, in certain cases, independent of the imperfections, and can be calculated from the ideal lattice alone. This applies in particular to the synchrotron sideband resonances of a first order betatron spin resonance. A formula giving the ratios of the synchrotron sideband resonance widths to the width of a first order betatron resonance was given by Yokoya [13], and a more detailed one in Ref. [14], where it was also shown that Yokoya's formalism and the SMILE algorithm are equivalent, in the domain where both formalisms are applicable.

From Ref. [14], the ratio $R$ of the width of the sideband resonance $\nu = k + Q_z, z + mQ_y$ to that of the first order resonance $\nu = k + Q_z, z$ is $R = W_m/W_0$, where
\[ W_{m}^{2} = \left[ 1 + \frac{J_s}{J_{x,z}} \left( mI + \alpha \frac{I[m+1(\alpha)]}{I[m](\alpha)} \right) \right] e^{-\alpha I[m](\alpha)} \quad (\alpha = \frac{\sigma_{e}^{2}(\gamma a)^{2}}{Q_{s}^{2}}) \quad (6) \]

\[ R \approx \left[ \left( 1 + \frac{J_s}{J_{x,z}} \right) \frac{\alpha}{2} \right]^{1/2} \quad (m = 1, \alpha \ll 1), \quad (7) \]

where \( \sigma_{e} \) is the energy spread and the \( J \)'s are damping partition numbers. The \( I \)'s are modified Bessel functions.

In Fig. 2 we show the width of the second order resonances \( \nu = Q_{x} + Q_{s} \) and \( \nu = Q_{z} + Q_{s} \) as a function of the width of the first order resonance \( \nu = Q_{s} \) for two sets of imperfections (crosses and circles) for a fifteen-fold symmetric planar ring with \( Q_{z} = 2.24 \), \( Q_{s} = 7.595 \) and \( Q_{s} = 0.041 \) — note that SMILE is not limited to synchrotron sideband resonances only. We shall discuss the latter resonance later in this section. We see that both graphs are straight lines through the origin, and the slope is independent of the imperfections. The slope of the graph, for the synchrotron sideband resonance, from Eq. (7) and Fig. 2, is

\[ R = 0.240 \quad (\text{Eq. (7)}), \quad R = 0.237 \quad (\text{Fig. 2}). \quad (8) \]

The values agree closely. In Fig. 3 we show the actual polarization curves for one set of resonances. Note that the linearity in Fig. 2 holds even when the resonances nearly overlap.

In Table 1 we present the value of \( R \) for several orders of synchrotron sidebands for LEP at 50 GeV (\( \alpha \approx 0.8 \) at 50 GeV). As expected, the ratios are larger than the lower energy model studied above.

### Table 1

Calculated values of \( R \) for resonances \( \nu = k \pm Q_{x,z} \pm mQ_{s} \) for LEP at 50 GeV.

<table>
<thead>
<tr>
<th>( m )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>0.51</td>
</tr>
<tr>
<td>4</td>
<td>0.36</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Fig. 2 Higher order resonance widths (denoted $W_1$) vs. first order resonance width ($W_0$) in arbitrary units, for two sets of imperfections.

Fig. 3 Polarization vs. $a \gamma$ curves for one set of imperfections in Fig. 2. The linearity in Fig. 2 works even for nearly overlapping resonances.

For higher order betatron resonances such as $\nu + k + Q_z + Q_z = 0$, the ratio to the width of the resonance $\nu + k' + Q_z = 0$ is also independent of imperfections, but the formula is not as simple as for synchrotron sideband resonances, where the formula for $R$ depends only on the energy spread, spin tune, synchrotron tune and damping partition numbers. We obtain the ratio by the following calculation. We decompose $\vec{\Omega}$ in Eq. (1) into $\vec{\Omega} = \vec{\Omega}_0 + \vec{\omega}$, where $\vec{\Omega}_0$ is the value on the closed orbit, and $\vec{\omega}$ contains the coupling to betatron and synchrotron oscillations. We then decompose

$$\vec{\omega} = a_{xz} \vec{\omega}_z + a_{xz} \vec{\omega}_x + a_{z} \vec{\omega}_z + \text{c.c.}$$

where $a_{j} = I_j^\frac{1}{2} e^{i(\theta_j - Q_j \theta)} / \sqrt{2}$, with an obvious notation. The widths of the above resonances are then determined by the integrals [3]

$$d_1 = -i \gamma \frac{\partial a_{x}}{\partial \gamma} \int_{-\infty}^{\theta} d\theta' \vec{\omega}_x \cdot \vec{k}_0$$

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\[ d_2 = \left( \gamma \frac{\partial a_z}{\partial \gamma} a_z + a_z \gamma \frac{\partial a_z}{\partial \gamma} \right) \int_{-\infty}^{\theta} d\theta' \tilde{\omega}_z \cdot \hat{n}_0 \int_{-\infty}^{\theta'} d\theta'' \tilde{\omega}_z \cdot \vec{k}_0, \]  

(10)

where \( \vec{k}_0 = \hat{l}_0 + i\hat{n}_0 \) and \( \{\hat{l}_0, \hat{n}_0, \hat{\nu}_0\} \) is a right-handed basis of orthonormal solutions of Eq. (1) on the closed orbit, with \( \hat{n}_0 \) as the periodic solution. Then we decompose into Fourier harmonics,

\[ \tilde{\omega}_z \cdot \vec{k}_0 = \sum_j A_j e^{i(j+\nu+Q_z)\theta} \]

\[ \tilde{\omega}_z \cdot \hat{n}_0 = \sum_j B_j e^{i(j+Q_z)\theta} \]  

(11)

and obtain

\[ d_1 \propto \sum_j \frac{A_j e^{i(j+\nu+Q_z)\theta}}{j + \nu + Q_z} \]

\[ d_2 \propto \sum_{j,j'} \frac{B_j A_{j'} e^{i(j+j'+\nu+Q_z+Q_{z'})\theta}}{(j+j'+\nu+Q_z+Q_{z'})(j'+\nu+Q_z)}. \]  

(12)

For the resonances mentioned above, we select \( j = k' \) in \( d_1 \) and \( j + j' = k \) in \( d_2 \). Assuming further that the bending is smooth and uniform around the ring, it can then be shown that the leading contribution to the ratio of resonance widths, \( R \), is given by

\[ R \simeq \left[ \frac{1}{2} \left( 1 + \frac{J_x}{J_z} \right) \frac{\epsilon_{\pi\beta}}{|A_k|^2} \sum_j \frac{|B_j A_{k-j}|^2}{(j+Q_z)^2} \right]^{1/2}. \]  

(13)

In the case of synchrotron sidebands, only the \( j = 0 \) term is nonnegligible, because \( Q_z \ll 1, B_0 = \gamma a, \epsilon_{\pi\beta} \rightarrow \sigma^2 \) and, putting \( k = k' \), we recover Eq. (7) for \( \alpha \ll 1 \), but for betatron sidebands we cannot make this simplification. To evaluate \( A_k \) and \( B_k \) we must know the LEP lattice, hence we shall not present a table of values for \( R \) here.

5. COMPARISON WITH SPEAR DATA

The above calculations can be applied to polarization measurements at SPEAR (Ref. [4] and Fig. 4). The synchrotron sidebands \( \nu = 3 + Q_z \pm Q_z \) and \( \nu = 3 + Q_z \pm 2Q_z \) of the horizontal betatron resonance \( \nu = 3 + Q_z \) are clearly visible. We find that, for the resonance \( \nu = 3 + Q_z - Q_z \),

\[ R = 0.23 - 0.37 \quad (\text{Fig. 4}), \quad R = 0.20 \quad (\text{Eq. (7)}). \]  

(14)

A fit to the SPEAR measurements using SMILE is shown in Fig. 5. (SMILE also contains additional less singular spin integrals which have not been included in Eq.
The width of the first order resonance \( \nu = 3 + Q \) is fitted to the data, and the widths of the sidebands then follow without further adjustment of parameters. The agreement appears to be satisfactory.

![Graph showing polarization measurements from SPEAR.](image)

**Fig. 4** Polarization measurements from SPEAR (from Ref. [4]).

![Graph showing theoretical fit of synchrotron sideband resonances.](image)

**Fig. 5** Theoretical fit of the synchrotron sideband resonances in Fig. 4 using SMILE.

6. **USE OF SMILE FOR LEP**

6.1 **General**

In this section, we shall discuss what the above results have taught us, and how SMILE may be used in conjunction with other programs in a procedure to calculate LEP polarization. The above results do in fact contain important information as to how to estimate absolute higher order resonance widths and to perform spin matching of higher order resonances.
6.2 Calculation of absolute resonance widths

We have noted that the spin resonances vanish in a perfectly aligned planar ring. This is because the first order spin integrals are of the form

\[ a_z \int_{-\infty}^{\theta} \bar{\omega}_z \cdot \bar{k}_0 \, d\theta' + a_z \int_{-\infty}^{\theta} \bar{\omega}_\perp \cdot \bar{k}_0 \, d\theta' + a_s \int_{-\infty}^{\theta} \bar{\omega}_s \cdot \bar{k}_0 \, d\theta' , \]

and for a planar ring, \( \bar{\omega}_z \cdot \bar{k}_0 = \bar{\omega}_\perp \cdot \bar{k}_0 = 0 \) and \( \langle |a_z|^2 \rangle = 0 \), and the higher order spin integrals are proportional to the first order ones. We have seen, however, that the ratio of the higher order integrals to the first order ones is independent of the imperfections, and depends on the ideal lattice alone. It has been verified that SMILE satisfies this analytical constraint on the spin integrals, the same ratio of resonance widths being obtained for different sets of imperfections, and that (for synchrotron sidebands) the ratio agrees with analytical calculations, and with polarization measurements at SPEAR [4].

Consequently, one need only perform a sophisticated statistical estimation of the absolute widths of the first order resonances. The widths of the higher order resonances can then be obtained by multiplying by the appropriate ratios. Note that this can be done even if the resonances overlap. We simply extrapolate the magnitudes of the spin integrals from a regime where the resonances do not overlap. The ratios can be calculated using SMILE, with just one set of imperfections (two to check for consistency). Strictly speaking, the above statements apply to the leading contributions to the resonances. There are corrections from less singular spin integrals, e.g. those proportional to the square of the closed orbit distortion, whose magnitudes also grow with increasing resonance strength. However, we can still expect expect an approximate linear relationship.

6.3 Spin matching of higher order resonances

We can also see how to extend the concept of spin matching beyond first order. For each resonance one wishes to eliminate, one determines the relevant integral over the optics which gives the ratio to the parent first order resonance. This integral is then made to vanish, by adjusting the lattice; this is the desired spin match. Since we are spin matching specific harmonics, this technique can be considered a “harmonic spin matching” procedure.

Of course, since the higher order integrals are proportional to the first order ones, a perfect spin match of the first order spin integrals will automatically eliminate all the higher order resonances. This point should not be forgotten — the first order spin match is the most important.
7. CONCLUSIONS

We have shown examples of polarization calculations of higher order spin resonances, up to fourth order, and have used them both to elucidate the behaviour of the polarization and to understand how to use SMILE as a practical tool. In particular, we have shown that the ratios of higher order resonances widths to the first order resonance widths are independent of the lattice imperfections, for synchrotron sidebands of a betatron resonance $\nu = k + Q_{z,z} + mQ_{z}$, and $z - z$ resonances $\nu = k \pm Q_{z} \pm Q_{z}$.

We have also shown that the SMILE results agree with analytical calculations and experimental measurements, for synchrotron sideband resonances, and we have presented some results for LEP. Finally, we have discussed how SMILE may be used in conjunction with other programs as part of a working procedure to calculate the polarization in LEP, in particular to calculate the widths of higher order spin resonances and, if desired, to perform higher order spin matching.

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REFERENCES


DEPOLARIZATION ENHANCEMENT DUE TO THE ENERGY SPREAD

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ABSTRACT
A new semiclassical and stochastic model of spin diffusion is used to obtain numerical predictions for depolarization enhancement due to beam energy spread. It confirms the results of previous models. For LEP, at $Z_0$ energy, the enhancement is of the order of a few units and increases very rapidly with the energy spread as it would be the case if wigglers would be used. This effect makes more difficult to obtain a high degree of polarization.

1. INTRODUCTION

The depolarization enhancement due to the beam energy spread is the main worry about the possibility to obtain a high degree of polarization in very high-energy electron storage rings like LEP.

Beam energy spread increases quadratically with energy and reaches a r.m.s. value of about 33 MeV at 46.5 GeV. On the other hand, the spacing between spin resonances stays constant with energy and each particular type of resonance occurs repeatedly with a 440 MeV separation. The distance from the operating energy to the nearest resonance can never be larger than about 100 MeV.

Electrons with sufficiently large energy deviation, i.e. with large synchrotron amplitude, may then approach a resonance energy, even if the mean energy of the beam is set as far as possible from nearby spin resonances. Larger depolarization than expected in a linear theory of small-amplitude oscillations can then occur.

However, the picture of a particle approaching a resonance is not very consistent as it mixes the time domain and the frequency domain. A more correct method to study the effect of energy spread is to consider the frequency modulation of spin precession produced by the synchrotron oscillations, as the precession frequency, measured by the spin tune, is proportional to the particle energy. This effect is similar to the well-known frequency modulation in RF waves. It leads to the appearance of satellite sidebands about the central line in the frequency spectrum. Similarly, for the spin motion, synchrotron satellites are generated about every spin resonance. These satellite resonances are regularly spaced by the synchrotron
tune $Q_s$. When the energy spread is sufficiently large, involving large synchrotron amplitudes, these satellites are wide and strong. Depolarization is enhanced everywhere in energy.

Already, at 3.7 GeV, in SPEAR, synchrotron satellites of the radial betatron resonance, at a spin tune $v = 3 + Q_x$, have been experimentally observed [1] (Fig. 1). The second-order and third-order satellites, $v = 3 + Q_x \pm Q_s$ and $v = 3 + Q_x \pm 2Q_s$, respectively, are clearly seen on figure 1, although the r.m.s. value of the energy spread is only about 3 MeV.

![Graph showing polarization versus beam energy]

Fig. 1 Measured relative polarization ($P_0 = 92.4\%$) versus beam energy in SPEAR. Resonant spin tune values are indicated above the figure, in particular the synchrotron satellites of the $v = 3 + Q_x$ resonance.

A quantitative estimate of the depolarization enhancement was first obtained by Ya. Derbenev and A. Kondratenko [2]. Their result has been rederived by C. Biscari et al. [3] for synchrotron resonances and by K. Yokoya [4] for synchrotron and betatron resonances. More recently, S. Mane [5] has found some corrections to apply to the Yokoya calculation for a betatron resonance. All these semiclassical models tend to calculate, by a perturbation method, incorporating the higher-order effects of energy spread, the spin deviation $\gamma \delta n/\partial \gamma$ from the equilibrium direction $\bar{n}$ following an energy jump $\delta \gamma$ when a photon is emitted by an electron. A statistical average, over photon emissions and eventually over amplitudes of particle oscillations, allows to derive the reduction in equilibrium polarization. For LEP at the $Z_0$ energy, with $0.7 \times 10^{-3}$ natural beam energy spread, they have indicated depolarization enhancements of the order of a few units, that increase very rapidly with the energy spread.
The author [6] has recently developed a new semiclassical and stochastic model of spin diffusion and depolarization enhancement that takes care of time correlation effects of successive photon emissions. This correlation plays a role since the particle oscillation amplitude and the spin perturbation result from many preceding photon emissions, and since spin motion is not the linear superimposition of individual perturbations when high-order effects are considered. The numerical predictions of this model, presented here, confirm the order of magnitude of the depolarization enhancements predicted by previous models.

2. A SEMICLASSICAL AND STOCHASTIC MODEL OF SPIN DIFFUSION

The model is briefly sketched here with only emphasis on its present limitations. A more detailed description is given in [6].

The spin-orbit coupling is responsible of a spin random walk as random photon emissions excite betatron and synchrotron oscillations. If the beam was initially polarized, its polarization would decay with a characteristic time $\tau_d$. This spin diffusion competes with the Sokolov-Ternov polarizing effect, that has a characteristic time $\tau_p$. The equilibrium degree of polarization $P$ is lower than the maximum $P_0$ allowed by this effect:

$$ P = \frac{P_0}{1 + \tau_p/\tau_d} $$

The spin random walk is a gaussian stochastic process as correlation times, i.e. the oscillation damping times, are much longer than the average interval between successive photon emissions.

In order to calculate the spin diffusion characteristic time $\tau_d$, one considers a stored beam, assumed to be initially polarized, and one ignores the Sokolov-Ternov effect, assuming no spin-flip asymmetry in synchrotron radiation.

The evolution of an electron spin component along the equilibrium polarization direction $\vec{n}$ is expressed at any order in spin motion as a time-ordered integral over a product of spin perturbations at different times. The time evolution of beam polarization is just obtained by taking the statistical average of this spin component. The obtained expression involves multi-time correlation functions of the spin perturbations. Formally, for a gaussian process, all these functions can be expressed in terms of the two-time correlation function which is finally enough to obtain the depolarization characteristic time.

To study depolarization enhancement due to energy spread, the spin motion is decomposed into a precession about the direction $\vec{n}$ and another precession about an axis that rotates in the plane orthogonal to $\vec{n}$. The latter precession is a stochastic perturbation, due to synchrotron and betatron motion, that bends the spin away from the equilibrium direction $\vec{n}$ and that is responsible of a spin resonance when the axis precesses about $\vec{n}$ at the same
frequency as the spin. The perturbation of the former precession about \( \vec{\alpha} \) is a stochastic modulation. Only the synchrotron contribution to the former is considered here in order to account for the effect of the energy spread and to obtain the synchrotron sidebands of the spin resonances.

This decomposition of the spin motion leads to a factorization such that depolarization due to each spin resonance is multiplied by an enhancement factor which represents the effect of the energy spread and which is independent of the resonance strength.

The spin precession about an axis orthogonal to \( \vec{\alpha} \) is only calculated to the first order of the perturbation, while the precession modulation about \( \vec{\alpha} \) is treated at all orders of perturbation, but in a smooth approximation, in order to obtain the full spectrum of synchrotron satellites. For simplicity, synchrotron radiation is assumed to be uniformly emitted all around the ring circumference. Then, interference effects between different spin resonances do not appear. The obtained depolarization rate \( \tau_0^{-1} \) is just an incoherent sum of resonant contributions. However, interference effects can give a significant contribution, especially when the operating energy is set in the middle between resonances and when strong wigglers are installed in a few places along the ring. In particular interference between one synchrotron resonance with another spin resonance is missed, although it gives a contribution to the synchrotron satellites of this spin resonance [7].

The betatron and synchrotron oscillations are assumed to be linear. In particular non-linear effects due to sextupoles are neglected. One must keep in mind that the depolarization enhancement is very sensitive to the tails of the energy distribution which are then gaussian in this model. It has also been proposed [3] to use non-linear wigglers to cut the tails and to reduce the enhancement factor.

With this model, an analytic expression of the enhancement factor is derived for either any betatron resonance (\( \nu = p \pm Q_{x,y} \)) or any synchrotron resonance (\( \nu = p \pm Q_s \)).

Finally, the obtained depolarization enhancement factor \( C \), for a given spin resonance, essentially depends on the parameter \( x \):

\[
x = \left( \frac{\nu \sigma(E)}{Q_s E} \right)^2
\]

where \( \sigma(E)/E \) is the r.m.s. value of the energy spread. This parameter measures the depth of the precession modulation and is known as the modulation index in frequency-modulated RF waves. It scales like \( E^4 \), and its rapid increase with the energy \( E \) explains why the energy spread effect becomes very severe at high energy.
Moreover, the enhancement factor $C$ depends on $\delta/Q_s$ which measures the distance $\delta$ of the operating energy to the resonance in units of $Q_s$. The synchrotron satellites correspond to integer values of $\delta/Q_s$ where $C$ is very large. The operating energy should be chosen in a middle between two successive satellites.

In this model, there is no other dependence of the factor $C$, apart on the ratio $J_y/J_y$ of the damping partition numbers for a radial ($y = x$) or vertical ($y = z$) betatron resonance.

The model has been compared with the polarization data of SPEAR (see fig.1.). For the $v = 3 + Q_x$ radial betatron resonance the model gives a 0.22 ratio of the first satellite ($v = 3 + Q_x - Q_s$) width to the central line width as compared to 0.21 measured. However, it predicts a width of the second satellites ($v = 3 + Q_x \pm 2Q_s$) much more narrow than observed. That may be due to an interference between a synchrotron resonance and another betatron resonance, contributing to these second satellites, as mentioned above [7].

3. NUMERICAL PREDICTIONS OF THE MODEL FOR LEP.

In principle, maximum polarization should be obtained when the operating beam energy is set such that the fractional part of the spin tune is half-integer. Then the operating point is at the longest possible distance from any spin resonance. This spin tune value should also be in the middle between two successive synchrotron satellites. This requirement is fulfilled when the synchrotron tune $Q_s$ is the inverse of an odd integer and when the betatron tunes $Q_x$ and $Q_z$ are multiple of $Q_s$.

The numerical predictions given here correspond to $Q_s = 1/9$, i.e. a relatively large synchrotron tune that minimizes the modulation index $x$ and that is compatible with the operation of dedicated wigglers. Then the vertical betatron tune is set to $Q_x = 2/9$ and the radial betatron tune to $Q_x = 0.34$, slightly different from $3/9$ to avoid the $1/3$ non-linear resonance of betatron motion. The partition number ratio is set at its natural value $J_y/J_x = 2$.

The spin tune is set at $v = 104.5$, corresponding to a beam energy close to the $Z_0$ mass energy.
Figure 2 shows the enhancement factor C of betatron and synchrotron resonances for three modulation index values \(\lambda = 0.446, 1.45\) and 2.87. They respectively correspond to the energy spread values of 0.71 \(\times 10^{-3}\) as natural, 1.28 \(\times 10^{-3}\) as given by the emittance and damping wigglers, and 1.8 \(\times 10^{-3}\) that new dedicated wigglers would give [8]. For betatron resonances these \(C\) values agree within a few percents with the values given by the Yokoyama’s calculation [4] and corrected by S. Mane [5].

Figures 3a, b and c show the variation of the relative degree of polarization \(P/P_0\) with the fractional part of the spin tune under the assumptions that only one spin resonance is significant and that the strength of this isolated resonance can be reduced to the point that the depolarization is only 10% at \(\nu = 104.5\) for very small energy spread. The latter assumption is compatible
with numerical simulations of polarization optimization by special correction procedures [9].
The polarization variation exhibits the occurrence of the synchrotron satellites and the depolarization increase when dedicated wigglers are turned on.

With these dedicated wigglers, at $v = 104.5$, the degree of polarization is only of the order of 20-30% under the preceding assumptions. It has not been tried to derive a degree of polarization when several spin resonances are significant since the model does not account for interference effects between them. However, it is unlikely that one could so find realistic situations where the degree of polarization may be much higher.

Without wigglers, at $v = 104.5$, the enhancement factors are relatively small such that a polarization of about 50% can reasonably be expected after optimization.

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A MATCHED SPIN ROTATOR FOR LEP

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ABSTRACT. This report describes a spin rotator installed in a straight section of LEP, its optical properties and its effect on polarization. It is found that a depolarization as small as $2 \times 10^{-3}$ is feasible.

Introduction

This report describes a Richter-Schwitters spin rotator [1] for LEP, which satisfies all the optical and the most important spin matching conditions. Chapter Layout of the LEP spin rotator gives the layout in detail. Chapter Optical and spin-matching describes how the spin matching conditions can be formulated such that standard beam-optical matching programs can be used for finding a solution. The matched spin rotator for LEP obtained by this method is discussed in Chapter Spin-matched solution.

Layout of the LEP spin rotator

The schematic layout of half of the spin rotator is shown in Figure 1. The other half is antisymmetric with respect to the interaction point IP. The string of bending magnets B1 between the quadrupoles QS2 and QS3 bends the beam vertically by 18.3459 mrad, and the bending magnet string B2 between the quadrupoles QS9 and QS10 bends the beam by $\psi = -3.46054$ mrad. The sum of the bending angles is $\phi = 14.88536$ mrad, such that the spin is rotated from the vertical direction into the longitudinal one at the $Z_\alpha$ energy of 46.5 GeV. Compared to an earlier version [2], the bending magnet string B2 is one half-cell closer to the IP. Therefore, the focusing direction of the quadrupole in front of B2 changes. This is thought to ease the spin matching.

![Figure 1: Layout of the Richter-Schwitters Spin Rotator](image)

The maximum vertical displacement of the design orbit with the spin rotator from the median plane in the standard LEP configuration occurs in the B1 dipoles. It amounts to ±564 mm and is caused by the distance of these dipoles from the IP. However, the B1 dipoles are inserted into the LEP lattice at the nearest place to the IP where a straight section of adequate length is available. But,
a greater distance from the IP makes it easier to shield the experiments from the synchrotron radiation emitted in the B1 dipoles. The properties of the dipoles in the spin rotator are summarized in Table 1.

Because of the vertical slope of the design orbit, each spin rotator increases the LEPI circumference by \(d = 8.7\, \text{mm}\). This effect has consequences on the exact positions where the bunch collisions occur along the beam axis. The four bunches in each beam are equidistant under all circumstances. Therefore, if the distance between two IP's becomes longer by the insertion of a spin rotator, the actual collision points are shifted from the IP's. With one spin rotator in LEPI, the collision points at the rotator and at the diametrically opposite IP are not shifted, while the other two collision points are shifted by \(\pm \frac{d}{4}\) which is small enough to be neglected. With two diametrically opposite spin rotators, the collisions occur at the IP's.

<table>
<thead>
<tr>
<th>Dipole</th>
<th>B1</th>
<th>B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eff. length</td>
<td>23.1</td>
<td>11.55</td>
</tr>
<tr>
<td>Bending angle</td>
<td>18.3459</td>
<td>3.46054</td>
</tr>
<tr>
<td>Field</td>
<td>0.123</td>
<td>0.046</td>
</tr>
<tr>
<td>Radiation loss</td>
<td>959.1</td>
<td>68.2</td>
</tr>
<tr>
<td>Radiation power at 3 mA</td>
<td>2877</td>
<td>205</td>
</tr>
<tr>
<td>Critical energy</td>
<td>177.1</td>
<td>66.8</td>
</tr>
</tbody>
</table>

*Table 1: Properties of Spin Rotator Dipoles at 46.5 GeV*

**Optical and spin-matching**

Since the spin rotator replaces a section of the standard LEPI lattice, it has to be matched optically to the LEPI lattice. The optical matching conditions are given in Section Optical matching conditions. In addition, the spin rotator has to satisfy spin-matching conditions. In Section Spin-matching conditions they are formulated such that they can be solved by standard beam-optics programs. The procedure and the computer files used for finding the spin-matched rotator for LEPI are described in Section Spin matching.

**Optical matching conditions**

The spin rotator is inserted into the straight part of the LEPI lattice occupied by the low-\(\beta\) and RF insertions. Therefore the insertion must be matched to the dispersion suppressor such that the values of \(\alpha_x, \alpha_y, \beta_x, \beta_y\) at the entrance of the dispersion suppressor remain unchanged when the spin rotator is installed. This yields four optical conditions. The new bending magnets bend the beam only in the vertical plane. Therefore there are no conditions on the horizontal dispersion \(D_x\) and its derivative \(D'_x\). Because of the vertical bending, the vertical dispersion \(D_y\) and its derivative \(D'_y\) do not vanish any longer. Because of the antisymmetry of the spin rotator, the dispersion at the IP vanishes, but its derivative does not. Without matching, the vertical dispersion will propagate through all the arcs. This is undesirable because of the increase in the vertical beam size due to vertical quantum excitation, and because of the coupling for off-momentum particles. Looking from the IP towards the arc, vertical dispersion matching implies 2 conditions, \(D_y = 0\) and \(D'_y = 0\), with a non-zero starting value for \(D'_y\) at the IP. Looking from the arc towards the IP, one can start with \(D_y = D'_y = 0\) at the end of the insertion, and impose the condition \(D'_y = 0\) at the IP. This is one condition less than needed for matching in the other direction. Hence there are five or six conditions for the beam optics alone.
Spin-matching conditions

The spin matching conditions which have to be satisfied by a Richter-Schwitters spin rotator in LEP were given by Buon [3] and Blondel [4]. The three extra conditions are caused by the quadrupoles between the dipole strings of the LEP spin rotator:

- The vertical phase advance $\mu_y$ from the IP to the centre of the B1 dipole string should be a multiple of $\pi$.
- The vertical phase advance $\mu_y$ between the centres of the B1 and B2 dipole strings should be a multiple of $2\pi$.
- Spin matching of the horizontal betatron oscillations implies the following condition [4]:

$$\int_{ip}^{n_2} K\sqrt{\beta} \cos\mu \, ds = \sin\frac{\pi}{n_1} \int_{ip}^{n_1} K\sqrt{\beta} \cos\mu \, ds$$

(1)

Here $\sin\frac{\pi}{n_1} = 0.3571154$ is the projection of the spin vector onto the orbit between B1 and B2, and $K$ is the quadrupole strength. As noted in [4], the integrals in (1) are proportional to the change of the slopes of a particle starting at the IP with $x \neq 0$ and $x' = 0$.

The two conditions on the vertical phase advance are familiar to standard beam-optics programs. The horizontal spin matching condition can be expressed as a relation between the $R_{21}$ elements of the $6 \times 6$ TRANSPORT [5] matrices $R$ from the IP to the centres of the dipole strings B1 and B2 which are routinely calculated by standard beam-optics programs:

$$R_{21}(B2) = \frac{1 + \sin\frac{\pi}{n_1}}{\sin\frac{\pi}{n_1}} R_{21}(B1)$$

(2)

Thus, eight or nine conditions must be satisfied in total. One condition is on matrix elements, the other conditions are on orbit functions, e.g. $\alpha$, $\beta$, $\mu$, $D$, or $D'$. The variables available to satisfy these matching conditions are the six gradients of the six independently powered quadrupoles in the low-$\beta$ insertion, and the two gradients of the four quadrupoles in the RF insertions which are powered in pairs. Hence, there are enough variables to satisfy all the conditions when matching towards the IP.

Spin matching

The MAD program [6] does not currently allow constraints on matrix elements. During matching, MAD only "knows" the matrix of the current element and the orbit functions, but not the accumulated matrix. Therefore, only the conditions on the orbit functions were satisfied and the quadrupole gradients obtained were used as starting values for matching with TRANSPORT [5] which allows simultaneous conditions on matrix elements and orbit functions. First matching attempts with TRANSPORT started at the IP and matched towards the arcs. However, when the orbit functions were launched with a non-zero value of $D_y'$ at the IP, TRANSPORT did not understand the constraints on $D_y$ and $D_y'$ at the end of the insertion.

There are relations between the elements of the $R$ matrix for a beam line and the elements of the $R$ matrix for its reflection. In particular, the $R_{21}$ and $R_{41}$ matrix elements are the same for a beam line and its reflection. Therefore, TRANSPORT can be used for matching from the dispersion suppressor towards the IP. This is advantageous because the number of constraints is smaller, and one gets around the TRANSPORT problem of launching the matching with a non-zero initial value of $D_y'$. Entering the constraints on the optical functions $\alpha$ and $\beta$, and on the dispersion $D_y$ is straightforward [7]. The conditions that the vertical phase advances are multiples of $\pi$ become conditions that the $R_{41}$
elements vanish for a matrix which is initialized at the centre of the B2 dipole string. The constraint on the matrix elements $R_{21}(B1)$ and $R_{21}(B2)$ is formulated as follows:

- Since the matching is done towards the IP, the linear matrices $A$ from B2 to B1 and $C$ from B2 to the IP are available.
- Using the properties of reflected and inverse $2 \times 2$ matrices with unity determinant, the spin matching condition (2) is expressed in terms of elements of the matrices $A$ and $C$:

$$C_{21} = \frac{1 + \sin^2 \gamma}{\sin^2 \gamma} (C_{21} A_{22} - C_{22} A_{21})$$  \hspace{1cm} (3)

- The elements of the $A$ and $C$ matrices needed are stored in TRANSPORT registers [7]. Arithmetic in those registers is used to obtain an expression which vanishes when the horizontal betatron oscillations are spin matched and which is also stored in a TRANSPORT register. The value in that register is constrained during the matching.

As a first step towards spin matching, a MAD data file was set up in which a spin rotator is inserted into the second superperiod, i.e. in Pit 4. After painful experience, none of the elements or beam lines in the LEF definition were redefine. Instead, all definitions of modified elements and beam lines were added using previously unused names. The new beam line definitions start at the very top with a definition of a modified line called LEPM, and then continue with shorter and shorter beam lines. All modified elements are defined by first copying the unmodified definitions from the LEF database [8], and then changing them by hand. Proceeding in this manner ensures that no changes are made involuntarily to beam lines and elements which are not involved in the spin rotator. The second step towards spin matching consists in preparing the data for TRANSPORT. The lattice description is translated by a program [9] from the TWISS file generated by MAD. The TRANSPORT 'vary codes' and constraints are inserted into the TRANSPORT data by hand.

**Spin-matched solution**

Using the procedure described above, an optically and spin-matched spin rotator was found with MAD and TRANSPORT. Below, its optical properties are described first, followed by a discussion of the results of a spin simulation.

**Optical properties of the solution**

The solution found with MAD and TRANSPORT satisfies all optical conditions and all spin-matching conditions. In particular, it has $R_{\alpha}(B1) = -0.0600$ and $R_{\alpha}(B2) = -0.2280$ which fulfills the spin matching condition (2) to the accuracy given. The quadrupole strengths in the standard LEF configuration and after spin matching as well as their relative changes are shown in Table 2. The largest change occurs in QS3M. The strength of the strongest MQA quadrupole, QS4M, is still low enough that the maximum gradient of the MQA quadrupoles, 10.9 T/m, is adequate for operation up to 78 GeV. Each spin rotator increases the tunes by $\Delta Q_x = 0.1135$ and $\Delta Q_y = 0.2441$, since the matching does not impose a constraint on the phase advance in a superperiod with a spin rotator. Such constraints can be satisfied later by varying quadrupoles in the neighbouring dispersion suppressor.

The orbit functions through half the spin rotator are shown in Figure 2. Despite the extra conditions due to the spin matching, the orbit functions do not differ much from those in the standard LEF configuration [10], apart from the vertical dispersion. The vertical dispersion is small in the vertical dipoles of the spin rotator because of the first two spin-matching conditions. Therefore the increase of the vertical emittance due to the emission of synchrotron radiation in these dipoles is also small. The vertical emittance was computed with the PETROC program [11] for the unlikely case of spin rotators near all four experimental pits. The ratio of the vertical and horizontal emittances was found to be
Table 2: Quadrupole Strengths for Spin Rotator

<table>
<thead>
<tr>
<th>Strength</th>
<th>After</th>
<th>Before</th>
<th>Rel. Change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KQSCM</td>
<td>-0.16373560</td>
<td>-0.16462428</td>
<td>-0.5</td>
</tr>
<tr>
<td>KQS1M</td>
<td>0.03392751</td>
<td>0.03522834</td>
<td>-3.7</td>
</tr>
<tr>
<td>KQS3M</td>
<td>0.02612732</td>
<td>0.01752874</td>
<td>49.1</td>
</tr>
<tr>
<td>KQS4M</td>
<td>-0.04215348</td>
<td>-0.0348306</td>
<td>21.0</td>
</tr>
<tr>
<td>KQS5M</td>
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</tr>
<tr>
<td>KQS6M</td>
<td>-0.03255052</td>
<td>-0.02796097</td>
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</tr>
<tr>
<td>KFSM</td>
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<td>KDSM</td>
<td>-0.03333682</td>
<td>-0.03284922</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Figure 2: Orbit Functions in the Spin Rotator
2.58×10⁻³, about a factor of two smaller than that expected from the misalignments in LEP [10], and more than an order of magnitude smaller than that needed for optimum luminosity. With fewer spin rotators, the emittance ratio should be smaller in proportion.

**Spin simulation results**

The magnitude of the depolarizing effects in LEP with one of the spin-matched Richter-Schwitters spin rotators was obtained by simulation with SLIM [12]. The results are displayed in Figure 3 and Figure 4. The MAD optics is calculated with the thick lenses, whereas SLIM works with thin lenses. Hence, the resulting optical matching is not perfect. In particular, a small (≈ 1cm) vertical dispersion propagates through the lattice. It may be seen, however, that the spin rotator does not excite integer resonances, and the horizontal betatron resonances very little. A small excitation of the vertical betatron and synchrotron resonances is present, because it is impossible to make the vertical dispersion vanish as well as to impose a spin matching condition over the entire length of the vertical dipoles B1 (23.1 m) and B2 (11.55 m).

*Figure 3:* Polarization degree in LEP with one spin rotator. The tunes are: \( Q_x = 70.5018, Q_y = 78.4713, Q_z = 0.0855 \). Machine equipped with polarization wigglers such that \( \tau_p = 36 \text{ minutes} \).
Altogether the result is almost perfect and depolarizing effects arising from the presence of the spin rotator are much smaller than those arising from lattice imperfections, as studied elsewhere in this report [13] [14]. A depolarization of a few per thousand or less can be obtained at nearly any energy given the flexibility offered by the optics to modify the vertical and horizontal tunes.

One should be cautious of the fact that neither solenoids nor twisted quadrupoles have been included in these simulations. The spin being parallel to the magnetic field in the solenoids, one would not need here to compensate the effect of the experimental solenoid by a mini-spin-rotator as suggested by Rossmanith [15] for the case of transverse polarization. The effect of the solenoidal field on particles travelling on betatron oscillations will very likely result in a modification of the above mentioned spin-matching conditions. This clearly remains to be analyzed in detail.

**Figure 4:** Depolarizing effects. Decomposition of the depolarizing effects in the same machine as Figure 3: dashed line: depolarization due to synchrotron oscillations, dash-dotted line: depolarization due to vertical betatron oscillations, dotted line: depolarization due to horizontal betatron oscillations, full line: their vector sum (note interference effects).
Conclusions

The Richter-Schwitters spin rotator is the simplest one that can be designed for LEP, and installed in the straight section(s) surrounding the interaction point(s). The small slope at the interaction point of about 15 mrad can presumably be handled by the experiments at little extra cost. A solution to the optical and spin-matching conditions has been found and is described in this paper. Its depolarizing effects are very small. The simplicity of this scheme makes it very attractive.

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PHOTON BACKGROUND AT THE LEP DETECTORS
FROM RICHTER-SCHWITTERS SPIN ROTATORS

D. Treille and G. von Holtey

1. Introduction

Richter-Schwitters spin rotators [1] located in the straignt section on each side of the experiment have several attractive features. They need only a few magnets and their cost is three times lower than for arc-rotators. The simplicity of the set-up and the global asymmetric character make them optically much better and allows for spin matching [2].

However, two features are potential problems: (i) the experiment has to be inclined by about 1°, (ii) the presence of dipoles close to the experiment increases strongly the flux of synchrotron radiation background.

The first problem will require some test, in particular about the behaviour of an inclined cryostat. We suppose here that this problem may be overcome. A possible solution for the second problem is described below.

2. Synchrotron Radiation from a Richter-Schwitters Rotator

Figure 1 shows the layout of the rotator considered here and the synchrotron radiation fans towards the experiment from the last rotator dipoles B2. Two characteristic features appear:

i) the rotator bends will absorb on their vacuum chamber all synchrotron radiation produced in the long straight sections [3] except for quadrupoles QSC and QS1;

ii) because the bending is in the vertical plane and the overall set-up is asymmetric, two vertically extended photon beams exist: above the axis in one direction, below it in the other direction.
In the 0.135 T field of the B2-dipole, a 46 GeV electron bunch of 1 mA radiates $2.23 \times 10^{11}$ photons with energy above 40 KeV per mrad of bend. The critical energy of this radiation is 190 KeV. In total $5.9 \times 10^{12}$ photons per bunch crossing are radiated towards the IP by the B2-dipoles with nominal luminosity beams (2 x 3 mA) at the $Z^*$-peak energy.

3. Collimator Protection System

The set of protection collimators one can conceive is following directly from the asymmetric photon beam pattern (Fig. 1):

i) They operate in the vertical direction.

![Diagram](image)

**Figure 1**
Photon Background from Richter-Schwitters Spin Rotators

ii) The two main collimators C1- and C1+ are set asymmetrically above and below the electron beam, ±15 m away from the IP. Their role is to prevent any direct illumination of the experimental vacuum chamber and of possible "nearby masks" in that
region. The non-intercepted beam of photon rays passes through the interaction region towards the downstream bend without interaction, since no symmetric counterparts of collimators C1 exist. The required opening of these collimators is about 15 mm, which leaves sufficient room (> 3σv) for the electron beams.

iii) The two collimators C2− and C2+, also at ±15 m from the IP and up/down antisymmetric to collimators C1− and C1+ must stay within the shadow of the latter, but when closed to about 40 mm, they will protect the experiment from backscattered photons from the B2 vacuum chamber.

iv) Auxiliary collimators C3− and C3+, further away from the IP, are set (still asymmetric) in such a way that no radiation from the upstream B2 can reach the experimental chamber with only one small angle scattering. All synchrotron photons are intercepted by the C3 or C1 collimator back-face so that at least two backward scatters are needed to continue forward. These collimators can be set to an opening larger than the primary collimator opening and can also be made less thick (2.5 cm of W) since their role is to suppress processes which anyway are not of first order.

4. Resulting Photon Backgrounds
With the typical beams in LEP (horizontal emittance = 25 times the vertical emittance) the electron beams in dipoles B2 have transverse dimensions of σH × σV = (2.4 × 0.3) mm² and the photon source can be considered a pencil beam in the vertical plane.

From the synchrotron spectrum, and a legitimate simplification of the photon reflection (an albedo of about one for small forward angle scattering and about 0.01 if large angles are involved) and the assumption of a pencil beam in the vertical plane, one can compute analytically the contribution of the various multiple scattering processes.

The result is that the main source of photons impinging on the unprotected experimental vacuum chamber are due to edge effects from collimators C1 and C3. A rate of 2.8 × 10⁴ photons above 40 keV
energy are scattered off the 5 cm long inner surface of collimators Cl^- and Cl^+ into a ± 2.0 m long experimental chamber of radius 60 mm for each crossing of two bunches of 1 mA current. A similar amount of photons arrives from collimators C3. The sum of these rates is still about two orders of magnitude higher than what can be accepted.

Several hardware solutions exist to improve this situation:

i) The edge effect is computed under the assumption that the collimator edge is strictly parallel to the beam axis while the photons come under an angle of about 1 mrad. To eliminate this geometrical effect, one can incline the edge by the same angle, an operation which has probably to be optimized by remote control. One is then left with a depth effect and one gets an order of magnitude improvement on the number of photons scattered to the chamber at IP.

ii) The exact configuration of the chamber at the time of polarization is unclear. There are attempts to reduce its dimensions [4], may be leading to an elliptical shape. It is likely that local masking near the IP may be necessary in this case. Figure 2 gives an example where such local masking shields the chamber against most photons from the collimators.

Fig. 2 - Protection by local masks
iii) An other possible improvement should consist in replacing the last mrad of bend (1.14 m) by a very low field dipole (e.g. 4 m of 0.04 T) with a critical energy such that scattered photons present much less danger and the rate of scattered photons from the Cl-edge is reduced by a further factor ten.

5. Conclusion

The two inner bending magnets of a Richter-Schwitters spin rotators located within 40 m from the experiments generate a very high synchrotron radiation background to LEP detectors. However, by taking advantage of the asymmetric character of the radiated photon beams, one can conceive a collimator system, that protects detectors from all direct and most single scattered photons. The primary source of background photons remaining is then from scattering off the collimators edges.

This rate, however, is still too high and can be reduced by either shielding the experimental vacuum chamber with additional local masks, and/or by adding a low field dipole for the last mrad of rotator bend.

It appears therefore, that the very high photon background from rotator magnets placed in the straight sections around LEP experiments can be controlled and reduced to acceptable levels.

More precise background simulation calculations, including the quadrupole radiation, should be performed to confirm this conclusion.

References