CHARGE DIFFUSION IN THE CAPACITIVE READ-OUT OF RESISTIVE CATHODES

Markku Ellilä 1)

Abstract

A quantitative description is given for the charge flow on a resistive cathode. The emphasis is on the implications on pad read-out of large calorimeters equipped with plastic streamer tubes. Measurements are presented which give support to the theory, and also reveal details of the internal construction of the streamer tubes.

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1) CERN, EP Division, CH-1211 Genève 23, Switzerland
1. Introduction

External capacitive pick-up electrodes were first used in wire chambers in which the discharge region was enclosed inside a glass tube by Maze [1]. Their use became a common technique for charge read-out in conjunction with the development of the limited streamer tubes in the late 1970's [2,3]. Studies were made on the transparency of cathodes made of a bulk isolator, covered by a poorly conductive material [4]. It was seen that the particle localization capability of a detector was highly dependent on the resistivity of the cathode surface.

It has been seen that it is very difficult to produce plastic streamer tubes with cathodes of a high uniform resistivity in quantities needed for large collider experiments. The higher the resistivity, the lower the yield of detectors operating in a stable manner [5]. One of the possible causes of the observed instabilities is the inhomogeneous coverage of the resistive material on the cathode [6].

This report presents a model for the charge dynamics on a surface resistive cathode.

A typical cathode geometry of a plastic streamer tube element is depicted in Figure 1. The rectangular cathode cell has dimensions 9 x 9 mm\(^2\). It is constructed of an extruded comb-like plastic profile which is closed from above by another extruded plastic component, a cover plate. The inner surfaces of both the cover plate and the profile are coated by a graphite varnish. This assembly is enclosed inside a rectangular plastic tube which is filled with the correct gas mixture for the limited streamer mode operation. A read-out electrode is placed directly on the outer surface of the tube. The tube couples the read-out electrode capacitively to the cathode surface. The read-out electrode is usually made of a printed circuit board material which is covered by thin copper foils on both surfaces. The read-out geometry is produced by removing some of the copper either chemically or mechanically.

In the experimental part of this report we have used plastic streamer tubes which were constructed for the DELPHI Hadron Calorimeter [7] at JINR Dubna from components supplied by the University of Helsinki and INFN Rome Sanità.
2. Diffusion equation

The limited streamer is produced in a process in which the electronic charge is released far away from the anode wire [8]. This leads to the fact that the fast component of the pulse, which is due to the electrons, carries an appreciable amount of the liberated charge. This distinguishes the limited streamer mode from the proportional mode in which the current due to the ions is predominant. Another characteristic of the streamer formation process is that a large volume of gas is ionized. Thus a typical streamer pulse has a high total charge (> 20 pC) and a fast rise time (5 – 20 ns).

A lumped-parameter model of the circuit in which the streamer is picked up externally is presented in Figure 2. The cathode is represented as a one-dimensional chain of resistors. The cathode-to-ground capacitance is often close to $C_r$, the capacitance between the read-out pad and the cathode, since the capacitance $C_r$ of the read-out pad is much higher.

The streamer produces a current in the circuit which can be described by considering the change of energy stored in the electric field between the anode and the cathode due to the drift of the released charge in the field. The basic equation for the current induced by a charge $q$ drifting at a velocity $v_o$ in an electric field $E$ due to a potential difference $U$, in which the space charge of the ions is ignored is

$$i = \frac{q E}{U} v_o. \tag{1}$$

(1)

There is a great difference between the current due to the electrons and the ions since the drift velocity of the latter is about 3 orders of magnitude lower than that of the former. If the space charge is included explicitly, the equation becomes

$$i = \frac{q v_o}{U} \left( \frac{U}{r \ln \left( \frac{r_o}{r_i} \right)} + \vec{E}(Q) \cdot \vec{r} \right) \tag{2}$$

where $r =$ radial distance of the charge from the anode wire, $r_o =$ outer diameter of the cathode cell, $r_i =$ radius of the anode wire, $\vec{E}(Q) =$ electric field due to the surrounding space charge $Q$ and $\vec{r} =$ radial unit vector. The cathode cell is approximated as a cylinder.
The equation must be integrated over the volume of the streamer to obtain the total current.

The current which is generated according to (2) flows first through the capacitors \( C_r \) and \( C_r \) since the cathode is highly resistive. This leaves these capacitors charged. The capacitor \( C_r \), i.e. the read-out pad, obtains the same charge as is deposited on the cathode. This charge flows onto the pad through the charge amplifier circuit, which normally has a low input impedance.

The capacitor \( C_r \) starts to be discharged through the resistive cathode. This also affects the charge on the read-out pad, which retains a charge equivalent to the charge remaining on that part of the cathode which is facing the pad.

The discharging of the cathode capacitance \( C_r \) occurs through a circuit which can be approximated by the circuit given in Figure 3. If we are studying a slice of length \( dx \) of the cathode, the resistance of the surface can be expressed as \( dR = r \, dx \), where \( r \) is given in \( \Omega/cm \). The capacitance to ground is \( dC = c \, dx \), where \( c \) is given in \( F/cm \). Thus, the voltage drop across \( dx \) can be written

\[
dU = dr \, dx,
\]

and the current flowing into the capacitance is

\[
dl = c \frac{dU}{dt} \, dx
\]

This leads to

\[
\frac{dl}{dt} = \frac{1}{rc} \frac{d^2 l}{dx^2}
\]

which is the one-dimensional diffusion equation.

If \( Q_0 \) is an impulse charge which is injected onto the cathode at \( x = 0 \) and \( t = 0 \) this equation has the solution [9]:

\[
q(x,t) = \frac{Q_0}{\sqrt{\pi}} \sqrt{\frac{rc}{t}} \exp[-\left(\frac{rc}{t}\right) x^2].
\]

This one-dimensional approximation is valid only if the width of the detector is clearly smaller than the length of the read-out pad, so that the charge spreads quickly across the cathode, and then proceeds longitudinally.
Figure 4 shows the time evolution of the charge distribution on a cathode according to (6).

As can be seen in Figure 2, there are two capacitors, \( C_i \) and \( C_r \), at opposite directions, through which the streamer current can flow. \( C_r \) can be used beneficially by placing another read-out electrode on that side. In this way it is possible to produce a very accurate two-dimensional read-out structure by placing boards with thin metal strip electrodes on both sides of a plane of plastic streamer tubes \([10,11]\).

The ratio \( C_i / C_r \) determines the average amplitude that can be collected on the read-out electrode. Therefore a large air gap should be used on the opposite side of the pad to maximize the charge collection in one-sided read-out.

3. Experimental set-up

The model presented above was tested by measuring the charge remaining on read-out pads as a function of the length of the charge integration gate. For this purpose the charge spectrum of streamers produced by traversing cosmic muons was measured. This corresponds to a uniform scan over the pad surface with a radioactive source or an accelerator beam of minimum-ionizing particles.

We used a set-up with three chambers of 8 streamer tubes. The two outer chambers were used for triggering. The system is shown in Figure 5. The gas mixture was argon—carbon dioxide—isobutane 1:3:1. The operating voltage was 3.8 kV. As the anode we used a copper—beryllium wire of 80 \( \mu \)m diameter. The charge was read out through copper—coated fiberglass pads of size 9 x 25 cm\(^2\) and a charge-sensitive ADC module (LeCroy 2249W). The gate was produced by a signal from a gate pad which was placed on top of the uppermost chamber. The capacitance of all the pads was about 750 pC, which means that the local cathode-ground capacitance was completely dominated by the small \( C_r \) between the cathodes and the pads, due to air and PVC. The cover plate of the chamber covered by the pads P1—P3 was turned towards the pads.

A software coincidence trigger was set up for excluding events in which charge diffusing from outside the pad region would contaminate the results. The pads T1 and T2 were used for this pur-
pose. They were posed onto the chambers on the top and at the bottom of the set-up.

The trigger pad T1 and the gate pad were placed above and T2 below the central read-out pad P2. For all triggered events the amplitudes of the three read-out pads P1, P2 and P3 were recorded. Also the highest amplitude among the three pads was selected, and a spectrum was established.

The selection of the maximum amplitude allowed us to reduce further the contamination from charge produced outside the region covered by the selected pad. The software trigger level was set at 8 pC. According to calculations based on integrals of (6), the spectra formed of the maximum amplitudes only contain < 1% of distant events at gate lengths ≤ 4 µs.

The average charge of the maximum spectrum was studied as a function of the charge collection gate width. This corresponds to the charge which remains on the pad at which the streamer was produced, measured at the gate closing time.

The cathode-to-ground capacitance, a series combination of \( C_c \) and \( C_r \), was varied by producing an air gap between the streamer tube and the read-out pad by using thin plexiglas rods. This gave us a possibility to decouple the \( r \) and \( c \) contributions to the time constant of (6).

4. Results

A typical maximum charge spectrum is presented in Figure 6 a), measured with a 500 ns gate. The effect of an increased gate width can be seen in Figure 6 b), where a 6 µs gate has led the spectrum to shift towards lower amplitudes.

The rise time of a typical streamer pulse which was read out through a pad was about 30 ns, and the falling slope met the zero level after about 250 ns, the time at which the collected charge reached its maximum. After 250 ns the charge diffusion on the cathode produces a small but long-lasting negative current. This means that the integral to infinity of the curve which describes the current as a function of time is zero, i.e. no charge remains on the pad after a long time.

The charge on the pad which was hit is plotted as a function of time in Figure 7 for different air gaps between the pad and the chamber (0, 3 mm and 6 mm). Included are curves describing fits to
integrals of (6). The integration was performed over the pad length for all possible impact points. The integrals were fitted to the experimental data with \( r_c \) as the free parameter. The fitted values of \( r_c \) are also indicated.

The time constant \( r_c \) is plotted in Figure 8 as a function of the air gap thickness. A fit was performed which gives the values \( r \) and \( c \) separated from each other. This was done by assuming that \( r \) remains constant as the air gap is varied, and that the additional series capacitance to \( C \), due to the air gap is 2.51 \( \text{pF/cm} \) for a 3 mm air gap. The average values of \( r \) and \( c \) in that part of the plastic streamer tube coinciding with the read-out pads were thus found to be

\[
c = 0.80 \pm 0.52 \text{pF/cm}
\]

\[
r = 5.52 \pm 2.71 \text{k\Omega/cm}.
\]

This linear resistivity corresponds to a surface resistivity of 46.7 \( \pm 22.9 \text{k\Omega/square} \).

The value of the capacitance is very low. One can evaluate the distance of the cathode charge from the read-out pad by translating this capacitance into the thickness of a hypothetical gas gap needed to produce that capacitance. Such a gas gap calculated from our results would be 9.4 mm thick. The capacitance of the 1.5 mm of PVC due to the cover plate and the outer tube of the chamber is so high that it can be neglected when calculating the series capacitance from the cathode to the pad. Our conclusion is that a gas gap was responsible for the low value of the capacitance, and that consequently the charge flowing on the cathode was, on the average, at a distance of 9.4 mm from the external read-out pad when no additional air gap was introduced.

The statistical error, translated into a gas gap thickness, corresponds to a variation of the gap thickness between 6 - 27 mm. The distance from the cover plate inner surface to the pad surface was less than 2.5 mm, and 1.5 mm of this was of PVC. This implies that the streamer charge does not spread to the cover plate inside the streamer tube but remains distributed over the cathode profile surface. This may be due to a bad mechanical contact between the graphite-varnished surfaces. Another possible cause for the small capacitance is a weak spreading of the charge in the transverse direction. This is possible if the tops of the profile walls are badly covered by the conductive varnish. To maxi-
mize the time constant $\tau_c$, i.e. to minimize the charge leakage from a read-out pad, the profile should be varnished uniformly enough to permit an isotropic charge diffusion, and the cover plate should be well pressed against the profile to make a good electrical contact.

5. Conclusions

We have shown that a one-dimensional solution of the diffusion equation can be used to describe the charge flow on the resistive cathode of a streamer tube. The measured time constants of the charge flow are consistent with the ones calculated from the model.

It can be seen that the amount of charge that can be collected on a charge-integrating amplifier is strongly influenced by the capacitance between the read-out pad and the cathode. This places stringent requirements on the varnishing of a cathode surface, as well as on the construction and the design of the cathode. The ratio of the pad-cathode capacitance to the opposite ground-cathode capacitance determines the ratio between the charge amplitudes which can be collected on the two sides.

The transparency of a resistive cathode depends on the time constant $\tau_c$ and the characteristics of the read-out electronics. A peak-sensitive charge collection gives a higher efficiency than a system with a fixed collection time in large systems where large local variations in capacitances and resistivities change the time behaviour of charge diffusion. If the time constant of charge diffusion is small compared to the rise time of the streamer pulse the transparency of the cathode is lost.
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Bibliography

FIGURE CAPTIONS

1. The internal geometry of an eight-cell plastic streamer tube element.

2. A lumped-parameter presentation of the one-sided streamer pulse pick-up circuit. $C_e = \text{capacitance between the cathode and an external ground}$, $C_d = \text{wire capacitance}$, $C_r = \text{capacitance between the read-out pad and the cathode}$, $C_p = \text{pad capacitance}$ and $R = \text{cathode surface resistivity}$. 

3. An infinitesimal slice of the cathode surface. $\sigma = \text{resistivity of the cathode surface}$ and $c = \text{capacitance between cathode and ground}$. 

4. The charge density due to an impulse charge deposited onto a cathode which has a time constant $\tau_c = 4.67 \text{ ns/cm}^2$, seen on a read-out board composed of pads of a 25 cm length in the longitudinal direction along the wires. The distribution is given at $t_1 = 500 \text{ ns}$, $t_2 = 2 \mu\text{s}$ and $t_3 = 6 \mu\text{s}$. 

5. The experimental set-up which was used in the measurements. The trigger pads are marked by $T1$ and $T2$, and the pads which were used for the read-out by $P1 - P3$.

6. Streamer charge spectra measured from the read-out pads. No air gap was produced between the streamer tube and the read-out pads $P1 - P3$. a) Charge collection gate length = 500 ns, b) Charge collection gate length = 6 $\mu$ns.

7. The charge remaining on a read-out pad as a function of time. The three curves are given for cases where the air gap produced by plexiglas rods was 0, 3 and 6 mm, respectively. $\tau_c = \text{time constant of (6)}$. 

8. The time constant $\tau_c$ of (6) plotted as a function of the air gap thickness. A curve is also given according to a fit with parameters $c = 0.8 \ \text{pF/cm}$ and $r = 5.52 \ \text{k}\Omega/\text{cm}$, converted into a surface resistivity 46.7 $\text{k}\Omega/\square$. 

Charge diffusion on resistive cathodes
Figure 1.
Figure 2.
Figure 3.
Figure 4.
Figure 6.
Figure 7.
Figure 8.

Fit parameters

\[ c = 0.80 \pm 0.52 \, \text{pF/cm} \]
\[ r = 46.7 \pm 22.88 \, \text{k\Omega/cm} \]