A PRELIMINARY STUDY OF A VOLTAGE MULTIPLYING STRUCTURE FOR ELECTRON ACCELERATION

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1. Introduction

In [1], a double cavity structure has been proposed which is capable of accelerating a short burst of electrons after excitation by a pumping beam of low energy. The structure does not only exhibit an impedance transformation as in the wake-field accelerator [2], [3], but also shows an appreciable measure of pulse compression.

2. Principle of Operation

![Diagram of cavity arrangement](image)

Fig. 1: Arrangement of cavities

The device consists of two coupled concentric cavities (fig. 1). The cavity traversed by the beam will be referred to as the interaction cavity, the other as the storage cavity. The accelerating cycle, which will be treated in more detail in part 5, can be divided into three phases:

1. The pumping beam is smoothly raised until it reaches its final value \( I_p \). During this time, it transfers energy to the interaction cavity. This cavity stores only a small part of this energy, but transports most of it to the storage cavity, until an equilibrium is reached, when power extracted from the beam only compensates for the cavity losses. The steady state voltage \( V_i \) across the interaction cavity and the ratio of the energies stored in both cavities depends on the arrangement of the coupling holes.

2. Now the pumping beam is switched off abruptly. This reverses the power flow between the cavities, and the stored energy starts to oscillate back and forth between the two. Half a period of this differential frequency later, all the energy has moved into the interaction cavity. As the voltage across the interaction cavity is proportional to the square root of the energy stored therein, this means a rise of the voltage to a much greater value \( V_i \). A voltage gain
$V_f/V_i$ of 10 to 40 is realistic.

3. Then the beam to be accelerated is injected as a short burst at this first peak of $V_f$.

3. Experimental Results

![Diagram](image)

**Fig. 2:** Test resonator

To demonstrate the operation, a model structure has been tested (fig. 2). The interaction cavity is operated in the $E_{o0}$ mode. It is surrounded by the storage cavity, which is shaped as a ring of rectangular cross-section. Its mode can be characterized as the $E_{oo}$ mode of an equivalent cylindrical resonator. Coupling occurs via 12 oval shaped holes of $11 \times 7.5$ mm. The other dimensions can be taken from fig. 3. The beam is replaced by a short antenna at the center of the interaction cavity. A second one at the opposite side allows probing of the electric RF field. It is essential that coupling be weak, so that the $Q$ value of the inner cavity is not too much reduced. Unfortunately, this requirement can only be met imperfectly because of sensitivity problems. Thus $Q_i$ is considerably reduced by the coupling to the probe (to about half its unloaded value). The measured data of the aluminium alloy cavities are:

- $Q$ value of storage cavity: $Q_s = 6000$
- $Q$ value of interaction cavity: $Q_i = 2430$
- Operating frequency: $f_0 = 3000$ MHz
- Frequency splitting: $\Delta f = 6.1$ MHz

Measurements have been done by a homodyne system, which is sketched in fig. 3. The signal from the voltage probe is mixed with the unmodulated generator signal. Thus the phase of the amplitude relative to the input signal is also available at the mixer output. The result with a square wave modulated input signal is shown in fig. 4. The upper trace is the drive, the lower trace the electric field in the interaction cavity as measured at the mixer output. The voltage overshoot is about 10.

It may be worth remarking that the cavity voltage features the 180° phase jump necessary to change from energy absorption to beam acceleration. The beam to be accelerated must therefore be injected with the same phase as the drive beam. Where both beams originate from the same gun, only an intensity change is required.
Fig. 3: Measurement of the transient behaviour

![Image of transient behaviour](image)

**drive voltage**

**gap voltage**

Fig. 4: Transient behaviour with square-wave modulated signal

![Image of transient behaviour with square-wave modulated signal](image)

**gap voltage**

**drive voltage**

Fig. 5: Transient behaviour with saw-tooth modulated signal

![Image of transient behaviour with saw-tooth modulated signal](image)
The first transient voltage peak at beam switch-on is of course undesirable. It is easily eliminated by turning up the drive intensity progressively as shown in fig. 5. The output signal is detected by a square-law detector instead of the mixer, polarity is negative. The upper trace is the detected signal, which is proportional to the negative square of the voltage, and the lower trace is the input wave. It can be seen, that the oscillations during switch-on are completely suppressed.

4. Design Considerations and Possibilities

It follows from the principle of operation, that the storage cavity must accumulate the maximum energy that the interaction cavity can be expected to accept without breaking down. Calling the assumed voltage limit $V_{\text{max}}$, this energy is (neglecting losses)

$$W_i = \frac{1}{2} \frac{V_{\text{max}}^2}{\omega_0 Z_0}$$

where $Z_0$ is the characteristic impedance of the interaction cavity and $\omega_0$ the operating angular frequency.

At the end of the charging period, this energy has accumulated in the storage cavity, and the beam must supply the corresponding losses

$$P_B = \frac{1}{2} \frac{V_B I_B}{Q_i} = \frac{\omega_0 W_i}{Q_i} = \frac{1}{2} \frac{V_{\text{max}}^2}{Q_i Z_0}$$

with $Q_i$ the $Q$-value of the storage cavity.

The impedance that the interaction cavity presents to the beam, i.e. the ratio $V_B/I_B$, can be adjusted over a wide range by the choice of the coupling between the cavities. It is convenient to express this coupling as the frequency splitting between the odd and even excitation of the cavities; this is the frequency with which the energy oscillates between both. The time from switch-off until the accelerating voltage reaches maximum is half its period.

For a pillbox, $Z_0 = 242 \Omega$, $h(\lambda/2) \approx 150 \Omega$ with $h$ the height of the cavity. Assuming a frequency of 3 GHz, a height of 3 cm, and a maximum RF field of 200 kV/cm, $V_{\text{max}} \approx 600$ kV.

Fig. 6 shows the required pumping beam characteristics for different values of $Q_i$, calculated by the more exact formulas derived in part 5. Minimum voltage $V_B$ and current $I_B$ of the pumping beam are given over the relative frequency splitting $\Delta\omega/\omega_0$. The $Q$-value of the interaction resonator, which does not appear in the formulas above, is assumed as $Q_i = 10000$. Scaling is possible by the proportionality

$$V_B \propto V_{\text{max}}, \quad I_B \propto \frac{V_{\text{max}}}{Z_0}.$$

Furthermore, the following proportionality are approximately valid:

$$V_B \propto \frac{1}{\Delta \omega \omega_0 Q_i}, \quad I_B \propto \frac{\Delta \omega}{\omega_0}.$$
Fig. 6: Data of pumping beam for $Q_s = 10000$

The $Q$-value $Q_s$ of the interaction resonator is of only minor influence if it is larger than, say, 5000. It determines, however, the storage time constant $\tau = \frac{4}{\varepsilon_0 v_s} \left( \frac{Q_s}{Q} + \frac{Q}{Q_s} \right)$, which is dominated by the smaller of the two $Q$-values. Thus it may be convenient to sacrifice some conversion efficiency for a shorter repetition rate. The pumping beam data for $Q_s = 10000$ are shown in fig. 7.

5. Detailed Analysis

For a quantitative discussion, a generalized arrangement of two cavities is treated. We will derive an equivalent circuit for this arrangement, and analyze its frequency response to pulsed input signals by Laplace transformation. The accessible port of the cavities is the aperture for the beams, and this is placed at a maximum of electric RF field in the interaction cavity. Hence a parallel type resonator is the appropriate equivalent circuit for the interaction cavity.

The storage cavity is coupled via holes in the side wall of the interaction cavity (fig. 1), i.e. at a maximum of magnetic RF field. Thus it may be represented by a series type resonator (fig. 8). It can also be regarded as a parallel type resonator coupled via a quarterwave line, which is established by the distance between beam axis and coupling hole in the interaction cavity. This also results in a series type resonance behaviour.
Fig. 7: Data of pumping beam for $Q_s = 1000$

The coupling strength is represented by the coupling coefficient $\beta$, which is referred to the $R \cdot Q$ of the interaction resonator. The input admittance as seen from the beam reads (fig. 8)

$$Y = \frac{1}{Q_s Z_0} + j \frac{1}{Z_0} \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) + \frac{\beta}{Z_0} \frac{1}{1 + j Q_s \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)} .$$  \hspace{1cm} (1)

The imaginary part of $Y$ has three zeros, one at center frequency $\omega = \omega_0$, and two $\pm \Delta \omega / 2$ apart, so we get

$$\beta = \frac{1}{Q_s} (1 + \Delta^2) \quad \text{with} \quad \Delta = \frac{\Delta \omega}{\omega_0} Q_s .$$  \hspace{1cm} (2)

for the coupling coefficient $\beta$. The frequency splitting $\Delta \omega$ can easily be measured.

To analyse the dynamic behaviour of the circuit in fig. 8, when excited by a modulated current source, we apply a band-pass to low-pass transformation, i.e. we replace the frequency $\omega$ by $\omega - \omega_0$. Hence a source of frequency $\omega_0$ is replaced by a DC source, a parallel type resonator by a capacitance $C' = \frac{2}{\omega_0} \sqrt{C/L}$, and a series type resonator by an inductance $L' = \frac{2}{\omega_0} \sqrt{L/C}$.
This results in the equivalent circuit of fig. 9, the current source $i(t)$ representing the envelope of the modulated beam.

Let us first consider a step function of $i(t)$ (fig. 10). Applying Laplace transformation, we get for the current and the voltage
\[ I(p) = \frac{I_B}{p}, \quad V(p) = \frac{I_B}{p} \left( \frac{Z_0}{1 + p \frac{2}{\omega_0} \frac{1}{Q_i}} + \frac{\beta}{1 + p \frac{2}{\omega_0} \frac{1}{Q_i}} \right). \] (3)

Retransforming to the time domain yields a damped, oscillating voltage

\[ v(t) = V_B \left[ 1 - \frac{1}{\cos \phi} e^{-\sigma t} \cos (\omega_r t + \phi) \right] \] (4)

with

\[ V_B = I_B Z_0 Q_i, \quad \frac{1}{Q_i} = \frac{1}{Q_i^*} + \frac{1 + \Lambda^2}{Q_i^*} = \frac{1}{Q_e^*} + \frac{\Lambda^2}{Q_i^*}. \]

\[ \omega_r = \frac{\omega_0 \Lambda}{2} \sqrt{\frac{1 + \Lambda^2}{Q_i^*} - \left( \frac{1}{2} \left( \frac{1}{Q_i^*} - \frac{1}{Q_i^*} \right) \right)^2}. \]

\[ \sigma = \frac{1}{\tau} = \frac{\omega_0}{2} \frac{1}{2} \left( \frac{1}{Q_i^*} + \frac{1}{Q_i^*} \right). \]

\[ \tan \phi = \frac{1}{\omega_r} \frac{\omega_0}{2} \left( \frac{1 + \Lambda^2}{Q_i^*} + \frac{1}{2} \left( \frac{1}{Q_i^*} - \frac{1}{Q_i^*} \right) \right). \]

The maximum voltage occurs at

\[ I_{\text{max}} = \frac{1}{\omega_r} \left( \pi - \arctan \frac{\sigma}{\omega_r} \right). \] (5)

and the voltage gain is

\[ \frac{V_{\text{max}}^r}{V_B} = 1 + k = 1 + \sqrt{\frac{1 + 2 \tan^2 \phi}{1 + (\sigma/m)^2}} e^{-\sigma t_{\text{max}}}. \] (6)

When switching off, the voltage gain is only given by \( k \) because the voltage approaches zero instead of \( V_B \) for \( t \to \infty \).

For high \( Q \)-factors, these equations can be simplified to

\[ k \approx \Lambda \frac{\Lambda_k}{\sqrt{\Lambda_k^2 + 1}} \exp \left( -\frac{1}{\Lambda_k} \arctan \frac{1 + \Lambda \Lambda_k}{\Lambda - \Lambda_k} \right) \] (7)

with

\[ \Lambda = \frac{\omega_0}{\omega_0} Q_i, \quad \Lambda_k \approx \frac{\Lambda}{\omega_0} Q_k, \quad \Lambda_k \approx \frac{2 \Lambda}{\omega_0} Q_k < \Lambda, \quad Q_e = \frac{Q_i Q_i}{Q_i + Q_i}. \]

- 8 -
Fig. 11: Switch-off of pumping beam

The second and third term in eq.(7) are somewhat smaller than 1 and depend only weakly on \( Q, Q', \) and \( \Lambda \omega. \) Hence the voltage gain \( k \) is mainly proportional to \( \Lambda: \)

\[
k \approx 0.7 \ldots 0.9 \cdot \Lambda.
\]

A typical example of switching is shown in fig. 11. The beam is switched off abruptly, and voltage gain is about 10.

This voltage overshoot is welcome after switching off the pumping beam, but it must not occur after switching on. If the induced voltage is greater than the beam voltage, the pumping beam would be reflected and soon blow up. Hence it must be switched on smoothly. This problem can be treated by Laplace transformation as well. Without discussing it in detail, it can be stated, that the switch-on time necessary to avoid excessive overshoot is in the order of the exponential decrement \( \tau \) of the exponential function in eq.(4). An example with a switch-on function of the pumping beam composed of two consecutive exponentials is shown in fig. 12.

Let us now consider the energy flow in the different phases of the accelerating cycle. For this purpose, we return to fig. 8. At the end of the first phase (storing energy from the pumping beam \( I_p \)), there is a voltage drop of \( V_s = I_p Z_0 Q. \) This yields for the stored energies (voltages and currents are given as peak values):
Fig. 12. Switch-on of pumping beam

interaction cavity: \( W^{(1)}_{i} = \frac{1}{2} \frac{I_{E}^{2}}{\omega_{0} Z_{0}} = \frac{1}{2} \frac{Z_{0}}{\omega_{0}} Q_{i}^{2} I_{R}^{2} \) .

storage cavity: \( W^{(1)}_{i} = (1 + \Lambda^{2}) \frac{1}{2} \frac{Z_{0}}{\omega_{0}} Q_{i}^{2} I_{R}^{2} = W^{(1)}_{i} (1 + \Lambda^{2}) \).

After switching off the pumping beam, the voltage rises to \( V_{res} \), which is given by eq.(6) or eq.(7). In this moment, all energy is stored in the interaction cavity, and this is

\[ W^{(2)}_{i} = k^{2} W^{(1)}_{i}. \]

This is the available energy for accelerating the electron bunch or bunches. To give a rough estimate for the possible current, we assume a burst of \( n \) rectangular shaped bunches of current \( I_{b} \) and length \( T/2 = \pi/\omega_{0} \), which is assumed to extract a fraction \( x \) of the available energy. For \( x \) well below 1, we may assume the voltage across the cavity constant during injection. The energy taken by the bunch is

\[ W_{b} = \frac{1}{2} \frac{k V_{b}}{\sqrt{2}} n I_{b} \frac{T}{2} = x W^{(2)}_{i}, \]

so that
\[ I_b = \frac{x}{n} \frac{\sqrt{2}}{\pi} k Q_c I_n \approx 0.4 \frac{x}{n} \frac{\epsilon_0}{\Lambda_0} I_n, \]  \hspace{1cm} (11)

i.e. for a frequency splitting of 0.1 \%, and a burst of 20 bunches extracting 5 \% of the available energy, the current of the beam to be accelerated may be as large as that of the pumping beam. Hence the cavity can be operated with a rather large beam during the short accelerating pulse.

References