PROTONS AND NEUTRONS CONTRIBUTIONS  
TO THE CHARGE LONGITUDINAL RESPONSE

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ABSTRACT
The charge longitudinal response of nuclei, as measured  
by inclusive inelastic electron scattering, is investigated  
in the semiclassical RPA framework. The proton and neutron  
contributions to the total response are explicitly separated  
out. It is found that a sizable neutron ejection can be  
induced by RPA correlations, particularly at low momentum  
and energy transfer. A comparison of our theory with the  
experimental data is presented, also in the case of  
asymmetric nuclei (N/Z). In particular, the differences in  
the neutron contributions between 40Ca and 48Ca are  
discussed.
In an additional mechanism for neutron excitation, work should also be investigated and taken into account as
a function of the number of neutrons excited. It is interesting to note that in our experiments, the effect appears
in a number of cases, even though the excitation is due to the presence of a neutron.

Finally it should be kept in mind that our experiments
in particular, the excitation of the neutron, is
1-Introduction
2. THE LONGITUDINAL POLARIZATION PROPAGATOR:
GENERALITIES AND NOTATIONS

One aims to calculate the longitudinal structure function measured in inelastic electron scattering at momentum \( q \) and energy transfer \( \omega \)

\[
R_L(q, \omega) = \sum_h \sum_i \sum_j \hat{O}_L(q, \omega, i) e^{i \hat{q} \cdot \hat{r}} |0\rangle \langle 0 | S(\varepsilon_n - \omega) \tag{1}
\]

where \(|0\rangle\) is the ground state of the nucleus and the summation runs over the nuclear excited states \(|n\rangle\) with excitation energy \(\varepsilon_n\). The one-body excitation operator \(\hat{O}_L(q, \omega)\) reads:

\[
\hat{O}_L(q, \omega, i) = \hat{F}_p(q, \omega) \hat{p}_i + \hat{F}_n(q, \omega) \hat{n}_i \tag{2}
\]

where \(\hat{p} = (1+\gamma)/2\) and \(\hat{n} = (1-\gamma)/2\) are the proton and neutron projection operators, \(F_p\) and \(F_n\) being the corresponding electromagnetic form factors.

The free neutron form factor \(F_n\) would be identically zero for point particles and experimentally is indeed very much smaller than \(F_p\). Therefore only the protons normally enter into the definition of the charge longitudinal operator. However, in the nuclear medium, the isovector and isoscalar form factors can become different, thus entailing \(F_n \neq 0\) (9).

In the calculation of response functions, the basic ingredient is the particle-hole Green's function

\[
G_{rs; r's'}(\omega) = \sum_n \left\{ \frac{\langle 0| \hat{E}_s^+ \hat{\mathcal{E}}_r^+ | n \rangle \langle n | \hat{E}_s^+ \hat{\mathcal{E}}_r | 0 \rangle}{\omega - \varepsilon_n - i\eta} \right\} \tag{3}
\]

where \(\hat{E}_r^+\) and \(\hat{E}_r\) are the operators annihilating or creating a particle in the single particle state \(r\).

We now introduce a polarization propagator relative to end-point operators \(\hat{A}\) and \(\hat{B}\) according to the definition

\[
D_{AB}(q, q'; \omega) = \sum_{rs, r's'} \langle s | \hat{A}^\dagger(q) e^{-i\hat{q}^\dagger \cdot \hat{r}} | r \rangle G_{rs; r's'}(\omega) \langle r' | \hat{B}(q') e^{i\hat{q} \cdot \hat{s'}} | s' \rangle \tag{4}
\]

The charge longitudinal response function can then be expressed in terms of the imaginary part of the polarization propagators corresponding to the \(\hat{p}\) and \(\hat{n}\) end point operators as follows

\[
R_L(q, \omega) = -\frac{i}{\pi} \text{Im} D_{qq}(q, q'; \omega) \tag{5a}
\]

\[
= -\frac{1}{\pi} \text{Im} \left\{ D_{pp}(q, q'; \omega) + D_{nn}(q, q'; \omega) + D_{pn}(q, q'; \omega) + D_{np}(q, q'; \omega) \right\} \tag{5b}
\]

We shall now calculate this response both within the mean field approximation and in RPA, using the semiclassical Thomas-Fermi framework. We should notice that in the absence of a direct photon-neutron coupling \(F_n = 0\) (5b) reduces to the well-known expression

\[
R_L(q, \omega) = -\frac{1}{\pi} F_p^2(q, \omega) \text{Im} D_{pp}(q, q'; \omega), \tag{6}
\]
When the proton-neutron coupling is ignored \[ (13) \]

\[
\{ (\gamma, \nu) (\nu', \nu) \} \quad \text{Tr} \quad \left( \begin{array}{cccc}
(\gamma, \nu) & (\gamma, \nu) & \cdots & (\gamma, \nu) \\
(\nu, \nu') & (\nu, \nu') & \cdots & (\nu, \nu') \\
\vdots & \vdots & \ddots & \vdots \\
(\nu, \nu) & (\nu, \nu) & \cdots & (\nu, \nu)
\end{array} \right)
\]

accordingly, Eq. (11) reduces to the familiar expression

\[
\text{or, for nuclear matter, } f_\delta \text{ is constant.}
\]

\[ (\nu) \text{ depending on } \nu \text{ by (10).} \]

\[ (12) \]

\[
0 = \left( \frac{1}{\alpha^2} \bar{\rho} (\gamma, \nu) \right) \left( \frac{1}{\alpha^2} \bar{\rho} (\gamma, \nu) \right)
\]

Eq. (11) gives the upper limit of interaction if \( \bar{\rho} \) is set by the

\[
\left\{ \left( \gamma, \nu \right) \left( \nu', \nu \right) \right\} \quad \text{Tr} \quad \left( \begin{array}{cccc}
\left( \gamma, \nu \right) & \left( \gamma, \nu \right) & \cdots & \left( \gamma, \nu \right) \\
\left( \nu, \nu' \right) & \left( \nu, \nu' \right) & \cdots & \left( \nu, \nu' \right) \\
\vdots & \vdots & \ddots & \vdots \\
\left( \nu, \nu \right) & \left( \nu, \nu \right) & \cdots & \left( \nu, \nu \right)
\end{array} \right)
\]

From (9) and (10) the mean field response function becomes replaced by the local one. New spin-dependent interaction is introduced, once the local momentum $\hbar$ has

\[
\{ (\gamma, \nu) (\nu', \nu) \} \quad \text{Tr} \quad \left( \begin{array}{cccc}
(\gamma, \nu) & (\gamma, \nu) & \cdots & (\gamma, \nu) \\
(\nu, \nu') & (\nu, \nu') & \cdots & (\nu, \nu') \\
\vdots & \vdots & \ddots & \vdots \\
(\nu, \nu) & (\nu, \nu) & \cdots & (\nu, \nu)
\end{array} \right)
\]

The particle-hole mean field response function

\[ (2) \]

\[
\bar{\rho} (\gamma, \nu) \theta \left[ \begin{array}{cccc}
\left( \gamma, \nu \right) & \left( \gamma, \nu \right) \\
\left( \nu, \nu' \right) & \left( \nu, \nu' \right)
\end{array} \right] = \left( \begin{array}{cccc}
\left( \gamma, \nu \right) & \left( \gamma, \nu \right) \\
\left( \nu, \nu' \right) & \left( \nu, \nu' \right)
\end{array} \right)
\]

In this section we evaluate the response function

\[ (6) \]

\[
\left\{ (\gamma, \nu) (\nu', \nu) \right\} \quad \text{Tr} \quad \left( \begin{array}{cccc}
(\gamma, \nu) & (\gamma, \nu) & \cdots & (\gamma, \nu) \\
(\nu, \nu') & (\nu, \nu') & \cdots & (\nu, \nu') \\
\vdots & \vdots & \ddots & \vdots \\
(\nu, \nu) & (\nu, \nu) & \cdots & (\nu, \nu)
\end{array} \right)
\]

The mean field approximation can be expressed in terms of the Mott's transformation, whose general derivation is given in (14). In the mean field framework, the $p$-polarization of the Fermi energy is different, in particular, for neutrons and protons (\( \gamma \) and \( \gamma' \)) in the single particle energy spectrum (\( \gamma \)).

\[ (7) \]

\[
\left\{ \left( \nu, \nu' \right) \right\} \quad \text{Tr} \quad \left( \begin{array}{cccc}
\left( \nu, \nu \right) & \left( \nu, \nu \right) & \cdots & \left( \nu, \nu \right) \\
\left( \nu', \nu' \right) & \left( \nu', \nu' \right) & \cdots & \left( \nu', \nu' \right) \\
\vdots & \vdots & \ddots & \vdots \\
\left( \nu, \nu \right) & \left( \nu, \nu \right) & \cdots & \left( \nu, \nu \right)
\end{array} \right)
\]

In this section we evaluate the response function.
2.2 - THE RPA RESPONSE FUNCTION

The RPA response function is calculated in the ring approximation. Correspondingly the RPA Green's function satisfies the equation

\[ G_{rs;rs'}(\omega) = G_{rs;rs'}^0(\omega) + \sum_{ss_1} G_{rs;rs_1}(\omega) \langle r_s s_1 | \hat{V} | s_1 s_r \rangle G_{r_s s_1;rs'}(\omega) \]  \hspace{1cm} (14)

\( \hat{V} \) being the residual particle-hole interaction. From the above equation a set of integral equations is obtained, which is satisfied by the various RPA polarization propagators \( \mathcal{D}_{AB}(\hat{R}, \hat{q}; \omega) \), defined from \( G \) according to eq. (4). The one-body end point operators (\( \hat{A} \) or \( \hat{B} \)) under consideration are again \( \hat{p} \) or \( \hat{n} \). Since in (14) we neglect antisymmetrization, the relevant p-h interaction is of the form

\[ U(1,2) = \left( \frac{d_\chi}{2\pi} \right)^3 [V_\sigma(q, q) + V_\tau(q, q) \hat{n}_1 \hat{n}_2] e^{i \hat{q} \cdot (\hat{r}_1 - \hat{r}_2)} \]  \hspace{1cm} (15)

where \( V_\sigma(q, q) = V_{\sigma p}^0 \) and \( V_\tau(q, q) = V_{\tau p}^0 \), the superscripts referring to the spin and isospin quantum numbers.

As the longitudinal response involves the third component of the isospin, we select from (15) only the piece relevant for the present investigation, which reads

\[ U(1,2) = \left( \frac{d_\chi}{2\pi} \right)^3 \left[ \{V_\sigma(q) + V_\tau(q)\} \hat{n}_1 \hat{n}_2 + \{V_\sigma(q) + V_\tau(q)\} \hat{n}_1 \hat{n}_2 + \{V_\sigma(q) - V_\tau(q)\} (\hat{n}_1 \hat{n}_2 + \hat{n}_2 \hat{n}_1) \right] e^{i \hat{q} \cdot (\hat{r}_1 - \hat{r}_2)} \]  \hspace{1cm} (16)

As in the mean field case we introduce the WT \( \Pi_{AB}(\hat{R}, \hat{q}; \omega) \) of the polarization propagator \( \mathcal{D}_{AB} \)

\[ \mathcal{D}_{AB}(\hat{R}, \hat{q}; \omega) = \int d_\chi e^{i (\vec{q} - \vec{q}_0) \cdot \vec{R}} \Pi_{AB}(\vec{R}, \vec{q}_0 + \vec{q}; \omega) \]  \hspace{1cm} (17)

The \( \Pi_{AB} \) are then determined using the semi-classical Thomas-Fermi theory. Thus the WT \( \Pi_{pp} \) and \( \Pi_{np} \) of the corresponding RPA polarization propagators are solution of the system of coupled equations

\[ \Pi_{pp} = \Pi_{pp}^0 + \Pi_{pp}^0 V_{pp} \Pi_{pp} + \Pi_{pp}^0 V_{pn} \Pi_{np} \]  \hspace{1cm} (18a)

\[ \Pi_{np} = \Pi_{nn} V_{nn} \Pi_{np} + \Pi_{nn} V_{np} \Pi_{pp} \]  \hspace{1cm} (18b)

where the arguments \( (\hat{R}, \hat{q}; \omega) \) have been omitted everywhere to simplify the notation. Inverting the role of \( \hat{p} \) and \( \hat{n} \) one obtains a similar set of equations for \( \Pi_{pn} \) and \( \Pi_{nn} \).

By comparing with (16) we define the interactions appearing in eqs. (18) as follows

\[ V_{pp}(q) = V_\sigma(q) + V_\tau(q) \]  \hspace{1cm} (19a)

\[ V_{nn}(q) = V_\sigma(q) + V_\tau(q) \]  \hspace{1cm} (19b)

and

\[ V_{np}(q) = V_{np}(q) = V_\sigma(q) - V_\tau(q). \] \hspace{1cm} (19c)

The choice of the density in the above interactions cannot be made on a purely theoretical ground. We have thus, euristically, taken the proton density in \( V_{pp} \), the neutron one in \( V_{nn} \) and the geometrical average \( V_{pn} = V_{pn}^0 \) for \( V_{np} \). Obviously these distinctions are unnecessary for symmetric nuclei as far as the proton and neutron densities are the same.
In the following section we deduce again the proton and neutron contributions to the charge longitudinal response. The following section we deduce again the proton and neutron contributions to the charge longitudinal response.

$$\frac{z}{Z} \cdot \frac{1}{(m'b)^2} \left\{ \frac{1}{F} \left[ \frac{\sum E^2 - \frac{T}{t} - \lambda}{\lambda} \right] \right\} \left( m'b \right) \frac{d\theta}{d\Omega} =$$

$$\frac{z}{Z} \cdot \frac{1}{(m'b)^2} \left\{ \frac{1}{F} \left[ \frac{\sum E^2 - \frac{T}{t} - \lambda}{\lambda} \right] \right\} \left( m'b \right) \frac{d\theta}{d\Omega} =$$

$$\frac{z}{Z} \cdot \frac{1}{(m'b)^2} \left\{ \frac{1}{F} \left[ \frac{\sum E^2 - \frac{T}{t} - \lambda}{\lambda} \right] \right\} \left( m'b \right) \frac{d\theta}{d\Omega} =$$

$$\frac{z}{Z} \cdot \frac{1}{(m'b)^2} \left\{ \frac{1}{F} \left[ \frac{\sum E^2 - \frac{T}{t} - \lambda}{\lambda} \right] \right\} \left( m'b \right) \frac{d\theta}{d\Omega} =$$

$$\frac{z}{Z} \cdot \frac{1}{(m'b)^2} \left\{ \frac{1}{F} \left[ \frac{\sum E^2 - \frac{T}{t} - \lambda}{\lambda} \right] \right\} \left( m'b \right) \frac{d\theta}{d\Omega} =$$
3- THE EFFECTIVE OPERATOR METHOD

In this section we present an alternative formalism which separates out in a more rigorous way the proton and neutron contributions to the charge response function.

To start with we exploit the completeness of states in isospace, which allows to write down a different expression for $\Pi_{\alpha\beta}(\mathbf{r}, \mathbf{q} \mid \omega)$ [the WT of the mean field polarization propagator relative to end point operators $\hat{A}$ and $\hat{B}$], namely:

$$\Pi_{\alpha\beta}(\mathbf{r}, \mathbf{q} \mid \omega) = \sum_{\alpha} \langle \alpha | \hat{A}^\dagger(\mathbf{q}) | \alpha \rangle \Pi_{\alpha\alpha}(\mathbf{r}, \mathbf{q} \mid \omega) \langle \alpha | \hat{B}(\mathbf{q}) | \alpha \rangle. \tag{26}$$

We next search for the WT of the Thomas-Fermi RPA polarization propagator $\Pi_{\alpha\beta}(\mathbf{r}, \mathbf{q} \mid \omega)$ in the form

$$\Pi_{\alpha\beta}(\mathbf{r}, \mathbf{q} \mid \omega) = \sum_{\alpha} \langle \alpha | \hat{A}^\dagger(\mathbf{q}) | \alpha \rangle \Pi_{\alpha\alpha}(\mathbf{r}, \mathbf{q} \mid \omega) \langle \alpha | \hat{B}(\mathbf{q}) | \alpha \rangle, \tag{27}$$

which differs from the corresponding mean field expression $\Pi_{\alpha\beta}^\alpha$ since the (position dependent) effective one-body operator acting on isospace, $\hat{\Lambda}(\mathbf{q} \mid \omega)$, replaces the bare one $\hat{\Lambda}(\mathbf{q})$. The operator $\hat{\Lambda}(\mathbf{q} \mid \omega)$ is calculable from the RPA equations.

Since for the longitudinal response both $\hat{\Lambda}$ and $\hat{\Lambda}$ coincide with the charge operator $\tilde{\sigma}_L$ defined in eq. (2), the mean field expression (26) for it becomes

$$\Pi_{\alpha\beta}^\alpha(\mathbf{r}, \mathbf{q} \mid \omega) = \langle p | \hat{\Lambda}(\mathbf{q}) | p \rangle \Pi_{pp}(\mathbf{r}, \mathbf{q} \mid \omega) \langle p | \hat{\Lambda}(\mathbf{q}) | p \rangle + \langle n | \hat{\Lambda}(\mathbf{q}) | n \rangle \Pi_{nn}(\mathbf{r}, \mathbf{q} \mid \omega) \langle n | \hat{\Lambda}(\mathbf{q}) | n \rangle \tag{28}$$

In the RPA framework, instead, the effective charge operator $\tilde{\sigma}_L$ writes, with obvious notations

$$\tilde{\sigma}_L(\mathbf{r}, \mathbf{q} \mid \omega) = F_p(q, \omega) \tilde{P}(\mathbf{r}, \mathbf{q} \mid \omega) + F_n(q, \omega) \tilde{N}(\mathbf{r}, \mathbf{q} \mid \omega). \tag{29}$$

The RPA equations (18) yield for the effective operators $\tilde{P}$ and $\tilde{N}$

$$\tilde{P} = \frac{1}{\omega} \left[ (d - V_{nn} \Pi_{nn}) \tilde{P} + \Pi_{pp} V_p \tilde{N} \right] \tag{30a}$$

$$\tilde{N} = \frac{1}{\omega} \left[ (d - V_{pp} \Pi_{pp}) \tilde{N} + \Pi_{nn} V_{np} \tilde{P} \right] \tag{30b}$$

being given by (21).

The RPA longitudinal response is then expressed in terms of the charge effective operator $\tilde{\sigma}_L$ as follows

$$R_L(q, \omega) = -\frac{1}{\omega} \left| \sum_{\alpha} \langle \alpha | \tilde{\sigma}_L(\mathbf{r}, \mathbf{q} \mid \omega) | \alpha \rangle \right|^2 \tag{31}$$

Expliciting the summation over $\alpha$, $\sum_{\alpha} | \alpha \rangle = | p \rangle + | n \rangle$ (as in (28)), we split the charge response into a proton and a neutron components

$$R_L(q, \omega) = R_L^{(p)}(q, \omega) + R_L^{(n)}(q, \omega) \tag{32}$$

with

$$R_L^{(p)}(q, \omega) = -\frac{1}{\omega} \left| \sum_{\alpha} \langle \alpha | \tilde{\sigma}_L(\mathbf{r}, \mathbf{q} \mid \omega) | \alpha \rangle \langle p | \tilde{\sigma}_L(\mathbf{r}, \mathbf{q} \mid \omega) | p \rangle \right|^2 \tag{33a}$$

$$R_L^{(n)}(q, \omega) = -\frac{1}{\omega} \left| \sum_{\alpha} \langle \alpha | \tilde{\sigma}_L(\mathbf{r}, \mathbf{q} \mid \omega) | \alpha \rangle \langle n | \tilde{\sigma}_L(\mathbf{r}, \mathbf{q} \mid \omega) | n \rangle \right|^2. \tag{33b}$$

The two pieces correspond to the emission of a proton and of a neutron in the continuum, respectively.