MEASUREMENTS OF STANDARD ELECTROWEAK MODEL PARAMETERS

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ABSTRACT

Determinations of the Standard Electroweak Model parameters are now available from complementary sources with a precision sufficient to be sensitive to electroweak radiative corrections. They permit an estimation of $\sin^2 \theta_W$, $\alpha$, $\Delta r$, and the setting of limits in particular on the mass of the top quark. The present value of Standard Model parameters is somewhat in disagreement with the minimal SU(5) prediction. In the near future, dramatic improvement should come from precision measurements at the SLC and LEP, for which the availability of longitudinal polarization at the Z pole is shown to be essential.

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1. **Introduction**

In spite of its success, the Standard ElectroWeak Model (SEM) leaves many questions unanswered. Leaving aside the origin of fermion masses and our ignorance of the top-quark mass, the existence and properties of the Higgs particle(s) are extremely uncertain. It is widely expected that the SEM will be embedded in a wider theory; a great number of extensions have been proposed -- Grand Unified Theories (GUTs), technicolour, compositeness, supersymmetry and superstring motivated models -- all invariably calling for new particles.

One can certainly hope that some of these objects will show up in the mass range available for experiments in the near future. Their mere existence, however, would imply process-dependent modifications to the SEM predictions through Electroweak Radiative Corrections (EWRCs), even if these particles are too heavy to be directly produced.

This is a strong motivation to perform precision measurements of SEM parameters, since they can be used to reveal the existence of new particles or to place strong constraints on the structure of a more general theory.

The aim of this paper is to review the present status of precision measurements, with some emphasis on the most precise ones and the assumptions related to them. A measurement of $\sin^2 \theta_W$, $\rho$, and $\Delta r$ can be extracted from the data. The agreement between the various experiments holds only for $m_t < 200$ GeV, showing that radiative corrections begin to give us means of investigating physics beyond the Standard Model.

These values will be dramatically improved in the era of the Stanford Linear Collider (SLC) and the Large Electron-Positron storage ring (LEP) with the measurements of i) the Z mass, ii) the W mass, and iii) the weak couplings of the various fermions to the Z, leading to sensitivity to the Higgs mass and much physics beyond the Standard Model.

2. **SEM parameters**

2.1 **Tree-level parameters**

In the SEM, based on the gauge group SU(2)$_L \times$ U(1)$_Y$, the interaction of the gauge bosons with fermions is given by the interaction Lagrangian:

$$ L = g(J \cdot W^+ + J \cdot W^- + J \cdot W^3) + g' J \cdot B , $$

(1)
with $g$, $W^+$, $W^3$ being the $SU(2)_L$ coupling constant and gauge fields, respectively, and $g'$ and $B$ being those of the $U(1)_Y$ hypercharge group.

Once the assignment of fermions to multiplets is made, any four-fermion interaction matrix element arising from interaction (1) can be calculated given the knowledge of $g$, $g'$, and of the masses of the gauge fields. These masses are acquired via the Higgs mechanism, leaving as physical particles the $W^\pm$, the massless photon, and the $Z$ bosons.

Four parameters,

$$g, \quad g', \quad m_W, \quad m_Z,$$

are therefore needed. They can be further constrained by experimental or theoretical input:

i) Identifying the electromagnetic current in the above Lagrangian leads to the relation

$$e^2 = \frac{g^2 g'^2}{g^2 + g'^2} = g^2 \sin^2 \theta_W. \quad (2a)$$

This equation constitutes the definition of $\sin^2 \theta_W$ at tree level.

ii) The muon lifetime relates $g$ and $m_W$ to the Fermi constant $G_\mu$:

$$m_W^2 = \frac{\alpha}{2G_\mu \sin^2 \theta_W} \frac{A}{\sin^2 \theta_W}. \quad (2b)$$

The fine structure constant $\alpha = e^2/4\pi$ and the Fermi constant $G_\mu$ are measured with errors negligible compared to the errors on the other parameters, which can safely be neglected.

iii) If the Higgs particle(s) belong to doublets of $SU(2)_L \times U(1)_Y$, as in the original version of the model [1] -- hereafter referred to as minimal SEM -- then the $W$ and $Z$ masses obey the relation:

$$m_W^2 = m_Z^2 \frac{g^2}{g^2 + g'^2} = m_Z^2 \cos^2 \theta_W. \quad (3)$$
Relation (3) appears quite different from the previous two, as it is based on a theoretical conjecture. In order to allow for a more general scalar sector of the theory, the \( q \) parameter has been introduced [2]:

\[
q = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W}. \tag{4a}
\]

If the Higgs particles belong to doublets and triplets, then

\[
q = 1 + \frac{\sum \langle \varphi_1^L (I_L^* + 1) \varphi_1^L \rangle - 3 \sum \langle \varphi_1^L I_{3L}^* \varphi_1^L \rangle}{2 \sum \langle \varphi_1^L I_{3L}^* \varphi_1^L \rangle}, \tag{4b}
\]

where the sum is made over Higgs fields \( \varphi_1 \), with weak isospin \( I_L \), third components of weak isospin \( I_{3L} \), and vacuum expectation values (v.e.v.'s) \( \langle \varphi_1 \rangle \). The \( q \) parameter can become different from unity if there exist triplets of Higgs particles with non-zero v.e.v.'s. If a triplet member \( I_L = 1, I_{3L} = 1 \) were to exist with a v.e.v. \( \langle \varphi_3 \rangle = \epsilon \langle \varphi_2 \rangle \), where \( \langle \varphi_2 \rangle \) is the doublet's v.e.v., then \( q \) would become equal to \( 1 - \epsilon^2 \); small deviations from the minimal SEM are thus perfectly plausible, and \( q \) will often be left as a free parameter in the following.

To summarize, the above four parameters can be re-expressed in terms of

\[ \alpha, \ G_\mu, \ \sin^2 \theta_W, \ q \]

or equivalently

\[ \alpha, \ G_\mu, \ \frac{m_W}{m_Z}, \ m_Z, \]

leaving only two free parameters. The minimal SEM (\( q = 1 \)) has only one free parameter, which can be chosen as \( \sin^2 \theta_W \), or \( m_W \), or \( m_Z \).

2.2 Electroweak Radiative Corrections (EWRCs)

The SEM is renormalizable and this ensures that higher-order diagrams are calculable and finite. The higher-order diagrams,
which can give substantial contributions, involve particles that are not known or not yet discovered, and this has recently prompted much activity [3].

These EWRCs can be divided into three classes:

i) QED photonic corrections which originate from the class of diagrams where any number of real or virtual photons have been added to the Born diagram. They are detector- and cut-dependent, and are often estimated by Monte Carlo techniques which simulate the detector response to the presence of additional photons. They have no important physics content, but lead to large corrections or even additional experimental errors.

ii) The corrections due to the self energy of the gauge bosons (often referred to as oblique corrections). They are responsible for the large shifts in the W and Z masses. To these loop diagrams contribute any particle coupled to the vector bosons: not only ordinary fermions and bosons, but also the Higgs meson(s) [4], new fermions, supersymmetric partners [5], Z-Z' mixing [6], etc.

iii) Box diagrams and vertex corrections (also called direct corrections). These are generally small and can be affected by any particle coupled to the fermions, such as a new Z' vector boson [7].

The effect of EWRCs is to modify the SEM predictions of physical quantities from the set of input parameters, and thus the above tree-level formulae. Results become dependent on other parameters, such as

i) fermion masses, and in particular the top-quark mass,

ii) Higgs particle(s) masses,

iii) 'new physics'.

The calculation of EWRCs involves the choice of a renormalization scheme. Most calculations are nowadays performed in the on-shell (OS) renormalization scheme [8]. The input parameters are the actual gauge boson masses and the tree-level formulae become

\[
m_W^2 = \frac{A}{\sin^2 \theta_W (1-\delta r)}
\]

\[
\delta = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W}.
\]
The quantity $\Delta r = \Delta r(\alpha, m_w, m_z, m_t, m_h, \ldots)$, the $O(\alpha)$ non-QED correction to muon decay in the OS scheme, depends most strongly on $m_t$ (Fig. 1).

In the case where $v = 1$ one recognizes in Eq. (5b) the familiar definition of $\sin^2 \theta_W$,

$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}. \quad (6)$$

This definition does not coincide with the tree-level definition of Eq. (2a). This has the consequence that $\sin^2 \theta_W$ measured from the neutral-current couplings (much closer to the tree-level meaning) must be redefined according to Eq. (6) in terms of the vector boson masses, thus acquiring a weak dependence on theoretical assumptions on the values of $v$ and $\Delta r$.

There exist other renormalization schemes and other definitions of parameters. The modified minimal subtraction scheme ($\overline{MS}$),
where a running $\sin^2 \theta^*_w(\mu)$ is defined according to Eq. (2a), is in particular useful when making a comparison with the predictions of GUTs. The numerical relation between $\sin^2 \theta^*_w$ and $\sin^2 \theta^*_w(m_W)$ is given by Stuart [9] and is dependent on $\varphi$, $m_t$, and $m_H$. A definition of a running $\sin^2 \theta^*_w$ of Eq. (2a) independent of the renormalization scheme is given by the $\varphi$-system of Lynn [10]. In either system, the $\varphi$ parameter defined in Eq. (5b) becomes different from 1 even if it is set to 1 at tree level.

Finally a word of caution should be given regarding the definition of the $\varphi$ parameter: EWRCs have so far been calculated in the minimal SEM with $\varphi = 1$, only one Higgs particle, and three families. The only free parameters left under those conditions are $m_Z$, $m_t$, and $m_H$. The possibility of having $\varphi \neq 1$ requires a more complicated model and new objects which have not been included in the loop calculations. One generally assumes that the tree-level effect of these possible new structures is larger than the loop effects. This might, or might not, be correct.

3. Present experimental results

In this section, the meaning in terms of $\sin^2 \theta^*_w$, $\varphi$, and $\Delta r$ of present measurements will be extracted with a short discussion of specific difficulties and limitations. Section 3.1 deals with neutrino experiments, Section 3.2 with the measurement of $m_W$ and $m_Z$, and Section 3.3 with $e^+e^-$ annihilation results.

3.1 Neutrino experiments

The general form of the neutral-current and charged-current scattering of a neutrino on an electron or a nuclear target is

$$
(\overline{\nu})^{\alpha}_{NC} = \frac{1}{m_Z^2} F_{NC}(I_\alpha, Q, \sin^2 \theta^*_w) \left( \frac{e^2}{\sin^2 \theta^*_w \cos^2 \theta^*_w} \right)^2
$$

(7)

$$
(\overline{\nu})^{\alpha}_{CC} = \frac{1}{m_W^2} F_{CC}(I_\alpha, Q) \left( \frac{e^2}{\sin^2 \theta^*_w} \right)^2
$$

(8)

where the functions $F_{NC, CC}$ depend on the specific choice of target of isospin components $I_\alpha$, $I_3$, on charge $Q$, on kinematical cuts, and on the neutrino spectrum. Often measured quantities are the NC/CC ratio in deep inelastic scattering on isoscalar targets.
\[
R_{VN} = \frac{\overline{\nu}_e}{\nu_e} = \frac{\sigma_{NC}^{\nu_e}}{\sigma_{NC}} = \frac{m_W^4}{m_Z^4 \cos^4 \theta_W} \frac{\tilde{F}_{NC}^N(I_3, Q, \sin^2 \theta_W)}{\tilde{F}_{NC}(I_3', Q)}
\]

the ratio of muon antineutrino to neutrino scattering on electrons:

\[
R_{\nu e} = \frac{\sigma_{NC}^{\overline{\nu}_e}}{\sigma_{NC}^{\nu_e}} = \frac{\tilde{F}_{NC}^N(I_3, Q, \sin^2 \theta_W)}{\tilde{F}_{NC}(I_3', Q, \sin^2 \theta_W)}
\]

Indices N stand for nuclear targets and e for interactions on electrons.

It appears from the above formulae that the neutral-to-charged-current ratio on nuclei \(R_{VN}\) is sensitive to the ratio \(m_W^4/m_Z^4 \cos^4 \theta_W = q^2\), whereas the ratio of antineutrino to neutrino neutral-current interactions on electrons is independent of q. In addition, it happens that \(R_{\nu e}\) is insensitive to \(\sin^2 \theta_W\). Combining \(R_{VN}\) and \(R_{\nu e}\) permits a determination of q and \(\sin^2 \theta_W\).

Precision measurements of \(R_{VN}\) on isoscalar targets have recently been reported [11-14]. Because they give the most precise value of the SEM parameters, it is useful to insist on the difficulties encountered in extracting \(\sin^2 \theta_W\) from measurements on nuclear targets.

The need for an exact knowledge of the nucleon structure is removed by using the Llewellyn Smith formula [15]:

\[
R_{VN} = \frac{1}{2} - \sin^2 \theta_W + \left(\frac{5}{9}\right) \sin^4 \theta_W (1+r)
\]

where r is the ratio of neutrino to antineutrino charged-current cross-sections on the same target and within the same experimental cuts. Relation (11) is valid for \(q = 1\) and for an isoscalar target only. The sea content of the nucleon and the magnitude of the longitudinal cross-section are automatically taken into account by the r correction, as well as any coherent or non-perturbative effect obeying strong isospin invariance. There remain, however, a number of assumptions and uncertainties:

1) No correction is made for higher-twist effects which have been controversial [16]. The justification is that the above
formula takes care of the isospin-conserving effects and that the possible remainder must be strongly suppressed by the large hadronic energy cuts applied to the data ($E_H > 10$ GeV for the CDHS experiment).

ii) The quark-mixing matrix is assumed to be unitary. In fact, the assumption made is that the mixing of the lower two families with possible new families beyond the third is small, according to our present theoretical belief. The results should be re-examined if a fourth family were to appear.

iii) As mentioned before, the radiative corrections were calculated in the minimal SEM, with $g = 1$, $m_t = 45$ GeV, $m_H = 100$ GeV. The $g$ dependence is very strong and will be treated below. The dependence of the value of $\sin^2 \theta_w$ extracted from $R_{VN}$ upon $m_t$ and $m_H$ is displayed in Fig. 2 and is clearly well below the experimental errors for $m_t < 350$ GeV. The dominant effect in the radiative corrections is the QED correction to the charged-current diagram, the emission of a photon by the outgoing muon having no counterpart in the neutral-current diagram.

![Graphs showing dependence of $\sin^2 \theta_w$ on $m_t$ and $m_H$.](image)

**Fig. 2** The dependence on $m_t$ and $m_H$ of the value of $\sin^2 \theta_w$ extracted from $R_{VN}$ in the CDHS experiment (from Ref. [25]).

iv) The major uncertainty in extracting $\sin^2 \theta_w$ from $R_{VN}$ comes from the charm-quark production threshold in the charged-current diagram. From the theoretical point of view, charm production is described by the slow rescaling model [17], which simply consists in calculating the quark-parton cross-section, giving the final-state quark a mass $m_C$. Two questions arise: one regarding the validity of this picture, and the
second concerning the value of the charm-quark mass; pending theoretical input [18], the experimental groups have used the results of the dimuon production by neutrinos [19] observing 'consistency with the slow rescaling model for $m_c = 1.5 \pm 0.3$ GeV'. This uncertainty affects not only the calculation of the total charm cross-section, but also the knowledge of the strange sea that contributes to the neutral-current reaction and is measured via the antineutrino production of charm. The dependence of $\sin^2 \theta_W$ on $m_c$ has been given separately in recent measurements.

The results of the most precise experiments on $\sin^2 \theta_W$ are summarized in Fig. 3 (for $m_c = 45$ GeV, $m_H = 100$ GeV, $q = 1$).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDHS 85</td>
<td>0.232 ± 0.012</td>
</tr>
<tr>
<td>CCCFRR</td>
<td>0.239 ± 0.010</td>
</tr>
<tr>
<td>FMM</td>
<td>0.245 ± 0.015</td>
</tr>
<tr>
<td>CDHS 86</td>
<td>0.225 ± 0.005</td>
</tr>
<tr>
<td>CHARM 86</td>
<td>0.236 ± 0.005</td>
</tr>
<tr>
<td>World average</td>
<td>0.232 ± 0.006</td>
</tr>
<tr>
<td>$\nu_\mu e$ scattering</td>
<td>0.212 ± 0.023</td>
</tr>
</tbody>
</table>

$\sin^2 \theta_W \equiv 1 - M_W^2 / M_Z^2$

($m_t = 1.5 \pm 0.3$, $m_H = 45$, $m_W = 100$, $p=1$)

![Graph showing $\sin^2 \theta_W$ values](image)

Fig. 3 Recent results on $\sin^2 \theta_W$ from neutrino-nucleon scattering [11-14]. For comparison the $\nu_\mu e$ scattering result [24] is also indicated.

Measurements of $q$ were not foreseen in the latest CERN experiments and only a small amount of antineutrino data were taken. The most precise existing measurements [20-24] are summarized in Table 1. The preliminary CDHS number [25] combines recent and older data [23]. The determination of $\sin^2 \theta_W$ and $q$ from $R_\nu$ and $R_{\nu}$ is graphically displayed in Fig. 4.
Table 1
Measurements of $q$ in neutrino-nucleon scattering experiments
(see also Ref. [20])

<table>
<thead>
<tr>
<th>Source</th>
<th>$q$ Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHARM [21]</td>
<td>1.034 ± 0.036</td>
</tr>
<tr>
<td>CFRR [22]</td>
<td>0.991 ± 0.025 ± 0.009</td>
</tr>
<tr>
<td>FMM [14]</td>
<td>1.027 ± 0.023 ± 0.026</td>
</tr>
<tr>
<td>CDHS (preliminary) [23, 25]</td>
<td>0.991 ± 0.020 ± 0.007</td>
</tr>
<tr>
<td>Average</td>
<td>1.002 ± 0.012 ± 0.023 (m_C=1.5)</td>
</tr>
<tr>
<td>$v$-e [24]</td>
<td>1.09 ± 0.09 ± 0.11</td>
</tr>
</tbody>
</table>

Fig. 4 Extraction of $q$ and $\sin^2 \theta_w$ from $R_v$ and $R_{\bar{v}}$ (CDHS data)

The result from neutrino data on isoscalar targets is then:

1) combined $q$, $\sin^2 \theta_w$ fit:

$$\sin^2 \theta_w = 0.234 \pm 0.015 \pm 0.023 \ (m_C - 1.5 \text{ GeV})$$

$$q = 1.002 \pm 0.012 \pm 0.023 \ (m_C - 1.5 \text{ GeV})$$

(correlation coefficient: 0.90).
ii) assuming $q = 1$ (minimal SEM):

\[
\sin^2 \theta_W = 0.232 \pm 0.003 \text{ (stat.)} + 0.003 \text{ (common theor. error)} \\
+ 0.013 \text{ ($m_C - 1.5$ GeV)}.
\]

Whereas the measurement of $R_{vN}$ has reached the useful limit given by the charm-mass uncertainty, some progress is clearly still possible on $R_{vN}$, and would improve the limits on $q$.

As mentioned earlier, the experiments with muon-neutrino scattering on electrons permit a value of $\sin^2 \theta_W$ to be obtained which is independent of $q$ and free of the above-mentioned theoretical uncertainties. The average of the present measurements is [24]:

\[
\sin^2 \theta_W = 0.212 \pm 0.021 \pm 0.009,
\]

in agreement with the previous result. The future should bring substantial improvement in this direction with the CHARM II experiment presently running at CERN [26] and the $v_e \bar{e}$ scattering experiment with a water Cherenkov detector now proposed at Los Alamos [27] promising errors on $\sin^2 \theta_W$ as small as $\pm 0.005$ and $\pm 0.002$, respectively.

3.2 Measurements of $m_W$ and $m_Z$

The current values of the vector-boson masses are summarized in Table 2 [28, 29]. The major experimental uncertainty comes from the absolute energy scale in the electromagnetic calorimeters. The $p\bar{p}$ environment does not provide an absolute calibration reaction (such as $e^+e^- \rightarrow e^+e^-$ in $e^+e^-$ colliders). The UA1 and UA2 Collaborations have devoted a large effort to the calibration of the calorimeters in a test beam, but the energy scale is ultimately limited by i) the interpolation of the calibration constants between test-beam runs and ii) the knowledge of the test-beam momentum itself.

The energy-scale uncertainty vanishes, in principle, when measuring the ratio $m_W^2/m_Z^2$. However, some uncertainty remains, which has to do with the different selection criteria for W and Z events and the different methods used in estimating the mass. The W events include a missing neutrino and this adds an uncertainty due to the missing-energy measurement.

The measurements of $m_W$ and $m_Z$ from the collider experiments can be averaged, since the dominant systematic errors due to the
<table>
<thead>
<tr>
<th>Quantity</th>
<th>UA1 [28]</th>
<th>UA2 [29]</th>
<th>Average</th>
<th>Small prints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_W$</td>
<td>$83.5 \pm 1.0$</td>
<td>$80.2 \pm 0.6$ + $0.5 \pm 1.3$ (stat) (p_{L}) (cal)</td>
<td>$80.9 \pm 1.3$</td>
<td></td>
</tr>
<tr>
<td>$m_Z$</td>
<td>$93.0 \pm 1.4$ + $3.0$</td>
<td>$91.5 \pm 1.2$ + $1.7$</td>
<td>$91.9 \pm 1.8$</td>
<td></td>
</tr>
<tr>
<td>$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}$</td>
<td>$0.194 \pm 0.031$</td>
<td>$0.232 \pm 0.025$ + $0.010$</td>
<td>$0.216 \pm 0.020$</td>
<td>no $\Delta r$ assumption</td>
</tr>
<tr>
<td>$\sin^2 \theta_W = \frac{A^2}{m_W^2(1 - \Delta r)^2}$</td>
<td>$0.214 \pm 0.005 \pm 0.006 \pm 0.015$</td>
<td>$0.232 \pm 0.003 \pm 0.008$</td>
<td>$0.228 \pm 0.008$</td>
<td>$m_t = 36 \text{ GeV}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_W$ from $A_{FB}$ $Z \rightarrow e^+e^-$ [28]</td>
<td>$0.18 \pm 0.04$</td>
<td></td>
<td></td>
<td>$m_H = 100 \text{ GeV}$</td>
</tr>
</tbody>
</table>

*) When two errors are present, the first one is statistical, the second is systematic, mostly calibration. The specific error due to $p_L$ assumption is singled out in UA2's $m_W$. If only one error is given, statistical and systematic errors have been combined.
energy scale are presumably independent for the two experiments: one finds

\[ m_Z = 91.9 \pm 1.8 \text{ GeV} \]
\[ m_W = 80.9 \pm 1.3 \text{ GeV} , \]

with \( m_W^2/m_Z^2 = 0.784 \pm 0.020 \).

The complete determination of \( \sin^2 \theta_W \) and \( \delta \) or \( \Delta r \), is possible from these two measurements alone: they can also be combined with the low-energy data. This will be discussed in Section 3.4.

The \( W \) mass gives a \( \Delta r \)-dependent measurement of \( \sin^2 \theta_W \) using formula (5a). Assuming \( m_H = 100 \), \( m_t = 30 \), then \( \Delta r = 0.0711 \pm 0.0013 \) (the remaining uncertainty comes from the light-quark-loops corrections) [30], one finds (\( \Lambda = 37.28 \)):

\[ \sin^2 \theta_W = 0.2286 \pm 0.0072 = \frac{(37.28)^2}{m_W^2 [1 - \Delta r (m_t = 30, m_H = 100)]} . \]

The \( \Delta r \)-independent value determined from formula (6), \( 0.216 \pm 0.020 \), is statistically less sensitive, but benefits from the cancellation of most of the systematic errors. These results can be combined to give a measurement of \( \Delta r \), \( \Delta r = 0.017 \pm 0.082 \).

Another possibility of measuring \( \sin^2 \theta_W \) has recently been brought to light by the UA1 Collaboration [29], using the angular distribution in \( Z^0 \) decays:

\[ p\bar{p} \rightarrow Z^0 \rightarrow x \to e^+e^- \]

shown in Fig. 5. The forward-backward asymmetry at the \( Z^0 \) pole for the \( q\bar{q} \rightarrow e^+e^- \) reaction is given by

\[ A_{FB} = \frac{3}{2} \frac{2v_q a_q}{v_q^2 + a_q^2} \frac{2v_e a_e}{v_e^2 + a_e^2} , \]

where \( v_f \) and \( a_f \) are the fermion (\( f = q, e \)) vector and axial-vector couplings to the \( Z \) (Table 3). The combination \( 2v_f a_f/(v_f^2 + a_f^2) = \gamma_f \) represents the parity violation in \( Z^0 \rightarrow f\bar{f} \). It is zero for pure
Table 3
Standard Model coupling constants
\( \text{numerical values for } \sin^2 \theta_W = 0.23 \)

\[
\begin{align*}
\alpha_f &= 2 \Gamma_f^\text{L} \\
\nu_f &= \alpha_f - 4 Q_f \sin^2 \theta_W
\end{align*}
\]

<table>
<thead>
<tr>
<th>( \alpha_f )</th>
<th>( \nu_f )</th>
<th>( A_f )</th>
<th>( \frac{dA_f}{d \sin^2 \theta_W} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>V +1</td>
<td>+1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>e -1</td>
<td>-0.08</td>
<td>+0.16</td>
<td>-7.9</td>
</tr>
<tr>
<td>u +1</td>
<td>+0.39</td>
<td>+0.67</td>
<td>-3.5</td>
</tr>
<tr>
<td>d -1</td>
<td>-0.69</td>
<td>+0.94</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

Fig. 5 Angular asymmetry in \( Z^0 \to e^+e^- \) (UA1 data)

Axial or pure vector couplings, 1 for \( V - A \) and -1 for \( V + A \) maximum parity violation. This provides a measurement of \( \sin^2 \theta_W(m_Z^2) \): with 40 events, one finds \( A_{FB} = 0.30 \pm 0.15 \), and

\[
\sin^2 \theta_W = 0.18 \pm 0.04.
\]

Although not yet very precise, this method prefigures the measurements of asymmetries at SLC/LEP.
3.3 Measurements in $e^+e^-$ annihilation

The impact on the Standard Model parameters of present measurements in $e^+e^-$ annihilation to lepton pairs or to hadrons will be discussed here. These measurements will become important in the near future, as discussed in Section 4.

The cross-section for $e^+e^- \rightarrow f\bar{f}$ is given by

$$\frac{d\sigma}{d\cos \theta} = \frac{\pi a_w^2 C_{QCD}}{2s} [C_1 (1 + \cos^2 \theta) + C_2 \cos \theta]$$

with

$$C_1 = Q_f^2 - 2v_e v_0 Q_f x_1 + x_2 (v_f^2 + a_f^2)(v_e^2 + a_e^2)$$

$$C_2 = -4x_1 Q_f a_e a_f + 8x_2 v_e v_f a_e a_f$$

$$C_{QCD} = \begin{cases} 1 & \text{for leptons} \\ 3[1 + (a_s/\pi) + O(a_s^2)] & \text{for quarks} \end{cases}$$

where $Q_f$ is the fermion charge, and $a_f$ and $v_f$ the axial and vector coupling constants of Table 3; the $x'$s are defined by

$$x_1 = \Re X(s), \quad x_2 = |X(s)|^2,$$

with

$$X(s) = \frac{s}{s - m_Z^2} \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W}.$$

At PETRA energies, one is far enough from the $Z$ pole and the terms in $\Gamma_Z$ can be neglected. Then

$$x_1 \approx \frac{s}{s - m_Z^2} \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W}, \quad x_2 \approx x_1^2.$$

One can derive from these expressions the measurable quantities, namely the total cross-section and the forward-backward asymmetry:

$$R_{f\bar{f}} = C_{QCD} [Q_f^2 - 2v_e v_0 Q_f x_1 + (v_f^2 + a_f^2)(v_e^2 + a_e^2)x_2]$$

and

$$A_{FB}^f = \frac{3}{8} (-4x_1 Q_f a_e a_f + 8v_e v_f a_e a_f x_2)/R_{f\bar{f}}.$$
The muon or tau forward-backward asymmetry is written to a
very good approximation as follows:

\[ \Lambda_{FB}^{\ell} = -\frac{3}{2} \frac{a_{e} a_{f}}{Q_{Z}} \chi_{1}, \quad \ell = \mu, \tau. \]

Since \( a_{e} \) and \( a_{f} \) are fixed by the isospin assignment in the Standard
Model, all the freedom is in \( \chi_{1} \).

There has been quite a bit of discussion regarding the 'best'
expression for \( \chi_{1} \). Neglecting \( \Gamma_{Z} \), one gets

\[ \chi_{1}^{G} = \frac{s}{s - m_{Z}^{2}} \frac{1}{4 \sin^{2} \theta_{W} \cos^{2} \theta_{W}}. \]

Since the aim here is to derive values of \( \sin^{2} \theta_{W} \), \( q \), and \( \delta \chi_{1} \),
the above parametrization does not help much. In addition, \( \sin^{2} \theta_{W} \) and
\( m_{Z}^{2} \) are two arbitrary parameters.

It has become customary to rewrite \( \chi_{1} \) in terms of \( a, G_{\mu}, m_{Z} \)
(and \( q \)), using the Born approximation formulae (2b) and (3):

\[ m_{Z}^{2} = \frac{m_{a}}{\frac{1}{2} G_{\mu} \sin^{2} \theta_{W} \cos^{2} \theta_{W} q} \]

leading to

\[ \chi_{1}^{B} = -\frac{\frac{1}{2} G_{\mu} q}{4m_{a}} \frac{s}{1 - (s/m_{Z}^{2})}. \]

Because \( s/m_{Z}^{2} \) is small (\( \lesssim 0.2 \) at presently reachable energies), \( \chi_{1}^{B} \)
has very little dependence on \( m_{Z}^{2} \); \( \sin^{2} \theta_{W} \) does not appear; thus,
\( \Lambda_{FB}^{\ell} \) at PEP and PETRA energies provides a direct measurement of the
\( q \) parameter (a review of these measurements can be found elsewhere [31]).

\[ q = \frac{\Lambda_{FB}^{\ell} \text{ (exp.)}}{\Lambda_{FB}^{\ell} \text{ (theor., } q = 1)} = 1.02 \pm 0.05, \quad \ell = \mu, \tau. \]

Radiative corrections have been calculated [32] for standard hy-
potheses on \( m_{t} \) and \( m_{H} \). This number should be substantially improved
when results from the 1986 data taking become available.
The total lepton-pair cross-section $R_{LL}$, $\ell = \mu, \tau$, 

$$R_{LL} = 1 + 2v^2_{e1} + (v^2_\phi + a^2_\phi)^2$$

depends, in principle, on $v^2_\phi = (1 - 4 \sin^2 \theta^2_w)$, which is very small for $\sin^2 \theta^2_w = 0.23$. The measurement of $R_{LL}$ therefore allows only weak bounds to be set on $\sin^2 \theta^2_w$.

In the total hadronic cross-section, on the other hand, with

$$R = \sum_{\text{quarks}} C_{QCD}(Q^2_f - 2Q_f v_e v_f X) + X_2 (v^2_\phi + a^2_\phi) (v^2_f + a^2_f)$$

owing to the numerical values of $Q^2_f$ and $v_e$, the situation is different: the $X_2$ term contributes to a change in $R$ of $\sim 5\%$ at the highest PETRA energy and becomes measurable and sensitive to $\sin^2 \theta^2_w$. A combined fit to $a_S$ and $\sin^2 \theta^2_w$ can thus be performed, if one uses data taken at different energies as in the impressive work of de Boer [33], with a careful analysis of the normalization errors in the various experiments. The result is that the electro-weak and strong effects decouple, and almost uncorrelated values of $a_S$ and $\sin^2 \theta^2_w$ can be obtained (Fig. 6a and b):

$$a_S = 0.165 \pm 0.03$$

$$\sin^2 \theta^2_w = 0.236 \pm 0.020$$.

Fig. 6 Determination of $a_S$ and $\sin^2 \theta^2_w$ from $e^+e^- \rightarrow$ hadrons (from Ref. [33]). a) Averaged $R$ values as a function of $\sqrt{s}$. (The errors include statistical and correlated normalization errors.) b) Error contours corresponding to the Standard Model fit for three different assumptions on the splitting of the systematic error into point-to-point and common normalization errors.
The analysis is performed assuming \( q = 1 \), and radiative corrections, claimed to be small, are not applied (the dependence upon the top-quark mass was not investigated).

### 3.4 Combined values of the electroweak parameters [34]

One can combine the various pieces of experimental information in a confidence region for \( \sin^2 \theta_W \) and \( q \) or \( \Delta r \). The following measurements can be used:

\[
m_W = 80.9 \pm 1.3 \text{ GeV} = \left[ \frac{(37.28)^2}{\sin^2 \theta_W (1 - \Delta r)} \right]^{1/2}
\]

\[
m_Z = 91.9 \pm 1.8 \text{ GeV} = \left[ \frac{(67.28)^2}{\sin^2 \theta_W (1 - \sin^2 \theta_W) (1 - \Delta r) q} \right]^{1/2}
\]

\[
\sin^2 \theta_W = 0.234 \pm 0.015 \pm 0.007 \quad \text{from } \nu N \text{ scattering}
\]

\[
q = 1.002 \pm 0.012 \pm 0.007 \quad \text{from PEP and PETRA data .}
\]

Setting \( q = 1 \), one finds

\[
\sin^2 \theta_W = 0.232 \pm 0.006
\]

\[
\Delta r = 0.0847 \pm 0.041 . \quad (12)
\]

Setting \( \Delta r = 0.07 \) (this is valid for \( m_t \leq 80 \text{ GeV} \) and \( m_H \leq 1000 \text{ GeV} \))

\[
\sin^2 \theta_W = 0.229 \pm 0.007
\]

\[
q = 0.997 \pm 0.008 . \quad (13)
\]

Finally, setting \( q = 1 \) and \( \Delta r = 0.07 \), one finds

\[
\sin^2 \theta_W = 0.230 \pm 0.005 . \quad (14)
\]

Some consequences of these measurements can be drawn:

1) The value of \( q \) supports a custodial-SU(2)-conserving Higgs mechanism \((q = 1)\) and is indeed the only positive experimental information available on the Higgs sector of the theory.
ii) The value of $\Delta r$ agrees well with the standard hypotheses: $m_t = 45$ GeV, $m_H < 1000$ GeV, no further families. This can be expressed as an upper limit to the top quark: $m_t \leq 200$ GeV ($q = 1$) at 90% confidence level. This assumes, of course, that no other phenomenon is there to contribute to $\Delta r$.

iii) It is not possible to give independently a value for $\Delta r$ and $q$ free of assumptions [35], even though the many measurements available provide enough equations. The physical origin of this indetermination lies in the fact that the dominant sensitivity of $\Delta r$ (the top-quark mass dependence) and a possible difference of $q$ from 1 (Higgs triplets) both result in $m_W^3 \neq m_W^+$, which is on what the experiment really puts limits. The precision needed to be sensitive to radiative correction effects respecting $m_W^3 = m_W^+$ (the Higgs mass for instance) is much higher and belongs to the future. Consequently a cancellation could very well occur between a heavy top quark and Higgs triplets, invalidating the statements (i) and (ii).

iv) The value of $\sin^2 \theta_W$ can be compared with the minimal SU(5) prediction [36], $\sin^2 \theta_W(m_W) = 0.215 \pm 0.004$. Extracting $\sin^2 \theta_W(m_W)$ from the data can be done using Eq. (14) and the relation given in Ref. [9], leading to $\sin^2 \theta_W(m_W) = 0.2284 \pm 0.005$ (for $m_t \leq 80, m_H \leq 1$ TeV, $q = 1$) 2.1σ away from SU(5) predictions. In an almost assumption-free way, using the results of Ref. [10], one can extract $\sin^2 \theta_W(m_W)$ from the W mass: $\sin^2 \theta_W(m_W) = 0.2286 \pm 0.0072$, a 1.6 standard deviation discrepancy.

v) The interpretation of the results will be made much easier once the top-quark mass is known.

4. The future

Forthcoming improvements in low-energy experiments, both in ve scattering and $e^+e^-$ collisions, have been mentioned before and will continue to provide invaluable information. A new era of precision is opening with the Antiproton Accumulator (ACOL), SLC, and LEP.

A real breakthrough should come soon from the measurement of the $Z$ mass at SLC [37] and LEP [38], which is expected with a precision of $\Delta m_Z = \pm 45$ MeV and $\Delta m_Z = \pm 20$ MeV, respectively. In both machines, the scale of masses will be given by the beam-energy calibration. Special experiments are foreseen to calibrate the beam energy: a special beam spectrometer at SLAC [39] and a spin resonance depolarization technique at LEP [40], provided transverse
polarization is available. These special experiments are important: without them, the mass resolution would be $\pm 100$ MeV at the SLC and $\pm 50$ MeV at LEP, from the a priori knowledge of the magnets' excitation curve.

As discussed before, the knowledge of the Z mass is not sufficient to describe the model completely, since predictions would still depend on the matter representations ($m_W$, $m_Z$) and on the assumption that there exists no further physics. Precision measurements of $m_W$ and of $\sin^2 \theta_W$ are of crucial importance.

Measurement of the W mass will come from the pp experiments. Statistics should be multiplied by a factor of $\sim 20$ with the use of ACOL, and the systematic error due to the energy scale can then be recalibrated using the Z mass measured in the SLC or LEP. Statistical errors of $\pm 150$ MeV on $m_W$ can then be reached. To which extent the remaining systematic error on the W mass measurement can be reduced using the $Z^0$ sample, mainly by understanding the $p_T^W$ distribution, is subject to debate. Estimates on the obtainable precision vary then from $\Delta m_W = \pm 150$ MeV to $\Delta m_W = \pm 350$ MeV [41].

At a later stage, the W-mass measurement can be performed at LEP II [42]. Distributing a total integrated luminosity of 500 pb$^{-1}$ over the W-pair-production threshold or accumulating the same amount of luminosity on the W-pair production, a precision of $\Delta m_W = \pm 100$ MeV can be obtained.

Having measured $m_W$ to $\pm 100$ MeV would lead to a direct measurement of $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$ to $\pm 0.002$, and, by comparison with $m_Z$, an estimate of $\Delta \tau$ to $\pm 0.006$ (assuming $\phi = 1$); this is not quite sufficient to be sensitive to the Higgs mass, the effect of which is $\Delta (\tau) \approx 0.006$, for 10 GeV < $m_H$ < 1 TeV.

Another direct measurement of $\sin^2 \theta_W$ can be obtained from precision measurement of the asymmetries at the $Z^0$ pole where the statistics will be very large, typically $10^6 Z^0$ a year; in Born approximation, the differential cross-section for $e^+e^- \rightarrow f\bar{f}$, at the $Z$ pole, neglecting the photon exchange (this is certainly justified for this crude demonstration since the photon exchange contributes 0.1% of the Z exchange) is [43]:

$$\frac{d\sigma(e^+e^- \rightarrow f\bar{f})}{d \cos \theta} = \sigma_u^f [ (1 + \delta A_e)(1 + \cos^2 \theta) + 2 \cos \theta (\delta A_e) A_f ] \frac{3}{8} (1 - P' P^+),$$

where $\sigma_u^f$ is the total cross-section for this channel at the top of the resonance, $P'$ and $P^+$ the longitudinal polarization of the e$^+$
and $e^-$ respectively ($P$ is positive when the spin is parallel to the particle velocity), $f$ is the polarization of the $e^+e^-$ system $f = (P - P')/(1 - P'P^2)$, $A_f = 2v_f^2a_f/(v_f^2 + a_f^2)$ is the combination of coupling constants of Table 3, and $\cos \theta$ the angle of the recoil fermion with the $e^-$ beam. This formula is valid for all fermions except $e$ and $v_e$ which are modified by $t$-channel exchange.

Measuring the cross-section for events in the forward ($\cos \theta > 0$) or backward ($\cos \theta < 0$) hemisphere, $\sigma_F$ and $\sigma_B$, one can construct the following quantities:

i) With unpolarized beams one can form the forward-backward asymmetry

$$A_{FB}^{f\bar{f}} = \frac{3}{4} A \sigma A_f = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}.$$  \hspace{1cm} (15)

ii) With polarized beams, by taking data with opposite beam helicities and measuring the corresponding cross-sections $\sigma'$ and $\sigma''$ and $\sigma_F^+$ and $\sigma_B^+$, one can measure the longitudinal polarization asymmetry (or left-right asymmetry) [44]

$$A_{LR} = \frac{1}{3} \frac{\sigma' - \sigma''}{\sigma' + \sigma''} = A_e$$  \hspace{1cm} (16a)

and the polarized forward-backward asymmetry [45]

$$A_{FB}^{pol, f\bar{f}} = \frac{1}{3} \frac{(\sigma_F^+ - \sigma_B^+)}{(\sigma_F^- + \sigma_B^-)} = \frac{3}{4} A_f.$$  \hspace{1cm} (16b)

iii) If the polarization of the final-state fermion can be measured, as in $\tau$ decays [46], its value, averaged over $\cos \theta$, is

$$P_\tau = A_\tau.$$

Measurements without polarization have been reviewed in 'Physics at LEP' [47]. The most reliable measurements are the muon forward-backward asymmetry and the $\tau$-polarization measurement, which lead to $\Delta \sin^2 \theta_W = 0.002$ for an exposure of 100 pb$^{-1}$ at the $Z^0$, which matches statistical and systematic errors. The lack of sensitivity of the muon-pair asymmetry is due to the smallness of $A_e = A_\mu$ for $\sin^2 \theta_W = 0.23$. 

21
The improvement obtainable with longitudinal polarization is clearly brought to light when comparing Eqs. (15), (16a) and (16b): one measures directly $\mathcal{A}_f$ and $\mathcal{A}_0$ instead of measuring their product. In addition a major breakthrough comes from the remark that $\mathcal{A}_{LR}$ is independent of the final-state fermion. All decay modes can thus be used, providing impressive statistical power. There is a very simple heuristic argument for this: $\mathcal{A}_{LR}$ is the asymmetry of producing a $Z^0$ from opposite beam helicities, and thus depends only on the electron coupling, and not on the decay mode of the $Z$.

Polarization in the SLC [48] is created in the electron gun, where the light of a circularly polarized laser is sent onto a photoemissive surface. The electron polarization $P$ can be reversed on a pulse-to-pulse basis (it is planned to do it randomly) by reversing the laser polarization, given the relatively low rate of the Linac (180 Hz). The positrons, on the other hand, cannot be polarized. The experiment will thus measure, at the same time, two cross-sections:

\[
\begin{align*}
\text{electron bunches} & \quad 1 \quad 2 \quad \text{etc.} \\
\text{positron bunches} & \quad 1 \quad 2 \quad \text{etc.} \\
\text{cross-section} & \quad \sigma_1 \quad \sigma_2 \\
\sigma_1 & = \sigma_u (1 + P \mathcal{A}_{LR}) \\
\sigma_2 & = \sigma_u (1 - P \mathcal{A}_{LR}).
\end{align*}
\]

A degree of polarization of about 45% is expected. In order to extract precisely $\mathcal{A}_{LR} = [(\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2)](1/P)$, the polarization $P$ must be known to $\Delta P/P < 1\%$, in order to match the statistical precision of $10^6 Z^0$ events. This represents a considerable challenge [49]. The SLC polarization project is very strongly supported and active.

Polarization at LEP was not considered as a priority. In view of its potential, a study was made [50] and the LEP Experiments Committee recommended a design study to be conducted by the Machine Division. Whereas, from the machine point of view, implementation of longitudinal polarization is rather delicate [51], from the experimental point of view the situation is very similar in LEP to that in the SLC: there are four bunches of both positrons and electrons in the machine; electrons and positrons acquire opposite
transverse polarizations by the Sokolov-Ternov effect in the bending arcs and wiggler magnets of the machine. The polarization is then rotated to be longitudinal. Whichever way this is done, the polarizations remain opposite, thus \( P' = P^- \) (since the velocities are opposite as well). In order to obtain non-zero helicity at the interaction point, any set of the eight bunches can be depolarized to a negligible polarization degree [52]. A possible choice has been proposed [53] such that each experiment will see the following succession of polarization combinations:

<table>
<thead>
<tr>
<th>electron bunches</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>positron bunches</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>cross-sections</td>
<td>( \sigma_1 )</td>
<td>( \sigma_2 )</td>
<td>( \sigma_3 )</td>
<td>( \sigma_4 )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\sigma_1 &= \sigma_u (1 + P'A'_{\text{LR}}) \\
\sigma_2 &= \sigma_u (1 - P'A'_{\text{LR}}) \\
\sigma_3 &= \sigma_u \\
\sigma_4 &= \sigma_u [1 - P'P' + A_{\text{LR}}(P' - P')] 
\end{align*}
\]

These four equations can be solved for \( A_{\text{LR}} \), \( \sigma_u \), \( P' \), and \( P^- \). This allows a direct measurement of the polarization from the data themselves.

An exposure of 40 \( \text{pb}^{-1} \) (\( \text{10}^6 \) \( Z^0 \) events) can be shown to yield a measurement of \( A_{\text{LR}} \) to \( \pm 0.003 \), and important gains on the forward-backward asymmetries. Since this scheme uses the fact that both the \( e^+ \) and \( e^- \) beams can be polarized (and depolarized), it is only applicable to LEP experiments.

This corresponds to measuring \( \sin^2 \theta_w(m_Z^2) \) to a precision of \( \Delta \sin^2 \theta_w = \pm 0.0004 \), and matches the main theoretical error, \( \Delta \sin^2 \theta_w = \pm 0.0004 \), encountered when predicting \( \sin^2 \theta_w \) from \( m_Z \). This uncertainty arises when estimating \( \Delta r \), \( \Delta(\Delta r) = \pm 0.0013 \) [30], and is due mostly to light-quark loops. The corresponding contribution to \( \Delta r \) can be related to the measurement of \( \sigma \) (\( e^+e^- \to \text{hadrons} \)); the error is dominated by the experimental error in this cross-section, especially at low energies. A more precise measurement of this quantity would improve the error on \( \Delta r \) by a factor of 2 [54], and would certainly be welcome.

The power of a combined measurement of \( m_Z', m_w' \), and \( A_{\text{LR}} \), with the above-mentioned precisions, is emphasized in Fig. 7.
Fig. 7 The dependence of $A_{LR}$ and $m_W$ on the top-quark and Higgs-boson masses. Also indicated is the error corresponding to the experimental information available at present and the experimental error obtainable from precision measurements at SLC/ACOL/LEP.
The order of magnitude of the Higgs mass and the top-quark mass (if it is not known then) would be severely constrained. The existence of physics beyond the Standard Model, Z', further families, and supersymmetric particles, would be likely to be visible [3-7].

5. Conclusions

Present experimental results are in impressive agreement with a SEM and permit an estimate of $\sin^2 \theta_w$ and $\alpha$ or $\Delta r$. The fact that EWRCs are sensitive to particles much heavier than the Z and W bosons permits already the setting of an upper limit on the top-quark mass.

The near future, with the SLC, ACOL, and LEP, promises of improvements by more than an order of magnitude (Fig. 7). The availability of longitudinal polarization in $e^+e^-$ machines at the Z pole will play an essential role.

Acknowledgements

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REFERENCES AND FOOTNOTES

   A. Salam, Proc. 8th Nobel Symposium, Aspenäsgården, 1968
   (1972) 1412.

   Batavia, 1972, eds. J.D. Jackson and A. Roberts (NAL, Chicago,
   1972), Vol. IV, p. 266.

[3] The field has bloomed since the early calculations of Veltman:
   Landmark publications are:
   b) B.W. Lynn, M.E. Peskin and R.G. Stuart, SLAC-PUB 3725
      (1985), and J. Ellis and R. Pececi (eds.), Physics at LEP
   c) A. Barroso et al., preprint CERN-EP/87-70 (1987), and
      references therein.


[5] The effect of supersymmetric particles was studied in
   Ref. [3b], and with somewhat different conclusions in

   P. Franzini and F. Gilman, Phys. Rev. D32 (1985) 237, and
   SLAC-PUB 3932 (1986).


   B.W.Lynn, these Proceedings.

     (1986) 298.

     446.

[13] F. Merrit (CCFR Collab.), Proc. 12th Int. Conf. on Neutrino
     Physics and Astrophysics, Sendai (Japan) 1986.

     1969.


26
2760.
see also comments in Ref. [15].
[20] C. Geveniger, Proc. 11th Int. Conf. on Neutrino Physics and
Astrophysics, Nordkirchen, 1984, eds. K. Kleinknecht and
[26] C. Busi et al., CHARM II proposal, CERN/SPSC/83-24, SPSC P 186
(1983).
[27] D.H. White (Spokesman), Large water Čerenkov proposal,
Los Alamos proposal (1986).
6th Topical Workshop on Proton-Antiproton Collider Physics,
Aachen, 1986.
[31] H.U. Martyn, these Proceedings.
D.H. Saxon, Rutherford Appleton Lab. preprint RAL 86-073
(1986).
[33] W. de Boer, to appear in Proc. 17th Int. Symp. on Multi-
400.
[34] A very detailed review on this subject has appeared while this written contribution was being prepared: U. Amaldi et al., Pennsylvania preprint UPR-0331T (1987). The results I have presented are very close to theirs, even though I used only the most precise experimental results. Point (iii) of subsection 3.4 of my paper is relevant to theirs as well.

[35] I have erred on this point in my talk; I am grateful to Bryan Lynn and Paul Langacker for clarifying it for me.


[37] For the SLC: P. Rankin, private communication.

K. Moffeit, SLAC-PUB 4325 and these Proceedings.


M. Della Negra and P. Derriulat, private communications.


J.K. Bienlein, ibid., p. 60.

D. Sivers, ibid., p. 82.


[45] A. Blondel, B.W. Lynn, F.M. Renard and C. Verzegnassi,


J. Chauveau, ibid., p. 177.

     J. Badier et al., ALEPH Note 87-17 (1987).
[54] C. Verzegnassi, private communication. I have since learned
     that such measurements are planned in Novosibirsk in the
     lowest energy range (|s| < 1 GeV); W. Hughes and N. May,
     private communication.