Numerical results on the effect of the parasitic multipoles
in the superconducting dipoles of the Large Hadron Collider

W. Scandale

ABSTRACT

Numerical simulations are presented, the results of which enlighten the behaviour of the dynamic aperture in the Large Hadron Collider when the higher order multipole imperfections are considered in the magnetic field of the superconducting dipoles.

Simplified test lattices are used, which limit this analysis to the on-momentum particles only, but allow to extend the computation of the dynamic aperture to a large number of machines with different samples of random errors.

Computations of the tune shift on amplitude are used to evaluate to what extent the systematic magnetic errors of different multipolar order are detrimental.

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1. Introduction

A recent study [1] shows how strongly the higher order non-linear deviations of the magnetic field expected [2] in the superconducting dipoles of the Large Hadron Collider (LHC) influence the betatron oscillations. The dynamic aperture of a test lattice with 120 m long FODO cells, sufficient to accommodate the proton beam at the injection energy in the presence of the sextupolar deviation of the magnetic field [3], becomes insufficient when the higher order multipoles are also considered.

We recently included the effect of the higher order multipoles in the dedicated computer program already used to simulate the beam trajectories in the LHC [3], [4] and we computed the dynamic aperture for a large number of machines with different samples of random errors. For the sake of comparison with the results of refs. [1] and [3] we used a test lattice with 120 m long FODO cells and 90 degrees phase advance per cell, but we extended our analysis to a test lattice with 100 m long FODO cells and 90 degrees phase advance per cell which seems more adequate to accommodate the beam at injection. The detailed description of our test lattices is given in ref. [4].

We also tried to disentangle the detrimental effect of the various systematic magnetic errors by evaluating the tune shift on amplitude for different sets of the multipole components.

As in ref. [4], only on-momentum particles have been tracked.

2. Magnetic field errors

In this study we only consider the normal multipole errors in the dipoles assuming that the skew multipole components are sufficiently small to be neglected.

We specify these errors using the three different conventions existing in the present literature, i.e.
multipole strength
(MAD convention) \[ k_n = \frac{B^{(n)}}{B^* \rho} [m^{-(n+1)}] \]

USA–SSC convention \[ b_n = \frac{B^{(n)}}{n! * B} [m^{-n}] \]

LHC convention \[ b(n+1) = \frac{B^{(n)} * r^n}{n! * B} [m^0] \]

here \( r \) is the reference radius where the errors are evaluated.

The multipole errors have two components:

i) a systematic constant component identical for all the dipoles due to the persistent current flowing in the superconducting wires after the descent of the magnetic field to some low value and its subsequent increase to the injection value;

ii) a random component varying from dipole to dipole due to the mechanical misalignment of the coils and to the random variation of the persistent current due to the non-uniform cross section of the superconducting wires;

These errors have been evaluated by R. Perin [2], at injection field, where the aperture problems are more critical, for dipoles with a coil inner diameter of 50 mm:
Table 1

Multipole errors in the LHC dipoles at the injection field.

<table>
<thead>
<tr>
<th>order of multipole n</th>
<th>component</th>
<th>b(n+1) at R = 1cm</th>
<th>bn [m^{-n}]</th>
<th>Kn [m^{-n}(n + 1)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>systematic</td>
<td>-3.6*10^{-4}</td>
<td>-3.6</td>
<td>-2.4*10^{-3}</td>
</tr>
<tr>
<td>2</td>
<td>random (rms)</td>
<td>1.5*10^{-4}</td>
<td>1.5</td>
<td>1.0*10^{-3}</td>
</tr>
<tr>
<td>3</td>
<td>systematic</td>
<td>0.05*10^{-4}</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>random (rms)</td>
<td>0.15*10^{-4}</td>
<td>0.15</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>systematic</td>
<td>0.56*10^{-4}</td>
<td>0.56</td>
<td>44.8</td>
</tr>
<tr>
<td>4</td>
<td>random (rms)</td>
<td>0.20*10^{-4}</td>
<td>0.20</td>
<td>16.0</td>
</tr>
<tr>
<td>6</td>
<td>systematic</td>
<td>-0.15*10^{-4}</td>
<td>-0.15</td>
<td>-3.6*10^{6}</td>
</tr>
<tr>
<td>6</td>
<td>random (rms)</td>
<td>0.02*10^{-4}</td>
<td>0.02</td>
<td>4.8*10^{5}</td>
</tr>
<tr>
<td>8</td>
<td>systematic</td>
<td>0.10*10^{-4}</td>
<td>0.10</td>
<td>1.34*10^{12}</td>
</tr>
<tr>
<td>8</td>
<td>random (rms)</td>
<td>0.005*10^{-4}</td>
<td>0.005</td>
<td>6.72*10^{10}</td>
</tr>
</tbody>
</table>

The Kn's are calculated for \( \rho = 3 \) km.

The sign of the systematic components follows from the requirement that the field deviation is minimum along the horizontal and the vertical axes in the dipoles.

The distribution of the random multipole components from dipole to dipole is assumed to be gaussian.

In the following sections we will only use the LHC and the MAD conventions.

3. Dynamic aperture

Given the lattice design and the multipolar magnetic field errors, our tracking program follows the particle motion typically for 100 revolutions. By varying the initial amplitude of the particle being simulated and examining its stability, machine apertures can be evaluated. The azimuth at which tracking is started is always chosen in the middle of a QF quadrupole at the beginning of an octant, where the horizontal \( \delta \)-function has a maximum. Initial amplitudes in the
horizontal and vertical planes normally scale as the square root of the $\beta$-functions and the slopes are zero.

Two kinds of aperture are examined with our tracking program:

i) the largest initial amplitude for which the motion remains bounded defines the **overflow aperture**;

ii) the largest initial amplitude for which the particle trajectory stays inside a set of collimators provides the **collimator aperture** or the **dynamic aperture**.

Collimators of point ii) consist of round transverse mechanical limitations of 40 mm inner diameter, located in the middle of each QF and QD.

Apertures resulting from the tracking have to be compared to the transverse dimension of the beam. The comparison is more critical at injection because of the bigger magnetic errors and of the larger beam dimensions. The most intense beams injected in the LHC are expected to have $20 \pi 10^{-6}$ m normalized emittance. Beam radius $R$ at maximum $\beta_x^*$, defined as 4 times the rms of the beam distribution is computed below:

<table>
<thead>
<tr>
<th>Cell length in the test lattice</th>
<th>100m</th>
<th>119m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ [mm]</td>
<td>5.32</td>
<td>5.82</td>
</tr>
<tr>
<td>$\sigma$ [mm]</td>
<td>1.33</td>
<td>1.45</td>
</tr>
</tbody>
</table>

The apertures previously defined strongly depend on the particular sample of the random magnetic errors selected in the dipoles of the test lattice. To have good statistics, 100 different
machines are considered which only differ by the particular sample of the random errors, the random components of the different multipoles being statistically uncorrelated. For any initial amplitude, the tracking of the particle motion is performed for 100 revolutions in each of the 100 machines. The different apertures are now defined by the largest initial amplitude for which 90% of the machines satisfy criteria i) or ii).

The tracking with increasing initial amplitudes is pursued in each machine up to its overflow amplitude.

Each sample of the random magnetic errors is obtained by changing the seed of the random number generator routine of our tracking program.

The dynamic aperture has been evaluated for the two test lattices of interest in the five cases listed below. The number of stable machines (for 100 revolutions) is plotted in figs. 1 and 2 as a function of the initial horizontal amplitude for the first four cases.

The limit amplitudes corresponding to stable motion in 90% of the machines are given in Table 3 for the 5 cases.

1 all the systematic and the random multipoles of Table 1 are active;

2 beside the random and the systematic sextupoles, only the systematic multipole of higher orders are active;

3 beside the random and the systematic sextupoles, only the random multipole of higher orders are active;

4 only the random components of all the multipoles are active.

5 only the systematic components of all the multipoles are active.
Table 3

Dynamic aperture for 90% of the machines (in terms of the relevant r.m.s. beam size).

<table>
<thead>
<tr>
<th>Case</th>
<th>Dynamic aperture / ( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cell length 100 m</td>
</tr>
<tr>
<td>1</td>
<td>9.5</td>
</tr>
<tr>
<td>2</td>
<td>9.9</td>
</tr>
<tr>
<td>3</td>
<td>9.4</td>
</tr>
<tr>
<td>4</td>
<td>9.6</td>
</tr>
<tr>
<td>5</td>
<td>10.9</td>
</tr>
</tbody>
</table>

In all these cases the overflow aperture practically coincides with the dynamic aperture.

Having taken into account the differences of the test lattices, the results relative to the 120 m long cell are compatible with those of ref. [1].

The test lattice with shorter cells has a more comfortable dynamic aperture.

Setting to zero the systematic component of the multipole errors results in a rather small change of the dynamic aperture for the on-momentum particles.

For comparison, the dynamic aperture due to the sextupole component only is also shown in figs. 3 and 4. Two cases are considered:

- curve 1 both the systematic and the random sextupole components are active;
- curve 2 only the random sextupole component is active.
The dynamic aperture for 90% of the machines is given in Table 4.

Table 4

Dynamic aperture for 90% of the machine due to sextupoles only.

<table>
<thead>
<tr>
<th>Case</th>
<th>Dynamic aperture / σ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cell length 100 m</td>
</tr>
<tr>
<td>1</td>
<td>9.9</td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
</tr>
</tbody>
</table>

By comparing the values of Tables 3 and 4 one can conclude that the reduction of the dynamic aperture due to the higher order multipoles is rather small for the on-momentum particles.

4. Tune shift with amplitude

The tune shift with amplitude gives an appropriate evaluation of the detrimental effect of the systematic multipole components in the dipoles. In computing it, care has to be taken to avoid the crossing of the diagonal in the working diagram which couples the horizontal and vertical motions and makes the horizontal and vertical tunes undefined. We performed simulations without crossing the diagonal by using the following initial conditions:

i) for horizontal tune: \( x_0 \) variable
   \( x'_0 = 0 \)
   \( y_0 = 0.1 * \sigma \)
   \( y'_0 = 0 \)

ii) for vertical tune: \( x_0 = 0.1 * s \)
    \( x'_0 = 0 \)
    \( y_0 \) variable
    \( y'_0 = 0 \)
The test lattice with 100 m long cells has been used. Several cases have been considered:

**case 1:** all the systematic multipoles are active;

**case 2:** all the systematic multipoles except the octupole are active, i.e. $b_4 = 0$;

**case 3:** all the systematic multipoles except the decapole are active, i.e. $b_5 = 0$;

**case 4:** all the systematic multipoles except the 18th-pole are active, i.e. $b_9 = 0$;

**case 5:** all the systematic multipoles except the 14th- and 18th-pole are active, i.e. $b_7 = 0$ and $b_9 = 0$;

**case 6:** only the systematic sextupole is active.

Results of a numerical evaluation of the horizontal and the vertical tune shift with amplitude are shown in figs. 5 and 6 respectively.

The following considerations hold:

i) when the higher order multipoles are added to the sextupole components the tune shift with amplitude increases substantially: the initial amplitude for which both the horizontal and the vertical tune shifts are below $5 \times 10^{-3}$ decreases from $7.2 \times \sigma$ (in cases 6) to $4.1 \times \sigma$ (in cases 1);

ii) the octupole component has a rather large effect; by setting it to zero a substantial decrease of the tune shift with amplitude is obtained;
iii) the lower order multipoles, namely $b_3$ and $b_4$, determine the tune shifts with amplitude at the lower values of the initial amplitudes;

iv) the higher order multipoles, namely $b_5$, $b_7$, $b_9$, influence the tune shifts with amplitude mainly at the higher values of the initial amplitude.

To confirm the last two considerations, two other cases are considered, the results of which are shown in figs. 7 and 8 namely:

case 7: only the systematic sextupole and octupole components are active;

case 8: only the systematic 10th-, 14th-, and 18th-pole components are active.

Comparisons of the numerical results with the analytical expectations are made in the appendix for a simplified test lattice in which only the systematic sextupoles or the systematic octupoles are active.

5. Effects of changing the sign of the octupolar systematic component

The results of the appendix show that sextupole and octupole systematic components of the same sign shift the tune with amplitude in opposite directions, and this may give to some extent a mutual compensation of their effects. Under these conditions it is interesting to evaluate if the stability of the particles motion in the LHC can be improved by changing the sign of the systematic octupole component of Table 1 to make it of the same sign as the systematic sextupole. The results of our simulations are discussed below.
When all the systematic multipoles are active, and the random components are switched off, the tune shift with amplitude changes sign under the effect of the octupoles, which overcome the effects of all the other multipoles. The resulting pattern is shown in figs. 9 and 10. Indeed the tune shift decreases in the horizontal plane and increases in the vertical one. Nevertheless, the initial amplitude for which both the horizontal and vertical tune shifts are below $5 \times 10^{-3}$ increases to $6.4 \times \sigma$.

On the contrary, when the random components are added and the dynamic aperture is computed, see fig. 11, the stability limit for 90% of the machines decreases to $9.1 \times \sigma$.

Because of these contradictory results, we believe it unnecessary to modify the octupole strength of Table 1.

6. Conclusions

Numerical simulations have been presented, the result of which allow to evaluate with a high statistical significance the dynamic aperture of the on-momentum particles. The higher order multipoles components, namely the 10th-, the 14th- and the 16th-poles only affect the higher emittance particles; the sextupole and the octupole components determine the quasi-parabolic shift of the tunes at the lower emittances.

7. Acknowledgements

We gratefully acknowledge the suggestions and the comment of M. Bassetti, R. Dilao, J. Gareyte and E. Keil.
APPENDIX

Tune shift with amplitude due to the sextupole only or to the octupoles only.

To check the consistency of our computations, rather simple cases have been considered for which analytical evaluations of the tune shift on amplitude are available. In this context, we used an 8 times shorter test lattice with 100 m long cell on which the strength of the chromatic sextupoles was set to zero. We considered two cases:

Case 1: only the systematic sextupole component is active:

In this case the analytic expression of the tune shift on amplitude is [5]:

\[ \Delta v = -\frac{5}{12} \frac{1}{(2\pi)^2} \frac{1}{\nu} \frac{1}{\beta^3} \left( \frac{K_2}{2} \right)^2 n d \ell_d \epsilon \]

The parameters of our test lattice are:

- horizontal tune: \( \nu_H = 7.785 \)
- average \( \beta' \): \( \bar{\beta} = 83.3 \) m
- pick \( \beta_x \): \( \hat{\beta}_x = 169.5 \) m
- sextupole strength: \( K_2 = -2.4 \times 10^{-4} \) m\(^{-3}\)
- total no. of dipoles: \( n_d = 220 \)
- dipole length: \( \ell_d = 10.04 \) m
- emittance: \( \epsilon = \frac{x_0}{\beta} \) with \( x_0 \) in mm

Thus the horizontal tune shift is

\[ \delta \nu_H = -55.09 \epsilon \]

By our numerical simulation we obtain

\[ \delta \nu_H = -58.49 \epsilon \]
A result in nice agreement with the analytical expectation. When the chromatic sextupoles are set on, the analytic expression given above is no longer valid due to the higher harmonics of the sextupolar driving term in the equation of motion.

Case 2: only the systematic octupole component is active.

In this case, the analytic expression of the horizontal tune shift on amplitude is:

\[ \delta \nu_H = \frac{1}{32 \pi} K_3 \bar{B}^2 n_d \alpha_d \epsilon \]

With \( K_3 = .01 \text{ m}^{-4} \) we obtain:

\[ \delta \nu_H = 15.23 \epsilon \]

By our numerical simulation we obtain:

\[ \delta \nu_H = 18.43 \epsilon \]

A result in nice agreement with the analytical expectation.

References


Dynamic aperture due to multipole
cell length 100 m, ph.advance 90 deg.

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n.of stable machines

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Fig. 1.
Dynamic aperture due to multipole cell length 120 m, pH advance 90 deg.
Dynamic aperture due to sextupoles

cell length 100 m, ph. advance 90 deg.

Fig. 3
Cell: 120 m long, 90 deg ph. advance.
Dynamic aperture with sextupoles only.
Cell length 100 m, ph. advance 90 deg.

Tune shift on amplitude
Tune shift on amplitude

cell length 100 m, ph. advance 90 deg.

all normal multipoles  b3 b4  b5 b7 b9

only  only

horizontal tune shift [10⁻³]
Tune shift on amplitude

cell length 100 m, ph.advance 90 deg.

all m-poles  \( b_4 = 0.01 \)

all m-poles  \( b_4 = -0.01 \)

horizontal tune shift \([10^{-3}]\)
Dynamic aperture due to multipole cell length 100 m, ph.advance 90 deg.

all m-poles \( b_4 = +0.01 \)

all m-poles \( b_4 = -0.01 \)

n.of stable machines

\( X_0 \) [mm]