SOME QUESTIONS CONCERNING THE LOW-ENERGY LIMIT
OF SUPERSTRING THEORIES

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ABSTRACT

Certain classical symmetries in superstring theories prevent us from understanding some relevant parameters like the magnitude of gauge coupling constants and the compactification scale. Resulting low-energy approximations are therefore not unambiguously defined. We investigate the breakdown of these symmetries and clarify which properties of the low-energy theory are artifacts of the classical approximation. Such a discussion has consequences for the relation between gauge coupling constants and the magnitude of the supersymmetry breakdown scale.

These lectures were devoted to a review of four-dimensional $N = 1$ supergravity models and their possible connection to a low-energy limit of superstring theories. Most of the material presented exists already in a written version\textsuperscript{1}), and I shall only expand here the discussion on the role of classical symmetries. The presence of these classical symmetries is related to open questions concerning the definition of a low-energy approximation of superstring theories. One question is the compactification of six spatial dimensions from $d = 10$ to $d = 4$. The simplest possibility would be a (twisted) torus compactification and this seems to be consistent with all presently-known constraints from the string theory. In the resulting $d = 4$ theory, one then obtains
several classical scale invariances due to the fact that the radii of
the torus are not fixed at the classical level, and this reflects
itself in a degeneracy of the vacuum state. The statement remains true
also in the case of more complicated ways of compactification, such as
Calabi-Yau manifolds, for example. An additional scale invariance
comes from the fact that in string theory the gauge coupling constant
is related to the vacuum expectation value of the dilaton field which
again, classically, remains undetermined.

At the classical level we thus do not understand the basic para-
eters of the low-energy limit of string theories, such as gauge coup-
ling constants and the compactification scale. Nonetheless, most
investigations of "superstring phenomenology" are based on this classi-
cal approximation, and one might ask the question whether there appear
some artifacts of the approximation which are not shared by the full
theory. Only through understanding the breakdown of these symmetries
shall we be able to answer these questions.

Here we shall investigate the breakdown of these classical scale
symmetries through quantum corrections and compare our results to
those obtained by strictly classical considerations. We shall see
that many aspects of the classical results change significantly. Our
analysis will be at the level of perturbation theory, and only at the
end shall we comment on non-perturbative effects. For well-known
reasons, we will restrict ourselves to the framework of N = 1 super-
gravity in four dimensions, since this, in addition, has a simple
structure which will allow us to make non-trivial statements. The
results from this analysis can be summarized as follows:

i) Several pseudoscalars receive axion-like couplings not present at
the classical level.
ii) While in the classical case the gauge coupling constants of the hidden and observable sectors are necessarily equal, they can be (significantly) different in the full theory \(^7\),\(^8\).

iii) Models with gravitino mass comparable to the Planck scale \(^9\) will not lead to traces of supersymmetry in the TeV region \(^8\).

iv) There does not exist a meaningful relation between the existence or breakdown of certain axial symmetries \(^10\) and the question of a vanishing or non-vanishing cosmological constant.

The statements (i-iii) had already been deduced by inspection of the anomaly cancellation terms \(^5\)-\(^8\),\(^11\)-\(^15\). This interpretation had been heavily rejected \(^16\) with arguments that are not necessarily convincing. The symmetry considerations presented here clarify this debate. A more general discussion of point (iv) can be found in Refs. 17 and 18).

Let us begin with the classical considerations. The relevant bosonic fields can be divided into three classes, S, T and C, where we use the notation of Refs. 3) and 7), i.e.,

\[
S = \phi^{-3/4} \exp(3\sigma) + i \theta
\]

\[
T = \phi^{3/4} \exp(\sigma) + CC^* + i \eta
\]

are gauge singlets and the C's are those fields that transform non-trivially under the gauge group (e.g., 27 of E\(_6\)). \(\phi\) is the dilaton field and \(\sigma\) is the fluctuation of the overall size of the compact manifold, i.e., \(\exp(\sigma)\) defines the radius of compactification in units of the Planck length. There could be several T fields, but we shall restrict our discussion to the case of only one; including more of them is straightforward. The same applies to the C field. The pseudoscalar fields (hereafter called axions) have their origin in the zero modes of the antisymmetric tensor field \(B_{MN}\). Since \(B\) only couples through its
field strength, Θ and η will only have derivative couplings, at least for perturbative considerations. This has important consequences. There will be two axial U(1) symmetries corresponding to a shift of S and T by an imaginary constant\(^5\). Furthermore, S and T cannot appear in the superpotential which is an analytic product of superfields and does not contain derivative couplings\(^19\). Finally we have to consider two classical scale invariances corresponding to the vacuum degeneracy discussed earlier. In \(d = 4\) they read\(^{20,4,1}\)

\[
\begin{align*}
\partial_{\mu} &\rightarrow t^{-4} \partial_{\mu}, \\
S &\rightarrow t^{-4} S, \\
T &\rightarrow T, \\

\end{align*}
\]

with the classical action scaling like

\[
\mathcal{L} \rightarrow t^{-4} \mathcal{L}
\]

and

\[
\begin{align*}
S &\rightarrow r^{-\frac{1}{2}} S, \\
T &\rightarrow r^{-\frac{1}{2}} T, \\
C &\rightarrow r^\frac{1}{4} C, \\

\end{align*}
\]

with

\[
\mathcal{L} \rightarrow r^{-\frac{1}{2}} \mathcal{L}
\]

These four symmetries give strong constraints for the \(d = 4, N = 1\) supergravity Lagrangian\(^4\). The general Lagrangian\(^21\), including terms up to two derivatives, is defined by two functions: the gauge kinetic function \(f(\phi_i)\), which is analytic in the left-handed superfields \(\phi_i\), and the Kähler potential
\[ C_i(\phi_i, \phi_i^*) = K(\phi_i, \phi_i^*) + \log |W(\phi_i)|^2 \]  

where \( W(\phi_i) \) is the superpotential. We limit ourselves here to the action up to two derivatives because these are the leading terms in an expansion of the slope parameter \( \alpha' \) and the dominant terms at low energies. New terms that appear in \( \alpha' \) perturbation theory are not expected to break the considered symmetries, which can be verified in first order. This, however, seems to be no longer true for non-perturbative effects, which we shall discuss later. The easiest way \(^7\) to extract \( f \) and \( G \) from an action given in component fields is given by the gauge kinetic terms

\[ e_4 \left( \text{Re} \, F_{\mu \nu} F^{\mu \nu} + \text{Im} \, \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu} F_{\rho \sigma} \right) \]

and the gravitino mass term

\[ e_4 \exp \left( \frac{G}{2} \right) \frac{1}{2} \gamma_{\mu} \gamma^5 \gamma^\nu \]

The transformations (2)-(5) then imply

\[ \begin{align*}
\text{Re} \, f & \quad \longrightarrow \quad t^4 \, r^{-1/2} \, \text{Re} \, f \\
\exp \left( \frac{G}{2} \right) & \quad \longrightarrow \quad t^{-2} \, r^{1/4} \, \exp \left( \frac{G}{2} \right)
\end{align*} \]

In addition, we deduce from the axial symmetries that the imaginary parts of \( S \) and \( T \) are unphysical and thus should not appear in \( \text{Re} \, f \) and \( G \). This then leads to \(^4\)

\[ f = S \]

and
\[ G = - \log (S + S^*) - n \log (T + T^* - 2|C|^2) + \log |W(C)|^2 + \bar{K} \left( \frac{C}{\sqrt{T + T^*}}, \frac{C^*}{\sqrt{T + T^*}} \right) \] (11)

with an arbitrary function \( \bar{K} \). The symmetries imply that the superpotential \( W \) only depends on the \( C \) fields and that it is homogeneous of degree \( n \). To fix \( n \), symmetry considerations are not sufficient. But since \( W \) comes from the trilinear part\(^2\) of the \( d = 10 \) Chern-Simons term, we know that \( n = 3 \). The reason for \( f \) and \( W \) being restricted so strongly by symmetry considerations comes from the fact that these functions are holomorphic in the left-handed superfields, which is not true for \( G \). In the absence of \( S \), (11) gives the most general \( G \) that leads to a potential with flat directions\(^3\). Including \( S \), this statement might also be true, but needs to be clarified. For \( \bar{K} = 0 \), one obtains a so-called no-scale model where the scalar kinetic terms exhibit \( SU(1,1) \times SU(n,1) \) symmetry\(^9\). These big symmetries are, however, broken by the presence of interactions, and the only relevant symmetries are those discussed earlier. Examples indicate that the case \( \bar{K} = 0 \) corresponds to orbifold compactification\(^3\). More complicated compact spaces, for example Calabi-Yau spaces, will lead to non-trivial \( \bar{K} \). In any case, \( \bar{K} \) will receive non-trivial contributions in \( \alpha' \) perturbation theory\(^{24} \).

One could now try to extract some information from (11), keeping in mind that such an approximation would not allow us to determine the compactification scale. If we just consider the truncated theory, we treat the limit \( M_\mathcal{C} \to \infty \) where \( M_\mathcal{C} \) is the compactification scale, inversely proportional to \( R_\mathcal{C} \sim \exp(\sigma) \). On the other hand, \( R_\mathcal{C} \) will be determined by the vacuum expectation values of \( S \) and \( T \). The consistency of the approximation therefore requires compatibility with \( M_\mathcal{C} \to \infty \), i.e., \( T \to 0 \). This is certainly true for the classical expression due to the vacuum degeneracy. It remains true even after including one-loop radiative corrections. The effective potential, however, becomes unbounded
from below at $T = 0$ in this truncated theory$^{25),26}$). This just tells us that the massive modes will be important for the understanding of the compactification scale. The truncated no-scale model is just inconsistent as a $d = 4$ field theory, and can only be regarded as an approximation to a more complete theory.

A discussion of the question of supersymmetry breakdown is also influenced by these vacuum degeneracies at the classical level. Given the gauge coupling constant, one would have SUSY breakdown by hidden-sector gaugino condensation and definite predictions for the parameters in the low-energy theory$^{22)}$. Since the gauge coupling constant is not determined in the classical approximation, the supersymmetry breakdown scale is undetermined$^{27)}$. Moreover, in this special case, the observable sector remains completely supersymmetric. This fact is at the basis of models with gravitino mass equal to the Planck mass and nonetheless observable-sector mass splittings in the TeV range$^9)$. The classical model, however, is inconsistent and one has to worry whether such things can happen. This accidental supersymmetry in the observable sector crucially depends on the form of the $f$-function$^7)$ and is only true for the classical result $f = S$. Any deviation from this form would lead to a transmission of SUSY breakdown to the observable sector with a scale that is not small compared to the gravitino mass. This situation is actually expected in general if one argues in the $d = 10$ theory. In principle we could imagine compactifications, which lead to $N = 4$ supergravity in $d = 4$. For the reasons explained above we are, however, interested in obtaining $N = 1$ supergravity, i.e., three of the four gravitini become massive at the Planck scale. The last one should have a smaller mass to be relevant at low energies.

We have seen that the classical symmetries prevent us from understanding some relevant parameters in the low-energy limit of the string. We thus have to understand how these symmetries are broken. While the axial symmetries are expected to survive the perturbative expansion and could possibly only be broken by non-perturbative effects, the situation for the symmetries (2)-(5) is different. Already at the
classical level the action is multiplied by a constant, indicating a breakdown at the quantum level. We can actually be more specific about the way this will happen \(^2\) in the loop expansion since the new counter-terms will behave homogeneously loop by loop. In this context the loop expansion parameter can be identified with \(ST^{-1}\) such that the new counterterms in the \(n\)-loop effective action will scale as

\[
\mathcal{L}_{n\text{-loop}} \rightarrow t^{4(1-n)} r^{n-\frac{1}{2}} \mathcal{L}_{n\text{-loop}}
\]

Thus the classical result for \(f\) and \(G\) as given in (10) and (11) will be modified in perturbation theory. For the behaviour of \(f\) and \(G\), (12) implies

\[
f_n \rightarrow t^{4(1-n)} r^{n-\frac{1}{2}} f_n
\]

\[
\exp\left(\frac{g}{2}r\right) \rightarrow t^{-2(1+2n)} r^{n+\frac{1}{4}} \exp\left(\frac{g}{2}r\right)
\]

as can be derived from (7) and (8) using the transformation properties (2) and (4). Let us first discuss \(f = \Sigma f_n\) with \(f_0 = S\), the classical result. At one loop \(f_1 \rightarrow t_0^{\frac{1}{2}} f_1\), and we see that \(S\) cannot appear since it is the only scalar particle that transforms non-trivially under (2). \(f_1\) has the same transformation behaviour as \(T\) and \(G^2\) possibly multiplied by a function that is inert under (2) and (4). The axial symmetry implies that the real part of \(f\) should be independent of the imaginary part of \(T\), and \(f\) should be an analytic function of the fields. This implies \(f_1 = aT + bc^2\) where \(a, b\) are constants that cannot be determined by the symmetry consideration. At higher loops, \(f_n\) scales with an inverse power of \(t\) and possible counterterms have to scale with \(S^{-n}\). The axial symmetries do not allow such terms and we obtain a non-renormalization theorem for \(f\) beyond one loop, i.e., its final form is given by
\[ f = S + \epsilon (T + \alpha C^2) \]

Observe that this non-renormalization theorem is not in contradiction with the running of the coupling constant. The coupling constant is not given directly by \( f \) but by its vacuum expectation value, which is determined from the minimum of the potential, and this receives non-analytic pieces even if \( f \) is unrenormalized.

Nonetheless, \( f \) receives new contributions at one loop, and this leads to significant physical consequences:

i) Unlike the classical case, \( T \) now couples to the gauge field strength. In particular, this implies that the pseudoscalar \( \eta \) couples as an axion. An investigation of the Calabi-Yau compactification scheme shows furthermore \(^6\),\(^7\),\(^11\) that \( \eta \) couples differently to the hidden and observable sectors, and thus the axion pair \( \theta \) and \( \eta \) could solve the strong CP problem of \( E_8' \) and QCD. Here, however, it has still to be checked whether the corresponding axial symmetries are only broken by gauge instantons.

ii) As well as the axions, the scalars in \( S \) and \( T \) do now couple differently to the hidden- and observable-sector gauge fields \(^7\),\(^8\). Since the corresponding gauge coupling constants are determined by the vevs of \( S \) and \( T \), they can now be different from each other, contrary to the classical case. This might have consequences for the condensation scale of \( E_8' \) and the magnitude of supersymmetry breakdown.

iii) There now exist two axion-dilaton pairs, and this might generalize the mechanism of relaxation of the cosmological constant to the observable sector in the same way as it happens in the hidden sector \(^11\).
iv) The new terms in (14) lead to an induced breakdown of supersymmetry in the observable sector once it is broken in the hidden sector, with magnitude $\epsilon m_{3/2}$. A gravitino mass of the order of the Planck mass thus will not lead to traces of supersymmetry in the low-energy region. This might also be relevant for the question whether superstring models lead to an intermediate scale for gauge symmetry breakdown.

As already indicated, most of these consequences had been deduced by an investigation\(^5\)-\(^8\),\(^11\)-\(^15\) of the anomaly cancellation counter-terms found by Green and Schwarz\(^28\). The symmetry consideration supports this interpretation.

While $f$ remained simple in the loop expansion, this is not expected for $G$ because of its non-analyticity. Using (2), (4) and (13), we obtain\(^2\)

$$G = -\log (S + S^*) - 3 \log (T + T^* - 2|c|^2) +$$

$$+ \log |W(c)|^2 + K \left( \frac{c}{T + T^*}, \frac{c^*}{T + T^*} \right) +$$

$$+ 2 \log \left( 1 + \sum_{n=1}^{\infty} a_n \epsilon^n \left[ \frac{S + S^*}{T + T^*} \right]^n \right)$$

(15)

where $\epsilon$ is the expansion parameter corresponding to a coupling constant and the $a_n$ are numerical coefficients. Because of its complexity, it is not yet clear what this implies for the scalar potential. Discussed in the context of unbroken supersymmetry, one might conjecture this to be the most general Kähler potential leading to a potential with two flat directions. While this has been checked at the one-loop level\(^8\), it might not necessarily be true at higher loops, and has to be clarified. The inclusion of supersymmetry breakdown changes the situation. Here one might see a chance to fix both the vacuum expectation values of $S$ and $T$ (in the classical case only one vev was fixed under these circumstances). This would imply that the gauge coupling constant and
compactification scale can only be understood once supersymmetry is broken. If we could handle the complexity of (15), some of these questions might be answered.

In any case, we have also to consider different mechanisms that lead to the breakdown of the symmetries in a non-perturbative way. Effects of gauge instantons are believed to break the axial symmetries and are also responsible for the breakdown of supersymmetry in the hidden sector\textsuperscript{22},\textsuperscript{27}. While the hidden $E_8$ plays this rôle for the symmetry connected to $S$, QCD-instantons might be the source of breakdown of the second axial symmetry and lead to a solution of the strong CP problem via the axion mechanism, barring some cosmological problems. Qualitatively, the influence of such instanton contributions can be described through the introduction of terms proportional to $\exp(-S)$ and $\exp(-T)$ in the superpotential. Superficially one might think that the inclusion of such terms destabilizes the scalar potential leading to a situation where some vev's run to infinity, but the simplest example shows that such effects can be compensated\textsuperscript{27}. Actually, in this case, we even find a mechanism for a relaxation of the cosmological constant by a sliding field, in a way similar to the relaxation of the $\theta$ angle through an axion. An explicit discussion of such a mechanism does, however, require a better understanding of the effective scalar potential. Observe only that such a mechanism is completely different from the one based on an axial $U(1)$ symmetry\textsuperscript{10}, which in any case seems not to work since the axial symmetries are broken.

The influence of world-sheet non-perturbative effects could also be significant\textsuperscript{13},\textsuperscript{14},\textsuperscript{29},\textsuperscript{30}. World-sheet instantons have consequences similar to those coming from gauge instantons, with the difference that they only affect the axial symmetries corresponding to the $T$ fields. This cannot happen for $S$, since its pseudoscalar originates from the antisymmetric tensor $B_{\mu\nu}$ with both indices in $d = 4$ Minkowski space. The breakdown of these axial symmetries depends strongly on special properties of the six-dimensional compact manifold\textsuperscript{14} and a general
statement cannot be made. For some manifolds these symmetries are already broken at the classical string level, while for others they are not broken at all.

Like the gauge instantons, such effects might give contributions to the superpotential proportional to \( \exp(-T) \). If all \( T \) fields receive such a contribution at the classical string level, one would expect the corresponding pseudoscalars to become heavy and they would no longer be candidates for the solution of the strong CP problem. But these questions have to remain open until we understand better the process of compactification and its influence on the breakdown of these axial symmetries.

One might fear that the terms proportional to \( \exp(-T) \) induced in the superpotential destabilize the vacuum in an undesired way\(^{14},30\). But such terms appeared already as a consequence of the gauge instantons. Whether such terms are dangerous or even desirable can only be decided after an inspection of the scalar potential. In any case, we need a destabilization of the classical vacuum to understand the relevant parameters of the low-energy limit, and it seems to us that a combination of loop effects and non-perturbative effects could lead to a better understanding of these questions. At the moment, it is the complexity of the Kähler potential (15) that prevents us from discussing these issues explicitly, and any progress in this direction is welcome.

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REFERENCES

9) For a review, see:


24) For a review, see:


