A Study of the Hadrons Produced in Deep Inelastic Muon Proton Scattering

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Abstract

Results are presented on the hadrons produced in 280 GeV $\mu^+\mu^-$ interactions. The experiment was performed at CERN using the E.M.C. spectrometer. The results presented in this thesis were obtained from a sample of $\approx 16,000$ events.

By studying the angular separation between hadrons in the plane transverse to the virtual photon it is shown that transverse momentum is locally conserved in the fragmentation chain. The values of the Lund model parameters $\sigma_q$ and $\sigma_{k_1}$ are estimated to have the values

$$\sigma_q = (0.41 \pm 0.02) \text{ GeV}/c,$$

and

$$\sigma_{k_1} = (0.27^{+0.12}_{-0.20}) \text{ GeV}/c.$$

A comparison is made between hadronic spectra obtained in deep inelastic scattering and $e^+e^-$ annihilation. The results support the parton model assumption of environmental independence. The differences observed are shown to be attributable in part to the production of heavy quarks in $e^+e^-$ annihilation, and in part to the QCD corrections to the parton model for these reactions. The effect that the target jet has on the determination of the jet axis is also investigated. Finally, a comparison is made between the properties of hadrons produced in deep inelastic $\mu\mu$ scattering and the predictions of a QCD branching model of hadronisation.
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As this work drew to a close I was forced to draw upon a number of people for information and papers from C.E.R.N. - I am grateful to all these people for their cooperation.

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INTRODUCTION.
The European Muon Collaboration (E.M.C.) is a group of about 130 European physicists. The collaboration formed in 1974 to perform muon scattering experiments using the muon beam at the CERN super-proton synchrotron (SPS). Various topics in deep inelastic muon nucleon scattering have been addressed in a series of experiments.

The first phase of the experiment (NA2) used a 6m hydrogen or deuterium target and a large magnetic spectrometer to study the nucleon's substructure. Measurements were also made on various nuclear targets (including iron, copper and carbon). In addition to measuring the structure functions of protons and neutrons the experiment observed unexpected behaviour in the structure functions measured in iron. This experiment also studied the hadrons produced in deep inelastic muon scattering. The experiment could only detect those hadrons produced with momentum greater than 6 GeV/c in the laboratory system. This corresponded roughly to the forward hemisphere in the centre of mass system.

The NA2 spectrometer suffered from two major problems as far as the study of hadronic final states is concerned. Firstly, the experiment could not detect all the charged hadrons produced in an event, and secondly the single Cherenkov counter used could only provide limited particle identification. The next phase of the experiment (NA9) was designed to meet these deficiencies. The aims of the NA9 experiment were to detect, and identify, as many of the charged particles as possible. To do this the 6m target was replaced by a 1m hydrogen or deuterium target, surrounded by a streamer chamber and a large superconducting magnet. In addition, the new spectrometer was instrumented with a series of wire chambers and Cherenkov counters. The results

1 European Organisation for Nuclear Research, Geneva, Switzerland.
presented in this thesis come from the NA9 experiment.

The E.M.C. has also been involved in measuring the structure functions of the nucleons in various nuclei when the muon is scattered through a very small angle (the NA28 experiment). At present the E.M.C. is engaged in measuring the structure functions of protons in polarised muon proton scattering.

This thesis is organised as follows. Chapter 2 contains a brief historical review of deep inelastic scattering, and an introduction to the leptoproduction of hadrons. A description of the apparatus is given in chapter 3. Chapters 4 and 5 give an introduction to the software used to reconstruct the events recorded by the apparatus, and the Monte Carlo simulation of the apparatus respectively. The criteria on which the final event sample is selected are described in chapter 6. In chapter 7, the assumption that transverse momentum is locally conserved is tested and various parameters of the Lund model are estimated. In chapter 8 properties of the hadrons produced in deep inelastic scattering are compared with the properties of hadrons produced in $e^+e^-$ annihilations, in order to test the parton model assumption of environmental independence. Departures from the quark parton model predictions are studied. Finally, in section 8.4 a comparison is made between the results of deep inelastic $\mu p$ scattering and a QCD based model for hadronisation.
2.1 Introduction.
This chapter contains an introduction to the theoretical results and ideas used in this thesis. In the first section a brief history of deep inelastic scattering is presented, followed by a discussion of the main results and their implications. There follows a sketch of the proton's substructure, leading to a discussion of hadron production from deep inelastic muon proton scattering.

2.2 Kinematics and cross-sections.
The use of leptons to study the physics of hadrons relies on the assumption that leptons are point-like Dirac particles obeying the standard electroweak theory. The interaction of charged and neutral leptons have been shown to be in good agreement with the standard electroweak theory. In particular, it has been shown that quantum electrodynamics may be used to explain the interactions of charged leptons.

![Feynman Diagram](image)

Figure 1: The first order Feynman diagram for muon-proton scattering.

The first order Feynman diagram for muon proton scattering is shown in figure 1. The muon is point-like, and couples directly to the virtual photon (γ*) at vertex 1. The proton is not point-like, and the virtual photon couples to its charged components. The details of the interaction between the photon and the
proton are represented by the shaded circle. With the particle momenta as shown in figure 1, the following Lorentz invariant quantities may be defined:

$$Q^2 = -q^2 = -(k' - k)^2 = 4E_\mu E'_\mu \sin^2(\theta/2) \quad (\text{for } Q^2 > m_e^2), \quad (2.1)$$

$$v = \frac{P \cdot q}{M} = E_\mu - E'_\mu, \quad (2.2)$$

where $M$ is the mass of the proton. The momentum transfer squared $q^2$ ($-Q^2$) is always negative (space-like), and in the laboratory system $v$ is the energy of the virtual photon and $\theta$ the scattering angle.

The invariant mass of the hadronic final state, $W$, is given by

$$W^2 = s = 2Mv + M^2 - Q^2. \quad (2.3)$$

For elastic scattering (i.e. $\mu p - \mu p$) $W^2 = M^2$, and so $Q^2 = 2Mv$. In inelastic scattering this no longer holds, and $Q^2$ and $v$ are independent, subject to the condition that $Q^2 < 2Mv$. This may be seen directly from equations 2.1 and 2.2 where $v$ is a function of $E_\mu'$ only, whereas $Q^2$ is a function of both $E_\mu'$ and $\theta$.

In the one photon exchange approximation, the matrix element for inclusive deep inelastic scattering (figure 1) may be written as follows

$$|M|^2 = e^2 L_{\mu\nu} (1/Q^4) e^2 W^{\mu\nu} \mu, \nu = 1 \ldots 4. \quad (2.4)$$

The virtual photon propagator contributes a factor $1/Q^4$, while the couplings at the two vertices each give a factor of $e^2$ (where $e$ is the electric charge of an electron). The kinematics of the scattering at the lepton vertex is contained in the tensor $L_{\mu\nu}$, while the nucleon structure functions are contained in $W^{\mu\nu}$.

For experiments in which the target protons are unpolarised, one need only consider the spin averaged cross-section. When averaged over incident lepton spins, and summed over final spin states, the lepton tensor may be written as

$$L_{\mu\nu} = 2(k' \cdot k_\mu k_\nu ' \cdot q^2 g_{\mu\nu}/2), \quad (2.5)$$

where $g_{\mu\nu}$ is the metric tensor ($g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$, and $g_{\mu\nu} = 0$ for $\mu \neq \nu$). Note that $L_{\mu\nu}$ contains no terms anti-symmetric in $\mu$ and $\nu$ since such terms would correspond to parity violation, or an explicit spin dependence.
The discussion of the structure of $W^\mu\nu$ may be begun by noting that $W^\mu\nu$ must be symmetric. One can form four symmetric Lorentz tensors from the available four vectors ($p_\mu$, $q_\mu$) and $g^\mu\nu$. However, only two of the three Lorentz scalars are independent when one imposes the condition that the electromagnetic current should be conserved (i.e. $q_\mu W^\mu\nu=q_\nu W^\mu\nu=0$). Since the Lorentz structure of $W^\mu\nu$ is obtained by construction, the two form factors must be Lorentz scalars. So, the form factors can only be functions of $Q^2$ and $v$ - the two independent (non-trivial) Lorentz scalars available. One obtains\[9\] the following expression for $W^\mu\nu$

$$W^\mu\nu = \left[-g^\mu\nu + \frac{q^\mu q^\nu}{Q^2}\right]W_1(Q^2,v) + \left[p^\mu - \frac{q^\mu}{Q^2}\right]\left[p^\nu - \frac{q^\nu}{Q^2}\right]W_2(Q^2,v). \quad (2.6)$$

Equations 2.4,2.5 and 2.6 may be used to derive an expression for the cross-section which may be written in the form\[9\]

$$\frac{d^2\sigma}{dQ^2dv} = \frac{4\pi\alpha^2}{Q^4} \frac{E\mu'}{E\mu} \left[2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2) \right]. \quad (2.7)$$

To obtain equation 2.7 the assumption has been made that the mass of the lepton may be neglected with respect to $\sqrt{Q^2}$.

Two form factors appear in equation 2.7 because there are two independent virtual photo-absorption cross-sections. Since the virtual photon has a mass, it has three polarisation states, and one may therefore define three virtual photon absorption cross-sections. However, the two longitudinal cross-sections must be equal since the electromagnetic interaction conserves parity. The total cross-section can therefore be written\[10\] in terms of the absorption cross-section for transversely and longitudinally polarised photons $\sigma_T(Q^2,v)$, $\sigma_L(Q^2,v)$ respectively as follows\[10\]

$$\frac{d^2\sigma}{dQ^2dv} = \psi[\sigma_T(Q^2,v) + \zeta\sigma_L(Q^2,v)] \quad (2.8)$$
where
\[ \psi = \frac{\alpha}{2\pi} \left( \frac{K}{Q^2 E^2} \right) \frac{1}{1 - \zeta} \],
\[ \zeta = \left[ 1 + 2(1 + \zeta) \tan^2(\theta/2) \right]^{-1} \]
\[ K = \frac{W^2 - M^2}{2M} \] (2.9)

\( \psi \) is the equivalent flux of transverse virtual photons, \( \zeta \) is the polarisation parameter and \( K \) is the equivalent real photon energy needed to excite the final state \( W \). One may write \( W_1 \) and \( W_2 \) in terms of \( \sigma_L(Q^2,\nu) \) and \( \sigma_T(Q^2,\nu) \) as follows
\[ W_1(Q^2,\nu) = \frac{K}{4\pi^2 \alpha} \sigma_T(Q^2,\nu) \],
\[ W_2(Q^2,\nu) = \frac{K}{4\pi^2 \alpha} \frac{Q^2}{(Q^2 + \nu^2)} \left[ \sigma_T(Q^2,\nu) + \sigma_L(Q^2,\nu) \right] \] (2.10) (2.11)

One may also define \( R \) the ratio of \( \sigma_T(Q^2,\nu) \) and \( \sigma_L(Q^2,\nu) \) as follows
\[ R \equiv \frac{\sigma_L(Q^2,\nu)}{\sigma_T(Q^2,\nu)}. \] (2.12)

2.3 The discovery of proton substructure.
Early electron scattering experiments (see reference 11) showed that the cross-section for elastic ep scattering was a swiftly decreasing function of \( Q^2 \) \( (d^2\sigma/dQ^2 d\nu \propto 1/Q^{12}) \). In the late 1960s experiments\( ^{12-13} \) at SLAC showed that for inelastic scattering with \( W \geq 2.5 \) GeV the cross-section was weakly \( Q^2 \) dependent. The dramatic difference between elastic and inelastic scattering above the resonance region is shown in figure 2. Here the double differential total cross-section divided by the point-like Mott cross-section \( (d\sigma_{\text{Mott}}/d\Omega)^2 \) is plotted as a function of \( Q^2 \). The \( Q^2 \) dependence introduced into the total cross-section by the virtual photon propagator is thus removed, and the ratio

\[ \sigma_{\text{Mott}} \] by\( ^{12} \)
\[ d\sigma_{\text{Mott}}/d\Omega = (a^2 \cos^2(\theta/2))/(4E^2 \sin^2(\theta/2)) \] where \( E \) is the beam energy.
becomes only weakly $Q^2$ dependent.

![Graph showing $d^2\sigma/d\Omega dE'$ versus $Q^2$. $E'$ is the energy of the scattered electron. The data are taken from reference 12. Also shown is the $Q^2$ dependence of the elastic cross-section for the mean beam energy and mean scattering angle.]

Figure 2: $d^2\sigma/d\Omega dE'$ versus $Q^2$. $E'$ is the energy of the scattered electron. The data are taken from reference 12. Also shown is the $Q^2$ dependence of the elastic cross-section for the mean beam energy and mean scattering angle.

It was known that such behaviour was a possibility.\cite{14} If the proton contains point-like constituents (partons), each parton would contribute a term to the cross-section. The $Q^2$ dependence of this contribution would be determined solely by the virtual photon propagator, since a point-like parton would have no form factor.
The existence of point-like constituents of the proton may also be inferred from the behaviour of $W_1$ and $W_2$ as $Q^2, v \to \infty$. In 1969\textsuperscript{[15]} Bjorken postulated that as $Q^2, v \to \infty$ with $x$ finite, the $W_i$ would become functions of $x$ only:

\begin{align*}
MW_1(Q^2, v) &= F_1(x), \quad (2.13) \\
vw_2(Q^2, v) &= F_2(x). \quad (2.14)
\end{align*}

Moreover, he suggested that $F_1$ and $F_2$ would be finite and non-zero. Such behaviour can only occur if there exist point-like scattering centres, since any non-point-like proton constituent would itself have a form factor with an explicit $Q^2$ dependence. Thus, the scale invariance (or scaling) of $W_1$, $W_2$, as $Q^2, v \to \infty$, implies that the partons have no structure, i.e. they are point-like. There remains the possibility that parton substructure may be revealed at very high $Q^2$.

In figure 3 $vw_2$ is plotted as a function of $x$ for various values of $Q^2$.\textsuperscript{[13]} The data appear to follow a universal curve, i.e. the data are a function of $x$ but not of $Q^2$.

![Figure 3: \(vw_2\) versus \(x\) for different values of \(Q^2\). Data taken from reference 13.](image-url)
Having established the existence of proton constituents, one may enquire into their properties. Their spin may be inferred from a determination of $R$. If the constituents have spin 0, they can not absorb a transverse photon i.e. $\sigma_T(Q^2,v)=0 \Rightarrow R=\infty$. Alternatively, if they have spin $\frac{1}{2}$ then $\sigma_L(Q^2,v)=0 \Rightarrow R=0$. The value of $R$ is found $^{[24]}$ to be close to zero, and to be independent of $Q^2$ and $v$ (see figure 4). One may conclude that partons have spin $\frac{1}{2}$ rather than spin 0.

![Figure 4: The structure function $R$. (a) As a function of $v$, and (b) as a function of $Q^2$. Data taken from reference 24.](image)

All the results which have been described in this section arise naturally in the quark parton model which will be described in section 2.6.

2.4 Validity of the one photon exchange approximation.

At the heart of all the results presented above, and indeed all the results presented in this thesis, is the one photon exchange approximation used to derive equation 2.7. The contribution to deep inelastic cross-sections of the two photon exchange graphs (see figure 5) is only calculable within the framework of some model. However, the interference between the one and two photon exchange amplitudes (figures 1 and 5) leads to a difference between the
cross-sections for the scattering of positive and negative leptons. The ratio of these two cross-sections may therefore be used as a measure of the deviation from the one photon exchange approximation.

Figure 5: Feynman diagrams of two photon exchange.

All measurements of the ratio to date have given values consistent with unity.\textsuperscript{[16-17]} One may conclude that, to a good approximation, the one photon exchange approximation holds.

2.5 QED radiative corrections to the one photon exchange approximation.

In addition to the possibility that two photons may be exchanged between the muon and the proton, the muon may radiate a real photon before, or after, the deep inelastic event (see figures 6(a) and (b)). In either case the values of $Q^2$ and $v$ calculated from the beam and scattered muon are systematically shifted from their true values. The effect is that $v$ is systematically increased and $Q^2$ systematically decreased. It is conventional to correct the results of deep inelastic charged lepton scattering experiments so that the results correspond to those which would have been obtained if the graph of figure 1 were the only contribution to the total cross-section. For this experiment, the calculation is performed using the exact formulae given in reference 18 as described in reference 19. An outline of how the calculation is performed is given below.
The calculation assumes that the one photon exchange approximation holds and that the interference between the bremsstrahlung from the hadrons and that from the muon is negligible. The only first order corrections considered are those shown in figures 6(a) and (b). To calculate the contribution of these processes to the observed cross-section ($\sigma_{\text{obs}}$), it is necessary to include the contributions from the two second order diagrams shown in figure 6(c) and (d). These two diagrams cancel divergences which arise in the calculation of the contributions from (a) and (b). The contribution of radiative elastic scattering ($\sigma_{E}$) and radiative inelastic scattering ($\sigma_{I}$) is calculated, the observed cross-section is then given by

$$\sigma_{\text{obs}} = \sigma_{\text{DI}} \ast \sigma_{E} \ast \sigma_{I}$$  \hspace{1cm} (2.15)

where $\sigma_{\text{DI}}$ is the deep inelastic cross-section obtained in the one photon exchange approximation (equation 2.7).
The size of the correction required is shown in figure 7 as a function of $y^+$ for several $x$ bins. In general the correction is small and slowly varying. The largest effects are seen at small $x$ and large $y$. The correction is particularly large for $y > 0.9$.

2.6 The quark parton model.

The quark parton model\cite{15,20} (QPM) views the proton as being composed of a number of point-like scattering centres, called partons (see figure 8). The partons may be associated with the quarks proposed to account for the spectroscopy of hadrons.\cite{21} Each parton is assumed to carry some fraction $\varepsilon$ of the proton's momentum. To describe the interaction of a high $Q^2$ virtual photon with a proton, one makes the following simplifying assumptions:\cite{20}

1. The partons do not interact with one-another during the collision. From the uncertainty principle the time scale over which the virtual photon interacts with the parton is

$$\tau = \frac{1}{q_0}$$

where $q_0$ is the energy of the virtual photon (i.e. $q_y = (q_0, q)$. A typical time scale for the interaction between partons is given by

$$\Gamma = \frac{(E_i + E_{\text{rest}} - E_p)^{-1}}$$

where $E_p$ is the energy of the proton, $E_i$ the energy of the $i^{th}$ parton, and $E_{\text{rest}}$ the energy of all partons excluding the parton $i$. If $\Gamma \gg \tau$, the partons may be regarded as free during the interaction.

2. The scattering from individual partons is incoherent. If $m_q$ is the mass of a parton, a virtual photon with $Q^2 \gg m_q^2$ will resolve the individual partons within the proton, and may be expected to scatter incoherently.

3. The parton mass is small, and does not change significantly (with respect to $Q^2$) during the collision.

$^\dagger y = \nu/E_\mu$. 
Figure 7: The size of the radiative correction to $\mu p$ scattering. The figure shows $\sigma_{\text{DI}}/\sigma_{\text{obs}}$ as a function of $\gamma$ for various $x$ bins.
4. The transverse momentum of the partons within the proton is small with respect to $\sqrt{Q^2}$.

One may now calculate the momentum fraction $\varepsilon$ as a function of the lepton variables $Q^2$ and $v$. The mass of the quark after the collision is given by

$$\left(\varepsilon P^+q\right)^2 = m_q^2$$

$$\Rightarrow \varepsilon^2 M^2 + 2\varepsilon P.q - Q^2 = m_q^2.$$  \hspace{1cm} (2.18)

Assuming both $M^2$ and $m_q^2$ are negligible with respect to $Q^2$

$$2\varepsilon P.q - Q^2 = 0$$

i.e. $\varepsilon = \frac{Q^2}{2P.q}$ \hspace{1cm} (2.19)

Equation 2.19 shows that one may associate the variable $x$ with the momentum fraction carried by the parton which absorbs the current.

Assuming that the proton is composed of point-like spin $\frac{3}{2}$ partons, and that these partons may be associated with the quarks of hadron spectroscopy, one may visualise the process of deep inelastic muon proton scattering as shown in figure 8. The cross-section for scattering from parton $i$, i.e. for the process $\mu q_i - \mu q_i'$, may be written$^9$
\[
\frac{d^2\sigma_i}{dQ^2 dv} = \frac{4\pi^2 a^2 E'_\mu}{Q^4 E'_\mu} \left[ e_i^2 \cos^2(\theta/2) + \frac{Q^2}{4m_i^2} - 2\sin^2(\theta/2) \right] x\delta(v - \frac{Q^2}{2m_i})
\]

(2.20)

where \(m_i\) is the mass of the parton \(i\) and \(e_i\) its charge.

To obtain the total cross-section one must sum the contributions from each parton. Assumption 2 allows one to do this incoherently. Since \(m_i = xM\), the \(\delta\) function in equation 2.20 may be expressed as

\[
\delta(v - \frac{Q^2}{2m_i}) = \frac{2Mx^2}{Q^2} \delta(x - \frac{Q^2}{2Mv}).
\]

(2.21)

Defining \(f_i(x)\) to be the probability density function for the \(i\)th parton, one may write

\[
\frac{d^2\sigma}{dQ^2 dv} = \Sigma_i \int dx f_i(x). \quad (2.22)
\]

The integral over \(x\) is performed using equation 2.21. By comparing the results of equation 2.22 and equation 2.7 one finds that \(W_1\) and \(W_2\) may be written as follows

\[
MW_1(Q^2, v) = F_1(x) = \frac{1}{2} \Sigma_i e_i^2 f_i(x)
\]

\[
vW_2(Q^2, v) = F_2(x) = \Sigma_i e_i^2 x f_i(x)
\]

(2.23)

The structure functions are therefore functions of \(x\) only. Furthermore equations 2.23 imply that one may write

\[
2xF_1(x) = F_2(x).
\]

(2.24)

This is the Callan-Gross relation. The form of the Callan-Gross relation is a consequence of the assumption that partons have spin \(1/2\).

The small values of \(R\) shown in figure 4 may be explained by writing \(R\) as a function of \(F_1\) and \(F_2\). Combining equations 2.10, 2.11, 2.12, 2.13 and 2.14 one may write

\[
R = \frac{F_2(x)}{2xF_1(x)} \left[ 1 + \frac{Q^2}{v^2} \right] - 1.
\]

(2.25)
Thus if the Callan-Gross relation holds, one may write

\[ R < \frac{4M^2}{Q^2}, \]  

(2.26)

since \( Q^2 < 2M^2 \). Therefore, the fact that \( R = 0 \), is consistent with the assumption that partons have spin \( \frac{3}{2} \).

To make the connection between partons and the quarks of hadron spectroscopy one may write \( u_p(x) \) for the distribution function of up quarks in the proton. With an obvious extension of the notation and the assumption that isospin is conserved one may write

\[
\begin{align*}
    u_p(x) &= d_n(x) \equiv u(x) \\
    d_p(x) &= u_n(x) \equiv d(x) \\
    s_p(x) &= s_n(x) \equiv s(x) \\
    \bar{s}_p(x) &= \bar{s}_n(x) \equiv \bar{s}(x)
\end{align*}
\]

(2.27)

where the subscript \( n \) indicates that the quark distributions of neutrons are being considered. Using these definitions, one may rewrite equations 2.23 as follows

\[ F_2^P = x[\frac{1}{2}(u(x) + d(x)) + \frac{1}{2}(d(x) + \bar{d}(x))s(x) + \bar{s}(x))], \]

(2.28)

For the neutron one may write

\[ F_2^n = x[\frac{1}{2}(d(x) + \bar{d}(x)) + \frac{1}{2}(u(x) + d(x))s(x) + \bar{s}(x))], \]

(2.29)

In equations 2.28 and 2.29 it has been assumed that the contribution of charmed and heavier quarks to the structure functions is negligible. [23] If one assumes that one can write \( u(x) \) etc. as the sum of a valence and a sea contribution \( u_v(x) \) and \( u_s(x) \) respectively, i.e.

\[ u(x) = u_v(x) + u_s(x) \]

(2.30)

then equations 2.28 and 2.29 may be rewritten

\[ F_2^P = x[\frac{1}{2}(d_v(x) + 4u_v(x)) + \frac{1}{2}(d_s(x) + \bar{d}_s(x))s_s(x) + \bar{s}_s(x))], \]

(2.31)

\[ F_2^n = x[\frac{1}{2}(u_v(x) + 4d_v(x)) + \frac{1}{2}(u_s(x) + \bar{d}_s(x))s_s(x) + \bar{s}_s(x))], \]

where

\[ K(x) \equiv u_s(x) = d_s(x) = \bar{d}_s(x) = s_s(x) = \bar{s}_s(x). \]
In writing equations 2.31 an appeal has been made to the quantum numbers of the nucleons to allow one to write $s_v = u_v = d_v = s_v = 0$. The relative contribution of the valence and sea distributions may now be inferred from the ratio

$$\frac{F_2^n}{F_2^p} = \frac{u_v(x) \cdot 4d_v(x) \cdot 12K(x)}{d_v(x) \cdot 4u_v(x) \cdot 12K(x)} \tag{2.32}$$

If $K(x)$ dominates then $\mathcal{R} \to 1$, whereas if $u_v \gg d_v$ and $u_v, d_v \gg K(x)$ then $\mathcal{R} \to \frac{1}{4}$ to $\frac{1}{3}$. The data,\footnote{2} see figure 9, indicate that

$$\begin{align*}
\mathcal{R} & \to 1 \quad \text{as } x \to 0, \\
\mathcal{R} & \to \frac{1}{4} \quad \text{as } x \to 1,
\end{align*} \tag{2.33}$$

i.e. sea quarks dominate at low $x$, while valence quarks dominate at high $x$.

The total fractional momentum carried by the charged partons in the nucleon is given by

$$\int x [u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + s(x) + \bar{s}(x)] dx = 2 \int [F_2^p + F_2^n] dx. \tag{2.34}$$

Experimentally\footnote{2} this is found to be about $\frac{1}{2}$. One may conclude that approximately $\frac{1}{2}$ of the proton's momentum is carried by neutral constituents.

2.6.1 Hadron production in the quark parton model.

To describe hadron production in the quark parton model the following assumptions are made:\footnote{20}

1. The hadrons produced in the fragmentation of an energetic quark are characteristic of the quark, not of how the quark was excited.

2. The time scale over which hadrons are produced is much larger than that of the hard scattering process which excited the quark. This leads to the factorisation hypothesis whereby the functions describing the fragmentation of quarks to hadrons are independent of the functions specifying how the quarks behave inside the proton.
Figure 9: The ratio of $F_2^n$ to $F_2^P$. Data taken from reference 2.

3. If one assumes that all masses may be neglected with respect to $Q^2$, and all momenta transverse to the virtual photon may be neglected with respect to $q$ then one may describe the production of hadrons from a quark in terms of the single variable

$$z = \frac{P_{hP}}{qP} = \frac{E_h}{v}. \quad (2.35)$$

Thus, one may express the cross-section for the production of a hadron $h$ in deep inelastic scattering as
\[
\frac{1}{\sigma_D} \frac{d\sigma}{dz} = \frac{\Sigma_i \epsilon_i^2 f_i(x) D_i^h(z)}{\Sigma_i \epsilon_i^2 f_i(x)}
\]  
(2.36)

where \(\sigma_D\) is the total deep inelastic cross-section. The fragmentation function, \(D_i^h(z)\), gives the probability that a hadron of type \(h\), with an energy such that \(z\) lies between \(z\) and \(z+\delta z\), is obtained from a quark of type \(i\). The variable \(x_F\) (\(=2p_{\mu}/W\) where \(p_{\mu}\) is the component of a hadrons momentum parallel to the virtual photon direction) has also been used to describe the fragmentation of quarks.

With the above assumptions and the additional assumption that isospin is conserved, one may obtain relations between hadron production cross-sections in various reactions. A review of the data (as given for example in chapter 12 of reference 21) shows that the assumptions are satisfied to a good approximation.

2.6.2 Beyond the quark parton model.

Though the parton model describes the gross features of the data well, it is found that its predictions are not absolutely obeyed. For example \(F_2\) is in fact a function of \(x\) and \(Q^2\).\(^{[2,25]}\) If one postulates that quarks have an additional three valued degree of freedom - colour - and that the neutral partons mentioned in section 2.6 are gluons, then one may describe the deviations from the quark parton model in terms of Quantum Chromodynamics, as outlined in section 2.7.

2.7 An Introduction to Quantum Chromodynamics.

The quark parton model is in good agreement with experimental data on the spectrum of hadrons, and with the results of deep inelastic scattering. However, hadrons exist and so there must be a binding force between quarks. The QPM offers no explanation for the existence of hadrons. It is believed that the force between quarks can be described by quantum chromodynamics (QCD). QCD is a gauge field theory in which the three types of coloured quark interact
with eight types of coloured gluon. The gluons are massless, electrically neutral vector bosons. Though the principles of QCD have not been tested, the theory has a number of successes to its credit. This section contains a brief summary of the arguments in favour of the principal assumptions of QCD, and mentions some experimentally verified consequences. The theoretical arguments presented below are explained more fully in the introductory reviews contained in reference 26.

2.7.1 Assumptions of QCD.

The main assumptions of QCD are that quarks are coloured, that they couple to vector gluons and that their interactions may be described by a gauge field theory. A fundamental feature of QCD is that the gluons interact among themselves. The arguments in favour of these assumptions are outlined below.

- Colour: That quarks should possess a three valued quantum number was originally postulated to explain the observed baryon spectrum. The lightest spin \( \frac{3}{2} \) baryons are assumed to have a symmetric space wavefunction corresponding to a state with no orbital angular momentum. In order that the \( \Delta^{++} \) (for example) can be described by a totally antisymmetric wave function the quarks must possess a quantum number in addition to their spin and flavour. This quantum number must have at least three values, and can be identified with colour. In addition, it is found that the quark parton model estimates of the \( \pi^0 \) meson life time\(^{[27]} \) and the ratio of the hadronic cross-section to the cross-section for the production of \( \mu^+\mu^- \) pairs in \( e^+e^- \) annihilation\(^{[28]} \) are not in agreement with experimental results unless the quarks are assumed to have three colours.

- Vector gluons: A meson is a bound state of a quark and an antiquark. To date there is no evidence for a bound two quark state. Hence, gluons must couple differently to quarks and antiquarks. If gluons have odd spin, their charge conjugation parity will be odd, and so they will couple differently to
quarks and antiquarks. Since it is only possible to construct a consistent
gauge field theory with spin 1 gauge bosons,[27] one must assume that
 gluons have spin 1. Other theoretical arguments have been advanced in
favour of the hypothesis that gluons have spin 1, for a review the reader is
referred to reference 26.

In reference 29 a comparison is made between the properties of the jets of
final state hadrons in $e^+e^-$ annihilation and the predictions of QCD type
theories with spin 0 and spin 1 gluons. The data prefer the spin 1 theory.

- The Self-Interaction of Gluons: The gluons of QCD are coloured. This means
  that they must interact with themselves. There is no direct experimental
evidence for such an interaction. However, a theory in which the self-
interaction of gluons is excluded violates unitarity,[27] i.e. cross-sections
  calculated using the theory would tend to infinity as the energy increases.

2.7.2 Experimental consequences of QCD.

The previous section gave arguments in favour of QCD which were largely
theoretical. It is the purpose of this section to outline experimental tests of
QCD.

- The strong interaction coupling constant: The effective coupling between a
  quark and a gluon depends on the momentum scale at which it is evaluated.
  This is true in any gauge theory. In QCD the specific form of the gluon
  self-interaction (the three gluon vertex see figure 10(c)) means that to
  leading order the coupling constant is given by[27]

$$
\alpha_s(Q^2) = \frac{12\pi}{(33-2N_F)\ln(Q^2/\Lambda^2)}.
$$

(2.37)

$Q^2$ is defined in equation 2.1 and $N_F$ is the number of quark flavours
contribute at a particular value of $Q^2$. $\Lambda^2$ is a parameter with dimensions
(mass)$^2$ giving the mass scale at which $\alpha_s$ was renormalised. Equation 2.37
exhibits two important properties.
Figure 10: First order Feynman diagrams in quantum chromodynamics.

Firstly, the value of $\alpha_s$ is chosen to have some value $\alpha_s(\Lambda^2)$ at a chosen value of $Q^2=\Lambda^2$. QCD may then be used to calculate the evolution of $\alpha_s$ as a function of $Q^2$, for $Q^2 \gg \Lambda^2$. QCD can not be used to calculate $\alpha_s$ and the renormalisation scale $\Lambda^2$ independently. In fact perturbative QCD may only be used to calculate the evolution of experimental quantities with $Q^2$.

More importantly, equation 2.37 exhibits the property of asymptotic freedom, i.e. $\alpha_s \to 0$, as $Q^2 \to \infty$. This means that when a proton is struck by a high $Q^2$ probe, the quarks appear to be free because $\alpha_s(Q^2)$ is small. The asymptotic freedom of QCD justifies the parton model assumption that the quarks within the proton behave as though they are free.

- Consequences of QCD for the structure functions: The prediction of the quark parton model is that at large $Q^2$ and $\nu$ the structure functions are independent of $Q^2$ at fixed $x$. The effect of the QCD corrections to the QPM result is to introduce a $Q^2$ dependence. An intuitive explanation of the violation of scaling may be given in terms of the length scale defined by $Q^2$. As $Q^2$ increases the length scale resolved by the virtual photon decreases. At low $Q^2$ the virtual photon 'sees' a quark. At a larger value of $Q^2$ the virtual photon sees a quark and a gluon, or a $q\bar{q}$ pair. Thus, as $Q^2$ increases the number of quarks resolved by the photon increases, and hence the fraction $x$.
of the momentum of the proton carried by a particular quark decreases. Since the \( x \) distribution of partons changes with \( Q^2 \), the structure functions must themselves be functions of \( Q^2 \). The data\([2,25]\) show a scaling violation which is compatible with that expected from QCD.

- Consequences of QCD for the hadronic final state: To describe the production of hadrons in the current jet of \( \mu p \) scattering in terms of the QPM one postulates that the fragmentation functions scale with \( Q^2 \), and that the cross-section factorises (equation 2.36). When the graphs of figure 10 are taken into account the scaling of the fragmentation functions is violated. If the QCD corrections to the parton model are calculated to second order the factorisation of the cross-section no longer holds.\([27]\) Scaling violation and the breakdown of factorisation have been observed in \( \nu d \),\([30]\) and \( \mu p \)\([31]\) scattering. Scaling violations have also been observed in \( e^+e^- \)\([32]\) annihilation. The trends of the data are consistent with the predictions of QCD.

The graphs of figures 10(a) and (b) lead to final states in which the average transverse momentum of hadrons rises as \( W^2 \) increases.\([33]\) Each parton will fragment into a jet of hadrons. (A jet is a collection of hadrons moving in roughly the same direction.) As \( W^2 \) increases the transverse momentum between these jets may be large enough for them to be distinguished. In \( e^+e^- \) annihilation each quark may radiate a gluon, and so one may expect to observe three jets in the hadrons from \( e^+e^- \) annihilations as well. Planar events with a three jet structure have been observed in both \( \mu p \) scattering\([34]\) and \( e^+e^- \) annihilations.\([28]\)

To date it has only been possible to perform reliable calculations with perturbative quantum chromodynamics. For soft processes in which \( \alpha_s \) is large one must resort to phenomenological models. One such process is the fragmentation of the struck quark into a jet of hadrons. The data presented in this thesis have been compared with two hadronisation models. The first is an
iterative cascade fragmentation model (the Lund model). The second model uses
the ideas of QCD to generate a shower of quarks and gluons from the struck
quark. These are then turned into colourless clusters which are allowed to
decay into hadrons.

2.8 An iterative cascade Monte Carlo - The Lund Model.
The basic assumption of this class of model is that the hadrons produced in the
fragmentation of an energetic quark arise in a series of steps in which a quark,
q, fragments into a hadron, h, and a new quark, q'; i.e. q → h+q'. The new
quark (q') may undergo a similar decay. The process continues until the
available energy has been dissipated in the hadrons h. Such a mechanism was
first suggested in 1972\cite{35} to explain hadron production in pp collisions. In
1974 the idea was applied to deep inelastic scattering.\cite{36} The model of Field
and Feynman\cite{37} generalised and extended the model, and the Lund model\cite{38}
post-dated the model of Feynman and Field. An outline of the ideas of
fragmentation developed in reference 37 will be given in the following
paragraphs.

2.8.1 The mechanism of fragmentation.
Suppose that an energetic quark, q₀, with energy E₀, has been created in e⁺e⁻
annihilation or deep inelastic scattering. There will be a colour field between it
and the antiquark in e⁺e⁻ annihilation, or between it and the remains of the
target proton in deep inelastic scattering. Quark-antiquark pairs created in the
colour field will be accelerated in opposite directions. Suppose that at some
point in the field a quark-antiquark pair, q₁\bar{q}_₁, is created. The antiquark, \bar{q}_₁,
will be attracted towards q₀. The colour of the q₁\bar{q}_₁ pair is such that a field
free region between q₁ and \bar{q}_₁ is produced as the pair separate. The pair q₀\bar{q}_₁
form a meson and carry off some fraction \xi₁ of the energy of q₀. The result is
that a meson with energy \xi₁E₀ and flavour q₀\bar{q}_₁ is formed. The quark q₁ now
assumes the role of $q_0$, and may in turn combine with the antiquark from a second $q_2\bar{q}_2$ pair created in the field. The process is iterative, as each new $q_i\bar{q}_i$ pair is created a meson $q_{i-1}\bar{q}_{i-1}$ is formed with a fraction $\xi_i$ of the remaining energy $E_{i-1}$, and the new quark $q_i$ takes the remaining energy. The result is a jet of hadrons as illustrated schematically in figure 11.

![Diagram showing the process of fragmentation.](image)

Figure 11: Schematic diagram of the process of fragmentation. Quark-antiquark pairs $q_1\bar{q}_1$, $q_2\bar{q}_2$, ... are produced and primary mesons are formed. The primary meson $q_0\bar{q}_0$ contains the original quark and is said to have rank 1, the second rank meson contains quarks $q_1$ and $\bar{q}_2$ etc. The decay products of unstable hadrons inherit the rank of their progenitors.

The following conventions will be used when discussing fragmentation. A jet system is a quark, an antiquark or a collection of quarks and antiquarks which give rise to a jet of hadrons as described above. The hadrons formed in the fragmentation chain, $q_i\bar{q}_{i+1}$ are called primary hadrons. Primary hadrons may be unstable (e.g. $\rho$, $\omega$, $\phi$ etc.) and decay to produce the hadrons observed in an experiment. The hadron formed from $q_0$ and $\bar{q}_1$ is said to have rank 1. The
rank of a hadron labels the position along the fragmentation chain at which it was produced (see figure 11). If hadron $h_1$ has rank $r_1$, and hadron $h_2$ rank $r_2$ then hadrons $h_1$ and $h_2$ are adjacent in rank if $|r_1 - r_2| = 1$. By convention the rank of a particle produced in the decay of an unstable primary hadron, $h_3$, is equal to the rank of $h_3$.

A model of hadron production must also reproduce the observed distribution of momentum transverse to the jet axis. Typically, the distribution of transverse momentum squared with respect to the jet axis falls exponentially and has an average value of $\sim (0.4 \text{ GeV/c})^2$ (see chapter 8). In the model this is obtained by assuming that the $q\bar{q}$ pairs created in the colour field are produced with some equal and opposite transverse momentum.

2.8.2 An introduction to the Lund model.

The main components of the Lund model are discussed below. Reference 38 contains a detailed description of the principles underlying the model.

- Longitudinal fragmentation scheme: In the Lund model the colour field between the struck quark and the target remnant system is assumed to lie within a thin string-like region. As the quark travels away from the proton it stretches the colour string. The jet systems at the ends of the string (the endpoints) are assumed to be massless. It is assumed that the energy density in the string is a constant per unit length ($\kappa$). The string breaks by the production of $q\bar{q}$ pairs which are pulled apart by the field.

The rank of a hadron may be defined in the same way as above. By considering the space-time evolution of the field between a quark and an antiquark travelling in the opposite direction$^{[38]}$ it is possible to show that on average the hadrons are ordered in rapidity$^3$, the particles with the smallest rapidities in any frame being produced first in that frame. A

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$^3$ The rapidity, $y$, is defined to be $y = \frac{1}{2} \ln \left( \frac{(E^+ p_n)}{(E - p_n)} \right)$, where $E$ is the energy of a particle and $p_n$ is momentum parallel to the axis of the jet. The centre of mass rapidity ($y^+$) is defined in the same way except that the quantities refer to the centre of mass.
consequence of this is that on average the ordering in rank will be reflected by an ordering of the hadrons in rapidity. Thus, since primary hadrons adjacent in rank contain the components of a common $q\bar{q}$ pair, one may expect that hadrons close in rapidity will exhibit properties that reflect the fact that certain quantities (e.g. charge, flavour and transverse momentum) are locally conserved in the fragmentation chain.

- Transverse momentum and flavour: If the quark and antiquark of a $q\bar{q}$ pair created in the colour field are to have mass or transverse momentum, they can not be produced at a single space-time point. They must materialise some distance apart so that the field energy in between can be converted into mass and transverse momentum. It is assumed that the production of a $q\bar{q}$ pair in the colour field is a quantum mechanical tunneling process. It can be shown\[38] that the probability of producing a quark of mass $m_q$ with a momentum, $p_\perp$ transverse to the string axis satisfies the following equation

$$P(p_\perp, m_q) \propto \exp(-mp_\perp^2/2\kappa)\exp(-nm_q^2/2\kappa).$$ \hspace{1cm} (2.38)

This equation has two consequences. Firstly, it may be used to calculate the relative production rates of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ and $c\bar{c}$ pairs. If constituent quark masses are used the result is that$^[38]$

$$u\bar{u}:d\bar{d}:s\bar{s}:c\bar{c} = 1:1:2/3:10^{-11}.$$ \hspace{1cm} (2.39)

Secondly, equation 2.38 states that the momentum of quarks transverse to the jet axis is Gaussianly distributed with a width of $\sigma_q$, where $\sigma_q = \sqrt{\kappa/\pi}$. The value of $\kappa$ has been estimated$^[38]$ to be $= 1$ GeV/Fm, and so $\sigma_q = 0.3$ GeV/c. The value of $\sigma_q$ is determined in section 7.2.

It is assumed that transverse momentum is conserved as each $q\bar{q}$ pair is created in the field, i.e. that if the quark has transverse momentum $p_\perp$, the antiquark has momentum $-p_\perp$. Thus the transverse momentum is locally conserved in the fragmentation chain. This assumption is tested in section 7.1.
In reference 38 it is argued that if the spin-spin interaction between a quark and an antiquark forming a hadron is taken into account when calculating the tunneling probability, vector mesons are suppressed relative to scalar mesons. It is argued in reference 38 that this has the effect of making the ratio of the production rate of scalar mesons to vector mesons, \( \alpha_S : \alpha_V = 1:1 \) (a ratio of 1:3 would have been expected on the basis of spin statistics alone). Experimentally\(^{[39]}\) the ratio has been found to be close to 1. Note that only the production of pseudoscalar and vector mesons is considered.

The ideas described above can be extended to the formation of baryons by the creation of diquark-antidiquark pairs in the colour field. This is discussed in detail in references 38 and 40. It is estimated\(^{[40]}\) that the ratio of diquark to quark pair production in the string is \( \approx 0.065 \). In the Lund model the baryons produced in the fragmentation chain are adjacent in rank, and one may expect strong correlations to be found between baryons.

- The Lund model applied to leptoproduction: The mechanism\(^{[41]}\) by which hadrons are produced in deep inelastic lepton proton scattering is summarised below.

As the struck quark (the \( I \) - for interacting - quark in figure 12) absorbs the virtual photon, it begins to stretch the colour field inside the proton into a colour string. A short time after the collision the situation is as shown in figure 12(b). A colour string has developed between the \( I \) quark and a second quark, \( J \) (for junction), in the proton. The remaining quark, \( L \), (for leading) stretches the string in a direction opposite to the direction of the virtual photon. The \( J \) quark comes to rest first as it is being retarded by the colour string attached to the \( I \) quark. The quarks \( I \) and \( L \) continue to stretch the string until all the available energy has been stored in the colour field. The final result is shown in figure 12(c). The colour flux lines run
from the I quark to the J quark, and from the L quark to the J quark, i.e. the field changes polarity at the J quark (see figure 12(d)). The system may be considered as consisting of two strings, I-J and J-L, each of which fragments in the normal way, except that the J quark always becomes part of a baryon. This is because the field is such that the quark of a \( q\bar{q} \) pair produced on either the I quark or the L quark side of the J quark is attracted towards the J quark. Though the J quark is always found in a baryon, the J and L quarks are only found in the same baryon in 60% of events. Further details of the fragmentation of a diquark may be found in references 38 and 41.

\[\text{(a)}\]

\[\text{(b)}\]

\[\text{(c)}\]

\[\text{(d)}\]

**Figure 12:** The creation of a jet system in \( \mu p \) scattering (a) The I quark absorbs the virtual photon and (b) stretches a colour string. (c) The quarks stop when all available kinematic energy is stored in the colour field. The final configuration of the colour field is shown in (d).
The uncertainty principle implies that the Fermi motion of quarks contained in the proton has some component transverse to the virtual photon direction. This is referred to as primordial transverse momentum. The longitudinal properties of the quarks inside a proton are parametrised by the structure functions. The structure functions are defined in a regime where quark masses and momenta can be ignored with respect to $\sqrt{Q^2}$ and $q$ respectively. Since one is unable to calculate the momentum distribution of the quarks inside a proton one must rely on models. In the Lund model the primordial transverse momentum of a quark inside the proton is taken to be Gaussianly distributed with a width of $\sigma_{k_t}$. From the uncertainty principle the parameter $\sigma_{k_t}$ should have a value of $\approx 0.4$ GeV/c. Further, it is assumed that the primordial transverse momentum of the struck quark results in a rotation of the event axis, as described in section 7.2. An estimate of the value of $\sigma_{k_t}$ is made in section 7.2.

- QCD and the Lund model: The first order QCD corrections to deep inelastic scattering give rise to two different final state configurations. The first is one in which the struck quark radiates a gluon, a $qg$ event. In the second type of event a gluon from the proton splits into a quark-antiquark pair (figure 10(b)) and either the quark or the antiquark absorbs the virtual photon, a $q\bar{q}$ event. In the Lund model it is assumed that such processes occur before the hadronisation starts. The fraction of $qg$ and $q\bar{q}$ events is calculated using first order QCD.\[38\] At large $x$, it is found that the contribution of $qg$ events to the total inelastic cross-section is large compared to the contribution of $q\bar{q}$ events. The reverse is true at small $x$.

In a $qg$ event it is assumed that a colour string is stretched from the target remnant diquark to the struck quark via the gluon. The gluon is represented by a kink in the string. This means that the colour string is divided effectively into two sections, one stretching from the struck quark to
the gluon, and one from the gluon to the target remnant. If the gluon has sufficient energy, each section of the string fragments as described above. The assumption is made that the fragmentation does not depend on whether a hadron contains the gluon kink or not. Such events are classed as three jet events, and the gluon is known as a hard gluon.

It is also possible that the gluon has too little energy to fragment, in this case it is called a soft gluon. The probability that a quark will radiate a soft gluon is large because $q_s$ is large. In addition the cross-section for gluon bremsstrahlung diverges if the gluons are soft or collinear. The effect of such soft and collinear gluon radiation is taken into account as follows. Gluons whose momentum is too small to give rise to a jet of hadrons may affect the way in which the fragmentation products are distributed in transverse momentum. In the Lund model first order QCD is used to calculate the effective gluon produced by the emission of soft gluons from the struck quark. This is done by summing the contribution of soft gluons within a rapidity interval of 1 unit. This is done throughout the available rapidity range. The effect is to produce a series of bumps on the string. The transverse momentum of the effective soft gluon is then added on to all hadrons close to the gluon in rapidity. Momentum is conserved by summing the transverse momentum of each effective soft gluon and assuming that the whole of the recoil is taken up by the quark which radiated the gluons - the struck quark in lepton hadron scattering. The effect of collinear gluon emission is to soften the longitudinal fragmentation functions.

The second class of events - the $q\bar{q}$ events - are dealt with as follows. A gluon has been radiated from the target, and split into a quark, $q_g'$, and an antiquark, $\bar{q}_g$, the target is therefore in an octet state of colour. The target is split into a quark and a diquark. Two colour triplet fields are formed, one between the diquark and $q_g'$, and the second between the quark and $\bar{q}_g$. These two strings are allowed to fragment independently.
The Lund model as used in this thesis: Three versions of the Lund model were used for the analysis presented in this thesis. The model used in the apparatus Monte Carlo (see chapter 5) is described in reference 42. A number of modifications were made to this model to take into account the effects of soft gluon radiation. In chapter 7 comparisons are made to the Lund model described in reference 43. The Lund model e⁺e⁻ annihilation [44] was used in chapter 8 where a comparison between the hadrons produced in deep inelastic lepton hadron scattering and e⁺e⁻ annihilations is presented.

2.9 A QCD branching model of hadronisation.

The branching model used for comparison with the data in chapter 8 was originally intended to simulate the hadronic final state produced in e⁺e⁻ annihilation. The way in which the model was used to simulate the lepto-production of hadrons is described in section 2.9.1. A description of the model for hadron production in e⁺e⁻ annihilation may be found in reference 45. The following paragraphs contain an introduction to the model.

The reaction e⁺e⁻ → γ* → q̄q produces a pair of quarks which have masses much larger than their rest masses, i.e. they are highly virtual, or far from mass shell. The quarks may radiate gluons, with a probability determined by q₂, thereby losing energy. These gluons may themselves split into two gluons or a quark-antiquark pair. The rate at which these processes occur is also determined by q₂, though q₂ must be evaluated at a different momentum scale. The result is that a shower of quarks and gluons is formed with each successive generation of quarks and gluons being less virtual than the last. The cascade must be terminated, and the quarks and gluons turned into hadrons. This is done by introducing masses for the quarks and an effective gluon mass. The shower is terminated when further branching would be inconsistent with the assumed quark and gluon masses. The model takes into account both the
singularities caused by soft and collinear gluon bremsstrahlung and the interference between the soft gluons themselves.

When the cascade has been terminated the gluons are split into $q\bar{q}$ pairs and colourless clusters are formed by grouping quarks and antiquarks together (see figure 13). The clusters are allowed to decay to give hadrons. Cluster decay occurs by one of two mechanisms. Most clusters (= 90%) undergo a two body decay in which the mass, flavour and spin of the decay products is determined by the phase space available to the decay. Clusters with masses greater than $4\text{ GeV/c}^2$ undergo an anisotropic string-like fragmentation.

![Diagram of an event produced with the QCD branching model. The event shown is an $e^+e^-$ annihilation event. The process has three stages: (a) A shower (or cascade) of quarks and gluons is produced from the original quarks, (b) colour neutral clusters are formed, (c) the clusters decay into the final state hadrons.](image)

Finally, the hadrons produced in the cluster decays may decay into the hadrons which may be observed by an experiment. This is done according to measured life times and branching ratios.
2.9.1 Implementation of the model for leptoproduction.

In order to compare the results of the Monte Carlo with the hadrons produced in deep inelastic muon proton scattering the model of reference 45 was implemented in the NA9 Monte Carlo as follows. The single arm kinematics are generated as described in section 5.2.1. The only differences are that QED radiative corrections are not considered, and a nominal muon beam energy of 280 GeV is used. Parametrisations[46] of the quark $x$ distributions in the proton are then used to choose randomly the flavour of the struck quark, $q_S$.

It is assumed that the target remnant system can be represented by the antiparticle of the struck quark, $\bar{q}_S$. The QCD model is now used to generate the final state hadrons from a $q_S\bar{q}_S$ initial state at a centre of mass energy $W$. After the hadrons have been generated, $q_S$ is aligned with the virtual photon axis, and the hadronic part of the event is rotated to take account of the primordial transverse momentum. This is done as described in section 2.8.2 assuming that $\sigma k_1 = 0.44$ GeV/c.

When considering the results given in section 8.4 one should bear in mind the limitations of the procedure described above:

1. The virtual photon in $e^+e^-$ annihilation is space-like ($q^2 > 0$), in deep inelastic scattering the virtual photon is time-like ($q^2 < 0$). The assumption has therefore been made that the evolution of a space-like pair of quarks is identical to that of a time-like pair.

2. No attempt has been made to simulate the proton fragments. The use of $\bar{q}_S$ to simulate the target jet allows the flavour and colour quantum numbers to be conserved but is not a realistic representation of the target remnants. The comparison must, therefore, be limited to hadrons produced in the forward hemisphere of the centre of mass system.
3.1 Introduction and general overview.

The aim of the NA9 experiment is to detect all final state hadrons produced in deep inelastic muon proton scattering. To detect all final state particles one must have a detector with good acceptance over the full kinematic range. Since the flavour of the hadrons is of interest, good particle identification is also required. In addition, since the muon-nucleon cross-section has a strong $Q^2$ dependence ($\propto 1/Q^4$), the spectrometer must have an efficient trigger selecting inelastic regions from the $Q^2$-$v$ plane (see figure 16). There follows a simplified overview of how the E.M.C. spectrometer[1,4] (figure 14) meets these requirements.

The incident muon is measured by hodoscopes BHA,BHB, while the scattered muon is identified by its ability to traverse the 2m iron absorber. The trigger is formed from coincidences in the hodoscopes H1 (before the absorber) and H3,4 (after the absorber) with the veto hodoscopes V1,V1.5,V2,V3 in anticoincidence.

The momentum of the final state particles is measured using the combined bending power of two magnets. The upstream 'vertex spectrometer magnet' (VSM) is a superconducting dipole magnet of bending power ($\int B.dl$) 4 Tm. The downstream magnet, the 'forward spectrometer magnet' (FSM), is a conventional dipole magnet and has a bending power of 5 Tm. Since the beam is used for an experiment downstream of NA9, the direction of the two fields is opposite, ensuring that the divergence of the beam is minimised.

Scattered muons, and hadrons with momentum $\gamma 5$ GeV/c ($x_F \geq 0$) enter the aperture of the FSM. A line before the FSM is measured in either the two drift chambers W1,2 or in the beam proportional chambers P0A,B,C. The
Figure 14: Diagram of the NA9 apparatus. The labels on the various pieces of apparatus are described in the text. The convention is that proportional chambers are labelled P, drift chambers are labelled W, and time of flight hodoscopes F, Cherenkov counters are labelled C, and hodoscopes H. H2 is the calorimeter. The lead glass array is labelled LG.
proportional chambers P1,2,3 inside the FSM allow particles to be tracked through the magnet. The lever arm behind the magnet is provided by the drift chambers W3 and W4,5. Muons traverse the absorber and are measured by the drift chambers W6,7.

Hadrons produced in the central region in the centre of mass system (x_F=0) are detected by the vertex spectrometer. This consists of proportional chambers PV1,2,3,P0C, and drift tube arrays WV1,2,3. The proportional chambers PV1,2,3, and P0C perform three functions. Firstly, they measure the momentum of tracks in the momentum range 0.5 < p < 20 GeV/c. Secondly, they allow the streamer chamber to be matched to the rest of the spectrometer. In addition, they aid pattern recognition in the vertex spectrometer. The drift tube arrays WV1,2,3 cover the region which is just outside the aperture of the FSM.

The experiment was designed to run in beam rates of up to 2×10^7 muons per second, so the central regions of all the large chambers in the beam (PV2,W1,2,P1,2,3,W3,4,5,6,7) are desensitised. Three small, fast proportional chambers P0A,B,C cover this region.

The backward region in the hadronic centre of mass system is covered by a streamer chamber situated inside the VSM and surrounding the 1m liquid hydrogen or deuterium target. Beam tracks passing through the chamber close in time to the trigger will be recorded on film. However, since the chamber can be triggered and has a memory time of ≈1μsec, this background is limited to about 10 tracks per frame.

Particles are identified by using four Cherenkov counters and four time of flight hodoscopes. The four threshold Cherenkov counters CA,C0,C1,C2 use different fillings, and in combination with the time of flight hodoscopes F1,2,3,4, provide π/K/µ identification over much of the momentum range. The combination C0-C1-C2 gives π/K/µ separation in the momentum range 2.5 GeV/c
to 80 GeV/c. For hadrons produced at large angles, C0 and CA may be used in conjunction with F1,2. The time of flight hodoscopes F3,4 measure tracks of momentum between 0.5 and 1.5 GeV/c.

3.2 The muon beam line.

The muon beam is derived from the decay in flight of pions and kaons, produced in the bombardment of a beryllium target by 400 GeV/c (more recently 450 GeV/c) protons from the CERN SPS. The muon beamline[48] has six main features. Firstly, a large range of beam momenta is available (50-320 GeV/c), with a high flux ($\geq 10^8$ muons/pulse at 200-250 GeV/c). The momentum spread of the beam is less than $\pm 10\%$, and the momentum of a single muon may be measured to an accuracy of $\Delta p/p_{\text{meas}} \leq 0.5\%$. In addition, the hadron content of the beam at the experimental hall is low ($\#n/\#\mu \leq 10^{-6}$) and the number of muons further than 5cm from the centre of the beam (halo muons) is small ($\leq 5\%$). The muon beam polarisation may be enhanced by selecting the momenta of the parent pions and kaons. The beamline has three stages.

The first section of the beamline (labelled 'front end' in figure 15) collects the secondaries produced in the bombardment of the target, and transports them to the decay channel. Pions and kaons are collected by a series of six quadrupole magnets parallel to the incident proton beam. After momentum dispersion is introduced via a horizontal bend of -18mrad, and a 20m drift distance, protons are removed by a magnetic lens and momentum slit. This arrangement allows the collection of a momentum band of $\Delta p/p_{\pi} \leq 10\%$. At this point the beam is below ground level, so a vertical bend of 9.6mrad is introduced at a vertical focus of the beam. A second horizontal bend of -7.8mrad matches the front end of the beam line to the decay channel. This bend also removes the dispersion introduced earlier.
Figure 15:

Diagram of the Muon Beam Line. EPB is the selected proton beam and T6 the primary target. BH and BV are horizontal and vertical bending magnets; CMH and CWV are the horizontal and vertical magnetic collimators; F and D represent focusing and defocusing elements of the FODO array respectively; and BMS is the beam momentum station.
The second stage of the beamline consists of a decay channel followed by a hadron absorber.

The decay channel consists of a series of quadrupole magnets of alternating gradient, forming a focusing-defocusing (FODO) array of 7.5 periods. The quadrupole fields are arranged so that the pions and kaons are transmitted and the wide range of momenta of the decay muons is contained with minimum losses. The polarisation of the muon beam may be enhanced by selecting muons from one of the extreme ends of this range.

At the end of the decay channel only 10% of the parent pions have decayed so a hadron absorber is required to reduce the pion contamination to $\leq 10^{-6}$ at the experimental hall. Only about 25% of the decay muons lie in the momentum range required by the experiment, and so it is arranged that these 'wanted' muons are focussed to a waist at the centre of the absorber. The absorber is chosen to be of low atomic number in order to minimise the effect of multiple Coulomb scattering.

The final stage of the beamline, labelled 'back end' in figure 15, serves to select the central muon momentum ($p_{\mu}^0$) and transport the muons to the experiment. A vertical bend of 24mrad for the selected central momentum, $p_{\mu}^0$, is performed by magnets which surround the absorber. Quadrupole magnets just downstream accept a momentum range of $\Delta p/p_{\mu}^0 = \pm 6\%$. Magnetised iron collimators further reduce the number of halo muons entering and the experiment. The beam is transported to the penultimate bend by alternating gradient quadrupole magnets forming a FODO array of 3 periods. A focusing spectrometer (the Beam Momentum Station - see section 3.3) is formed by a -24mrad vertical bend at the end of the third period. A final bend of -9.6mrad makes the beam horizontal and brings it to a focus around the experimental target.

Two factors contribute to the reduction in the number of halo muons at the experimental hall. Firstly, the magnetic collimators reduce the number of halo
muons entering the apparatus by bending them outside an area of $4 \times 4 \text{m}^2$ around the beam in front of the experimental target. In addition, the beam transport of the back end contains all but a small number of muons, and so is a source of few halo muons.

3.3 The beam momentum station.

The beam momentum is measured by the beam momentum station (BMS), using the focusing and bending magnets of the penultimate bend of the beam line (24mrad), instrumented with two fast hodoscopes close to each of the conjugate points (see figure 15). The over-constrained momentum measurement, provided by the four hodoscopes planes, is essential to remove the confusion caused by high beam rates and high rates of knock on electrons.

Every plane consists of 64 scintillator elements, each with a width of 5mm. To provide a high efficiency, the elements overlap, and in the central region the scintillators are divided so that no element counts faster than $3 \times 10^7/\text{s}$ even for the highest beam rates. The scintillators are 2cm deep in order to provide a large pulse for accurate timing. During running the relative timing between the hodoscopes is checked to an accuracy of $\approx 100\text{ps}$ by laser light piped to each scintillator by a fibre optic light guide. In the off-line reconstruction the internal timing of the BMS is again checked (to an accuracy of $\approx 100\text{ps}$). The BMS is timed to one hodoscope in the forward spectrometer (H3V) to an accuracy of $\approx 300\text{ps}$.

The momentum and horizontal and vertical slopes of a beam muon are determined from parametrisations of sets of hodoscope hits produced in a Monte Carlo simulation of the beam and the magnets of the BMS. A test is made to ensure that the four hits used are compatible. This method allows the momentum of each muon to be determined to an accuracy of $\approx 0.5\%$. Agreement to within $1\%$[^49] was found when the calibration of the BMS was checked against beam momenta as measured by the forward spectrometer of NA2.
3.4 The trigger hodoscopes and the trigger.

The aim of the NA9 experiment is to collect a large sample of deep inelastic events. For this to be possible the apparatus must select events of the desired type. The energy and momentum of the scattered muon may be used to tag deep inelastic events. It is convenient to represent the phase space available to the scattered muon in terms of $Q^2$ and $v$, as shown in figure 16. Lines of constant laboratory scattering angle, $\theta$, and centre of mass energy, $W$ are shown in figure 16. The cross-section is proportional to $1/Q^4$, and so is large when $Q^2$ is small. Since $Q^2 \propto \sin^2(\theta/2)$, a small value of $Q^2$ is obtained if the scattering angle is small. For a deep inelastic event one must have $Q^2 > 2 \text{ (GeV/c}^2)^2$ and $W > 2 \text{ GeV/c}^2$. Thus, deep inelastic events may be selected by requiring that the scattered muon has a scattering angle $> \frac{\pi}{2}$ and an energy greater than 15 Gev. The muon is identified by its ability to traverse the absorber (i.e. the presence of a hits in H3 and H4).

Using the hodoscopes H1, H3 and H4 alone to trigger the apparatus would lead to several problems. Firstly, halo muons would fire the apparatus since they are outside the beam envelope and at an angle to the nominal beam axis. The rate of such triggers is reduced by including a signal to veto the event if a particle is found outside the beam upstream of the target. A second source of fake triggers is electromagnetic and hadronic showers. Showers can be initiated in any piece of material. Hence they may generate a set of hodoscope hits which could have been produced by a deep inelastically scattered muon. This background is reduced by requiring that the candidate scattered muon points back to the target in both the horizontal and the vertical projections before and after the absorber. Another source of fake triggers is pion and kaon decay in the spectrometer. Such muons have a momentum which is low compared to that of the true scattered muon. One may, therefore, reduce this background by imposing the requirement that the muon have an energy greater than some minimum.
Figure 16: The $Q^2$-v plane. The phase space available to the scattered muon for a beam energy of 280 GeV. The dashed lines show the region selected by the ideal trigger described in the text.

In the following paragraphs a brief outline of the hardware of the trigger will be given. This will be followed by a description of the logic used to select events.

3.4.1 The trigger hardware.

The experimental trigger was formed from allowed coincident combinations of elements of the hodoscopes H1,3,4 with the veto hodoscopes V1,1.5,2,3 in anti-coincidence. Other triggers were also taken in order to monitor or calibrate various parts of the apparatus. Only the main experimental triggers will be described below. Details of all the triggers may be found elsewhere. [47] A brief description of some of the triggers mentioned in this thesis may be found in table 1.
Table 1: A selection of the NA9 triggers.

<table>
<thead>
<tr>
<th>Trigger</th>
<th>Requirements</th>
<th>Apparatus read out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$^{1/2}_0$ muon</td>
<td>All apparatus except streamer chamber</td>
</tr>
<tr>
<td>2</td>
<td>$^{1/2}_0$ muon</td>
<td>All apparatus including streamer chamber</td>
</tr>
<tr>
<td>10</td>
<td>Random</td>
<td>BMS hodoscopes, BHA,B and veto hodoscopes</td>
</tr>
</tbody>
</table>

The hodoscopes H1 and H3 have both horizontal and vertical strips, while H4 has only horizontal strips. The width of each strip is determined by the requirement that the angle cut in the trigger may be varied in about $^{1/2}_0$ steps. The horizontal strips of H3 and H4 are divided in two, so that no element is longer than 5m. The scintillator light is collected from both ends of the elements by photomultiplier tubes. A 40cm thick iron wall was erected in front of H4 to absorb electromagnetic showers.

H1 and H3 measure the horizontal and vertical coordinates of the scattered muon at two positions along the beam, and hence may be used to estimate both the scattering angle and the momentum of the scattered muon. H3 and H4 can be used to measure the vertical angle of the muon after the absorber, helping to reduce the number of fake triggers coming from multiple hits caused by hadron punch-through or electromagnetic showers.

The outputs from both ends of the scintillator elements pass through a mean timer. The output from the mean timer is independent of the particle's position along the strip to $\pm 1$ns, and forms the input to the trigger matrices. The vertical strips of H3 are also connected to TDCs,\(^4\) in order to measure

\(^4\) Time to digital converters.
accurately the time of the trigger with respect to the beam hodoscopes (BMS hodoscopes, BHA, B and H5).

At the end of the spectrometer, a small hodoscope (H5) is placed in the beam to measure the beam intensity. It has two planes split into 5 and 6 elements. There is one photomultiplier tube per element. H5 does not participate in trigger 1 or 2.

The veto signal is derived from a logical OR of the four sets of veto hodoscopes V1,1.5,2,3. The hodoscopes of the 'veto wall', V2 and V3 are, placed upstream of the target between BHA and BHB. The rate in V2 and V3 due to knock on electrons and soft muons bent out of the beam, is reduced by a 40cm thick iron wall just upstream of V2. An area of 36×36cm² is covered by V2. The beam hole in V2 is circular, and may be adjusted by remote control. The accepted beam is defined by the setting of V2. V3 extends the area of the veto wall to 6.5×4m². V1 is situated between the last elements of the beam line, and serves to reject muons at small angles to the beam which pass through the beam hole in V2,3. A small veto hodoscope V1.5 is included just downstream of the iron wall. Its function is to veto muons close to the beam hole in the veto wall which would otherwise scrape the edges of the hole, producing knock on electrons.

3.4.2 The half degree trigger.

One may summarise the requirements to be imposed on an event by the trigger as follows:

\[ \theta > \frac{1}{2}^0 \]  
\[ E'_\mu > 15 \text{ GeV} \]

To ensure the event is inelastic and that \( Q^2 \gg 2 (\text{GeV}/c^2)^2 \).

To ensure that the event is inelastic and that the apparatus is not triggered by a decay muon.

Target pointing

The set of hits forming the trigger must point back to the target to ensure that the trigger was not caused by the association of random hits caused by showers or 'noise'.
No veto To ensure that the trigger is not caused by a halo muon a veto signal must not have been recorded.

A schematic diagram of the trigger logic is shown in figure 17. The matrices \(^{[50]}\) (M0 to M7 - see figure 17) are 36×25 arrays of programmable coincidence units. An allowed coincidence between the \(i^{th} \times j^{th}\) element is passed to the \(i^{th}\) address in the output. The output, of 36 bits, is thus a copy of the allowed coincidences satisfied by the 36 element input line. This parallel column output may be used as input to another matrix for further conditions to be imposed.

The matrices \(^{[1,51]}\) M0, M1, M3 and M5 ensure that the candidate muon points back to the target. M0 and M1 are used as one large matrix (36×50) and receive inputs from H1V and H3V. Thus, M0 and M1 ensure that the scattered muon points back to the target in the horizontal projection. The vertical target pointing requirement is applied by matrix M5 which receives inputs from H1H and H3H. M3 receives inputs from H3H and H4 and checks that the candidate muon track points back to the target (in the vertical projection) after the absorber.

The angle cut is applied by forming allowed coincidences from H3V and H3H in M2. M7 receives inputs from H1V and H3H and reinforces the angle cut. It differs from M2 in that H3V is replaced by H1V and so it is less easily satisfied by electromagnetic showers. Implicit in the angle cut is the restriction that the muon energy must be greater than 15 GeV.

Two \(^{\frac{1}{2}}\) degree triggers were recorded (see table 1). Trigger 1 was activated by a muon satisfying the above criteria. All the apparatus was read out for this trigger. Trigger 2 is also a \(^{\frac{1}{2}}\) degree trigger. It differs from trigger 1 in that the trigger requires that the streamer chamber is live, i.e. that a picture may be taken. Thus, triggers 1 and 2 are the same except that trigger 1 occurs in the dead time of the streamer chamber, whereas trigger 2 occurs only when the streamer chamber is active.
Figure 17: The trigger logic. M0-M7 are the eight trigger matrices, the shaded areas represent the 'enabled' nodes (the anomalies near the centre are due to the split hodoscope elements). The notation 'H3H.M5' is used to represent the parallel column outputs from M5 corresponding to the hits in H3H (see text).

3.4.3 Measurement of the muon flux - trigger 10.

An accurate measurement of the available beam phase space and flux is required in order that a Monte Carlo simulation may be made of the experiment. In addition, a knowledge of the available flux allows cross-sections to be calculated.
The Apparatus.

Trigger 10\textsuperscript{[52,53]} randomly starts the TDCs of the BMS hodoscopes and the hodoscopes BHA, BHB. Any hits in the beam hodoscopes and veto counters are recorded during the trigger 10 gate of 50ns. Beam tracks are then reconstructed off line in the same way as for the physics trigger. The number of accepted beam tracks in a short (10ns) window is counted, and the momentum and position of each is written to tape. The beam flux may be calculated from the random trigger rate and the average number of valid tracks found within the window, while the tapes contain a record of the beam phase space.

3.5 Momentum measurement.

Final state particle momenta are measured by exploiting the combined bending power of the magnets, and the precise position measurements obtained from a series of proportional and drift chambers.

The process of gas amplification close to the anode wire of a proportional chamber leads to a voltage pulse on the wire with a short rise time.\textsuperscript{[54]} A well defined short pulse may be obtained by differentiating this pulse with a low input impedance amplifier. If the gas mixture contains a small amount of an electronegative gas, the sensitive region around the wire can be limited. Ideally, the gas mixture would be chosen such that only electrons produced closer to the wire than half the wire spacing will be detected. In this case, the accuracy of the position measurement is limited only by the wire spacing. The memory time is short since the number of positive ions is limited by the gas composition, gas flow and operating voltage. Proportional chambers may therefore be operated successfully in high flux rates. In addition, since most of the electrons are produced close to the anode wire where the electric field is large, the performance of a proportional chamber is not affected badly by the presence of a magnetic field.
The main disadvantage of the proportional chamber is that a large number of wires is required to achieve high position resolution over a large area. Since each wire requires its own electronics (amplifiers, discriminators etc.) the cost of proportional chambers rises quickly with their size. In regions of low flux, the drift chamber may be used to circumvent this problem by using a large wire spacing and measuring the arrival time of the electrons at the anode. A knowledge of the drift velocity of electrons in the gas of the drift chamber allows the calculation of the position of the charged track.

With these considerations in mind, the system of proportional and drift chambers in the NA9 apparatus will now be reviewed. The coordinate system is such that the X axis is along the beam direction, the Z axis points vertically upwards, and the Y axis completes a right handed coordinate system.

3.5.1 The magnets.
The momentum dispersion necessary for the measurement of final state particle momenta is provided by two magnets.

The downstream forward spectrometer magnet (FSM) is a conventional water cooled magnet. It has two coils wound round an iron former defining a field volume of $4.3 \times 2 \times 1 \text{m}^3$. The front aperture subtends angles of $8^\circ$ horizontally and $4^\circ$ vertically at the target.

The superconducting vertex spectrometer magnet has circular poles of radius 1m giving a field which is largely uniform in the centre of the magnet. However, the iron return yoke, designed to maximise the angular 'emittance' of the magnet ($\pm 60^\circ$ in the bending plane and $\pm 10^\circ$ in the vertical plane), is asymmetric. This leads to a field of increasing non-uniformity towards the downstream window.

The field of the FSM was accurately mapped for the experiment NA2.\[1\] For this experiment, a Hall probe was used to measure the magnetic field of the VSM (with a mean error of 10G), including the region where the fields of the two magnets overlap.\[4\]
3.5.2 Measurement of large momenta.

The momenta of particles with momentum $> 5$ GeV/c are determined by measuring their trajectory before, inside and beyond the FSM. Since these particles are produced at small angles to the $X$ axis, the only vertex spectrometer detectors which contribute to the determination of the track parameters are PV2 and P0C.

The coordinates of the track before the FSM are measured in W1,2 and, where possible, PV2 (see sections 3.5.4 and 4.7). If the particle is close to the beam, in the dead regions of W1,2, the proportional chambers P0A,B and C are used. The drift chambers W1,2 have a sensitive area of $1.2 \times 2.2 \text{m}^2$ covering the acceptance of the FSM. The chambers are built in two modules with separate gas volumes, each having four measuring planes (see table 2). The arrangement of field and sense wires is shown in figure 18, and some of the chamber parameters are collected in table 2. The central regions are made insensitive by electroplating the wires with silver.\[^{[55]}\] The dead regions have a diameter of 12cm. By building P0A,B with a roughly circular sensitive area of diameter 16cm these dead regions were well instrumented. Both P0A and P0B have 6 planes of 144 wires with a spacing of 1mm. Consecutive parallel planes are displaced by 0.5mm, resulting in an effective wire separation of 0.5mm.

The lever arm behind the FSM is provided by the drift chambers W3 (directly behind the FSM) and W4,5 (5m downstream of the FSM). The construction of W3,4,5 is similar to that of W1,2 (see figure 18 and table 2). To cover the acceptance of the FSM, W3 must have a sensitive area of $1.2 \times 2.4 \text{m}^2$, and W4,5 a sensitive area of $2.6 \times 5.2 \text{m}^2$. Polyamide foils glued to the wires serve to desensitise the W4,5 chambers in the beam region. The central region of W3 is desensitised by electroplating with silver.

Position measurement inside the FSM is provided by the proportional chambers P1,2,3. These chambers fill the magnet aperture and have a sensitive area of $1.8 \times 0.8 \text{m}^2$. Each chamber has three planes. The cathode planes are
made from horizontal copper wires of spacing 1mm. The central areas are desensitised with 5.7cm diameter kapton boxes. [56]

The large drift chambers W6,7 behind the absorber are used to identify muons and are described in detail in section 3.6.1.

3.5.3 Measurement of low momenta.
A streamer chamber placed inside the vertex spectrometer magnet and surrounding the target is used to measure momenta in the range 0.1 to 10 GeV/c.

The streamer chamber is a visual detector in which the tracks of charged particles are made visible by short electric discharges. [54] If the avalanche, caused by the application of a high electric field to a region through which a charged particle has just passed, is arrested after a very short time, a small cylindrical streamer will be formed. Typically, if the avalanche is allowed to
Table 2: Main parameters of the forward spectrometer wire chambers.

<table>
<thead>
<tr>
<th>Wire chamber</th>
<th>Sensitive area $Z \times Y$ (m²)</th>
<th>Plane structure</th>
<th>Wire or drift space (mm)</th>
<th>Drift velocity (mm/ns)</th>
<th>Anode-cathode gap (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0A</td>
<td>$\phi 14.4$ cm</td>
<td>$\Theta^-, Z, Z, Y, Y, \Theta^+$</td>
<td>1</td>
<td>-</td>
<td>3.2</td>
</tr>
<tr>
<td>P0B</td>
<td>$\phi 14.4$ cm</td>
<td>$\Theta^-, Z, Z, Y, Y, \Theta^+$</td>
<td>1</td>
<td>-</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Theta^+=\pm30^\circ/Y$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1</td>
<td>0.8$\times$1.8</td>
<td>$Z, Y, \Theta$</td>
<td>2</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td>P2</td>
<td>0.8$\times$1.8</td>
<td>$\Theta, Y, Z$</td>
<td>2</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td>P3</td>
<td>0.8$\times$1.8</td>
<td>$\Theta, Y, Z$</td>
<td>2</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Theta=20^\circ/Y$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W1</td>
<td>1.2$\times$2.2</td>
<td>$\Theta^-, \Theta^+, Z, Z, \Theta^-, Z, \Theta^+$</td>
<td>10</td>
<td>0.053</td>
<td>6</td>
</tr>
<tr>
<td>W2</td>
<td>1.2$\times$2.2</td>
<td>$Z, \Theta^-, \Theta^+, \Theta^-, \Theta^+, \Theta^+, Z$</td>
<td>10</td>
<td>0.053</td>
<td>6</td>
</tr>
<tr>
<td>W3</td>
<td>1.2$\times$2.4</td>
<td>$Z, \Theta^-, \Theta^+, \Theta^-, \Theta^+, \Theta^+, Z$</td>
<td>10</td>
<td>0.053</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Theta^+=30^\circ/Y$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4</td>
<td>2.6$\times$5.3</td>
<td>$YY^-, Z'Z, \Theta^-, \Theta^-, \Theta^-, \Theta^-, \Theta^-$</td>
<td>20</td>
<td>0.050</td>
<td>10</td>
</tr>
<tr>
<td>W5</td>
<td>2.6$\times$5.3</td>
<td>$ZZ', \Theta^-, \Theta^-, YY', Z'Z$</td>
<td>20</td>
<td>0.050</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Theta^-=30^\circ/Y$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W6A</td>
<td>3.5$\times$2.9</td>
<td>$ZY\Theta^-, ZY'\Theta^-$</td>
<td>60</td>
<td>0.051</td>
<td>7.5</td>
</tr>
<tr>
<td>W6B</td>
<td>3.5$\times$2.9</td>
<td>$ZY\Theta^-, ZY', ZY\Theta^-$</td>
<td>60</td>
<td>0.051</td>
<td>7.5</td>
</tr>
<tr>
<td>W6C</td>
<td>3.5$\times$2.9</td>
<td>$ZY\Theta^-, ZY'\Theta^-$</td>
<td>60</td>
<td>0.051</td>
<td>7.5</td>
</tr>
<tr>
<td>W7A</td>
<td>4.3$\times$3.5</td>
<td>$Z, ZY'\Theta^-$</td>
<td>60</td>
<td>0.051</td>
<td>7.5</td>
</tr>
<tr>
<td>W7B</td>
<td>4.3$\times$3.5</td>
<td>$ZY\Theta^-, ZY'\Theta=Z''\Theta^-$</td>
<td>60</td>
<td>0.051</td>
<td>7.5</td>
</tr>
<tr>
<td>W7C</td>
<td>4.3$\times$3.5</td>
<td>$Z, ZY'\Theta^-$</td>
<td>60</td>
<td>0.051</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Theta^-=60^\circ/Y$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Gas mixtures (%)

<table>
<thead>
<tr>
<th>Argon</th>
<th>Isobutane</th>
<th>Methane</th>
<th>Ethane</th>
<th>Freon</th>
<th>Methyilal</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0A/B</td>
<td>75</td>
<td>25</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>P1/2/3</td>
<td>71.5</td>
<td>23.8</td>
<td></td>
<td>0.7</td>
<td>4.0</td>
</tr>
<tr>
<td>W1/2/3</td>
<td>81.5</td>
<td>1.5</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4/5</td>
<td>77</td>
<td>2</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W6/7</td>
<td>65</td>
<td></td>
<td></td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>
develop for ≈10ns a self-luminous streamer of radius ≈0.5mm and length ≈5mm will be formed. Thus with a good optical system a streamer chamber can be used to give three dimensional position measurements of an accuracy comparable to those obtained using wire chambers.

The high-voltage pulse is formed by a Marx generator. The Marx generator is followed by a Blumlein line to shape the pulse. Figure 19 shows a schematic diagram of four stages of one of the NA9 Marx generators. The capacitors $C_0$ are charged in parallel and discharged in series, giving an output pulse of magnitude $nV_0$ (where $n$ is the number of stages).

![Schematic diagram of part of the NA9 Marx generator.](image)

There are eight capacitors (C) connected in parallel in each stage, and there are 21 stages for each polarity. The capacitors are charged to 19kV via the resistors R. The charging time is ≈150ns, an extra dead time of 50ns is imposed to reduce sparking. The generator discharges by arcing across the spark gaps, G, when the gap $G_1$ is caused to break down by a trigger pulse. The discharge is triggered after the experiment has been read out in order to reduce chamber oscillations.
The output high-voltage pulse from the Marx generator is ≈100ns long. This is reduced to ≈10ns by a Blumlein line. In principle, a Blumlein line is a wave guide with a streamer chamber between the two output electrodes. [54] A schematic diagram of the NA9 double Blumlein line is shown in figure 20.

Figure 20: Schematic diagram of the NA9 Blumlein line. The central plates are charged to a voltage ±V by the HV pulses from the Marx generators. The spark gaps, G, break down and voltages pulses travel down the waveguides to the right. At A the pulses are partially transmitted, and partially reflected, and some energy travels along the waveguide B. Each Blumlein is terminated with its characteristic impedance (Z). The lengths of the sections B and C are chosen so that the pulses reflected from D and from the spark gaps cancel. The active volume of the streamer chamber is contained within the dashed box.

The streamer chamber used in the experiment NA9 has three gaps. The gaps are formed with two high-voltage planes so that there need be no electrode in the median plane where the particle flux is large. This allows the target to be inserted as shown in figure 21.
Figure 21: Sketch of the vertex magnet together with the streamer chamber. 1: data box; 2: optical grid; 3: foil and grid window; 4: fiducial marks; 5: grid electrodes; 6: inner adaptor; 7: spark gaps; 8: capacitors of the Marx generator; 9: diffusion pump.

The chamber is viewed through three cameras. The optical axes of the cameras are inclined towards the centre of the chamber, and are tilted with respect to one another. Each camera has a single lens giving a demagnification of ≈66. In order that the operating voltage of the chamber be reduced, and that the streamers be as small as possible, the cameras are equipped with image intensifiers. The lower operating voltage also results in a greater uniformity of streamer density over the chamber volume.
The fiducial system is made up of two parts. Illuminated crosses etched onto a thin lucite disk at the bottom of the chamber allow the film to space transformation to be adjusted frame by frame. The determination of optical distortions is performed by measuring a grid of 36 nylon wires coated with fluorescent paint and illuminated with an ultra-violet lamp.

The chamber is a box of dimension 2.0×1.2×0.72m³ made of 5cm thick Rohacell with 1mm lexan coating. The top is made from a 120μm thick mylar foil of good optical quality. The electrodes are formed from grids of phosphor bronze wires, giving 80% light transmission. The outside electrodes are at ground potential, the inner electrodes receive a voltage pulse of ±350kV. Hence, the central gap of 36cm is twice as wide as the two outer gaps. Radio frequency interference is excluded by electrically closing the whole system.

The Marx generator has 21 stages for each polarity, the 8 capacitors of each stage being charged to 19kV. The double Blumlein line then feeds the high-voltage pulse to the two inner electrodes via an impedance-matching adaptor to ensure that there is no reflection at the chamber entrance.

3.5.4 Measurement of intermediate momenta.

The lever arm of the vertex system may be used to improve the measurement of tracks seen in the streamer chamber. For example, a fast track has a large radius of curvature. This results in a large error in the measured momentum if it is determined by the streamer chamber alone. In addition, once the primary vertex has been determined, additional tracks may be found by searching for lines in the vertex detectors which have not been associated to a track in the streamer chamber. Thus, the vertex system is used to measure the momentum of tracks in the momentum range 1 < p < 20 GeV/c.

The extension of the streamer chamber measurement for tracks at large angles is given by PV1,3. As shown in figure 14, these chambers sit in the wings of the VSM. PV2, in conjunction with the drift tube arrays WV1,2,3,
measures particles in the higher part of the vertex spectrometer range. Again, since the central region of PV2 is desensitised, a small beam chamber P0C is provided.

PV1,2,3 are very similar in construction. The sensitive areas of PV1 and PV3 are $1.3 \times 0.8 \text{m}^2$. PV2 must cover the front of the VSM, and so has a sensitive area of $2.8 \times 1.0 \text{m}^2$. Each chamber has six planes (see table 3). The wire spacing is 2mm. In PV1 and PV3, where the particle flux is low, pairs of wires are internally connected together to give an effective wire spacing of 4mm. The design of cathode planes was determined by the requirements of low mass and rigidity. It was found that with cathodes made of graphited mylar foils attached to 1cm thick Rohacell foam boards, the wire tension could be supported with acceptably thin frames.

To cover the dead region of PV2, a small proportional chamber P0C was installed just downstream of PV2. P0C has an octagonal sensitive area of minimum diameter 15cm, and is similar in construction to P0A,B. It has eight planes of sense wires, whose orientations are given in table 3.

In order to extend the lever arm available on tracks passing through PV2, but not entering the acceptance of the forward spectrometer, the drift tube arrays WV1,2 and 3 were built. WV1,2 lie in the wings of the vertex spectrometer, while the two halves of WV3 measure tracks passing above and below W1. Drift tubes were preferred to drift chambers in the vertex spectrometer because drift tube arrays can be built as self supporting structures thus reducing the size of their frames. This results in a minimisation of the dead space between the drift tube arrays and other vertex spectrometer detectors.

WV1,2 have 11 layers of 36 tubes covering an area of $2 \times 2 \text{m}^2$. The tubes are alternately vertical and horizontal and have an inner diameter of 5cm. A resistive cathode is made by coating the bakelite paper walls with carbon black.
Successive planes of tubes of the same orientation are staggered so that the centres of the tubes lie on a circle of radius 2m when viewed from the end. This reduces the dead space caused by the tube walls, and helps remove left-right ambiguities.

To help to remove left right ambiguities in multiparticle events, the longitudinal position of a hit along the outer tubes is measured by a delay line. A delay line is essentially a wave guide. Since the cathode of a drift tube is capacitively coupled to the anode, the voltage pulse on the anode is 'mirrored' by a voltage pulse on the tube wall. The cathodes of the outer tubes are made highly resistive so that a signal may be induced in the delay lines held against them. One delay line picks up from two tubes and is read out at both ends.

The two halves of WV3 have 7 layers of tubes each. They are alternately horizontal and vertical, covering an area of 2.0x0.5m². The construction of the tubes is identical to those in WV1 and 2.

A summary of some of the parameters of the wire chambers in the vertex spectrometer is given in table 3.

3.6 Particle identification.

3.6.1 Muon identification.

The muon is identified by the presence of a track downstream of the absorber which matches a track found in W4,5. The absorber is made of 2m (10 absorption lengths) of iron. Possible backgrounds from electromagnetic showers and hadron punch-through are rejected partly by demanding an in time hit in H3 and H4, and partly by requiring that the muon candidate forms a valid track in the drift chambers W6,7 which can be linked to a track upstream of the absorber. The absorber is complemented by the 0.5m of iron in the hadron calorimeter H2[1] (see figure 14). The calorimeter information was not used in this analysis.
Table 3: Main parameters of the vertex spectrometer wire chambers

<table>
<thead>
<tr>
<th>Wire chamber</th>
<th>Sensitive area (m²)</th>
<th>Plane structure</th>
<th>Wire or drift space (mm)</th>
<th>Drift velocity (mm/ns)</th>
<th>Anode-cathode gap (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0C</td>
<td>φ 15cm</td>
<td>Θ⁺, Θ⁺⁺, Z, Z, Y, Y, Θ⁻, Θ⁻⁻</td>
<td>1</td>
<td>-</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Θ⁻⁻ = ±30°/Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV1</td>
<td>1.3x0.8</td>
<td>Z, Θ⁺⁺⁺, Θ⁺⁺⁺⁺, Θ⁻⁻⁻, Θ⁻⁻⁻⁻, Z</td>
<td>4⁺</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>PV2</td>
<td>2.8x1.0</td>
<td>Z, Θ⁺⁺⁺, Θ⁺⁺⁺⁺, Θ⁻⁻⁻, Θ⁻⁻⁻⁻, Z</td>
<td>2</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>PV3</td>
<td>1.3x0.8</td>
<td>Z, Θ⁺⁺⁺, Θ⁺⁺⁺⁺, Θ⁻⁻⁻, Θ⁻⁻⁻⁻, Z</td>
<td>4⁺</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Θ⁺⁺⁺⁺⁺ = ±45°/Y</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
<td>Θ⁺⁺⁺⁺⁻ = ±18°/Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WV2</td>
<td>2.0x2.0</td>
<td>Y, Z, Y, Z, Y, Z, Y, Z, Y, Z, Y, Z, Y, Z</td>
<td>25</td>
<td>0.043</td>
<td>25</td>
</tr>
<tr>
<td>WV3</td>
<td>2.0x0.5</td>
<td>Z, Y, Z, Y, Z, Y, Z</td>
<td>25</td>
<td>0.043</td>
<td>25</td>
</tr>
<tr>
<td>WV4</td>
<td>2.0x0.5</td>
<td>Z, Y, Z, Y, Z, Y, Z</td>
<td>25</td>
<td>0.043</td>
<td>25</td>
</tr>
</tbody>
</table>

† Wires have 2mm pitch but are connected internally in pairs

<table>
<thead>
<tr>
<th>Gas mixtures (%)</th>
<th>Argon</th>
<th>Isobutane</th>
<th>Methane</th>
<th>Ethane</th>
<th>Freon</th>
<th>Methylal</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0C</td>
<td>70</td>
<td>24</td>
<td>0.8</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>70</td>
<td>24</td>
<td>0.8</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WV</td>
<td>80</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

To cover the aperture of the FSM W6,7 must have a sensitive area of 44m².

In order that the chamber sizes be manageable, the chambers were built in three sections A, B and C, and each section was split into two modules (see figure 14). Monte Carlo studies showed that the rejection of halo muons and knock on electrons, and hence the track finding efficiency, was higher if the chamber modules were staggered as shown in figure 14.

The orientations of the eleven planes of sense wires in W6,7 A and C, and the 16 planes of sense wires in W6,7 B are given in table 2. A larger number of planes was required in the B chambers because of the higher flux. The beam
region in the centre of W6,7 B is desensitised by 18×24cm² kapton boxes.\textsuperscript{[56]}

Figure 22 shows the organisation of potential and sense electrodes in W6,7. A position resolution of 0.4mm is achieved using a drift distance of 6cm and electronics with a time resolution of 4ns.

![Diagram](image)

Figure 22: Schematic diagram of an electrode cell in the drift chambers W6,7. All distances are in mm and all voltages in kV.

3.6.2 Hadron identification.

Two different physical principles are exploited to identify a hadron.

For a particle of known momentum, the time of flight over a known distance depends only on its mass. However, as the velocity of the particle approaches the speed of light ($\beta - 1$) the time of flight over a given distance approaches a constant value, independent of the particle's mass. Consequently, it is no longer possible to resolve the difference in the time of flight of unlike particles. The difference in the time of flight to F3 or F4 for pions and kaons with a momentum of 3 GeV is $\approx 0.2$ns, and for kaons and protons is $\approx 0.6$ns. Thus with a time resolution $\approx 0.4$ns the time of flight hodoscopes F1,2,3,4 can not be used to identify particles much above 3-4 GeV/c.

When a charged particle passes through a dielectric, the amount of energy deposited at a distance $r$ away from the trajectory decreases exponentially, with an exponential slope $\lambda$ ($e^{-\lambda r}$).\textsuperscript{[58,59]} Thus if $\lambda$ is purely imaginary there is no damping, and energy may escape as radiation. This is the Cherenkov effect. It
is found that \( \lambda \) depends on the dielectric constant \((\varepsilon(\omega))\), and light will be emitted if \( \beta^2\varepsilon(\omega) > 1 \), or, equivalently in a transparent medium, \( \beta n > 1 \), where \( n \) is the refractive index. Since the medium behaves coherently, Huygens principle may be used to show that the light is emitted in a cone around the particle trajectory of opening angle \( \theta = \cos^{-1}(\beta/n) \). Integration of the spectrum of light emitted leads to a velocity-dependent deposition of energy above threshold. Since this energy is detected by photomultiplier tubes, it is convenient to express the energy liberated in terms of the number of photoelectrons produced in the first stage of the photomultiplier tube

\[
N_{pe} = LN_0 \left[ 1 - 1/n^2\beta^2 \right] \tag{3.1}
\]

where \( L \) is the length of the radiator traversed by the particle. \( N_0 \) is a constant characteristic of an individual Cherenkov counter, and depends on details of its design and operation. To maximise the sensitivity of the counter, mirrors of high reflectivity must be used. In addition, a light funnel may be used to channel as much light to the photomultiplier tube as possible.

Particle identification over the full kinematic range can not be achieved using one Cherenkov counter, and measurement of the time of flight of a particle only yields good mass resolution for particles of low momenta. However, a series of Cherenkov counters with different thresholds may be used to identify particles over a wide momentum range. The NA9 apparatus uses four threshold Cherenkov counters, whose thresholds are shown in figure 23 II, and four time of flight hodoscopes. These counters may be used singly, or in conjunction, to give information on a particle's mass. Figure 23 indicates the momentum regions in which these detectors, or various combinations of them, may be used to identify hadrons.

C2: The Cherenkov counter, C2, was designed to cover the aperture of the FSM. Hence, it contributes to the identification of particles with
Figure 23: Summary of particle identification regions. II shows threshold values of detectors; I is for outer region in the horizontal plane ($\pm 90^\circ$ to $320^\circ$) and TOF up to $60^\circ$) up to 16 GeV/c momentum; III is for central region in the horizontal plane ($-90^\circ$ to $90^\circ$) from 2 to 100 GeV/c momentum; I and III show the identification capability: full line implies complete identification for corresponding particle; dotted line implies ambiguity between corresponding pair of particles. The logical combination signal (C) is explained in each case.

momenta greater than 5 GeV/c. C2 was filled with neon (n = 6.7x10^{-8}) and may be used to separate pions, kaons and protons of momenta between 20 and 70 GeV/c. As shown in figure 24, C2 is split vertically into two symmetric halves. Each half has 39 cells, each cell having a spherical mirror of radius 1.5m focusing light into a light funnel viewed by one photomultiplier tube. The
radiator length is 4m and the mirrors have dimension 36×40cm² - larger than the Cherenkov cone at the mirror (which is ≈21cm). The mirrors have a reflectivity of 80% at a wavelength of 210nm. With this arrangement 30%, of tracks cause signals from more than one photomultiplier tube, and 20% of photomultiplier tube signals are caused by more than one track. In order that the beam, and light associated from material in the beam should not contaminate the light seen in the central cells, a 15cm diameter hole was made in the mirrors close to the beam and a 15cm diameter black tedlar tube was inserted along the beam axis. The sensitivity of the photomultiplier tubes was doubled by coating their entrance windows with a thin layer of wavelength shifter. The photomultiplier tube signals are read out via ADCs⁵ and pattern units.⁶

Figure 24: Diagram of the Cherenkov counter C2.

⁵ Analogue to digital converter.
⁶ Parallel to serial converters.
C1: The Cherenkov counter, C1, was designed to cover the gap in the momentum range in which C0 and C2 can be used to identify particles. Filled with purified nitrogen (n-1 = 3×10^{-4}), the thresholds for pions, kaons and protons are 5.6, 20 and 38 GeV/c respectively. The horizontal aperture of C1 was determined by the FSM while the vertical aperture was determined by the VSM. C1 subtends an angle of ±10° at the target in both the vertical and horizontal projections. As shown in figure 25, C1 is symmetric about the median plane. There are 32 spherical mirrors above and below the median plane of dimension 26.5×20cm², and radii of 190, 210 and 230cm depending on the cell's position. The light is collected by Hintberger-Winston[60] light funnels viewed by one photomultiplier tube each. The tubes must be shielded from the fringe fields of the FSM and the VSM. This is done by surrounding each tube with three cylinders of mu-metal and one cylinder of soft iron. The entrance windows of the photomultiplier tubes are coated with a thin layer of wavelength shifter, resulting in an increased efficiency. The photomultiplier tube outputs are read out via ADCs and TDCs. The TDCs were intended to be used to help reject tracks not associated with the event but producing light in C1. This was found to be an inefficient method of reducing the background light in the Cherenkov counter. The beam killer in C1 is a blackened tube, as shown in figure 25.

C0: The Cherenkov counter, C0, was required to identify hadrons with low momentum. To do this a counter with a single tank of volume 7m³ was designed. To cover the aperture of the VSM, C0 subtends an angle of ±10° vertically, and ±33° horizontally at the target. C0 was filled with neopentane (n-1 = 1.71×10^{-3}). C0 contains a 12cm diameter tube to remove contamination from particles in the beam region. The active volume is divided into 12 cells arranged symmetrically above and below and to either side of the beam axis (see
figures 14 and 26). In each cell the light is collected via two spherical mirrors, and guided to the photomultiplier tubes by light funnels as shown in figure 26. In the central region the cells are viewed by 4x4 arrays of tubes, while in the wings, the reduced particle flux allows the use of 2x2 arrays. Lack of space means that the light funnels are quadratic at the end, becoming Hintberger-Winston cones as they approach the photomultiplier tubes. With this arrangement, 18% of tube signals originate from two particles in one cell. Since C0 is closer to the VSM than C1, passive shielding of the photomultiplier tubes from the magnetic field is no longer sufficient. The component of the field perpendicular to the tube axis is shielded by a mu-metal cylinder around the tube, and a low carbon steel shield around the light funnel matrices. The longitudinal component of the field is shielded by coils wound round each matrix of tubes. C0 is read out via ADCs and TDCs.
Figure 26: Section through the vertex spectrometer. The figure shows the relative positions of PV2, C0, CA, F2 and WV2.

CA: The Cherenkov counters, CA, fill the momentum gap left by the identification ranges of C0 and the time of flight hodoscopes. The radiator is aerogel (n-1 = 0.030) and the thresholds for pions, kaons and protons are 0.56, 1.98 and 3.76 GeV/c respectively. The two Cherenkov counters CA are placed symmetrically with respect to the beam, and subtend a vertical angle of
±9.8° at the target. Horizontally, the sensitive area extends from 10° to 32°. Each half of CA has 10 cells, five above the median plane and five below. Each cell is rotated by 4.3° horizontally with respect to its neighbours to reduce the chance that a particle will pass through two cells. With this arrangement, 10% of the cells have light coming from more than one particle. The aerogel radiator is 18.5cm deep, and is followed by a 48cm deep diffusing box, the internal faces of which are coated with highly reflective paper. Each cell is instrumented with 5 photomultiplier tubes read out via ADCs only. The tubes are shielded from stray magnetic fields by coils wound around each group of five photomultiplier tubes.

The time of flight hodoscopes: The time of flight hodoscopes, F1,2,3,4, extend the particle identification down to =0.8 GeV/c. The dimensions of the hodoscopes were determined by the aperture of the VSM. All hodoscopes, therefore, subtend ±10° vertically at the target. F1 and 2 extend from 9.75° to 34° horizontally, complementing the measurements made by CA. F3 and 4 complete the acceptance for particles of momenta less than 1.5 GeV/c, or those produced at large angles, by covering the horizontal angular range 32° to 60°. F1 and 2 are composed of 21 plastic scintillator strips of 1.6×0.1m². The eight elements nearest to the beam are 4cm thick with no overlap, while the other thirteen are 1.5cm thick and overlap by 0.2cm. This arrangement gives larger signals where particle momenta are higher. F3 and 4 have 17 elements of plastic scintillator of dimension 1.6×0.15m². They are 2cm thick and do not overlap. A fish-tail light guide couples the elements to photomultiplier tubes at each end. The tubes are shielded by two layers of mu-metal and a further cylinder of soft iron. Both the time and size of the photomultiplier tube pulse are recorded. The start signal for the TDCs comes from the BMS and has a mean resolution =125ps. Combination of the stop times from the two ends of an
element, together with a correction factor coming from the pulse heights, gives a resolution of \( \pm 200\) ps on the stop time.

**Monitoring the hadron identification counters:** Some parameters of the counters are collected in table 4. During the data taking, each counter is monitored by an LED system. The LED signals are used to follow the calibration of the photomultiplier tubes which may drift with time. Absolute calibration of the Cherenkov ADCs is obtained from particles giving light on the plateau. A detailed description of the Cherenkov calibration procedures maybe found in reference 47.

**Table 4: Main parameters of the particle identification detectors.**

<table>
<thead>
<tr>
<th>Detector</th>
<th>Horizontal angle coverage (degrees)</th>
<th>Sensitive area (cm²)</th>
<th>Cell size</th>
<th>Sensitive radiator (n=refrac. index)</th>
<th>Threshold values ( \pi/K/p ) (GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1, F2</td>
<td>( \pm (10-34) )</td>
<td>160×206</td>
<td>160×10</td>
<td>NE 110</td>
<td>( \pi/K &lt; 1.5 )</td>
</tr>
<tr>
<td>F3, F4</td>
<td>( \pm (32-60) )</td>
<td>160×252</td>
<td>160×15</td>
<td>NE 110</td>
<td>( K/p &lt; 2.5 )</td>
</tr>
<tr>
<td>CA</td>
<td>( \pm (10-32) )</td>
<td>150×130</td>
<td>65×30</td>
<td>aerogel n-1=0.030</td>
<td>0.6/2.1/3.8</td>
</tr>
<tr>
<td>C0</td>
<td>( \pm 32 )</td>
<td>300×100</td>
<td>12×14</td>
<td>neopentane n-1=0.00171</td>
<td>2.4/8.4/16</td>
</tr>
<tr>
<td>C1</td>
<td>( \pm 9 )</td>
<td>109×143</td>
<td>14×18</td>
<td>nitrogen n-1=0.000282</td>
<td>5.6/20/38</td>
</tr>
<tr>
<td>C2</td>
<td>( \pm 7 )</td>
<td>150×300</td>
<td>23×25</td>
<td>neon n-1=0.65×10(^{-4})</td>
<td>12/45/90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected number of photoelectrons above threshold</th>
<th>CA</th>
<th>C0</th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>14</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

\(^\dagger\) each module if any

\(^{\dagger\dagger}\) perpendicular to the front surface
3.6.3 Photon detection.

A photon detector, optimised to accept photons produced forward in the centre of mass, was included between the two halves of the calorimeter H2 (see figure 14). It is a 23×20 array of lead glass blocks, each 14 radiation lengths long, and of dimension 40×8×8 cm$^3$. In front of the blocks is a 1.2 cm thick lead plate (2 radiation lengths). By starting the electromagnetic shower before the glass it increases the resolution of the detector for high energy photons at the expense of the low energy sensitivity.

On-line monitoring and time dependent calibration are provided by laser light piped to each block by a fibre optic light guide. The absolute calibration of the lead glass is done using halo muons. During these calibration runs the photomultiplier tubes are run at a higher voltage, because of the small amount of light released by a muon. The laser then allows the connection to be made between this calibration and the readings made at the normal operating voltage. This is done by comparing the signal generated by laser light passed through a filter of known attenuation with that produced by halo muons. The calibration is accurate to $\pm 10\%$.

The energy resolution is limited by the granularity of the ADCs ($\approx 250$ MeV per channel), and the limited accuracy of the calibration. The response of the lead glass is non-linear, the non-linearity being especially pronounced for low energy depositions. This decreases the resolution for low energy photons. The energy resolution is $=0.06+0.2/\sqrt{E}$ GeV.

3.7 Data Acquisition and on-line monitoring.

The task of recording and monitoring the experiment is performed by four PDP 11/70 computers. One computer is used for data acquisition[$^{[1,61]}$] (DAC) and may pass events, via a PDP 11/10 computer, to any of the other three experimental computers (U0, U1, U2) or the CERN IBM for further analysis.
During the spill, events are read into a buffer on the DAC via the experimental CAMAC. Simultaneously events are written to tape, any data remaining in the buffer at the end of the spill are also written to tape. The data acquisition software is flexible enough to allow 16 triggers to be taken in parallel and written independently in any combination to each of the four tape drives. Outside the spill, the DAC satisfies the requests for events from monitoring tasks running on the other three computers.

The three computers U0,U1,U2 are used to monitor the experiment. Monitoring tasks either check the status of the hardware, or they accumulate events in order to display the performance of a particular piece of apparatus. The monitoring tasks check that the high and low voltage power supplies are set to the correct voltage, that no chamber is drawing too much current either when the beam is on or when it is off, and that the trigger and readout are working correctly. The response of the readout is checked by other tasks which send pulses to the wire chambers' amplifiers, or pulse the LEDs of light sensitive detectors. These tasks check that the signals produced are consistent with those expected. In addition, other tasks check that the distribution of hits on a particular detector is reasonable by sampling the data being recorded by the experiment. Error messages from these tasks are recorded on a central teletype. An alarm system, used in conjunction with television screens on which error messages are displayed, ensures a swift response to faults.

At regular intervals a sample of a few thousand events is transferred to a disk file on the IBM. When the accumulation is complete, programmes are run which reconstruct these events. The output is in the form of hit maps, efficiencies etc. This allows a more detailed and complete check to be made on the performance of the apparatus.

---

7 CAMAC is a modular instrumentation system for data handling and is described in reference 62.
It is possible to set and check the elements of the beam line from a terminal connected to one of the computers controlling the beam line. This terminal also allows access to the information recorded by multiwire proportional chambers situated upstream and downstream of the experiment. These chambers, situated in the beam, allow the position of the beam to be checked accurately.

The streamer chamber performance is also monitored. This is done with a television monitoring system and by the exposure of strips of film which are developed immediately.
4.1 Introduction.

The transformation of the measurements made by the various pieces of apparatus to particle momentum and energy is performed in four stages. Each stage is preceded by the determination of the parameters germane to that stage. A flow chart of the analysis chain is shown in figure 27, and a brief summary of each stage is given below.

In the first stage, the pattern recognition of muons and hadrons in the forward spectrometer is performed. Before this can be done, the parameters of the relevant proportional and drift chambers must be tuned. This forward spectrometer 'alignment' is described in section 4.2.

The line segments found above must then be geometrically fitted to give a first estimate of the vertex position, particle momentum and event topology. This step is preceded by an additional tuning of the forward spectrometer wire chamber parameters.

The reconstruction of tracks in the vertex spectrometer begins with the measurement and reconstruction of tracks in the streamer chamber. After a detailed alignment of the vertex spectrometer detectors, tracks may be reconstructed using all the information available from the vertex spectrometer. At this stage, tracks measured in the the vertex spectrometer and streamer chamber are fitted and linked, where possible, to tracks found in the forward spectrometer. Also, the final primary vertex is defined, and a search is made for secondary vertices.

Finally, the information gained from the Cherenkov counters, time of flight hodoscopes and lead glass photon detector is used to identify charged and neutral particles. A knowledge of the calibration of the detectors is essential for this stage.
Simulate deep inelastic scattering by a Monte Carlo programme

Generate pseudo raw data

Forward spectrometer alignment

I/p raw data from experiment

Pattern recognition in the forward spectrometer

Geometrical reconstruction in the forward spectrometer

Simulate streamer chamber measurement

I/p streamer chamber picture measurements

Pattern recognition and geometrical reconstruction in the streamer chamber

Vertex spectrometer alignment

Pattern recognition in the vertex spectrometer and geometrical reconstruction of whole event

Event and track selection

Analysis and acceptance correction

RESULTS

--- Data
--- Long chain Monte Carlo
--- Short chain Monte Carlo

Figure 27: The E.M.C. analysis chain.
The coordinate system is such that the X axis is along the spectrometer axis, the Z axis points vertically upwards, and the Y axis completes the right handed coordinate system.

4.2 Alignment of the forward spectrometer wire chambers.

To reconstruct tracks in the spectrometer it is crucial to have an accurate knowledge of the chamber positions, and the parameters by which their signals may be converted into points in space. This knowledge is obtained in the 'alignment procedure'. The apparatus is surveyed, and an 'alignment file' created which includes the position of each chamber. This file also includes all the necessary chamber and counter parameters, for example, wire spacings, hodoscope element widths, timing parameters, drift velocities and trigger matrix settings. Data taken with special triggers are now used to tune these parameters.

4.2.1 Alignment triggers.

Alignment data for the forward spectrometer is taken with both magnets switched off. Special alignment tapes are written using three triggers. To align counters in the beam a beam trigger is formed by taking H5 in anticoincidence with the veto hodoscopes. Other chambers are aligned using 'near' or 'far' halo triggers. These triggers are activated by muons outside the 'beam' as defined by V2. Coincident signals from V3, H3 and H4 define the far halo trigger, while V1.5 and H1 taken in coincidence define the near halo trigger.

4.2.2 Forward spectrometer alignment procedure.

Modified versions of the forward spectrometer pattern recognition programme (see section 4.3) and geometrical track fitting programme (see section 4.4) are used to calculate small corrections to the parameters contained in the alignment file. The first stage involves checking the timing and calibration of the H3V TDCs. When this is done, the time given by H3V is used as the reference time for all the detectors, except the time of flight hodoscopes.
The pattern recognition programme is used to find lines in W1,2,3,4,5 in the data taken with the halo triggers. The relative position of planes within the chamber modules W1,2,3,4,5, P1,2,3 and PV2 is varied until the distribution of hits around the tracks (the 'residuals') is centred on zero. The process is iterative. A reduction in the width of the drift chamber residual distributions is achieved by tuning their timing parameters (drift velocity, \( t_0 \), and non-linearity parameters).

The position and timing parameters of the W6,7 wire chambers are determined using a similar method. Lines are found in W6,7 and changes made to the chamber parameters until their residual distributions are narrow and well centred.

The next stage is to use data taken with the beam deflected from its normal trajectory and with the current in the VSM at its normal working value of 5000 A. The FSM remains off. A modified near halo trigger is used. Tracks found in W4,5 and W3 are projected back to POA and B. The positions of POA and B are determined by an iterative procedure similar to that described above.

A second deflected beam configuration is used, with the VSM and FSM off, to align the beam hodoscopes (BHA,B) and POC with respect to the forward spectrometer (W1,2,3,4,5). Tracks found in events recorded using the beam trigger are now used to align BHA and BHB, both with respect to one another and to the rest of the spectrometer. For this data both experimental magnets are off and the beam passes along the spectrometer axis. At the same time a fine tuning of the position of POA,B,C is performed.

The modified alignment file is now used by the geometrical track fitting programme to reconstruct a small fraction of the data. Further small changes to the parameters may be required in order to get narrow well centred residual distributions and Gaussian error distributions. All forward spectrometer chambers are considered and multiple scattering is taken into account. This
allows the link between W6,7 and the detectors upstream of the absorber to be tuned. The changes are normally small.

When the procedure is complete, the position of each plane in the forward spectrometer detectors (BHA,B, W1,2,3,4,5,6,7, P1,2,3, and, P0A,B) and PV2 is known to an accuracy of \( \pm 10\% \) of the plane's resolution.

4.3 Pattern recognition in the forward spectrometer.

This section describes how a track is found from the hits in the forward spectrometer wire chambers (W1,2,3,4,5,6,7, P1,2,3, P0A,B). The pattern recognition programme begins by searching for lines in the drift chambers W6,7. Hits in W6,7 are projected into the XZ and XY planes. Considering only wire planes in W6,7 relevant for a particular projection, each hit in the first plane is paired with each hit in the last plane. A straight line is drawn between these two points, and if more than a minimum number of the relevant planes have hits within a certain distance of this line, it is kept as a candidate muon line. The information from the theta planes is used to find 'real' tracks from pairs of lines found in the projections. If no muon candidate is found, the event is dropped.

The large drift gap means that many out of time hits may be recorded. These are removed, either because they do not form a valid line, or because there is no coincident hit in the hodoscopes H3,4. Once a muon candidate has been found, the event is only removed from further analysis if no tracks are found upstream of the absorber.

An attempt is now made to find lines in W4,5 associated with muon lines in W6,7. The algorithm used to do this is based on the assumption that the scattering material of the absorber may be considered to be concentrated in its central plane. The muon line is projected to the central plane of the absorber, intersecting it at a point \( a_{abs} \). A search is now made for hits in the planes of W4,5 close to a straight line drawn from the centre of the target to the point
A line is fitted to these hits if the number of planes offering a hit to the line is greater than some minimum.

The line found in W4,5 is projected to W3. A search is then made for hits in W3 close to the extrapolated line. If more than a minimum number of hits is found in W3 a line is fitted through them.

Hits must now be found inside and upstream of the FSM. In order to simplify the problem it is assumed that the VSM field has a negligible effect on the track, and that the FSM field may be approximated by that of a box magnet. The line found in W3,4,5 is projected to the central plane of the magnet, intersecting it at a point a_{FSM}. A line is then drawn from the target centre to a_{FSM} and a circle is drawn joining the line before the FSM to that beyond it. The circle is determined by the requirement that the lines before and after the FSM be tangential to it. A search is now made for hits close to this line in P1,2,3. Again, certain minimum plane requirements must be satisfied. If this is the case, the circle is refitted using the hits found in P1,2,3.

A tangent to the new circle is now used to tag hits in W1,2 and P0A,B. Again, if certain minimum plane requirements are satisfied, a line is fitted through the hits.

Finally, the procedure for finding muon lines in W6,7 is applied to hits in W4,5 in order to find hadron tracks. Pattern recognition in the rest of the forward spectrometer is carried out as described above.

4.4 Geometrical reconstruction in the forward spectrometer.

A large number of triggers arise from events in which no muon line can be reconstructed in W6,7, or in which the scattered muon could not fulfil the trigger conditions by itself. The largest number of fake triggers are caused by the association of hadron hits before the absorber, with the hits produced in the hodoscopes behind the absorber by a muon which would not have satisfied
the trigger by itself. Many fake triggers are caused by electromagnetic showers and hadrons which are not fully absorbed by the absorber. Some hits in H1 are caused by tracks not passing through the absorber. These hits, in conjunction with the hits in H3,4 caused by the electromagnetic and hadronic punch through, may satisfy the trigger conditions. A number of fake triggers are caused by halo muons which have not produced a signal in one of the veto hodoscopes. A small number of fake triggers arise from muons scattered from material in the beam other than the target, and muons from pion and kaon decay in the spectrometer. As a result, roughly 1 in 4 streamer chamber pictures contains a deep inelastic event. Thus, a reduction in the time spent measuring film is achieved if a list of such events can be prepared. This is the main aim of the geometrical reconstruction of events in the forward spectrometer. A preliminary determination of particle momenta allows a first estimate of the primary vertex position to be made. The list of events to be measured is now obtained by demanding that certain minimum requirements be satisfied.

The programme starts with the reconstruction of beam muons in the beam hodoscopes BHA and BHB. For each beam track found, the hits in the hodoscopes of the BMS are interrogated to determine the beam momentum. Straight line fits are performed, and a beam muon is accepted if five conditions are satisfied. Firstly, at least five hodoscope planes must have hits used in the fit. Secondly, if there is more than one fitted beam track then pairs of tracks whose time separation is less than 2ns are rejected. This criterion is designed to remove muons associated with knock-on electrons. Thirdly, pairs of beam tracks separated by less than the beam hodoscope, H3V time window (7ns) and whose transverse separation at the target is too small (< 1cm) are also rejected. This cut removes pairs of beam tracks which could cause ambiguities in the determination of the primary vertex. Fourthly, if the beam track has too large a \( \chi^2 \) from the line fit, it is dropped. Finally, if there remains more than one
beam track after these cuts (a situation which occurs in ≈ 7% of events), the programme uses the positive muon with the highest momentum (determined by the fitting procedure described below), to define a $\chi^2$. This $\chi^2$ is based on the closest distance of approach of the beam and scattered muon and the difference between the H3V time for this muon and the beam hodoscope time for the relevant beam track. The beam track giving the best $\chi^2$ is accepted.

The conditions imposed on beam tracks in trigger 10 events are identical to those described above. Thus, the corrections for wrongly rejected or fake beam tracks may be made using the trigger 10 beam tracks, as described in section 5.2.1.

The programme now proceeds to determine the momenta of final state particles as follows. In each chamber independently, a line is fitted to the sets of hits associated with a particular track by the pattern recognition programme. These fits are used to give space points along the track from P0A to W4,5. Hits in W6,7 are treated separately as described below. A quintic spline fit[^63] is now performed to estimate the particle's momentum and trajectory.[^64] For tracks of momentum less than 100 GeV/c the effect of multiple Coulomb scattering in all material along the track is taken into account. This is done by increasing the errors on the points during the error propagation in the quintic spline fit,[^64] i.e. effectively by modifying the $\chi^2$.

Muons are now identified. Through each set of hits in W6,7 a straight line is fitted and extrapolated upstream to the centre of the absorber, taking account of the multiple scattering. Each track in W4,5 is extrapolated downstream to the centre of the absorber taking account of the multiple scattering and a $\chi^2$ test is used to decide whether the line in W6,7 is consistent with the track fitted as far as W4,5.

A vertex is now fitted using the selected beam muon and every positive scattered muon whose momentum is larger than 15 GeV/c. The fit minimises the
\( \chi^2 \) for the minimum distance of approach without improving the track parameters.

A list of events to be reconstructed by measuring the streamer chamber film may now be obtained by making loose cuts on the results of the above track and vertex fits. These cuts only require that at least one muon satisfy the trigger itself (i.e. without the help of hadron hits in H1 etc.), and that the vertex lie within 5 standard deviations of the target.

4.5 Alignment of the vertex spectrometer.

The alignment of the vertex spectrometer includes the determination of the distortion corrections for the streamer chamber optics. To determine the corrections needed to reconstruct light rays in the streamer chamber, pictures are taken with the grid of wires at the top of the chamber illuminated. This is done both with the FSM field on, and with it off. The wires are then measured (see section 4.6.2) and the known position of the grid of wires is used to determine position dependent corrections for optical distortions introduced by the lens and image intensifier of each camera separately. These corrections are further adjusted using straight line tracks reconstructed in the vertex spectrometer alignment procedure described below. The determination of the optical constants of the streamer chamber is complicated by the presence of a magnetic field within the image intensifiers in later data taking. Consequently, the corrections for data taken with the field on are slightly different to those for data taken with the field off.

4.5.1 Vertex spectrometer alignment triggers.

In order to align the vertex spectrometer detectors a large number of straight tracks at large angles is required. These are recorded by means of a pion beam and a special trigger (the 'VS alignment trigger'). The beam line is modified to transport a beam of 100 GeV/c pions to a thin lead target placed in front of the
VSM. With the VSM switched off, a trigger is formed from coincident hits in hodoscopes placed immediately upstream and downstream of the lead target and hodoscopes placed just downstream of the drift tube arrays WV1,2,3.

A special trigger (the WV drift time calibration trigger) is formed by taking the far halo trigger in coincidence with hodoscopes behind the WV drift tube arrays. Tracks reconstructed in data taken with this trigger are used to determine the drift time parameters and the timing parameters of the delay lines.

4.5.2 The vertex spectrometer alignment procedure.

A modified version of the programme described in section 4.7 is now used to determine small corrections to the surveyors file of position measurements. In addition, chamber and counter parameters are tuned. The procedure is similar to that used for the forward spectrometer alignment described in section 4.2.

The timing parameters of the drift tubes and their delay lines are determined using the WV drift time calibration trigger. Then, data taken using the pion beam and the VS alignment trigger are used to align the drift tubes and the streamer chamber with respect to PV2.

The chambers PV1 and PV3 must be aligned with respect to the streamer chamber. Since the chambers are not illuminated with the VS alignment trigger, this is done using tracks recorded with the normal magnet configuration and the $^{90}_2$ trigger.

4.6 Reconstruction in the streamer chamber.

Rolls of film exposed during data taking periods are developed at CERN and then distributed among the member institutes. The institutes which measured the streamer chamber film for the data presented here are Aachen, Budapest, Hamburg, Mons, Munich, Orsay, Oxford, and Torino.
4.6.1 Measuring the film.

The pictures taken of the events on the scan list are measured on semi-automatic film measuring devices. The film measuring hardware is different in the different laboratories. Since the resolution and accuracy varies, a set of 'scan rules' has been agreed upon in order to try to minimise the systematic differences between laboratories.

A typical streamer chamber picture is shown in figure 28. The effect of out of time beam tracks may be clearly seen downstream of the target (which is outlined by the dashed line). To avoid the danger of using streamers not belonging to a particular track, no measurements are made within the confused beam region. Some tracks would then be measured over a distance too short for their momenta to be measured accurately, so, a track is only measured if it has a measurable length of at least 20 cm outside the confused beam region.

To avoid measuring low energy electrons, a minimum ionising track is not measured if its momentum is less than 100 MeV/c. Dark tracks (i.e. non-minimum ionising tracks) are measured irrespective of their momenta.

Attempts to use the ionisation information available from the streamer density were unsuccessful.\[65-67]\ It was found that the streamer density and shape were not uniform enough for an algorithm to be developed which would reliably separate protons from pions and kaons. Moreover, most dark tracks appeared to be dark simply because they were produced with a large vertical slope (dZ/dX).

In addition to the above track selection criteria, some frames were considered unmeasurable. Unmeasurable pictures fall into three categories. Either a frame was too faint, or it was covered with 'snow', or it had too many 'flares'.

If a frame was covered with a large number of small spots it was labelled as 'snow'. This was believed to have been caused by x-rays emitted from electrons rapidly accelerated in the high fields at the entrance of the chamber. In later
Figure 28: A typical streamer chamber picture. The position of the target is indicated by the dashed line. Many straight tracks are seen leaving the target. These are the out of time beam tracks mentioned in the text.

runs, the chamber entrance was optically sealed with foam. This significantly reduced the number of frames covered with 'snow'.
Flares are very dark spots on the film. They are assumed to be caused by electrons spiralling upwards or downwards in the magnetic field. Since the electron produces a large amount of ionization, much light is emitted. This saturates the image intensifier and causes a dark spot on the film. Unfortunately, tracks in the vicinity of a flare are distorted as a result of the overloading of the image intensifier. The severity of this problem was reduced by installing two stage image intensifiers in later running.

4.6.2 Geometrical reconstruction in the streamer chamber.

The parameters of the coordinate transformation connecting points measured on film to lines in space must be calculated for each frame independently. This is done by comparing the measured fiducial positions on the film with the reference points determined by direct measurement. An image point measured on the film now defines a 'light ray' originating from the object point somewhere within the chamber. If the same point is measured on all three views (e.g. a neutral decay vertex seen in the streamer chamber), then the point of intersection of the three light rays gives the position of the point in space.

The track of a charged particle appears as a curved line on the film (see figure 28). A parametrisation of a track is made in the film plane and this is then used to define a curved surface in space, on which the track must lie. An attempt is now made to pair tracks between views. This is done by projecting light rays from three points on each track (in each of the other two views) in turn into the chamber and calculating the points of intersection with the curved surface defined by the track on the first view. If no point on the track segment defined by this procedure lies outside the chamber, the line is kept as a track candidate. The intersection of the surfaces defined by the tracks on the two views is used to define a line in space, and a $\chi^2$ is calculated from the points measured on the tracks. Tracks measured on the third view are offered in turn to the 'doublet' defined above. The third track with the
smallest contribution to a new $\chi^2$ is included in a 'triplet'. A triplet with a $\chi^2$
per degree of freedom greater then 5 is rejected. Any doublets remaining are
retained as possible track candidates. The final list of tracks to be stored is
made by selecting the combination of triplets which gives the largest multipl-
licity.

Lastly, a vertex fit is made. Secondary vertices measured in the streamer
chamber are used to constrain tracks seen to be emerging from them. The
primary vertex position is estimated by finding the closest distance of approach
of all the tracks not associated with secondary vertices, without updating the
track parameters.

4.7 Geometrical reconstruction of the complete event.
Tracks reconstructed separately in the streamer chamber and the forward
spectrometer must now be collected together. By making use of the information
provided by the vertex spectrometer detectors, the track parameters of existing
tracks can be improved, and additional tracks may be found. In addition, the
full event topology remains to be determined. These are the tasks performed by
the vertex spectrometer geometrical reconstruction programme.

The first step is to link streamer chamber tracks, where possible, to tracks
reconstructed in the forward spectrometer. To do this, a forward spectrometer
track is extrapolated into the streamer chamber (taking account of the multiple
scattering), and a search is made for a streamer chamber track with which it
may be linked. If a link is found, the forward spectrometer track is used, and
the streamer chamber track deleted, since the forward spectrometer track is
more accurate. Any hits belonging to the track found in P0C or PV2 are used,
and the track is refitted.

Streamer chamber tracks not linked to forward spectrometer tracks are now
extrapolated downstream. Again, account is taken of the effect of multiple
scattering, and a search is made for hits close to the track in the detectors through which the track passed. If any hits are deemed to belong to the track, the track is refitted using this extra information.

A similar procedure is applied to the forward spectrometer tracks for which no matches were found in the streamer chamber. They are extrapolated upstream and an attempt is made to find extra hits in POC or PV2 which may be associated with the track. If any hits are found which are consistent with having been caused by this track, the track is refitted.

The pattern recognition of hits close to an extrapolated track in the vertex detectors is done as follows.

In PV1,2,3 two methods are used. If all six planes have candidate hits then the Y coordinate and the Y slope (dY/dX) are determined directly from the two outer planes. Two of the inner planes are now used to determine the Z coordinate and Z slope (dZ/dX), the last two 8 planes being used to make a consistency check. If less than 6 planes are found to have candidate hits, then the slopes and intercepts are calculated by solving a set of simultaneous equations. In POC, a weighted mean of clusters of hits in two adjacent parallel planes is calculated. These are then used to associate groups of hits to give an estimate of the position and slopes of the impact. In the drift tube arrays W1,2,3 lines are found in Y and Z projections and then associated using the delay line signals. The method used to find lines in W1,2 is the same as that used for W4,5 and W6,7 (see section 4.4). Finally, the method used to associate hits in POC is similar to that described for P0A,B.

The vertex fitting procedure begins by using the improved track parameters to refit the vertices formed by the beam muon and positive final state muons (type 1 vertices - see table 5). Three further types of vertex fit are then performed as described below.
Table 5: Vertex types.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Beam and one same sign muon.</td>
</tr>
<tr>
<td>2</td>
<td>Beam and more than one same sign muon.</td>
</tr>
<tr>
<td>3</td>
<td>Type 1 or 2 with hadrons associated.</td>
</tr>
<tr>
<td>4</td>
<td>Two oppositely charged hadrons ($V_0$).</td>
</tr>
<tr>
<td>5</td>
<td>Beam and oppositely charged muon.</td>
</tr>
<tr>
<td>6</td>
<td>Charged decay ($V^\pm$).</td>
</tr>
<tr>
<td>7</td>
<td>Secondary vertex (not $V^\pm$) with incoming track.</td>
</tr>
<tr>
<td>8</td>
<td>Secondary vertex (not $V^\pm$) without incoming track.</td>
</tr>
</tbody>
</table>

The primary hadron vertex is formed by associating tracks with the type 1 vertex. This is done by the vertex processor. If a track passes within a certain normalised distance of closest approach (i.e. the distance of closest approach divided by the error on this distance) to the type 1 vertex, the vertex is refitted, including the new track, but without updating any track parameters. An attempt is made to associate each track in turn, the vertex being refitted each time. The process is then repeated until a stable vertex position is determined, and each remaining unassociated track fails to satisfy the test. The beam muon and the scattered muon are labelled 'safe', and are never disconnected from the vertex.
If a vertex has been observed in the streamer chamber, for example a $\Lambda_S^0$ decay, the vertex as measured in the streamer chamber is used as a first estimate of the vertex position. Tracks seen to be connected to this vertex during scanning are marked as safe. Tracks are associated, and the vertex is fitted using the procedure described for associating tracks at the primary vertex. Again, the safe tracks are never disconnected.

To complete the event topology the programme attempts to group pairs of associated 'unsafe' and unassociated tracks (in all combinations). The criterion to be satisfied is that the tracks pass within a certain normalised closest distance of approach to one another. Extra tracks may be added to give vertices of higher multiplicity. Neutral decay vertices ($V_0$s) are rejected unless the invariant mass of the two charged tracks is consistent with the hypothesis that the parent particle was a photon, a kaon, or a lambda. $V_0$s must also 'point' back to the primary vertex.

Before storing the event, the programme uses the pattern recognition procedures described above (generalised so that a search may be made throughout a chamber module) to try to find tracks measured only in the vertex spectrometer. Since there is only a very small field outside the magnet, the momentum of these tracks is determined by assuming that they came from the primary vertex. Their track parameters are calculated using a quintic spline fit constrained by the primary vertex and the measured hits in the vertex spectrometer.

4.8 Particle identification and other matters.

The last programme in the E.M.C. analysis chain makes use of the information provided by the Cherenkov counters, time of flight hodoscopes and lead glass photon detector. It is also used to calculate the efficiencies of the vertex spectrometer wire chambers (see section 5.4.1). A brief outline of the tasks performed by the programme is given below.
4.8.1 The beam processor.
The function of the beam processor is to reject events with a badly recon-
structed beam particle. After checking that the beam tracks pass the cuts
described in section 4.4, additional cuts are made on the beam position and
phase space. The cuts are designed to ensure that the beam would have passed
through the beam hole in V2 and entered the target. In order to remove an
event in which the trigger may be spurious, limits are placed on the slope of
the beam track in both the Y and Z projections. A cut is also made on the beam
momentum. These cuts must be tuned for each data taking period independently.

4.8.2 The Cherenkov and time of flight processors.
The measurements made by the Cherenkov counters and time of flight
hodoscopes are used to calculate probabilities for a particle to be an electron,
muon, pion, kaon, or a proton - the particle hypotheses. The aim is to
identify hadrons. The Cherenkov threshold for electrons is low, and therefore,
for the Cherenkov analysis, electron tracks are considered to be a background.

If a reconstructed track passes through a Cherenkov counter, the
probability that it could have caused the observed number of photoelectrons is
calculated. Since the probability depends on the mass of the particle, the
calculation is performed five times, once for each particle hypothesis. These
probabilities are then manipulated to yield a number proportional to the chance
that the particle was an electron (or a muon, pion, kaon or proton) given the
observed number of photoelectrons. The calculation of the probabilities takes
into account the measured background light as described in reference 47.

The background light is of two types. One type of background is light
which is observed when the particle entering the counter is below the muon
threshold; this is called track correlated background. The second type of
background is light unassociated with any track - uncorrelated background. It
is much larger in photomultiplier tubes close to the beam than in those in the
wings of the counter. A large contribution to the track correlated background is due to electrons from photon conversions in the target. The uncorrelated background is less well understood. Possible sources include: tracks not found by the reconstruction programmes, tracks passing through the photomultiplier tubes themselves, and scintillation light produced in the active volume of the counter.

Particles entering the aperture of the time of flight hodoscopes are identified as follows. The probability that, if the track had been a pion (electron, muon etc.) the measured time of flight would have been obtained is calculated. This is used to give a measure of the probability that the particle was a pion (electron, muon etc.) given the observed time of flight. The calculation takes into account the time resolution of the hodoscopes as described in reference 47.

A complete description of the treatment of the Cherenkov counters and time of flight hodoscopes may be found in reference 47. In addition, this reference describes the calibration procedure in detail, and the methods used to deal with cases where more than one track gives light in a particular Cherenkov cell.

4.8.3 The match processor.
If more than one 'identification counter' (C0, C1, C2, CA, or time of flight hodoscope) can be used to give information on a track's identity, the probabilities must be combined to give a single probability that the track has a particular mass. This is done by assuming that the measurements made by each detector are independent.

4.8.4 The lead glass processor.
The algorithm starts by searching the lead glass photon detector (section 3.6.3) for the block in which the largest amount of energy has been deposited. If this is larger than some minimum (typically 2 GeV), it is taken to be the 'nucleus' of a possible 'neutral cluster'. Adjacent blocks are now associated with the cluster if the energy deposited in them is greater than some minimum (typically
Data reduction.

0.75 GeV). This process is iterative, starting with adjacent (not diagonally adjacent) blocks, and continuing until no further blocks may be associated to the cluster. The process is now repeated to look for additional clusters. Blocks associated with a cluster are not considered in subsequent searches.

The primary cluster energy is determined by summing the energy deposited in each block associated with the cluster. The energy deposited in any blocks immediately surrounding the cluster is added to the primary cluster energy, giving the final cluster energy. In order to determine cuts on the cluster energy, Monte Carlo studies were made using the NA9 Monte Carlo to simulate the apparatus, and EGS[68] to simulate the electromagnetic shower development in the lead glass. Clusters of less than some maximum energy, $E_{\text{cut}}$ (typically 25 GeV) are considered to have been caused by a single photon, and are classed as 'single clusters'. If the final cluster energy is greater than $E_{\text{cut}}$ it is not considered to have been caused by a single photon, and a moments analysis is made to find the centroid, or centroids, of the cluster. Clusters with one centroid are classed as single clusters, while those with more than one centroid are classed as overlapping clusters. Single clusters are paired, and the mass of the pairs is calculated. The moments analysis may be used to determine a mass for the overlapping clusters. Cuts on the mass may now be made to classify the clusters as photon clusters, or $\pi^0$ clusters. A cluster is rejected if a reconstructed charged track passes through a block associated with the cluster.
5.1 Introduction.

Neither the apparatus, nor the software used to reconstruct the events is perfect, so corrections must be made to remove the artefacts introduced in the data by these imperfections. The method used to calculate this 'acceptance correction' relies on a Monte Carlo simulation of the apparatus.

Conceptually the method is simple. A deep inelastic muon-proton scattering event is generated in the target. After the leptonic part of the final state has been generated according to the measured $\mu p$ cross-section, the final state hadrons are generated using the Lund model. Each generated event is stored. A set of such events is called a 'short-chain Monte Carlo data set'. Since the positions of all the detectors are known, and the magnetic fields of the two magnets have been accurately measured, the final state particles may be tracked through the apparatus. The impact positions in each chamber may be calculated and used to simulate the signals which would have been recorded had the event occurred in the real data.

In simulating the signals produced by impacts on a detector, account is taken of both its efficiency and its resolution. It is also important to include a simulation of any systematic effects known to be in the data. To this end, the energy loss experienced by final state hadrons as they pass through the target material is calculated, and the reinteraction of hadrons in the target is simulated. The effect of energy loss and multiple scattering on the trajectories of final state particles are calculated as they traverse the lead glass detector, the calorimeter or the absorber. A simulation of electron pair production by photons is made, taking into account all the material in the apparatus. The decays of unstable final state particles is simulated using the known decay rates
and branching ratios. The effect of the radiative corrections to the muon-photon vertex is simulated as described in section 5.2.1.

The simulated data generated as outlined above is now passed through the same reconstruction programmes as used for the real data. Two specific modifications to the analysis chain are required. Firstly, the streamer chamber picture measurement must be simulated. This is described in section 5.4.2. Secondly, the simulated raw data for the Cherenkov counters, time of flight hodoscopes and lead glass detector are generated after the event has been processed through the whole analysis chain - this is described in section 5.4.2.

5.2 Event generation.

5.2.1 Generation of the scattered lepton.

Before the final state hadrons can be generated the kinematic variables of the scattered muon must be determined. The distribution of events in $Q^2$ and $v$ will be affected by the distribution of the beam muons with respect to both momentum and position. The sample of beam tracks recorded using trigger 10, and accepted by the beam processor (see section 4.8), is used as the starting point of the generation of an event. The parameters of the beam track for each event are taken from one of the beam tracks in this sample. This ensures that the beam phase space and position are correctly simulated.

The momentum of the beam muon obtained above determines the region in the $Q^2$-$v$ plane accessible to the event. A $Q^2_{\text{meas}}$-$v_{\text{meas}}$ point is chosen within this region and is used to determine the momentum and energy of the scattered muon. (This is the value of $Q^2$ and $v$ which would be calculated from the beam and scattered muon, and is therefore the 'measured' $Q^2$-$v$ point - hence the subscript meas.) Using the formulae contained in reference 18 and a parametrisation of $F_2$, the effect of the radiative corrections to the muon-photon vertex (figure 6) may be calculated.\[19\] Radiative photons are generated according to
the QED cross-section and the values of $Q^2_{real}$ and $v_{real}$ of the real virtual photon calculated. The parameters of the virtual photon-proton centre of mass system may now be determined from $Q^2_{real}$ and $v_{real}$ and the hadronic final state may be generated as described below.

The real values of $Q^2$ and $v$ are used to define an event weight based on the measured muon-proton cross-section. This ensures that in the subsequent analysis, events in kinematic regions in which the cross-section is small enter only with a small weight.

5.2.2 Generation of the final state hadrons.

The values obtained above for $Q^2_{real}$ and $v_{real}$ determine $x$, and hence the probability that a particular flavour of quark will absorb the current. Parametrisations of the quark distribution functions, taken from reference 46, are used to determine which quark absorbed the virtual photon. This 'struck quark' now forms the current jet, and a target jet is formed from the quarks remaining in the target fragments. This is done using the procedure outlined in section 2.8.2 and reference 41. The hadronisation of the jets is then performed using the Lund model of reference 42. The model includes hard gluon bremsstrahlung from the struck quark (leading to three jet events), primordial $k_T$ and soft gluon emission before the hadronisation. In addition, hadrons decaying strongly or electromagnetically (for example the baryon resonances and $\pi^0$s) are allowed to decay according to the measured branching ratios.

5.3 Background simulation.

In addition to losing tracks, through hardware or software inefficiency, the apparatus may 'invent' or distort tracks. Various backgrounds may also cause losses of tracks preferentially in certain regions of phase space. In order to correct for such effects various background processes have been included in the Monte Carlo and are described in sections 5.3.1 to 5.3.3 below.
5.3.1 Hadron decays.

The particles allowed to decay as they are tracked through the apparatus are: $K^\pm$, $K^0_S$, $K^0_L$, $\pi^\pm$, $\Sigma^\pm$, $\Lambda$ and $\bar{\Lambda}$. The distance covered by the particle before it decays is determined by its measured lifetime, and the decay is generated according to the measured branching ratios.

Decays yielding two charged particles (e.g. $K^0_S \rightarrow \pi^+\pi^-$, $\Lambda \rightarrow p\pi^-$, etc.) may be reconstructed in the vertex fit. However, the kinks in tracks at points where charged particles have decayed (e.g. $K^+ \rightarrow \mu^+\nu$) may cause two problems. Firstly, they may fake low energy muons, and secondly, the kink may cause the track to be lost or wrongly reconstructed.

5.3.2 Photon conversion.

Real photons are produced in hadronic decays and also arise as bremsstrahlung photons from the incident or scattered muon (see section 5.2.1). The pair production of electrons in the material of the apparatus may cause a number of problems. Among the problems are fake tracks and fake neutral decay vertices. In addition, a large amount of the background light detected in the Cherenkov counters is believed to be caused by electrons.

All the material in the apparatus is considered as a possible source of photon conversions. It is assumed that the electrons produced are massless and share randomly the photon’s energy and momentum.

5.3.3 Secondary interactions.

The hadrons produced at the primary vertex may reinteract in the material of the apparatus. The target is the only material in which the reinteraction of hadrons is considered. There are two main reasons for this. Firstly, the target is the largest 'mass' through which the particle must travel. Secondly, hadrons produced are more likely to be included in the primary vertex if the secondary interaction occurred in the target. Tracks produced by a secondary interaction will be preferentially slow and at large angles to the beam. Since
wide angle tracks fix the $X$ position of the vertex, any tracks from secondary interactions associated to the primary vertex, in the vertex fit (see section 4.7), will strongly bias the vertex position. In addition, the secondary hadrons will have low momenta, and hence, will bias the fragmentation functions to low $z$, and large negative $x_F$ if fitted to the primary vertex. The procedure to be described below was used to give an estimate of the effect of secondary interactions using a relatively simple and fast programme.

The following particles are considered to be the dominant contributors to the secondary scattering in the target: $\pi^\pm$, $K^\pm$, $K^0_S$, $K^0_L$, $p$, $\bar{p}$, $n$, $\bar{n}$. The interaction length in liquid hydrogen, $\lambda^h(p)$, is calculated for each hadron from a momentum ($p$) dependent parametrisation\textsuperscript{[69]} of the hadron-proton cross-section. The potential path length, $d$, of a particular hadron in the target was chosen randomly according to the probability density function

$$P(d) = \exp\left(-\frac{d}{\lambda^h(p)}\right).$$ \hspace{1cm} (5.1)

If the interaction point is within the target a secondary interaction is generated as follows.\textsuperscript{[70]}

A momentum dependent parametrisation\textsuperscript{[69]} of the ratio of the elastic cross-section to the total cross-section is used to choose whether an elastic or an inelastic scattering event is to be generated.

**Elastic scattering events:** Elastic events are generated by choosing the momentum transfer squared, $t$, from the distribution

$$P(t) = \exp\left(-b|t|\right)$$ \hspace{1cm} (5.2)

with\textsuperscript{[71]} $b = 7.5 \text{ (GeV/c)}^2$. The momentum transfer is from the incident hadron to the proton involved in the secondary interaction. The azimuthal angle about the collision axis is chosen randomly between 0 and $2\pi$.

**Inelastic scattering events:** Inelastic events are generated using the Lund model. Natural consequences of this procedure are the inclusion of resonance production, a rapidly falling $p_T$ distribution (where $p_T$ the momentum
transverse to the secondary interaction collision axis) and an increase in multiplicity with the centre of mass energy vs. However, the leading particle effect is not simulated accurately by this procedure. In order to generate plausible longitudinal momentum distributions, the inelastic events are split into three classes, namely: fragmentation, diffractive and annihilation events. Fragmentation events are generated according to a slightly modified version of the procedure described in reference 72. Diffractive events are generated specifically to simulate the observed leading particle effect. The final third class of event is pp annihilation. A brief description of each class is given below.

- Fragmentation secondary scattering events: Two parton scattering cross-sections (e.g. qq-qq) are strongly peaked to low momenta (α ∝ 1/s²). If a hadron-hadron interaction is viewed as being due to one quark from the projectile 'hitting' the target, the interacting quark is most likely to have only a small fraction of the projectile's momentum. The remaining quark or quarks are essentially undeviated and carry most of the projectile's momentum. In reference 72 the interaction is viewed as taking place by the amalgamation of the projectile hadron 'bag' and the target 'bag'. The bag is stretched into a string, absorbing all the projectile's momentum into the colour field. Either approach leads to a jet system which may be hadronised using the Lund model. [42]

These ideas are implemented as follows. For non-strange mesons, the quark stopped by the proton is chosen from the valence quarks in the projectile with equal probability. In the case of strange mesons the non-strange quark has a 60% chance of interacting. [73] The undeviated 'leading' quark forms the forward jet system. The fraction of the projectile's momentum taken by the leading quark, x_{1q}, is chosen from the probability density function

\[ P(x_{1q})dP = k \left[ x_{1q}^2/(1-x_{1q}) \right] dx_{1q}. \]  

(5.3)
shown in figure 29. The particular form of $P(x_{lq})$ was chosen to that $P(x_{lq}) = 0$ rapidly as $x_{lq} \to 0$. A hadron ($h_2$) and a backward jet system are formed from the quarks in the proton as shown in figure 30. Some intrinsic momentum is now given to the components of the target in order to simulate the effect of $k_\perp$, and to give the jet system enough mass to fragment. [42]

![Graph](image)

**Figure 29:** Probability density function for the leading quark.

In some cases (where $\sqrt{s}$ is small), the programme is unable to fragment the jet system. In this case, the forward and backward jets are combined into a hadron, $h_2$. If $h_1$ and $h_2$ are not identical to the initial hadrons, the kinematics of the two new hadrons are calculated in the same way as for an elastic scattering event.

- **Diffractive secondary scattering events:** The leading particle effect produces a final state with one fast particle and a low mass remainder system. A certain fraction, $r$, of inelastic events was chosen to be diffractive.

The scheme for generating diffractive events is shown in figure 31. The scattered hadron is taken to be the same particle as the projectile. In the centre of mass of the projectile and proton, $p_n^*$ is the momentum of the
Figure 30: Generation of a fragmentation secondary interaction. Four methods were used to generate the jet system. (a) The antiquark (\( \bar{q} \)) from a meson is stopped by the proton. By combining it with a quark (\( q \)) from the proton a meson is formed. (b) The quark from a meson, picks up a diquark from the proton to form a baryon. (c) A quark from a baryon is stopped by the proton. A baryon is formed by combining it with a diquark from the proton. (d) An antiquark from an antibaryon picks up a quark from a proton to form a meson.

In each case two jet systems are formed from the remaining quarks and antiquarks.

(e) If \( \sqrt{s} \) is small and the programme is unable to fragment the jet system a hadron, \( h_3 \) (not identical to the target or projectile), is formed from the forward and backward jet systems.
scattered hadron parallel to the collision axis, and \( p_b^* \) that of the projectile. Defining \( x_1 \) as

\[
x_1 = \frac{p_\perp^*}{p_b^*},
\]

(5.4)

the value of \( p_\perp^* \) is determined by choosing a value for \( x_1 \) in some range \( \Delta(x_1) \). Both \( \Delta(x_1) \) and \( r \) were varied until the measured \( x_1 \) distribution was reproduced. To reproduce the data it was found to be necessary to subdivide the diffractive events into three subsets. Each subset contains a fraction \( r_i \) (\( i=1,2,3 \)) of the inelastic events (\( \Sigma r_i = r \)). The range \( \Delta x_{\perp_i} \) had to be chosen independently for each subset.

![Diagram](image)

Figure 31: Schemes for generating diffractive events. There are two possibilities for the orientation of the jets in the excited system. They are indicated in (a) and (b) above.

A quark-diquark jet system is then formed from the quarks in the proton, as shown in figure 31. The momentum and energy of the system are determined by \( p_\perp^* \) and \( \sqrt{s} \). The Lund procedure is used to fragment the jet.
system. If the mass of the excited system is too small, a baryon which is not allowed to be a proton, is formed from the jet system. Finally the event is rotated to give some $p_{\perp}$ to the leading hadron and the excited system.

The values of $\Delta(x_{\perp})$ and $r$ required to fit the $x_{\perp}$ distribution for 16 GeV/c positive pions taken from reference 74 are shown in table 6.

\begin{table}[h]
\centering
\begin{tabular}{lll}
Subset & Fraction (%) of inelastic events ($r_i$) & $\Delta x_{\perp i}$ \\
\hline
1 & 10 & $0 < x_{\perp} < 0.35$ \\
2 & 2 & $0.4 < x_{\perp} < 0.7$ \\
3 & 2.5 & $0.8 < x_{\perp} < 1$ \\
Total & & $r = 14.5\%$
\end{tabular}
\end{table}

- Anti-proton annihilation: The annihilation cross-section is a large contribution to the anti-proton - proton total cross-section. It may be written [74]

$$\sigma_{\text{ann}} = \sigma_{\bar{p}p} - \sigma_{pp} = a + b$$

(5.5)

with $a=61\text{mb}$ and $b=0.61(\pm0.02)$.

A jet system is formed by allowing two quarks to annihilate with two antiquarks. Since there is only one $d$ ($\bar{d}$) quark in a proton (antiproton) it is four times as likely that the jet system to be fragmented contains two $u$ quarks.

After each event has been generated, any unstable particles are allowed to decay.

Performance of the generator: In order to show that the results produced by the generator are a good approximation to the data, distributions produced with the algorithm as described above were compared to data for the
inelastic scattering of 16 GeV/c positive pions on protons. [74] Figure 32(a) shows the inclusive $x_1$ distribution generated by the Monte Carlo compared to the data from reference 74 (open squares). Agreement is good except in the central region. The depletion in the central region is caused by diffractive events in which the system recoiling against the leading hadron is taken by few particles.

Figure 32: Simulated distributions of hadrons from secondary interactions. The generator of secondary interactions was used to simulate π⁺p scattering for $p_\parallel$=16 GeV/c. The Monte Carlo results are shown as crosses, and the data (from reference 74) as open squares. (a) Inclusive distribution of $x_1$, (b) Inclusive distribution of $P_t^2$. See text for the definition of the kinematic variables.
The inclusive $p_t$ distribution is shown in figure 32(b). Agreement is satisfactory over two orders of magnitude.

In figure 33 the average charged multiplicity is plotted versus $s$. Again the agreement is satisfactory.

![Graph](image)

**Figure 33:** Average multiplicity from simulated secondary interactions. The generator was run for three beam energies, corresponding to three values of $s$. The figure compares the Monte Carlo results (full circles) for $\pi^+p$ scattering with the data from reference 74 (open squares).

The size of the effect: Using the generator described above the following results[75] were obtained. 28% of events contained a secondary interaction. After the Monte Carlo data had been passed through the analysis chain, it was found that 5% of tracks associated to the primary vertex originated from secondary interactions.

The contribution of 'faked' tracks to the tracks fitted to the primary vertex is a strong function of $x_F$ and $y^*$. As can be seen from figures 34(a) and 34(b)
for $x_F < -0.2$ or $y^* < -2$ the contamination is larger than 10%. In contrast, fake tracks contaminate $p_t$ distributions to a much smaller extent. Figure 34(c) shows that the contamination of fake tracks in $p_t$ is never more than 10%.

Figure 34: Contamination due to hadrons from secondary interactions. The ratio ($R_s$) of the number of hadrons from secondary interactions fitted to the primary vertex to the number of hadrons fitted to the primary vertex (excluding those from secondary interactions) is plotted as a function of: (a) $x_F$, (b) $y^*$, (c) $p_t^2$. Note that the pion mass was assumed throughout and that the kinematic variables are referred to the virtual photon axis.

5.4 Apparatus simulation.

Once the final state has been generated, the programme begins to track the final state particles through the apparatus. The first particle to be tracked is the scattered muon. The programme calculates the impact positions of the muon on the planes of the wire chambers and hodoscopes. The hodoscope and
chamber signals are simulated taking into account their resolution and efficiency. Before any other final state particles are tracked through the apparatus, a check is made to see whether the simulated hodoscope signals would have passed the trigger conditions; i.e. that the muon would have satisfied the trigger itself without the help of any possible background hits. This is a possible systematic error in the apparatus simulation, because the pattern recognition and geometrical reconstruction programmes might have found a valid muon track from the set of hits produced by the hadrons. Studies of the error made by not including events in which the trigger was satisfied with the help of hadron hits were made for the experiment NA2. It was found that the error was negligible.

If the trigger is satisfied, the programme tracks all remaining particles through the apparatus. Extra particles created, for example by hadron decays are also tracked. The simulated detector digitizations are created as follows.

- **Streamer chamber digitization**: The Monte Carlo records points in space along the track of a particle inside the streamer chamber. These correspond to streamers which might have been produced. These points form the input to the streamer chamber Monte Carlo, see section 5.4.2.

- **Wire chambers and hodoscopes**: The wire, or hodoscope element, giving a signal due to the passage of a simulated particle, is chosen to lie on a Gaussian the width of which is given by the chamber's resolution. The number of times the wire is said to have fired depends on its efficiency, which is determined as described in section 5.4.1.

5.4.1 Efficiency determination.

The efficiencies of the forward spectrometer hodoscopes are calculated using the pattern recognition programme described in section 4.3. A single efficiency for each element is calculated[76] by reconstructing lines in the wire chambers W3, W4,5 and W6,7, and counting how many times a hodoscope hit is found to correspond to this line.
The efficiency of the forward spectrometer wire chambers is calculated\cite{76} using the tracks determined by the spline fit performed by the programme described in section 4.4. For each plane through which the track passes, a check is made to see whether a corresponding hit was recorded. Parametrisations of each plane's efficiency are then made with respect to the wire number and the radial distance from the chamber centre.

The efficiencies of the vertex spectrometer detectors are determined in the same way, using the programmes described in sections 4.7 and 4.8. After counting the number of times a hit is found to correspond to a fitted track, a parametrisation is made for each plane as described above.

A large contribution to the inefficiency is made by the 'software inefficiency'; that is, the reconstruction software fails to find all possible tracks, finds false tracks or mis-associates hits. This inefficiency is taken into account twice, when the Monte Carlo-generated data is passed through the analysis chain. Monte Carlo studies have shown\cite{76,77} that, for a particular wire plane the correction is in error by only 2-4%.

5.4.2 The analysis chain.

The Monte Carlo data generated above is now reconstructed using the E. M. C. analysis chain as described in chapter 4. With the exceptions mentioned below, all programmes are the same as those used for the reconstruction of the data. By comparing the results of this 'long-chain Monte Carlo' to the short chain Monte Carlo, corrections can be made for all the effects included in the generation.

The simulation of the film measurement: The streamer chamber Monte Carlo programme projects the space points generated by the Monte Carlo on to the film plane. In doing so, it takes into account the measured optical distortions and the effect of the high voltage pulse on the streamer position. Points along the tracks are now chosen on the film plane and 'smeared' according to
the estimated resolution of a film measuring machine. These points are then fed into the streamer chamber geometry programme (section 4.6).

Simulation of hadron identification: The scheme used to simulate the signals recorded in the Cherenkov counters and time of flight hodoscopes is conceptually the same. The number of photoelectrons produced by a hadron generated by the Monte Carlo is chosen randomly from a Poisson distribution determined by the hadron's momentum and mass. To this is added the number of photoelectrons produced by the background light. This is obtained by sampling randomly from the measured background distribution. The result is added to the pedestal for the tube excited by the light and this is then recorded as the simulated raw data. The particle is then identified as described in section 4.8.

Simulation of the lead glass photon detector: The electromagnetic shower simulation programme EGS\(^{[68]}\) was used to generate showers in the lead glass. The deposition of energy was tabulated as a function of photon energy, position and y and z slopes. In addition, a parametrisation of the signals produced by the passage of a charged particle passing through the glass was obtained from the data. Surrogate data may now be obtained by a method analogous to that described in section 5.4.2. The photon energy is determined as described in section 4.8.4.
Selection of the Data Sample.

- Chapter 6 -

SELECTION OF THE DATA SAMPLE.

6.1 Data-taking.

The data presented in this thesis were taken in six SPS periods between September 1981 and May 1982. The beam conditions are summarised in table 7. In total 361,000 trigger 1 events were recorded, and 272,000 events were recorded using trigger 2. This is about half the number of events available from the experiment NA9 taken with a hydrogen target. The other half of the data (taken in 1982 and 1983) was not available for this thesis.

Table 7: Beam Conditions During the Running.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPS primary beam energy</td>
<td>400 GeV</td>
</tr>
<tr>
<td>SPS primary beam intensity</td>
<td>(3.5-4.5) x 10^{12} protons/spill</td>
</tr>
<tr>
<td>SPS cycle length</td>
<td>12 seconds</td>
</tr>
<tr>
<td>Spill length</td>
<td>1.4-1.5 seconds</td>
</tr>
<tr>
<td>Muon beam polarity</td>
<td>μ⁺</td>
</tr>
<tr>
<td>Nominal muon beam energy</td>
<td>280 GeV</td>
</tr>
<tr>
<td>Average muon beam energy</td>
<td>277 GeV</td>
</tr>
<tr>
<td>Muon beam intensity†</td>
<td>(5-6) x 10⁶ muons/spill</td>
</tr>
<tr>
<td>Muons/proton</td>
<td>1.5 x 10⁻⁶</td>
</tr>
<tr>
<td>Target</td>
<td>Hydrogen</td>
</tr>
<tr>
<td>Trigger Rate (T1)</td>
<td>(4-5) x 10⁻⁷ per muon</td>
</tr>
</tbody>
</table>

† muons accompanied by in-time hits in the veto counters are excluded.

6.2 Event selection.

There are a large number of events in which the trigger is satisfied by a combination of contributions from various sources of background (see section 4.4). Hence, deep inelastic events must be selected from the background. The principle aim of the selection criteria to be described below, is to restrict the sample to those events having a beam and scattered muon which are well
determined, so that the single arm kinematic quantities \((k_\mu, k'_\mu, q_\mu, \text{and } W)\) are well determined. A secondary aim is to produce a sample of events whose characteristics may be well simulated by the Monte Carlo procedure described in chapter 5. The preliminary reduction of the data, performed as described in sections 4.3 and 4.4, removes a large number of events. Five additional selection criteria are applied to this sample of events as follows:

1. The event must have one beam track only. This requirement removes any ambiguity in deciding which beam muon caused the event.

2. The event must contain at least one reconstructed scattered muon which by itself fulfills the trigger conditions.

3. The primary vertex (type 3 if it exists, otherwise type 1) must lie within the fiducial volume of the target. This cut is designed to remove events in which the muon scattered from the wall of the vessel containing the target. An event was accepted if the primary vertex position \((x_v, y_v, z_v)\) satisfied the following criteria

\[
x_t - (x_{lt}/2) - 5\sigma < x_v < x_t + (x_{lt}/2) + 5\sigma,
\]

\[
\sqrt{[(y_v-y_t)^2 + (z_v-z_t)^2]} < 0.9 r_t,
\]

where \(\sigma\) is the error on the \(X\) position of the vertex, \(x_{lt}\) is the length of the target, \(x_t, y_t, z_t\) are the coordinates of the centre of the target and the radius of the target is \(r_t\).

4. The event must not contain any reconstructed vertices (type 8) upstream of the primary vertex. In such events studies have shown that there is a high probability that the primary vertex is wrong.

5. There must be a good streamer chamber picture of the event. This is necessary since \(= 67\%\) of tracks are measured in the streamer chamber.
The number of events satisfying these criteria individually is shown in Table 8. Also shown in Table 8 is the effect of applying all the cuts simultaneously.

Table 8: Events passing the various event selection criteria.
The table shows the number of events passing each cut applied on its own. The last line gives the effect of all cuts applied simultaneously.

<table>
<thead>
<tr>
<th></th>
<th>P6C81</th>
<th>P7A81</th>
<th>P7B81</th>
<th>P7C81</th>
<th>P1A82</th>
<th>P1B82</th>
</tr>
</thead>
<tbody>
<tr>
<td>After analysis chain</td>
<td>6888</td>
<td>5361</td>
<td>7751</td>
<td>12350</td>
<td>3818</td>
<td>10549</td>
</tr>
<tr>
<td>Good beam</td>
<td>6030</td>
<td>5117</td>
<td>7262</td>
<td>11559</td>
<td>2629</td>
<td>7572</td>
</tr>
<tr>
<td>Good Muon</td>
<td>4928</td>
<td>4341</td>
<td>6076</td>
<td>9196</td>
<td>2961</td>
<td>7350</td>
</tr>
<tr>
<td>Good primary vertex</td>
<td>5951</td>
<td>4402</td>
<td>6426</td>
<td>10336</td>
<td>3465</td>
<td>9640</td>
</tr>
<tr>
<td>No upstream vertex</td>
<td>5828</td>
<td>4291</td>
<td>6247</td>
<td>10068</td>
<td>3377</td>
<td>9422</td>
</tr>
<tr>
<td>Good streamer chamber picture</td>
<td>3120</td>
<td>3517</td>
<td>5363</td>
<td>7103</td>
<td>2160</td>
<td>5073</td>
</tr>
<tr>
<td>Satisfying all criteria</td>
<td>2273</td>
<td>2662</td>
<td>4017</td>
<td>5401</td>
<td>1309</td>
<td>3317</td>
</tr>
</tbody>
</table>

6.3 Track selection.

The aim of the selection of tracks from those reconstructed in the analysis chain is to produce a sample of reliable tracks each with a well determined momentum. It is also important that the main characteristics of the selected tracks are well reproduced by the Monte Carlo simulation. Where possible, cuts have been made on quantities which are directly measured by the apparatus (e.g. the position of hits in a chamber). The selection criteria described below are based on the results of studies made by many members of the collaboration.

The reconstruction software defines four kinds of track as follows:
• FS tracks: Forward spectrometer tracks are tracks for which a full spline fit to the hits found in the forward spectrometer detectors has been possible.

• FS partial tracks: These tracks are FS tracks for which no hits have been found in the forward spectrometer detectors upstream of the FSM. Their momentum is determined on the assumption that the track originated from the primary vertex.

• SC tracks: Streamer chamber tracks are tracks for which streamer chamber measurements have been used in the determination of the track parameters. Note that if an FS track matches an SC track the FS track is used since the track parameters of an FS track are more accurate.

• VS tracks: Vertex spectrometer tracks are determined from the measurements of the vertex spectrometer proportional and drift chambers only. Such tracks are seen in at most two detectors. Their momentum is determined assuming that the track originated from the primary vertex. This assumption, and the fact that the magnetic field outside the VSM is small means that the VS tracks are less well measured than other kinds of track. Thus, particular care must be taken in their selection.

In addition, the following classes of track are defined:

• Primary tracks: Tracks associated to the primary vertex.

• Secondary tracks: Tracks associated to a secondary vertex.

• Close tracks: Tracks not associated to any vertex.

FS and SC tracks may be of any class, whereas FS partial and VS tracks can only be primary tracks since their momentum is determined on the assumption that they originate at the primary vertex. The selection criteria depend on the kind and class of track.

6.3.1 Selection criteria applied to all tracks.

The only selection criterion applied to all tracks is that a track must have momentum greater than 0.2 GeV/c in the laboratory system. In the Monte Carlo
simulation charged hadrons are not tracked out of the hydrogen target unless
their momenta are greater than 0.2 Gev/c. This cut must therefore be imposed
so that the data and Monte Carlo may be compared directly.

6.3.2 Selection criteria applied to forward spectrometer tracks.
Primary FS tracks are considered to be reliable well measured tracks and are
never rejected.

Since FS partial tracks lack hits before the FSM, a check is made to see
whether a hit can be found in PV2 or POC which may be associated to the
track. If no suitable hit is found, the track is rejected. This cut removes = 87%
of FS partial tracks.

FS close tracks are accepted if the distance of closest approach to the
primary vertex \(d_{\text{min}}\) is less than 3cm. Various studies indicated that FS
tracks were being lost. Some of them were found to be present in the data as
VS tracks. If a spurious hit is used in the determination of the parameters of
an FS track, it may be distorted in such a way that it can no longer be
associated to the primary vertex. In this case it is classified as a close track.
The effect is not realistically simulated by the Monte Carlo. The selection
criterion for FS close tracks was determined from the distribution of \(d_{\text{min}}\)
(figure 35). The distribution has a peak at \(d_{\text{min}} = 1.5\)cm. The dip at low
\(d_{\text{min}}\) corresponds to FS tracks which are associated to the primary vertex. The
cut on close tracks retains a large proportion of close tracks which should have
been associated to the vertex. This cut removes = 75% of close FS tracks.

6.3.3 Selection criteria applied to streamer chamber tracks.
There are two principle problems to be met in the selection of SC tracks.
Firstly, since the streamer chamber surrounds the target, electrons from photon
conversions, and hadrons from secondary interactions may contaminate the
sample of SC tracks. Secondly, if the curvature of a track is measured over a
distance of < 0.2m, large errors may be made in the determination of the track's
momentum.
Figure 35: Distance of closest approach for FS tracks. (a) Distribution of distance of closest approach ($d_{\text{min}}$) to the primary vertex for FS tracks. (b) Distribution of $d_{\text{min}}$ for close FS tracks. FS close tracks are accepted if $d_{\text{min}} < 3\text{cm}$.

By reconstructing secondary vertices the vertex processor minimises the contamination of background hadron and electron tracks in the SC track sample. Since streamer chamber tracks are often at large angles to the beam, the point at which they intersect the beam track may be accurately determined. Hence, if a track is not associated to the primary vertex by the vertex processor one may be fairly confident that it did not originate at the primary vertex. For this reason close SC tracks are excluded. There remains a small electromagnetic background caused by knock-on electrons and electrons from photon conversions. To reduce this contamination SC tracks are rejected if their distance of closest approach is greater than 1cm and their Z slope (the Z component of momentum divided by the X component of momentum) is less than 0.01.

A track's momentum ($p$) is determined from its curvature in the magnetic field of the VSM. Hence one measures $1/p$ and calculates $p$. Differentiating $1/p$ to find the error, $\Delta p$, on $p$, one finds that $\Delta p/p \propto p$. In order to remove tracks whose momenta are badly determined one may make a cut on $\Delta p/p$. Unfortunately, this is implicitly a cut on the momentum of the track. However,
one may argue that since $\Delta p/p \propto p$, tracks with a large error ($\Delta p$) are biased towards high momenta and should be removed. Hence, SC tracks are rejected if $\Delta p/p > 0.1$. The distributions are well reproduced by the Monte Carlo (see figure 36), so that the acceptance correction (see section 6.4) may be made with confidence.

![Graphs showing distributions of $\Delta p/p$ versus $p$.](image)

**Figure 36:** Distributions of $\Delta p/p$ versus $p$. Distributions of $\Delta p/p$ (a) for FS tracks, (b) for long-chain FS tracks, (c) for FS partial tracks, (d) for long-chain FS partial tracks, (e) for SC tracks, (f) for long-chain SC tracks, (g) for VS tracks, and (h) for long-chain VS tracks.

6.3.4 Selection criteria applied to vertex spectrometer tracks.

Vertex spectrometer tracks (VS tracks) are the least well measured of all the classes of track, and hence must satisfy the most rigorous selection criteria. The VS tracks may be divided into two sub-classes. The first sub-class contains those tracks for which information from only one detector is available to
constrain the track parameters, one detector VS tracks. The second sub-class
contains tracks determined from hits in two detectors. The two sub-classes of
track behave differently. One indication of this difference is shown in figure
36(g) and (h). The double banded structure of the graph of $\Delta p_p$ versus $p$
indicates that the errors on the track parameters of one detector VS tracks are
larger than for two detector VS tracks.

In order to be accepted, a one detector VS track must satisfy the following
criteria. Five or more hits must be present in the detector in which the VS
track was found, and the track fit probability must be greater than 0.01. If the
track is in the acceptance of the time of flight hodoscopes, W2, or P1 a hit must
be found in W2 or P1 corresponding to the track.

In addition, all VS tracks must satisfy the following criteria. A VS track is
rejected if, when extrapolated, it should have been seen in the streamer
chamber, but no line segment in the streamer chamber can be found to match
the track. If the track is within 0.3 radians of the horizontal plane it is
rejected. The distribution of VS tracks in the plane perpendicular to the beam
axis shows an excess in the horizontal plane. Furthermore, this excess is not
reproduced by the Monte Carlo. The excess arises because a set of hits found
in the bending plane of the magnet may always be fitted to a line passing
through the primary vertex, simply by adjusting the track's momentum, so,
these tracks must be removed. Tracks with a large error on the measured
momentum are rejected by making a cut on $\Delta p_p$ as for SC tracks. VS tracks
are rejected if $\Delta p_p > 0.2$.

6.3.5 Selection criteria to remove the double counting of tracks.

In each stage of the analysis chain care is taken to ensure that no track
appears twice, i.e. is reconstructed in two different categories. Nevertheless it
was found that some tracks do appear twice. Several methods were proposed to
solve this problem. The following method was chosen because the Monte Carlo
simulation of the effect it produced was the most satisfactory.
The distance between pairs of tracks in the plane to which measurements in PV2 and P0C are referred is calculated ($d_{sep}$). The pairs of track kinds which are considered are FS-SC, FS-VS, SC-VS. If $d_{sep} < 2$cm, one track is rejected. The rejected track is always the one from the class of tracks whose parameters are less well determined than its double. Thus, a VS track is rejected in favour of an SC track, and an FS track is preferred to an SC track. The result is that $\approx 3\%$ of tracks are rejected by this cut.

6.3.6 The effect of the selection criteria.

The effect of the track selection criteria is summarised in table 9. With these cuts the average charged multiplicity ($<n_{ch}>$) in the kinematic region to be defined in section 6.5 in the data is $= 6.5$. In the long chain Monte Carlo $<n_{ch}> = 6.47$. The Monte Carlo appears to be a good representation of the data.

Some problems remain. For example, studies of FS tracks indicate that there is a loss of up to 0.25 FS tracks per event. These tracks are not recovered either as close FS tracks or as VS tracks. This loss of tracks contributes to the systematic error, and is taken into account as described in section 6.6.

<table>
<thead>
<tr>
<th>Kind/class of track</th>
<th>Number of tracks before cuts</th>
<th>Number of tracks after cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS</td>
<td>13406</td>
<td>13406</td>
</tr>
<tr>
<td>FS close</td>
<td>3980</td>
<td>1012</td>
</tr>
<tr>
<td>FS partial</td>
<td>1500</td>
<td>183</td>
</tr>
<tr>
<td>SC</td>
<td>74654</td>
<td>68044</td>
</tr>
<tr>
<td>SC close</td>
<td>10597</td>
<td>0</td>
</tr>
<tr>
<td>VS</td>
<td>24141</td>
<td>9753</td>
</tr>
</tbody>
</table>
6.3.7 Assigning masses to particles.

Hadron production from deep inelastic $\mu p$ scattering is most conveniently studied in the centre of mass system of the proton and virtual photon. However, the momenta of hadrons are measured in the laboratory system. In order to obtain the momentum of a particle in the centre of mass system a Lorentz transformation must be made. This transformation requires that the particle's energy be known. In order to calculate the energy, the mass of a particle must be known. Since the experiment NA9 has an extensive particle identification system it would be possible only to consider hadrons which have been directly identified. This, however, would greatly reduce the size of the sample. An alternative possibility is to calculate a particle's energy according to some algorithm, and correct for any errors using the Monte Carlo. This method has the disadvantage that the correction which has to be made, depends, to some extent, on the model used to generate the Monte Carlo events.

The error made in calculating the energy of a particle with the incorrect mass is a function of the momentum of the particle. For momenta large compared with the particle mass the error is small. Thus for hadrons with $p > 6$ GeV/c ($x_F > 0$) the error is small. The largest problem arises for hadrons with low momentum where a large number of protons are to be found. If the boost to the centre of mass system for a slow proton is performed using the pion mass a systematic error will be made. The method of assigning masses to particles, to be described below, is an attempt to minimise the correction which has to be made for this effect.

The method for assigning masses to particles to be described below, relies on two premises. It is assumed that the proportion of protons is large for $x_F < 0$. This is supported by data taken from this experiment.\textsuperscript{78} Secondly, the method rests on the fact that if a pion is boosted using the proton mass the value of its centre of mass momentum is always less (i.e. more negative) than it
would be had it been boosted using the pion mass. There follows a brief
description of the method. [79]

All negative particles are treated as pions. All positive particles with \( x_{FP} \)
\((x_F \text{ calculated with the pion mass})\) greater than -0.2 are also treated as pions.
If a positive particle with \( x_{FP} < -0.2 \), has \( x_{FP} \) \((x_F \text{ calculated with the}
proton mass)\) less than -0.9 it is also treated as a pion. A positive particle with
\( x_{FP} < -0.2 \text{ and } x_{FP} > -0.9 \) is treated as a proton. The assignment criteria are
summarised in table 10.

Table 10: Summary of method used to assign masses to hadrons.

<table>
<thead>
<tr>
<th>Class of particles considered</th>
<th>Mass used</th>
</tr>
</thead>
<tbody>
<tr>
<td>all -ve particles</td>
<td>pion mass</td>
</tr>
<tr>
<td>+ve particles with ( x_{FP} &gt; -0.2 )</td>
<td>pion mass</td>
</tr>
<tr>
<td>+ve particles with ( x_{FP} &lt; -0.2 \text{ and } x_{FP} &gt; -0.9 )</td>
<td>proton mass</td>
</tr>
<tr>
<td>+ve particles with ( x_{FP} &lt; -0.2 \text{ and } x_{FP} &lt; -0.9 )</td>
<td>pion mass</td>
</tr>
</tbody>
</table>

The correction procedure described in section 6.4.2 relies on the assumption
that the distribution of particle masses (i.e. the relative production rates of
pions, kaons and protons) in the data is well described by the Monte Carlo. A
comparison of the data from this experiment[78] has been made with the Lund
Monte Carlo,[41] and shows good agreement. The \( x_F \) distribution of tracks
assigned the proton mass by the method described above was compared to the
proton yield from this experiment,[79] and again the agreement was good. Monte
Carlo studies[79] indicate that 65% of pions and 85% of protons are correctly
treated by this method.
6.4 Acceptance correction.

If the results are to be directly compared to data from other experiments, or with the predictions of theoretical models, the artefacts introduced into the data by the apparatus and the reconstruction software must be removed. This is done by comparing the events generated by the Monte Carlo (these are called short-chain data see chapter 5), with the data obtained by passing the generated events through the complete analysis chain (long-chain data). Thus, the corrected results correspond to hadron production from 280 GeV \( \mu \bar{\mu} \) scattering (in the kinematic region to be defined in section 6.5) where no particles are lost and no errors are made in the measurement of momenta, vertex position or event topology. The sample to which the corrected distributions correspond is defined below.

1. The data have been corrected for QED processes up to first order. Higher order QED radiative corrections are assumed to be negligible. This means that the corrected distributions are those which would have been obtained if the muon scattered from the proton by the exchange of a single virtual photon.

2. The effect of smearing, finite detector resolution, multiple coulomb scattering and software errors on the determination of a particle's momentum are corrected for. Hence, the corrected distributions are those which would have been produced by tracks with no errors on their track parameters.

3. A correction has been made for the loss of primary tracks, or the inclusion of fake tracks at the primary vertex. The fake tracks may arise from secondary interactions or the decays of hadrons. Thus, the distributions presented would have been obtained if all charged primary
hadrons had been detected. Hadrons produced in the strong or electromagnetic decays of hadrons are included as primary hadrons. The decay products of hadrons decaying weakly are not included. For some parts of the analysis the hadrons from $\Lambda$, $\bar{\Lambda}$ and $K^0_S$ decays are also included as primary hadrons.

Some problems remain after the correction has been made. A minor problem is that the beam energy to which the data is corrected is not a $\delta$ function at 280 GeV. The measured beam phase space is used to generate the Monte Carlo beam track. The corrected beam energy has a width of 20 GeV centred on 277 GeV. A second problem lies in the simulation of background. Secondary interactions are only simulated in the hydrogen target. A more serious problem lies in the simulation of the errors on track parameters and position measurements. Though care was taken to parametrise the characteristics of the wire chambers, the partition of tracks between the various classes in the long chain Monte Carlo is not identical to that found in the data. Such omissions lead to possible systematic errors in the results.

6.4.1 Acceptance correction procedure.
The acceptance correction procedure\cite{79,80} requires that the distribution to be corrected must be obtained from the data, the short-chain and the long-chain Monte Carlo data. The 'acceptance' is calculated by dividing the 'long-chain histogram' by the 'short-chain histogram'. This is done separately for each bin of the histogram. The relative error on the acceptance is taken to be equal to the relative error on the long-chain Monte Carlo data. Figure 37 shows the acceptance as a function of $\theta$, $Q^2$, $v$ and $W^2$.

Finally, the acceptance correction is performed by dividing the 'data histogram' by the acceptance. Again the calculation is done bin by bin. The statistical error on the final result is obtained by adding the relative errors on the data and acceptance in quadrature.
Figure 37: Acceptance as a function of (a) $\theta$, (b) $Q^2$, (c) $v$, and (d) $W^2$.

6.4.2 Correction for unknown particle masses.

A correction is made for the misidentification of particles identified by the method described in section 6.3.7. In the short-chain Monte Carlo the true particle masses are used. In the long chain Monte Carlo the identification procedure of section 6.3.7 is applied in the same way as for the data. The correction for mistakes in the assignment of masses to particles is then automatically made in the procedure described in section 6.4.1.
6.5 Cuts on single arm variables.

In this section the kinematic region from which events are accepted is defined. The aim is to accept events from as large a kinematic region as possible, while maintaining a large and smoothly varying acceptance. The cuts used are summarised in table 11.

Table 11: The kinematic cuts applied to the data. For each cut an indication of why the cut was made is given. Details may be found in section 6.5.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta &gt; 0.75^\circ$</td>
<td>Acceptance</td>
</tr>
<tr>
<td>$E_\mu &gt; 20$ GeV</td>
<td>Resolution</td>
</tr>
<tr>
<td>$4 &lt; Q^2 &lt; 200$ (GeV/c$^2$)$^2$</td>
<td>Radiative corrections and Monte Carlo</td>
</tr>
<tr>
<td>$20 &lt; v &lt; 260$ GeV</td>
<td>Resolution</td>
</tr>
<tr>
<td>$W^2 &gt; 20$ (GeV/c$^2$)$^2$</td>
<td>Acceptance, resolution and Monte Carlo</td>
</tr>
<tr>
<td>$y &lt; 0.9$</td>
<td>Radiative correction</td>
</tr>
<tr>
<td>$x &gt; 0.01$</td>
<td>Radiative correction and Monte Carlo</td>
</tr>
</tbody>
</table>

The restriction to events in which the muon scattering angle is greater than $0.75^\circ$ is made to remove the region in which the trigger is inefficient (see figure 37(a)).

The energy of the scattered muon must be greater than 20 GeV. This cut together with the requirement that $v$ satisfy $20 < v < 260$ GeV is made in order to ensure that the single arm variables are well defined. The effect of multiple
scattering and finite detector size is to smear the measured momenta about the true value. The Monte Carlo generates scattered muons with energy greater than 15 GeV. A cut in $E'_\mu$ slightly larger than this value ensures that the effects of resolution are properly accounted for by the acceptance correction procedure. The origin of the upper $v$ cut is to reinforce the $E'_\mu$ cut for low energy beam muons.

Low values of $v$ are obtained by subtracting two large numbers. The lower cut on $v$ must therefore be applied so that the error on $v$ from the errors on $E'_\mu$ and $E'_\mu$ does not become large compared to $v$.

To reduce the size of the radiative corrections the following cuts are made

$$Q^2 > 4 \, (\text{GeV/c}^2)^2, \quad y < 0.9, \quad x > 0.01. \quad (6.2)$$

As explained in section 2.5 radiative corrections are large at high $y$ and low $x$. This is shown graphically in figure 7.

The cuts on $Q^2$ and $x$ are also necessary to avoid problems with the comparison of the data to the Monte Carlo. The Monte Carlo begins generating at a $Q^2$ value of 2 $(\text{GeV/c}^2)^2$. A larger $Q^2$ cut is therefore required so that the resolution of the spectrometer is properly accounted for (cf. the $E'_\mu$ cut).

The QCD matrix elements, for the processes shown in figures 10(a) and (b), are calculated for a grid of $x$ and $W^2$ values. The lowest $x$ and $W^2$ values are 0.01 and 20 $(\text{GeV/c}^2)^2$ respectively. For this reason cuts in $x$ and $W^2$ must be applied. The Monte Carlo generates no events above $Q^2 = 200 \, (\text{GeV/c}^2)^2$ - hence the upper $Q^2$ cut.

The $W^2$ cut is made for three reasons. Firstly the acceptance is falling rapidly below $W^2$ of 20 $(\text{GeV/c}^2)^2$. Secondly the lower $v$ cut is reinforced for higher values of $Q^2$. The third reason has to do with the calculation of QCD matrix elements and was outlined above.
The region in the $Q^2$-$v$ plane to which the data is restricted is shown in figure 38.

Figure 38: The kinematic region covered by the data. The scatter plot shows the distribution of data in the $Q^2$-$v$ plane. The solid lines show the region selected from the $Q^2$-$v$ plane by the cuts described in section 6.5

6.6 Estimation of the systematic errors.

The acceptance correction procedure described above can only remove artefacts which were foreseen when the Monte Carlo programme was written. This section describes the method used to estimate the error on the results caused by the sources of error not taken into account in the Monte Carlo, and by the correction method itself.
The method used to estimate the systematic errors was based on the assumption that the largest systematic uncertainties arise from three sources. In order of decreasing importance these are:

1. The simulation of the partition of tracks between the various track classes.

2. The exclusion of background tracks.

3. The removal of the double counting of tracks.

Modifications of the track selection criteria were used to study each of the three sources of systematic error independently. For each of the five sets of modified selection criteria described below, acceptance corrected results were obtained. The difference between the new result and the result obtained with the selection criteria described above was taken to be the systematic error attributable to the source under study. The total systematic error on a particular quantity was calculated on the assumption that the result obtained using the five sets of cuts are independent estimates of that quantity. Hence, the systematic errors are added in quadrature. Table 12 summarises the sets of cuts used in the estimation of the systematic error. Each set of cuts is discussed below.

To estimate the effect of the failure to generate the correct partition of tracks in the Monte Carlo, a certain fraction of FS tracks were reclassified as VS tracks. This was done for long-chain Monte Carlo data only. The fraction of FS tracks \( R_K^- \) to be reclassified as VS tracks is determined by comparing the ratio of the number of FS to VS tracks in the data and long-chain Monte Carlo. \( R_K^- \) is adjusted until this ratio is the same in the data and the long-chain Monte Carlo. \( R_K^+ \) is determined separately for positive \( (R_K^+)^+ \) and negative \( (R_K^-)^- \) hadrons. The track parameters of the FS track are then smeared
Table 12: The sets of cuts used to determine the systematic error. Each set of cuts is designed to study a different systematic effect. This table outlines the cuts used to study various systematic effects. Details may be found in section 6.6.

<table>
<thead>
<tr>
<th>Set</th>
<th>Source under study</th>
<th>Cuts used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Partition</td>
<td>Move tracks from the class of FS tracks to VS tracks in the long-chain Monte Carlo.</td>
</tr>
<tr>
<td>2</td>
<td>Partition Loss of FS tracks</td>
<td>Remove 0.25 data FS tracks per event.</td>
</tr>
<tr>
<td>3</td>
<td>Double counting</td>
<td>Deselect tracks on the basis of the difference between their momenta.</td>
</tr>
<tr>
<td>4</td>
<td>Background rejection</td>
<td>Increase severity of track selection criteria.</td>
</tr>
<tr>
<td>5</td>
<td>Background rejection</td>
<td>Decrease severity of track selection criteria.</td>
</tr>
</tbody>
</table>

See text for details of the cuts used.

according to the average value of the error on the momentum of VS tracks. The values of $R_K$ and the VS track errors are given in table 13.

A related problem is the possible loss of $\approx 0.25$ FS tracks per event. In order to estimate the error made by not simulating this effect in the Monte Carlo a further 0.25 FS tracks per event were removed from the data. The tracks to be removed were chosen randomly.

A certain arbitrariness is involved in determining the exact values of the cuts designed to remove background tracks. The uncertainty in the values chosen for these cuts is another possible source of systematic error. To estimate the size of this error the selection criteria were modified to accept (a) more background tracks, and (b) less background tracks. To accept more background the following cuts were used
\[ \Delta p^x_p < 0.15 \text{ for SC tracks,} \]
\[ \Delta p^y_p < 0.3 \text{ for VS tracks,} \]
angle from y axis $> 0.2$ r for vs tracks, \hspace{1cm} (6.3)
\[ d_{\text{min}} < 4.5\text{cm for FS close tracks,} \]
\[ d_{\text{min}} < 1.5\text{cm and Z slope} > 0.005 \text{ for SC tracks.} \]

To accept less background the following cuts were used
\[ \Delta p^x_p < 0.05 \text{ for SC tracks,} \]
\[ \Delta p^y_p < 0.1 \text{ for VS tracks,} \]
angle from y axis $> 0.4$ r for vs tracks, \hspace{1cm} (6.4)
\[ d_{\text{min}} < 1.5\text{cm for FS close tracks,} \]
\[ d_{\text{min}} < 0.5\text{cm and Z slope} > 0.015 \text{ for SC tracks.} \]

Table 13: Constants used in the reclassification of FS tracks.
FS tracks are reclassified as VS tracks in order to estimate the systematic error caused by the partition of tracks between the various classes of track.

\[
\begin{align*}
R^+ K & \quad 0.41 \\
R^- K & \quad 0.38 \\
\text{Error on } 1/p & \quad 0.016 \text{ (GeV/c)}^{-1}
\end{align*}
\]

To estimate the error caused by the rejection of doubly counted tracks a second method of rejecting this background was used. In addition to the cut on the separation of two tracks in PV2,POC, if the difference between the momenta of two tracks $(\Delta x, \Delta y, \Delta z)$ satisfied the following criteria
\[ \Delta p^x < 1.\text{ GeV/c} \]
\[ \Delta p^y < 0.1\text{ GeV/c} \]
\[ \Delta p^z < 0.1\text{ GeV/c} \] \hspace{1cm} (6.5)
then one would be rejected. All pairs of tracks are considered. If the two tracks are of different kinds, the member of the pair to be rejected is chosen in the same way as before. If both tracks are of the same kind, then the track with the highest momentum is rejected.
In all figures the systematic errors determined by this procedure are indicated by the dashed extensions to the solid (statistical) error bars. Ideally, the systematic error would be determined by extensive Monte Carlo studies. In these ideal studies each of the parameters affecting the generation of the event would be changed. Indeed, it would be desirable to repeat the whole acceptance correction using a different model of hadron production. In addition, the ideal calculation would vary each of the parameters determining the response of the apparatus (e.g. detector resolutions and efficiencies). Such a procedure would be too expensive in both computer time and manpower, and one is forced to use a method similar to the one proposed in this section.
- Chapter 7 -

TRANSVERSE MOMENTUM AND THE LUND MODEL.

One of the main assumptions of the Lund fragmentation model (see section 2.8.2 and reference 38) is that when a quark-antiquark pair tunnel through the colour string, they acquire an equal and opposite momentum transverse to the string axis; i.e. the transverse momentum is locally conserved. One aim of the analysis presented in this chapter is to see whether the assumption that transverse momentum is locally conserved is justified.

It is also of interest to study the parameters of the Lund model which govern the transverse momentum of final state hadrons. In the Lund model the transverse momentum of a quark produced in the string is taken to be Gaussianly distributed with a width of $\sigma_q$. In section 7.2 an estimate of $\sigma_q$ is made. The quarks, confined inside the proton, must have some intrinsic momentum. The longitudinal properties of this momentum are parametrised by the structure functions. To take into account the effect of the degrees of freedom transverse to the virtual photon axis, additional assumptions have to be made. In the Lund model this primordial transverse momentum is taken to be Gaussianly distributed with a width of $\sigma_{k_t}$. An estimate of $\sigma_{k_t}$ is also made in section 7.2.

The data presented in this chapter are interpreted in terms of the phenomenology of the Lund model. All results have been corrected for artefacts of acceptance by the method described in section 6.4. The results presented in this chapter have been corrected to correspond to a sample of charged tracks containing no hadrons from the decay of $K^0_S$, $\Lambda$ or $\bar{\Lambda}$ particles.
7.1 Local conservation of transverse momentum in the fragmentation chain.

To study the local conservation of transverse momentum one may resolve the momenta of hadrons into the plane perpendicular to the virtual photon. Let $\Delta \phi$ be the angle between the projections of the momenta of two hadrons in this plane (see figure 39).

![Diagram of hadrons and virtual photon](image)

**Figure 39:** The definition of $\Delta \phi$. The figure shows the definition of the angle, $\Delta \phi$, between two hadrons in the plane transverse to the virtual photon. The virtual photon axis is perpendicular to, and directed into, the paper. Two hadrons are shown, $H_1$ and $H_2$. $H_1$ has rapidity $y_1^*$, $H_2$ has rapidity $y_2^*$. The momenta $p_{11}$ and $p_{12}$ are the momenta of hadrons $H_1$ resolved into the plane perpendicular to the virtual photon. The coordinate system is such that the $x$ axis lies along the virtual photon and the $y$ and $z$ axes complete a right handed coordinate system.

It is not possible to select only those pairs of hadrons which are adjacent in rank since most hadrons are the decay products of short lived primary hadrons. Therefore, it is necessary to find a set of cuts which will select a sample of pairs of tracks rich in hadrons adjacent in rank. Note that, since each
quark-antiquark pair is assumed to be produced independently of any other qq pair, the correlation in $\Delta \phi$ only exists between hadrons adjacent in rank.

Studies of hadron production in $e^+e^-$, hadron-hadron and lepton-nucleon scattering have shown that short range correlations in rapidity ($y$) exist between final state hadrons (for a review see reference 81). As an example, consider the two particle correlation function

$$\Phi(y, y') = \frac{\Lambda(y, y')}{\int \Lambda(y, y') dy}, \quad (7.1)$$

where

$$\Lambda(y, y') = \frac{1}{\Delta y \Delta y'} \langle (1/n) \Sigma_{k=1}^{n} e_{i} \delta_{iy} e_{j} \delta_{iy'} \rangle. \quad (7.2)$$

In equation 7.1 $e_{i}$ is the charge of the $i^{th}$ hadron. The multiplicity of the event is $n$ and the symbols $\langle \rangle$ represent an average over all events. $\delta_{iy}$ is 1 if the hadron lies in the rapidity interval $\Delta y$, and zero otherwise. $\Phi$ may be interpreted as the probability that the charge of the particle $k$ in the rapidity range $\Delta y'$ is compensated in the range $\Delta y$ by other hadrons in the same event. A peak is observed in $\Phi$ for values of $y$ close to $y'$. This enhancement is rather broad and extends over $\pm 2$ units of rapidity (see figure 40). This short range correlation is evidence that if two hadrons are close in rapidity (i.e. within $\pm 1$ unit of one another) they are likely to have been produced close in rank to one another in the fragmentation chain.

The phenomenology of fragmentation described in section 2.8.1 leads to the conclusion that if two charged hadrons are adjacent in rank, they will have opposite charges. To enrich the sample of pairs of hadrons with pairs that are adjacent in rank one may select pairs of tracks separated by less than one unit of rapidity ($|y_{1}^{*} - y_{2}^{*}| < 1$) and whose charges are opposite ($e_{1} e_{2} = 0$). Pairs of hadrons with the same charge can not be adjacent in rank. One may compare the distribution of $\Delta \phi$ for oppositely charged pairs of tracks close in rapidity
Figure 40: Rapidity correlation of hadrons produced in $e^+e^-$ annihilation. The charge compensation probability $\Phi(y, y')$ is plotted as a function of $y$ for a particle produced at $y'$. The data (taken from reference 82) show a short range correlation over $|y-y'| < 1$.

($|y_1^*-y_2^*| < 1$), with same sign pairs. If the proposition that transverse momentum is locally conserved is true, one would expect to see an enhancement at $\Delta \phi = 180^\circ$ in the opposite charge case over the same sign case.

Other effects may be present in the data which would lead to a difference between the behaviour of same and opposite charge pairs. A large contribution to any observed effect may be attributable to hadrons from the decays of short lived particles. In order to study this effect, two different modifications to the
Lund model were made. In the first no vector mesons were produced in the fragmentation chain, leading to fewer hadrons originating from the decays of primary hadrons. The second modification forced the Monte Carlo to produce only vector mesons in the fragmentation chain, i.e. no primary scalar mesons were produced.

In order to conclude that an effect, if observed, is due to the local conservation of transverse momentum in the Lund model, one must compare the data to a model in which the transverse momentum is not locally conserved. To do this a Monte Carlo with 'randomised $p_t$' was used. In this model the transverse momentum of the quark and the antiquark were chosen independently, i.e. the transverse momentum of the quark and antiquark were chosen randomly from independent Gaussian probability distributions of the same width. This leads to a model in which the quark and antiquark do not have equal and opposite $p_t$. Though transverse momentum is not exactly conserved in a particular event, it is conserved when averaged over many events. Any angular correlation remaining in this model must be attributed to the hadrons from the decay of unstable particles.

7.1.1 Results.

Figure 41 shows the distribution of $\Delta \phi$ for pairs of particles satisfying $|\gamma^*_1 - \gamma^*_2| < 1$. The normalisation is such that $\int (dN/d\Delta \phi) d\Delta \phi = 1$. In figure 41(a) $\Delta \phi$ is calculated for oppositely charged pairs, while figure 41(b) contains like sign pairs (both positive pairs and negative pairs). The two graphs have a different shape. The data points in figure 41(a) tend to rise as $\Delta \phi$ increases, whereas, the distribution for like sign pairs is relatively flat. Thus, if a pair of hadrons is produced close in rapidity, the angle between them is more likely to be large if they are oppositely charged, than if they have the same charge.

Also shown in figure 41 are the predictions of the standard Lund model (solid line), and the randomised $p_t$ model (dashed line). There is a clear
+ Data − Δy' < 1.

---------- Randomised p_{\perp} model.

----------- Standard Lund model.

**Figure 41:** Differential distributions of Δφ. The inclusive distributions of pairs of tracks for which |y_{1}^* − y_{2}^*| < 1 and the charges of which are (a) opposite, and (b) the same. The solid lines show the predictions of the standard Lund model, while the dashed lines show the prediction of the randomised p_{\perp} model.

difference between the two predictions in figure 41(a). The data are in agreement with the standard Lund model, but differ markedly from the Lund model with randomised p_{\perp}. A further point to note is the similarity between the trends of the two models and the data in figure 41(b). This indicates that the shape of the distribution of Δφ for like sign tracks is an estimate of the shape that would be obtained from uncorrelated pairs of tracks.
The effect of decays is investigated in figure 42. The solid lines in figures 42(a) to (d) show the predictions of the standard Lund model. The results obtained if scalar mesons only are produced as primary hadrons are shown in figures 42(a) and (b). Also shown are the predictions of a model in which only vector mesons are produced as primary mesons. In both cases the lines lie close to the prediction of the standard Lund model and the data. For comparison figures 42(c) and (d) show the effect of randomising the transverse momentum of the quark-antiquark pair, and allowing only vector or only scalar mesons to be produced in the fragmentation chain as primary hadrons. The standard Lund model gives a better representation of the data than any of the modified models.

7.1.2 Conclusion.

The results presented in section 7.1.1 indicate that the Lund model, which includes the assumption that transverse momentum is locally conserved, reproduces the trends of the data. The disagreement between the results obtained with the randomised $p_\perp$ Monte Carlo and the data allow one to conclude that, within the framework of Lund model phenomenology, $p_\perp$ is locally conserved.

7.2 An estimation of $\sigma_q$ and $\sigma_{k_\perp}$.

In this section an estimate of the Lund model parameters $\sigma_q$ and $\sigma_{k_\perp}$ is presented. The assumption is made that $p_\perp$ is locally conserved (see section 7.1). The significance of the parameters $\sigma_q$ and $\sigma_{k_\perp}$ is described in detail in section 2.8.2 and reference 38.

The size of the parameter $\sigma_q$ determines the size of the relative transverse momentum of two primary hadrons. Thus, the difference, $\delta q = | p_{1\perp} - p_{2\perp} |$, between the transverse momenta of two primary hadrons which are adjacent in rank is sensitive to the value of $\sigma_q$. As above a sample of tracks must be chosen which is rich in hadrons which are adjacent in rank. In the analysis
Figure 42: Contributions to the angular correlation. The distribution of $\Delta \phi$ for pairs of tracks for which the rapidities differ by less than one unit, are compared to various Monte Carlo models. For (a) and (c) oppositely charged hadrons were used, while (b) and (d) contain data for like sign pairs. In each plot the result of the standard Lund model is shown as the solid line. The dashed lines show the predictions of models in which no scalar mesons are produced as primary mesons in the fragmentation chain. The dash-dotted lines show the results obtained if no vector mesons are produced as primary mesons. In figures (a) and (b) $p_\perp$ is locally conserved, whereas in figures (c) and (d) the randomised $p_\perp$ model was used to calculate the dashed and the dotted lines.
presented below the quantity $\delta q$ is calculated for tracks with opposite charge and for which $|y_1^*-y_2^*| < 1$.

An ideal method to determine the value of $\sigma_q$ would be to adjust the value $\sigma_q$ until the distribution of $\delta q$ produced by the Monte Carlo is the same as that found in the data. This is impracticable because it would require a large amount of computer time. The method used in this analysis is to take the mean of the $\delta q$ distribution ($\langle \delta q \rangle$), and the mean of the $\delta q^2$ distribution ($\langle \delta q^2 \rangle$) to be characteristic of the $\delta q$ and $\delta q^2$ distributions respectively.

A quark confined within a proton has some Fermi momentum. Thus, the momenta of hadrons produced by the fragmentation of the struck quark will contain a contribution $k_\perp$ due to the primordial transverse momentum. The net effect is that a transverse momentum $k_\perp$ is added to the momentum of each hadron in the forward hemisphere in the centre of mass system, and a contribution $-k_\perp$ is added to the momentum of each hadron in the backward hemisphere. Thus, the overall effect of the Fermi motion of the quarks inside the proton is to rotate the event axis with respect to the virtual photon axis.

Previous studies of hadron production in muon proton scattering$^{[83]}$ yielded results in agreement with the Lund model, but were unable to distinguish between the standard Lund model including soft gluon effects, and the Lund model without soft gluon radiation but with a large value of $\sigma_{k_\perp}$ (=0.88 GeV/c). Subsequent results$^{[84,85]}$ have shown that to reproduce the data the Lund model must include the effects of soft gluon radiation. The value of $\sigma_{k_\perp}$ remains to be determined.

In order to study the effect of $k_\perp$ on the event axis, a four vector $P$ was formed from the sum of the momenta of all hadrons with $x_F < 0$. $P$ is defined as follows

$$P = (E_T, P)$$

(7.3)

where
\[ E_T = \Sigma E_i \]
\[ P = \Sigma p_i. \]

(7.4)

E_i and \( p_i \) are the energy and momentum of the \( i^{th} \) hadron, the sum runs over all hadrons with \( x_F < 0 \). The direction defined by the vector \( P \) may be taken as an estimate of the event axis. The component of \( P \) transverse to the virtual photon axis (\( K_A \)) is calculated and distributions of \( K_A \) and \( K_A^2 \) are plotted in bins of the fraction, \( f (=2E_T/W) \), of the available centre of mass energy observed as charged particles. These distributions may be used to calculate the average values of \( \langle K_A \rangle \) and \( \langle K_A^2 \rangle \).

The next stage is to make an estimate of the values of \( \langle K_A \rangle \) and \( \langle K_A^2 \rangle \) which would be obtained if no energy was lost as neutral particles (\( \langle K_A \rangle_A \) and \( \langle K_A^2 \rangle_A \) respectively). This is done by plotting \( \langle K_A \rangle \) (\( \langle K_A^2 \rangle \)) against \( f \). A straight line is fitted through the points and extrapolated to \( f=1 \). The values of \( \langle K_A \rangle_A \) and \( \langle K_A^2 \rangle_A \) may be obtained from the fit.

To extract the values of \( \sigma_Q \) and \( \sigma_{K_A} \), the Lund Monte Carlo was used to study the variation of \( \langle \delta q \rangle \) (\( \langle \delta q^2 \rangle \)) and \( \langle K_A \rangle_A \) (\( \langle K_A^2 \rangle_A \)) with \( \sigma_Q \) and \( \sigma_{K_A} \). Monte Carlo calculations show that \( \langle K_A \rangle_A \) is not independent of \( \sigma_Q \) and \( \langle \delta q \rangle \) is not independent of \( \sigma_{K_A} \). So, the values of \( \sigma_Q \) and \( \sigma_{K_A} \) must be extracted simultaneously from the data. To do this a graph of \( \langle \delta q \rangle \) versus \( \langle K_A \rangle \) is plotted. The Monte Carlo is used to generate a grid of points corresponding to different \( \sigma_Q \), \( \sigma_{K_A} \) values. These points may be used to define contours of constant \( \sigma_Q \) and \( \sigma_{K_A} \). The values of \( \sigma_Q \) and \( \sigma_{K_A} \) allowed by the data may now be obtained from the graph.

7.2.1 Results.

The distributions of \( \delta q \) and \( \delta q^2 \) are shown in figure 43. The systematic errors on the means of the distributions were calculated from the means of the distributions obtained under the conditions described above, using the same method as that described in section 6.4.1 for calculating the systematic errors of distributions. The mean of the \( \delta q \) distribution is
\[ \langle \delta q \rangle = (0.539 \pm 0.001 \pm 0.020) \text{ GeV/c.} \]  
(7.5)

The first error is statistical, the second systematic. The mean of the \( \delta q^2 \) distribution is

\[ \langle \delta q^2 \rangle = (0.415 \pm 0.003 \pm 0.030) \text{ (GeV/c)}^2. \]  
(7.6)

Figure 43: The distributions of \( \delta q \) and \( \delta q^2 \). The figure shows normalised distributions of (a) \( \delta q \), and (b) \( \delta q^2 \).

Histograms of \( K_z \) and \( K_z^2 \) were plotted in bins of \( f \). The mean was calculated from the acceptance corrected distribution. The systematic error on the mean was calculated in the same way as that used to determine the systematic error on \( \langle \delta q \rangle \) above. Figure 44(a) shows the variation of \( \langle K_z \rangle \) with \( f \). A straight line was fitted to the points and extrapolated to \( f=1 \). The value of \( \langle K_z \rangle_z \) may be obtained from the fit:

\[ \langle K_z \rangle_z = (0.643 \pm 0.009 \pm 0.014) \text{ GeV/c.} \]  
(7.7)
Figure 44(b) shows the variation of $\langle K_4^2 \rangle$ with $f$. Using the same method as that used to determine $\langle K_4 \rangle_1$ one obtains:

$$\langle K_4^2 \rangle_1 = (0.56 \pm 0.01 \pm 0.03) \text{ (GeV/c)}^2.$$ (7.8)

The statistical errors on $\langle K_4 \rangle_1$ and $\langle K_4^2 \rangle_1$ were obtained from the fit. The systematic errors were obtained by calculating $\langle K_4 \rangle_1 (\langle K_4^2 \rangle_1)$ for each of the five sets of cuts described in section 6.4.1, and then applying the method of section 6.4.1 to the values of $\langle K_4 \rangle_1 (\langle K_4^2 \rangle_1)$ obtained.

![Graph](image)

Figure 44: $\langle K_4 \rangle$ and $\langle K_4^2 \rangle$ versus the fractional visible energy, $f$. The figure shows the variation with $f$ of (a) $\langle K_4 \rangle$ and (b) $\langle K_4^2 \rangle$. The solid line is the result of a straight line fit. The bars on the abscissa $f=1$ show the result obtained for $\langle K_4 \rangle_1 (\langle K_4^2 \rangle_1)$ together with statistical and systematic error.

In figure 45 graphs of $\langle \delta q \rangle$ versus $\langle K_4 \rangle_1$ (figure 45(a)) and $\langle \delta q^2 \rangle$ versus $\langle K_4^2 \rangle_1$ (figure 45(b)) are shown. The data points are shown as crosses. The error bars include both statistical and systematic errors. For clarity the bars drawn on the axes of the figures show the position of the relevant experimental
measurement. The Monte Carlo generated points are shown as circles. Straight lines were drawn through these points to give contours along which $\sigma_q$ is fixed while $\sigma_{k_\perp}$ is varying and vice-versa.

\begin{center}
\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure45.png}
\caption{The simultaneous estimation of $\sigma_q$ and $\sigma_{k_\perp}$. The figure shows graphs of (a) $\langle \delta q \rangle$ versus $\langle k_{\perp}^2 \rangle$ and (b) $\langle \delta q^2 \rangle$ versus $\langle k_{\perp}^2 \rangle$. The circles are the points obtained using the Lund Monte Carlo, the lines are contours of constant $\sigma_q$ and $\sigma_{k_\perp}$. The data points are shown as crosses. The bars on the axes indicate the experimental measurements.}
\end{figure}
\end{center}

Using a linear extrapolation between the contours drawn on figure 45(a) the following values for $\sigma_{k_\perp}$ and $\sigma_q$ are obtained:

$$\sigma_{k_\perp} = (0.27 \pm 0.12) \text{ GeV/c},$$

(7.9)

and

$$\sigma_q = (0.41 \pm 0.02) \text{ GeV/c}.$$  

(7.10)
7.3 In conclusion.

To conclude, it has been shown that:

1. An angular correlation exists between final state hadrons separated by less than one unit of rapidity and having opposite charge. In terms of Lund model phenomenology this has been shown to correspond to the local conservation of transverse momentum in the fragmentation chain, i.e. that a quark and antiquark produced in the fragmentation chain are produced with equal and opposite momentum transverse to the string axis.

2. The Lund model parameter $\sigma_{k_4}$ has been determined to be

$$\sigma_{k_4} = (0.27^{+0.12}_{-0.20}) \text{ GeV/c.}$$  \hspace{1cm} (7.11)

3. Finally, the Lund model parameter $\sigma_q$ has been estimated to be

$$\sigma_q = (0.41 \pm 0.02) \text{ GeV/c.}$$  \hspace{1cm} (7.12)
A Comparison Between $\mu p$ and $e^+e^-$ Scattering.

- Chapter 8 -

A COMPARISON BETWEEN $\mu p$ AND $E^+E^-$ SCATTERING.

When quark fragmentation is described in terms of the parton model, the assumption is made that the fragmentation does not depend on the process by which the quark was excited. It is assumed (see section 2.6.1 and reference 20) that the spectrum of hadrons is determined solely by the energy and flavour of the fragmenting quark. It is the purpose of this chapter to test this assumption. The observed differences from the quark parton model predictions are investigated in order to determine their origin. In addition, in section 8.4, a comparison is made between the results of this experiment and the predictions of the QCD based Monte Carlo described in section 2.9.

In muon proton scattering the virtual photon provides a natural event axis to which the momenta of hadrons may be referred. There is no such natural event axis for hadrons produced in $e^+e^-$ annihilations. Event axes must, therefore, be defined using the final state hadrons themselves. The sphericity and thrust axes were chosen, and are defined in section 8.1.

Before a comparison between the properties of the hadronic final state of $e^+e^-$ and $\mu p$ scattering is made, it is instructive to compare hadronic distributions referred to the different event axes (virtual photon, sphericity, and thrust). This is done in section 8.2 and allows one to assess the effect that the choice of event axis has on the hadronic distributions.

The comparison of hadron production from deep inelastic $\mu p$ scattering and hadron production from $e^+e^-$ annihilations is presented in section 8.3. The $e^+e^-$ results were taken from reference 86. In order that this comparison can be made, most of the distributions presented in this chapter have been corrected to correspond to distributions in which the hadrons from the decays of $K^0_S$, $\Lambda$ and $\bar{\Lambda}$ particles are included as primary hadrons. The only exceptions to this are
the results presented in section 8.4 where the hadrons from \( K^0 \), \( \Lambda \) and \( \bar{\Lambda} \) decays are excluded. In this case the results are to be compared with the Monte Carlo of reference 45 and it is, therefore, inappropriate to include such particles.

The results presented in this chapter have been obtained from a restricted sample of events. In addition to the standard event selections described in section 6.5 the following cuts have been made:

- Each event must have a charged multiplicity of at least four. This cut is made so that the sphericity and thrust axes can be meaningfully determined.
- \( 10 < W < 18 \text{ GeV/c}^2 \): The restriction to this range of \( W \) is made because the \( e^+e^- \) data used for comparison were obtained at a fixed centre of mass energy of 14 GeV. The average value of \( W \) in the sample of events used for the comparison is 14.2 GeV.

The data presented in section 8.4 contain events from the full range of \( W^2 \).

8.1 Determination of the sphericity and thrust axes.

The sphericity axis is defined\(^{[87]}\) as the major axis of the momentum tensor

\[
M_{\alpha\beta} = \Sigma_{i} p_i^\alpha p_i^\beta,
\]

where \( \alpha, \beta \) represent the \( x, y, z \) components of momentum (\( p \)) in the centre of mass system, and the sum runs over all final state particles. \( M_{\alpha\beta} \) has eigenvectors \( n_1, n_2 \) and \( n_3 \). The eigenvalues are given by

\[
Q_k = \Sigma_{i} (p_i \cdot n_k)^2/(\Sigma p_i^2).
\]

The sphericity axis is the eigenvector corresponding to the largest eigenvalue.

The sphericity is defined to be

\[
S = \frac{3}{2} (Q_1 + Q_2),
\]

where \( Q_1 Q_2 Q_3 = 1 \) and \( Q_1 < Q_2 < Q_3 \) i.e. \( n_3 \) is the sphericity axis. Note that equation 8.3 may be written

\[
S = \frac{3}{2} \left( \Sigma p_{\perp}^2 \right)/(\Sigma p^2),
\]

(8.4)
where $p_{\perp S}$ represents the momentum transverse to the sphericity axis. Thus the sphericity axis is the axis for which $\Sigma p_{\perp}^2$ is a minimum. Spherical events have $S=1$, while a linear event would have $S=0$. The sphericity, $S$, satisfies the inequality $0 \leq S \leq 1$.

The thrust axis is defined as that axis for which

$$T = (\Sigma |p_{\perp t}|)/(\Sigma |p|),$$

is a maximum (the subscript $t$ indicates that the momentum is referred to the thrust axis). Spherical events have $T=3/2$, while linear events have $T=1$. The thrust, $T$, satisfies the inequality $3/2 \leq T \leq 1$.

For the analysis described in reference 86 the data were corrected so that the distributions obtained correspond to those that would have been obtained if all charged and neutral particles were used to determine the event axis. The results presented in section 8.2 have been obtained using charged particles only. When the comparison with the data for $e^+e^-$ annihilation is made, the distributions are corrected to correspond to distributions in which charged and neutral ($\pi^0$, $K_L^0$, $n$ and $\bar{n}$) particles have been used to determine the jet axis.

8.2 Comparison of event axes.

In this section a comparison of hadronic distributions referred to the virtual photon, sphericity and thrust axes is presented.

8.2.1 Comparison of $p_{\perp}^2$ distributions.

The $p_{\perp}^2$ distribution plotted with respect to the sphericity axis is compared to the $p_{\perp}^2$ distribution plotted with respect to the virtual photon ($\gamma_V$) axis in figure 47(a). The distributions are different in shape. Since the sphericity axis is the axis for which $\Sigma p_{\perp}^2$ is a minimum, the distribution of $p_{\perp}^2$ with respect to the sphericity axis is shifted to smaller values of $p_{\perp}^2$. As a result, the distribution of $p_{\perp}^2$ with respect to the sphericity axis is larger in the first bin and steeper than the $p_{\perp}^2$ distribution with respect to the virtual photon axis.

The $p_{\perp}^2$ distribution plotted with respect to the thrust and $\gamma_V$ axes are
Figure 47: $p_z^2$ distributions plotted with respect to different axes. (a) The distribution of $p_z^2$ with respect to the virtual photon axis (circles) is compared to the $p_z^2$ distribution referred to the sphericity axis (squares). (b) The distribution of $p_z^2$ referred to the $\gamma_V$ axis (circles) is compared to that plotted with respect to the thrust axes (triangles).
compared in figure 47(b). Again the distributions are different in shape. Since the thrust axis is the axis for which $\Sigma |p_{lt}|$ is a maximum, the $p_\perp^2$ must be smaller than that with respect to the $\gamma_{v}$ axis. The result is that the $p_\perp^2$ distribution with respect to the thrust axis is shifted to smaller $p_\perp^2$, in a similar way to the $p_\perp^2$ distribution referred to the sphericity axis. The effect is less pronounced than in the case of the distribution referred to the sphericity axis because the sphericity axis is the axis for which $\Sigma p_{s}^2$ is a minimum.

8.2.2 Comparison of longitudinal distributions.

The longitudinal distributions plotted with respect to the various axes are shown in figure 48. The distributions are similar. Some points are worthy of comment.

In figure 48(a) the $x_F$ distribution plotted with respect to the sphericity axis is smaller than that plotted with respect to the $\gamma_{v}$ axis in the bins around $x_F = 0$. For large values of $|x_F|$ the distributions agree well. The differences between the longitudinal distributions are shown more clearly in figure 48(c), where the distributions of $\gamma^*$ are plotted with respect to the sphericity and $\gamma_{v}$ axes. The $\gamma^*$ distribution with respect to the sphericity axis is smaller in the central bins but extends to larger values of $|\gamma^*|$. The distributions of $\gamma^*$ with respect to the sphericity axis ($\gamma_s^*$) is slightly skewed to positive $\gamma^*$, i.e. $-4 < \gamma_s^* < 4.5$. These graphs show how the minimisation of $\Sigma p_{s}^2$ tends to increase the component of a hadron's momentum parallel to the event axis. The effect is to populate the wings of the $x_F$ or $\gamma^*$ distribution at the expense of the central bins. The skewed sphericity distribution is a reflection of the fact that there is more transverse momentum in the forward hemisphere in the centre of mass system than in the backward hemisphere.

In figures 48(b) and (d) the longitudinal distributions plotted with respect to the thrust and $\gamma_{v}$ axes are compared. Again, the central bins of the $x_F$ distributions are lower in the distribution plotted with respect to the thrust axis than that plotted with respect to the $\gamma_{v}$ axis. The effect is much more pronounced than for the sphericity axis, as indicated by the large dip in the
Figure 48: Longitudinal distributions plotted with respect to different axes. The distributions plotted with respect to the $\gamma V$ axis are shown as circles, those with respect to the sphericity axis as squares and those with respect to the thrust axis as triangles. The $x_F$ distribution plotted with respect to the $\gamma V$ axis is compared to that plotted with respect to the sphericity axis in (a), and with respect to the thrust axis in (b). The distributions of $y^*$ are compared in (c) and (d).
central region in the rapidity distribution (see figure 48(d)). The thrust axis is the axis for which $\Sigma |p_{lt}|$ is a maximum. Thus, when particle momenta are referred to the thrust axis the wings of the distributions of $x_F$ and $y^*$ are populated at the expense of the central bins. The close agreement of the $y^*$ distributions for $y^* < -1.5$ in figure 48(d), indicates that the thrust axis is largely determined by the more pencil-like jet in the target fragmentation region.

8.2.3 Comparison of seagull plots.

In figure 49(a) a comparison of graphs of $\langle p_z^2 \rangle$ versus $x_F$ (the seagull plot) referred to the sphericity and $\gamma_V$ axes is made. Though the differential distributions presented above are rather similar, the seagull plot with respect to the sphericity axis is very different from the seagull plot with respect to the $\gamma_V$ axis. The major difference is that the seagull plot with respect to the sphericity axis is symmetric, whereas the seagull plot with respect to the $\gamma_V$ axis is asymmetric. The values of $\langle p_z^2 \rangle$ with respect to the sphericity axis are always lower than the values calculated with respect to the $\gamma_V$ axis. This is expected from the definition of the sphericity axis.

The seagull plot with respect to the thrust axis is compared to the seagull plot with respect to the $\gamma_V$ axis in figure 49(b). Again, the major difference is that the seagull plot with respect to the thrust axis is symmetric. The close agreement between the two distributions for $x_F < 0$ again indicates that the more pencil-like target jet strongly affects the determination of the thrust axis.

8.2.4 Conclusion.

The major difference observed between the properties of final state hadrons when referred to hadronic event axes and the properties when referred to the $\gamma_V$ axis is the loss of the asymmetry in the seagull plot. In terms of the phenomenology of the Lund model a large contribution to the asymmetry has been shown\(^{[84]}\) to be caused by soft gluon radiation. One may conclude that the hadronic event axis effectively averages out the effect of soft gluon radiation.
Figure 49: Comparison of seagull plots plotted with respect to various axes. The points for variables referred to the \( y_v \) axis are shown as circles, squares are drawn for points referred to the sphericity axes and, the points referred to the thrust axis are shown as triangles. (a) The seagull plots plotted with respect to the \( y_v \) and sphericity axis are compared. (b) A comparison between the seagull plots referred to the \( y_v \) and thrust axis.

8.3 Comparison with e\(^+\)e\(^-\).

In this section a comparison is made between the hadrons produced in deep inelastic \( \mu \mu \) scattering and those produced in e\(^+\)e\(^-\) annihilations.\(^{[86]}\) The differential inclusive distributions are compared at a centre of mass energy of 14 GeV. While reading the following section, the reader should bear in mind
that the average multiplicity in $e^+e^-$ annihilation at 14 GeV is $\approx 9$, while for the deep inelastic $\mu p$ scattering data presented here it is $\approx 7$.

The comparison is made only for hadrons produced in the fragmentation of the struck quark, i.e. for hadrons with $x_F > 0$. In order that a direct comparison may be made, the differential cross-sections for hadron production from $e^+e^-$ annihilation have been divided by two so that the latter refer to a single jet. The comparison is made to distributions plotted with respect to the sphericity axis unless a statement is made to the contrary. All differential distributions are normalised to the total number of events satisfying the cuts described above.

8.3.1 Comparison of $p_\perp^2$ distributions.

The properties of the final state hadrons transverse to the event axis are compared in figure 50(a). The agreement between the two data sets is, in general, good. For very small values of $p_\perp^2$ ($p_\perp^2 < 0.1 \text{ (GeV/c)}^2$) the $p_\perp^2$ distribution for $e^+e^-$ is larger than that for $\mu p$ scattering. The two distributions are very similar for $p_\perp^2$ in the range $0.1 < p_\perp^2 < 0.4 \text{ (GeV/c)}^2$. For $p_\perp^2 > 0.4 \text{ (GeV/c)}^2$ the $e^+e^-$ data lie above the data from $\mu p$ scattering. The fact that the $p_\perp^2$ spectra of hadrons produced in $e^+e^-$ annihilations is harder than that for $\mu p$ scattering is confirmed by plotting $\langle p_\perp^2 \rangle$ as a function of $W^2$ (see figure 50(b)).

8.3.2 Comparison of longitudinal distributions.

The longitudinal distributions are compared in figure 51. Figure 51(a) shows the differential $x_F$ distributions. The $e^+e^-$ data lies above the $\mu p$ data in the first two or three bins for $x_F > 0$, and the distribution for hadrons produced in $e^+e^-$ annihilation is steeper than the distribution for $\mu p$ scattering. The same effects are observed when the $y^*$ distributions are compared (figure 51(b)). There is a marked difference between the two distributions in the central bins. Care must be taken when considering the points at large values of $|y^*|$. It is
Figure 50: Comparison of the transverse properties. The results from $\mu p$ scattering are shown as circles, while the results of $e^+e^-$ annihilations are shown as squares. (a) The differential distribution of $p_t^2$ with respect to the sphericity axis. (b) The $\langle p_t^2 \rangle$ with respect to the sphericity axis is plotted as a function of $W^2$. 
important to remember that the e\(^{+}\)e\(^{-}\) data were collected at a fixed value of W (\(\approx 14\) GeV/c\(^{2}\)), whereas the value of W for the \(\mu p\) data lies in the range \(10 < W < 18\) GeV/c\(^{2}\). The fact that the longitudinal momentum distributions are softer in e\(^{+}\)e\(^{-}\) annihilation than in \(\mu p\) scattering is confirmed as a function of \(W^{2}\) by plotting \(\langle p_{T} \rangle\) versus \(W^{2}\) in figure 51(c) (\(p_{T}\) is the component of momentum parallel to the event axis).

8.3.3 Comparison of seagull plots.

The seagull plots, plotted with respect to the sphericity and thrust axes, are shown in figure 52. The seagull plot with respect to the sphericity axis (figure 52(a)) shows that the hadrons from e\(^{+}\)e\(^{-}\) annihilation have a harder \(p_{T}^{2}\) distribution (ie a larger value of \(\langle p_{T}^{2} \rangle\)) for \(x_{F}\) in the range \(0.05 < x_{F} < 0.4\). For higher values of \(x_{F}\) the data tend to coalesce.

A similar trend is observed in figure 52(b) where the seagull plots with respect to the thrust axis are compared. Here the e\(^{+}\)e\(^{-}\) data have a harder \(p_{T}^{2}\) distribution for \(0.05 < x_{F} < 0.6\).

8.3.4 Comparison of sphericity and thrust distributions.

The differential distributions of sphericity and thrust are compared in figures 53(a) and (b). The \(\mu p\) data is less spherical (figure 53(a)) and more thrusting (figure 53(b)) than the e\(^{+}\)e\(^{-}\) data. This difference persists as a function of \(W^{2}\) as shown in figures 53(c) and (d), where the average values of the sphericity (\(\langle S \rangle\)) and thrust (\(\langle T \rangle\)) are plotted as a function of \(W^{2}\).

8.3.5 Conclusion.

The comparison made in the previous section indicates that hadron spectra produced in e\(^{+}\)e\(^{-}\) annihilation have softer longitudinal momentum spectra, and harder transverse momentum spectra than the hadrons produced in the current jet from deep inelastic \(\mu p\) scattering. In addition, e\(^{+}\)e\(^{-}\) events are more spherical (less thrusting) than leptonproduction events. In general these differences are small, and one may conclude that the parton model assumption of environmental independence is reasonably well satisfied.
Figure 51: Comparison of the longitudinal properties. The leptonproduction results are shown as circles, the results from $e^+e^-$ annihilations are shown as squares. The figure compares: (a) the inclusive $x_F$ distributions, (b) the inclusive $y^*$ distributions and (c) the $<p_t>$ as a function of $W^2$. All quantities are referred to the sphericity axis.
Figure 52: Comparison of the seagull plots. The data points for $\mu\mu$ scattering are shown as circles, for $e^+e^-$ they are shown as squares. The figure compares the seagull plots obtained in $\mu\mu$ scattering and $e^+e^-$ annihilation when plotted with respect to: (a) the sphericity axis and (b) the thrust axis. The solid lines show the prediction of the standard Lund model for $e^+e^-$ annihilation, the dashed curves were obtained using only $u$ quark jets. The dot-dashed curve shows the predictions of a version of the Lund model in which hard QCD effects have been ignored, and the dotted line was obtained from the Lund model using only $u$ quark jets and neglecting hard QCD.
Figure 53: Comparison of sphericity and thrust. The $\mu\mu$ data are shown as circles, the $e^+e^-$ points are shown as squares. The differential sphericity distribution is shown in (a), while the differential thrust distribution is shown in (b). (c) shows the variation of $\langle S \rangle$ with $W^2$, and (d) shows the variation of $\langle T \rangle$ with $W^2$. The solid lines show the prediction of the standard Lund model for $e^+e^-$ annihilation, the dashed curves were obtained using only $u$ quark jets. The dot-dashed curve shows the predictions of a version of the Lund model in which hard QCD effects have been ignored, and the dotted line was obtained from the Lund model using only $u$ quark jets and neglecting hard QCD.
It is interesting to enquire into the origin of the differences observed. There are two main differences between the jets produced in $e^+e^-$ annihilation and the current jet in $\mu\mu$ scattering. Firstly, the proportion of primary heavy quarks is different. Neglecting threshold effects, the ratio of $u:d:s:c:b$ quarks in $e^+e^-$ annihilation is $4:1:1:4:1$. In $\mu\mu$ scattering the current jet is most likely to have been initiated by a $u$ quark. The second difference arises from the fact that in $e^+e^-$ annihilation there is roughly twice the chance per event that a hard gluon will have been radiated from the fragmenting quark. Both these effects may lead to the observed differences. In addition, there is no analogue of the photon-gluon fusion process in $e^+e^-$ annihilation.

In order to separate these two contributions Monte Carlo events were generated using the Lund model for $e^+e^-$ annihilations. The solid curves in figures 52 and 53 represent the predictions of the standard Lund model. The dashed line is the result obtained from the model if only $u$ quark jets are generated. The dot-dashed line represents the predictions of a model in which the effect of (hard) QCD corrections to the process $e^+e^- \rightarrow q\bar{q}$ have been neglected. Note that soft gluon emission as parametrised in the Lund model is included and that all quarks ($i.e.$ $u, d, s, c, b$) are considered. The dotted curves represent the predictions of a version of the Lund model in which only $u$ quark jets are generated and for which the effects of hard QCD have been excluded.

The seagull plot obtained from $e^+e^-$ annihilation at 14 GeV is well reproduced by the standard Lund model (see figure 52). The contribution from the decays of hadrons containing heavy quarks to the seagull plot is shown by the difference between the solid and dashed lines in figures 52(a) and (b). A reduction in $\langle p_T^2 \rangle$ is seen for values of $x_F = 0.2$. The $\langle p_T^2 \rangle$ is also reduced when one ignores hard QCD. This is shown by the difference between the dash-dotted and solid lines in figures 52(a) and (b). The seagull plot obtained
attempt to simulate the target jet, the comparison of data to Monte Carlo is made only for hadrons with $x_F > 0$. All results presented in this section have been normalised to the number of events satisfying the cuts. The error bars plotted below are statistical only.

8.4.1 Comparison of inclusive distributions.

The inclusive $p_T^2$ and $x_F$ distributions are shown in figure 54(a) and (b). The $p_T^2$ distribution (figure 54(a)) is reproduced by the Monte Carlo in both shape and normalisation. The $x_F$ distribution is less well reproduced. The shape of the $x_F$ distribution produced by the Monte Carlo is too large at $x_F = 0.5$, and then falls too steeply. In general the shape of the $x_F$ distribution resembles the $x_F$ distribution obtained with hadrons produced in $e^+e^-$ annihilations.

8.4.2 Comparison of seagull plots.

The seagull plots plotted with respect to the $\gamma_V$, sphericity, and thrust axes are shown in figure 55. Agreement is poor between the model and the data when plotted with respect to the $\gamma_V$ axis. However, when plotted with respect to the sphericity or thrust axes the seagull plot obtained from the data agrees well with that obtained from the Monte Carlo.

8.4.3 Conclusion.

One consequence of the results presented in section 8.2 is that the seagull plots depend on the axis to which they are referred. Thus, one may conclude from the plots presented in figure 55 that a major problem with the QCD branching Monte Carlo, set up as described in section 2.9.1, is that the event axis has not been correctly simulated.

It has been pointed out[90] that the $p_T$ of final state hadrons depends on the event axis, and that the event axis will depend in turn on the details of diquark fragmentation. Thus, the results presented above should not be regarded as tests of the concept of QCD branching. Rather, one should regard them as a study, indicating where more work can profitably be expended.
Figure 54: Comparison of the inclusive distributions. The data points are plotted as circles, the points obtained using the Monte Carlo are shown as squares. Both (a) and (b) are plotted with respect to the $y_\nu$ axis. (a) The distribution of $p_z^2$. (b) The distribution of $x_F$.

Indeed, the large differences found indicate that lepton-hadron scattering may be a sensitive reaction by which to test mature QCD branching models.
Figure 55: Comparison of seagull plots with the QCD branching Monte Carlo. The data points are shown as circles, and the Monte Carlo points are plotted as squares. The figure compares the seagull plots with respect to (a) the $\gamma_v$ axis, (b) the sphericity axis and (c) the thrust axis.
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