THE N DEPENDENCE OF Cd MEAN SQUARE CHARGE RADII

(54 ≤ N ≤ 72) AND THE NUCLEAR MOMENTS OF $^{103}\text{Cd}$

F. Buchinger 1), P. Dabkiewicz 2), H.-J. Kluge 3), A.C. Mueller 4),

E.-W. Otten

Institut für Physik, Universität Mainz

D-6500 Mainz, RFA

and the ISOLDE Collaboration, CERN, Geneva, Switzerland

(to be submitted to Nucl. Phys. A)

1) Present address: Foster Radiation Laboratory, McGill University, Montreal, Québec, Canada H3A 2B2.
2) Present address: Technische Universität, D-2000 Hamburg-Harburg.
4) Present address: CANIL,F-44021 Caen, B.P. 5027, France.
KEY WORD ABSTRACT

NUCLEAR STRUCTURE, 102-105, 107, 111, 113, 115, 117-120 Cd; measured isotope shifts, 103 Cd hyperfine constants A(5s5p 3P1), B(5s5p 3P1), deduced rms charge radii, deformation β, μ(103 Cd), quadrupole moment (103 Cd).
ABSTRACT:

The isotope shift of the radioactive Cd isotopes $^{102}$Cd, $^{103}$Cd, $^{104}$Cd, $^{105}$Cd, $^{107}$Cd, $^{111}$Cd, $^{113}$Cd, $^{115}$Cd, $^{117}$Cd, $^{118}$Cd, $^{119}$Cd, $^{120}$Cd and the hyperfine structure splitting of $^{103}$Cd are determined in the CdI $5s^2 \, 1S_0 - 5s5p \, 3P_1$, $\lambda=326.1$ nm line by fluorescence laser spectroscopy using a pulsed laser system and frequency doubling. The unstable Cd isotopes are produced at the ISOLDE mass separator at CERN and are investigated on-line in a resonance vessel.

Changes of mean square charge radii are derived from the isotope shift data. The curve of the radii as a function of the mass number, shows on top of the regular $A^{1/3}$ expansion a collective contribution of parabolic shape which peaks at $N=66$ in the middle of the shell. This contribution is considerably stronger than calculated from the BE2$^+_1$ -values solely. Isotonic and isobaric shifts between $Z=37$ and $Z=56$ are obtained in a combined analysis of isotope shifts ($\delta<r^{-2}>$), muonic $<r^{-2}>$ -values and BE2$^+_1$ values. The shifts agree with the predictions of the droplet model showing the buildup of a neutron skin along isobaric chains.

The nuclear moments of $^{103}$Cd have been determined as $\mu_1 = -0.81(3)\mu_n$ and $Q_s = -0.79(66)$ b.
I  INTRODUCTION

In this paper, we report on a systematic investigation of the atomic hyperfine structure (hfs) and optical isotope shift (IS) of short-lived Cd isotopes in the mass region 102≤A≤120 performed at the on-line mass separator ISOLDE at CERN. The experiments are a continuation of the work which started with the study of radioactive Hg isotopes/1/. Preliminary results of this Doppler-limited spectroscopy of Cd atoms confined in resonance cells are published in Ref. /2/.

The experimental technique and the measurements are described in the following two sections. Details on the data analysis are given in Section 4. The nuclear moments and changes in mean square charge radii (δ<r²>) of the Cd isotopes are derived from the hfs and IS data in Section 5.

In Section 6 we discuss the spins and moments of the Cd nuclei. The particle plus rotor model is applied to calculate the magnetic and quadrupole moments of the I=5/2 ground states. The observed change in sign as well as the magnitude of the spectroscopic moments are accounted for by this model.

A systematic of δ<r²> is presented in Section 7. The integral IS exhibits an almost parabolic shape starting at N=54 near the closed neutron shell up to N=72 far above the middle of the shell. It is shown that the two parameter models of the isotope shift, based on the spherical droplet model /3/ for the volume expansion and the experimental BE2 values as a measure for the collective contribution to δ<r²> is insufficient in the case of Cd. In the middle of the shell, the data asks for a much stronger collective or surface contribution as offered by this model. Differential irregularities in the radial change may be attributed to subshell closures.

The Cd charge radii are correlated to those of the neighbouring elements (Rb, Sn, Cs, Ba) whose IS's have been determined for very long chains in recent years. This large array of data allows for the first time to extract isotonic and isobaric shifts over considerable distances (37≤Z≤56, 54≤N≤72) and to compare them to nuclear models.
II THE METHOD

The measurements of the hfs and IS of Cd are carried out in a resonance fluorescence experiment. A tunable dye laser is used to excite the CdI, \( 5s^2 \, ^1S_0 - 5s5p \, ^3P_1 \), \( \lambda = 326.1 \) nm transition. The method applied is essentially the same as in the case of the extensive investigations performed on short-lived Hg isotopes reported by Dabkiewicz et al. \(^1\). As in the case of Hg, the sample of radioactive Cd isotopes is obtained on-line at the mass separator ISOLDE II at CERN and is prepared as an atomic vapour in a resonance vessel. Fig. 1 shows schematically the experimental arrangement. Details of the experimental technique are briefly described in the following where we put the emphasis mainly onto the differences between the set-ups used in the Hg and Cd experiments.

2.1. Isotope Production: At ISOLDE, Cd isotopes are produced from a molten Sn target by spallation with 600 MeV protons and are on-line separated. Isotopically clean ion beams with an intensity of up to \( 10^9 \) ions per second and per mass number are available from a 115 g/cm\(^2\) target irradiated with a 2 \( \mu \)A proton beam.

2.2. The Resonance Cell: Similar to that one used in the case of Hg, it consists of a cylindrical quartz tube (Suprasil, \( \phi = 20 \) mm \( l = 15 \) cm) with sealed, flat windows (Fig. 1b). In contrast to the set-up used for the investigation of Hg, the Mo foil which collects the ion beam of ISOLDE is mounted in a side arm of the vessel. The ion beam is focussed through a vacuum valve and a second side arm directly on the Mo foil. As indicated in Fig. 1, the entire quartz system is heated by an oven. Furthermore, additional heating elements keep the tube through which the ion beam is led in and the side arm of the vessel where the Mo foil is mounted, at a temperature of 700\(^\circ\)C - 800\(^\circ\)C whereas the cell itself is heated to about 400\(^\circ\)C.

2.3. Preparation of the Sample: The ion beam is collected for times up to two half-lives of the radioactive isotope under investigation. During this time the foil is cooled by pressured air in order to keep the Cd isotopes implanted in the foil. Then, the valve separating the resonance cell from the ISOLDE vacuum system is closed. He gas with a pressure of about 175 Torr is filled into the resonance
cell and the catcher foil is heated to 1400°C in order to release the radioactive isotope as neutral atoms. The He buffer gas prolongs the diffusion time of the Cd atoms to the cold parts of the system (near valves), where they finally condense. The He pressure chosen is a compromise between the longest possible diffusion time and a minimum of pressure broadening of the spectral line. Depending on the experimental conditions such as the contamination of the cell walls by evaporated foil material, the settings of the temperatures, etc., the Cd atoms can be observed in the vapour phase for times reaching from about 25 sec up to several minutes (compare 4.2.).

2.4. Light Source: Radiation at \( \lambda=326.1 \text{ nm} \) is obtained by frequency doubling the output of a dye laser (Molelectron DL 400) pumped by a pulsed nitrogen laser. The laser efficiency at 652 nm is optimized by mixing the dyes chrysolevioletperchlorate (CVP) and Rhodamine 6G in ethanol. A peak power of 2.5 kW with a spectral width of 1.2 GHz is obtained. After frequency doubling in a KDP crystal with angle matching (conversion efficiency about 10%) a peak power of 250 W is available for 3 ns at a repetition rate of 20 Hz. The laser bandwidth at \( \lambda=326.1 \text{ nm} \) is then 1.9 GHz, slightly less than twice that of the fundamental frequency due to the narrow doubling profile of the KDP crystal.

2.5 Frequency Calibration: The UV laser beam is split in order to pass simultaneously the radioactive sample and a reference cell. The latter is filled with stable \(^{114}\text{Cd} \) and is placed in the gap of an electromagnet. The Zeeman split resonances of \(^{114}\text{Cd} \) provide a convenient frequency scale by use of the known \( g_J \) factor (\( g_J=1.499835(5) / 4 / \)). The magnetic field ranging from 0.14 T to 0.25 T is measured with a mean accuracy better than \( 10^{-4} \) by a Hall probe. The inhomogeneity of the magnetic field over the observation volume of the reference cell (1 cm\(^3\)) reaches up to \( 10^{-3} \) T. The frequency scan of the laser, accomplished by synchronously tilting a grating and an etalon, exhibits a nonlinearity of the frequency scale of about 3%. However, the error caused by this effect can be kept small by choosing a magnetic field where the position of the resonance line searched for coincides with the Zeeman manifold for calibration. Moreover this procedure minimizes the error arising from the pulse-to-pulse frequency jitter of the laser.
2.6. Signal Detection: The fluorescence light emitted by the excited Cd atoms in the two resonance cells is observed by 56 DUVP photomultipliers which are blocked with UG 11 colour filters. The currents of the photomultipliers are digitized by ADC's. In case of very small signals from the least abundant radioactive isotopes, the photon counting technique is applied instead. The background of the laser light scattered from the walls of the resonance cell can be cut off by gating the output of the photomultiplier signals (delay between laser pulse and the start of the detection: 800 ns, duration of the detection: 8 µs). This is possible because the lifetime of the $^5s^5p_1^3$ state ($\tau=2.2$ µs) is long compared to the length of the laser pulse. Furthermore, the photomultiplier is blanked during the laser shot in order to avoid noise and afterpulsing arising from the high load on the photocathode during the laser pulse. For this purpose, a -200 V pulse of about 20 ns width is applied to the focussing electrode of the photomultiplier. A photodiode monitoring the laser intensity is used to calibrate the individual pulses, thus eliminating the pulse-to-pulse fluctuations of the light intensity of the observed spectra. The signals from the photomultipliers are recorded by a NOVA 1220 minicomputer which stores the data.

Tests with stable Cd isotopes showed a lower detection limit of the fluorescence signal at $10^{10}$ atoms collected on the foil in the resonance vessel. With the yields available at ISOLDE, we thus could observe signals for all isotopes between $^{102}$Cd ($T_{1/2}=330$ s) and $^{120}$Cd ($T_{1/2}=60$ s).

III MEASUREMENTS

Measurements on the radioactive Cd isotopes $^{102}$Cd, $^{103}$Cd, $^{104}$Cd, $^{105}$Cd, $^{107}$Cd, $^{111}$Cd, $^{113}$Cd, $^{115}$Cd, $^{117}$Cd, $^{118}$Cd, $^{119}$Cd and $^{120}$Cd have been carried out during a single run of our days. Fig. 2 shows the spectrum of $^{103}$Cd representing the sum of 30 single scans which are obtained with one laser pulse per channel. The content of each channel is normalized to constant laser intensity. The signal to noise as well as the signal to background ratio in the observed spectra were in the order of 20 for the strongest signals and decreased to about 3 for the weakest signal observed ($^{102}$Cd).
The experimentally observed absorption line shape can be described in the presence of a buffer gas by a convolution of a Gaussian and a Lorentzian (Voigt profile). The Gaussian part has a linewidth of $\Delta \nu_G = 2.6$ GHz which can be determined from the unbroadered signals obtained in the evacuated calibration cell. Decomposing into the absorption Doppler width of $\Delta \nu_D = 1.7$ GHz and the laser width according to the relation $\Delta \nu_G = (\Delta \nu_D^2 + \Delta \nu_L^2)^{1/2}$ yields $\Delta \nu_L = 1.9$ GHz. The collision induced Lorentzian adds another 1.3 GHz to the total linewidth of $\Delta \nu_{\text{Voigt}} = 3.9$ GHz.

Unfortunately, this linewidth is comparable to the hfs splitting of the odd isotopes. Thus, the hfs pattern is barely resolved (see Fig. 2) and great care has to be taken in fitting the data. The situation becomes even more complicated when an isomeric state or a contamination of the spectrum by a stable or long-lived isotope has to be taken into account.

IV DATA EVALUATION

4.1. Fitting Procedure: The Voigt profile is fitted to the resonance signals of the radioactive Cd isotopes on the CDC 6600/700 computer at CERN, using MINUIT / 5 / for the least-square fit procedure. This routine allows not only the determination of the fit parameters themselves but offers in addition an analysis of the errors and correlations of the parameters. A priori, free fitting parameters are the background of the signal as well as the height, the width and the position of each line in the spectrum. For hfs manifolds, however, identical line-shapes are attributed to each hfs component of the pattern. The large number of free parameters for the odd isotopes ($I \geq 1/2$) and those isotopes with isomers (ground state $I=1/2$, isomeric state $I=11/2$) can be reduced if the hfs splitting or, in the case of the isomers, the IS either of the ground state or of the isomeric state are known.

The errors of the parameters are defined by a change of $\chi^2_{\text{min}}$ to $\chi^2_{\text{min}} + \chi^2 / F$ with $F$ the number of degrees of freedom. The fit of the signals of the isotopes $^{102}$Cd, $^{104}$Cd, $^{105}$Cd, $^{107}$Cd, $^{113}$Cd, $^{115}$Cd, $^{118}$Cd and $^{120}$Cd yields $\chi^2 / F$ between 1.0 and 1.8. A $\chi^2 / F=7.4$ is obtained for $^{111}$Cd which indicates a strong unknown contamination (probably $^{105}$Cd which
was the preceding isotope under investigation in the resonance vessel. In the case of $^{117}$Cd and $^{119}$Cd, a fit is not feasible because of the unresolved spectra and the complete lack of data on the hfs and IS of the ground as well as of the isomeric state. In order to give a very rough impression about the averaged IS of the unresolved spectrum, the center of gravity (CG) of the entire hfs pattern is evaluated according to $\text{CG}_{\text{avg}} = \left( \Sigma c_i x_i \right) / (\Sigma c_i)$ with $c_i$ the background corrected counts of channel $x_i$.

4.2. Line Shifts: The position of the spectral lines of the Cd isotopes as determined by the fitting procedure must be corrected for two effects: (i) for the pressure shift caused by the buffer gas in the resonance vessel and (ii) for the shift due to the significant decrease of the number of free Cd atoms in the observation region during the laser scan.

Recent measurements of pressure effects in the CdI, $\lambda=326.1$ nm line by Dietz et al. /6/ show a red shift of the line by $\beta = -0.22(10)$ MHz/Torr for the He-Cd system. No systematic dependence of $\beta$ on the temperature has been observed by the authors. Using this value for the pressure shift we obtain a correction of the center of gravity of $38(18)$ MHz corresponding to He pressures of roughly 175(2) Torr.

Even for the most short-lived isotopes the decrease of the resonance signal during the laser scan is found to be dominated by the diffusion of the Cd atoms to cold parts of the system and absorption at the walls of the cell. Detailed measurements for each case show that this decrease can be described by an exponential with one single time constant $\tau_D$ which varied under the influence of the experimental conditions between 25 s and several minutes. At a scanning velocity of $+1$ GHz/s, $\tau_D$ thus causes frequency shifts in the order of $-100$ MHz to $-10$ MHz, for which the final results are corrected.

The values for the IS as determined directly from the experiment and after correction for the pressure shift and the decay of the signal during the laser scan are compiled in Table 1 column 2 and column 3, respectively. The final values are furthermore plotted in Fig. 3. In this Figure, we also include earlier
measurements in the transition investigated here. Furthermore, the hfs components of $^{103}$Cd (determined for the first time during this experiment) are indicated.

V

DETERMINATION OF MOMENTS and RADII

5.1. Nuclear Moments of $^{103}$Cd: A nuclear spin of $I^P = 5/2^+$ was assigned to the ground state of $^{103}$Cd as a result of systematics in the nuclear decay scheme /10/. With this spin value and the positions of the hfs components as given in Fig. 3, the following hfs interaction parameters are found:

\[
\begin{align*}
A^{(103\text{Cd})} &= -1129(40) \text{ MHz} \\
B^{(103\text{Cd})} &= 188(160) \text{ MHz}
\end{align*}
\]  

Using the precisely known A factor and $\mu_\perp$ of $^{109}$Cd ($A=-1148.748(7)$ MHz /11/ and $\mu_\perp = -0.827846(2) \mu_n$ /12/ we obtain for the magnetic dipole interaction constant:

\[
\mu_\perp^{(103\text{Cd})} = -0.81(3) \mu_n
\]

whereby the hyperfine anomaly, estimated to be in the order of $10^{-3}$ /12/ is neglected.

The spectroscopic quadrupole moment is determined with help of the data of $^{109}$Cd ($B=-165.143(5)$ MHz and $Q_S=0.69(7)$ b /11/ as:

\[
Q_S^{(103\text{Cd})} = -0.79(66) \text{ b}
\]

5.2. Evaluation of $\delta< r^2>$: The change of the nuclear charge distribution between two isotopes with mass number A and A' is connected with their IS by:

\[
\begin{align*}
IS^{AA'} &= \nu^{A'} - \nu^A = \delta\nu_F^{AA'} + \delta\nu_M^{AA'} \\
&= -\sum_{k=1}^{\infty} C_k/C_{-1} \delta<r^{2k}>^{AA'} + \delta\nu_M^{AA'}
\end{align*}
\]

$\delta\nu_F^{AA'}$ reflects the change in the binding energy of electrons when the radial distribution of the nuclear charge is changed. It is the product of the electronic field shift factor F and a power series of nuclear radial moments. For Cd, moments higher than $<r^2>$ contribute less than 4% and are neglected hereafter. $\delta\nu_M^{AA'}$ describes the mass
shift which is split into normal (N) and specific (M) mass shift:

\[ \delta v_{M}^{AA'} = (N+M) \frac{A' - A}{AA'} = \frac{v}{1836.12} (1 + M) \frac{A' - A}{A'A} \]  (5)

The normal mass shift amounts to 81.4 MHz in the \( \lambda = 326.1 \) nm line for the isotope pair \((A,A') = (110,112)\). The difficulties of reliably calculating the specific mass shift M and the electronic field shift factor F increases with the complexity of the optical spectrum. Therefore, it is advantageous to calibrate them via a King-plot / 13 / (i) with data from simpler spectra or (ii) with results from sources other than optical spectra. Under (ii) one numerates isotope shifts from electronic X-ray - or muonic spectra, and from electron scattering.

V. Eijk et al. / 14 / have measured electronic X-ray shifts for the Cd pairs \((110,112), (114,112), (116,114), (116,110)\) with the results \(\delta <r^2>_N/fm^2 = 0.135(22), 0.132(20), 0.142(26), 0.42(2)\), respectively. The total shift is in fair agreement with the most recent analysis of optical data by Bauche et al. / 15 /, yielding

\(\delta <r^2>_{116-110} = 0.36(5) \text{ fm}^2\). The errors are experimental ones in the former and purely systematic (due to F and M) in the latter case. In principle, one could determine F and M from a King-plot of the optical against the X-ray data. To be meaningful, this would require, unfortunately, a much higher experimental accuracy of the differential X-ray shifts, at least by a factor of 10. The same argument applies to \(\delta <r^2>\) values obtained from electron scattering / 16 /. But again, the total shift \(\delta <r^2>_{116-110} = 0.56(11) \text{ fm}^2\) is distinctly higher than the optical value.

Recently muonic shifts have been measured for the stable Cd isotopes by Shera et al. / 17 /. A King-plot with the transition investigated here \((\lambda = 326.1 \text{ nm})\) yields \(F_{326} = 6.82(1.0) \text{ GHz/fm}^2\) and \((M/N)_{326} = 7.28(2.0)\). Both values, especially the latter one, are much too high to be acceptable (see below). The total muonic shift is \(\delta <r^2>_{116-110} = 0.459(10) \text{ fm}^2\) which is in agreement with the electron scattering results.

Considering the alternative (i), we relate the IS's for the stable Cd isotopes in the \(\lambda = 326.1 \text{ nm}\) line from Kelly and
Tomchuk / 7 / to those in the 5s $^{2}S_{1/2} - 5p^{2}P_{1/2}$, $\lambda = 226$ nm transition of the alkali like CdII-spectrum / 18 /. Fig. 4 shows a least square fit to the King-plot using all isotopes measured. It yields a slope $F_{326}/F_{226} = 0.640(6)$ and the intersection on the ordinate $\nu M_{\lambda = 326,1110} = 5.7(4.0)$ MHz with $\chi^2 = 2.0$ per degree of freedom. The fit shows slight incompatibilities of the data which has to be attributed to the rather old IS measurements in the $\lambda = 326.1$ nm line / 7 / with underestimated error bars. (Note that the precision of our data is in the same order as in Ref. / 7 / due to the Doppler limited resolution of our method). We have reanalyzed the IS of the two odd-even pairs using as additional input parameters the A factors which are known precisely from later sources / 11 /. The results are $IS_{326}^{110,111} = 60(21)$ MHz, $IS_{326}^{112,113} = 60(12)$ MHz.

Adopting from / 15 / the parameters for the ionic resonance line:

$$F_{226} = 6.11(74) \text{ GHz/fm}^2 \text{ and } \nu M_{226} = (1-0.26)N_{112,110} = 87(60) \text{ MHz}$$

yields with the fitted King parameters for the $\lambda = 326.1$ nm line

$$F_{326} = 3.91(46) \text{ GHz/fm}^2 \text{ and } \nu M_{326}^{112,110} = 61(39) \text{ MHz}$$

We consider the choice of $F_{226}$ and hence $F_{326}$ to be ambiguous within the errors given. Recent Dirac-Fock calculations yield $F_{326}(DF) = 5.58 \text{ GHz/fm}^2$ and $F_{326}(DF) = 4.16 \text{ GHz/fm}^2$ / 20 / these values are contained in our error bars.

Using the values of $F_{326}$ and $\nu M_{326}$ from (7) and the available IS data, leads to the $\delta <r^2>$ values compiled in Table 2 (the poorly analyzed cases of $^{117}$Cd and $^{119}$Cd have been omitted). In this Table we also include the $\delta <r^2>$ of isotopes which were not investigated in this experiment. The $\delta <r^2>$ values of the stable

Footnote: The value of $F_{226}$ is slightly higher as that one given in / 15 / since we, (i) subtracted for the $P_{1/2}$ contribution to the IS instead of 3% only 1.6% (compare Refs. / 19,20 /) and (ii), take a revised $f(2)$. The error has been enlarged in order to include the uncorrected semiempirical value (see Appendix).
isotopes are calculated from the precise IS data in the 4d\(^{10}\) 5p\(^{2}\) \(P_{3/2}\) - 4d\(^{9}\) 5s\(^{2}\) \(D_{5/2}\), \(\lambda = 441\) nm transition. We have used the data and the King parameters of Brimicombe et al. / 18 / and related this transition to the ionic resonance line, \(\lambda = 226\) nm, with \(F_{226}\) and \(\nu_{M226}\) as given in (6). For the evaluation of the \(\delta<r^2>\) values of the radioactive \(^{109}\)Cd and \(^{115m}\)Cd, we have used the IS data in the \(\lambda = 326\) nm line as reported by Hull et al./ 9 / and Moskowitz et al. / 8 /, respectively.

The errors assigned to the \(\delta<r^2>\) values in the first bracket are experimental ones only. In the second bracket, we include a possible systematic error according to the errors of \(F_{326}\) and \(\nu_{M326}\) as given in (7).

VI DISCUSSION OF THE MOMENTS OF \(^{103}\)Cd

Meyer et al. / 21 / have explained successfully the level structure of the odd Cd-isotopes in the range \(^{103}\)As\(^{111}\) in the frame of a rotor plus particle model. This model was also chosen by Ekström / 22 / for the interpretation of the moments of Cd nuclei and their isotones. His calculations were performed with a computer code by Larsson / 23 /. The single particle structure of these moderately deformed nuclei is characterized by strong mixtures of several Nilsson states with different projections \(\Omega\), i.e., the weak coupling case is realized. The Nilsson states originate predominantly from the \(2\) \(d_{5/2}\) and \(1\) \(g_{7/2}\) shells.

Table 3 gives the amplitudes \(a_{IV}^{\Omega}\) of the \(5/2^+\) wave functions in the series according to Ref. / 22 / and the nuclear moments derived. The physical situation has been simplified in these calculations by keeping the deformation at a fixed value \(\beta_2 = 0.16\) for all isotopes. The comparison with the experimental moments included in Table 3 shows satisfactory agreement for \(\mu_I\) and the correct, increasing trend for \(Q_S\). Recalling the projection formula between the spectroscopic \((Q_S)\) and intrinsic quadrupole moment \((Q_o)\)

\[
Q_S = Q_o \frac{\sum \nu (a_{IV}^{\Omega})^2 - I(I+1)}{(I+1)(I+3)}
\]  

(8)
this trend is explained by a shift of $\Omega$ values from smaller ones to larger ones throughout the series (compare Table 3). Ekström has found a similar behaviour of $Q_S$ also for the $h_{11/2}$ isomers of Cd and for several other transitional nuclei. It should be noted that the systematic variation of $Q_S$ in the series of $i_{13/2}$ isomers of Hg / 1 / has already been interpreted under the same aspect, leading to a quantitative reproduction of the results / 24,25 /. Concerning the $5/2^+$ series in Cd, the qualitative agreement between experiment and theory is so far on its own not a conclusive argument for the prolate intrinsic shape of the nuclei, since oblate shapes as well as a $j^n$ shell model configuration would exhibit the same trend in $Q_S$. However, the level structure and here especially, the decoupled bands observed in the Cd series / 21 / clearly indicate prolate shape.

VII DISCUSSION OF THE ISOTOPE SHIFT DATA

7.1. Parameterization of the Integral Isotope Shift: Fig. 5 shows the integral changes in mean square charge radii of the Cd isotopes. The isomer shifts which are zero within the present limits of error are not shown. The data covers an extensive part of the neutron shell between 50≤N≤82 exhibiting a smooth variation with N. The predictions of the spherical droplet model / 3 / is added for comparison. It describes quite well the average slope of the $\delta <r^2>$ values around A=114, i.e. in the middle of the shell. Towards lower as well as towards higher mass numbers, the experimental values deviate from the droplet predictions, indicating a change in the nuclear shape mainly caused by a decrease in nuclear deformation.

The contribution to $<r^2>$ from the changes in nuclear volume and deformation are usually expressed in terms of a two parameter model / 26 /.

$$<r^2>_A = <r^2>_\text{Sph} + <r^2>_{\text{quad}}$$
$$= <r^2>_\text{Sph} (1 + 5/(4\pi) <\beta_2^2>)$$

and corresponding to first order

$$\delta <r^2> = \delta <r^2>_{\text{Sph}} + 5/(4\pi) \delta <r^2>_{\text{Sph}} \delta <\beta_2^2>$$
where \( <r^2>_{\text{Sph}}^{A,A'} \) and \( \delta<r^2>_{\text{Sph}}^{A} \) account for the nuclear charge volume and its change at spherical shape. According to any nuclear model with saturated short range binding, \( \delta<r^2>_{\text{Sph}}^{A} \) should be a very smooth function of \( A \), i.e. it should be rather linear over the limited mass range considered here. The droplet prediction in Fig. 4, yielding \( \delta<r^2> = 0.055 \text{ fm}^2/N \cdot 3 \), represents this behaviour. The curvature observed in the course of \( \delta<r^2> \) would thus, according to (10), be attributed to the collective contribution which is represented in (10) only by its leading term \( \delta<s^2> \), the change of the mean square quadrupole deformation. The collective contribution has apparently a parabolic shape peaking in the middle of the shell at \( N=66 \) or \( A=114 \). This is confirmed by a parabolic fit to the data with:

\[
\delta<r^2>_A^{114} = \delta<r^2>_{\text{Sph}}^{A,114} + \delta<r^2>_{\text{quad}}^{A,114} = a(A-114) + b(A-114)^2 + c + 1/2d(1-(-1)^A) \tag{11}
\]

where \( c \) is the offset of the fit at \( A=114 \); \( d \) accounts for the odd even staggering, assumed to be constant as explained below. The parameters found from the fit are:

\[
a = 0.0472(9) \text{ fm}^2 \quad b = -2.64(12) \times 10^{-3} \text{ fm}^2 \tag{11a}
\]

\[
c = -0.0129(53) \text{ fm}^2 \quad d = -0.031(10) \text{ fm}^2
\]

The data have not been weighted and the standard deviation of the data from the fit is \( \sigma = 0.02 \text{ fm}^2 \). The fitted parabola is plotted in Fig. 5, together with the data. The fit represents the general trend of the data quite well. The linear term is in the range expected from the droplet model, though it is distinctly smaller. This could be easily cured by developing around a point of larger slope, say at \( A=112 \), for instance. But, this would destroy the symmetry with respect to the shell closures and leads to difficulties in the interpretation (see below).

In the absence of sharp shape transitions (as occurring for Rb at \( N=58 \) / 27 /) such a parabolic shape in between neutron shells is frequently observed. It can be found in the well investigated \( 1f_{7/2} \) shell (Ca to Ni region / 28,29,30 /) as well as in higher shells (e.g. Yb / 31 /). Note that traditionally the IS data has
been plotted differentially as $\delta r^2_{N,N-2}$ as shown for Cd in Fig. 7. This kind of plot is still used for exhibiting $\delta r^2$ data from muonic X-ray and electron scattering experiments. In this representation the parabolic contribution to $\delta r^2(N)$ is reflected in a linear sloping down of the $\delta r^2_{N,N-2}$ curve. Since mesic and electron scattering experiments yield absolute $r^2$ values irrespective of the Z or N in question, they can also be interpreted in terms of isotonic shifts (N=const.). In the $\d_7/2$ shell as well as in the subsequent one, differential isotonic shifts behave quite analogous to the isotopic ones / 30 / i.e. $\delta r^2(Z)$ has a parabolic contribution which reaches its maximum in the middle of the shell.

Coming back to the region, discussed here, it should be mentioned that the recently measured isotope chains of Sn / 32,33 / and In / 34 / exhibit the same parabolic feature but with a shrinking curvature when Z approaches the closed proton shell at Z=50.

Theoretically a parabolic shape of the collective quadrupole contribution to $r^2$ can be explained in a core polarization model / 35 / . For a pure $\nu j^n$ shell, the dependence of $r^2_{\text{quad}}$ on the occupation number n is:

$$r^2_{\text{quad}}(n) = \frac{2|\langle j||T^{(2)}_{N}\rangle||j\rangle|^2}{(2j+1)(2j-1)} \left[ n(2j+1-n) - \frac{(1-(-1)^N)}{2} (j+1/2) \right]$$  (12)

The factor in front of eq. (12) is the reduced matrix element of the quadrupole operator acting on a single neutron in a state $\nu j$. The second term in the bracket describes the odd even staggering as a constant reduction of $r^2$ over the whole shell. The first quadratic term in the bracket is proportional to the number of particles times the number of holes in a shell. For $n=(2j+1/2)$ this expression for the quadrupole deformation is equivalent to the one in eq. (11) as can be seen by rewriting it in respect to $N=50$ as reference isotope

$$\delta r^2_{50,N} = a(N-50) + b(N-50)(82-N)$$

$$= \delta r^2_{50,Sph} + \frac{5}{(4\pi)}r^2_{Sph}$$

$$<g^2_{2}>_{Sph} = 0$$

where $<g^2_{2}>_{Sph} = 0$ by definition of the model (12). Equation (13) is a generalization of (12) in so far as the maximum
occupation number $(2j+1)$ of a single $j$-shell has been replaced by the total span of 32 of the major shells. This may be justified by considering that the pairing interaction "dissolves" the individual $j$-character of the subshells in the major shell. In the framework of a generalized seniority model Talmi has derived a more general form of (12) /35,36/

$$
\delta<r^2> = a \text{ge} n + b \text{ge} \frac{1}{2} n(n-1) - d \text{ge} \frac{1}{2} (1-(-1)^n)
$$

(14)

where $n$ is now the number of particles in the open major shell.

Equation (14) holds at least for even semi-magic nuclei under certain conditions also for odd semi-magic ones. In the latter case, the odd even staggering would again be constant over the shell.

Equation (14) preserves the quadratic character of (11) and (12). But, since the parameters $a$, $b$, $c$, and $d$ are undetermined, it gives no information about the form of the parabola. Physically it is evident that it still should have its vertex in the middle of the shell. Namely, if the collective amplitude is made up by some coherent, residual interaction of the pairs in the valence shell, then it should be proportional to their number. This explains $<a^2>$ to be a quadratic function of that number. If, moreover, this interaction is limited to the valence space of the major shell in question and obeys the Pauli principle, then it should be symmetric in particles and holes, and hence, should peak in the middle of the shell.

### 7.2. Ground State Deformations of Cd Isotopes

Inserting the fitted $b$ from (11a) into (13) we obtain for the center isotope $^{114}_{48}$Cd a ground state deformation of:

$$
^{114}_{48}<a^2>^{1/2} = 0.28(1)
$$

(15)
where the error comes only from the parabolic fit. For \( \beta_2^2 \), in (13) we have used the droplet value \( \beta_2^2 \text{Sph, DM} = 21.128 \text{ fm}^2 / 3 \) /\(^\ast\). Starting from (15) we can calculate \( \beta_2^2 \) from the IS for all other isotopes with the help of (10) by inserting the experimental \( \delta \beta_2^2 \text{66, N} \). The values are given in Table 2 and plotted in Fig. 6.

The deformation parameters found in this way \( \beta_2^2 \text{IS} \) may be compared with those calculated from BE2-values according to:

\[
\beta_2^2 \text{BE2} = \left( \frac{4\pi}{32R_0^2} \right)^2 \eta \beta_2 \text{BE2} (E2, 2^+_n \to 0^+) \quad (16)
\]

which is the closure relation for the quadrupole operator / 37 /.

For the static rotor as well as for the harmonic oscillator, the sum is exhausted by the transition strength to the first excited \( 2^+ \text{n} \) state already, which makes it useful in practice since information about higher \( n \) is sparse.

The comparison (see Table 2 and Fig. 6) clearly show the \( \beta_2^2 \text{BE2} \) values / 38 / to be considerably smaller than those derived from the IS. One might conclude, therefore, that a substantial part of the ground state collectivity is missed by truncating (16) to the first BE2-value. This may be expected in particular for transitional nuclei whose potential energy surface is likely to be anharmonic. In some stable Cd isotopes, the \( \beta_2 \text{BE2}(2^+_2 \to 0^+) \) values are also known and contribute with \( 4\% \) to \( 7\% \) to (16), an amount which does not change the situation drastically.

A similar discrepancy between \( \beta_2^2 \text{IS} \) and \( \beta_2^2 \text{BE2} \) has also been observed in Kr / 39 /, Sr / 40 /, and Sn / 33 /. However, for heavier elements, like those in the rare earth region, they seem to agree

\(^\ast\)Footnote: Note that this calculation of \( \beta_2^2 \) is based alone on the IS data, treated with the schematic model, as input. Usually \( \beta_2^2 \) was always related to one particular reference isotope, whose \( \beta_2^2 \) was calculated from other data such as BE2 values or simply set close to zero when a magic nucleus was concerned. In the Cd case the latter way is excluded so far.
rather quantitatively / 41,42 /.

Close to magic numbers the collective zero point motion will not be restricted to the quadrupole mode; the octupole contributes definitively. Effects from this source have been considered in the charge radii of Ca by Andl et al. /28/ and in a more general form by Drechsel and Reinhard /43/ and explain most of the discrepancy observed here.

In analysing the IS of stable Cd isotopes, Wentz et al. /44/ have proposed to attribute the missing radius to small variations of the surface thickness. The same group has discussed this possibility before for the Kr case /39/. The authors conclude that the semi-magic \(^{86}\)Kr has a smaller skin thickness than the lighter isotopes. An analog interpretation was also given for the Sr isotopes below the N=50 shell closure /40/.

The effect of a variable skin thickness has definitively been observed in measurements of the radial charge distribution by electron scattering experiments and is discussed in detail by Friedrich and Voegler /45/. Still this observation does not conclusively prove that the surface thickness of the nuclear charge distribution would change when measured in the intrinsic frame of the nucleus. Since it is only the monopole form factor which is measured in elastic electron scattering, the experiments are not discriminating between the intrinsic skin thickness and the one simulated by averaging the collective nuclear motion over all angles. For this reason, we leave the question open whether the "extra radius" of non magic nuclei is totally and always due to collective motion or whether a change of the intrinsic surface distribution partially contributes.

We would like to mention that the discrepancy between \( <\varepsilon_2^2>_{IS} \) and \( <\varepsilon_2^2>_{BE2} \) values is not due to possible deficiencies of the parabolic model, but persist in any kind of conventional analysis. This can be seen by calculating \( <\varepsilon_2^2> \) from (10) with respect to \(^{114}<\varepsilon_2^2>_{BE2} = 0.0351(3) /38/ and the droplet slope of \( a = 0.055 \text{fm}^2/\text{N}, \)

resulting in a unphysical value of \( ^{102}<\varepsilon_2^2>_{IS} = -0.0038(51). \)
The situation gets even worse when extrapolating the slope of the experimental $\delta<\bar{r}^2>$ curve towards the semimagic $^{98}$Cd, yielding $\delta_{^{98}Cd}^{2} = -0.0334(63)$. In this case, one could argue that the droplet slope has to be increased as proposed recently by Møller and Myers ($a = 0.084 \text{ fm}^2/N / 46$) and Tondeur and Berdichevsky /47/, who assume in the region considered here a non linear dependence

$$\delta<\bar{r}^2>_{\text{Sph}} = 0.067(N-66) - 1.08 \times 10^{-3}(N-66)^2$$

(17)

which is also steeper. The use of this approach, however, transfers the problem to the heavy end of the shell with the consequence of negative $\delta_{^{182}Cd}^{2}$ values near $N=82$. Also, a rescaling of $\delta<\bar{r}^2>$ by use of different $F$-values does not help. The persisting fact is that the observed curvature in the $\delta<\bar{r}^2>$ values of the Cd isotopes, and especially a further extrapolation of $\delta<\bar{r}^2>$ towards higher and lower mass numbers, asks for a higher collective or surface contribution to $<\bar{r}^2>$ than offered by BE2 values.

7.3. Differential Changes in $\delta<\bar{r}^2>$: A plot of the differential changes in mean square charge radii $\delta<\bar{r}^2>_{N,N-2}$ is shown in Fig. 7. This plot accentuates finer details beyond the parabolic model discussed above. Despite the large error bars of the unstable isotopes, it shows a distinctive structure. We observe a linear decrease of $\delta<\bar{r}^2>_{N,N-2}$ between $^{102}$Cd and $^{106}$Cd, followed by a plateau up to $^{112}$Cd and again a linear decrease towards higher masses. The mean slope is $4b = -11.4 \pm 1.4 \times 10^{-3} \text{ fm}^2$ which is consistent, of course, with the result from fitting (11) to the integral IS. Interpreted in terms of deformation, the plot demonstrates again that starting at low $N$, $\delta_{^{182}Cd}^{2}$ grows somewhat faster than expected in the parabolic model, remains almost constant and decreases subsequently.

Wenz et al. /44/ have attributed the kink in the differential IS at $A=112$, which is underlined by our data on $A=118$ and $A=120$, to a transition from the $(d_{5/2},g_{7/2})$ to the $(s_{1/2},d_{3/2},h_{11/2})$ subshells at $N=64$. It is not obvious to explain the discontinuity at $A=106$ in an analogous way by
another subshell closure. In this context it should be pointed out, that the signature of actually both discontinuities does not resemble the clear and unique signature of a major shell closure (compare e.g. Ref. / 30 /). Moreover, we would like to remind that the wave functions of the light Cd isotopes calculated in the frame of a rotor-plus-particle model (see Section 6) show a strong mixture of shell model states over the mass range considered here. Similarly, the experimental magnetic moments and hyperfine anomalies of the odd isotopes with \( ^{107}\text{As}^{113} \), interpreted by McDermott et al. / 48 / using the configuration mixing model of Arima and Horie / 49 /, can be reproduced only under the assumption of highly mixed configurations. We therefore leave the question open if, and in what form, possible subshell closures are expressed in the present data.

7.4 Isotone and Isobar Shifts of Medium Mass Nuclei: The isotonic shifts reflect the variation of the nuclear charge radii as a function of the absolute nuclear charge at constant neutron number \( (N=\text{const.}) \), whereas the isobaric shifts reveal the influence experienced by the nuclear charge distribution upon the exchange of neutrons and protons \( (A=\text{const.}) \). Concerning the gross behaviour of nuclear charge radii the available IS data in long isotopic chains now allows for the first time a systematic check of the descriptive capacity of nuclear models in those important dimensions of the nuclear chart. We concentrate here our attention on the region of medium mass nuclei where during the last years in addition to Cd systematic measurements of optical IS have been carried out for Rb / 27 /, In / 34 /, Sn / 33 /, Cs / 50 / and Ba / 41 /. The evaluation of isotonic and isobaric shifts uses as input parameters the optical isotope shifts and in addition, absolute \( \langle r^2 \rangle \) -values for at least one isotope in each chain serving as a link between the different elements. The latter are taken from muonic spectra of stable nuclei. Absolute charge radii determined in this way for eight series of isotones and isobars are tabulated in column 3 of Table 6.

The isotonic shift, calculated from these numbers is plotted in Fig. 8a for the example \( N=66 \). The curve includes data at both sides
of the proton shell closure at Z=50. As in the case of isotopic shifts, the shell closure is pronounced by a distinct kink in the $\delta<r^2>$ curve, which indicates in analogy to the commonly applied interpretation of isotopic shifts in a two parameter model (see eq. 10), a decreasing nuclear deformation when the closed proton shell is approached from the low as well as from the high Z side. We note that a similar discontinuity is observed at Z=40 in the isotonic shifts of N=54,56,58.

The deformation effect on the isotone shift can be further verified by correcting the experimental data for it according to eq. (10). For this purpose, we used $\beta_2^2$ values calculated from BE2 data with the help of eq. (16). In cases where no BE2 data was available, the deformation was estimated from the energies of the first excited $2_1^+$ state according to Ref. / 59 /; for the odd Z elements $\beta_2^2$ was interpolated from their even neighbours and an error of 30% was assigned to $\beta_2^2$. Although BE2-values seem to underestimate the full collective contribution to $r^2$ (compare 7.2.), we have not used values determined from isotope shifts in order to avoid correlations between the interpretation of isotope shift data on one hand and isotone (isobar) shift data on the other hand. The deformation parameters are listed in column 4, the corrected "spherical" $r^2$ values in column 5 of Table 3. Fig. 8b shows the deformation corrected isotone shift at N=66, Fig. 9 the corrected isobaric shift for A=118. The available data crosses the valley of stability for the first time over a large distance (exchange of seven nucleons).

The corrected isotonic and isobaric shifts are compared with the predictions of the droplet model / 3 / in column 7 of Table 4 and in Figs. 8b and 9. The agreement is very good with the few exceptions of $^{95}$Rb and $^{120}$Cs where the deviations slightly exceed the error bars. This agreement underlines strongly the quality of the droplet model in describing nuclear gross properties.

Especially instructive is the correct description of the isobaric shift by the droplet model demonstrating its superiority over the simple liquid droplet model. The latter assumes a uniform nuclear charge and matter distribution up to a sharp radius
\[ R = r_oA^{1/3} \]. Hence, the isobaric shift would be identical to zero in this model. We could abandon the \( A^{1/3} \) proportionality of \( R \) in an isobaric series but keep strictly the relation \( \rho_z(r) = Z/A \rho_A(r) \) between the charge and matter distribution. Under this condition, the isobaric shift would imply a strong variation of matter density which is certainly excluded on the ground of the very small nuclear compressibility (the breathing mode is observed at 15 MeV excitation energy / 60 /). In conclusion, we are forced to assume that the proton and neutron distribution shift relative to each other along an isobaric chain. In particular a neutron skin will develop for high neutron excess.

We demonstrate this effect under the schematic assumption that the mean square nuclear matter radius \( \langle r^2 \rangle_A \) is constant for isobars (incompressible nuclear matter). From the definition:

\[
\langle r^2 \rangle_X = \frac{4\pi}{3} \int_0^\infty \rho_X(r) r^4 dr
\]  
(18)

with \( X=Z,N,A \), follows immediately the relation between the mean squared radii of matter, charge and neutron distribution

\[
\langle r^2 \rangle_A = \frac{Z}{A} \langle r^2 \rangle_Z + \frac{N}{A} \langle r^2 \rangle_N
\]  
(19)

Under the above conditions one derives from (19) the following relations between changes of \( \langle r^2 \rangle_N \) and \( \langle r^2 \rangle_Z \) in an isobaric chain:

\[
\langle r^2 \rangle_{N_1} - \langle r^2 \rangle_{N_0} = \frac{Z_1}{N_1} \left( \langle r^2 \rangle_{Z_1} - \langle r^2 \rangle_{Z_0} \right)
\]  
(20)

\[
-\frac{A}{N_0 N_1} (N_1 - N_0) \left( \langle r^2 \rangle_A - \langle r^2 \rangle_{Z_0} \right)
\]

with \( N_1 + Z_1 = N_0 + Z_0 = A \). The first term in (20) dominates. Neglecting the second one we calculate from Table 4 for \( A=118 \) a swelling of the neutron distribution by

\[
\langle r^2 \rangle_N(^{118}_{48}\text{Cd}^{70}) - \langle r^2 \rangle_N(^{118}_{55}\text{Cs}^{63}) = 0.59 \text{ fm}^2
\]  
(21)

on cost of a shrinking of the charge distribution by 0.86 \( \text{fm}^2 \). Defining the RMS radius \( r_X = (\langle r^2 \rangle_X)^{1/2} \), we can express the result (21) by an increase of \( r_N \) of the neutron distribution by an amount of
δr_N = 0.063 fm. Correspondingly, we calculate δr_Z = -0.092 fm. These numbers may be compared to results from Hartree-Fock BCS calculations by Angeli et al. /61/. From their Fig. 9, we read for the pair of stable isotopes 116_{48} Cd, 116_{52} Te, where four nucleons are exchanged: δr_N - δr_Z = 0.12 fm. For a chain of seven isobars, we obtain from our analysis δr_N - δr_Z = 0.155 fm. Reducing this value by a factor 4/7 yields δr_N - δr_Z = 0.088 fm in fairly good agreement with the calculated value. Thus, the calculations of Angeli et al. are supported as is our assumption <r^2>_A = const. for isobars, by which we got access to δ<r^2>_N. The development of a neutron skin is also inherent to the droplet model by allowing for a potential which depends on the asymmetry between proton and neutron density.

SUMMARY and CONCLUSIONS

The isotope shifts of the radioactive Cd isotopes 102_{54}As120 and the hyperfine structure of 103_{53}Cd were measured on-line by pulsed laser spectroscopy.

The negative spectroscopic quadrupole moment of the 5/2^+ ground state of 103_{53}Cd which implies a change in sign as compared to the heavier isotopes of the 5/2^+ series, may be explained by a change in the occupation of the sublevels from a d_{5/2} shell model state due to an increase in the Fermi energy as neutrons are added within a shell.

A smooth variation of the nuclear shape is indicated from the changes in mean square charge radii. We have extracted the deformation parameters for the Cd isotopes through a parametrization of the δ<r^2> data by a second order polynomial plus an odd-even term. This novel parametrization, which was chosen on the grounds of a core polarization model of Talmi /35/, allows the extraction of deformation parameters from changes in mean square charge radii without relating them to the deformation of a particular reference isotope. In this way, the IS data becomes a complete independent source of information on nuclear shape effects. The comparison between the deformation parameters from the IS measurements and those obtained from BE2_1^+ values reveals a systematic discrepancy which asks for a higher collective or surface contribution to the nuclear shape than offered by the BE2_1^+ values.
We have used the Cd data in order to evaluate isotonic and isobaric changes in mean square charge radii in the medium heavy mass region between Rb and Ba. The shell closures at Z=50 and Z=40 are pronounced in the data. The comparison of experimental shifts with the predictions of the droplet model and Hartree-Fock BCS calculations show in general a good agreement demonstrating the effect of a variable neutron skin thickness along isobaric series.

Measurements in the Cd series will be taken up again at the ISOLDE mass separator by the method of collinear laser spectroscopy in order to improve the accuracy, especially on the yet unresolved structures of the odd isotopes. Moreover, one can hope for a considerable extension of the measurements towards the shell closures. Together with the recent measurements on the tin /33/ and indium series /34/, this would strengthen our considerations on the gross structure of nuclei in this region.

ACKNOWLEDGEMENTS:

We would like to thank Dr. C. Ekström for his contributions to the discussion of the nuclear spins and moments and for making available to us some of his unpublished calculations in Cd. This work has been supported by the Bundesministerium für Forschung und Technologie and by the Deutsche Forschungsgemeinschaft.
APPENDIX

Evaluation of the Electronic Field Shift Factor

In the semiempirical analysis the electronic field shift factor $F$ is defined as:

$$ F = (\Delta|\psi_o|^{2} / a^3_o) \times f(Z) \quad (A1) $$

where the first factor is the change of the nonrelativistic electron charge density at the nucleus in atomic units and $f(Z)$ is a relativistic correction factor accounting for 505.2 MHz/fm$^2$ for $Z=48 / 62,15$. (The authors of Ref. / 15 / use 498 MHz/fm$^2$). The $(5s^2 S_{1/2} - 5p^2 P_{1/2})$ transition density is related to the ground state density by:

$$ \Delta |\psi_o|^{2} = \beta(1-a) \left| \psi_o \right|^{2}_{5s} = 1.1(1-0.16) \left| \psi_o \right|^{2}_{5s} \quad (A2) $$

$$ = 1.082 \left| \psi_o \right|^{2}_{5s} $$

in perfect agreement with the direct Dirac-Fock calculation (1.082 $|\psi_o|^{2}_{5s}$) from Ref. / 20 /. $\beta$ is the screening factor taken from / 15 / and $a$ is the $P_{1/2}$ contribution to $\Delta |\psi_o|^{2}$ calculated according to / 19 /.

Two semiempirical sources lead to $|\psi_o|^{2}_{5s}$ / 63 /. It can be evaluated

(i) from the quantum defect $\sigma$ (Goudsmit-Fermi-Segré formula):

$$ \left| \psi_o \right|^{2}_{5s} = \frac{Z_i Z_a}{(n-\sigma)^2} \left( 1 - \frac{d\sigma}{dn} \right) = 12.42 \quad (A3) $$

with $Z_i = 2$, $Z_a = 48$, $n-\sigma = 2$, and $\frac{d\sigma}{dn} = \sqrt{\frac{R_y}{T}} = 1.794$, $d\sigma/dT = (d\sigma/dT)/(d\sigma/dT - (n-\sigma))/2T = -0.173$

(ii) from the hyperfine splitting

$$ \left| \psi_o \right|^{2}_{5s} = \frac{a_s^{111} \times 1836.12}{8\pi R_y^3} \frac{111}{(1-\delta)(1-\epsilon)g_I^{111} 8\pi R_y^3 a^2} = 12.65 \quad (A4) $$

with $a_s^{111} = 16.571(15)$ GHz / 18 / \( F_z(48) = 1.276 \) (relativistic correction,

*Footnote: Bauche et al. / 15 / probably used a number close to $\Delta\sigma/\Delta n \approx 0.073$ instead, yielding a density of 11.36 (they give 11.65).
(1-5) = 0.97 (Breit Rosenthal correction, = 0.991 (Bohr Weisskopf effect) and $g_{111} = 1.190$. Taking as usual the average of (A3) and (A4) which agree quite well yields

$$F_{226} = 6.85 \text{ GHz/fm}^2$$  \hspace{1cm} (A5)

Bauche et al. / 15 / started from (A4) and then applied a correction of -12\% for the contribution of spin exchange core polarization to the hfs which then leads to a $F_{226}$ as given in eq. (6). We have adopted this correction. The number of 12\% is in accordance with a calculation by T.P. Das and co-workers / 64 / yielding 10\% core polarization effect in the magnetic hfs interaction of conduction electrons in metallic Cd, which has predominantly s-wave character.

The application of a core polarization correction is a novelty in the standard semiempirical analysis. First one notices that in the Goudsmit Fermi Segré procedure (A3) the core polarization effect seems to be implicitly included, for reasons not known so far. Namely, the agreement between (A3) and (A4) is not fortuitous but observed for almost every s electron. It is quite conceivable that the core polarization simply scales with $|\psi_0|^2$ and hence can be treated as a renormalization of the hfs operator, because it is due to configuration mixing far out in the continuum. Secondly, one has to ask how much similar far configuration interactions contribute to the IS and whether they compensate each other. So far, very little is known about this: it has to be answered before definite conclusions can be drawn as to how the field shift can be evaluated.
REFERENCES


/ 5 / MINUIT, CERN Program Library.


G. Fricke, E.B. Shera, private communication.


TABLE CAPTIONS

Table 1
Isotope shifts of the investigated Cd isotopes in the \( \lambda = 326 \) nm line relative to \(^{114}\)Cd. The second column contains the shifts as determined directly from the observed spectra. The third column gives the shifts after the correction for the pressure shift and the decay of the resonance signal during the laser scan. For \( A=117 \) and \( A=119 \) the shifts of the centers of gravity for the unresolved signals of the nuclear ground and isomeric states are listed.

Table 2
Changes in mean square charge radii \( \langle \delta r^2 \rangle \) of the Cd isotopes with \( 102 \leq A \leq 120 \) (2nd column). The error indicated in the first bracket is solely due to the experimental error of the IS data used for the derivation of the \( \delta r^2 \) values. The 2nd bracket indicates a possible systematic error due to the uncertainty in the specific mass shift and the calibration factor \( F_{126} \) (see Section 5.2). The 3rd and 4th column contain the deformation parameters for the Cd isotopes as obtained from the IS measurements \( \langle \delta_2^2 \rangle _{IS} \) and BE2 values \( \langle \delta_2^2 \rangle _{BE2} / 38 \rangle / \). The error in \( \langle \delta_2^2 \rangle _{IS} \) is based on the error of \( \delta r^2 \) given in the first bracket of column 2.

Table 3
Wave functions, at the deformation \( \beta_2 = 0.16 \), of 5/2\(^+\) ground states in Cd together with the derived nuclear moments. The experimental values are given in the last columns.

Table 4
Compilation of isotonic and isobaric shifts in the medium heavy mass region. The mean square charge radii as obtained from muonic and IS data are listed in column in column 3. In column 4 the mean square deformation \( \delta_2^2 \) is compiled. The following column contains the mean square charge radii connected for deformation effects according to eq. (9). The corresponding changes in mean square charge radii, relative to Cd, are given in column 6 and the droplet model prediction in column 7.
References and Details of the Data Evolution:

$E_{2+}$ values and lifetimes of the $2_1^+$ states for calculating the BEZ values according to Ref. / 52 / are, if not otherwise stated, obtained from Ref. / 51 /.

a): Extrapolation from $\langle r^2 \rangle _{114}$Cd with $\delta \langle r^2 \rangle _{114}$Cd from Table 2.

b): Ref. / 53 /

c): Ref. / 54 /

d): Extrapolation from $\langle r^2 \rangle _{87}$Rb / 55 / and $\delta \langle r^2 \rangle _{87}$Rb from Ref. / 50 /

e): From $E_{2+}^{11}$ value

f): From BEZ value

g): Estimation from $E_{2+}^{92}$Sm and $E_{2+}^{90}$Kr

h): Extrapolation from $\langle r^2 \rangle _{94}$Zr / 54 / and $\delta \langle r^2 \rangle _{94}$Zr from Ref. / 56 /

i): Estimation from $E_{2+}^{94}$Sr

j): Extrapolation from $\langle r^2 \rangle _{115}$In / 57 / and $\delta \langle r^2 \rangle _{115}$In from Ref. / 34 /

k): Ref. / 38 /

l): Estimation from BEZ of $^{106}$Cd / 38 / and $E_{2+}^{108}$Sn

m): Estimation from $E_{2+}^{96}$Sr

n): Ref. / 58 /

o): Extrapolation from $\langle r^2 \rangle _{133}$Cs / 53 / and $\delta \langle r^2 \rangle _{133}$Cs from Ref. / 27 /.

p): Extrapolation from $\langle r^2 \rangle _{138}$Ba / 53 / and $\delta \langle r^2 \rangle _{138}$Ba from Ref. / 41 /

q): Estimation from BEZ $^{114}$Cd / 38 / and BEZ $^{116}$Sn

r): Estimation from $E_{2+}^{122}$Ba and BEZ $^{120}$Xe

s): Estimation from $E_{2+}^{124}$Ba and BEZ $^{122}$Xe
tn): Estimation from BE2 $^{126}\text{Ba}$ and BE2 $^{124}\text{Xe}$

u): Estimation from BE2 $^{128}\text{Ba}$ and BE2 $^{126}\text{Xe}$

v): Estimation from $E + ^{116}\text{Xe}$, $^{118}\text{Xe}$ and $^{120}\text{Ba}$

w): Derived from the spectroscopic quadrupole moment $Q_S$
<table>
<thead>
<tr>
<th>( A )</th>
<th>( \text{IS}^{114}\text{A} \text{ (MHz)} )</th>
<th>( \text{IS}^{114}\text{A}_\text{Corr} \text{ (MHz)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>3301 (92)</td>
<td>3454 (170)</td>
</tr>
<tr>
<td>103</td>
<td>3016 (128)</td>
<td>3075 (160)</td>
</tr>
<tr>
<td>104</td>
<td>2420 (104)</td>
<td>2595 (184)</td>
</tr>
<tr>
<td>105</td>
<td>2302 (107)</td>
<td>2349 (131)</td>
</tr>
<tr>
<td>107</td>
<td>1624 (93)</td>
<td>1666 (115)</td>
</tr>
<tr>
<td>111m</td>
<td>831 (370)</td>
<td>871 (390)</td>
</tr>
<tr>
<td>113m</td>
<td>400 (246)</td>
<td>440 (266)</td>
</tr>
<tr>
<td>115</td>
<td>-152 (134)</td>
<td>-63 (174)</td>
</tr>
<tr>
<td>117/117m</td>
<td>(-480)</td>
<td></td>
</tr>
<tr>
<td>118</td>
<td>-461 (39)</td>
<td>-378 (85)</td>
</tr>
<tr>
<td>119/119m</td>
<td>(-1010)</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>-570 (120)</td>
<td>-497 (164)</td>
</tr>
</tbody>
</table>

**TABLE 1**
<table>
<thead>
<tr>
<th>A</th>
<th>$\delta r^2$</th>
<th>$\langle \beta^2 \rangle_{1S}$</th>
<th>$\langle \beta^2 \rangle_{BE2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>-0.982 (43) (169)</td>
<td>0.028 (13)</td>
<td></td>
</tr>
<tr>
<td>103</td>
<td>-0.876 (41) (151)</td>
<td>0.035 (12)</td>
<td></td>
</tr>
<tr>
<td>104</td>
<td>-0.745 (47) (131)</td>
<td>0.045 (13)</td>
<td></td>
</tr>
<tr>
<td>105</td>
<td>-0.673 (34) (118)</td>
<td>0.048 (11)</td>
<td></td>
</tr>
<tr>
<td>106</td>
<td>-0.5345 (2) (960)</td>
<td>0.059 (7)</td>
<td>0.0281 (3)</td>
</tr>
<tr>
<td>107</td>
<td>-0.481 (29) (86)</td>
<td>0.060 (10)</td>
<td></td>
</tr>
<tr>
<td>108</td>
<td>-0.3920 (2) (700)</td>
<td>0.065 (7)</td>
<td>0.0290 (3)</td>
</tr>
<tr>
<td>109</td>
<td>-0.377 (40) (65)</td>
<td>0.061 (11)</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>-0.2555 (2) (460)</td>
<td>0.070 (7)</td>
<td>0.0297 (3)</td>
</tr>
<tr>
<td>111</td>
<td>-0.2348 (2) (400)</td>
<td>0.066 (6)</td>
<td></td>
</tr>
<tr>
<td>111m</td>
<td>-0.246 (100) (41)</td>
<td>0.066 (18)</td>
<td></td>
</tr>
<tr>
<td>112</td>
<td>-0.1206 (2) (220)</td>
<td>0.075 (6)</td>
<td>0.0329 (3)</td>
</tr>
<tr>
<td>113</td>
<td>-0.0989 (2) (160)</td>
<td>0.072 (6)</td>
<td></td>
</tr>
<tr>
<td>113m</td>
<td>-0.120 (68) (16)</td>
<td>0.069 (14)</td>
<td></td>
</tr>
<tr>
<td>114</td>
<td>-</td>
<td>0.078 (6)</td>
<td>0.0351 (3)</td>
</tr>
<tr>
<td>115</td>
<td>0.023 (45) (7)</td>
<td>0.075 (11)</td>
<td></td>
</tr>
<tr>
<td>115m</td>
<td>0.030 (12) (7)</td>
<td>0.076 (8)</td>
<td></td>
</tr>
<tr>
<td>116</td>
<td>0.0941 (2) (190)</td>
<td>0.078 (6)</td>
<td>0.0345 (3)</td>
</tr>
<tr>
<td>118</td>
<td>0.125 (22) (29)</td>
<td>0.070 (9)</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0.169 (42) (42)</td>
<td>0.065 (12)</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2**
<table>
<thead>
<tr>
<th></th>
<th>[431 \frac{1}{2}]</th>
<th>[420 \frac{1}{2}]</th>
<th>[422 \frac{3}{2}]</th>
<th>[411 \frac{3}{2}]</th>
<th>[413 \frac{5}{2}]</th>
<th>[402 \frac{5}{2}]</th>
<th>(\mu_1[\mu_n])</th>
<th>(Q_s[^b])</th>
<th>(\mu_1[\mu_n])</th>
<th>(Q_s[^b])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{103}\text{Cd})</td>
<td>-.599</td>
<td>-.354</td>
<td>.534</td>
<td>-.375</td>
<td>-.214</td>
<td>-.190</td>
<td>-0.88</td>
<td>-0.28</td>
<td>-0.81(3)</td>
<td>-0.79(66)</td>
</tr>
<tr>
<td>(^{105}\text{Cd})</td>
<td>-.467</td>
<td>-.318</td>
<td>.511</td>
<td>-.464</td>
<td>-.299</td>
<td>-.327</td>
<td>-0.93</td>
<td>-0.12</td>
<td>-0.7393(2)</td>
<td>0.43(4)</td>
</tr>
<tr>
<td>(^{107}\text{Cd})</td>
<td>.319</td>
<td>.246</td>
<td>-.429</td>
<td>.468</td>
<td>.412</td>
<td>.509</td>
<td>-0.92</td>
<td>0.15</td>
<td>-0.615055(1)</td>
<td>0.68(7)</td>
</tr>
<tr>
<td>(^{109}\text{Cd})</td>
<td>.187</td>
<td>.160</td>
<td>-.318</td>
<td>.392</td>
<td>.470</td>
<td>.678</td>
<td>-0.85</td>
<td>0.43</td>
<td>-0.827846(2)</td>
<td>0.69(7)</td>
</tr>
</tbody>
</table>

**TABLE 3**
<table>
<thead>
<tr>
<th>$^A_X$</th>
<th>$&lt;r^2&gt;(fm^2)$</th>
<th>$&lt;s^2&gt;$</th>
<th>$&lt;r^2&gt;_o(fm^2)$</th>
<th>$\delta&lt;r^2&gt;_o(fm^2)$</th>
<th>$\delta&lt;r^2&gt;_{o,Dr}(fm^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=54</td>
<td>102 Cd</td>
<td>20.28 (17)$^a$</td>
<td>0.027 (9)$^e$</td>
<td>20.07 (24)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>96 Mo</td>
<td>19.21 (4)$^b$</td>
<td>0.031 (2)$^f$</td>
<td>18.98 (5)</td>
<td>-1.09 (25)</td>
</tr>
<tr>
<td></td>
<td>94 Zr</td>
<td>18.64 (9)$^c$</td>
<td>0.007 (1)$^f$</td>
<td>18.59 (10)</td>
<td>-1.48 (26)</td>
</tr>
<tr>
<td></td>
<td>91 Rb</td>
<td>18.20 (6)$^d$</td>
<td>0.037 (12)$^g$</td>
<td>17.94 (14)</td>
<td>-2.13 (28)</td>
</tr>
<tr>
<td>N=56</td>
<td>104 Cd</td>
<td>20.52 (13)$^a$</td>
<td>0.030 (10)$^e$</td>
<td>20.28 (21)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>98 Mo</td>
<td>19.42 (4)$^b$</td>
<td>0.030 (1)$^f$</td>
<td>19.19 (5)</td>
<td>-1.09 (22)</td>
</tr>
<tr>
<td></td>
<td>96 Zr</td>
<td>18.81 (10)$^h$</td>
<td>0.007 (4)$^f$</td>
<td>18.76 (12)</td>
<td>-1.52 (24)</td>
</tr>
<tr>
<td></td>
<td>93 Rb</td>
<td>18.42 (6)$^d$</td>
<td>0.035 (15)$^i$</td>
<td>18.16 (17)</td>
<td>-2.12 (27)</td>
</tr>
<tr>
<td>N=58</td>
<td>106 Cd</td>
<td>20.73 (10)$^a$</td>
<td>0.028 (1)$^k$</td>
<td>20.50 (10)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>107 In</td>
<td>20.69 (10)$^j$</td>
<td>0.020 (10)$^l$</td>
<td>20.53 (18)</td>
<td>-0.03 (21)</td>
</tr>
<tr>
<td></td>
<td>100 Mo</td>
<td>19.74 (4)$^b$</td>
<td>0.054 (3)$^f$</td>
<td>19.33 (5)</td>
<td>-1.17 (11)</td>
</tr>
<tr>
<td></td>
<td>95 Rb</td>
<td>18.74 (8)$^d$</td>
<td>0.035 (15)$^m$</td>
<td>18.48 (19)</td>
<td>-2.02 (22)</td>
</tr>
<tr>
<td>N=66</td>
<td>114 Cd</td>
<td>21.25 (1)$^h$</td>
<td>0.035 (1)$^k$</td>
<td>20.97 (2)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>115 In</td>
<td>21.26 (9)$^j$</td>
<td>0.024 (12)$^q$</td>
<td>21.07 (19)</td>
<td>0.10 (19)</td>
</tr>
<tr>
<td></td>
<td>116 Sn</td>
<td>21.42 (1)$^b$</td>
<td>0.013 (1)$^f$</td>
<td>21.31 (2)</td>
<td>0.34 (3)</td>
</tr>
<tr>
<td></td>
<td>121 Cs</td>
<td>22.83 (1)$^o$</td>
<td>0.055 (18)$^r$</td>
<td>22.34 (17)</td>
<td>1.37 (17)</td>
</tr>
<tr>
<td></td>
<td>122 Ba</td>
<td>23.20 (3)$^p$</td>
<td>0.069 (23)$^e$</td>
<td>22.58 (23)</td>
<td>1.61 (23)</td>
</tr>
<tr>
<td>N=68</td>
<td>116 Cd</td>
<td>21.35 (2)$^a$</td>
<td>0.035 (1)$^k$</td>
<td>21.06 (3)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>118 Sn</td>
<td>21.52 (1)$^b$</td>
<td>0.013 (1)$^f$</td>
<td>21.41 (2)</td>
<td>0.35 (4)</td>
</tr>
<tr>
<td></td>
<td>123 Cs</td>
<td>22.88 (1)$^o$</td>
<td>0.054 (18)$^s$</td>
<td>22.40 (17)</td>
<td>1.34 (17)</td>
</tr>
<tr>
<td></td>
<td>124 Ba</td>
<td>23.23 (1)$^p$</td>
<td>0.058 (19)$^e$</td>
<td>22.71 (17)</td>
<td>1.65 (17)</td>
</tr>
<tr>
<td>N=70</td>
<td>118 Cd</td>
<td>21.39 (3)$^a$</td>
<td>0.031 (10)$^e$</td>
<td>21.13 (11)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>120 Sn</td>
<td>21.67 (1)$^b$</td>
<td>0.012 (1)$^f$</td>
<td>21.57 (2)</td>
<td>0.44 (11)</td>
</tr>
<tr>
<td></td>
<td>125 Cs</td>
<td>22.93 (1)$^o$</td>
<td>0.050 (17)$^t$</td>
<td>22.48 (16)</td>
<td>1.35 (19)</td>
</tr>
<tr>
<td></td>
<td>126 Ba</td>
<td>23.27 (3)$^p$</td>
<td>0.057 (11)$^f$</td>
<td>22.75 (13)</td>
<td>1.62 (17)</td>
</tr>
</tbody>
</table>

TABLE 4
| N=72 | 120\text{Cd} | 21.43 (4)\text{a} | 0.028 (9)\text{e} | 21.19 (12) | 0 | 0 |
| 122\text{Sn} | 21.77 (1)\text{b} | 0.010 (1)\text{f} | 21.68 (2) | 0.49 (12) | 0.40 |
| 127\text{Cs} | 22.99 (1)\text{g} | 0.051 (14)\text{u} | 22.53 (13) | 1.34 (18) | 1.39 |
| 128\text{Ba} | 23.30 (4)\text{p} | 0.065 (14)\text{f} | 22.71 (17) | 1.52 (21) | 1.59 |
| A=118 | 118\text{Cd} | 21.39 (3)\text{a} | 0.031 (10)\text{e} | 21.13 (11) | 0 | 0 |
| 118\text{Sn} | 21.52 (1)\text{b} | 0.013 (1)\text{f} | 21.41 (2) | 0.28 (11) | 0.29 |
| 118\text{Cs} | 22.58 (1)\text{g} | 0.068 (23)\text{v} | 21.99 (20) | 0.86 (23) | 1.01 |
| A=120 | 120\text{Cd} | 21.43 (4)\text{a} | 0.028 (9)\text{e} | 21.19 (12) | 0 | 0 |
| 120\text{Sn} | 21.67 (1)\text{b} | 0.012 (1)\text{f} | 21.57 (2) | 0.38 (12) | 0.29 |
| 120\text{Cs} | 22.97 (1)\text{g} | 0.108 (11)\text{v} | 22.02 (11) | 0.83 (16) | 1.01 |
FIGURE CAPTIONS

Figure 1  Experimental setup used for on-line laser spectroscopy of Cd isotopes.  
Upper part: schematic view.  
(GR = grating, E = etalon, T = telescope, DC = dye cell,  
P = polarizer, m = Mirror, L = lens, FD = frequency  
doubling crystal, S = beam splitter, PM = photomultiplier,  
F = UC-11 colour filter, $H_o$ = magnetic field for the  
reference cell, BS = ion beam stop, V = valve, MO = laser  
intensity monitor, O = oven).  
Lower part: resonance vessel and oven system.

Figure 2  Signal of $^{103}$Cd. The hyperfine components are indicated by  
arrows. The solid line represents the best fit to the signal  
by a Voigt profile.

Figure 3  Integral isotope shifts of Cd in the $\lambda = 326$ nm line. The  
shifts are given relative to $^{114}$Cd. Results of this work are  
indicated by crosses, others /7,8,9/ are shown as open  
circles. Isomers are marked by open quadrangles. Triangles  
indicate hfs components.

Figure 4  King plot of the CdII, $\lambda = 226$ nm and the CdI, $\lambda = 326$ nm  
transitions. The solid line represents a least square fit  
to the data.

Figure 5  $\delta <r^2>$ $^{114}$A for the Cd isotopes. The errors indicated are  
experimental ones only. The solid line represents the  
predictions of the spherical droplet model /3/. The  
broken line connects the values obtained by a fit to the  
data using the parametrization of $\delta <r^2>$ given in eq. 11.
Figure 6  Comparison between the deformation parameters $\beta_2^2$ of Cd ground states as obtained from IS measurements (full dots) and BE2 values / 38 / (open circles).

Figure 7  Plot of the differential changes $\delta <r^2>_{N,N-2}$ of the Cd isotopes. Full dots indicate $\delta <r^2>_{N,N-2}$ values of nuclear ground states, open circles those of isomeric states. The prediction of the spherical droplet model / 3 / is given by the broken line.

Figure 8  
(a) $\delta <r^2>$ in the isotonic series $N=66$ relative to the closed proton shell isotope $^{116}_{50}$Sn. The broken line connects the data points for guiding the eye.

(b) Same as 8a but corrected for the contribution to $<r^2>$ from quadrupole deformation according to eq. 9. The solid line indicates the prediction of the spherical droplet model / 3 /.

Figure 9  $\delta <r^2>$ in the isobaric series $A=118$ corrected for quadrupole deformation according to eq. 9. The predictions of the spherical droplet model / 3 / are shown as a solid line.
$^{103}\text{Cd}$

INTENSITY (A.U.)

$V^{114}$ [GHz]

F 7/2 5/2 3/2

1 GHz

Fig. 2