THE LEADING EFFECT EXPLAINS THE CHARGED PARTICLE MULTIPLICITY
DISTRIBUTION OBSERVED AT THE CERN pǐ COLLIDER.

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ABSTRACT
The assumption that the leading effect, and the charged particle
multiplicity distributions, measured at the ISR, scale with energy, and that the
mean charged multiplicity, measured at the ISR, can be extrapolated up to
Collider energy using the leading subtraction method, allows a prediction of the
charged multiplicity distribution at the CERN pǐ Collider.

This prediction is found to be in excellent agreement with the experimental
data obtained at the Collider by the UA5 group.

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Nobody has so far succeeded in fitting, with known facts, the charged particle multiplicity distribution measured, in the full rapidity range, at the CERN p\bar{p} Collider\(^1\).

This is shown in Fig.1, where the experimental results, obtained by the UA5 collaboration, are compared with two attempts proposed in order to find out a way to understand the charged multiplicity distribution at the highest energy available so far in hadronic machines.

The main trend was to interpret this lack of agreement, between theoretical models and experimental results, as an example of scale breaking\(^1\) in the large energy range from fixed target, to ISR, to Collider, experiments: i.e. from \(\approx 10\) up to 540 GeV.

We have reported\(^2\) that scale invariance\(^3\) holds in the charged multiplicity distributions measured at ISR once the leading effects are taken in due account. We also know that the leading effects scale in the ISR energy range\(^4\). It is therefore reasonable to extrapolate the ISR results to Collider energies. This allows a prediction of the charged multiplicity distribution expected at the CERN p\bar{p} Collider.

We will see that this prediction is in excellent agreement with the experimental data.

Let us start with the charged multiplicity distributions measured in (pp) interactions at the ISR\(^2\), once the effective energy available for particle production, \(\sqrt{q^\text{had}_{\text{tot}}}\), is used as the correct energy parameter\(^5-7\).

The best fit to our data\(^2\) is shown in Fig.2. If scaling is true at ISR energies, it is reasonable to assume that it will remain valid even at higher energies, for example at Collider energies. We will assume that the best fit shown in Fig.2 holds true at Collider energies.

The second step is to know the basic ingredient which produces, in any hadronic interaction, the correct energy available for particle production, i.e. the quantity \(\sqrt{q^\text{had}_{\text{tot}}}\). This is shown in Fig.3\(^8,9\), where \(x_F\) is the fractional momentum of the leading proton and \(d\sigma/dx_F\) is the inclusive cross-section for the reaction:
As reported in earlier works, at ISR, $d\sigma/dx_F$ scales, if we go from minimum to maximum ISR energies\(^4\); if instead of (pp) we take (p\bar{p}), it is reasonable to expect only minor, if any, change in $d\sigma/dx_F$. An exponential fit ($d\sigma/dx_F \sim \exp(-x_F/x_0)$) to the data (Fig.3) in the range 0.15 $< x_F < 0.9$ gives for the slope the value $1/x_0 = 0.23 \pm 0.07$.

Let us assume that the best fit to the data shown in Fig.3 holds true at Collider energy for (p\bar{p}) interactions. Given this best fit, it is possible to calculate the distribution function $F[\sqrt{(q_{\text{had}}/q_{\text{tot}}})^2]$ of the quantity $\sqrt{(q_{\text{had}}/q_{\text{tot}}})^2$, (indicated from now on as $\sqrt{s}$ for simplicity):

\[
F(\sqrt{s}) \cdot d(\sqrt{s}) = \frac{2\sqrt{s} \cdot d(\sqrt{s})}{x_0^2[1-\exp(1/x_0)]^2} \int_1 \frac{q^2+(1-x_F)^2s}{q^2/s} \cdot d(1-x_F)/(1-x_F)
\]

(1)

where $s$ is the square of the nominal c.m. energy of the initial state.

The distribution function (1) is valid when the leading particles in the final state are uncorrelated. At ISR we have established that the two leading protons show no correlation\(^10\). We will therefore assume that the uncorrelation remains valid at Collider energies.

For the dependence of the average charged multiplicity\(^6\) on $\sqrt{s}$, the following analytic form\(^11,12\) is taken:

\[
<n_{\text{ch}}(\sqrt{s})> = A \cdot \exp[B \cdot \sqrt{\ln(q^2/A^2)}]
\]

(2)

where $A=0.2$ GeV.

Following our method of leading particle subtraction, the average charged multiplicity in (pp)\(^13,14\), (p\bar{p})\(^1\) interactions, at fixed nominal energy $\sqrt{s}$, is obtained from the knowledge of the distribution function (1):
\[ <n_{\text{ch}}(\sqrt{s})> = \int_{\sqrt{q_{\text{min}}^2}}^{\sqrt{q_{\text{max}}^2}} F(\sqrt{q^2}) \cdot <n_{\text{ch}}(\sqrt{q^2})> \cdot d(\sqrt{q^2}) + n \quad (3) \]

where \( \sqrt{q_{\text{min}}^2} \) and \( \sqrt{q_{\text{max}}^2} \) are respectively the minimum and the maximum values allowed for \( \sqrt{\frac{q_{\text{had}}^2}{\text{tot}}^2} \) and \( n \) is the number of charged particles in the initial state. For the sake of simplicity in what follows we take \( \sqrt{q_{\text{min}}^2} = 0 \), and \( \sqrt{q_{\text{max}}^2} = (\sqrt{s})_{\text{pp}} \). This corresponds to the full \( x_F \) range (0\%1) of the leading particles. The quantity \( n \) is taken to be 2 in the (pp) case and in the (p\( \bar{p} \)) case, on the basis of the very small contribution from charge exchange and annihilation expected in (p\( \bar{p} \)) interactions at the Collider on the basis of extrapolated ISR data. The fit to the data of Fig.4 gives the following values for \( A \) and \( B \): \( A = 0.082 \pm 0.002 \), \( B = 1.56 \pm 0.01 \).

All the quantities needed in order to calculate the charged multiplicity distribution expected at the p\( \bar{p} \) Collider now exist. In fact the probability of observing a charged multiplicity, \( n_{\text{ch}} \), at a value of \( \sqrt{s} \), \( P[n_{\text{ch}}; \sqrt{s}] \), is given by the integral of the probability, \( P[n_{\text{ch}} - n; \sqrt{q^2}] \), of observing the multiplicity, \( n_{\text{ch}} - n \), at \( \sqrt{q^2} \). This function \(^2\) is the best fit to the data in Fig.2.

\[ P[n_{\text{ch}}; \sqrt{s}] = \int_{\sqrt{q_{\text{min}}^2}}^{\sqrt{q_{\text{max}}^2}} F(\sqrt{q^2}) \cdot P[n_{\text{ch}} - n; \sqrt{q^2}] \cdot d(\sqrt{q^2}) \quad (4) \]

The result of this calculation is the curve in Fig.5. Notice that the comparison between the experimental data and formula (4) gives a \( x^2/\text{point} = 1.3 \).

We wish to stress that this prediction is based on the hypothesis that:

i) the charged particle multiplicity distributions (Fig.2),

ii) and the leading proton effect (Fig.3),

scale from ISR up to Collider energies;

and

iii) the average charged particle multiplicity is a function of \( \sqrt{\frac{q_{\text{had}}^2}{\text{tot}}^2} \) as shown in Fig.4.
Conclusion.

If we assume that the leading effect, together with the multiplicity distributions, measured at ISR, scale up to Collider energy, and that the mean charged multiplicity, measured at ISR, can be extrapolated up to Collider energy using the leading subtraction method, the ISR data allow one to predict the charged multiplicity distribution measured by UA5 at the CERN $p\bar{p}$ Collider. The agreement between this prediction and the experimental data, as shown in Fig.5, is impressive.
REFERENCES

2) M. Basile et al., Scaling in the charged multiplicity distributions at the ISR and comparison with \((e^+e^-)\) data, preprint CERN-EP/84-94 (Submitted to N. C. Lett.)
6) M. Basile et al., N.C. 67A (1982) 244.
9) From a \(p_L\) integration of data by:
FIGURE CAPTIONS

Fig.1: The charged particle multiplicity distribution measured by the UA5 collaboration\(^1\) at \((\sqrt{s})_{p\bar{p}} = 540\) GeV. The dashed curve is the KNO\(^3\) scaling prediction based on the fit by P. Slattery\(^{15}\). The solid curve is the UA5 fit\(^1\).

Fig.2: The \(n_{ch}/\langle n_{ch} \rangle\) distributions measured in our ISR experiment\(^2\) in \((pp)\) interactions, for two different intervals of \(\sqrt{(Q_{\text{had}}/Q_{\text{tot}})^2}\) [open circles: \(10 \leq \sqrt{(Q_{\text{had}}/Q_{\text{tot}})^2} \leq 15\) GeV; solid circles: \(25 \leq \sqrt{(Q_{\text{had}}/Q_{\text{tot}})^2} \leq 30\) GeV]. The curve is the best fit to the data.

Fig.3: Inclusive differential cross-section \(d\sigma/dx_F\) as a function of \(x_F\) \((x_F = 2p_T/\sqrt{s})\) measured\(^8,9\) in the reaction \(pp \rightarrow p + X\), at ISR energies. Open circles are Ref.8; solid circles are Ref.9.

Fig.4: The average charged multiplicities measured\(^6\) in \((pp)\) interactions at the ISR versus \(\sqrt{(Q_{\text{had}}/Q_{\text{tot}})^2}\), i.e. using the leading subtraction method\(^6\) (open circles). The average charged multiplicities measured\(^{13,14}\) in \((pp)\) interactions versus \(\sqrt{s}_{pp}\), i.e. using the standard method (solid circles). The open triangle is the average charged multiplicity measured by UA5 collaboration\(^1\) at the CERN p\bar{p} Collider, again using the standard method. The dashed curve corresponds to formula (2) and the solid curve to formula (3) in the text.

Fig.5: The charged particle multiplicity distribution measured by UA5 collaboration\(^1\) at \((\sqrt{s})_{p\bar{p}} = 540\) GeV. The curve is our own prediction based on formula (4).
\[
\sqrt{s}_{pp} = 62 \text{ GeV} \left\{ \begin{array}{l}
10 \leq \sqrt{(q_{\text{tot}}^{\text{had}})^2} \leq 15 \text{ GeV} \\
25 \leq \sqrt{(q_{\text{tot}}^{\text{had}})^2} \leq 30 \text{ GeV}
\end{array} \right.
\]

\[\langle n_{ch} \rangle \cdot P(n_{ch})\]

\[n_{ch}/\langle n_{ch} \rangle\]

Fig. 2
$d\sigma/dx_F [\text{mb}]$

$x_F = 2p_L / \sqrt{s}$

Fig. 3