TWO-JET PRODUCTION IN p\bar{p} COLLISIONS*)

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Abstract: The production of two quark/gluon jets in p\bar{p} collisions is analyzed. We determine the cross section dependence on the transverse momentum, the rapidities, the two-jet invariant mass, and on the CM/LAB angles, and we evaluate the sensitivity of the results on the theoretical input assumptions and the experimental cuts. We extrapolate these results into the TeV energy region.

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1. INTRODUCTION

The recent experimental observation \([1,2]\) of hadron-jets at the CERN p\bar{p} collider \([3]\), and in particular the considerable number of 2-jet events with a large jet transverse momentum which grows as the overall CM-energy increases, permits a detailed study of the constituent dynamics. Jets are doubtlessly the strongest piece of evidence in favour of perturbative quantum chromodynamics (QCD) \([4]\). Their dynamical behaviour is described by this theory and the quark/gluon momentum distributions in the nucleon, which are known from deep-inelastic scattering. A small number of predictions of this general framework, with mass factorization \([5]\) as its basic input, have so far been experimentally verified in p\bar{p} collisions \([6,1,2]\). From the experimental results already available now, we soon may expect further detailed analyses on the 2-jet dynamics.

These facts motivate us to carry out a systematic analysis of all hadronic 2-jet (constituent) processes searching for their particularities and their distinctions in the kinematical distributions, and looking in particular for kinematical differences between the gg, qq and qq subprocesses. We determine the integrated cross sections, the transverse momentum distributions, the dependence on the jet rapidities giving information on the jet-jet correlations, the cross section behaviour as a function of the 2-jet invariant mass, and the two-jet center-of-mass (CM) and laboratory (LAB) angular distributions.

Earlier theoretical analyses \([7]\) have partially covered this program. Besides presenting more detailed new distributions, we evaluate with the present study the influence of the theoretical input assumptions on the results, and assess the effect of changes in the 'input momentum distributions' and/or the perturbative QCD 'scale \( \Lambda \)'. Since we soon may expect experimental results of much higher precision, there is doubtlessly a need to know the uncertainties in the theoretical predictions. Our study is limited to the leading-log approximation (without any K-factor), and we ignore the primordial transverse momentum of the partons in the nucleon. Furthermore, we do not account for the fragmentation of the final state quanta into hadrons.

The experimental results are limited by the acceptance of the detectors. Jets can only be identified if their transverse momenta are above 10–15 GeV. The analyses presented so far by the UA1 and UA2 groups are limited to the rapidity ranges: \(-2.5 \leq \eta \leq +2.5\) (UA1) and \(-1.5 \leq \eta \leq +1.0\) (UA2), respectively, whereby the uncertainty in the definition of a jet should be kept in mind. We therefore also study the influence of a cutoff in the tran-
sverse momentum, the jet-jet invariant mass, and in the jet rapidities on
the integrated and the differential cross sections.

Several $pp$ and $p\bar{p}$ colliders, which will reach much higher energies, are
at present under construction or in the planning stage. We therefore con-
sider it fruitful to extend our analysis into the TeV-range. We evaluate, by
extrapolation, the cross sections and the kinematical distributions of the
2-jet (constituent) processes at 2 and 20 TeV using the presently accepted
calculation scheme. We are aware that nature, however, could well behave
differently as one reaches the TeV-scale [8]!

The paper is organized as follows: in section 2 we introduce the calcu-
lation scheme; we discuss its assumptions and we focus on the size and
shape of the lowest order QCD 2-constituent cross sections. In section 3 we
determine the overall-energy and $p_T$ cutoff dependence of the integrated
cross sections for 2-jet production in $p\bar{p}$ collisions; we assess the relative
influence of the different constituent processes, and we vary the input
assumptions and the kinematical cuts. Section 4 presents a discussion of
the inclusive single jet transverse momentum and rapidity distributions. In
section 5 we focus on the jet-jet rapidity correlations. The cross section
dependence on the jet-jet invariant mass (M) is evaluated in section 6. At a
fixed jet-jet invariant mass, we also vary the jet transverse momentum or
the jet rapidities and observe in the latter case an rising cross section with
increasing rapidity values. The constituent CM- and LAB-angular distrib-
butions and their sensitivity on the theoretical parameters and the experi-
mental cuts are presented in section 7. In section 8 we summarize our
results.

2. CALCULATION SCHEME AND PARTON CROSS SECTIONS

We introduce the calculation scheme for hadronic 2-jet production, discuss
its input assumptions, and give details on the size and the behaviour of
the parton cross sections.

The cross section for $p\bar{p}$ 2-jet production in the framework of perturba-
tive QCD reads

$$ \frac{d\sigma}{dt^2dy_1dy_2} = x_1x_2 \cdot $$

$$ \Sigma_{q/g} \left[ \frac{d\sigma_0}{dt_0} u(x_1)\bar{u}(x_2) + \frac{d\sigma_0}{dt_0} u_0 \bar{u}(x_1)u(x_2) \right] \quad (2.1) $$
The \( u(x_i) \) are the (scale dependent) quark/gluon momentum distributions in the antiproton and proton with the momentum fractions \( x_i \). In this notation \( x_1 \) is always associated with the proton and \( x_2 \) always with the antiproton. The incident proton momentum defines the \( z \)-direction of the spatial coordinate system. The \( d\sigma_0/dt_0 \) is the parton differential cross section. Note, all quantities on the parton level carry the index \( 0 \), whereas in the figures they are given the superscript \( \Lambda \). The squared \( pp \) center-of-mass energy is \( s=(E_{CM})^2 \). The \( y_1, y_2 \) are the rapidities of the two final state jets and \( q_t \) is the transverse momentum of each of them. Since all primordial transverse momentum of the nucleon constituents is ignored, the transverse plane momentum vectors of the two jets point in opposite directions and their sum vanishes. The summation over the corresponding quark/gluon contributions is shown explicitly. The momentum fractions are given by [9]

\[
x_1 = \left( x_t/2 \right) ( e^{-y_1} + e^{-y_2} ) , \quad x_2 = \left( x_t/2 \right) ( e^{y_1} + e^{y_2} )
\]

(2.2)

where \( x_t \equiv (2q_t/\sqrt{s}) \) is the transverse scale variable. The Mandelstam variables on the constituent level read

\[
s_0 = x_1 x_2 s \quad (2.3)
\]

\[
t_0 = (s/2) ( -x_1 x_2 + \sqrt{x_1 x_2 (x_1 x_2 - x_t^2)} ) \quad (2.4)
\]

\[
u_0 = (s/2) ( -x_1 x_2 - \sqrt{x_1 x_2 (x_1 x_2 - x_t^2)} ) \quad (2.5)
\]

The sign of the square-root in Eqs. 2.4 and 2.5 is determined in the CM-system. In the right (left) half-side of the momentum plane \( q=(q_t,q_L) \) (\( q_t \) and \( q_L \) are the transverse and longitudinal vector components) with \( \Theta_{CM} \rightarrow 90^\circ \) (290\(^\circ\)), the sign in \( t_0 \) is positive (negative) and that in \( u_0 \) is negative (positive). As \( \Theta_{CM} \rightarrow -180^\circ \), this definition assures that \( t_0=-s_0 \) and \( u_0=0 \). The rapidities allow for a definition of these signs in a Lorentz-invariant manner: if \( y_1 \leq y_2 \) the sign in \( t_0 \) is positive, but negative otherwise.

The integrated 2-jet cross section is obtained by integrating first over the rapidities and subsequently over the transverse momentum. The integration boundaries are [9]

\[
y_1^- \leq y_1 \leq y_1^+ , \quad y_1^{+-} \equiv \ln \left[ (\sqrt{s}/q_t) - \exp(\mp y_2) \right] \quad (2.6)
\]

\[
y_2^- \leq y_2 \leq y_2^+ , \quad y_2^{+-} \equiv \ln \left[ (E \pm p_L^{\max})/q_t \right] \quad (2.7)
\]

\[
p_t \leq q_t \leq E=\sqrt{s}/2 , \quad p_L^{\max} \equiv \sqrt{(E^2-q_t^2)} \quad (2.8)
\]
Note, $q_t$ is the transverse momentum of each jet, whereas $p_t$ stands for the (imposed) lower cutoff when carrying out the $q_t$ integration. The boundaries in Eq.2.6 are obtained from Eqs.2.2 by expressing $y_2$ as a function of $x_2$ and by assuming $x_1 (or x_2) = 1$. The boundaries in Eq.2.7 follow from the definition of the jet rapidity where $p_L^{\text{max}}$ is defined as the maximum longitudinal energy remaining after the jet transverse energy has been subtracted.

The parton cross sections are evaluated in the lowest order of the strong coupling constant. All higher order (leading-log) contributions are included in the renormalization group summation of perturbative QCD resulting in the scale dependence of the momentum distributions. The next-to-leading corrections lead to another $\alpha_s$-dependent factor whose influence is ignored since we limit ourselves in this study to the leading-log approximation. The subsequent results therefore do not include any K-factor.

The $Q^2$-evolution of the momentum distributions is determined by the 'input distributions' which were measured in deep-inelastic scattering at the input point $Q_0^2$, and the solution of the renormalization group equation specifying the $Q^2$-evolution. The actual calculation of their $Q^2$-dependence is involved. Since there exists no simple parametrization sufficiently general as to allow for a continuous variation of the input distributions we use the simplified forms: BGOR [10], GROR [11], BETAL [12], GHR [13] and CDHS [14] which offer a variety of distinct input $x$-dependences leaving the $Q^2$-evolution however unchanged. The capital letters, characterizing the different parametrizations, are synonyms for the collaborations.

We discuss their characteristics [15]. The valence input distributions of all four parametrizations lead to similar results. The main differences appear in the sea and the gluon distributions. With increasing $x$ values, their input distributions either fall-off very rapidly ('soft') or they extend further into the $x$ range ('hard'). Such behaviour is often characterized by the exponents $n$(sea), $n$(gluon) of the counting rule parametrization: $x\cdot u = A (n+1)(1-x)^n$. The importance of the gluon/sea distribution is governed by the coefficients $A = \langle x \cdot u \rangle$, which represent the averaged momentum fraction. The coefficient of the sea distribution changes significantly as we go from one parametrization to another. In the gluon distribution it remains (almost) unchanged $A_G = 0.5$. In Table 1, we give a summary of the main parameters of the scale dependent momentum distributions to be used in our subsequent numerical analysis. The $-$ sign indicates that the input distributions do not exactly follow a counting rule parametrization and the given
numbers are meant as a guide. The GHR [13] input gluon distribution is 'hard', despite of its large counting rule exponent, with a maximum around $x=0.1$ and a fall-off as $x=0$, whereas the influence of the BGOR [10] gluon distribution is reduced due to its softness and its smaller contribution to the momentum sum rule.

In our subsequent analysis we use the running coupling constant

$$\alpha_s(Q^2) = \frac{12\pi/(33-2n_f)}{\ln^{-1}(Q^2/\Lambda^2)}$$

(2.9)

and limit ourselves to the three low-mass flavours $u,d,s$ and the charmed quark $c$; thus $n_f=4$. The contributions from the higher mass flavours hardly change the results since their parton cross sections are suppressed due to the non-negligible quark masses. Since the precise value of the scale parameter $\Lambda$ is not known we determine the cross section spread whilst changing $\Lambda$ in the range $0.1 \text{ GeV} \leq \Lambda \leq 0.5 \text{ GeV}$. All such cross section variation results from changes in $\alpha_s$ and the scale dependent momentum distributions. In any viable parametrization of the scale dependent momentum distributions, the $Q^2$ dependence enters via the variable $s = \ln\left(\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)\right)$. The $Q^2$ dependence is thus affected by the scale parameter $\Lambda$ such that back-extrapolation to the input point $Q_0^2$ leaves the input distributions unchanged. We analyze in the following the $\alpha_s$ and $s$ variations with changing $\Lambda$ (in the above indicate range) at three typical values of the scale $Q^2$. We find:

$$Q^2 = (1.8 \quad 100 \quad 2500) \text{ GeV}^2$$

$$\alpha_s = (0.29 - 2.50, 0.16 - 0.33, 0.12 - 0.19)$$

(2.10)

and conclude that the strongest $\alpha_s$ variation is at small $Q^2$ values. The strongest $\Lambda$ sensitivity of $s$ is in the large $Q^2$ range, where we expect the kinematical distributions to decrease as $\Lambda$ increases. The $\Lambda$ dependence of the cross section follows from an interplay between the $\Lambda$ dependence of $\alpha_s$, and $s$ in the momentum distributions.

In order to demonstrate the influence of the QCD corrections we frequently resort to the scale independent momentum distributions at $Q^2=Q_0^2$. Since $Q_0^2$ is not the same value for all parametrizations (see Table 1) one may expect slight differences. The cross section normalization therefore shows significant changes from one parametrization to another. In order to prevent such ambiguity we set $\alpha_s=0.3$. 
Mass factorization is understood as the separation of the mass singularities from the Feynman diagrams; they are absorbed in the momentum distributions. Part of the remaining dependence on the dynamical variable $Q^2$ can be identified with the solution of the renormalization group equation which gives rise to the $Q^2$ dependence of the momentum distributions. The left-over $Q^2$ dependence is absorbed in the coefficient function where an expansion in $\alpha_s(Q^2)$ is assumed, giving rise to the next-to-leading logarithmic Feynman diagram contributions. As long as $\alpha_s(Q^2)$ is small such procedure is well justified; if $\alpha_s(Q^2)$ however is no longer small we are faced with a problem.

Once the dynamical variable $Q^2$ is chosen, the leading-log approximation is well defined. Mass factorization, however, involves an ambiguity concerning the next-to-leading contributions where the coefficients of the lowest order terms in a $(\log)^n$-expansion have to be specified. This ambiguity in the next-to-leading contributions leaves some freedom in the choice of the dynamical variable $Q^2$. Changing one definition into another changes the next-to-leading terms, which, in the leading-log approximation, however are assumed to be small. How well this assumption is satisfied, and in which kinematically region, is not totally clear.

Due to this ambiguity, several definitions for $Q^2$ have been proposed. The proponents argue that in one or the other choice the next-to-leading contributions were unimportant. The most commonly used forms are:

$$Q^2 = 2s_0 t_0 u_0 / (s_0^2 + t_0^2 + u_0^2)$$  \hspace{1cm} (2.11)

$$Q^2 = \sqrt[3]{s_0 t_0 u_0}$$  \hspace{1cm} (2.12)

$$Q^2 = t_0 u_0 / s_0 = q_t^2$$  \hspace{1cm} (2.13)

For sufficiently large $q_t$ (and $s_0$) they should not differ very much. In our subsequent numerical analysis of the parton cross sections we determine the influence of the $Q^2$ choice as a function of the kinematical variables. In the following sections of this paper we mainly will use the form (2.11).

There are altogether eight quark/gluon subprocesses

\begin{align*}
1 & : q_1 + q_2(\bar{q}_2) & - & q_1 + q_2(\bar{q}_2) \\
2 & : q_1 + q_1 & - & q_1 + q_1 \\
3 & : q_1 + \bar{q}_1 & - & q_2 + \bar{q}_2 \\
4 & : q_1 + \bar{q}_1 & - & q_1 + \bar{q}_1 \\
5 & : q + \bar{q} & - & g + g \\
6 & : g + g & - & q + \bar{q} \\
7 & : q + g & - & q + g \\
8 & : g + g & - & g + g
\end{align*}

(2.14)
with their cross sections (in lowest order QCD, see Fig.2.1) listed in Ref.16. The correct colour factors due to averaging over the initial and summing over the final colour states are included in the listed parton cross sections. They however do not include the summation over the possible quark flavour indices (indices in Eq.2.14) as is needed for the subprocesses (3),(6) for instance. All quark rest masses are assumed to vanish: $m_q = 0$. Instead of repeating the exact expressions, we give their characteristics in the large energy and small (and analogously backward) angle limit. Some of the order-$g^2$ Feynman diagrams contributing to the above processes involve $t_0$ (or $u_0$)- channel quantum number exchanges. For such graphs, the parton cross sections behave according to the exchanged spin quantum numbers. In the large $s_0$ and small $t_0$ (and similarly $u_0$) regime, they show the following characteristics:

$$
\frac{d\sigma_0}{dt_0} = \left( \frac{1}{t_0^2}, \frac{1}{(t_0s_0)}, \frac{1}{s_0^2} \right) \quad (2.15)
$$

where the exchange of a gluon, quark or no quantum number at all is assumed.

We have numerically evaluated the differential distributions and the size of the integrated parton cross sections. In Fig.2.2a we show the differential cross section: $d\sigma_0/dt_0$ at $\sqrt{s_0}=300$ GeV as a function of $t_0$, where $\alpha_s=0.3$ was assumed. The indicated values of $\cos\theta_{cm}$, where $\theta_{cm}$ is the center-of-mass forward-angle between the in- and out-going partons, orient the reader on the CM angular regions. The maximal and minimal values of $t_{01}$ are $s_0$ and $0$. In Fig.2.2b we introduce the scale dependence $\alpha_s(Q^2)$ where the scale $Q^2$ is determined by Eq.2.11 and the scale parameter is set to $\Lambda=0.5$ GeV. When going from the curves a) to those in b) we notice a suppression due to $\alpha_s(Q^2)$. At small (medium) $t_{01}$ values it is a factor $-0.4$ ($-0.14$). Note the $t_0-u_0$ symmetry of Eq.2.11 which gives similar $Q^2$ values in the low and very large $t_{01}$ range. $Q^2$ is biggest in the medium $t_{01}$ region which explains the stronger suppression. These changes are obviously the same for all subprocesses since they result from $\alpha_s(Q^2)$. Defining $Q^2$ by Eq.2.12 leads to a change of (at most) a factor 2, whereas almost no cross section difference is observed if $Q^2$ is fixed by Eq.2.13 instead of Eq.2.11. The subprocesses (1,4,7) lead to an asymmetric angular distribution whereas all other subprocesses reveal a $t_0-u_0$ symmetric angular dependence. The reason lies in the contributing Feynman diagrams. Note the predominance of $gg(8)$ and $qq(7)$ scattering due to the large colour
factor. The $qq(1)$ and $q\bar{q}(4)$ subprocesses are in the small angular region roughly a factor $-4$ below, whereby these results do not account for the production of several flavours; this is taken into account when the momentum distributions are considered. The results are obtained by averaging over the initial and summation over the final colour states. The forward (and backward) peaking of the angular distributions results from the spin-1/2 and spin-1 exchanges which diverge differently as $t_0$ (or $u_0$) vanishes.

These exchanges are also at the origin of the different energy dependences of the eight subprocesses in Eq.2.14. They involve $t_0$-channel gluon exchanges, or $t_0$-channel quark exchanges, or no $t_0$ (or $u_0$)-channel exchanges at all; there is no mixture between gluon and quark exchanges. The subprocess (3):$q_1^+q_2^-q_2^--q_1^-$, for instance, has no $t_0$-pole contribution, whereas the subprocess (2):$q_1^+q_2^-q_1^--q_2^-$ involves only a $t_0$-channel gluon exchange. In the subprocess (5):$q^+q^-g^+g^-$ there is only a $t_0$-channel quark exchange (see Fig.2.1). At large $s_0$ values the differential cross section behaves as

$$\frac{d\sigma_0}{dt_0} \sim (s_0)^0, (s_0)^{-1}, (s_0)^{-2}$$ \hspace{1cm} (2.16)

corresponding to the set of subprocesses: $1, 2, 4, 7, 8$, $(5, 6)$, (3). The relative size of the eight subprocesses as well as their $t_0$-dependence now becomes obvious. The subprocess (3), for example, has no $t_0$-pole contribution, and for large $s_0$ it decreases as $(s_0)^{-2}$. Since a $t_0$-channel quark exchange contributes to the subprocess (6), this process strongly rises as $t_0$ (or $u_0$) $\rightarrow 0$, and for large $s_0$ values the parton cross section falls as $-(s_0)^{-1}$. Similar arguments apply to the subprocess (5). As $s_0$ gets very large, the differential cross sections of all other processes remain almost constant and their $t_0$ dependence diverges almost universally as $(t_0)^{-2}$ as a result of the $t_0$-channel gluon exchange. When comparing the size of the different subprocesses one should keep in mind that the flavour factors were ignored and, above all, that the momentum distributions (or parton luminosities) can still considerably change the relative size of the subprocesses contributing in the $p\bar{p}$ collisions. Due to the relation $t_0=(s_0/2)(1+\cos\Theta_{CM})$, the CM angular distributions of all subprocesses are practically the same as the above discussed $t_0$ dependence, apart from a change in the overall normalization.

The $t_0$ and $\cos\Theta_{CM}$ distributions give insight into the angular dependence of the parton cross sections and hopefully will provide a mean
to distinguish between different subprocesses. However, the dependence on the transverse momentum $q_t$ is, from an experimental viewpoint, of foremost interest. Using the relations

$$\frac{d\sigma_0}{dq_t} = (2q_t)(\sqrt{s_0}/2)(1/q_L) \frac{d\sigma_0}{dt_0}, \quad q_L = \sqrt{(s_0/4-q_t^2)} \quad (2.17)$$

we show in Fig.2.3 the $q_t$ dependence of the parton cross sections. The continuous (dotted) curves are obtained by using $a_s(Q^2)$ ($a_s = 0.3$). The scale $Q^2$ is given by Eq.2.11 and $\Lambda = 0.5$ GeV. As $q_t$ gets large the damping influence of $a_s(Q^2)$ becomes apparent. At $q_t = 140$ GeV $a_s(Q^2)$ depresses the cross section by $-1$ order of magnitude. The cross section rise at the upper end of the $q_t$ spectrum is a consequence of the vanishing longitudinal momentum $q_L \to 0$. $q_t^2$ gets so large that it becomes comparable to $s_0/4$. Due to the variable transformation $t_0 \to q_t$, $q_L$ appears in the denominator of Eq.2.17 and leads to the cross section divergence.

We have analyzed the sensitivity of the $q_t$ distribution on changes in $\Lambda$ and the scale $Q^2$. For demonstration purposes we have chosen the curves (1,2,4) and the curve (6); the subsequent insight however applies to all curves in the same way. The shaded region of the (1,2,4) curves indicates the (little) cross section changes as the scale parameter varies within the region $0.3 \text{ GeV} < \Lambda < 0.5 \text{ GeV}$. The shaded area of curve 6 illustrates the cross section variation if the $Q^2$ definition is changed from Eq.2.11 (upper curve) to Eq.2.12 (lower curve); the $Q^2$ definitions Eq.2.11 and Eq.2.13 lead to almost identical results. We maintain that around $q_t = (20,60,120)$ GeV there is an uncertainty of a factor $\sim (5,2,1.2)$ and conclude that the choice of the scale $Q^2$ introduces a considerable uncertainty in the lower $q_t$-region. Note again the several orders of magnitude suppression of the subprocesses (3,5,6) in the lower $q_t$ region. This feature was already observed in Figs.2.1. As $q_t$ gets large we reach the $\Theta_{CM} = 90^\circ$ region where the difference between this set and all other subprocesses is smaller, since the angular distribution of the (3,5,6) subprocesses is flatter.

We have determined the energy ($\sqrt{s_0}$) dependence of the integrated parton cross section $\sigma_0(s_0, p_t)$ by integrating Eq.2.17 over $q_t$ within the boundaries $p_t \leq q_t < \sqrt{s_0}/2$. The lower transverse momentum cutoff was chosen at $p_t = 30$ or 60 GeV. The curves in Fig.2.4a show the energy dependence of the eight subprocesses in Eq.2.14 where we have assumed that $a_s = 0.3$. After the initial threshold onset, we notice that there is an (almost) constant $\sqrt{s_0}$ dependence of all those subprocesses which involve a $t_0$ (or $u_0$) gluon exchange. The subprocesses (5,6) receive contributions from the $t_0$-channel quark (but not gluon) exchanges leading to the $(s_0)^{-1}$ decrease,
whereas the subprocess (3) decreases as \((s_0)^{-2}\) since there is no \(t_0\) (and \(u_0\))-exchange at all. The predominance of \(gg(8)\) scattering, -it is a factor -5 above its closest competitors (2,7) - , is explained by the large colour factor. In Fig.2.4b we introduce \(\alpha_s(Q^2)\) and vary the scale parameter \(\Lambda\). We notice a factor -4 suppression from the running coupling constant. Since the integrated cross section is mostly composed of the \(q_t\) area just above the (fixed) cutoff \(p_t\), the numerical value of \(p_t\) determines the scale in \(Q^2\). The reduction factor is therefore practically \(\sqrt{s_0}\) independent. By increasing \(p_t\) we shift the threshold onset \(\sqrt{s_0}(2p_t)\) and reduce the size of the integrated cross section. Fig.2.4b shows also the cross section change as the scale parameter varies within: 0.2 GeV\(\leq\Lambda\leq0.5\) GeV (shaded areas). All features remain practically unchanged between the \(p_t=30\) GeV and 60 GeV curves, apart from the later threshold onset.

In the preceding figures we have focused on the \(\sqrt{s_0}\) energy dependence of the integrated parton cross sections, and we now consider the integrated \(p_t\) spectrum. We choose a fixed energy, typically \(\sqrt{s_0}=300\) GeV, and vary the lower transverse momentum cutoff \(p_t\). In Fig.2.5a we show the cross section decrease of all eight subprocesses assuming \(\alpha_s=0.3\) (dotted curves) and \(\alpha_s(Q^2)\) (solid curves) with \(Q^2\) defined by Eq.2.11 and \(\Lambda=0.5\) GeV. One notices three distinct groups of curves due to the subprocesses (1,2,4,7,8), (3,5) and (6). At small \(p_t\) values the first group reveals a large cross section with a rapid fall-off for growing \(p_t\) values. The fall-off of the second group is more moderate whereas the subprocess (6) shows a flat \(p_t\) dependence. The reason for these characteristics can be traced back to the \(t_0\) (or \(u_0\))-channel exchanges as explained above. The straight-line and parallel \(p_t\) fall-off of the first group of subprocesses is therefore a consequence of the \(t_0\) (or \(u_0\)) channel gluon exchanges. If \(\sqrt{s_0}\) were given larger values, the second and third group of subprocesses would be, even at larger \(p_t\) values, of considerably less significance.

In Fig.2.5b we focus, once more, on the \(Q^2\) and \(\Lambda\) choice. We have selected two representative subprocesses and show the sensitivity of the integrated \(p_t\) spectrum for the \(Q^2\) choices of Eqs.2.11-2.13; these results may then be compared with the analogous curves where \(\alpha_s=0.3\) was assumed. We similarly have indicated the (small) cross section changes if the scale parameter is varied in the region: 0.2 GeV\(\leq\Lambda\leq0.5\) GeV (shaded regions). All these results are obviously a reflection of the differential cross section \(d\sigma/dq_t\) characteristics shown in Fig.2.3.

We summarize this section as follows: We have introduced the differential and the integrated jet-jet cross sections, and we have discussed five parametrizations for the scale dependent momentum distributions. The un-
certainty of the scale $Q^2$ appears mostly at transverse momenta below 20 GeV. The uncertainty of the scale parameter $\Lambda$ appears at low $Q^2$ via $\alpha_s(Q^2)$ and at large $Q^2$, to a significant degree, via the momentum distributions.

We have analyzed the differential and the integrated parton cross sections of the eight quark/gluon subprocesses listed in Eq.2.14. Their relative size is to a considerable degree a reflection of the contributing $t_0$(or $u_0$)-channel gluon-, quark-, or no-exchanges. They determine the energy dependence of the differential and the integrated parton cross sections, and they control the (non)divergence of the (differential) parton cross sections in the small $t_0$(or $u_0$) or small $q_t,p_t$ kinematical regions. The colour quantum numbers are furthermore responsible for the predominance of the $gg$ and $qg$ scattering processes; the $qq$ and $qg$ parton cross sections are a factor $\approx 3-4$ below. The subprocesses $(3,5,6)$ in Eq.2.14 contribute little.

3. HADRONIC JET-JET PRODUCTION

In the preceding section we have specified our calculation framework. In this section we fold the parton cross sections with the momentum distributions and analyze the eight constituent processes contributing to the integrated cross section of $p\bar{p}$ jet-jet production. The influence of kinematical cuts on the transverse momentum, the rapidity and the jet-jet invariant mass distributions are determined.

Fig.3.1 shows the energy dependence of the fully integrated jet-jet cross section for each of the eight constituent processes listed in Eq.2.14. The calculation was performed using the BGOR [8] parametrization with $Q^2$ determined by Eq.2.11, $\Lambda=0.5$ GeV, and $p_t=10$ GeV. At $\sqrt{s}=540$ GeV the $qg(7)$ and $gg(8)$ subprocesses dominate by a factor $\approx 5$ over $qq(1)$ and by an order of magnitude over $qg(4)$ scattering. As we go to $\sqrt{s}=20$ TeV the $gg(8),qg(7)$ subprocesses continue to grow due to the strongly rising gluon distribution at low $x$-values. The cross section for $qq(1)$ scattering is here an order of magnitude smaller and the subprocesses $(2,4)$ are again a factor $\approx 4$ below. The small influence of the subprocesses $(3,5,6)$, as already noted on the level of the parton cross sections, is here unchanged. The constituent cross sections $(7)$ and $(8)$ in Fig.2.4b differ by almost an order of magnitude whereas their jet-jet cross sections are of the same size. The reason lies in the momentum distributions. In $qg(7)$ scattering a gluon in one nucleon interacts with all valence and sea quarks of the other
nucleon, which adds up to a significant contribution as compared to gg(8) scattering. One should further keep in mind that the QCD corrections increase the sea-quark distributions strongly at small x values of $x_t^{-10^{-2}}-10^{-3}$. For $pp$ energies around $\sqrt{s}=540$ GeV with $x_t^{-10^{-2}}-10^{-2}$, the contribution of the gluon structure function in gg(8) scattering is considerably reduced, whereas the valence component on the quark side in qg(7) scattering is enforced. This latter feature is at the origin of the qg(7) over gg(8) predominance. The opposite applies to the qq(1) and q$\bar{q}$(4) subprocesses which differ by a factor $\sim 2-5$ at $\sqrt{s}=20$ TeV due to the predominance of sea-quark scattering in the former process as compared to valence-quark annihilation in the latter process. As one reaches the 20 TeV energy region all jet-jet cross sections rise logarithmically in almost the same way.

The uncertainties in the low x region affect the predictions in the TeV region. At $\sqrt{s}=20$ TeV, typically $x_t \approx (2p_t/\sqrt{s}) \cdot 10^{-3}$, which is an order of magnitude below the x range of applicability of the scale dependent parametrizations of the momentum distributions. Since at small x values the input distributions behave as $-x^{-\alpha}$, we do not expect any pathologies apart from a possibly incomplete inversion of the Mellin transformation. Such short comings should then manifest themselves in a transparent way for the different subprocesses and for the different parametrizations we are considering. We therefore analyze the energy dependence of the three typical subprocesses (1,7,8) by varying the QCD parameters. In Fig.3.2a, we show the cross section changes for the previously introduced five parametrizations of the scale dependent momentum distributions (see section 2). The curves (a-e) illustrate in particular the cross section sensitivity on their input x dependence. We first consider the gg(8) subprocess. At $\sqrt{s}=20$ TeV the cross section spread is around a factor 2 if the BETAL [12] parametrization (curve c) is ignored; its gluon distribution is known to suffer from an incomplete Mellin inversion which usually manifests itself in the low x region. At $\sqrt{s}=540$ GeV the cross section variation is still around a factor 2. At $\sqrt{s}=100$ GeV the 'hard' parametrizations lie almost an order of magnitude above the 'soft' parametrizations. The analogous cross section curve for the qq(1) subprocess, which is mostly sensitive to the sea distribution, shows remarkably little cross section spread, apart from the BGOR [10] parametrization (curve a) which is distinct by its 'soft' but influential sea. Using the BGOR [10] parametrization, we vary in Fig.3.2b the scale parameter in the range: 0.1 GeV $\Lambda$ 0.7 GeV, and notice at extreme energies a cross section change of about one order of magnitude. A change in $\Lambda$ can be interpreted as a redefinition of the scale. One
therefore may infer from these results some idea of the influence of a $Q^2$
change, and in turn of the next-to-leading corrections. Most other parame-
trizations give similar results. In Fig.3.2c we switch off all QCD correc-
tions and set $\alpha_s=0.3$. The observed variations at lower and extreme ener-
gies are considerable. It is interesting to notice that the QCD corrections
diminish the discrepancies. Since $p_t$ and in turn $Q^2$ are relatively small we
may not expect a big difference between the scale dependent and the scale
independent cross sections. The above analyses were done with $p_t=10$ GeV.
At extreme energies this $p_t$ (cutoff) value is quite small. By choosing lager $p_t$
values the cross section spread does not increase since $gg$(8) and
$qg$(7) scattering, which introduce most of the uncertainties, loose their
influence with growing $q_t$ against the $qq$(1) and $q\bar{q}$(4) subprocesses. From
the Figs.3.2 we thus conclude that the jet-jet cross section extrapolations
to extreme energies are uncertain to about one order of magnitude.

With these uncertainties in mind we now analyze the influence of a lower
transverse momentum cutoff $p_t$, of an upper cutoff in the rapidity variable
$y_{cut'}$ and of a lower cutoff in the jet-jet invariant mass $M_{cut'}$ on the cross
section. In Fig.3.3, we show the integrated $p_t$ spectrum at $\sqrt{s}=(0.54,2,20)$
TeV for the most important subprocesses. There is no cutoff in the rapidi-
ties nor in the invariant mass. Focusing on the $\sqrt{s}=0.54$ TeV curves, we
notice that as $p_t$ grows there is an enormous loss in rate. In the region
$p_t\lessgtr 30$ GeV, the cross section is dominated by $gg$(8) and $qg$(7) scattering,
whereas in the region $p_t\lesssim 80$ GeV the $qq$(1) and $q\bar{q}$(4) subprocesses become
most influential. The strong fall-off of $gg$(8) scattering as compared to the
other contributions is due to the 'soft' gluon distribution in the BGOR [10]
parametrization. In the same figure we show the analogous curves at $\sqrt{s}=(2,
20)$ TeV where the fall-off of the most influential subprocesses (1,4,7,8) is
much smaller. The $Q^2$ values are almost unchanged, but the scale of the
transvers scaling variable $x_t=(2p_t/\sqrt{s})$ is considerably stretched. As a
result, much larger $p_t$ values are needed for an appropriate comparison. At
$p_t=60$ GeV, the cross section values, corresponding to the three analyzed
energies, are $\sigma(p_t=30 \text{ GeV})=(2.10^{-33}, 3.10^{-31}, 3.10^{-29}) \text{ cm}^2$. We notice a
gain in rate of roughly two orders of magnitude as one increases the CM
energy from 540 GeV to 2 and 20 TeV. Whereas at $\sqrt{s}=540$ GeV the gluon
and quark scattering processes are of comparable size, we notice at 2 TeV
a predominance of $qg$(7) and to a lesser degree $gg$(8) scattering, and at
$\sqrt{s}=20$ TeV the latter two processes contribute at equal rates.

We now fix $p_t$ and impose an upper cutoff on the jet rapidities:
$y_1,y_2 \lesssim y_{cut'}$. For sufficiently small $y_{cut}$ the cross sections are reduced with
a reduction factor which is shown in Fig.3.4. In Fig.3.4a we vary
$p_t=(10-40)$ GeV and show the $y_{\text{cut}}$ dependence of the reduction factor of two typical subprocesses. The $gg(8)$ subprocess is less suppressed as compared to $qq(1)$ scattering since its rapidity distribution decreases faster with growing $y_1,y_2$ values. An upper rapidity cut therefore introduces changes only at relatively low $y_{\text{cut}}$ values. As $p_t$ diminishes, the $y_{\text{cut}}$ reduction extends to larger values, since smaller $p_t$ (and in turn $q_t$) values allow for a more extended rapidity range. The $y_{\text{cut}}$ reduction is therefore felt at larger values. In Fig.3.4b, we analyze the cross section reduction as experienced by the constituent processes $(1,4,7,8)$. At $p_t=10$ GeV the $qq(4)$ scattering obviously extends farthest into the large rapidity range, whereas at $p_t=40$ GeV (leaving $(8)$ aside) there is little difference among the subprocesses. We have carried out the analogous analyses at $\sqrt{s}=2$ and 20 TeV and present our results in Figs.3.5 and 3.6. The $y_{\text{cut}}$ values used here are larger since the increased CM-energy manifests itself in an extension of the rapidity ranges. At $\sqrt{s}=(0.54,2,20)$ TeV the largest rapidity cutoff values are $y_{\text{cut}}=(3,4,6)$. Choosing a fixed $p_t$ value, we have also placed a lower cutoff on the jet-jet invariant mass: $M_{\text{cut}}<M$, without any further phase space restrictions. For sufficiently large $M_{\text{cut}}$ the cross sections are reduced. The reduction factors at $\sqrt{s}=0.54$ TeV $pp$ energy are shown in Fig.3.7. In Fig.3.7a, we display their $M_{\text{cut}}$ dependence for each of the eight subprocesses. The $q\bar{q}(4)$ subprocess experiences the smallest reduction whereas $gg(8)$ scattering shows a gradually increasing suppression with $M_{\text{cut}}$. The reason lies in the flatter (steeper) $M$ distribution of the former (latter) subprocess. Increasing $p_t$ leads, for growing $M_{\text{cut}}$ values, to a later and slower cross section reduction. In Fig.3.7b, we give details on this feature limiting ourselves to the most important subprocesses in the range $p_t=(10-40)$ GeV. The curves belonging to the same $p_t$ are, for the sake of clarity, connected by a perpendicular line at the high mass end.

We similarly have carried out such an analysis at $\sqrt{s}=2$ TeV and 20 TeV $pp$ energy, and we present in Fig.3.8 and Fig.3.9 the analogous curves. Since the available energy is now substantially increased, the $M_{\text{cut}}$ values are also larger, and the fall-off of the curves is slower. Note that at $\sqrt{s}=20$ TeV the gluon subprocesses $(7,8)$ and the quark subprocesses $(1,4)$ all experience roughly the same reduction. The reason is obvious, $M_{\text{cut}}\approx 600$ GeV is, for this energy, a rather small value.

We summarize the findings of this section. We have determined the fully integrated $pp$ cross section and evaluated the relative size of the eight subprocesses up to extreme energies. By varying the QCD parameters we have found that the uncertainties of the leading-log predictions at extreme
energies are below one order of magnitude. The influence of the next-to-leading contributions should, however, not be forgotten. We have evaluated the integrated $p_T$ spectrum. We have determined the cross section reduction due to an upper cut in the rapidities $\gamma_{cut}$. And finally, we have analyzed the reducing influence of a lower cut in the jet-jet invariant mass, $M_{cut}$. All these investigations were carried out at the $p\bar{p}$ energies: $\sqrt{s}=(0.54,2,20)$ TeV.

4. SINGLE JET DISTRIBUTIONS

This section aims at a study of the single-jet distributions of hadronic jet-jet production. The dependence on the jet transverse momentum and the jet rapidity are is shown, and the cross sections for quark and gluon jet production are compared.

We first focus on the transverse momentum dependence of the single jet cross section without regard to the jet quantum numbers. In Fig.4.1, we show the differential cross section $d\sigma/(dydq_t)$ at $y=0$ with the $p\bar{p}$ energy at $\sqrt{s}=(0.54,2,20)$ TeV in the diagrams a)-c). The momentum distributions are specified by the BGOR [10] parametrization with $Q^2$ defined in Eq.2.11 and $\Lambda=0.5$ GeV. We first concentrate on Fig.4.1a where $\sqrt{s}=0.54$ TeV. The fat and fine solid lines indicate the total sum (SUM) as well as the individual contributions (1-8) of the subprocesses in Eq.2.14. The cross section rapidly decreases with increasing $q_t$. At $q_t=(20,100)$ GeV we typically find $d\sigma/dq_t=4\times10^{-3},2.5\times10^{-3}$ cm$^2$/GeV. At lower transverse momentum values ($q_t<60$ GeV), the gluon subprocesses (7),(8) dominate, whereas at larger $q_t$ values ($q_t>60$ GeV) the constituent processes involving quarks only become predominant. The rapid decrease of $gg(8)$ scattering is due to the strong fall-off of the gluon x-distribution. In the same figure we include two additional curves. The dashed curve indicates the cross section (SUM) behaviour in the absence of any scale dependence in the momentum distributions with $Q_s=0.3$. At $q_t=100$ GeV the difference between the scale independent and the scale dependent curves is almost three orders of magnitude. The dashed-dotted curve indicates the gain in rate if $q_t$-integration of the curve (SUM) within the limits $p_T, q_t, \sqrt{s}/2$ is carried out. The lower cutoff $p_T$ is varied as a free parameter as indicated near the horizontal axis. The shown units on the vertical axis obviously have to be replaced by $d\sigma/dy$ [cm$^2$].

In Fig.4.1b,c we show the analogous distributions at $\sqrt{s}=2,20$ TeV. The general effect of increasing the $p\bar{p}$ energy is to increase the cross section
and to flatten the $q_t$ distribution. At $\sqrt{s}=0.54$ TeV the cross section size of $d\sigma=10^{-36}$ cm$^2$/GeV is reached around $q_t=100$ GeV whereas at $\sqrt{s}=2(20)$ TeV this occurs at $q_t=250(750)$ GeV. Let us similarly compare at these $p\bar{p}$ energies the cross section values in the $q_t$-region where quark jet production (subprocesses (1,4)) dominates. At $\sqrt{s}=0.54$ TeV we notice still appreciable rates whereas at $\sqrt{s}=2$ TeV, and even more so at $\sqrt{s}=20$ TeV, they are substantially suppressed. Note, as the $p\bar{p}$ energy increases the difference between the fat solid and the dashed curves (SUM), at a fixed $q_t$ value, diminishes. This feature is partially due to the changed $x_t$ scale (resulting in different values for $x_t^*$), and it partially follows from using $\alpha_s=0.3$ in the "scaling" curves.

In order to show the importance of the QCD corrections, we have plotted in Fig.4.2 the distribution $q_t^{-3}d\sigma/dq_t$ versus the transverse scaling variable $x_t=(2q_t/\sqrt{s})$. The dashed curve is evaluated in the absence of any scale, and it therefore must remain unchanged if $\sqrt{s}$ is varied. The solid curves, evaluated at the $p\bar{p}$ energies $\sqrt{s}=(0.063,0.54,0.62,2,20)$ TeV, involve scale dependence. Their deviations from the "scaling" curve therefore result from the QCD corrections. When going from 0.54 TeV $p\bar{p}$ energy to 2 TeV, the solid curves in Fig.4.2 are almost uniformly suppressed by a factor $\approx 2$ as a result of the $Q^2$ dependence in the running coupling constant and in the structure functions. We infer that the substantial rise of the distributions in Fig.4.1b,c as compared to those in Fig.4.1a is mainly due to a shift of $x_t$ to lower values.

The curves in Fig.4.1 and Fig.4.2 were obtained for a fixed set of QCD parameters. We now consider the sensitivity of these distributions to changes in the QCD parametrization. We limit this study to the presently accessible energy region of $\sqrt{s}=0.54$ TeV, and we reevaluate the single-jet $q_t$ distribution using the previously introduced five parametrizations for the scale dependent momentum distributions (see section 2). The scale parameter is in the range: 0.1 GeV$\ll\lambda<0.7$ GeV. In Fig.4.3 we show the cross section spread in the sum of all contributions (SUM), as well as in $gg(8)$ and $qq(1)$ scattering. Note the changes in the vertical scale which are indicated by its slight side displacement. We first focus on the subprocess (8): $gg\rightarrow gg$. At large $q_t$ values, the BETAL [12] parametrization (curve c) with its relatively 'hard' gluon distribution shows a significantly larger cross section than all other parametrizations. The 'soft' BGOR [10] parametrization (curve a), on the other hand, gives the most conservative estimate. At $q_t=100$ GeV the predictions vary by about 1-2 orders of magnitude between the different parametrizations. Reliable measurements at $\sqrt{s}=0.54$ TeV could, in principle, help to fix the gluon distribution. A major
difficulty however comes from the fact that in the large $q_t$ range the measured cross section is likely to be no longer gluon dominated (see above) since the $qq(1)$ and $q\bar{q}(4)$ subprocesses become gradually predominant. Note the considerable cross section spread due to the $\Lambda$ variation. In Fig.4.3 we also show the cross section spread of the subprocess (1):$q_1\ast q_2 - q_1\ast q_2$ for different momentum distributions and $\Lambda$ values. Apart from the CDHS [14] parametrization, there is remarkably little change when going from one parametrization to another, whereas the $\Lambda$ variation is more important. The set of curves at the top of Fig.4.3 (SUM) indicates the cross section spread in the sum of all contributions. Ignoring the BETAL [12] curve c, we find fairly small cross section changes which are explained by the $qq(1)$ and $q\bar{q}(4)$ predominance at large $q_t$ values.

In the preceding analyses we have varied the transverse momentum at $y=0$. By fixing $q_t$ at a few selected values, we now analyze the rapidity dependence of the single-jet distribution. We initially keep the $p\bar{p}$ energy at $\sqrt{s}=0.54$ TeV and use the BGOR [10] parametrization with $Q^2$ defined in Eq.2.11 and $\Lambda=0.5$ GeV. In Fig.4.4a we fix $q_t=30$ GeV and we show the rapidity $y_1$ dependence of all constituent processes (1-8), as well as the sum of all contributions (SUM). In the range $-2\leq y_1 \leq 2$ the cross section is almost flat, but beyond these values it falls off drastically. The rapid decrease of the $gg(8)$ subprocess is again due to the 'soft' gluon distribution. The large rapidity region is dominated by $qg(7)$ scattering. The reason for this features will be given later in section 5. The dashed curve in Fig.4.4a, involving the sum of all contributions, results from a calculation without any higher order QCD corrections and $a_s=0.3$. It is almost parallel to the analogous curve involving QCD corrections. In Fig.4.4b we present the rapidity dependence of $d\sigma/(dydq_t^2)$ (solid curves) at increasing transverse momentum values, whereby the shrinking of the rapidity range becomes transparent. We have performed $q_t$ integration of the above distribution in the range $p_t \leq q_t \leq \sqrt{s}/2$. The dashed curves in Fig.4.4b thus indicate the rapidity dependence of $d\sigma/dy$ [cm$^{-2}$] whereby the lower cutoff $p_t$ was chosen at similar values as $q_t$. In Fig.4.4c we show the cross section variation for different choices of the scale dependent momentum distributions. The shaded regions result from varying the scale parameter in the BGOR [10] parametrization. As we change from one parametrization to another, the discrepancy gets smaller in the extended rapidity range, which is a consequence of the reduced influence of the $qg(7)$ subprocess.

We carried out the same analysis at large and extreme $p\bar{p}$ energies: $\sqrt{s}=2.20$ TeV, and we present in Fig.4.5a,b the single-jet rapidity distribution (solid curves) at selected $q_t$ values. Similarly, we indicate the size of
the $q_t$ integrated rapidity distribution $d\sigma/dy$ [cm$^2$] (dashed curves) which is evaluated and presented as described above.

When evaluating the preceding results we did not constrain the phase space integration region of the second jet. Since some detectors are limited in their jet rapidity acceptance, we also investigate the influence of an upper rapidity cutoff on the (thus far fully integrated) second jet. Furthermore, we carry out $q_t$ integration within the limits $p_t \leq q_t \leq \sqrt{s}/2$ where the lower transverse momentum cutoff $p_t$ is varied as a parameter. We thus consider

$$\frac{\sqrt{s}}{2} d\sigma/dy_1[y_2\text{-cut}] \equiv \int dq_t \int dy_2 \left[ d\sigma/(dy_1dy_2dq_t) \right]$$

(4.1)

In Fig.4.6a we choose in particular $|y_2| \leq 1$ and we show the ratio of the $y_2$ constrained cross section to the $y_2$ unconstrained cross section whereby the rapidity $y_1$ (on the horizontal axis) is varied freely. The curves are evaluated for several $p_t$ values in the range: $p_t = 10-50$ GeV. In Fig.4.6b, instead, we set $y_1 = 0$ and use the rapidity cutoff of the second jet $y_{\text{cut}}$ as the free variable, whereby the parameter $p_t$ is given different values.

So far no distinction between quark and gluon jets was made. In the following we therefore determine the single-jet cross section $d\sigma/(dydq_t)_{|y=0}$ for gluon jets and quark jets of different flavours. The solid lines in Fig.4.7 show their fall-off with increasing transverse momentum values at $y=0$. In the region $q_t \leq 60$ GeV, the gluon jets dominate, whereas at large $q_t$ values, the sum of all quark jets is predominant. The difference between the gluon jets and the sum of all quark jets is in both regions not very drastic, a factor $2-5$ at most. At larger $q_t$ values, the cross sections for the various flavours obey the inequality: $g \geq u(=\bar{u}) \geq d(=\bar{d}) \geq s(=\bar{s}) \geq c(=\bar{c})$. We have carried out $q_t$ integration within the limits $p_t \leq q_t \leq \sqrt{s}/2$. The dotted curves in Fig.4.7 indicate $d\sigma/dy$ [cm$^2$] where the cutoff $p_t$, with its values indicated near the horizontal axis, is varied as a free variable. At larger $q_t, p_t$ values, an increase of one order of magnitude is evidenced as $q_t$ integration is carried out.

The fraction of each jet type is shown in Fig.4.8a. The presented curves correspond to the solid lines of Fig.4.7 whereby the relative frequency of jets with a specific flavour is compared to the importance of the gluon jets. The same results are shown in Fig.4.8b with different scale dependent momentum distributions being used. We infer that these predic-
tions vary significantly as we change from one parametrization to another. The changes due to \( \Lambda \) variation are negligible.

In Fig.4.7 and 4.8 we have fixed the rapidity at \( y=0 \), and we have analyzed the \( q_t \) dependence. We now switch variables fixing \( q_t=30 \) GeV and varying the rapidity. The result is shown in Fig.4.9 where, at first, the dotted lines should be ignored. The solid lines illustrate the rapidity dependence of the cross section \( d\sigma/(dydq_t) \) and correspond to the solid curves in Fig.4.7. The particular jet type is indicated near the curves. Note the predominance of the gluon jets in the central region and the importance of the quark jets beyond \( |y|>1.5 \). The positive (negative) rapidity values correspond to longitudinal momenta in the flight direction of the incident proton (antiproton). The cross sections for \( u,d \)-jet production are asymmetrical in the rapidity variable, whereas those for gluon and \( s,c \)-quark jets are symmetrical. The reason lies in the valence-quark contribution to \( qq(7) \ldots \) scattering which becomes more influential as the rapidity \( y \), and in turn the corresponding momentum fraction \( x_i \) gets larger. We now pay attention to the dotted curves. They result from \( q_t \) integration within the limits \( p_t<q_t<\sqrt{s}/2 \) leading to \( da/dy \) [cm\(^2\)]. The dotted curves in Fig.4.9 show the rapidity dependence of this cross section, with \( p_t=30 \) GeV being used. The scale on the vertical axis also applies to this distribution, but the indicated cross section and the units are obviously different.

In Fig.4.10 we show the percentage of the gluon contribution and of the various quark flavours as a function of the rapidity \( y \). The curves presented in Fig.4.10a correspond to the solid lines in Fig.4.9 with \( q_t=30 \) GeV. Note in particular the dominance of the \( u \)-jet (\( \bar{u} \)-jet) for \( y>2 \) (\( y<-2 \)). In Fig.4.10b we analyze this feature by selecting different transverse momentum values in the range: \( q_t=10-90 \) GeV. The gradually increasing fraction (with growing \( q_t \)) of the quark jets as compared to the gluon jets becomes apparent. These results depend quite sensitively on the chosen scale-dependent momentum distributions. The variations are indicated in Fig.4.10c. At large jet rapidities the BGOR [10] parametrization under rates the importance of the gluon contribution as compared to the BETAL [12] parametrization which gives higher cross section values.

Summarizing this section, we have determined the inclusive single-jet transverse momentum and rapidity dependence at \( \sqrt{s}=(0.54, 2, 20) \) TeV. At \( \sqrt{s}=0.54 \) TeV the cross section rapidly decreases for small \( q_t \) and flattens out for larger \( q_t \) values. In the former (latter) region, it is dominated by the gluon (quark) scattering processes. The QCD uncertainties are below a factor \( \sim 5 \). In the extended rapidity range, the \( qg \) subprocess dominates.
The relative contribution of the gluon and quark jets of different flavours, as a function of the transverse momentum and the rapidity (as well as of the QCD parameters), were determined.

5. TWO JET DISTRIBUTIONS

We have so far evaluated the cross section characteristics by integrating over one or several of the kinematical variables of the differential cross section in Eq.2.1. We now analyze \( d\sigma/(d\gamma_1 dy_2 dq_t) \).

In Fig.5.1a we show the rapidity correlations at selected transverse momentum values. We vary the rapidity of the first jet and define the rapidity of the second jet as: \( \gamma_2=0 \) or \( \gamma_2=-\gamma_1 \) or \( \gamma_2=+\gamma_1 \). In the \( \gamma_1,\gamma_2 \) rapidity plane this corresponds to three straight lines which pass through the origin at the angles \( 0,-\pi/4,+\pi/4 \). The corresponding distributions in Fig.5.1a (solid curves a,b,c) are shown for a set of \( q_t \) values. The \( \gamma_2=0 \) curves (labelled a) extend to larger rapidity values, whereas the \( \gamma_2=+\gamma_1 \) curves (labelled c) fall much more steeply at large \( \gamma_1 \). The \( \gamma_2=-\gamma_1 \) curves lie between the two extremes. This pattern repeats itself at each \( q_t \) value.

Before we discuss these characteristics, we show in Fig.5.1b the \( q_t \) dependence in the central region: \( \gamma_2=\gamma_2=0 \). The fat curve 4 involves the usual leading-log QCD corrections. The scale \( Q^2 \) is given by Eq.2.11, and the BGOR [10] parametrization for the momentum distributions is used. This curve should be compared with the analogous \( q_t \) distribution for single-jet production in Fig.4.1a (fat curve: SUM). With this distribution we also demonstrate the cross section dependence on the scale \( Q^2 \) via \( \alpha_s(Q^2) \) and the scale dependent momentum distributions. The curve 1 (in Fig.5.1b) is calculated with \( \alpha_s=0.3 \) and scale independent momentum distributions (at \( Q^2=1.8 \text{ GeV}^2 \) for BGOR). It represents the well-known scaling behaviour: \( (1/q_t)^4 f(x_t) \). Introducing the running coupling constant \( \alpha_s(Q^2) \) reduces the cross section at large \( q_t \) values by a factor \( \leq 6 \) (curve 2), whereas at smaller \( q_t \) values the reduction is smaller. The strongest cross section suppression results from the scale dependence of the momentum distributions. When going from curve 1 to curve 3 there is a suppression of a factor \( -1/20 \) at \( q_t=100 \text{ GeV} \); at lower \( q_t \) values the suppression is substantially smaller.

We now return to Fig.5.1a. Choosing the fixed values \( q_t=(15,30) \text{ GeV} \), we show in Fig.5.2 the connection between the rapidities and the momentum fractions: \( \gamma_1,\gamma_2=x_1,x_2 \) as it is defined in Eq.2.2. On the horizontal axis we vary the rapidity \( \gamma_1 \), and on the vertical axis we show the momentum
fraction $x_1$. The $y_2$ values are parameters of the curves. The analogous curves for $x_2$ are obtained by replacing: $y_1, y_2 \to -y_1, -y_2$. Since the rapidity is defined as $y = (1/2) \ln (E+q_T)/(E-q_T)$ the definition of the z-axis fixes its sign via the sign of $q_T$. The spatial z-axis was defined in section 2 by the flight direction of the incoming proton. Large positive rapidities therefore correspond to large longitudinal momenta $q_T$ parallel to the incident proton momentum. $x_1$ was defined as the parton momentum fraction in the proton, and similarly $x_2$ in the antiproton. We now focus on the $q_T=30$ GeV curves and determine $x_1$ as a function of $y_1$. We increase $y_1$ from negative to positive values and fix $y_2$ by the correlation constrains of the curves $a,b,c$ in Fig.5.1a. These constrains lead to the fat curves (again denoted by $a,b,c$) in Fig.5.2. The momentum fraction $x_2$ follows from the same curves whereby the rapidities here however must be given the opposite sign. We therefore vary $y_1$ from positive to negative values, instead, and we read from the fat curves in Fig.5.2 the values for $x_2$. Whilst in case (a) (or (c)) $x_1$ grows from small (very small) values in the region $y_1<0$ to reach 1 for $y_1>0$, $x_2$ decreases from 1 to small (very small) values. The constrain (b) imposes $x_1=x_2$ for all $y_1$ values. These $x_1,x_2$ characteristics explain the different behaviours of curves $a,b,c$ in Fig.5.1a. The rapid fall-off of curve (c) is due to $x_1=1$ for $y_1=0$ (phase space limit). The large rapidity values reached by curves (a) follow from two factors. First, the phase space limit $x_1=1$ is reached later at $y_1=2.7$. Second, the kinematical region $x_1$-large, $x_2$-small is dominated by $gg(7)$ scattering where the 'medium-$x_1$' valence quarks interact with the 'soft-$x_2$' gluons and vice versa. The strong suppression of the curves (b) with respect to the curves (a) follows from the fact that at large $y_1$ ($\geq 2.7$) values, $x_1$ and $x_2$ reach the phase space limit: $x_1=x_2=1$. The suppression due to the momentum distributions is hence therefore double.

We show in Fig.5.3 the $q_T$ integrated cross section $d\sigma/(dy_1dy_2)$ with the lower cutoff $p_T=30$ GeV, and evaluate the contribution of each of the eight subprocesses (1-8) in Eq.2.14. The rapidities $y_1,y_2$ are correlated as: $y_2=y_1$ (Fig.5.3a), $y_2=-y_1$ (Fig.5.3b) and $y_2=0,y_1=\text{variable}$ (Fig.5.3c). Interchanging $y_1-y_2$ leads to no changes. In the central region the cross section is dominated by the $gg(8), qg(7)$ subprocesses. In the extended rapidity domain, closer to the phase space boundary, $qg(7)$ scattering essentially dominates in Fig.5.3a,c. The correlation constrain $y_2=-y_1$ of Fig.5.3b is particularly fruitful since for $y_2>2$, $qq(1)$ and $q\bar{q}(4)$ scattering dominate by a factor ~2-5 over the $qq(7)$ subprocess. For all rapidity correlations in Fig.5.3a, one notices a rapid decrease of $gg(8)$ scattering which is a consequence of the strong fall-off of the gluon distribution. For
other parametrizations it is, however, less pronounced. Fig.5.2 shows that
the predominance of the quark subprocesses in Fig.5.3b follows from the
large values of $x_1$ and $x_2$ as $y_1$ reaches the phase space boundary. Due to
the slower fall-off of the quark momentum distributions, the quark subpro-
cesses, as compared to the gluon scattering processes, extend to larger
rapidity values.

So far, we have been concerned with the dynamical properties of the
jet-jet correlations. We now analyze the sensitivity of the $q_t$ and the $y_1$
distributions to variations of the QCD parameters. In Fig.5.4a, we focus on
the $q_t$ fall-off at $y_1=0$. At $q_t=80$ GeV there is a 1 order of magnitude dif-
ference between the BETAL [12] and the CDHS [14] parametrizations. There
is a factor 2 spread between the other predictions with the CDHS curve
falling off more strongly and the GHR curve being rather flat. Similarly we
indicate (in the BGOR parametrization) the cross section spread for vari-
ations of the scale parameter in the range: 0.1 GeV/$\Lambda$≤0.7 GeV. At larger
$q_t$ values it leads to a factor 3 uncertainty which becomes smaller as $q_t$
becomes smaller. Similarly, the uncertainties in the $y_1$ dependence of the
$y_2=y_1$ correlation at $q_t=30$ GeV are shown in Fig.5.4b. These results are
representative for the other types of correlations. The 'hard' BETAL par-
ametrization dominates over the full rapidity range. Apart from this fact,
we note a fairly strong spread in the cross section predictions of the cen-
tral region. At large rapidities, the predictions of the different parametri-
izations are very similar. Varying the scale parameter within the above
indicated range leads to a uniform, almost rapidity independent, cross sec-
tion spread.

In the preceding sections we have evaluated the fully integrated cross
section and the single-jet cross section. We are now in the position to com-
pare the size of their kinematical distributions with those determined in this
section. In Fig. 5.5a we therefore show the $q_t$ (and $p_t$) dependence of the
differential (and the transverse momentum integrated) cross section, and
we compare their influence. We note that: (i) carrying out the $q_t$ integra-
tion leads, at larger $p_t$ (cutoff) values, to a stronger cross section
increase than at smaller $p_t$ values; (ii) integration over one rapidity vari-
able results, at $q_t=30$ GeV, in a factor 3 cross section increase, whereas
the additional increase due to integration over the second rapidity is
smaller; (iii) the gain in rate due to integration over one/both rapidities
gets smaller as we go to larger $q_t$ values. We have evaluated the analo-
gous curves at $\sqrt{s}=2.20$ TeV and show the results in Fig.5.5b,c. One not-
ices quite a strong increase in rate if integration over the rapidities/trans-
sverse momentum is carried out which is not surprising in view of the large
$p_p$ energy.
We summarize the results of this section. We have determined the rapidity and the transverse momentum dependence of $d\sigma/(dy_1dy_2dq_t^2)$ giving insight into the jet-jet rapidity correlations. We have, in particular, analyzed the three constraints: $y_2=0$, $y_2=-y_1$, $y_2=+y_1$, which lead to different cross section fall-offs in the extended $y_1$ rapidity region. The first (third) constrain, with $x_1$=large and $x_2$=small (very small), favours $qg(7)$ scattering, whereas the second constrain, with $x_1=x_2$=large, favours quark (and antiquark) scattering. We also have determined the $q_t$ integrated correlations with the relative size of all constituent processes being shown. At $\sqrt{s}=(0.54,2,20)$ TeV $p\bar{p}$ energy, we have determined the differential ($q_t$) and the integrated ($p_t$) transverse momentum fall-off of the total cross section, and of the single-jet and the two-jet distributions. The influence of the QCD parameters on the rapidity and the transverse momentum dependence of the correlation cross section is not very different from the single-jet and the fully integrated cross sections.

6. JET-JET INvariant Mass Distributions

The jet-jet invariant mass ($M$) distribution and its transverse momentum and rapidity dependence are analyzed in this section.

The jet-jet mass dependence has been evaluated using

$$
\frac{d\sigma}{dx_1dx_2dq_t^2} = \sqrt{s_0(s_0-4q_t^2)} \left( \frac{d\sigma_0}{dt_0u_1\bar{u}_2} + \frac{d\sigma_0}{dt_0\bar{u}_1u_2} \right) x_1x_2
= \sqrt{s_0(s_0-4q_t^2)} \frac{d\sigma}{(dy_1dy_2dq_t^2)}
$$

(6.1)

where $u_1 \equiv u(x_1)$. The mass distribution is now obtained by the transformation of $x_2 \rightarrow s_0$, and by the integration over $q_t$. Taking the Jacobians into account we obtain

$$
\frac{d\sigma}{dM} = \int dq_t \int dx_1 \frac{1}{(2M/x_1s)(M/\sqrt{(M^2-4q_t^2)})} \frac{d\sigma}{dy_1dy_2dq_t^2}
$$

(6.2)
The momentum fraction $x_2$ is fixed by

$$s_0 \equiv M^2 = x_1 x_2 s$$  \hspace{1cm} (6.3)$$

and the rapidities follow from the inversion of Eqs.2.2,

$$y_1 = \ln \left( \frac{x_2}{x_t} + \sqrt{\nu} \right)$$  \hspace{1cm} (6.4)$$

$$y_2 = \ln \left( \frac{x_2}{x_t} - \sqrt{\nu} \right)$$  \hspace{1cm} (6.5)$$

$$\nu \equiv (x_2/x_t)^2 - x_1/x_2 \ , \ x_t \equiv 2q_t/\sqrt{s}$$  \hspace{1cm} (6.6)$$

The constituent variables $s_0, t_0, u_0$ are given by Eqs.2.3-2.5. Subsequently we also analyze $d\sigma/(dM_dq_t)$ which follows from Eq.6.2 by suppression of the $q_t$ integration. The cross section $d\sigma/(dMdy_1dy_2)$ is obtained from Eq.2.1 via the differential form of

$$M^2 = q_t^2 \left( 2 + e^{y_1-y_2} + e^{-(y_1-y_2)} \right)$$  \hspace{1cm} (6.7)$$

The jet-jet invariant mass dependence is shown in Fig.6.1a where the importance of the eight constituent processes is also shown. The transverse momentum cutoff is set to $p_t=30$ GeV. Due to this relatively large $p_t$ value, the influence of the subprocess $gg(8)$ is limited. Its rapid decrease with growing jet-jet mass $M$ is a consequence of the 'soft' gluon distribution. At lower $M$ values $qg(7)$ scattering dominates whereas above $M=200$ GeV the $qq(1), q\overline{q}(4)$ subprocesses are most influential. The influence of the other subprocesses can be ignored. The sum of all contributions (SUM) shows an almost exponential decrease. These curves were determined using the BGOR parametrization with $Q^2$ defined in Eq.2.11 and $\Lambda=0.5$ GeV. In Fig.6.1b we determine the analogous curves with no scale dependence at all ($Q^2=Q_0^2=1.8 \text{ GeV}^2$) and $\alpha_s=0.3$. The dominance of $qq(1)$ and $q\overline{q}(4)$ scattering now sets in at larger $M$ values, and the decrease of the cross section is more moderate. The uncertainties of these results that follow from $\alpha_s$ not being fully specified should however still be kept in mind.

We now consider the sensitivity of the curves in Fig.6.1a on the QCD parameters by varying the momentum distributions and the scale parameter. In Fig.6.2a, we indicate the cross section spread for the $gg(8), qq(1)$ subprocess and the sum of all contributions (SUM); in the former cases the gluon resp. sea momentum distributions are most influential. Note, in particular, the enormous variations in $gg(8)$ scattering (-2 orders of magni-
tude!) whereas much less uncertainty results from the qq(1) subprocess. The moderate difference in the sum of all contributions (SUM) is due to the small influence of gg(8) scattering at larger M values. Using the BGOR parametrization we similarly show in Fig. 6.2a the cross section spread due to A variation in the range : 0.1 GeV/\AA \leq 0.7 GeV. At small M values there is little change, whereas at larger M values one notices a factor -3 variation. In Fig. 6.2b, we simulate the experimental (UA2) situation by imposing |y| \leq 0.85 with p_T = 15 GeV, and we again show the variation of the predicted M dependence of the cross section for different parametrizations of the scale dependent momentum distributions.

The experiments being currently carried out are limited in their rapidity ranges to the central (or almost central) region. We therefore study the influence of this constrain on the jet-jet mass distribution. In Fig. 6.3a we have chosen p_T = 30 GeV, (as in Fig. 6.1a) and, in addition, we have imposed the condition that both jet rapidities are confined to the region |y| \leq 0.85. The full phase space region is considerably reduced which manifests itself in a strong decrease of the cross section. Comparing Fig. 6.3a with Fig. 6.1a we notice at M = 150 GeV (300 GeV) a reduction of more than one (two) orders of magnitude. The cross section reduction due to the rapidity cut therefore becomes stronger as we go to larger M values. The M decrease is more pronounced and it no longer follows an exponential fall-off. In the early measurement periods, the UA1 analysis was done over a limited rapidity range. One of the two jet rapidities could reach the maximum value |y_1| \leq 2.5 whereas the other was limited to |y_2| \leq 1 (and 1\rightarrow 2). In the y_1, y_2 rapidity plane one thus covered the region of two centered rectangles where one of them is rotated by 90° with respect to the other - the shape of a 'swiss cross'. In Fig. 6.3b we show the mass distribution under the early restrictions of the UA1 analysis and we find a much smaller cross section reduction as compared to Fig. 6.3a. We show the influence of the rapidity constraints on all subprocesses and conclude that the changes are quite similar for all of them.

We now consider the cross section variation as the rapidity cutoff, \gamma_{\text{cut}}', takes different values. At fixed M values, we therefore constrain both rapidities |y| \leq \gamma_{\text{cut}} and gradually increase the \gamma_{\text{cut}}. The resulting increase of d\sigma/dM, at four chosen values of M, is indicated in Fig. 6.3c. Only the most important subprocesses are shown since all others lead to much smaller contributions. From these results we conclude that large jet-jet masses are preferentially produced at large rapidities.

As the transverse momentum cutoff p_T is lowered, the threshold onset in the jet-jet invariant mass distributions is shifted to lower values. This fea-
ture is shown in Fig. 6.4a (solid curves). For changing values of $p_t$ we notice that the jet-jet invariant mass distributions are almost parallel displaced. As $p_t$ is increased from 10 GeV to 30 GeV, we read at $M=300$ GeV a cross section reduction of $1.1/2$ orders of magnitude. The dotted curves indicate the size and shape of the cross section if scale independent momentum distributions with $\alpha_s=0.3$ are used. In Fig. 6.4b, we consider the cross section behaviour near and off the threshold region for different values of the transverse momentum cutoff $p_t$, assuming a limited rapidity range. As soon as the rapidity cut $|y|<0.85$ is applied there remains only one universal, meaning almost $p_t$ independent, (fat) curve. In the threshold region a cross section depletion develops.

We now discuss the following features: (i) parallel displacement of curves as $p_t$ increases, (ii) cross section reduction for upper cut in rapidity $y_{cut}$ with $p_t$ insensitivity, (iii) cross section increase with growing $y_{cut}$. We therefore analyze the $q_t$ dependence of the differential cross section $d\sigma/(dMdq_t)$ at a fixed invariant mass $M$, and we also determine the influence of a cut on the jet rapidities. In Fig. 6.5a we set $M=100$ GeV and we show the $q_t$ dependence of all constituent processes. Since the $q_t$ dependence is primarily determined by the parton cross section $d\sigma/dq_t$ we expect the characteristics of Fig. 2.3 to appear. The exchanged quantum numbers in the $t_0$ (or $u_0$) channel of the subprocess thus determine the fall-off. Note, in particular, in Fig. 6.5a the behaviour of the subprocess (3) which consists only of an $s_0$-pole (similar to W decay); it shows the typical shape of a Jacobian peak close to the upper phase space boundary. The peak is traced back to the Jacobian $[s_0(s_0-4q_t^2)]^{-1/2}$ in Eq. 6.2 which, for $s_0 -(2q_t^2)$, diverges. The physical interpretation of the above insight is: at any fixed jet-jet mass $M$, the subprocesses (1,2,4,7,8) are produced at smallest possible transverse momenta whereas the subprocess (3) tends to large $q_t$ values. Due to the interference of the $s_0$ and $t_0$ poles, the subprocesses (5,6) lie in between these two extrema. In Fig. 6.6b, we indicate the sensitivity of this distribution to changes in the QCD parametrization. Statement (i) is understood as follows: imagine $q_t$ integration of the curve (SUM) in Fig. 6.5a with $p_t$ as its lower cutoff. If $p_t$ grows, the cross section $d\sigma/dM$ falls rapidly. Since $q_t$ integration primarily involves the parton cross section and since $M$ enters via the kinematics only, the reduction is roughly the same for all $M$, as seen in Fig. 6.4a. The effect of a rapidity cut is explained by the solid curves in Fig. 6.5c. We choose $M=100$ GeV and reconsider the $q_t$-dependence of $d\sigma/(dMdq_t)$. If no cut is imposed, we recover the curve (SUM) in Fig. 6.5a. We now assume that the rapidities of both jets are confined to the
region \( |y_1| \leq y_{\text{cut}} \), where we successively choose \( y_{\text{cut}} = (2.5, 2.0, 1.5, 0.85) \). As the rapidity cut becomes more stringent, the \( q_t \) distribution is constrained to larger and larger \( q_t \) values. This is understood by Eq.6.7 with \( M^2 = \text{fixed} \); we let the rapidities get smaller and realize that \( q_t \) cannot fall anymore below a certain value.

We now assume that \( q_t \) integration is carried out within the region \( p_t \leq q_t \leq M/2 \). The resulting \( p_t \) dependence of \( d\sigma/dM \) is shown by the dashed curves in Fig.6.5c. They give us an understanding of the depleted threshold onset in Fig.6.4b when the rapidity cut is applied. The dashed curves also explain why the stringent rapidity cut in Fig.6.4b leads to an (almost) universal mass distribution, independent of \( p_t \), apart from a shift of the threshold onset. In Fig.6.5c, we note that for \( |y| \leq 0.85 \), \( d\sigma/dM \) has a flat \( p_t \) dependence. Since, as is shown in Fig.6.5c, the rapidity cut limits \( q_t \) in the differential distribution to larger values (solid curves), the flat behavior of the integrated distribution (dotted curves) is no surprise. The apparently universal M fall-off in Fig.6.4b is thus explained by this flat \( p_t \) dependence.

We analyze the rapidity correlations at fixed values of the jet-jet invariant mass \( M \). The cross section \( d\sigma/(dMdy_1dy_2) \) is obtained from Eq.2.1 by using Eq.6.7. We first show in Fig.6.6 its variation with changing \( M \) values at \( y_1 = y_2 = 0 \). Due to Eq.6.6 this distribution essentially reflects the \( q_t \) fall-off of \( d\sigma/(dy_1dy_2dq_t) \). On the horizontal axis we plot the jet-jet mass \( M \) and, in addition, we show the corresponding \( q_t \) and \( x_1, x_2 \) values. The fat lines (SUM) represent the sum of all subprocesses, and the numbered solid curves show the influence of the individual subprocesses (1-8). These curves were obtained using the BGOR parametrization for the scale dependent momentum distributions. The dashed, dotted, etc. curves illustrate the changes (in SUM) for different parametrizations.

The rapidity correlations \( d\sigma/(dMdy_1dy_2) \) at the jet-jet mass \( M=100 \) GeV are shown in Fig.6.7. We investigate the constrains: \( y_2 = +y_1 \) (Fig.6.7a), \( y_2 = -y_1 \) (Fig.6.7b), and \( y_2 = 0 \), \( y_1 = \text{variable} \) (Fig.6.7c). On the horizontal axis we vary the rapidity \( y_1 \) and similarly show \( x_1, x_2, q_t \). The fat line (SUM) represents the sum of all contributions. We also indicate the individual cross sections for the constituent processes (1-8) of Eq.2.14. The distributions may be compared with the analogous curves \( d\sigma/(dy_1dy_2) \) in Fig.5.3 where no jet-jet mass constrain was imposed. The unfamiliar feature shown here is the cross section rise (1) with growing rapidities \( y_2 = -y_1 \) (as shown in Fig.6.7b). This is a reflection of the fact that the transverse momentum, related by Eq.6.7 to the jet-jet invariant mass and the rapidities, is decreasing. A massive jet-jet system is thus preferentially produced.
back-to-back at smallest possible transverse momenta and at largest possible rapidities. The (unimportant) subprocess (3) however does not follow these characteristics due to the absence of any $t_0$ or $u_0$ pole graph. Note the predominance of qg(7) scattering for all correlation constrains. For $y_2=+y_1$ (Fig.6.7a) the transverse momentum remains fixed. This is consistent with the fact that a Lorentz transformation to $y_1=y_2=0$ leaves the transverse energy unchanged. It however is distinct from the case $y_2=-y_1$ (Fig.6.8b) where the momentum fractions $x_1=x_2$ remain fixed and only $q_t$ varies.

What is the consequence of an increase in the overall $p\bar{p}$ energy? In Fig.6.8 we compare $d\sigma/dM$ at the CM energies $\sqrt{s}=(0.54,2,20)$ TeV keeping the transverse momentum cutoff at $p_t=30$ GeV. The very much enlarged M ranges of the distributions at higher energies become immediately apparent. One should also note the increasing predominance of the qg(7) and, to a lesser extent, of the gg(8) subprocesses.

In Fig.6.9a we set $\sqrt{s}=2,20$ TeV. The jet-jet invariant mass is chosen to be at $M=300$ GeV and we again analyze the influence of the previously defined rapidity correlations: $y_2=+y_1$, $y_2=-y_1$ and $y_2=0$. Note the considerable difference in the rapidity ranges. In Fig.6.9b, we show a comparison of the distributions $d\sigma/(dMdy_1dy_2)$ and $d\sigma/dM$. There is an enormous difference as we go from $\sqrt{s}=540$ GeV to $\sqrt{s}=2,20$ TeV which, to a large extent, is due to the rise seen in Fig.6.7b.

We summarize the findings of this section: We have evaluated the jet-jet invariant mass M dependence at $\sqrt{s}=(0.54,2,20)$ TeV, and we show the sensitivity of this distribution on the QCD parameters. The contributions involving the gluon structure function reveal, at large mass, considerable uncertainties. The strong influence of quark-scattering moderates to a large degree, the predictions for the total sum. We have also analyzed the influence of transverse momentum and rapidity cuts whereby the latter reduce considerably the counting rates. These features are partially understood by the differential and the integrated $q_t$ dependence of the jet-jet mass distribution. It clearly reveals the $y$ and $q_t$ connection: 'small rapidity' implies 'large transverse momentum' (at fixed mass). This is the essential clue to understand the almost $p_t$ independent M fall-off when a sufficiently stringent rapidity cut is imposed. We similarly have analyzed the rapidity correlations at fixed M and find, for $y_2=-y_1$, a rising cross section since $q_t$ diminishes. Large invariant jet-jet masses are therefore preferentially produced back-to-back, at smallest possible transverse momenta, and at largest possible rapidities.
7. CM AND LAB ANGULAR DISTRIBUTIONS

In this section we discuss the jet angular distributions in the center-of-mass (CM) and the laboratory (LAB) systems. The relative importance of the constituent processes is determined for various transverse momentum, jet-jet mass, and rapidity cutoffs. The LAB angular distributions of inclusive 1-jet and 2-jet production are compared.

The CM angular distribution trivially follows from

\[ \frac{d\sigma}{d\cos\theta_{\text{cm}}} = \int dx_1 dx_2 (d\sigma_0/dt_0 \ u_1 u_2 + d\sigma_0/du_0 \ u_1 u_2) s_0/2 \]  

(7.1)

with the integration boundaries: \( t_M \equiv M_C^2/s \leq x_1, x_2 \leq 1 \) where \( M_C^2 = \text{Max}(M_{\text{cut}}^2, (2p_t/\sin\theta_{\text{CM}})^2) \). \( M_{\text{cut}} \) is a lower cutoff in the jet-jet invariant mass, and \( p_t \) is the usual lower cutoff in the transverse momentum.

In Fig.7.1a we show the CM angular distribution of the constituent processes (1)-(8) in Eq.2.14, at \( p_t = 15 \text{ GeV} \) and \( M_{\text{cut}} = 0 \). The total sum is dominated by \( qg(7) \) scattering. The distribution in Eq.7.1a has two parameters, \( M_{\text{cut}} \) and \( p_t \), whose values are varied in Figs.7.1b,c. The \( p_t \) cut essentially affects the forward and backward regions and \( M_{\text{cut}} \) causes the dip around \( \cos\theta_{\text{CM}} = 0 \). The observed characteristics are a reflection of kinematics and not dynamics.

Figs.7.2 give insight into the cross section dependence on \( M_{\text{cut}} \), \( y_{\text{cut}} \) and on the QCD parametrizations. In Fig.7.2a we show the dependence \( M_{\text{cut}} \) of the dip around \( \cos\theta_{\text{CM}} = 0 \) (and also \( \cos\theta_{\text{CM}} = 0.9 \)). The sensitivity of the CM angular distribution on the scale dependent momentum distributions and the \( \Lambda \) parameter is illustrated by Fig.7.2b. From Fig.7.2c we learn that the influence of a rapidity cutoff is quite similar to a cutoff in the transverse momentum.

The LAB angular distributions are given by the rapidity distributions since \( y = -\ln \tan \Theta_{\text{LAB}}/2 = -\ln \sqrt{(1+z)/(1-z)} \) with \( z \equiv \cos\Theta_{\text{LAB}} \). In Fig.7.3 we show the 1-jet and 2-jet \( z \) dependence at a fixed transverse momentum (dotted curves), and we compared with the analogous \( q_t \) integrated distributions (solid curves). The three curves for the 2-jet angular distribution correspond to the correlations: \( z_1 = z_2, z_1 = -z_2 \) and \( z_2 = 0 \), \( z_3 \) = variable. The typical peaking near the forward and backward directions results from the Jacobian when going from the rapidity to the LAB angular distributions. The transverse momenta \( q_t, p_t \leq 30 \text{ GeV} \) enforce the cross section fall-off in the extreme forward and backward direction.

We summarize the findings of this section: We have determined the CM angular distribution and we have shown its dependence on the mass and
transverse momentum cutoffs. $M_{\text{cut}}$ diminishes the cross section mostly around $\Theta=90^\circ$ whereas $p_t$ reduces the cross section around the forward and backward regions. The effect of a rapidity cut is similar to a cut in the transverse momentum. The sensitivity on the QCD parameters is moderate. We have similarly shown the single- and double-jet LAB angular distributions for fixed transverse momentum values.

8. CONCLUSIONS

In this paper we have carried out a systematic analysis of the kinematical distributions for $p\bar{p}$ two-jet production. The QCD corrections were taken into account in the leading-log approximation. We determined the energy dependence of the integrated cross sections and analyzed the most important differential distributions of the single- and two-jet production. In particular we analyzed the transverse momentum-, the rapidity-, and the jet-jet invariant mass dependence as well as the CM/LAB angular distributions. The main goal of this study was threefold: to show the cross section sensitivity on different parametrizations of the scale dependent momentum distributions and the scale parameter $\Lambda$; to evaluate the effect of the kinematical restrictions due to the limitations of the experimental detectors; and to extrapolate the predictions of the (almost) established present-day calculation scheme into the TeV energy region.

Our results are mostly contained in the figures of this paper and we limit ourselves here to a few qualitative observations: (i) the cross section evaluation suffers considerably from the imprecision of the gluon distribution. The rapidity correlations favor in some cases $qg$ and in others $qq$ scattering at the high end of the rapidity distribution and thus might give a way for clear measurements of the $qq/q\bar{q}$, or the $qg$ subprocesses; (ii) the limitations in the rapidity ranges of the UA2 and early UA1 analyses reduce the rates by orders of magnitude and prevent measurements in the rapidity regions where the physics looks promising (even though this will involve a considerable experimental effort); (iii) the cross section extrapolation into the TeV energy range suffers from an uncertainty of less than an order of magnitude. At $\sqrt{s}=(0.54, 2, 20)$ TeV typical cross section values are $\sigma=(10^{-28}, 10^{-27}, 10^{-28})$ cm$^2$ for $p_t=30$ GeV.
ACKNOWLEDGEMENTS

I would like to thank K. Eggert, J. Ellis, P. Jenni, M. Swartz and Prof. J. Zakrzewski for their reading of this paper at several stages of its development. Without the kind help of Prof. C. Joseph this work could not have been completed.
REFERENCES


TABLE CAPTION

Table 1 : Characteristic features of the parametrizations used for the scale dependent momentum distributions.

FIGURE CAPTIONS

Fig.2.1 : The lowest order QCD graphs for the constituent processes in Eq.2.14 contributing to jet-jet production.

Fig.2.2 a) : Differential parton cross sections of the constituent processes in Eq.2.14 at $\sqrt{s_0}=300$ GeV with $\alpha_s=0.3$.
   b) : Differential parton cross sections of the constituent processes in Eq.2.14 at $\sqrt{s_0}=300$ GeV; $\alpha_s(Q^2)$ with $Q^2$ specified by Eq.2.11.

Fig.2.3 : Transverse momentum ($q_t$) dependence of the constituent processes in Eq.2.14 at $\sqrt{s_0}=300$ GeV; $\alpha_s=0.3$ (dotted curves) and $\alpha_s(Q^2)$ (solid curves). The shaded regions indicate the sensitivity on the $\Lambda$ parameter and the $Q^2$ choice.

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b)  Energy dependence of the integrated cross section for $p\bar{p} -jj^*X$ using the BGOR [10] parametrization; the sensitivity on $\Lambda$ is shown. (Note the three vertical scales, indicated by the 'break', which apply to their closest $gg$, $qg$, and $qq$ curves).

c)  Energy dependence of the integrated cross section for $p\bar{p} -jj^*X$ using scale independent momentum distributions with $\alpha_s=0.3$. (Note the three vertical scales, indicated by the 'break', which apply to their closest $gg$, $qg$ and $qq$ curves).

Fig. 3.3  Transverse momentum cutoff ($p_t$) dependence of the integrated cross section for $p\bar{p} -jj^*X$ at $\sqrt{s}=(0.54, 2, 20)$ TeV. BGOR [10] parametrization with $\Lambda=0.5$ GeV.

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b)  Cross section reduction (at $\sqrt{s}=540$ GeV) due to a cut in the jet rapidities $y_1, y_2 \leq \text{cut}$; the most relevant subprocesses are compared.

Fig. 3.5 a)  Cross section reduction (at $\sqrt{s}=2$ TeV) due to a cut in the rapidities $y_1, y_2 \leq \text{cut}$; the transverse momentum cutoff ($p_t$) is varied within $p_t=10-75$ GeV.

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b) : Cross section reduction (at $\sqrt{s}=540$ GeV) due to a lower cut in the jet-jet invariant mass $M_{cut} \leq M$; the transverse momentum cutoff ($p_T$) is varied within $p_T=10$-40 GeV.

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b) : Transverse momentum ($q_t$) dependent (at $\sqrt{s}=2$ TeV) with rapidity $y=0$ of the subprocesses contributing to $p\bar{p} \rightarrow jj+X$; the influence of the scale (in-)dependence and the $q_t$ integration is shown.

c) : Transverse momentum ($q_t$) dependent (at $\sqrt{s}=20$ TeV) with rapidity $y=0$ of the subprocesses contributing to $p\bar{p} \rightarrow jj+X$; the influence of the scale (in-)dependence and the $q_t$ integration is shown.

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b) : Single jet rapidity distribution at $\sqrt{s}=0.54$ TeV and for different $q_t$ values (solid curves); $q_t$ integration in the range $p_t \leq q_t \leq \sqrt{s}/2$ (dashed curves).

c) : Single jet rapidity distribution at $\sqrt{s}=0.54$ TeV and for $q_t=30$ GeV; the sensitivity on the scale dependent momentum distributions and on the $\Lambda$ parameter is shown. For clearness of presentation some curves are lowered by three orders of magnitude.

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is shown.

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For clearness of presentation one set of curves has been
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c) : Cross section comparison of the $q_t$ non-integrated and the
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b) : Transverse momentum dependence of the jet-jet invariant mass distribution at $M = 100$ GeV; cross section sensitivity on the scale dependent momentum distributions and the $\Lambda$ parameter. For clearness of presentation one set of curves is lowered by three orders of magnitudes.

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Fig.6.8 : Jet-jet invariant mass distribution at $\sqrt{s}=(0.54,2,20)$ TeV. The numbers near the curves indicate the contributions of the subprocesses listed in Eq.2.14.

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   c) : The CM angular distribution with a cut on the jet rapidities.
Fig. 7.3: Cross section comparison of the $q_t$ non-integrated (dotted curves) and the $q_t$ integrated (solid curves) LAB angular distributions. For clearness of presentation some curves have been shifted by one order of magnitude.
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<th>(n_G)</th>
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Validity Range

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| BGOR| \(
\begin{align*}
0.02 \leq x,
\end{align*}
\) | \(1.8 \text{ GeV}^2 \leq Q^2 \leq 4 \times 10^3 \text{ GeV}^2\) | \(4.0 \text{ GeV}^2 \leq Q^2 \leq 4 \times 10^4 \text{ GeV}^2\) |
| GROR| \(
\begin{align*}
0.02 \leq x,
\end{align*}
\) | \(3.0 \text{ GeV}^2 \leq Q^2 \leq 4 \times 10^4 \text{ GeV}^2\) | \(4.0 \text{ GeV}^2 \leq Q^2 \leq 4 \times 10^4 \text{ GeV}^2\) |
| BETAL| - | - | - |
| GHR | \(
\begin{align*}
0.01 \leq x \leq 0.8,
\end{align*}
\) | \(4.0 \text{ GeV}^2 \leq Q^2 \leq 4 \times 10^4 \text{ GeV}^2\) | \(4.0 \text{ GeV}^2 \leq Q^2 \leq 4 \times 10^4 \text{ GeV}^2\) |
| CDHS| - | - | - |

Table 1
Differential parton cross sections

\[ \sqrt{s} = 300 \text{ GeV} \]

\[ \alpha_s = 0.3 \]

\[ d\sigma / dt \quad \text{[cm}^2/\text{GeV}^2] \]

\[ \cos \theta = +1 \quad \cos \theta = 0 \quad \cos \theta = -1 \]

\[ (-t / 10^4) \quad \text{[GeV}^2] \]

\[ d\sigma / dt \quad \text{[cm}^2/\text{GeV}^2] \]

\[ \cos \theta = +1 \quad \cos \theta = 0 \quad \cos \theta = -1 \]

\[ (-t / 10^4) \quad \text{[GeV}^2] \]

Fig. 2.2
Differential parton cross sections
(q_t - dependence)

\[ \sqrt{s} = 300 \text{ GeV} \]

\[ \alpha_s = 0.3 \quad \text{and} \quad \alpha_s(Q^2), \Lambda = 0.5 \text{ GeV} \]

\[ \Lambda\text{-Variation} \]

\[ Q^2\text{-Variation} \]

1: q_1 + q_2 \rightarrow q_1 + q_2
2: q_1 + q_1 \rightarrow q_1 + q_1
3: q_1 + \bar{q}_1 \rightarrow q_1 + \bar{q}_1
4: q_1 + \bar{q}_1 \rightarrow q_1 + \bar{q}_1
5: q\bar{q} \rightarrow gg
6: gg \rightarrow q\bar{q}
7: qg \rightarrow qg
8: gg \rightarrow gg

Fig. 2.3
Integrated parton cross sections
(Energy dependence)

Fig. 2.4
Integrated parton cross sections

(Transverse momentum cutoff)

\[ \sqrt{s} = 300 \text{ GeV} \]

\( \alpha_s = 0.3 \)

\( \alpha_s(Q^2), \Lambda = 0.5 \text{ GeV} \)

\[
\frac{\delta \hat{s}(p_T)}{\text{cm}} = \begin{cases}
8 & \text{gg} \rightarrow \text{gg} \\
7 & \text{gg} \rightarrow \text{gg} \\
6 & \text{gg} \rightarrow \text{gg} \\
5 & \text{gg} \rightarrow \text{gg} \\
4 & \text{gg} \rightarrow \text{gg} \\
3 & \text{gg} \rightarrow \text{gg} \\
2 & \text{gg} \rightarrow \text{gg} \\
1 & \text{gg} \rightarrow \text{gg}
\end{cases}
\]

Fig. 2.5
$p\bar{p} \rightarrow \text{jet+jet+X}$

[Transverse momentum cutoff]

$\sqrt{s} = 540 \text{ GeV, } 2 \text{ TeV, } 20 \text{ TeV}$

BGOR, $\Lambda = 0.5 \text{ GeV}$

$\sqrt{s} = 540 \text{ GeV}$

$\sqrt{s} = 2 \text{ TeV}$

$\sqrt{s} = 20 \text{ TeV}$

Fig.3.3
\[ \frac{\sigma_{y \text{-cut}}}{\sigma_{\text{total}}} \]

\[ p\bar{p} \rightarrow \text{jet} + \text{jet} + X \]

[Rapidity cutoff]

\[ \sqrt{s} = 540 \text{ GeV} \]

BGOR, \( \Lambda = 0.5 \text{ GeV} \)

Fig. 3.4
Fig. 3.5
$p\bar{p} \rightarrow \text{jet+jet+X}$

[Rapidity cutoff]

\[ \sqrt{s} = 20 \text{ TeV} \]

BGOR, $\Lambda = 0.5 \text{ GeV}$

Fig. 3.6
$p\bar{p} \rightarrow \text{jet+jet+X}$

(Mass cutoff)

$\sqrt{s} = 540 \text{ GeV}$

BGOR, $\Lambda = 0.5 \text{ GeV}$

$p_T = 10 \text{ GeV}$

$p_T = 50 \text{ GeV}$

Fig. 3.7
$p\bar{p} \rightarrow \text{jet+jet+X}$

(Mass cutoff)

$\sqrt{s} = 2 \text{ TeV}$

BGOR, $\Lambda = 0.5 \text{ GeV}$

$R = \frac{\sigma_{N\text{-cut}}}{\sigma_{\text{total}}}$

$p_T = 10 \text{ GeV}$

$p_T = 50 \text{ GeV}$

$M_{\text{cut}} \text{ [GeV]}$

Fig. 3.8
$p\bar{p} \rightarrow \text{jet+jet+X}$

[Mass cutoff]

$\sqrt{s} = 20 \, \text{TeV}$

BGOR, $\Lambda = 0.5 \, \text{GeV}$

Fig. 3.9
Fig. 4.1
\( p\bar{p} \rightarrow \text{jet} + \text{jet} + X \)

\[ \sqrt{s} = \begin{cases} 63, 540, 620 \text{ GeV} \\ 2, 20 \text{ TeV} \end{cases} \]

BGOR, \( \Lambda = 0.5 \text{ GeV} \)

\[ \left. \frac{d^3 \sigma}{dy dq_t} \right|_{y=0} \]

[no units]

\[ x_t \equiv 2q_t / \sqrt{s} \]

- Scaling, \( \alpha_s = 0.3 \)
- 63 GeV
- 540 GeV
- 620 GeV
- 2 TeV
- 20 TeV

Fig. 4.2
Fig. 4.3
Fig. 4.4
$\sqrt{s} = 2\,\text{TeV}$
BGOR, $\Lambda = 0.5\,\text{GeV}$

$\sqrt{s} = 20\,\text{TeV}$
BGOR, $\Lambda = 0.5\,\text{GeV}$

Fig. 4.5
\[ p\bar{p} \rightarrow \text{jet} + X \]

[y_2\text{-cutoff dependence}]

\[ \sqrt{s} = 540 \text{ GeV} \]
BGOR, \( \Lambda = 0.5 \) GeV

\[ R = \frac{\text{d}^2\sigma / \text{d}y_1 \text{d}y_2 \text{cut}}{\text{d}^2\sigma / \text{d}y_1 \text{d}y_2 \text{total}} \]

\( y_1 \)

\( p_T = 50, 30, 20, 10 \) GeV

\[ \sqrt{s} = 540 \text{ GeV} \]
BGOR, \( \Lambda = 0.5 \) GeV

\[ R = \frac{\text{d}\sigma / \text{d}y_1 \text{d}y_2 \text{cut}}{\text{d}\sigma / \text{d}y_1 \text{d}y_2 \text{total}} \]

\( y_{\text{cut}} \)

Fig. 4.6
\( p\bar{p} \rightarrow \text{jet+X} \)

\( \sqrt{s} = 540 \text{ GeV} \)

BGOR, \( \Lambda = 0.5 \text{ GeV} \)

\( \cdots \cdots \frac{d\sigma}{dy} | y = 0 \)

\( u+\bar{u}+d+\bar{d}+s+\bar{s} \)

\( c=\bar{c} \)

\( s = \bar{s} \)

\( d = \bar{d} \)

\( u = \bar{u} \)

\( \frac{d\sigma}{dy} \big|_{y=0} \big| \text{(cm}^2\text{/GeV)} \)

\( q_T, p_T \) [GeV]

Fig. 4.7
$p\bar{p} \rightarrow$ jet$+X$

\[ \sqrt{s} = 540 \text{ GeV} \]

BGOR, $\Lambda = 0.5 \text{ GeV}$

$\frac{d\sigma}{dy} \frac{dq_t}{|y|} = 0$

\begin{itemize}
  \item $g$
  \item $u+\bar{u}$
  \item $s+\bar{s}$
  \item $d+\bar{d}$
  \item $c+\bar{c}$
\end{itemize}

$\frac{d\sigma}{dy} \frac{dq_t}{|y|} = 0$

\begin{itemize}
  \item BGOR, $\Lambda = 0.5 \text{ GeV}$
  \item GHR
  \item BETAL
  \item CDHS
\end{itemize}

Fig. 4.8
$p\bar{p} \rightarrow \text{jet} + X$

$\sqrt{s} = 540$ GeV
BGOR, $\Lambda = 0.5$ GeV
$q_t = 30$ GeV

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$\frac{d\sigma}{dy}$
\[ p\bar{p} \rightarrow \text{jet+jet+X} \]

**Rapidity correlations**

\[ \sqrt{s} = 540 \text{ GeV} \]

BGOR, \( \Lambda = 0.5 \text{ GeV} \)

\( q_T = 10 \text{ GeV} \)

\( q_T = 30 \text{ GeV} \)

\( q_T = 50 \text{ GeV} \)

\( q_T = 70 \text{ GeV} \)

- a: \( y = y_1, \ y_2 = 0 \)
- b: \( y = y_1 - y_2 \)
- c: \( y = y_1 = y_2 \)

\[ \frac{d\sigma}{dy_1 \ dy_2 \ dq_T} \text{ [cm}^2/\text{GeV]} \]

\[ y \]

**Scale dependence**

\[ \sqrt{s} = 540 \text{ GeV} \]

\( y_1 = y_2 = 0 \)

BGOR, \( \Lambda = 0.5 \text{ GeV} \)

1: \( Sc + \alpha_s = 0.3 \)

2: \( Sc + \alpha_s(Q^2) \)

3: \( AF + \alpha_s = 0.3 \)

4: \( AF + \alpha_s(Q^2) \)

\[ \frac{d\sigma}{dy_1 \ dq_T} \text{ [cm}^2/\text{GeV]} \]

\[ q_T \text{ (GeV)} \]

**Fig. 5.1**
$p\bar{p} \rightarrow \text{jet + jet + X}$

$\sqrt{s} = 540$ GeV

- a: BGOR, $\Lambda = 0.5$ GeV
- b: GROR
- c: BETAL
- d: GHR
- e: CDHS

$\frac{d^2 \sigma}{dy_1 dy_2 dq_t}$ [cm$^2$/GeV]

$q_t$ [GeV]

$\Lambda = 0.1$ GeV
$\Lambda = 0.7$ GeV

$\frac{d^2 \sigma}{dy_1 dy_2 dq_t}$ [cm$^2$/GeV]

$y = y_1 = y_2$

$\Lambda = 0.1$ GeV
$\Lambda = 0.7$ GeV

Fig. 5.4
\( p\bar{p} \rightarrow \text{jet+jet+X} \)

[Parametrizations]

\( \sqrt{s} = 540 \text{ GeV} \)
\( p_T = 30 \text{ GeV} \)

- a: BGOR, \( \Lambda = 0.5 \text{ GeV} \)
- b: GROR, \( \cdots \cdots \cdots \cdots \)
- c: BETAL, \( \cdots \cdots \cdots \cdots \)
- d: GHR, \( \cdots \cdots \cdots \cdots \)
- e: CDHS, \( \cdots \cdots \cdots \cdots \)

\( q_1 + q_2 \rightarrow q_1 + q_2 \)

\[ M \text{ [GeV]} \]

\( d\sigma/dM \text{ [cm}^2/\text{GeV}] \)

\( a: \Lambda = 0.7 \)
\( b: \Lambda = 0.1 \)

\( p\bar{p} \rightarrow \text{jet+jet+X} \)

[Parametrizations]

\( \sqrt{s} = 540 \text{ GeV} \)
\( p_T = 15 \text{ GeV} \)
\( \gamma y = 0.85 \)

- a: BGOR, \( \Lambda = 0.5 \text{ GeV} \)
- b: GROR, \( \cdots \cdots \cdots \cdots \)
- c: BETAL, \( \cdots \cdots \cdots \cdots \)
- d: GHR, \( \cdots \cdots \cdots \cdots \)
- e: CDHS, \( \Lambda = 0.25 \text{ GeV} \)

Fig. 6.2
Fig. 6.3
Fig. 6.4
p\bar{p} \rightarrow \text{jet+jet+X}

[Correlations at M]

\sqrt{s} = 540 \text{ GeV}

y_1 = y_2 = 0

- a: BGOR, \Lambda = 0.5 \text{ GeV}
- - - c: BETAL
- - - d: GHR
- - - e: CDHS

1: q_1 \bar{q}_2 \rightarrow q_1 \bar{q}_2
2: q_1 \bar{q}_1 \rightarrow q_1 \bar{q}_1
3: q_1 \bar{q}_1 \rightarrow q_2 \bar{q}_2
4: q_1 \bar{q}_1 \rightarrow q_1 \bar{q}_1
5: q\bar{q} \rightarrow gg
6: gg \rightarrow q\bar{q}
7: qq \rightarrow gg
8: gg \rightarrow gg

\text{Fig. 6.6}
$p\bar{p} \rightarrow \text{jet+jet+X}$

(Mass dependence)

$\sqrt{s} = 540$ GeV, 2 TeV, 20 TeV

$p_T = 30$ GeV

BGOR, $\Lambda = 0.5$ GeV

$\sqrt{s} = 2$ TeV

$\sqrt{s} = 540$ GeV

$\sqrt{s} = 20$ TeV

$\frac{d\sigma}{dM}$ [cm$^2$/GeV]
$p\bar{p} \rightarrow \text{jet+jet+X}$

[CM-Angular distribution]

$\sqrt{s} = 540 \text{ GeV}$

$p_t > 15 \text{ GeV}$

BGOR, $\Lambda = 0.5 \text{ GeV}$

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$M > 30 \text{ GeV}$

$M > 40 \text{ GeV}$

$M > 50 \text{ GeV}$

$M > 60 \text{ GeV}$

$M > 80 \text{ GeV}$

$M > 100 \text{ GeV}$

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$M > 2 \text{ GeV}$

$M > 60 \text{ GeV}$

$p_t = 0 \text{ GeV}$

$p_t = 15 \text{ GeV}$

$p_t = 20 \text{ GeV}$

$p_t = 30 \text{ GeV}$

$p_t = 50 \text{ GeV}$

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Fig. 7.1
**p\bar{p} \rightarrow jet+jet+X**

(Lab-angular distribution)

$\sqrt{s} = 540$ GeV
$q_t, p_t = 30$ GeV
BGOR, $\Lambda = 0.5$ GeV

a: $z_1 = z_2$
b: $z_1 = -z_2$
c: $z_2 = 0$

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Fig. 7.3