Model-independent measurement of 
\( CP \) violation parameters in 
\( B^{\pm} \to (K_S^{0}h^{+}h^{-})DK^{\pm} \) decays

The LHCb collaboration

Abstract

The following \( CP \) violating observables are measured in a model-independent analysis of \( B^{\pm} \to (K_S^{0}h^{+}h^{-})DK^{\pm} \) (where \( h = \pi, K \)) decays reconstructed in a data sample corresponding to an integrated luminosity of 2.0 fb\(^{-1}\) of \( pp \) interactions collected by the LHCb experiment at a centre-of-mass energy of 8 TeV in 2012:

\[
\begin{align*}
x_+ &= (-8.7 \pm 3.1 \pm 1.6 \pm 0.6) \times 10^{-2}, \\
x_- &= (5.3 \pm 3.2 \pm 0.9 \pm 0.9) \times 10^{-2}, \\
y_+ &= (0.1 \pm 3.6 \pm 1.4 \pm 1.9) \times 10^{-2}, \\
y_- &= (9.9 \pm 3.6 \pm 2.2 \pm 1.6) \times 10^{-2},
\end{align*}
\]

where the first uncertainty is statistical, the second arises from systematic effects from the method or detector considerations and the third from external strong-phase measurements used in the fit.

These results are combined with those obtained in the equivalent analysis of LHCb data collected in 2011. The best-fit value of the CKM unitarity triangle angle \( \gamma \) is determined to be \((57 \pm 16)^\circ\) and the parameters \( r_B \) and \( \delta_B \) are determined to be \((8.8^{+2.3}_{-2.4}) \times 10^{-2}\) and \((124^{+15}_{-17})^\circ\), respectively.

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\^[1]Conference report prepared for the 14th international conference on \( B \)-physics at hadron machines, Bologna, Italy, 8–12 April 2013. Contact authors: Sneha Malde, [s.malde1@physics.ox.ac.uk](mailto:s.malde1@physics.ox.ac.uk) and Christopher Thomas, [c.thomas2@physics.ox.ac.uk](mailto:c.thomas2@physics.ox.ac.uk)
1 Introduction

A precise measurement of the Cabibbo-Kobayashi-Maskawa (CKM) angle $\gamma$ is an important goal in flavour physics. Complementary measurements of $\gamma$ using tree and loop diagrams will provide a powerful test for the presence of any BSM physics. In addition, measurements of $\gamma$, together with the other two CKM angles $\alpha$ and $\beta$, are important to overconstrain the Standard Model Unitarity Triangle, providing stringent tests of unitarity and the three-generation quark model.

Decays of the form $B^{\pm} \rightarrow DK^{\pm}$, where $D$ indicates a coherent sum of $D^{0}$ and $\bar{D}^{0}$ mesons, can be used to measure $\gamma$ by exploiting the interference between $b \rightarrow u\bar{c}s$ and $b \rightarrow e\bar{c}s$ transitions. The $D$ mesons must decay to the same final state for this to be possible.

In this note we consider decays to the three-body self-conjugate final state $K^{0}_{s}h^{+}h^{-}$ ($h = K, \pi$). Such a state can be analysed by studying the distribution of candidates across the $K^{0}_{s}h^{+}h^{-}$ Dalitz plot. It is essential to take into account the variation of the $D$ strong phase across the Dalitz plot. This variation is encapsulated in the strong-phase difference $\delta_{D}$. We present a model-independent measurement of the CP violation parameters that exploits measurements made by the CLEO-c experiment with quantum-correlated $D^{0}\bar{D}^{0}$ data. CLEO-c measured $c_{i}$ and $s_{i}$, the amplitude-weighted cosine and sine of $\delta_{D}$ in bins across the $K^{0}_{s}h^{+}h^{-}$ Dalitz plane [1].

The $K^{0}_{s}h^{+}h^{-}$ Dalitz plots are divided into bins that have been chosen to maximise the statistical sensitivity. The bins are divided into symmetric halves along the line $m_{-}^{2} = m_{+}^{2}$, where $m_{\pm} \equiv m_{K^{0}_{s}h^{\pm}}$. Opposite bins are labelled $\pm i$ where the positive sign is taken for the bin in which $m_{-}^{2} > m_{+}^{2}$. Fig. [1] shows the binning of the $K^{0}_{s}\pi^{+}\pi^{-}$ and $K^{0}_{s}K^{+}K^{-}$ Dalitz plots used in the analysis.

The amplitude ratio and relative CP-conserving strong phase difference between favoured and suppressed $B^{\pm} \rightarrow DK^{\pm}$ decays are denoted $r_{B}$ and $\delta_{B}$, respectively. These quantities are combined with $\gamma$ to produce the observables $x_{\pm}$ and $y_{\pm}$

$$x_{\pm} \equiv r_{B} \cos(\delta_{B} \pm \gamma), \quad y_{\pm} \equiv r_{B} \sin(\delta_{B} \pm \gamma).$$

These quantities are henceforth referred to as the ‘CP observables’.

The yields of $B^{\pm}$ events in the Dalitz plot bin labelled $i$ are

$$\Gamma_{\pm i}(B^{-}) = n^{-}(K_{\pm i} + r_{B}^{2}K_{\mp i} + 2\sqrt{K_{i}K_{-i}}(x_{-}c_{i} \pm y_{-}s_{i})), \quad (2)$$

$$\Gamma_{\pm i}(B^{+}) = n^{+}(K_{\mp i} + r_{B}^{2}K_{\pm i} + 2\sqrt{K_{i}K_{-i}}(x_{+}c_{i} \mp y_{+}s_{i})), \quad (3)$$

where $K_{\pm i}$ is the efficiency-corrected yield of flavour-tagged candidates in bin $\pm i$ and the $n^{\pm}$ are normalisation constants. The formalism is identical for flavour-tagged $K^{0}_{s}\pi^{+}\pi^{-}$ and $K^{0}_{s}K^{+}K^{-}$ decays but the values of $c_{i}$, $s_{i}$ and $K_{i}$ differ between the two modes.

A preliminary analysis of a data sample corresponding to an integrated luminosity of 2.0 fb$^{-1}$ in $pp$ collisions at a centre-of-mass energy of 8 TeV collected in 2012 by LHCb is presented in this note. The four CP observables are measured using a simultaneous fit to the $B^{\pm} \rightarrow (K^{0}_{s}\pi^{+}\pi^{-})_{D}K^{\pm}$ and $B^{\pm} \rightarrow (K^{0}_{s}K^{+}K^{-})_{D}K^{\pm}$ Dalitz plots. This method has
previously been pursued by LHCb\cite{2} using a data sample corresponding to an integrated luminosity of 1.0 fb$^{-1}$ at a centre-of-mass energy of 7 TeV collected in 2011. The current analysis largely follows the same procedure as that of the 2011 study, but benefits from improvements in the selection strategy.

The LHCb detector\cite{3} is a single-arm forward spectrometer covering the pseudorapidity range 2 < $\eta$ < 5, designed for the study of particles containing $b$ or $c$ quarks. The detector includes a high precision tracking system consisting of a silicon-strip vertex detector (VELO) surrounding the $pp$ interaction region, a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes placed downstream. The combined tracking system has momentum resolution $\Delta p/p$ that varies from 0.4% at 5 GeV/$c$ to 0.6% at 100 GeV/$c$, and impact parameter (IP) resolution of 20 $\mu$m for tracks with high transverse momentum. The IP of a track is defined as the distance of closest approach of that track to a primary vertex (PV) that has been produced in a proton-proton interaction. Charged hadrons are identified using two ring-imaging Cherenkov (RICH) detectors\cite{4}. Photon, electron and hadron candidates are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter and a hadronic calorimeter. Muons are identified by a system composed of alternating layers of iron and multiwire proportional chambers.

The trigger\cite{5} consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage which applies a full event reconstruction. In the analysis presented in this note several systematic uncertainties are evaluated using Monte Carlo simulated data in which heavy flavour decays have been generated, propagated through the detector, and processed with the LHCb reconstruction software.

Figure 1: (Left) division of $K^0_S\pi^+\pi^-$ Dalitz plot into eight pairs of bins and (right) division of $K^0_SK^+K^-$ Dalitz plot into two pairs of bins. These plots have been produced using bitmaps of the $D$ decay amplitude that were provided by the BaBar collaboration.
2 Selection

In this section we describe the procedure used to select and reconstruct the $B^\pm \rightarrow (K^0_S h^+ h^-)D h^\pm$ candidates that are used to determine the $CP$ observables. The $h^\pm$ particle produced directly in the $B^\pm$ decay is denoted the ‘bachelor’ hadron and is either a kaon or a pion. The decay $B^\pm \rightarrow D\pi^\pm$ is an essential control mode in this analysis, as detailed in Sect. 4.

Candidates considered in the analysis are required to pass both hardware and software trigger requirements. At least one of the two following criteria must be satisfied at the hardware stage: either the $B^\pm$ candidate leaves a deposit with high transverse energy in the hadronic calorimeter, or another object in the event fulfils any of the hadronic or leptonic trigger requirements. At the software selection stage, the $B^\pm$ candidate must consist of at least one track with transverse momentum larger than 1.7 GeV/c and an IP $\chi^2$ with respect to the PV of at least 16. The IP $\chi^2$ is defined as the difference between the $\chi^2$ of the PV reconstructed with and without the considered track. The decay vertex of the candidate is required to be displaced from the PV and the sum of the transverse momenta of its constituent tracks must be large.

We consider $K^0_S$ candidates in which both pions have reconstructed segments in all the tracking stations (‘long long’ or LL) and candidates for which the pions have not been reconstructed in the VELO (‘downstream downstream’ or DD). We require that the flight distance $\chi^2$ of LL $K^0_S$ candidates with respect to the $D$ vertex is larger than 100. The mass of the $D$ candidate must lie within 25 MeV/$c^2$ of the known $D$ mass [6]. We require that information on the bachelor particle from the RICH detectors is present and that the bachelor has momentum smaller than 100 GeV/c, which is the value above which the RICH detectors are unable to perform reliable particle identification.

When determining the distribution of candidates across the $K^0_S h^+ h^-$ Dalitz plot we impose an additional fit to the full $B^\pm$ decay chain. The fit constrains the $B^\pm$ candidate to point towards the PV and the $D$ and $K^0_S$ candidates to have their known masses. The constrained fit improves the $B^\pm$ mass resolution and therefore provides greater discrimination between signal and background; additionally, it ensures that all candidates lie within the physical region of the Dalitz plot. An additional fit in which only the $B^\pm$ pointing and $D$ mass constraints are used is employed to aid discrimination between genuine and background $K^0_S$ candidates. After this fit has been applied we require that the mass of the $K^0_S$ meson lies within 15 MeV/$c^2$ of the known $K^0_S$ mass. Both of these fits must converge for a candidate to be used in the analysis.

A multivariate approach is employed to improve the selection relative to that used in 2011. We use a Boosted Decision Tree [7] (BDT) trained on simulated signal and background taken from the high $B^\pm$ mass sideband (5800–7000 MeV/$c^2$) of data collected in 2011. The BDT uses the following variables: the logarithm of the IP $\chi^2$ of the $D$ daughters and also of the bachelor particle, the logarithm of the IP $\chi^2$ of the $K^0_S$ daughter pions (LL only), the logarithm of the $D$ IP $\chi^2$ with respect to the PV, the $B^\pm$ IP $\chi^2$ with respect to the PV, the logarithm of the $B^\pm$ flight distance $\chi^2$ from the PV, the $B^\pm$ and $D$ momenta, a variable designed to ensure the $B$ candidate is well isolated from other tracks...
in the event and the $\chi^2$ of the decay chain fit in which $D$ and $K^0_S$ masses are constrained.

Additional selection criteria are applied to the samples before choosing the BDT working point. The cosine of the angle between the $D$ momentum vector and the vector connecting its decay vertex to the PV must be larger than 0.99. The same requirement is placed on $K^0_S$ candidates; for $B^\pm$ candidates with a LL (DD) daughter $K^0_S$ this variable must be larger than 0.9999 (0.99995). The displacement along the beamline between the $D$ and $B^\pm$ decay vertices is required to be positive in order to remove charmless $B^\pm$ decays. Particle identification (PID) requirements are placed on the bachelor to separate $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ candidates. PID criteria are also applied to the $D$ daughter kaons for the final state $K^0_SK^+K^-$. A proton veto is placed on the bachelor candidate in the $B^\pm \to DK^\pm$ sample to suppress possible contamination from $b$-baryon and combinatorial decays.

The BDT working point is chosen to maximise purity (defined as the ratio of the number of signal candidates to the total number of candidates) while retaining the same signal yield per pb$^{-1}$ as the selection criteria that were used in the analysis of 2011 data. The BDT is particularly effective at removing combinatorial backgrounds; we retain a similar signal yield per pb$^{-1}$ as for the 2011 analysis but reduce the level of combinatorial background by $\sim 40\%$. Pseudo-experiments indicate this choice of BDT working point is more efficient for the measurement of the $CP$ violation parameters than another possibility with the same level of combinatorial background as in 2011 and $\sim 10\%$ higher signal yield.

In events with multiple candidates, the candidate with the lowest $\chi^2$ of the fit to the full decay chain, as described above, is retained. Such events only occur below the per-mille level.

### 3 Invariant mass spectrum fit

An unbinned extended maximum-likelihood fit is performed on the $B^\pm$ mass to discriminate between the signal and the various categories of background. The samples have been divided into $B^\pm \to D\pi^\pm$ and $B^\pm \to DK^\pm$ candidates using the bachelor PID requirement described in the previous section.

The signal is parameterised by a Gaussian distribution with asymmetric tails; all parameters are common to the $K^0_S\pi^+\pi^-$ and $K^0_SK^+K^-$ fits. The mean of the distribution and the tail parameters (denoted $\alpha_{L,R}$) are shared between both modes ($B^\pm \to D\pi^\pm$, $B^\pm \to DK^\pm$) and $K^0_S$ types (LL, DD). The ratio of widths of Gaussian functions between the $D\pi^\pm$ and $DK^\pm$ distributions is shared between the two $K^0_S$ samples and fixed to 1.07 based on fits to simulation. The ratio of yields between the $DK^\pm$ and $D\pi^\pm$ distributions is shared between the two $K^0_S$ samples.

$B^\pm \to D\pi^\pm$ candidates in which the pion is misidentified as a kaon form a significant background to the $B^\pm \to DK^\pm$ signal. These candidates are modelled with a Crystal Ball distribution. All shape parameters (mean, width, $n$ and $\alpha$) are shared between both $K^0_S$ types and between the decay modes $K^0_S\pi^+\pi^-$ and $K^0_SK^+K^-$. The yield is constrained relative to the $B^\pm \to D\pi^\pm$ signal yield according to determined particle
identification efficiencies.

The parameterisation of low-mass candidates is determined from an admixture of simulated backgrounds weighted by their branching fractions. This shape is shared between both $K_S^0$ types and between the decay modes $K_S^0\pi^+\pi^-$ and $K_S^0K^+K^-$. The parameterisation of low-mass candidates is determined from an admixture of simulated backgrounds weighted by their branching fractions. This shape is shared between both $K_S^0$ types and between the decay modes $K_S^0\pi^+\pi^-$ and $K_S^0K^+K^-$. Purely combinatorial candidates are parameterised by a first-order polynomial. In the $B^\pm \to (K_S^0K^+K^-)_D K^\pm$ sample, the slope is fixed to zero for both $K_S^0$ types because there are not enough candidates to fit the shape reliably. Figs. $2\,3$ show the invariant mass distributions of the selected candidates.

![Figure 2](image1.png)

![Figure 3](image2.png)

Figure 2: Fits to $B^\pm \to (K_S^0\pi^+\pi^-)_D K^\pm$ and $B^\pm \to (K_S^0\pi^+\pi^-)_D \pi^\pm$ invariant mass spectra in data for (top row) LL $K_S^0$ candidates and (bottom row) DD $K_S^0$ candidates. The total fit PDF is shown by a solid blue line and the various components are: $B^\pm \to DK^\pm$ (red, dark), $B^\pm \to D\pi^\pm$ (green, light), low mass background (pink dashed), and combinatorial background (blue dotted).

We define the ‘signal region’ to cover the $B^\pm$ mass range of 5247–5317 MeV/c$^2$. The yields of each signal and background category in the signal region are shown in Table $1\,2$. Table $2$ shows the purity, defined as the ratio of the signal yield to the total yield in the
Figure 3: Fits to $B^\pm \to (K^0_S K^+ K^-)_D K^\pm$ and $B^\pm \to (K^0_S K^+ K^-)_D \pi^\pm$ invariant mass spectra in data for (top row) LL $K^0_S$ candidates and (bottom row) DD $K^0_S$ candidates. The total fit PDF is shown by a solid blue line and the various components are: $B^\pm \to DK^\pm$ (red, dark), $B^\pm \to D\pi^\pm$ (green, light), low mass background (pink dashed), and combinatorial background (blue dotted).

signal region. The Dalitz plots for each $B^\pm \to DK^\pm$ decay mode are shown in Figs. 4 and 5. All candidates in the signal region are shown.

4 Fit to determine CP observables

The $D$ meson kinematics, the values of $K_i$ and the $CP$ parameters $x_\pm$ and $y_\pm$ govern the distribution of $B^\pm \to DK^\pm$ candidates across the Dalitz plots. Mass fits are simultaneously performed to the $B^\pm \to DK^\pm$ candidates in each Dalitz plot bin and the $x_\pm$ and $y_\pm$ parameters are determined. In conjunction with this we fit the $B^\pm \to D\pi^\pm$ candidates; the observed yields of this decay determine the misidentified yield in the corresponding $B^\pm \to DK^\pm$ bin as well as the efficiency corrected $K_i$ parameters.
Table 1: Yields of each signal and background category in the signal region. The category ‘$DK^\pm$ mis-ID’ indicates $B^\pm \to D\pi^\pm$ candidates that are misidentified as $B^\pm \to DK^\pm$ signal.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$D \to K^0_S \pi^+\pi^-$</th>
<th>$D \to K^0_S K^+K^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LL DD</td>
<td>LL DD</td>
</tr>
<tr>
<td>$DK^\pm$ signal</td>
<td>422 ± 14 964 ± 32</td>
<td>61 ± 3 140 ± 5</td>
</tr>
<tr>
<td>$DK^\pm$ mis-ID</td>
<td>31 ± 5 67 ± 8</td>
<td>4 ± 2 10 ± 3</td>
</tr>
<tr>
<td>$DK^\pm$ combinatorial</td>
<td>13 ± 4 22 ± 5</td>
<td>1 ± 1 3 ± 1</td>
</tr>
<tr>
<td>$DK^\pm$ low mass</td>
<td>22 ± 2 60 ± 3</td>
<td>4 ± 1 8 ± 1</td>
</tr>
<tr>
<td>$D\pi^\pm$ signal</td>
<td>6709 ± 85 15276 ± 136</td>
<td>961 ± 31 2211 ± 46</td>
</tr>
<tr>
<td>$D\pi^\pm$ combinatorial</td>
<td>50 ± 5 201 ± 11</td>
<td>19 ± 3 31 ± 4</td>
</tr>
<tr>
<td>$D\pi^\pm$ low mass</td>
<td>63 ± 1 145 ± 2</td>
<td>9 ± 1 21 ± 1</td>
</tr>
</tbody>
</table>

Table 2: Purity for each decay type in the signal region.

<table>
<thead>
<tr>
<th>$B^\pm$ decay mode</th>
<th>$D \to K^0_S \pi^+\pi^-$</th>
<th>$D \to K^0_S K^+K^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LL DD</td>
<td>LL DD</td>
</tr>
<tr>
<td>$B^\pm \to DK^\pm$</td>
<td>(86.4 ± 1.3)% (86.6 ± 0.9)%</td>
<td>(86.0 ± 2.8)% (87.1 ± 1.9)%</td>
</tr>
<tr>
<td>$B^\pm \to D\pi^\pm$</td>
<td>(98.4 ± 0.1)% (97.8 ± 0.0)%</td>
<td>(97.2 ± 0.1)% (97.7 ± 0.1)%</td>
</tr>
</tbody>
</table>

We split the data in categories depending on the decay type ($D\pi^\pm$ or $DK^\pm$), $K^0_S$ type (LL or DD), $B$ charge (plus or minus) and which Dalitz plot bin the event falls into. The log likelihood is the sum of the log likelihoods for each category of candidates in every bin of the $D^0$ Dalitz plot

$$\log L = \sum_{\text{charge}} \sum_{\text{LL,DD}, K^0_S} (\log L_{D\pi^\pm} + \log L_{DK^\pm}).$$

Figure 4: Dalitz plots for $B^\pm \to (K^0_S \pi^+\pi^-)_D K^\pm$ decays; (left) $B^+$, (right) $B^-$. 

LHCb preliminary
$|L| dr = 2.0 fb^{-1}$

LHCb preliminary
$|L| dr = 2.0 fb^{-1}$
The log likelihood for $D\pi^{\pm}$ candidates is determined by summing the log likelihoods over all the bins in Dalitz space (labelled $-8$ to $+8$)

$$\log \mathcal{L}_{D\pi^{\pm}} = \sum_{i=-8,\neq0}^{8} \log \left( N_{D\pi^{\pm},\text{sig}}^{i} S_{D\pi^{\pm}}(m_{D\pi^{\pm}}) + \sum_{j=1}^{2} N_{D\pi^{\pm},\text{bkg},j}^{i} B_{D\pi^{\pm},j}(m_{D\pi^{\pm}}) \right),$$  \hspace{0.5cm} (5)

where $S_{D\pi^{\pm}}$ is the signal shape, $B_{D\pi^{\pm},\{1,2\}}$ are the two background shapes and the yields of these three components, $N_{D\pi^{\pm},\text{sig}}^{i}$ and $N_{D\pi^{\pm},\text{bkg},\{1,2\}}^{i}$, are varied independently in each bin. The log likelihood for $B^{\pm} \rightarrow D K^{\pm}$ candidates is

$$\log \mathcal{L}_{DK^{\pm}} = \sum_{i=-8,\neq0}^{8} \log \left( N_{DK^{\pm},\text{sig}}^{i} S_{DK^{\pm}}(m_{DK^{\pm}}) + \sum_{j=1}^{3} N_{DK^{\pm},\text{bkg},j}^{i} B_{DK^{\pm},j}(m_{DK^{\pm}}) \right),$$  \hspace{0.5cm} (6)

where in this case there are three background components, and the signal yield is determined as follows. The yield of $B^{\pm} \rightarrow D K^{\pm}$ candidates in each bin is

$$Y_{-i}^{\pm} \propto N_{D\pi^{\pm},\text{sig}}^{i} + r_{B}^{2} N_{D\pi^{\pm},\text{sig}}^{i} + 2 \sqrt{N_{D\pi^{\pm},\text{sig}}^{i} N_{D\pi^{\pm},\text{sig}}^{i}} (x_{-i} \pm y_{-i}),$$  \hspace{0.5cm} (7)

$$Y_{+i}^{\pm} \propto N_{D\pi^{\pm},\text{sig}}^{i} + r_{B}^{2} N_{D\pi^{\pm},\text{sig}}^{i} + 2 \sqrt{N_{D\pi^{\pm},\text{sig}}^{i} N_{D\pi^{\pm},\text{sig}}^{i}} (x_{+i} \mp y_{+i}),$$  \hspace{0.5cm} (8)

for $B^{-}$ and $B^{+}$, respectively, where we have used the $D\pi^{\pm}$ yield in each bin to represent $\varepsilon_{i} K_{i}$; we assume efficiencies in opposite bins are the same and that there is no interference in the $B^{\pm} \rightarrow D\pi^{\pm}$ system (i.e. the value of $r_{B}(D\pi^{\pm})$ is zero). The normalised yield $N_{DK^{\pm},\text{sig}}^{i}$ is then

$$N_{DK^{\pm},\text{sig}}^{i} = N_{DK^{\pm},\text{tot}} \frac{Y_{i}^{\pm}}{\sum_{i=-8,\neq0}^{8} Y_{i}^{\pm}},$$  \hspace{0.5cm} (9)
where \( N_{DK^\pm, \text{tot}} \), the total \( DK^\pm \) yield, is allowed to vary in the fit.

In the binned simultaneous fit the following parameters are allowed to vary freely: background yields, yields of \( B^\pm \rightarrow D\pi^\pm \) candidates in each Dalitz plot bin, the total \( B^\pm \rightarrow DK^\pm \) yield and \( x_\pm \) and \( y_\pm \). The shape parameters determined as described in Sect. 3 are fixed; the systematic uncertainty related to this procedure is described in Sect. 5.

### 5 Systematic uncertainties

In this section we list the sources of systematic uncertainty in the analysis. The systematic uncertainties are summarised in Table 3 and explained in more detail in the following text.

<table>
<thead>
<tr>
<th>Component</th>
<th>( \sigma(x_+) )</th>
<th>( \sigma(x_-) )</th>
<th>( \sigma(y_+) )</th>
<th>( \sigma(y_-) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>3.1</td>
<td>3.2</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Strong-phase systematic</td>
<td>0.6</td>
<td>0.9</td>
<td>1.9</td>
<td>1.6</td>
</tr>
<tr>
<td>Interference effects in control mode</td>
<td>1.6</td>
<td>0.6</td>
<td>1.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Global fit shape parameters</td>
<td>0.2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Efficiency effects</td>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Migration</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Partially reconstructed background</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>PID efficiency</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Shape of misidentified ( B \rightarrow D\pi )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Bias correction</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Total experimental systematic</td>
<td>1.6</td>
<td>0.9</td>
<td>1.4</td>
<td>2.2</td>
</tr>
</tbody>
</table>

There is a systematic uncertainty related to the imprecise knowledge of \( c_i \) and \( s_i \), denoted the ‘Strong-phase systematic’ in the summary table. To determine the magnitude of this uncertainty we smear the \( c_i \) and \( s_i \) central values by their correlated uncertainties and repeat the fit many times, taking the resulting widths of the \( x_\pm \) and \( y_\pm \) distributions as the uncertainties. The magnitude of this systematic uncertainty is reduced compared to 2011. As has been verified in pseudo-experiments, the stochastic variation of the yields in each Dalitz plot bin induces a sample-dependent variation in the precise value of this uncertainty [2] but this variation, and the overall magnitude of the uncertainty, is expected to decrease as the yield of \( B^\pm \) candidates increases.

The dominant source of systematic uncertainty on \( x_\pm \) and \( y_- \) is the assumption of no interference in the \( B^\pm \rightarrow D\pi^\pm \) system, denoted ‘Interference effects in control mode’ in the summary table. The magnitude of this uncertainty is evaluated by modelling some interference and \( CP \) violation in the \( D\pi^\pm \) system and refitting with the initial assumption. To model the interference we use a conservative value of \( r_B(D\pi^\pm) = 0.02 \).
The systematic uncertainty related to the global fit shape is evaluated by varying the shape parameters, taking into account their uncertainties and correlations, before performing the fit to determine $x_\pm$ and $y_\pm$. This is repeated many times and the corresponding width on the distributions is taken to be the uncertainty.

Uncertainties related to the selection efficiency across the Dalitz plot and migration between different bins are evaluated using simulated data and are found to be small in magnitude. There are additional uncertainties related to partially reconstructed background, misidentified $B^\pm \to D\pi^\pm$ candidates, the efficiency of identification of the bachelor particle and correction for any inherent bias in the fitter are all found to have very minor influence on the final result.

The ‘Total experimental systematic’ entry in the table has been determined by adding the individual sources of systematic uncertainty, except the strong-phase systematic, in quadrature.

6 Results

Using a data sample corresponding to an integrated luminosity of 2.0 fb$^{-1}$ collected by the LHCb experiment in 2012 we have measured the following CP violating observables in a model-independent analysis of $B^\pm \to (K^0_s h^+ h^-) D K^\pm$ decays

$$x_+ = (-8.7 \pm 3.1 \pm 1.6 \pm 0.6) \times 10^{-2},$$
$$x_- = (5.3 \pm 3.2 \pm 0.9 \pm 0.9) \times 10^{-2},$$
$$y_+ = (0.1 \pm 3.6 \pm 1.4 \pm 1.9) \times 10^{-2},$$
$$y_- = (9.9 \pm 3.6 \pm 2.2 \pm 1.6) \times 10^{-2},$$

where the first uncertainty is statistical, the second arises from systematic effects from the method or detector considerations and the third from strong-phase measurements used in the fit. These are the most precise measurements of $x_\pm$ and $y_\pm$ to date.

The confidence intervals on the $(x_+, y_+)$ and $(x_-, y_-)$ planes, using the statistical uncertainties and correlations only, are shown in Fig. 6.

7 Combination with 2011 and implications for the CKM angle $\gamma$

The measurements of $x_\pm$ and $y_\pm$ from the current analysis are compatible with those found in 2011, and so a combination of the two sets of results is performed. This combination takes account of the known correlations of the systematic uncertainties between the two analyses. Full correlation is assumed between the strong-phase uncertainties in the two data sets. In other cases where this information is not available, full correlation
Figure 6: Confidence intervals on the $(x, y)$ plane for $B^+$ and $B^-$ data collected in 2012 using the statistical uncertainties and correlations only. The star indicates the central value and the contours indicate the $1\sigma$, $2\sigma$ and $3\sigma$ boundaries moving from the centre outwards.

is conservatively assigned. The results that were obtained in 2011 are

\[
\begin{align*}
x_+ &= (-10.3 \pm 4.5 \pm 1.8 \pm 1.4) \times 10^{-2}, \\
y_+ &= (-0.9 \pm 3.7 \pm 0.8 \pm 3.0) \times 10^{-2},
\end{align*}
\]

The following results are obtained for the combined $CP$ parameters

\[
\begin{align*}
\langle x_+ \rangle &= (-8.9 \pm 3.1) \times 10^{-2}, \\
\langle y_+ \rangle &= (-0.1 \pm 3.7) \times 10^{-2},
\end{align*}
\]

The correlation matrix for the combined parameters is given in Table 4.

<table>
<thead>
<tr>
<th></th>
<th>$x_+$</th>
<th>$x_-$</th>
<th>$y_+$</th>
<th>$y_-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_+$</td>
<td>1.000</td>
<td>-0.136</td>
<td>0.106</td>
<td>-0.186</td>
</tr>
<tr>
<td>$x_-$</td>
<td>-0.136</td>
<td>1.000</td>
<td>-0.031</td>
<td>-0.053</td>
</tr>
<tr>
<td>$y_+$</td>
<td>0.106</td>
<td>-0.031</td>
<td>1.000</td>
<td>-0.074</td>
</tr>
<tr>
<td>$y_-$</td>
<td>-0.186</td>
<td>-0.053</td>
<td>-0.074</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The results can be interpreted in terms of the underlying physics parameters $\gamma$, $r_B$ and $\delta_B$. This is done using the frequentist approach described in Ref. 2. The results are shown in Fig. 7 which show the two-dimensional projections of the confidence regions onto the $(\gamma, r_B)$ and $(\gamma, \delta_B)$ planes.

The solution for the physics parameters has a two-fold ambiguity: $(\gamma, \delta_B)$ and $(\gamma + 180^\circ, \delta_B + 180^\circ)$. Choosing the solution that satisfies $0 < \gamma < 180^\circ$ yields $\gamma = (57 \pm 16)^\circ$, $r_B = (8.8_{-2.4}^{+2.3}) \times 10^{-2}$ and $\delta_B = (124_{-17}^{+15})^\circ$.  

Table 4: Correlation matrix between $CP$ parameters in combination of 2011 and 2012 results.
8 Conclusions

Using a data sample corresponding to an integrated luminosity of 2.0 fb$^{-1}$ collected by LHCb in 2012 we have measured the CP violation parameters $x_\pm$ and $y_\pm$ with the decays $B^\pm \rightarrow (K^0 h^+_\pm)_{D} K^\pm$. We find

$$\begin{align*}
x_+ &= (-8.7 \pm 3.1 \pm 1.6 \pm 0.6) \times 10^{-2}, \\
x_- &= (5.3 \pm 3.2 \pm 0.9 \pm 0.9) \times 10^{-2}, \\
y_+ &= (0.1 \pm 3.6 \pm 1.4 \pm 1.9) \times 10^{-2}, \\
y_- &= (9.9 \pm 3.6 \pm 2.2 \pm 1.6) \times 10^{-2}
\end{align*}$$

where the first uncertainty is statistical, the second arises from systematic effects from the method or detector considerations and the third from strong-phase measurements used in the fit.

Combining these results with those obtained using the 2011 dataset we find the following:

$$\begin{align*}
\langle x_+ \rangle &= (-8.9 \pm 3.1) \times 10^{-2}, & \langle x_- \rangle &= (3.5 \pm 2.9) \times 10^{-2}, \\
\langle y_+ \rangle &= (-0.1 \pm 3.7) \times 10^{-2}, & \langle y_- \rangle &= (7.9 \pm 3.8) \times 10^{-2}.
\end{align*}$$

The resulting best-fit values for the CP violating parameters $\gamma$, $r_B$ and $\delta_B$ are $(57 \pm 16)^\circ$, $(8.8^{+2.3}_{-2.4}) \times 10^{-2}$ and $(124^{+1.2}_{-1.5})^\circ$, respectively.
References

[1] CLEO collaboration, J. Libby et al., Model-independent determination of the strong-phase difference between $D^0$ and $\bar{D}^0 \to K_{S,L}^0 h^+ h^- (h = \pi, K)$ and its impact on the measurement of the CKM angle $\gamma/\phi_3$, Phys. Rev. D 82 112006.

[2] LHCb collaboration, R. Aaij et al., A model-independent Dalitz plot analysis of $B^\pm \to D K^\pm$ with $D \to K_{S,L}^0 h^+ h^- (h = \pi, K)$ decays and constraints on the CKM angle $\gamma$, Phys. Lett. B718 (2012) 43, arXiv:1209.5869.


Figs. 8–9 show the selected $B^\pm$ candidates with a logarithmic scale on the $y$-axis.

Figure 8: Fits to $B^\pm \to (K^0_{s}\pi^+\pi^-)_D K^\pm$ and $B^\pm \to (K^0_{s}\pi^+\pi^-)_D \pi^\pm$ invariant mass spectra in data for (top row) LL $K^0_{s}$ candidates and (bottom row) DD $K^0_{s}$ candidates. The total fit PDF is shown by a solid blue line and the various components are: $B^\pm \to DK^\pm$ (red, dark), $B^\pm \to D\pi^\pm$ (green, light), low mass background (pink dashed), and combinatorial background (blue dotted).
Figure 9: Fits to $B^\pm \rightarrow (K^0_S K^+ K^-)_D K^\pm$ and $B^\pm \rightarrow (K^0_S K^+ K^-)_D \pi^\pm$ invariant mass spectra in data for (top row) LL $K^0_S$ candidates and (bottom row) DD $K^0_S$ candidates. The total fit PDF is shown by a solid blue line and the various components are: $B^\pm \rightarrow DK^\pm$ (red, dark), $B^\pm \rightarrow D\pi^\pm$ (green, light), low mass background (pink dashed), and combinatorial background (blue dotted).