Angular analysis of the $B^0 \rightarrow K^{*}\mu\mu$ decay: status and prospects

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Third Workshop on Flavour Physics in the LHC Era: Theoretical and Experimental Views

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• The Standard Model is very successful (e.g. $B_s \rightarrow \mu \mu$)

Important to look for complementary decays, e.g. $B_d \rightarrow K^* \mu \mu$
Introduction

- The decay $B_d \rightarrow K^* \mu \mu$ is a laboratory to search for New Physics

Sensitive to $C_{(7,9,10)}$ and their right-handed counterparts ($C'_{(7,9,10)}$)
The angular decay rate

This should be interpreted for a given $q^2$ bin

$$\frac{1}{\Gamma} \frac{d^3(\Gamma + \bar{\Gamma})}{d \cos \theta_\ell \, d \cos \theta_K \, d\phi} = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell 
- F_L \cos^2 \theta_K \cos 2\theta_\ell + 
S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + S_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + 
S_5 \sin 2\theta_K \sin \theta_\ell \cos \phi + S_6 \sin^2 \theta_K \cos \theta_\ell + 
S_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + 
S_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right]$$

We will concentrate on the “blue” terms now
Observables where form factors cancel out have been proposed (see talk by J. Virto)

\[ F_L = \frac{A_0^2}{A_\parallel^2 + A_\perp^2 + A_0^2} = 1 - F_T \]

\[ P_1 = \frac{A_\perp^2 - A_\parallel^2}{A_\perp^2 + A_\parallel^2} + L \rightarrow R \]

\[ P_2 = \frac{\Re(A_\perp^* A_\parallel)}{|A_\perp|^2 + |A_\parallel|^2} - L \rightarrow R \]

\[ P_3 = \frac{\Im(A_\perp^* A_\parallel)}{|A_\perp|^2 + |A_\parallel|^2} - L \rightarrow R \]

\[ P_1 = \frac{2 S_3}{1 - F_L} = A_T^2 \]

\[ P_2 = \frac{S_6}{2(1 - F_L)} = \frac{1}{2} A_T^{Re} \]

\[ P_3 = -\frac{S_9}{(1 - F_L)} = -\frac{1}{2} A_T^{Im} \]
Early measurements

B-factories and CDF used angular projections

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\vartheta_K} = \frac{3}{2} F_L \cos^2 \vartheta_K + \frac{3}{4} (1 - F_L) \sin^2 \vartheta_K
\]

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\vartheta_l} = \frac{3}{4} F_L \sin^2 \vartheta_l + \frac{3}{8} (1 - F_L)(1 + \cos^2 \vartheta_l) + A_{FB} \cos \vartheta_l
\]

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\phi} = \frac{1}{2\pi} \left\{ 1 + \frac{1}{2} (1 - F_L) A_T^{(2)} \cos 2\phi + A_{\text{Im}} \sin 2\phi \right\}
\]

(early measurements showed moderate tension wrt the SM predictions at low $q^2$)

BaBar: S.Akar Lake Louise (2012)


LHCb strategy

\[ \phi \rightarrow \phi + \pi \text{ if } \phi < 0 \]

\[ \phi \text{ otherwise} \]

Variable transformation

Terms with \( \cos(\phi) \) and \( \sin(\phi) \) cancel out

Terms with \( \cos(2\phi) \) and \( \sin(2\phi) \) survive

\[
\frac{1}{\Gamma} \frac{d^3(\Gamma + \bar{\Gamma})}{d \cos \theta_\ell d \cos \theta_K d \phi} = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell \right. \\
- F_L \cos^2 \theta_K \cos 2\theta_\ell + S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + S_6^s \sin^2 \theta_K \cos \theta_\ell + \\
S_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \]

3d angular analysis \( \rightarrow \) better sensitivity
Analysis strategy (LHCb, 1fb⁻¹)

- S/B is about 0.25 in (5230, 5330)
- 900 ± 34 signal events
- BDT studied to keep the acceptance in the angles as flat as possible

Analysis performed in 6 $q^2$-bins

Simultaneous fit to the B-invariant mass and the three angles

<table>
<thead>
<tr>
<th>$q^2$ (GeV²/c⁴) range</th>
<th>Signal Yield</th>
<th>Background Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4m_{\mu}^2 &lt; q^2 &lt; 2.00$</td>
<td>162.4 ± 14.2</td>
<td>27.7 ± 3.8</td>
</tr>
<tr>
<td>$2.00 &lt; q^2 &lt; 4.30$</td>
<td>71.4 ± 10.7</td>
<td>37.1 ± 4.1</td>
</tr>
<tr>
<td>$4.30 &lt; q^2 &lt; 8.68$</td>
<td>270.5 ± 18.8</td>
<td>58.8 ± 5.5</td>
</tr>
<tr>
<td>$10.09 &lt; q^2 &lt; 12.90$</td>
<td>167.0 ± 14.9</td>
<td>41.7 ± 4.5</td>
</tr>
<tr>
<td>$14.18 &lt; q^2 &lt; 16.00$</td>
<td>113.0 ± 11.7</td>
<td>17.1 ± 3.0</td>
</tr>
<tr>
<td>$16.00 &lt; q^2 &lt; 19.00$</td>
<td>115.0 ± 12.4</td>
<td>23.9 ± 3.6</td>
</tr>
<tr>
<td>$1.00 &lt; q^2 &lt; 6.00$</td>
<td>195.2 ± 16.9</td>
<td>75.8 ± 6.0</td>
</tr>
<tr>
<td>$4m_{\mu}^2 &lt; q^2 &lt; 19.00$</td>
<td>900.0 ± 34.4</td>
<td>206.2 ± 10.3</td>
</tr>
</tbody>
</table>
Acceptance effects: LHCb

- Tuning of the LHCb simulation using data driven techniques

- Quality of the simulation verified by using the control channel $B_d \rightarrow K^*J/\psi$

- Using the simulation as a function of the three angles and $q^2$ to correct on an event-by-event basis
**Differential BR**

**LHCb Collaboration: LHCb-CONF-2012-008**

**Theory prediction from**

**Consistent with SM predictions**

BaBar: S. Akar Lake Louise (2012)
Consistent with SM predictions

Theory prediction from

BaBar: S.Akar Lake Louise (2012)

3rd Workshop on Flavour Physics in the LHC Era
Theory prediction from C. Bobeth, G. Hiller, D. van Dyk, JHEP 07, 067 (2011)


Consistent with SM predictions
LHCb Collaboration: LHCb-CONF-2012-008

Consistent with SM prediction

N.B. The LHCb angular basis is different than the one used by CDF, therefore this is the CP average observable
**LHCb Collaboration: LHCb-CONF-2012-008**

**Theory prediction from**

Consistent with SM predictions

**3rd Workshop on Flavour Physics in the LHC Era**
Zero-crossing point

$q_0^2$ extracted through a 2D fit to $q^2$ and $M(K\pi\mu\mu)$ of forward- and backward-going events separately.

**LHCb Collaboration:** LHCb-CONF-2012-008

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World's first measurement of the ZCP

$q_0^2 = 4.9^{+1.1}_{-1.3} \text{GeV}^2$

Good agreement with SM

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SM predictions: $4 - 4.3 \text{GeV}^2$

Some experimental details
Boundaries

\[ \begin{align*}
A_{FB} &\leq \frac{3}{4}(1 - F_L), \\
A_{Im} &\leq \frac{1}{2}(1 - F_L) \quad \text{and} \\
S_3 &\leq \frac{1}{2}(1 - F_L)
\end{align*} \]

• We give confidence levels in the physical region

• The confidence intervals quoted on the angular observables are constructed treating the other angular observables as nuisance parameters \( \rightarrow \) The full likelihood cannot be obtained as the simple product

NB: these are the same transformations needed to go to the transverse variables \((A_T^{Re}, A_T^{Im} \text{ and } A_T^2)\)
How about the Ps

- For a given $q^2$ value $S_3 = \frac{1}{2} (1-F_L) P_1$
- Why cannot you easily fit for $P_1$?

\[
\frac{1}{\Gamma} \frac{d^3(\Gamma + \bar{\Gamma})}{d \cos \theta_\ell \, d \cos \theta_K \, d \phi} = \frac{9}{16\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell \\
- F_L \cos^2 \theta_K \cos 2\theta_\ell + \frac{1}{2} (1 - F_L) P_1 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi \\
+ 2(1 - F_L) P_2 \sin^2 \theta_K \cos \theta_\ell - (1 - F_L) P_3 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right]
\]

- The relation "$S_3 = \frac{1}{2} (1-F_L) P_1$" is valid for a given $q^2$ point, but we are integrating over a relatively large $q^2$-bin
- However you can of course still fit for this Pdf
How about the $P_i$?

We measure the observables in relatively large $q^2$-bins

$$\langle F_L \rangle = \frac{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} F_L(q^2) \frac{d\Gamma}{dq^2} dq^2}{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} \frac{d\Gamma}{dq^2} dq^2}$$

- Different for products of two $q^2$-dependent observables

$$\langle A(q^2)B(q^2) \rangle \neq \langle A(q^2) \rangle \langle B(q^2) \rangle$$

- For the transverse observables we can write:

$$\langle (1 - F_L(q^2))P_1(q^2) \rangle = \langle 1 - F_L(q^2) \rangle \langle \widetilde{P}_1(q^2) \rangle$$

$$\langle \widetilde{P}_1(q^2) \rangle = \frac{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} P_1 \frac{d\Gamma}{dq^2} (1 - F_L(q^2)) dq^2}{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} \frac{d\Gamma}{dq^2} (1 - F_L(q^2)) dq^2}$$

Transverse rate-averages which are theoretically clean have been given in S. Descombes-Genon et al., JHEP, 1301:048, 2013
So far all experiments have neglected lepton masses.

This assumption clearly breaks for $q^2 < 1 \text{GeV}^2/c^4$

Moreover:

$$J_1^C \neq -J_2^C$$

$$J_1^S \neq 3J_2^S$$

Using SM simulation the effect of neglecting lepton masses is 10-20% in the region $4m_\mu^2 < q^2 < 2.0 \text{ GeV}^2/c^4$

See talk by J.Camalich and by S. Jaeger

S. Jarger and J.Camalich arXiv:1212.2263

3rd Workshop on Flavour Physics in the LHC Era
We consider a 200MeV/c² mass window.

Possible interference with other higher $K\pi$ resonances (e.g. $K^*(1430)$ or non-resonant S-wave?)

Interference between S-wave and P-wave in $B \rightarrow J/\psi \ K^*$ seen by B-factories and LHCb
S-wave

\[
\frac{1}{\Gamma} \frac{d^3 \Gamma}{d \cos \theta_{\ell} \, d \cos \theta_K \, d \phi} = \frac{9}{36\pi} \left[ (1 - F_s) P_{df,K^*}(\cos \theta_K, \cos \theta_{\ell}, \phi) + P_{df,S}(\cos \theta_K, \cos \theta_{\ell}, \phi) \right]
\]

\[
P_{df,S} = \left[ F_S \sin^2 \theta_{\ell} + A_s \sin^2 \theta_{\ell} \cos \theta_K + A_S^{(4)} \sin \theta_K \sin 2\theta_{\ell} \cos \phi + A_S^{(5)} \sin \theta_K \sin \theta_{\ell} \cos \phi + A_S^{(7)} \sin \theta_K \sin \theta_{\ell} \sin \phi + A_S^{(8)} \sin \theta_K \sin 2\theta_{\ell} \sin \phi \right]
\]

\[
F_s = \text{S-wave fraction}
\]

\[
A_s^i = \text{Interference between the S-wave and the P-wave}
\]

\[
e.g. A_s \text{ is the interference between the S-wave and } A_{0L,R}^i
\]

J. Matias PRD 86, 094024 (2012),
D. Becirevic and A. Tayduganov arxiv:1207.4004

T. Blake et al., arxiv:1210.5279 for experimental point of view
S-wave

\[
\frac{1}{\Gamma} \frac{d^3 \Gamma}{d \cos \theta_\ell d \cos \theta_K d \phi} = \frac{9}{36\pi} \left[ \left( 1 - F_s \right) P_{df_{K^*}}(\cos \theta_K, \cos \theta_\ell, \phi) + P_{df_S}(\cos \theta_K, \cos \theta_\ell, \phi) \right]
\]

Normal \( B \rightarrow K^* \mu \mu \) Pdf

\[
P_{df_S} = \left[ F_S \sin^2 \theta_\ell + A_s \sin^2 \theta_\ell \cos \theta_K + A_S^{(4)} \sin \theta_K \sin 2\theta_\ell \cos \phi + A_S^{(5)} \sin \theta_K \sin \theta_\ell \cos \phi + A_S^{(7)} \sin \theta_K \sin \theta_\ell \sin \phi + A_S^{(8)} \sin \theta_K \sin 2\theta_\ell \sin \phi \right]
\]

After folding

\[
P_{df_S} = \left[ F_S \sin^2 \theta_\ell + A_s \sin^2 \theta_\ell \cos \theta_K \right]
\]

In LHCb-CONF-2012-008 systematic for S-wave associated by using \( B \rightarrow J/\psi K^* \)

We have been working to try to extract this directly from \( B \rightarrow K^* \mu \mu \)
(Very) Near Future

• Fit for the “clean” observables (as well)

• Account for lepton mass terms (in the lowest $q^2$ bin)

• Extract the S-wave contribution from $B \rightarrow K^* \mu \mu$
Future Prospects
The other observables

The red observables are also sensitive to New Physics... how much data do you need to measure them?

\[
\frac{1}{\Gamma} \frac{d^3(\Gamma + \bar{\Gamma})}{d \cos \theta_\ell d \cos \theta_K d\phi} = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell \right. \\
- F_L \cos^2 \theta_K \cos 2\theta_\ell + \\
S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + S_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + \\
S_5 \sin 2\theta_K \sin \theta_\ell \cos \phi + S_6^8 \sin^2 \theta_K \cos \theta_\ell + \\
S_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + \\
S_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right]
\]

\[
P_4' = \frac{2S_4}{\sqrt{(1-F_L)F_L}} \quad P_6' = -\frac{S_7}{\sqrt{(1-F_L)F_L}}
\]

\[
P_5' = \frac{S_5}{\sqrt{(1-F_L)F_L}} \quad P_8' = -\frac{2S_8}{\sqrt{(1-F_L)F_L}}
\]

Observables $H^{(i)}_T$ clean at high $q^2$

(C Bobeth et al. JHEP 1007 (2010) 098)
Using the transformations

\[ \phi \rightarrow -\phi \quad \text{if } \phi < 0 \]
\[ \phi \rightarrow \phi \quad \text{otherwise} \]

\[ P_{df} = \frac{9}{8\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_{\ell} - F_L \cos^2 \theta_K \cos 2\theta_{\ell} + \right. \]
\[ S_3 \sin^2 \theta_K \sin^2 \theta_{\ell} \cos 2\phi + \sqrt{F_L(1 - F_L)} P_5 \sin 2\theta_K \sin \theta_{\ell} \cos \phi \]

With similar techniques we can extract all observables.

Not all of them extracted simultaneously, i.e. global SM P-value or fitting of the Wilson coefficient requires some care.
When can LHCb do a full angular (not folding) fit?

Still under study!
Not in a very short timescale.
Future prospects

• Extract all $S_i$ and $P_i$ with folding techniques

• We can fit for the $S_i$ or equivalently for $P_i$ in $q^2$ bins (with no folding)

• We can try to fit for the amplitudes (?)

• Parametrize the $q^2$ dependence:
  – Easier experimental treatment of acceptance
  – Easier experimental treatment of lepton mass terms
  – Slightly more complicated comparison with the theory

• Complete fit of the S-wave contribution using also the $K\pi$ invariant mass

• Extract the CP asymmetry angular observables
Conclusions

- The decay \( B_d \rightarrow K^* \mu \mu \) is a sensitive probe for New Physics
- Several angular observables
- Up to now agreement with SM predictions
- We still have too measure more observables (some of them have never been measured)
- Statistically limited and several improvements can be made
Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism

George A. Akerlof

and Robert J. Shiller
Backup Slides
Differential partial rate:

\[
\frac{d\Gamma}{d\cos\Theta_K d\cos\Theta_\ell d\phi} = \frac{9}{32\pi} \left[ I_1^s \cdot (1 - \cos^2 \Theta_K) + I_1^c \cdot \cos^2 \Theta_K + (I_2^s \cdot (1 - \cos^2 \Theta_K) + I_2^c \cdot \cos^2 \Theta_K) \cdot (2 \cos^2 \Theta_\ell - 1) + I_3^s \cdot (1 - \cos^2 \Theta_K) \cdot (1 - \cos^2 \Theta_\ell) \cos 2\phi + I_6^s \cdot (1 - \cos^2 \Theta_K) \cdot \cos \Theta_\ell + I_9^s \cdot (1 - \cos^2 \Theta_K) \cdot (1 - \cos^2 \Theta_\ell) \sin 2\phi \right]
\]

\[
I_1^s = \frac{3}{4} \left( |A_\perp|^2 + |A_\parallel|^2 \right) \left( 1 - \frac{x}{3} \right) + \frac{x}{2} \left( |A_\perp|^2 + |A_\parallel|^2 \right) = \frac{1}{4} \left( |A_\perp|^2 + |A_\parallel|^2 \right) (3 + x)
\]

\[
I_1^c = |A_0|^2 \cdot (1 + x)
\]

\[
I_2^s = \frac{1}{4} \left( |A_\perp|^2 + |A_\parallel|^2 \right) (1 - x)
\]

\[
I_2^c = -|A_0|^2 \cdot (1 - x)
\]

\[
I_3^s = \frac{1}{2} \cdot (1 - x) \cdot \left( |A_\perp|^2 - |A_\parallel|^2 \right)
\]

\[
I_6^s = 2\sqrt{1-x} \cdot \text{Re} \left( A_{\parallel L}^* A_{\perp L} - A_{\parallel R}^* A_{\perp R}^* \right)
\]

\[
I_9^s = (1 - x) \cdot \text{Im} \left( A_{\parallel L}^* A_{\perp L} + A_{\parallel R}^* A_{\perp R}^* \right)
\]

\[
x = \frac{4m_\ell^2}{q^2}
\]

Sensitive to $C_7$

Sensitive to $C_7'$
\[
\begin{align*}
\frac{1}{\Gamma_1} I_1^S &= \left( \frac{3}{4} (1 - F_L) \times \left( 1 - \frac{4m^2}{3q^2} \right) + \frac{1}{\Gamma} \frac{4m^2}{q^2} \Re \left( A_L A_{L}^* R + A_{||} A_{||}^* R \right) \right) \sin^2 \theta_K \\
\frac{1}{\Gamma_1} I_1^C &= \left( F_L + \frac{1}{\Gamma} \frac{4m^2}{q^2} \times \left( |A_t|^2 + 2 \Re(A_0 A_0^* R) \right) \right) \cos^2 \theta_K \\
\frac{1}{\Gamma_2} I_2^S &= \frac{1}{4} (1 - F_L) (1 - \frac{4m^2}{q^2}) \sin^2 \theta_K \\
\frac{1}{\Gamma_2} I_2^C &= - F_L (1 - \frac{4m^2}{q^2}) \cos^2 \theta_K \\
\frac{1}{\Gamma} I_3 &= \frac{1}{2} (1 - F_L) A_T^2 \left( 1 - \frac{4m^2}{q^2} \right) \times \sin^2 \theta_K \\
\frac{1}{\Gamma} I_6 &= 2 A_T^R (1 - F_L) \sqrt{\left( 1 - \frac{4m^2}{q^2} \right)} \times \sin^2 \theta_K \\
\frac{1}{\Gamma} I_9 &= \frac{1}{2} (1 - F_L) A_T^m \left( 1 - \frac{4m^2}{q^2} \right) \times \sin^2 \theta_K
\end{align*}
\]

New amplitude terms

New kinematical terms

Frank Krüger, Joaquim Matias