MUON-ELECTRON UNIVERSALITY AND PCAC
IN MUON CAPTURE IN HYDROGEN

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This lecture is about muon capture in hydrogen in general; the discussion also takes into account the recent experimental results obtained by the Saclay-Bologna-CERN Collaboration\(^1\) working at Saclay on the capture process

\[ \mu^- + p = n + \nu_\mu \quad (t_n = 5.2 \text{ MeV}) \] \(\text{(1)}\)

observed by stopping negative muons in liquid hydrogen.

A negative muon stopped in hydrogen will disappear mostly through the decay channel

\[ \mu = e + \nu_\mu + \nu_e , \] \(\text{(2)}\)

the rate of which, \(\lambda_\mu\), has recently been accurately determined at Saclay\(^1\) by stopping positive muons. The Saclay results combined with all the results obtained in other laboratories give

\[ \frac{1}{\lambda_\mu} = \tau_\mu = 2197.093 \pm 0.052 \text{ ns} . \] \(\text{(3)}\)

According to the currently accepted theoretical frame, process (1) is described by the matrix element \(\text{ME}^2\):\(^2\)

\[ \text{ME} = (G/\sqrt{2}) \cos \theta_C \ell_\chi \cdot \ell_h , \]

where

\[ G \cos \theta_C = 1.395 \times 10^{-49} \text{ erg cm}^2; \]

\(\ell_\chi\) is the leptonic current, here assumed to be well known;

\(\ell_h = \ell_{hv} + \ell_{hA}\) is the hadronic current with

\[ \ell_{hv} = \bar{u}_n \left[ f_v(q^2) \gamma^\alpha + f_M(q^2) \sigma_{\alpha\beta} q_{\beta} - iq_{\alpha} f_S(q^2) \right] u_p \]

\[ \ell_{hA} = \bar{u}_n \left[ f_A(q^2) \gamma^\alpha + f_T(q^2) \sigma_{\alpha\beta} q_{\beta} - iq_{\alpha} f_P(q^2) \right] \gamma^5 u_p \] \(\text{(4)}\)

in which

\[ q^2 = -0.8767(m_\mu)^2 , \] \(\text{(5)}\)

where \(m_\mu\) is the muon mass.

Using these expressions, the rate \(\lambda(F)\) for process (1) is written

\[ \lambda(F) = A \cdot B(F) \text{ s}^{-1} , \quad F = 1, 0 , \]
with $A = 29.2$; $F$ is the total spin of the initial up system in process (1), and

$$B(0) = (G_V - 3G_A + G_P)^2$$

$$B(1) = (G_V + G_A)^2 - \frac{2}{3}(G_V + G_A)G_P + G_P^2,$$

where $G_V$, $G_A$, and $G_P$ are well-known expressions of the form factors $f_V$, $f_A$, $f_M$, $f_S$, $f_P$, and $f_T$ which can be found, for example, in Ref. 2.

Assuming vector current conservation (CVC), we arrive [at $q^2$ given by formula (5)] at

$$2Mf_M(q^2) = 3.5875 \pm 0.052,$$

$M$ being the nucleon mass, and

$$f_V(q^2) = 0.9748 \pm 0.0009$$

$$f_S(q^2) = 0.$$  

Assuming now the muon-electron universality, we get from the neutron beta-decay$^5$)

$$n = p + e + \nu_e,$$  

$$f_A(0) = -1.250 \pm 0.009.$$  

Moreover, from experiments on neutrinos and antineutrinos scattering on nucleons, we get$^5, 6$)

$$f_A(q^2) = f_A(0)/\left[1 - (q/m_A)^2\right]^2,$$  

with $m_A = 930 \pm 30$.

Using these assumptions, the rates $\lambda(F)$ now remain only a function of the combination

$$F = m_\mu f_P(q^2) + 2Mf_M(q^2),$$

i.e. of the sum of the pseudoscalar form factor and the tensor form factor.

One of the main objectives of the experiments on muon capture in hydrogen is to collect information about the two above-mentioned quantities $F$ and $f_A(0)$.

Regarding $m_\mu f_P$, this is quantitatively predicted starting from the partial conservation of the axial current (PCAC); we have$^2$

$$m_\mu f_P(q^2) = -\frac{2Mm_\mu}{\left[1 - (q/m_\mu)^2\right]} \left[\frac{f_A(q^2)}{m_A^2 + \frac{f_{\pi p}(q^2)f_{\pi p} - f_A(q^2)}}\right],$$  

(9)
where

\[ m_\pi \] is the pion mass;

\[ f_\pi \] is the pion decay constant, the value of which is

\[ f_\pi = 0.94 \pm 0.01 \quad \text{. (10)} \]

\[ f_{\pi \nu} \] is the pion-nucleon coupling constant, which for \( q^2 = m_\pi^2 \) is

\[ f_{\pi \nu}(m_\pi^2) = 1.41 \pm 0.01 \quad \text{.} \]

From Ref. 7 we obtain

\[ r(q^2) = f_{\pi \nu}(q^2)/f_{\pi \nu}(0) = 1/[1 - (q/m_A)^2] \quad \text{, (11)} \]

with \( m_A = 1250 \text{ MeV} \). From this we deduce that

\[ f_{\pi \nu}(0) = 1.39 \pm 0.01 \quad \text{. (12)} \]

From expression (12) evaluated at \( q^2 = 0 \) we get the Goldberger-Treiman (GT) relation

\[ f_A^{GT}(0) = 1.32 \pm 0.02 \quad \text{ (13)} \]

which, compared to Eq. (7), shows a disagreement of about 6%. For this reason we write Eq. (9) as

\[ m_\mu f_p(q^2) = -\frac{2Mm_\mu}{[1-(q/m_\pi)^2]} f_A(q^2) \left[ \frac{1}{m_\pi^2} + \frac{r(q^2)[1-(q/m_A)^2]^2 - 1}{q^2} \right] \]

\[ = C(q^2)f_A(0) \quad \text{, (14)} \]

with \( f_A(q^2) \) given by Eq. (8): using for \( f_A(0) \) the value \([\text{Eq. (7)}]\) given by the neutron beta-decay, one obtains

\[ m_\mu f_p(q^2) = -8.1 \pm 0.1 \quad \text{. (15)} \]

for the \( q^2 \) given by Eq. (5).

As far as \( f_T \) is concerned, experimentally there is no evidence of its contribution: so for the moment we put \( f_T = 0 \).

Let us now look at the experimental results shown in Tables 1 and 2.
Table 1
Capture in hydrogen gas

<table>
<thead>
<tr>
<th>Group</th>
<th>Capture rate $\lambda_s$ (s$^{-1}$)</th>
<th>Particle detected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dubna$^9$</td>
<td>686 ± 88</td>
<td>Neutron</td>
</tr>
<tr>
<td>CERN-Bologna$^9$</td>
<td>651 ± 57</td>
<td>Neutron</td>
</tr>
</tbody>
</table>

We get from Table 1 an average capture rate obtained experimentally by stopping negative muons in hydrogen gas:

$$\lambda_s^{\text{exp}} = 661 \pm 47 \text{ s}^{-1}.$$  \hspace{1cm} (16)

Table 2
Capture in liquid hydrogen

<table>
<thead>
<tr>
<th>Group</th>
<th>Capture rate $\lambda_s$ (s$^{-1}$)</th>
<th>Particle detected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago$^{10}$</td>
<td>428 ± 85</td>
<td>Neutron</td>
</tr>
<tr>
<td>CERN-Bologna$^{11}$</td>
<td>450 ± 50</td>
<td>Neutron</td>
</tr>
<tr>
<td>Columbia I$^{12}$</td>
<td>515 ± 85</td>
<td>Neutron</td>
</tr>
<tr>
<td>Columbia II$^{13}$</td>
<td>464 ± 42</td>
<td>Neutron</td>
</tr>
<tr>
<td>Saclay Coll.$^{11}$</td>
<td>444 ± 19</td>
<td>Electron</td>
</tr>
</tbody>
</table>

In the case of capture in liquid hydrogen, we cannot make an average of the measured rates, because of the different experimental conditions.

It should be remembered that stopping negative muons in $H_2$ will form the atomic and molecular muonic processes schematically presented in Fig. 1 with $\lambda_{pp}$ proportional to the hydrogen pressure. For liquid hydrogen $\lambda_{pp} \gg \lambda_0$; therefore, by stopping negative muons in liquid hydrogen we essentially observe capture from a molecular pup system, the time for the $\nu p \rightarrow \nu p$ process being quite small$^{14})$. On the other hand, stopping negative muons in gaseous hydrogen, where $\lambda_{pp} < \lambda_0$, we essentially measure capture from the atomic system $\nu p$, in particular from the singlet state$^{14}$), the rate of which is $\lambda_s$. 
The system pup\textsuperscript{15}) can be in a state of rotational angular momentum \( L = 1 \), the ortho state, or \( L = 0 \), the para state, the energy difference between the two states being 148 eV. It has been calculated that\textsuperscript{16}) from the \( \mu \)p neutral atomic system in a singlet state, at a liquid-hydrogen temperature, initially the formation of a \( (\mu \mu) \) in a para state is only 3 per mile of the ortho state. Therefore we will assume that initially all pup systems (as shown in Fig. 1) are in an ortho state. However, the para state\textsuperscript{15, 17}) can later be reached via an ortho-para transition, the rate of which will be called \( \lambda_{OP} \). It should be mentioned that the hydrogen to be used in a muon capture experiment must be quite free from deuterium or any other impurity (y) in order to avoid the transfer processes which, however, can take place only in a neutral \( (\mu \mu \) or \( \mu d \)) system:

\[
\mu p + d = \mu d + p ,
\]

\[
\mu p + y = \mu y + p .
\]

From the theory and using values (7), (14), and (15), we get the following predictions for the capture rates:

a) From the neutral \( \mu p \) atom:

Singlet \( \lambda(0) = \lambda_s = 673 \text{ s}^{-1} \)

Triplet \( \lambda(1) = \lambda_t = 12 \text{ s}^{-1} \) \hspace{1cm} (17)

Let \( R \) be the ratio \( \lambda_t / \lambda_s \).

b) From a pup molecule:

Ortho-state \( \lambda_{OM} = (2\gamma_0 \lambda_s / 4)\{3 + R\} = 513 \text{ s}^{-1} \) \hspace{1cm} (18)

Para-state \( \lambda_{PM} = (2\gamma_p \lambda_s / 4)\{3R + 1\} = 276 \text{ s}^{-1} \) \hspace{1cm} (19)

where\textsuperscript{17})

\[
2\gamma_0 = 1.009 \pm 0.001 ,
\]

\[
2\gamma_p = 1.143 \pm 0.001 .
\]

It is obvious that from either of the systems \( \mu p \) or \( p p p \) the negative muons will also decay at a rate, given by Eq. (3), \( \lambda_0 = 455147 \text{ s}^{-1} \), i.e. much bigger than any of the capture rates \( \lambda_s, \lambda_t, \lambda_{OM}, \lambda_{PM} \).

Until recently the situation was a bit confused, since it was not possible for all the experimental results obtained from the liquid hydrogen (Table 2) to be unambiguously analysed and compared with the theory, owing to the lack of knowledge, experimental as well as theoretical, of the rate of ortho-para transition \( \lambda_{OP} \). For negative muons stopped in gas, this problem clearly does not exist.
At this point, therefore, and since the information is needed later, let us find out from the hydrogen-gas data of Table 1, what can be said regarding the quantity $R = \frac{\lambda_s}{\lambda_0}$.

Fixing $f_A(0) = -1.250 \pm 0.009$ [Eq. (7)] (and $f_\pi = 0$), i.e. assuming muon-electron universality, we show in Fig. 2 how the singlet capture rate $\lambda_s$ and the quantity $R$ depend on the pseudoscalar coupling constant $\mu f_p$.

From the figure we see that in these conditions the experimental value (16) defines for $R$ a possible range of values given by

$$R_{\text{min}} = 0.01 \leq R \leq 0.04 = R_{\text{max}}.$$  

We can also proceed in a different way. Alternatively, we can analyze the result (16), assuming the PCAC expression (14), without however fixing the value of $f_A(0)$ (therefore not assuming muon-electron universality), and again see what range of $R$ and of $f_A(0)$ is fixed in this case. It happens that $R$ remains fixed within an interval contained in the region (20). This conclusion will be useful for what follows.

As we have already pointed out, in order to interpret the liquid-hydrogen data of Table 2 we need to know the ortho-para rate $\lambda_{\text{OP}}$.

In order to arrive at a value of $\lambda_{\text{OP}}$, as in Bleser et al.\textsuperscript{18}) an experiment has been performed at Saclay\textsuperscript{1}) in which the time distribution $dN_n/dt$ of the captured neutrons from Eq. (1), which were emitted after negative muons were stopped in liquid hydrogen, has been accurately measured.

It we neglect the short formation time of the process $\mu^- + p \rightarrow \pi^- + n$ and, with respect to $\lambda_0$, neglect the rates $\lambda_s$ and $\lambda_t$, it can easily be shown that we have with very good approximation\textsuperscript{14})

$$dN_n/dt = \left(1 + \frac{\lambda_{\text{OM}} - \lambda_{\text{PM}}}{\lambda_{\text{PM}}} e^{-\lambda_{\text{OP}} t}\right) e^{-\lambda_0 t},$$  

i.e. the distribution $dN_n/dt$ is completely determined by the quantities $\lambda_0$, $R$, and $\lambda_{\text{OP}}$.

In fact, since $\lambda_{\text{OP}} < \lambda_0$, the ortho-para decay makes the yield (21) similar to an exponential, with the decay parameter $\tau_n$ given by

$$\frac{1}{\lambda_n} = \tau_n = \frac{1}{\lambda_0} \left(1 - \frac{\lambda_{\text{OP}} \lambda_{\text{OM}}}{\lambda_0 \lambda_{\text{OM}}}ight).$$

If $\lambda_{\text{OP}} = 0$, then $\tau_n = 1/\lambda_0$ as expected.
The experimental result from the Saclay-Bologna-CERN Collaboration is\(^1\)

\[
\frac{1}{\lambda_n} - \frac{1}{\lambda_0} = 0.119 \pm 0.028 \ \mu s ,
\]

which indicates that \(\lambda_{OP} \neq 0\).

Let us see how it is possible to deduce the rate \(\lambda_{OP}\) from this experimental result. We have already said that the quantity (22), independent of any hypothesis contained in the process (1), is a function of \(R, \lambda_0\), and \(\lambda_{OP}\).

Assuming for \(\lambda_0\) the value given in Eq. (3), Fig. 3 shows all possible pairs of values \(R\) and \(\lambda_{OP}\) consistent with the experimental value (22).

To arrive at a value for the ortho-para transition rate \(\lambda_{OP}\) we must at this point use the results (20) obtained from the analysis of the gas data (which are independent of \(\lambda_{OP}\)).

Using also the limitation (20) from the experimental value (22) we get\(^1\)

\[
\lambda_{OP} = (4.6 \pm 1.4) \times 10^8 \ \text{s}^{-1} .
\]

Recently, the Dubna Group\(^1\) have analysed the stability of the pup molecule in the ortho state; they arrived at the following conclusions.

i) The ortho-pup system will form the complexes given in Table 3.

<table>
<thead>
<tr>
<th>Complex</th>
<th>([(\text{pup})\text{pe}]^+)</th>
<th>([(\text{pup})\text{e}])</th>
<th>([(\text{pup})2\text{p}2\text{e}]^+)</th>
<th>([(\text{pup})2\text{p}2\text{e}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_{OP}^i)</td>
<td>(10^8 \ \text{s}^{-1}) 4.3</td>
<td>6.3</td>
<td>7.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Population</td>
<td>(Q_1)</td>
<td>(Q_2)</td>
<td>(Q_3)</td>
<td>(Q_4)</td>
</tr>
</tbody>
</table>

ii) From each of these complexes they determine a value of the ortho-para conversion rate; these also are given in Table 3 and shown in Fig. 3.

iii) Taking for the muonic complexes of Table 3 a relative population \(Q_i\) as suggested by the corresponding electronic complexes, they obtain

\[
\lambda_{OP}^{\text{th}} = (7.1 \pm 1.2) \times 10^8 \ \text{s}^{-1} .
\]

The conclusions of the Dubna Group are in good agreement with the experimental result (23), and we will use this value in the following analysis.
From the capture values of Tables 1 and 2 and taking for \( f_A(0) \) the neutron beta-decay value (7) (\( \mu - e \) universality), we get

\[
F = m_{\mu} f_p + 2Mf_T = -8.7 \pm 1.9.
\] (25)

Comparing this with the expected value (15), we get

\[
2Mf_T = -0.6 \pm 1.9,
\] (26)

which shows that \( f_T \) is compatible with zero when consistency with PCAC is required. This agrees with the results obtained by Roesch et al.\(^{19}\),

\[
2Mf_T = -1.1 \pm 1.9.
\] (27)

and the limit obtained by Baker et al.\(^{6}\)

\[
|2Mf_T| < 2.
\] (28)

Hence we will assume \( f_T = 0 \). From Tables 1 and 2 we obtain\(^{1}\)

\[
\frac{m_{\mu} f_p}{F} = -8.7 \pm 1.9
\] (29)

at \( q^2 = 0.87 \ m^2 \), in very good agreement with the PCAC value given in Eq. (15)*).

On the other hand, we can assume only the PCAC expression (14) -- which is independent of the muon-electron universality, leaving \( f_A(0) \) free -- and use the experimental results of Tables 1 and 2 with the value given in Eq. (23) (with \( f_T = 0 \)) in order to extract a value for \( f_A^{(0)}(0) \). In this way we find

\[
f_A^{(0)}(0) = -1.240 \pm 0.04.
\] (30)

Of course, the error given in Eq. (30) does not take into account the possible systematic uncertainty due to the theoretical assumption of the PCAC expression (14) relating \( m_{\mu} f_p \) to \( f_S \); it must be said that if relation (14) is not exact, but approximated within up to 10%, as low-pion-energy physics seems to indicate, conclusion (30) remains valid. Inserting \( f_A^{(0)}(0) \) in expression (14) gives \( m_{\mu} f_p = -8.12 \pm 0.26 \).

Value (30) agrees very well with value (7), thus giving experimental confirmation of the muon-electron universality in the two low-energy processes (1) and (6) to a level of about 3\%, I think that this is the most important result that can be obtained from the data of Tables 1 and 2, using (22). As expected, value (30) disagrees with the Goldberger-Treiman \( f_A^{(1)}(0) \) value (13). The two main conclusions of this discussion, Eqs. (29) and (30), would be changed if a value of \( 2Mf_T > 2 \) was assumed.

*) Taking for \( \lambda_{OP} \) the value zero, as was done in the past, one would get

from Table 2 \( m_{\mu} f_p(q^2) = -14.5 \pm 1.8 \).
Regarding the induced pseudoscalar coupling constant, it is important to realize that muon-capture processes are the most sensitive to the $m_\mu E_\mu$ term, because at the $q^2$ given by Eq. (5) its contribution to process (1) is very near to its maximum: the contribution becomes quite negligible for values of $q^2$ much lower than the one given by Eq. (5), as in high-energy neutrinos scattering.

The unambiguous conclusions (29) or (30) are the main results that can at present be drawn from the muon-capture experiments of Tables 1 and 2. Their particular importance stems from the fact that no nuclear physics considerations are involved when process (1) is observed, stopping negative muons in hydrogen.

REFERENCES

See also G. Bardin, Thèse Doctorat d'Etat (1982), Univ. Paris-Sud
2) We will follow closely the theoretical exposition of H. Primakoff,
and use the same symbols.
4) Particle Data Group, Review of particle properties, Rev. Mod. Phys.
52 (1980).
See also J.H. Doede and R. Hildebrand, Proc. Int. Conf. on High-
11) E. Bertolini et al., Proc. Int. Conf. on High-Energy Physics, Geneva,
C. Rubbia, Proc. Int. Conf. on Fundamental Aspects of Weak Interactions,
Brookhaven, 1963, p. 278.
p. 219.
See also E. Zavattini, "in Proc. TRIUMF Muon Physics Facility Workshop,
Fig. 1 Atomic and molecular processes following the stopping of a negative muon in $H_2$.

Fig. 2 Relation between $\lambda_S$ and $m_\mu f_p$ assuming $\xi_T = 0$ and value (7) for $f_A(0)$ (muon-electron universality). It also shows, in the same conditions, the relation between $R$ and $m_\mu f_p$. Horizontal full lines define the region given by Eq. (20) obtained using the experimental value for the muon capture in gas (see Table 1): $661 \pm 47 \, s^{-1}$.
Fig. 3 Region of values for $R$ and $\lambda_{OP}$ between lines (a) and (b) allowed by the experimental value (22): the four arrows are placed at the values of $\lambda_{OP}^k$ for the four complexes of Table 3. The dashed line shows the region of $\lambda_{OP}$ determined by Eq. (24); the dotted line shows the region of $R$ defined by Eq. (20).