RAPID PHASE TRANSITIONS IN LOCAL SUSY GUTS

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ABSTRACT

We propose a class of SU(5) local SUSY GUTs exhibiting the following properties: i) local SUSY and the SU(5) gauge group are spontaneously broken when a field in the adjoint representation of SU(5) takes a VEV of the order of the grand unification scale $M_X$; ii) the non-zero VEV of that field follows because of finite temperature effects, independently of possible confining properties of the GUT; iii) the phase transition is extremely fast and the critical temperature is of the order of $10^3$ GeV; and, iv) the monopole production is suppressed.

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Supersymmetric grand unified theories (SUSY GUTs) are, nowadays, the only realistic framework to solve the gauge hierarchy problem\(^1\). However, there were unresolved difficulties common to all proposed global SUSY models. In particular, i) how to take into account the gravitational effects when the scale of SUSY breaking is \(M_3 = 10^{10} \text{ GeV}\) and ii) how to break SUSY so as to get a realistic low energy spectrum? Both problems were solved by the introduction of local SUSY GUTs, i.e., by coupling GUTs to \(N = 1\) supergravity. In that case, gravitational effects are automatically considered. Moreover, the super-Higgs phenomenon\(^2\) induces a soft breaking of global SUSY in agreement with low energy phenomenology\(^3\).

The super-Higgs mechanism is usually supposed to be driven by a singlet field\(^4\) (Polonyi field) which is completely decoupled in the flat limit (hidden sector) and takes a VEV of the order of the Planck mass \(M_p\). In this picture the hidden singlet is put in by hand just to break local SUSY, without any relation with the dynamics of the expanding Universe. On the other hand, the finite temperature induced GUT phase transition in the early Universe has been recently analyzed in the context of SU(5) global SUSY GUTs\(^5\)-\(^8\). It is thus tempting to identify in local SUSY GUTs both phenomena, spontaneous breaking of local SUSY and GUT phase transition, as finite temperature effects. In this context, the SU(3) \(\times\) SU(2) \(\times\) U(1) singlet contained in the adjoint representation \(E\) of SU(5) should play the role of the Polonyi field, and the breaking of local SUSY could be understood in terms of the dynamics of the cooling down early Universe. Since \(N = 1\) supergravity is non-renormalizable, and the use of non-renormalizable interactions is able to solve many of the standing problems in GUTs\(^9\)-\(^12\), we are allowed to use a non-renormalizable superpotential \(f(E)\). A first step along this direction was recently given by some of us in Ref. 13), where local SUSY and the SU(5) gauge symmetry were spontaneously broken by a field taking a VEV of the order of \(M_X = 10^{16} \text{ GeV}\). Also, once more\(^9\)-\(^12\) the necessity of introducing non-renormalizable interactions was proven in that work. The zero temperature potential exhibited two degenerate minima with zero cosmological constant corresponding to the symmetric and broken phases. Due to this degeneracy the confining properties\(^5\)-\(^8\) of the GUT had to be invoked for the phase transition to happen.

In this paper, we propose a class of models with the following properties:

i) local SUSY and the SU(5) gauge group are spontaneously broken when the adjoint of SU(5) takes a VEV of order of the grand unification scale \(M_X\);

ii) the phase transition is very fast and independent of possible confining properties of the GUT;
iii) the critical temperature can be computed very accurately and is of the order of $10^8$ GeV;

iv) monopole production is suppressed.

Since scalar fields in supermultiplets are complex fields, we must study the potential in the whole complex plane. Before going into the SU(5) potential we shall consider the simplest case of a scalar supermultiplet $(z,\bar{z})$ breaking local SUSY. The potential is given, in units of $M = M_p/\sqrt{8\pi}$, by

$$V(z,\bar{z}) = e^{\frac{|\bar{z}|^2}{2}} \left\{ |F_z|^2 - 3|f|^2 \right\}$$

(1)

where $F_z = \partial f/\partial z + z^* f$, and $f(z)$ is the superpotential. We are looking now for a class of superpotentials with the following properties:

a) the potential $V$ has at some point $z = A$, which can be taken to be real without loss of generality, a minimum in the complex $z$ plane with vanishing cosmological constant. $A$ is a free parameter and can be of the order of $M$ (as in usual models) or less, without any physical consequence.

Supersymmetric minima are potentially dangerous, since they have usually a negative cosmological constant, $\Lambda_{\text{cos}} = -3|f|^2 e|\bar{z}|^2$, becoming thus easily the global minimum of the theory. Therefore, we must require:

b) No supersymmetric minima with negative cosmological constant, i.e., no $F_z = 0, f \neq 0,$ in the complex plane.

A class of superpotentials satisfying the above two conditions is given by

$$f(z) = \lambda (b + \bar{z})^\gamma e^{c\bar{z}}$$

(2)

where $b, c, \gamma$ and $\lambda$ are real parameters. Condition (a) is satisfied provided the following relations hold

$$\gamma = \frac{\bar{z}/a}{\bar{z}/a}$$

$$b = \sqrt{\bar{z}} - A$$

$$c = \sqrt{\bar{z}} - \sqrt{\bar{z}} - A$$

(3)

where $A$ is the VEV of $z$ ($\langle z \rangle = a$) and $a$ is bounded by $0 < a < 4/\sqrt{3}$ (i.e., $3/4 < \gamma < \infty$) and gives the mass of $R$ and $\bar{I}$, $z = R + i\bar{I}$, as
\[
\begin{align*}
    m_R^2 &= \sqrt{3} \alpha \ m_{3/2}^2 \ > \ 0 \\
    m_Z^2 &= (4 - \sqrt{3} \alpha) \ m_{3/2}^2 \ > \ 0 
\end{align*}
\]
(4)

The gravitino mass \( m_{3/2} \) is given by
\[
    m_{3/2} = \sqrt{2} \, \gamma \, \alpha^{3/2} \, \lambda \, \exp \left\{ \frac{1}{2} \, \lambda \, A^2 \right\} \]
(5)

Let us observe that \( A \) is a free parameter in Eqs. (3) and (5) and can take any value.

On the other hand, the equation \( F_z = 0 \) has a solution with vanishing cosmological constant at \( z = -b \) if \( \gamma > 1 \). Now, for \( z \neq -b \), the equation \( F_z = 0 \) can be written as
\[
    \gamma + (c + b) \, \rho \, \cos \theta + \rho^2 + c \, b = 0 \\
    (c - b) \, \rho \, \sin \theta = 0 
\]
(6)

where \( z = \rho \, \exp(i \theta) \). Equation (6) has no solution in the complex plane provided that Eqs. (3) and (4) are satisfied. In this way we have proven that condition (b) holds for superpotentials given by Eqs. (2)-(3).

Let us now make some comments about the scalar potential provided by the superpotential (2). If \( \gamma > 1 \) there are two minima on the real axis: one supersymmetric minimum at \( z = -b \) and one non-supersymmetric minimum at \( z = A \). Both of them have vanishing cosmological constant. The distance between them does not depend on the parameters of the superpotential but only on the curvature at \( z = A, \ b + A = 2/a. \) If \( \gamma = 1 \) there only remains the non-supersymmetric minimum at \( z = A. \)

Next we apply the above ideas to the breaking of the \( SU(5) \) gauge group. Now the Higgs supermultiplet \( \Sigma \) is in the adjoint representation of \( SU(5) \) and the \( SU(3) \times SU(2) \times U(1) \) singlet contained in \( \Sigma \) will break spontaneously local SUSY and \( SU(5) \) when taking a non-vanishing VEV. The class of superpotentials we propose is given by
\[
    f(\Sigma) = \frac{m_{3/2}}{\sqrt{2} \, \gamma} \left( b + \sqrt{\frac{4}{\gamma} \, \Sigma^2} \right) \, \exp \left\{ c \left( \sqrt{\frac{4}{\gamma} \, \Sigma^2} - A \right) - \frac{1}{2} \, A^2 \right\} 
\]
(7)

where \( b, c \) and \( \gamma \) are real parameters satisfying Eq. (3).
Notice that the function $(\tau^{\frac{2}{3}})^{1/2} = \exp(\frac{1}{2} \text{log} \, \tau^{\frac{2}{3}})$ is analytic in the complex $\tau$ plane. When $\Sigma$ takes a VEV, the function $(\text{tr} \, \Sigma^{2})^{1/2}$ can be expanded in a power series around the VEV and, in the flat limit, the series is cut off and the effective potential is a simple polynomial. Since $f$ is a function of $\text{tr} \, \Sigma^{2}$, all directions are equivalent, so one can choose the VEV along the direction

$$\Sigma = \sqrt{\frac{3}{15}} \, \sigma \cdot \text{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}) \tag{8}$$

with $\sigma$ a complex scalar field and $\text{tr} \, \Sigma^{2} = \sigma^{2}$.

The scalar potential, using Eqs. (7) and (8), can be explicitly written as

$$V(\sigma, \sigma^*) = \frac{m_{\Sigma}^{2}}{\tau^{\frac{2}{3}}} \exp \left\{ \frac{1}{2} (x+A)^{2} + \frac{1}{2} y^{2} + \frac{1}{2} c x \right\} \cdot \left[ (x + \sqrt{3} \tau)^{2} + y^{2} \right]^{\frac{1}{\gamma - 1}} \cdot \left\{ \tau^{2} \left( \sqrt{3} a \gamma + 2 \tau^{2} x + x^{2} \right) + y^{2} \left( 4 - \sqrt{3} a \gamma + 2 \tau^{2} x + x^{2} + y^{2} \right) \right\} \tag{9}$$

where $\sigma = x + A + iy$. Now the value of $A$ is no longer arbitrary but has a precise physical meaning: the grand unification scale. Thus we take $A = M_{X} \approx 10^{-2}$ as usual.

In the flat limit, the effective scalar potential corresponding to the superpotential (7) becomes

$$V_{\text{eff}}(\Sigma, \Sigma^*) = \frac{1}{2} \, m_{k} \, x^{2} + \frac{1}{2} \, m_{l} \, y^{2} + \frac{1}{2} \, M_{4} \, \Sigma \Sigma^* + \frac{1}{2} \, M_{4} \, \Sigma^* \Sigma \tag{10}$$

where

$$m_{k}^{2} = \sqrt{3} \, a \, m_{3/2}^{2}, \quad m_{l}^{2} = (4 - \sqrt{3} a) \, m_{3/2}^{2}$$

$$M_{4} = m_{3/2} \left( \frac{M_{X}}{\sqrt{3}} \right)^{2} \left( 1 + \Theta \left( \frac{M_{X}}{M} \right) \right)$$

$$M_{4}^{2} = m_{3/2} \left( \frac{M_{X}}{\sqrt{3}} \right)^{2} \cdot \Theta \left( \frac{M_{X}}{M} \right)$$

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and $\bar{\tau}$ means that the $SU(3) \times SU(2) \times U(1)$ singlet, Eq. (8), has been suppressed from the trace, i.e., in $SU(3) \times SU(2) \times U(1)$ notation

$$\bar{\tau} \Sigma \Sigma^* = \tau \left( \Theta \Theta^* + TT^* + D \bar{D}^* + \bar{D} D^* \right)$$

$$\bar{\tau} \Sigma^2 = \tau \left( \Theta^2 + T^2 + D \bar{D} + \bar{D} D \right)$$

(12)

where $\Theta = (8,1)(0)$, the colour octet, $T = (1,3)(0)$, the $SU(2)$ triplet and $D = (3,2)(-5)$, $\bar{D} = (\bar{3},2)(5)$ the triplet-doublets contained in $\Sigma$. Using Eq. (10) one can easily see that the point $\sigma = A$, $\Theta = T = D = \bar{D} = 0$ is a true (global) minimum in the 24-dimensional complex $\Sigma$ plane. The interactions in $V_{\text{eff}}$ are suppressed by powers of $m_{3/2}^{-N_X}$. Thus trilinear interactions have coupling constants of the order of $10^{-14} m_{3/2}$ and quartic interactions coupling constants of the order of $10^{-28}$.

The shape of the potential $V(\sigma, \bar{\sigma})$ for $\gamma = 1$ is shown in Fig. 1 and for $\gamma = 2$ in Fig. 2. The section of both potentials at $\gamma = 0$ is drawn in Figs. 3 and 4. The value of the potential at $\sigma = 0$ (i.e., $x = A$, $\gamma = 0$) is given by

$$V(0) = \sqrt{\frac{3}{2}} \left( m_{3/2} M_X \right)^2 \sim \left( 10^9 \text{GeV} \right)^4$$

(13)

Since at very high temperature the VEV is $\sigma = 0$, the symmetric phase, when the temperature is cooling down the field, will be driven to the broken phase $M_X$ at the critical temperature $T_c$. From Fig. 4 we see that even in the case $\gamma = 2$, when there exists a supersymmetric minimum at $\sigma = -\sqrt{2} M$, it is impossible for the field to be driven to that minimum. We also see that the transition is very rapid. Next, we shall consider the finite temperature corrections to one-loop of the potential $V(\sigma, T=0)$ of Eq. (9).

For VEVs of $\sigma$ so that all masses are much larger than $T$, the finite temperature corrections drop off like $e^{-|\sigma|/T}$. For VEVs so that some mass eigenvalues are much smaller than $T$, the high temperature expansion becomes valid and gives

$$\Delta V_T(\sigma) = \frac{1}{24} T^2 \sum \bar{\tau} \sum \Theta^2 m_j^2(\sigma) - \frac{1}{90} N(T) T^4$$

(14)

where $N(T) = N_B + 7/8 N_F$ is the number of light degrees of freedom at the temperature $T$.

At temperatures $T >> M_X$, the factor $e^{-|\sigma|/T} = 1$ for $|\sigma| < M_X$, and the potential $V(\sigma, T) = V(\sigma, 0) + \Delta V_T(\sigma)$ has only a minimum at $\sigma$ around zero, corresponding to the symmetric phase.
At temperatures $T \ll M_X$, the exponential factor $e^{-|\sigma|/T}$ is small for $\sigma \approx M_X$ and the local minimum $SU(3) \times SU(2) \times U(1)$ at $<\sigma> = 0$ appears. However, in the region $\sigma \ll T$ the temperature factor $e^{-|\sigma|/T}$ does not drop off, and the potential can be approximated by

$$V(\sigma, T) \approx V(\sigma, 0) - \frac{B^2}{9\sigma} N(\tau) T^{-4}$$

since the mass terms $m_J^2(\sigma)$ are small as compared with $T^2$.

Here, we shall consider the minimal model with three generations of fermions, one Higgs in the adjoint $24$, and one pair of WS Higgses $\frac{5}{2} + \bar{\frac{5}{2}}$. Then, in the $SU(5)$ phase, at $T \ll M_X$, $N_B = N_F = 208$. Similarly, in the $SU(3) \times SU(2) \times U(1)$ phase $N_B = N_F = 148$, where we have taken into account that after the breaking of $SU(5)$ the colour octet $\phi$ and the $SU(2)$ triplet $T$ remain with masses of the order of $(M/M_X)^{3/2}$. If we normalize the potential $V(\sigma, T)$ to be zero at the $SU(3) \times SU(2) \times U(1)$ phase, we can write

$$\tilde{V}(\sigma, T) = V(\sigma, 0) - \frac{5}{4} \pi^2 T^{-4}$$

The critical temperature $T_c$, defined by $\tilde{V}(0, T_c) = 0$, is given by

$$T_c = \frac{1}{\sqrt{\pi}} \left( \frac{48}{25\pi} \right)^{1/8} \left( m_{3/2} M_X \right)^{1/2}$$

so that using $m_{3/2} = 10^2$ GeV and $M_X = 10^{16}$ GeV we obtain $T_c = 0.61 \times 10^9$ GeV for $\gamma = 1$ and $T_c = 0.56 \times 10^9$ GeV for $\gamma = 2$. In fact, the critical temperature depends on the Higgs content of the model. If we consider a model with $n_H$ pairs of $\frac{5}{2} + \bar{\frac{5}{2}}$ Higgses and $n_S$ pairs if light singlets coming from $\frac{10}{2}$, $\frac{10}{2}$, Eq. (17) can be generalized to

$$T_c = \frac{1}{\sqrt{\pi}} \left( \frac{3}{8\pi} \right)^{1/8} \left( m_{3/2} M_X \right)^{1/2}$$

where $d = (1 + n_H/4 - n_S/12)^2$.

The potential $\tilde{V}(x, T)$ is represented in Figs. 5 and 6. We see that the phase transition is extremely fast since the local minimum of the finite temperature potential effectively disappears, and neither tunnelling from a metastable false vacuum nor thermal fluctuations are necessary.

Let us now see how standard cosmology and GUT problems are solved for our model to become a realistic theory.
In the SU(5) phase transition that we have proposed there might not be enough inflation because the transition is extremely rapid, as we remarked above. However, primordial inflation can be produced by a singlet field which takes a VEV of the order of the Planck mass without breaking local SUSY. This mechanism has been recently proposed by Nanopoulos, Olive, Srednicki and Tamvakis in the context of $N = 1$ supergravity. It is able to provide enough inflation and correct energy density fluctuation for formation of galaxies.

One must worry about gravitinos with masses of the order of $10^2$ GeV since their decays can produce too much entropy and drown out the cosmological baryon asymmetry. A way out to the gravitino problem was suggested by Ellis, Linde and Nanopoulos in inflationary scenarios, using the fact that gravitinos and light hidden singlets are depleted during the inflationary epoch. Recently, Nanopoulos, Olive and Srednicki have analyzed the problem of cosmological baryon asymmetry (CBA) and gravitino suppression in the primordial inflationary scenario proposed in Ref. 16). The effective Hawking temperature is given by $T_H = 0(10^{-3} - 10^{-4}) m_\phi$, where $m_\phi$, the inflaton mass, is restricted by the requirement of CBA and gravitino suppression, to $10^{-5} M \leq m_\phi \leq 10^{-3} M$.

The above mechanism will also work in the model presented in this paper, and, in general, in other cosmologically acceptable models recently proposed with gravitinos and/or light singlets, provided that the SU(5) phase transition happens at the end of the inflationary era, i.e., if $T_H = T_C = 10^{9-10}$ GeV.

If the inflaton is to yield a density fluctuation spectrum compatible with galaxy formation one derives that $m_\phi \approx 10^{-5} M$ which solves the CBA generation and gravitino problems. It is surprising that this value of $m_\phi$ provides a Hawking temperature $T_H = 0(10^9 - 10^{10})$ GeV which is of the order of magnitude of the critical temperature for the SU(5) phase transition in all cosmologically acceptable models. We conclude then that the inflaton can save the gravitino/light scalar problem in the general class of local SUSY models compatible with cosmological requirements.

Monopole production accompanying the SU(5) phase transition is sufficiently suppressed since the critical temperature is $T_C = 10^9$ GeV and thus making the ratio $n_m/n_\gamma \sim (T_C/T_F)^3 \sim 10^{-27}$ safely below present experimental upper bounds, but still not far below them.

Non-vanishing baryon asymmetry can be generated by "light" Higgs colour triplets, $m_\phi = 10^{10}$ GeV. This intermediate scale can be generated in a natural way by the use of non-renormalizable terms in the superpotential using the mechanism recently introduced by the present authors with M. Srednicki. These terms are of the form $\lambda/M \delta H \Sigma^2 + \lambda'/M \delta H \Sigma^2 + \delta H \delta \phi$ where $\delta = \bar{5}, \bar{\phi} = \bar{\delta}, H = 5$ and $\bar{\Sigma} = \bar{\delta}$. 
When $\Sigma$ takes a VEV along the direction (8) the WS doublets remain massless as there are not doublets in $\vartheta$ and $\vartheta$ to be coupled to them. However, the triplets $H_3$ and $\vartheta_3$ get a mass through the mass matrix

$$
\begin{pmatrix}
M & M_X^2/M \\
M_X^2/M & 0
\end{pmatrix}
$$

whose eigenvalues are $M_{H_3}^2 = M^2$, $M_{\vartheta_3}^2 = M_X^2/M^2 = 10^{10}$ GeV. We obtain in this way a natural intermediate scale $\Lambda = 10^{10}$ GeV. The potentially dangerous light singlet $^{20)}$ from $\Sigma$ is harmless since it is not coupled to $H_3 H_3$ and $H_3 H_3$ because there is no term like $\bar{\vartheta} H H$ in the superpotential.

In conclusion, we presented here a class of local SUSY models which enjoy a rapid but overdue phase transition from SU(5) into SU(3) $\times$ SU(2) $\times$ U(1) in a natural way. Things look very much like ordinary GUTs, thanks to the existence of non-renormalizable terms. In contrast to previously proposed scenarios no need of SU(5) confinement is advocated, while the critical temperature ($T_c$) of the phase transition is determined by the two basic scales in the problem $M_X$ and $m_{3/2}^2$: $T_c = (m_{3/2}^2)^{1/2}$. They also determine the two intermediate scales in the theory: $\Lambda = M_X^2/M^2 = 10^{10}$ GeV, the mass of "light" triplets, and $\Lambda' = M/M_X m_{3/2}^2 = 10^6$ GeV, the mass of colour octet and SU(2) triplet contained in the adjoint of SU(5).

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FIGURE CAPTIONS

Fig. 1 : The potential (9), for $\gamma = 1$, as a function of the complex variable $\sigma$.

Fig. 2 : The same as in Fig. 1, for $\gamma = 2$.

Fig. 3 : A section at $\text{Im} \sigma = 0$ of the potential (9), for $\gamma = 1$.

Fig. 4 : The same as in Fig. 3, for $\gamma = 2$.

Fig. 5 : a) The full potential including temperature corrections given by Eq. (16), at the section $\text{Im} \sigma = 0$, for $\gamma = 1$.
       b) A different view of Fig. 5a.

Fig. 6 : The potential of Eq. (16) as a one variable function, $\text{Re} \sigma$, for different values of the temperature around the critical temperature.