Towards a high precision $CP$ violation measurement with the ATLAS Detector

by

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For Stephen, Barbara and Kim de Mora.
ATLAS is a general purpose detector designed to study the high energy proton-proton collisions produced at the Large Hadron Collider. The channel $B_s^0 \rightarrow J/\psi \phi$ offers a measurement of standard model $\mathcal{CP}$ violating parameters, and has the potential to be sensitive to new physics. The measurement of $\mathcal{CP}$ violation in the $B_s^0 \rightarrow J/\psi \phi$ channel at ATLAS is largely determined by three factors: statistics, lifetime resolution and flavour tagging. This study concentrates on improving the lifetime measurement and the implementation of flavour tagging. Flavour tagging has a strong influence on the sensitivity of $\mathcal{CP}$ violation studies. The jet charge flavour tagging method was optimised to increase its effectiveness. The lifetime of a $B$-hadron is calculated using the $p_T$, mass and vertexing. A deformation in the inner detector can produce a detectable change in these observables.
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Chapter 1

Introduction

The Standard Model of particle physics is a triumph of both experiment and theory, it allows for accurate and testable predictions for the behaviour of fundamental particles. There are many areas of ongoing research within the framework of the Standard Model, including the search for the Higgs Boson, the study of $CP$ violating effects and the observation of neutrino masses. Chapter 2 contains a brief description of the Standard Model, the state of $B$-physics research, $CP$ violation and the statistical methods used in this thesis.

The Large Hadron Collider (LHC) is a proton-proton collider built at the European Centre for Particle Physics, CERN. It was constructed for a nominal centre of mass energy of 14 TeV and to attain a luminosity of $10^{34}$ cm$^{-2}$ s$^{-1}$. Chapter 3 contains a description of the LHC. The ATLAS detector is one of six experiments that use the LHC. ATLAS is a general purpose detector built to record the products of the proton-proton collisions. Chapter 4 contains a description of the ATLAS experiment. ATLAS is expected to produce up to three petabytes of data every year, in at least five different data formats, which are distributed around the world for the use of its 2000 collaborators. The specifics of the computation model are described in chapter 5.

The purpose of the LHC is not only to probe fundamental physics beyond the limits of the Standard Model, but also to make high precision measurements within the framework of the Standard Model. The uncertainty of the measurement of $CP$ violation in the $B^0_s \rightarrow J/\psi\phi$ channel at ATLAS is largely determined by three factors: statistics, lifetime resolution and flavour tagging. For a given luminosity and recording efficiency, larger statistics can be achieved with time. However, the lifetime resolution can be improved through a better understanding the material deformations of the ATLAS inner detector and the optimisation of flavour tagging algorithms can decrease the uncertainty of a $CP$
violation measurement. Chapter 6 contains a description of the optimisation of the jet charge flavour tagger.

Lifetime is calculated using the transverse momentum, reconstructed mass and vertexing. Any deformation in the inner detector can produce an observable change in these observables. The impact of global systematic deformations in the ATLAS inner detector on $B$-physics performance is described in chapter 7. Finally, chapter 8 contains a study of the impact of extra material and misalignment on an inclusive $B$-lifetime measurement.
Chapter 2

$\mathcal{CP}$ violation in Standard Model

$B$-physics

This chapter introduces some of the theory used in modern particle physics. Firstly, section 2.1 contains a brief explanation of the fundamental particles and forces in the Standard Model. Section 2.2 is an introduction to flavour changing charged currents and the Cabibbo Kobayashi Maskawa matrix. Section 2.3 introduces the charge conjugation, parity inversion and time reversal operations, then shows a condition for $\mathcal{CP}$ to be conserved in the Standard Model. Section 2.4 introduces the concept of $B^0_s - \bar{B}^0_s$ mixing via the box diagram and section 2.5 discusses the means by which $\mathcal{CP}$ violation can enter $B$-physics. Finally, section 2.6 describes how the interference of mixing and decay amplitudes can induce $\mathcal{CP}$ violation in the decay $B^0_s \rightarrow J/\psi \phi$.

2.1 The Standard Model

The Standard Model of particle physics is the best current theory of three of the four fundamental forces, all particles observed so far and their observed interactions. The three forces described by the Standard Model are electromagnetism, the strong force and the weak force. The Standard Model does not account for the fourth force, gravity.

2.1.1 Bosons

The fundamental forces are mediated by the exchange of bosons. The bosons have integer spin and take the role of force carriers. The electromagnetic force is mediated by the exchange of massless photons. Similarly, the strong force is mediated by the
exchange of massless gluons. The weak force is mediated by the exchange of massive $W^\pm$ and $Z$ bosons. The gravitational force is expected to be mediated by the as-yet unobserved spin 2 graviton. The graviton is not generally considered to be a member of the Standard Model. Table 2.1 shows a summary of the properties of bosons.

Gauge symmetry breaking gives mass to particles, this makes an additional scalar field and generates a scalar boson, the Higgs Boson, $H$. The Higgs boson is the only particle within the Standard Model that has yet to be observed. For more details about the Higgs boson, a summary of Higgs theory and searches is available in reference [1].

The range of these interactions is determined by the mass of the force carrier, with a greater mass limiting the range. The weak force has a range of $10^{-18}$ m and the electromagnetic force and gravitational forces have infinite range. Although gluons are massless, the strong force has a range of $10^{-15}$ m. This short range is due to the gluon-gluon self-interaction which leads to quark confinement.

![Table 2.1: The properties of the force carrying bosons: mass, charge, spin and interaction. This table taken from reference [1].](image)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Mass</th>
<th>Charge</th>
<th>Spin</th>
<th>Interaction</th>
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<td>$\gamma$</td>
<td>Photon</td>
<td>$&lt;10^{-18}$ eV</td>
<td>0</td>
<td>1</td>
<td>Electromagnetism</td>
</tr>
<tr>
<td>$W^\pm$</td>
<td>W-boson</td>
<td>$80.398\pm0.025$ GeV</td>
<td>$\pm1$</td>
<td>1</td>
<td>Weak</td>
</tr>
<tr>
<td>$Z$</td>
<td>Z-boson</td>
<td>$91.1876\pm0.0021$ GeV</td>
<td>0</td>
<td>1</td>
<td>Weak</td>
</tr>
<tr>
<td>$g$</td>
<td>Gluon</td>
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<td>0</td>
<td>1</td>
<td>Strong</td>
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<tr>
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<td>0</td>
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<td>Gravity</td>
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<td>Higgs boson</td>
<td>$114.4$ GeV $&lt;m_H &lt;200$ GeV</td>
<td>0</td>
<td>0</td>
<td>Electro-weak</td>
</tr>
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2.1.2 Fermions

Matter is composed of fundamental particles called fermions. Fermions are point-like and have half-integer spin. The Standard Model has two groups of fermions: quarks and leptons. Both of these groups interact via the weak force. A principal difference between them is that quarks carry colour charge whereas leptons are colourless.

The colour charge carried by quarks allows them to interact via the strong force. As such, quarks are only observed in nature as constituents of composite objects known as hadrons. A hadron is a colourless object composed of many quarks and gluons with a number of valence quarks. In the case that the valence quarks are a quark and an anti-quark, the hadron is a meson. In the case that the valence quarks are three quarks or three anti-quarks, the hadron is a baryon or an anti-baryon. Evidence for penta-quarks
Chapter 2: CP violation in Standard Model B-physics

and other exotica has been reported from time to time, but the balance of evidence does not confirm the claims thus far.

There are six observed “flavours” of quark, which are divided into two types and three generations. The up-type consists of the quarks: up, charm and top, where up is a member of the first generation, charm is second generation and top is third generation. The down-type consists of the quarks: down, strange and bottom, where down is a member of the first generation, strange is second generation and bottom is third generation. Up-type quarks have an electric charge of $+\frac{2}{3}$ of the charge of a proton and down-type quarks have an electric charge of $-\frac{1}{3}$ of the charge of a proton.

Leptons, like quarks, interact via the weak force, the gravitational force. Some but not all leptons can also interact via the electromagnetic force if they carry electric charge. Unlike quarks, leptons do not carry colour charge and as such are not subject to strong confinement. This means that leptons can be observed as point-like objects. There are six observed “flavours” of lepton, which are divided into two types and three generations. The three electron-type leptons are the electron $e$, the muon $\mu$ and the tau $\tau$, where the electron is a member of the first generation, the muon is second generation and the tau is third generation. The electron-type leptons are massive and have an electric charge of $-1$ in units of $[\text{electron charge}]$ or $1.602176487 \times 10^{-19}$ Coulombs. The three neutrino-type leptons are the electron-neutrino $\bar{e}$, the muon-neutrino $\bar{\mu}$ and the tau-neutrino $\bar{\tau}$, where the $\bar{e}$ is a member of the first generation, the $\bar{\mu}$ is second generation and the $\bar{\tau}$ is third generation. The neutrino-type leptons are uncharged and do not interact electromagnetically.

The properties of the quarks are summarised in the tables 2.2 and 2.3. Each of the twelve particles listed in these tables also has an anti-matter partner. A fermion and its anti-matter partner have the same mass, but opposite electric charge. By convention, anti-particles are denoted with an over-line, for instance an anti-up quark is notated $\bar{u}$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Mass</th>
<th>Charge</th>
<th>Spin</th>
<th>Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>down</td>
<td>$1.5$-$3.3$ MeV</td>
<td>$-1/3$</td>
<td>$1/2$</td>
<td>$1$</td>
</tr>
<tr>
<td>$u$</td>
<td>up</td>
<td>$3.5$-$6.0$ MeV</td>
<td>$2/3$</td>
<td>$1/2$</td>
<td>$1$</td>
</tr>
<tr>
<td>$s$</td>
<td>strange</td>
<td>$104^{+26}_{-34}$ MeV</td>
<td>$-1/3$</td>
<td>$1/2$</td>
<td>$1$</td>
</tr>
<tr>
<td>$c$</td>
<td>charm</td>
<td>$1.27^{+0.07}_{-0.11}$ GeV</td>
<td>$2/3$</td>
<td>$1/2$</td>
<td>$2$</td>
</tr>
<tr>
<td>$b$</td>
<td>bottom</td>
<td>$4.2^{+0.17}_{-0.07}$ GeV</td>
<td>$-1/3$</td>
<td>$1/2$</td>
<td>$3$</td>
</tr>
<tr>
<td>$t$</td>
<td>top</td>
<td>$171.2 \pm 2.1$ GeV</td>
<td>$2/3$</td>
<td>$1/2$</td>
<td>$3$</td>
</tr>
</tbody>
</table>

Table 2.2: The properties of the six Standard Model quarks: mass, charge, spin and generation number. This table taken from reference [1].
Table 2.3: The properties of the six Standard Model leptons: mass, charge, spin and generation number. This table taken from reference [1].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Mass</th>
<th>Charge</th>
<th>Spin</th>
<th>Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>electron</td>
<td>0.510998910±0.000000013 eV</td>
<td>-1</td>
<td>1/2</td>
<td>1</td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>electron-neutrino</td>
<td>$&lt; 2$ eV at 90% C.L.</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
</tr>
<tr>
<td>$\mu$</td>
<td>muon</td>
<td>105.658367 ±0.000004 MeV</td>
<td>-1</td>
<td>1/2</td>
<td>2</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>muon-neutrino</td>
<td>0.19 eV at 90% C.L.</td>
<td>0</td>
<td>1/2</td>
<td>2</td>
</tr>
<tr>
<td>$\tau$</td>
<td>tau</td>
<td>1776.84 0.17 MeV</td>
<td>-1</td>
<td>1/2</td>
<td>3</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>tau-neutrino</td>
<td>$&lt; 18.2$ MeV at 95 % C.L.</td>
<td>0</td>
<td>1/2</td>
<td>3</td>
</tr>
</tbody>
</table>

The helicity of a particle is the projection of its spin onto the direction of the motion, but is not Lorentz invariant. The chirality of a particle is similar to its helicity, but chirality is Lorentz invariant. For massless particles, helicity is the same as chirality. A massless particle is right-handed if its momentum and its spin are in the same direction and left-handed if the directions of spin and momentum are opposite. The handedness is denoted with a L or R subscript, for instance $q_L$ denotes a left-handed quark, $q$.

### 2.1.3 Current limitations of the Standard Model

The Standard Model cannot be the complete and final theory, as it does not account for the gravitational force. Furthermore, the Standard Model does not account for the matter anti-matter imbalance of the universe, nor does it offer an explanation for the existence of dark matter. Dark matter has been postulated due to the discrepancy of “missing mass” in many astronomical models relative to what is observed. It is postulated to be composed of stable massive matter that does not interact electromagnetically with photons. There is no such particle in the Standard Model, but many extensions of the Standard Model have dark matter candidates. A summary of dark matter in theory and experiments is available in reference [1].

Matter-antimatter asymmetry arises in the Standard Model through $\mathcal{CP}$ violation. However, the expected Standard Model asymmetry is not sufficient to account for the observed asymmetry in the universe. Standard Model $\mathcal{CP}$ violation is described in section 2.4.
2.2 The Cabibbo Kobayashi Maskawa matrix and the unitarity triangle

Unlike the strong and electromagnetic interactions, the weak interaction allows for flavour and generation changing interactions. For instance, the flavour changing decay $b \rightarrow cW^-$ occurs in nature, but the decays $e \rightarrow \mu\gamma$ and $g \rightarrow u\pi$ are not observed at tree level. The weak flavour changing neutral current, e.g. $Z \rightarrow u\pi$, is also not observed at tree level. The Cabibbo Kobayashi Maskawa (CKM) matrix [2] allows a calculation of the probability of a flavour changing interaction to occur, as a function of the flavour of the interacting quark anti-quark pair.

A general $n \times n$ complex matrix has $2n^2$ parameters. Due to the constraint of unitarity, this drops to $n^2$ free parameters. As the phases are only relative to each other, the matrix can be rotated to remove $2n - 1$ relative phases leaving $(n - 1)^2$ parameters. Of these $(n - 1)^2$ parameters, $\frac{1}{2}n(n - 1)$ are real angles and the remaining $\frac{1}{2}(n - 1)(n - 2)$ are complex phases. A $3 \times 3$ complex unitary matrix such as the CKM matrix has three mixing angles and one complex phase.

As such, the Kobayashi and Maskawa parametrisation has three angles ($\theta_1, \theta_2, \theta_3$) and a complex phase $\delta$. In equation 2.1 cosines and sines of the angles are denoted $c_i$ and $s_i$, respectively.

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
= 
\begin{pmatrix}
c_1 & -s_1 c_3 & -s_1 s_3 \\
s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\
s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta}
\end{pmatrix}
\]  

(2.1)

The measured values (as of 2008) of the CKM matrix are shown in equation 2.2. The CKM is nearly a diagonal matrix as the diagonal elements are close to unity and the off diagonal elements are much smaller. This indicates that same generation transitions occur more readily than generation changing transitions. The values shown here were reported in reference [1].

\[
\begin{pmatrix}
0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\
0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\
0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.99913^{+0.000044}_{-0.000043}
\end{pmatrix}
\]  

(2.2)

A simplified parametrisation of the CKM matrix was introduced by Lincoln Wolfenstein in reference [3]. In this parametrisation, the complex phase $\eta$ only occurs in the $V_{ub}$ and $V_{td}$ elements. The Wolfenstein parametrisation of the CKM matrix, to order $\lambda^3$, is
shown in equation 2.3.

\[
\begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}
\]  
(2.3)

The best determination of the Wolfenstein variables is \( \lambda = 0.2257^{+0.0009}_{-0.0010} \), \( A = 0.814^{+0.021}_{-0.022} \), \( \rho = 0.135^{+0.031}_{-0.016} \) and \( \eta = 0.349^{+0.015}_{-0.017} \) [1].

It is also worth noting that the elements \( V_{ub} \) and \( V_{tb} \) both have complex elements and that \( V_{ub} \neq V_{ub}^\dagger \), and hence the transitions \( b \to W^- u \) and \( \bar{b} \to W^+ \bar{u} \) are not completely symmetrical.

The diagonal elements of the CKM are larger than the off-diagonal elements. This means that same generation transitions are more likely to occur than generation changing transitions. A generation changing transitions such as \( W^+ \to s\bar{u} \) has a much smaller value of the CKM matrix element. For this reason, these transitions are sometimes called “Cabibbo-suppressed”. The transitions that change two generations, \( V_{ub} \) and \( V_{td} \), are known as “doubly Cabibbo-suppressed”.

This matrix is also constrained by unitarity. A unitary matrix is an \( n \times n \) complex matrix, \( U \), that satisfies the condition in equation 2.4.

\[ U^\dagger U = UU^\dagger = I_n \]  
(2.4)

where \( I_n \) is the \( n \times n \) identity matrix. This leads to twelve unitarity conditions, which are two sets of six equations. The first set of six equations describes the weak universality relation. Written out in full, these six weak universality conditions are shown in equations 2.5-2.10.

\[
\begin{align*}
V_{ud}^2 + V_{cd}^2 + V_{td}^2 &= 1 \\
V_{us}^2 + V_{cs}^2 + V_{ts}^2 &= 1 \\
V_{ub}^2 + V_{cb}^2 + V_{tb}^2 &= 1 \\
V_{ub}^2 + V_{us}^2 + V_{ud}^2 &= 1 \\
V_{cb}^2 + V_{cs}^2 + V_{cd}^2 &= 1 \\
V_{tb}^2 + V_{ts}^2 + V_{td}^2 &= 1
\end{align*}
\]  
(2.5-2.10)

These tests reflects the completeness of the three quarks generations system. If any of these equation were untrue, then this would imply that there are more than three quark generations.
The off-diagonal elements of the unitarity condition of equation 2.4 lead to a further six orthogonality equations. These conditions are described in equations 2.11-2.16.

\[ V_{ud}V_{cd}^{*} + V_{us}V_{cs}^{*} + V_{ub}V_{cb}^{*} = 0 \]  \hspace{1cm} (2.11)
\[ V_{ud}V_{td}^{*} + V_{us}V_{ts}^{*} + V_{ub}V_{tb}^{*} = 0 \]  \hspace{1cm} (2.12)
\[ V_{cd}V_{td}^{*} + V_{cs}V_{ts}^{*} + V_{cb}V_{tb}^{*} = 0 \]  \hspace{1cm} (2.13)
\[ V_{ud}V_{us}^{*} + V_{cd}V_{cs}^{*} + V_{td}V_{ts}^{*} = 0 \]  \hspace{1cm} (2.14)
\[ V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0 \]  \hspace{1cm} (2.15)
\[ V_{us}V_{ub}^{*} + V_{cs}V_{cb}^{*} + V_{ts}V_{tb}^{*} = 0 \]  \hspace{1cm} (2.16)

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{unitarity_triangle.png}
\caption{A Diagram of the unitarity triangle. This figure is normalised such that the longest side has unity length. The angle of the triangle have been named according to two different conventions: numbered $\phi$ angles and Greek letters. This image taken from reference [1].}
\end{figure}

These relationships can be drawn in the Argand plane and they form six triangles of equal area. The area of the triangle can be used to quantify the Standard Model $\mathcal{CP}$ violation. Equation 2.15 forms a triangle which has angles of order unity. For this reason, it is often referred to as “the unitarity triangle”. Unitarity triangles are often used to illustrate our knowledge of the CKM matrix and to test $\mathcal{CP}$ violation in the Standard Model. A diagram of this triangle is shown in figure 2.1. The sides of the triangle are normalised such that the longest side of the triangle has a side of length 1.

The combination of the independent measurement of many CKM parameters is shown in the unitarity triangle in figure 2.2. In this figure, the closed contours at 68% and 95% probability are shown. The full lines correspond to 95% probability regions for the constraints. The best fit for the peak of the triangle is $\rho = 0.155 \pm 0.022$ and $\eta = 0.342 \pm 0.014$. This plot was published by the UTfit group in reference [4], using measurements published from a range of experimental collaborations.
2.3 $\mathcal{CPT}$ operators

The combination of the time-reversal, charge-conjugation and parity inversion operators is known as the $\mathcal{CPT}$ operator. $\mathcal{CPT}$ theorem requires the preservation of $\mathcal{CPT}$ by all phenomena. $\mathcal{CPT}$ symmetry is considered to be a fundamental symmetry of physics and no observations have been made where it is violated. Although $\mathcal{CPT}$ theorem requires the preservation of $\mathcal{CPT}$ by all physical phenomena, each of the symmetries $\mathcal{C}$, $\mathcal{P}$ or $\mathcal{T}$ can be individually violated in the Standard Model.

2.3.1 Time reversal

The time reversal operator, $\mathcal{T}$, is an operator that reverses time in a process.

$$\mathcal{T} : t \mapsto -t. \quad (2.17)$$

2.3.2 Charge conjugation

The charge conjugation operator, $\mathcal{C}$, is an operator that transforms a particle into its anti-particle.

$$\mathcal{C} |\psi\rangle = |\bar{\psi}\rangle \quad (2.18)$$
Charge conjugation replaces some charge, \( q \), of a particle, with its opposite charge \(-q\), but does not change chirality, linear momentum or mass. For instance, applying the charge conjugation operator to a left-handed positron results in a left-handed electron. Another example would be to replace a left-handed electron-neutrino \( \nu_{L,e} \) with a left-handed anti electron-neutrino, \( \bar{\nu}_{L,e} \). Left-handed anti-neutrinos and right-handed neutrinos are not experimentally observed. For this reason, \( C \)-symmetry is maximally violated in the lepton sector.

### 2.3.3 Parity inversion

The parity inversion operator, \( \mathcal{P} \), is an operator that flips the sign of the spatial coordinates of a particle.

\[
\mathcal{P} : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}
\]

In essence, it replaces a left-handed particle with its right-handed counterpart. For instance, parity inversion acting on a left-handed electron-neutrino \( \nu_{L,e} \) results in the unobserved right-handed electron-neutrino \( \nu_{R,e} \). As such, parity conservation is maximally violated in the Standard Model. The violation of parity conservation was first suggested in 1956 and observed in 1957 in the beta-decay of Cobalt 60 in references [5, 6].

### 2.3.4 Charge conjugation and parity inversion

The combination of the charge conjugation and parity inversion operators forms the \( CP \) operator, which replaces a left-handed particle with its right-handed anti-particle. Both \( C \) and \( P \) are violated maximally by the weak interaction, but are conserved in gravitational, strong and electromagnetic interactions. \( CP \) violation occurs when some interaction do not conserve \( CP \) symmetry and was first observed in 1964 in neutral Kaon mixing. \( CP \) violation is introduced into the Standard Model through the complex phases in the CKM matrix.

The amplitude for the interaction between the \( W^\pm \) field and a quark anti-quark pair is shown in equation 2.20.

\[
M_{Wq\bar{q}} = -\frac{e}{\sqrt{2} \sin \theta_W} \left( u_L^\dagger \ c_L^\dagger \ t_L^\dagger \right) \gamma^\mu V_{CKM} \left( \begin{array}{c} d_L \\ s_L \\ b_L \end{array} \right) W_\mu^\dagger + HC
\]
where $V_{CKM}$ is the CKM matrix as described in section 2.2, $\gamma^\mu$ is the gamma-matrix, $e$ is the charge of an electron and $\theta_W$ is a parameter of the electro-weak model. The terms $\left(\begin{array}{ccc} u_L & c_L & t_L \end{array}\right)$ and $\left(\begin{array}{ccc} d_L & s_L & b_L \end{array}\right)$ represent the six quarks. HC is the Hermitian conjugate of the first part of the Lagrangian. The derivation of this equation can be found in reference [7]. As the CKM matrix is not diagonal, the $W^\pm$ bosons can couple to quarks of different generations with the coupling strength determined in the CKM matrix. Furthermore, charged weak current interactions must change the flavour of the quarks. Incidentally, tree level flavour changing neutral currents are not yet observed in nature and only enter the Standard Model at loop level. A summary of FCNC searches can be found in reference [1].

When the $\mathcal{C}\mathcal{P}$ operator is applied to the Lagrangian for the interaction between the $W^\pm$ field and leptons described above, equation 2.20 now becomes:

$$\mathcal{L}_{qW} = \frac{e}{\sqrt{2} \sin \theta_W} \sum_{i,j} \left[ u^\dagger_L i \sigma^\mu V_{ij} d_L j W^\pm_j + d^\dagger_L i \sigma^\mu V^*_j i u_L j W^\mp_i \right]$$  (2.21)

where $i,j$ are the three generations of up-type and down-type quarks, $V_{ij}$ is the CKM matrix element, $\sigma$ are the Pauli spin matrices. Under a $\mathcal{C}\mathcal{P}$ transformation, this Lagrangian becomes:

$$\mathcal{C}\mathcal{P}\mathcal{L}_{qW}\mathcal{C}\mathcal{P}^\dagger = \frac{e}{\sqrt{2} \sin \theta_W} \mathcal{C}\mathcal{P} \sum_{i,j} \left[ u^\dagger_L i \sigma^\mu V_{ij} d_L j W^\pm_j + d^\dagger_L j \sigma^\mu V^*_j i u_L i W^\mp_j \right] \mathcal{C}\mathcal{P}^\dagger$$

$$= \frac{e}{\sqrt{2} \sin \theta_W} \sum_{i,j} \left[ -u^\dagger_L i (\sigma^\mu)^T V_{ij} d^\dagger_L j W^\mp_j - d^\dagger_L j (\sigma^\mu)^T V^*_j i u_L i W^\pm_j \right]$$

$$= -\frac{e}{\sqrt{2} \sin \theta_W} \sum_{i,j} \left[ d^\dagger_L j \sigma^\mu V_{ij} u_L i W^\mp_j + u^\dagger_L i \sigma^\mu V^*_j i d_L j W^\pm_i \right]$$  (2.22)

Thus, in order for $\mathcal{C}\mathcal{P}$ to be conserved, equation 2.22 must be equal to equation 2.20, which is only true if

$$V_{ij} = V^*_j i$$  (2.23)

Hence, if all elements of the CKM matrix are real, $\mathcal{C}\mathcal{P}$ violation does not occur in the Standard Model. However, the observation of a complex phase in the CKM implies that $\mathcal{C}\mathcal{P}$ violation occurs in the Standard Model.

### 2.4 Neutral $B$-meson mixing

Neutral $B$-mixing refers to the oscillation between a neutral $B$-meson and its anti-particle. This mixing is a result of flavour non-conservation in weak decays. While mixing occurs in both $B^0_s-B^0_s$ and $B^0_s-B^0_s$, this section focuses on mixing in the $B^0_s-B^0_s$
system. This mixing results in non-negligible mass and lifetime differences between the mass eigenstates. Mixing occurs via the “box diagrams”, as in figure 2.3. This image taken was taken from reference [8]. This process is kinematically allowed in the $B^0_s$-$\bar{B}^0_s$ systems as the $B$-meson and its anti-particle have identical mass.

\[ |\Psi\rangle = a(t)|B^0_s\rangle + b(t)|\bar{B}^0_s\rangle \]  

(2.24)

where $a(t)$ and $b(t)$ describe the relative proportion of each state at time $t$. The time evolution of this state is governed by the time-dependent Schrödinger equation:

\[ i\hbar \frac{\partial}{\partial t} \Psi = H\Psi \]  

(2.25)
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where $H$ is a $2 \times 2$ complex Hamiltonian and can be expressed as a sum of two Hermitian matrices, $M$ and $\Gamma$. A Hermitian or “self-adjoint” matrix is a square matrix with complex entries that is invariant under the Hermitian conjugate operator. The Hermitian conjugate operator is the combination of the transpose operator followed by the complex conjugate operators.

$$H = M + i \Gamma$$

(2.26)

These matrices can be written out explicitly.

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix}, \Gamma = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}$$

(2.27)

The best way to solve the time dependent Schrödinger equation (equation 2.25) is to diagonalise the matrix $H$. The solutions to these decoupled equations represent the two mass and lifetime eigenstates. The two states are named $B_H$ and $B_L$, where $H$ and $L$ represent the Heavy and the Light eigenstates. These eigenstates are shown in equations 2.28 and 2.29.

$$|B_H\rangle = p|B_0^0\rangle + q|B_0^{0'}\rangle$$
$$|B_L\rangle = p|B_0^{0'}\rangle - q|B_0^0\rangle$$

(2.28)  (2.29)

where $p$ and $q$ are normalised such that

$$|p|^2 + |q|^2 = 1$$

(2.30)

The relationship between the eigenstates and the Hermitian matrices above are:

$$M_L - \frac{i}{2} \Gamma_L = M - \frac{i}{2} \Gamma + \frac{q}{p} \left(M_{12} - \frac{i}{2} \Gamma_{12}\right)$$
$$M_H - \frac{i}{2} \Gamma_H = M - \frac{i}{2} \Gamma - \frac{q}{p} \left(M_{12} - \frac{i}{2} \Gamma_{12}\right)$$

(2.31)  (2.32)

where $M_H$ and $M_L$ are the heavy and light mass eigenstates and $\Gamma_H$ and $\Gamma_L$ are the heavy and light decay width eigenstates. The relationship between $p$ and $q$ is

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}} = \frac{H_{21}}{H_{12}}$$

(2.33)

where $q/p$ is in general a complex number. The real part of $q/p$ can be positive or negative but the positive solution is usually chosen by sign convention.
These states can be written as Heavy or Light states, but it is also possible to write the mean mass, $M$, and the difference in mass $\Delta M$:

$$M = \frac{M_H + M_L}{2} \quad (2.34)$$

$$\Delta M = M_H - M_L \quad (2.35)$$

similarly for the lifetime,

$$\Gamma = \frac{\Gamma_H + \Gamma_L}{2} \quad (2.36)$$

$$\Delta \Gamma = \Gamma_L - \Gamma_H \quad (2.37)$$

where $\Gamma_L > \Gamma_H$.

### 2.4.1 Time evolution of neutral $B$-mesons

The time evolution of a $B$-meson which was pure at time $t = 0$ is given by:

$$|B_s^0(t)\rangle = g_+ (t) |B_s^0\rangle + \frac{q}{p} g_- (t) |\overline{B_s^0}\rangle \quad (2.38)$$

$$|\overline{B_s^0}(t)\rangle = g_+ (t) |\overline{B_s^0}\rangle + \frac{q}{p} g_- (t) |B_s^0\rangle \quad (2.39)$$

where

$$g_\pm (t) = \frac{1}{2} e^{-\frac{t}{2}} e^{-iM_s t} \left( \cosh \left( \frac{\Delta \Gamma}{2} t \right) \pm \cos (\Delta M_s t) \right) \quad (2.40)$$

A pure state at $t = 0$ becomes mixed when $t > 0$. The probability of finding the various states at time $t$ given the initial state as pure $|B_s^0\rangle$ or $|\overline{B_s^0}\rangle$ is then:

$$P \left( B_s^0 \rightarrow B_s^0; t \right) = |\langle B_s^0 | B_s^0(t) \rangle|^2 = |g_+(t)|^2 \quad (2.41)$$

$$P \left( B_s^0 \rightarrow \overline{B_s^0}; t \right) = |\langle B_s^0 | \overline{B_s^0}(t) \rangle|^2 = \left| \frac{q}{p} \right|^2 |g_-(t)|^2 \quad (2.42)$$

$$P \left( \overline{B_s^0} \rightarrow B_s^0; t \right) = |\langle \overline{B_s^0} | B_s^0(t) \rangle|^2 = |g_+(t)|^2 \quad (2.43)$$

$$P \left( \overline{B_s^0} \rightarrow \overline{B_s^0}; t \right) = |\langle \overline{B_s^0} | \overline{B_s^0}(t) \rangle|^2 = \left| \frac{p}{q} \right|^2 |g_-(t)|^2 \quad (2.44)$$

where

$$|g_\pm (t)|^2 = \frac{1}{4} \left( e^{-\Gamma_H t} + e^{-\Gamma_L t} \pm 2 e^{-\Gamma_L t} \cos (\Delta M_s t) \right) \quad (2.45)$$

In the case that $\mathcal{CP}$ is conserved, at time $t$, the probability of an initial $B_s^0$ to mix into a $\overline{B_s^0}$ is equal to the probability of an initial $\overline{B_s^0}$ to mix into a $B_s^0$. This means that equations 2.42 and 2.44 would be be equivalent. This constraint leads to the condition
$q^2 = p^2$ which then implies that $M_{12} - \frac{i}{2}\Gamma_{12} = M_{12}^* - \frac{i}{2}\Gamma_{12}^*$. This means that the mass and lifetime eigenstates are identical when $\mathcal{CP}$ is conserved.

The last term of equation 2.45 describes the oscillation between $B^0_s$ and $\bar{B}^0_s$ in an initially pure beam. The period of oscillation, $T$, is equal to:

$$T = \frac{2\pi}{|\Delta M_s|}$$

(2.46)

The Particle Data Group quotes the value of $|\Delta M_s|$ to be $17.72\pm0.12$ ps$^{-1}$, which is equivalent to $117.8\pm0.8\times10^{-10}$ MeV, [1]. The period of oscillation is $\sim350$ fs. In order to make a measurement of the mixing oscillation, the lifetime resolution will need to be smaller than the oscillation period. Ideally, the lifetime resolution should be sufficient to measure five or more points per period.

A study of mixing induced $\mathcal{CP}$ violation is a priority for the ATLAS $B$-physics working group. In order to do so, the capacity to measure mixing is essential, which requires a good lifetime resolution. In order to improve the lifetime resolution of the measurement a good understanding of the inner material, the alignment and the magnetic field map will be required. The chapters 7 and 8 look at the impact of extra material and alignment on exclusive and inclusive lifetime measurements.

A sensitive determination of the $B$-meson flavour at production and at decay is also required to measure mixing induced $\mathcal{CP}$ violation. The process of determining the flavour is called “flavour tagging”. The jet charge flavour tagging method is described in chapter 6. The determination of the flavour is strongly dependent on the effectiveness of the flavour tagger. Chapter 6 also contains a study performed to maximise the quality of the jet charge tagger.

### 2.5 $\mathcal{CP}$ violation in $B$-physics

The decay amplitudes for the decay of $B^0$ and $\bar{B}^0$ into two final state $f$ and $\bar{f}$ are defined:

$$A_f = \langle f | H | B^0(0) \rangle \quad \bar{A}_f = \langle f | H | \bar{B}^0(0) \rangle$$

$$A_{\bar{f}} = \langle \bar{f} | H | B^0(0) \rangle \quad \bar{A}_{\bar{f}} = \langle \bar{f} | H | \bar{B}^0(0) \rangle$$

(2.47)

(2.48)

There are three ways in which $\mathcal{CP}$ violation can manifest in neutral $B$-mesons; direct, mixing induced and through the interference of mixing and decay amplitudes.
2.5.1 Direct $\mathcal{CP}$ violation

Direct $\mathcal{CP}$ violation could occur in the case that there was no difference between the mass and width of the eigenstates. If direct $\mathcal{CP}$ violation occurred then the

$$|A_f| \neq |\bar{A}_f|$$

(2.49)

This direct $\mathcal{CP}$ violation is illustrated in figure 2.4. Figures 2.4, 2.5, 2.6 and 2.7 were taken from reference [8] and use the notation $P^0$ and $\bar{P}^0$ to denote the flavour eigenstates.

![Figure 2.4: Direct $\mathcal{CP}$ violation. This image taken from reference [8].](image)

2.5.2 Mixing induced $\mathcal{CP}$ violation

$\mathcal{CP}$ violation can still occur when the direct decays are $\mathcal{CP}$ symmetric but the mixing decays are asymmetric. This means that there is a difference in the decay amplitudes $P\left(B_s^0 \to B_s^0 \to f \right)$ and $P\left(B_s^0 \to B_s^0 \to \bar{f} \right)$. Numerically, this condition is written in equation 2.50 and is illustrated in figure 2.5.

$$\frac{|q|}{|p|} \neq 1$$

(2.50)

![Figure 2.5: Mixing induced $\mathcal{CP}$ violation. This image taken from reference [8].](image)

2.5.3 $\mathcal{CP}$ violation induced by the interference of mixing and decay amplitudes

Interference $\mathcal{CP}$ violation can occur when that both initial hadrons decay to the same final state, $f$. As such, the initial hadron can mix into its anti-particle and still decay to the same final state. This is illustrated in figure 2.6.
The CP asymmetry parameter $\xi_{CP}$ is defined

$$\xi_{CP} = \frac{q A_f}{p A_f}$$

such that when $\xi_{CP} \neq 1$, CP asymmetry is present. The CP asymmetry parameter is sometimes written $\lambda_f$. This condition is illustrated in figure 2.7.

2.6 CP violation in the decay $B_s^0 \rightarrow J/\psi \phi$

The decay $B_s^0 \rightarrow J/\psi \phi$ is a specific case of the mixing and CP violating systems described in sections 2.4 and 2.5. The $B$-meson and its anti-particle both decay to the same final state ($f$), such that their decay amplitudes can be written as in equations 2.52 and 2.53.

$$A_{B_s^0(t) \rightarrow f} = g_+(t) A_f + \frac{q}{p} g_-(t) \overline{A_f}$$

$$A_{\overline{B_s^0(t)} \rightarrow f} = g_+(t) \overline{A_f} + \frac{p}{q} g_-(t) A_f$$

The decay rates for these processes are denoted $\Gamma_{B_s^0(t) \rightarrow f}$ and $\Gamma_{\overline{B_s^0(t)} \rightarrow f}$. As in previous sections, these decay rates are calculated by taking the modulus squared of their decay
amplitude.

\[ \Gamma_{B^0_s(t) \to f} = N_f e^{-\Gamma t} \left[ \left| A_f \right|^2 + \left| \frac{q}{p} A_f \right|^2 \right] \cosh \left( \frac{1}{2} \Delta \Gamma t \right) + \left| A_f \right|^2 - \left| \frac{q}{p} A_f \right|^2 \cos (\Delta M t) + 2 \Re \left( \frac{q}{p} A_f A_f^\dagger \right) \sinh \left( \frac{1}{2} \Delta \Gamma t \right) - 23 \Re \left( \frac{q}{p} A_f A_f^\dagger \right) \sinh (\Delta M t) \] \tag{2.54}

\[ \Gamma_{B^0_s(t) \to f} = N_f e^{-\Gamma t} \left[ \left| A_f \right|^2 + \left| \frac{p}{q} A_f \right|^2 \right] \cosh \left( \frac{1}{2} \Delta \Gamma t \right) - \left| A_f \right|^2 - \left| \frac{p}{q} A_f \right|^2 \cos (\Delta M t) + 2 \Re \left( \frac{p}{q} A_f A_f^\dagger \right) \sinh \left( \frac{1}{2} \Delta \Gamma t \right) - 23 \Re \left( \frac{p}{q} A_f A_f^\dagger \right) \sinh (\Delta M t) \] \tag{2.55}

where \( N_f \) is a time independent normalisation factor, \( \Im \) is the imaginary operator and \( \Re \) is the real operator.

As the \( B_s^0 \) and its anti-particle \( \bar{B}_s^0 \) both decay to the same final state, \( \mathcal{C}\mathcal{P} \) violation enters through the interference between the mixing and the decay amplitudes via the process described in section 2.5.3. This interference \( \mathcal{C}\mathcal{P} \) violation can be observed using the asymmetry of the neutral meson decays into final \( \mathcal{C}\mathcal{P} \) eigenstates, \( f_{CP} \).

\[ A_{f_{CP}}(t) = \frac{\frac{d}{dt} [B^0_s \to f_{CP}] - \frac{d}{dt} [\bar{B}_s^0 \to f_{CP}]}{\frac{d}{dt} [B^0_s \to f] + \frac{d}{dt} [\bar{B}_s^0 \to f]} \] \tag{2.56}

This can be re-written as

\[ A_f(t) = -\frac{A_{mix_{CP}} \sin (\Delta M t)}{\cosh \left( \frac{1}{2} \Delta \Gamma t \right) + A_{\Delta \Gamma} \sinh \left( \frac{1}{2} \Delta \Gamma t \right)} \] \tag{2.57}

For the case of the decay \( B_s^0 \to J/\psi \phi \),

\[ A_{mix_{CP}} (B_s^0 \to J/\psi \phi)_{CP\pm} = \pm \sin(\phi_s) \] \tag{2.58}

\[ A_{\Delta \Gamma} = \mp \cos(\phi_s) \] \tag{2.59}

where

\[ \phi_s = 2 \arg \left( \frac{V_{ts}^* V_{ts}}{V_{tb} V_{cs}^*} \right) \] \tag{2.60}

The decay \( B_s^0 \to J/\psi \phi \) contains two vector mesons that do not have well defined orbital momentum and therefore cannot be \( \mathcal{C}\mathcal{P} \) eigenstates. As such, an angular analysis is required to distinguish \( \mathcal{C}\mathcal{P} \)-odd from \( \mathcal{C}\mathcal{P} \)-even eigenstates. This method was proposed in reference \([9]\). This angular analysis is beyond the scope of this thesis, but has been described and performed elsewhere, for instance in references \([8, 10]\).
2.6.1 New physics potential in $B_s^0 \rightarrow J/\psi\phi$

In the decay $B_s^0 \rightarrow J/\psi\phi$, the behaviour of the $b \rightarrow c\bar{c}s$ transition has Standard Model tree contributions and is unlikely to be affected by new physics contributions. However, the $B_s^0\overline{B_s^0}$ mixing is a place for potential new physics to manifest itself.

In the Standard Model, the main contribution to the quark sides of the box diagram in figure 2.3 arises from $t\bar{t}$. In the case that heavy non-Standard Model particles were observed, they would have an impact on the rate that mixing occurs.

In general, new physics would change the phase $\phi_s$ such that

$$\phi_s = \phi_s^{SM} + \phi_s^{NP} \quad (2.61)$$

where

$$\phi_s = -2\delta\gamma = -2\lambda^2\eta \simeq -0.03 \quad (2.62)$$

If new physics is observed here, it would affect the value of $M_{12}$, which leads to a change in the difference in decay width between the heavy and light states.

$$\Delta\Gamma = \Delta\Gamma^{SM}\cos 2\theta \quad (2.63)$$

The expected Standard Model value of $\Delta\Gamma$ is $\simeq 15\%$ of the $B_s^0$ lifetime[11]. If present, new physics would decrease $\Delta\Gamma$ below this value, depending on the size of its contribution. For this reason, a high precision measurement of the both $B_s^0$ lifetimes is required in order to check for the presence of new physics. This is discussed in detail in reference [12].
Chapter 3

The Large Hadron Collider

The Large Hadron Collider (LHC) is a proton-proton beam accelerator and collider built at the European Organisation for Nuclear Research, CERN, near the city of Geneva, Switzerland. A 26.659 km in circumference circular tunnel houses the LHC at a depth that varies between 50 m and 175 m underground and passes under both Switzerland and France, as shown in figure 3.1. It was constructed for a nominal centre of mass energy of 14 TeV and luminosity of $10^{34}$ cm$^{-2}$ s$^{-1}$. The accelerator complex in the tunnel holds two parallel proton beams that travel in opposite directions around the LHC ring and intersect at 4 interaction points. The beams are confined and bent by approximately 1600 superconducting magnets, each of which operates at liquid Helium temperatures of 1.9 K. As such, the LHC is both the largest superconducting system and the largest cryogenic system in the world. The LHC is also very power intensive, consuming 120 MW, which is approximately the same as the household power consumption of Geneva Canton.

The LHC was built inside the pre-existing circular LEP tunnel and uses much of the same infrastructure, notably the injector accelerator complex of the previous generation accelerator. Many of the tunnels from the LEP accelerator complex were also pre-existing and built for the previous generation of experiments. This chapter is a summary of the LHC technical design report [13].

3.1 Physics motivation for building the LHC

The motivation for the construction of the LHC was to further understanding of fundamental experimental particle physics. It was built as a discovery machine with the expectation that it sees the first direct signs of some electro-weak symmetry breaking mechanism, and with the hope that it will uncover physics beyond the Standard Model.
The Higgs Boson is the only fundamental particle of the Standard Model that has not been observed, although some ranges of mass have been excluded. The Higgs mass is currently limited to with 95% confidence greater than 114 GeV from LEP, not between 160 and 170 GeV from Tevatron and less that 190 GeV from theoretical constraints. A summary of current Higgs searches and properties can be found in reference [1].

The top quark will be produced at a rate of order 10 Hz at the LHC, allowing the world’s best measurement of the top mass. A good understanding of top quark physics is required as they are an important background to many of the potential discovery channels for Higgs bosons and new physics. For an introduction to top quark physics at the LHC, see reference [14].

The $b$-quark has been studied with high statistics at BaBar and Belle experiments. These $B$ factories produced great numbers of $B$-hadrons in the form of $B^0 \overline{B^0}$ pairs and $B^+ B^-$ pairs. The heavy $B_s^0$ meson was first discovered at LEP and was observed at the Tevatron with relatively small statistics. As such, many questions remain, especially for those topics that can only be addressed by the $B_s^0$ with large event samples. Furthermore, the study of $B$-physics is useful for the calibration and validation of the general purpose detectors. Some details of $B$-physics theory were presented in chapter 2.

The LHC also has plans to accelerate heavy ions, expected to occur roughly one month per year and will be starting approximately after one year of proton-proton running. The centre of mass collision energy will be 1150 TeV, which is 2.75 TeV per nucleon. ALICE is a dedicated heavy ion detector with the goal of studying a strongly interacting...
new phase of matter, quark-gluon plasma. The existence of quark-gluon plasma and its properties are key issues in order to better understand QCD and quark confinement. Details of the ALICE experiment can be found on their website and in reference [15].

3.2 Accelerator physics for a proton proton collider

To address its varied physics programme, the LHC detectors will need for the LHC to deliver large numbers of events at unprecedentedly high collision energies. The rate at which a specific interaction occurs in a collision experiment, \( R \), is determined by two major factors: \( \sigma_{\text{int}} \), the cross section of the interaction, and \( \mathcal{L} \), the luminosity of the collider as described in equation 3.1.

\[
R = \sigma_{\text{int}} \cdot \mathcal{L}
\] (3.1)

The cross section is an absolute measurement of the probability for a process to occur, measured in units of inverse area. The cross section of a given physical process, for instance \( b\bar{b} \) production at the interaction point is a function of the centre of mass energy, \( \sqrt{s} \), as shown in figure 3.2. The centre of mass energy is the energy with which the two protons collide, and is an indicator of the energy available to produce new particles in a collision. Figure 3.2 shows the cross sections for some contributing important hard processes expected in high-energy pp collisions, as well as the inclusive cross section. The dominant elastic scattering is not shown in figure 3.2. The production cross section that is relevant in this thesis, the \( b \)-quark production cross section \( \sigma_b \), is expected to rise with increasing \( \sqrt{s} \). Due in part to the expected increase of many significant cross sections as a function of \( \sqrt{s} \), the LHC will deliver an unprecedentedly large centre of mass energy in order to produce a large number of interesting physics events.

The upper beam energy limit of the LHC can be derived from first principles, as a combination of the equations for a charged particle moving in a field

\[
F = q\mathbf{v} \times \mathbf{B}
\] (3.2)

and the centripetal equations:

\[
F = \frac{m\mathbf{v}^2}{R} = \frac{\mathbf{p}^2}{mR}
\] (3.3)

where \( \mathbf{F} \) is the force exerted on the charged particle, \( q \) is the charge, \( \mathbf{v} \) is velocity and \( \mathbf{B} \) is the magnetic field strength, \( \mathbf{p} \) is the momentum and \( R \) is the bending radius. Together,
they can be solved for momentum \( (p = mv) \);

\[
\frac{p^2}{mR} = qvB
\]  

(3.4)

becomes

\[
p^2 = q \frac{vm}{p-im} RB
\]  

(3.5)

and finally, if it can be assumed that the protons are highly relativistic, the substitution \( p = \frac{E}{c} \) where \( c \) is speed of light is appropriate.

\[
E = qRBc
\]  

(3.6)

The values used are \( q = 1eV/CJ^{-1} \), the maximal bending radius is 2806 m, and the maximal field strength is 8.33 T, resulting in the value \( E = 7.01 \) TeV per beam.

The specific luminosity, \( \mathcal{L} \), is the number of particles per beam cross sectional area per unit time. In essence, luminosity is a measure of the frequency of opportunity for an interaction to occur. This means that the number of interactions per second increases with the luminosity of the collider. As such, luminosity is considered a figure of merit for the comparison of colliders and is given by

\[
\mathcal{L} = f \frac{n_1n_2}{4\pi \sigma_x \sigma_y}
\]  

(3.7)

where \( n_1 \) and \( n_2 \) are the number of protons per bunch for each beam, \( f \) is the frequency of bunch crossings and \( \sigma_x \) and \( \sigma_y \) describe the beam profile in the horizontal and vertical directions, respectively, such that \( 4\pi \sigma_x \sigma_y \) describes the cross sectional area of the beams. \( \sigma_x \) and \( \sigma_y \) are of dimension length.

The integrated luminosity is an indication of the maximum number of events that have been delivered to an experiment; the actual number of recorded events depends on data-taking efficiency and acceptances. The integrated luminosity has units of inverse area, and at a collider like the LHC, it is often measured in nanobarns, where one nb= \( 10^{-37} \) m\(^2\) or in picobarns (pb) where one pb= \( 10^{-40} \) m\(^2\).

The LHC, like any synchrotron, can be thought of as an accelerating device and a magnetic field to return the protons back to the accelerating device. Both the luminosity provided by the LHC and the centre of mass collision energy are uniquely determined by the properties of the accelerator. The physical properties of the accelerator were restricted by the size of the LEP tunnel and the limitations of magnetic technology and budget. The LHC machine performance was limited by the following 7 effects [13]:
Figure 3.2: A diagram of the LHC production cross section for various interesting processes as a function of centre of mass collision energy. At $\sqrt{s} = 14\,\text{TeV}$, $\sigma_{\text{total}} = 99.4\,\text{mb}$ and $\sigma_{b} = 0.633\,\text{mb}$. This plot was produced in reference [16], using a NNLO parton distribution known as MRST after its authors, Martin, Roberts, Stirling and Thorne.

- Beam-beam interaction
- Mechanical aperture
- Maximum dipole field and magnet quench limits
- Energy stored in the circulating beams
- Heat load
- Field quality and dynamic aperture
- Beam instabilities
These seven effects limit the LHC machine performance, which in turn puts an upper limit on the centre of mass energy and luminosity as shown in section 3.2. The maximum beam energy of the LHC is 14 TeV, but the actual field in the storage ring is dependent on heat load, temperature margins inside cryo-magnets and beam losses. Although the heat load due to protons is tiny compared to electron synchrotron radiative losses, it still imposes limits on LHC performance as synchrotron radiation needs to be absorbed into the cryogenics system.

Due to the risk of magnet quenching, the centre of mass energy is expected to be kept lower than the nominal value until the quench protection systems and safety systems are installed for all sectors and properly commissioned. A magnet quench occurs when a magnet loses its superconductive properties and enters a normal resistive state. When a section of the magnet goes from superconducting state to a normal resistive state in a quench, that section of the magnet still undergoes the same current, but is no longer perfectly conductive. This can cause catastrophic resistive heating if the quench is confined to a small area of a magnet. Magnet quenches occur more readily with higher field strength. As such, in order to prevent quenching in the early data-taking period, the magnetic field will remain lower than nominal, and the centre of mass collision energy will be 3.5 TeV per beam.

3.3 The eight sectors

The LHC is divided into eight sectors, which are separated by eight access points, each of which has a separate usage. There are six LHC experiments that surround the four LHC collisions points. They are ATLAS, CMS, LHCb, ALICE, TOTEM and LHCf. Both ATLAS and CMS are considered general purpose detectors, LHCb is a dedicated B physics experiment and ALICE is a heavy ion experiment. TOTEM is a total elastic and diffractive cross section measurement experiment and the LHC forward detector LHCf measures the energy and numbers of neutral pions to improve understanding of high energy cosmic rays. The other four points are dedicated to beam maintenance: injection, RF cavity, beam cleaning and beam dump.

With such a high energy density, many levels of beam safety have been implemented to protect the magnets, detectors and other sensitive apparatus. These safety procedures include collimators to clean the beam and a beam-dump to eject it from the LHC. Collimators are made of absorbing material that is placed close to the beam to intercept any stray protons. The bulk of the collimating material has been collected together at points 3 and 7, and can be as close as 0.25 mm from the beam.
A beam dump is located at point 6 of the LHC. This is where the high energy protons are ejected from the LHC in the case of some problem that could endanger the magnets or some other apparatus. The beam is also dumped at the end of a run when the differential luminosity begins to drop. This is expected to occur maybe once every day. The beam is dumped from the LHC using extremely fast magnets. These “kicker” magnets attain maximum field strength in less time than it takes protons to complete a loop. The kicker magnets push the beam away from the LHC and further magnets de-focus the beam so it can be safely spread evenly on a large block of concrete and graphite.

### 3.4 Beam properties

Relative to the previous generation of collider experiments, the LHC was designed to have more bunches, more population per bunch, more collisions per bunch, tighter packed bunches, a smaller beam spot, a smaller collision angle, and more frequent collisions. During the acceleration process, the intended default scheme is such that the protons are grouped into 2808 bunches at capacity, which are separated by 25 ns. Nominal bunches are expected to contain $11.5 \times 10^{10}$ protons, and are expected to be completely symmetric between the two colliding beams. The beam size at the interaction point will be 16 microns. Due to the LHC using a separate beam pipe for each particle beam, the beams meet at the collision point with a crossing angle of 285 $\mu$rad. The total energy of the nominal beam is 362 MJ per beam. Inside the beam-pipe, a vacuum of roughly $10^{-10}$ Torr or $1.3 \times 10^{-7}$ Pascals ($N/m^2$) is maintained. This is equivalent to 3 million molecules per cm$^3$. Table 3.1 has a summary of the beam properties at nominal running energy.

| Bunch Intensity | $1.15 \times 10^{11}$ |
| Number of bunches | 2808 |
| Beam size at IP | 16 $\mu$m |
| Crossing angle | 285 $\mu$rad |
| Bunch length | 1.06 ns |
| Luminosity | $10^{34}$ cm$^{-2}$s$^{-1}$ |
| Total Beam energy | 362 MJ per beam |
3.5 The proton injection complex

The LHC protons originate from ionised Hydrogen through a duoplasmatron. The duoplasmatron receives Hydrogen from a bottle of Hydrogen gas and produces a beam of ionised protons that is then subjected to a series of accelerators to increase their energy. The first accelerating system is the linear particle accelerator LINAC2. It emits 50 MeV protons. Subsequently, the Proton Synchrotron Booster accelerates protons to 1.4 GeV, and the Proton Synchrotron accelerates them up to 26 GeV. Then, protons are accelerated to 450 GeV by the Super Proton Synchrotron, from which energy they can be injected into the main ring. The main ring accelerates them up to 3.5 TeV or eventually 7 TeV. Figure 3.3 shows the complete acceleration complex. This image is taken from reference [17].

![CERN Accelerators diagram](image)

**Figure 3.3:** A diagram of the LHC accelerator complex.

3.6 LHC magnets and RF cavity

There are two main types of magnet at the LHC. The quadrupole magnets constrict the beam and the dipole magnets bend the beam. The radio frequency (RF) cavities increase the energy in the beam by increasing the revolution frequency of the bunches.
3.6.1 Dipole magnets

Dipole magnets are used in particle accelerators to bend charged particles. The LHC has 1232 dipole magnets creating a maximal bending field of 8.33 Tesla and a bending radius of 2805.95 m. Each dipole magnet is 14.3 m long. The superconducting material is Niobium Titanium (NbTi) carries a current of 11850 Amperes. These magnets are cooled to 1.9 K using liquid Helium. Figure 3.4 shows a diagram of an LHC dipole magnet, and a photograph is shown in 3.5. These images are taken from reference [18, 19].

![Figure 3.4: A diagram of the LHC cryo-dipole magnets.](image)

![Figure 3.5: A photograph of an LHC Dipole magnet.](image)
3.6.2 Quadrupole magnets

Quadrupole magnets are used in particle accelerators to focus beams of charged particles. The 392 quadrupole magnets of the LHC are used for strong focusing and confining of the beam. The focusing scheme used at the LHC can be likened to a series of alternating focusing and defocusing lenses. Quadrupole magnets will focus the beam in one transverse plane while defocusing in the direction perpendicular to that plane. As a consequence, the LHC quadrupoles are lined up in alternate directions so the beam is continually focused and defocused in both x and y directions. Figure 3.6 shows an illustration of the cross section of a quadrupole magnet. Figure 3.7 is a photograph in which the 4 magnets of each beam-pipe can easily be seen. The final focusing of the beam is provided by a triplet of quadrupole magnets on each side of the collision point. These images are taken from reference [18, 19].

\[ \text{Figure 3.6: A diagram of an LHC Quadrupole magnet.} \]

3.6.3 Radio Frequency cavity

The “ramping-up” of beam energy in the LHC from the injection energy of 450 GeV up to 7 TeV is done by the Radio Frequency (RF) cavity at point 4. The RF cavity contains an oscillating voltage which increases the energy of the particles in the ring with every pass. In order to accelerate the proton bunches from 450 GeV up to 7 TeV, the voltage in the LHC RF cavity is gradually increased up to design voltage of 16 MV or 5.5 MV m$^{-1}$ at a frequency of 40 MHz. A photograph of the LHC Radio Frequency Cavity is shown in Figure 3.8. This image is taken from reference [19].
The principle behind an RF cavity is based on the principle of phase stability and can be thought of as a sinusoidal restoring force. The net acceleration, $A_e$, on a particle of charge $e$ in a magnetic field $B$ is

$$A_e = eV \sin (\omega \Delta t) \quad (3.8)$$

The peak acceleration and deceleration occur when $\omega \Delta t = \frac{\pi}{2}$ and $\omega \Delta t = \frac{3\pi}{2}$. The voltage in the cavity oscillates with some frequency, $f$ and a period $T = \frac{1}{f}$. As such, the amount of energy a passing proton would gain is a sinusoidal function of period $T$. If a proton passes the RF cavity at time $t = 0$, the beginning of a voltage oscillation, but takes a little longer than $T$ to circulate the ring, by the next time it returns to the
RF cavity, it will arrive a little later than the rest of its bunch. Due to the sinusoidal restoring force, it will receive slightly more acceleration than it would had it arrived at $t = T$. In other words, if a proton arrives late it receives more acceleration, and if it arrives early, it receives less acceleration. In addition to uniform beam energy, the RF cavity acceleration has the added bonus that it causes bunches to form.

3.7 First beam

On the morning of the 10th of September 2008, the first beam was circulated in the LHC and the first pass was in the clockwise direction. Two beam-spots could be seen clearly in the x-y plane in the beam monitoring tools, as shown in figure 3.9. This image was taken from reference [20]. The two spots were made by the beam on its first and second pass around the LHC ring. Later on the same day, beam was successfully circulated in the anti-clockwise direction as well. Many elements of the LHC were successfully tested with beam for the first time, including some collimator systems, the dump block, the beam kickers and the CMS and ATLAS detectors. Testing progressed extremely quickly and before the end of the first day of beam, a beam was measured to successfully complete at least 300 turns. Due to these reasons, first beam was considered by many at CERN to be a day of triumph, and many bottles of champagne were opened around CERN.

Figure 3.9: The first time that the beam completed a full turn of the LHC. The two spots show the beam on its first and second passes.
Chapter 3: *The large hadron collider*

3.8 Incident of the 19\textsuperscript{th} of September 2008

In contrast to the enthusiastic success of the first beam events, only 9 days after first beam, catastrophe stuck the LHC. The following explanation was given in reference [21]. During a power test on the 19\textsuperscript{th} of September 2008, a resistive zone developed in a sector 34 dipole bar splice, specifically between Q24 R3 and its neighbour. A dipole bar splice is shown in figure 3.4, on the left hand side of the picture, and is labelled as spool piece bus bars and quadrupole bus bars. The resistance was found to be about 220 n\(\Omega\) and was due to bad electrical and thermal contacts. This led to an electrical arc which punctured the Helium enclosure. Then, Helium was allowed into the insulating vacuum. The relief valves that allow He venting actuated as they were supposed to, but could not handle the flow of Helium, as between 6 and 15 tonnes of liquid Helium were released before the leak could be contained. Enormous pressure waves travelled both ways along the accelerator, causing collateral damage over hundreds of metres. The magnets in the damaged section were ripped off their support jacks and moved by many cm and the vacuum chamber was severely contaminated. In addition to the mechanical damage, the vacuum in 59 magnets from sector 3-4 was contaminated with soot.

The incident caused massive damage to the LHC and a further delay of roughly one year. This delay was not due only to repairs to the damaged area, but also due to the time it would take to heat up other sectors, perform checks, implement new safety procedures, and subsequently cool down the magnets again. The repairs and new safety measures caused by the incident were very involved, including over 2000 new Helium relief safety valves, new bus bar slice construction, new monitoring systems, an upgrade to the quench protection system, new floor jacks, new support posts and new vacuum barriers.

After these repairs and new safety equipment installation had been completed, beam circulation was continued in November 2009. For the first time beam was circulated in both LHC beam-pipes and all four detector experiments recorded collision candidate events. On the 30\textsuperscript{th} of November 2009, the LHC achieved the highest beam energy world record for with 1.18 TeV per beam [22]. Then, 3.5 TeV per beam collisions began in March 2010.
Chapter 4

The ATLAS detector

The ATLAS detector is a multi-purpose detector built to probe the boundaries of the Standard Model. The acronym stands for “A Toroidal LHC ApparatuS”. ATLAS was built at point 1 of the LHC. ATLAS is one of four main LHC detectors, the other three being the Compact Muon Solenoid (CMS), LHCb and A Large Ion Collider Experiment (ALICE). ATLAS is the largest of the four with external dimensions of a 44 m long cylinder with a diameter of 25 m. At a weight of 7000 tonnes, ATLAS is slightly more than half the weight of the second largest LHC detector, CMS which weighs 12500 T.

The ATLAS collaboration was formed from two proto-collaborations, ASCOT and EAGLE, and its letter of intent was published in 1992 [23]. Construction began in 1999 while beam was still circulating in the LEP tunnel and first beam passed through the detector on the 10th of September 2008. The first beam-beam collisions occurred in November 2009. ATLAS is fully described in the two ATLAS Technical Design Reports [24] and [25], and this chapter is based on those reports. Unless otherwise stated, the photographs in this chapter were taken from the official ATLAS public web site [26].

As a general-purpose detector on the LHC, ATLAS was built to work towards the following physics tasks:

- Search for a Higgs boson over a range of possible masses and potential theories.
- Search for signs of physics beyond the Standard Model.
- Perform precise measurements of $W^\pm$ and top masses.
- Perform precise measurements of $CP$ violating and CKM parameters, expanding our understanding of the Standard Model.
In order to achieve the physics goals of each different group, the ATLAS detector was designed to have the following capabilities:

- A large pseudo-rapidity acceptance and full azimuthal angle acceptance.
- Efficient tracking for full event reconstruction at low luminosity and identifying leptons, hadronic jets and photons at high luminosity.
- Electromagnetic calorimetry for lepton and photon identification.
- Hadronic calorimetry for jet and missing energy reconstruction.
- An advanced filtering and data acquisition system to pick out and save the interesting events out of the millions of background events.

These capabilities are implemented into ATLAS through four main detector systems each of which is described in this chapter.

- The inner detector tracks the paths of electrically charged particles.
- The liquid Argon calorimeter and the tile calorimeter measure the energy of charged and neutral particles and also stop most hadrons and non-muonic leptons.
• The muon spectrometer identifies muons, then measures the path they take to
determine their momenta.

• The magnet system bends charged particle trajectories to allow the measurement
of the particle momenta.

In the barrel region, each sub-detector is built in concentric cylindrical layers, and in
the end-cap region, the sub-detectors are formed of successive disks. Figure 4.1 shows
an illustration of the detector. This image is taken from reference [26].

The proton-proton interactions of LHC collisions will create an enormous quantity of
data which can not all be recorded, as it would quickly overwhelm any local computing
site. The Trigger and Data Acquisition (TDAQ) system is in place to filter out and
record the interesting physics events. The trigger system selects 200 Hz of interesting
events out of 40 million others every second. The data acquisition system reads the
data from the detector, and the event filter transmits it to Castor input buffers in the
computer centre via a 1 GB s\(^{-1}\) link.

The distributed computing system distributes the data around the world, performs re-
processing and analysis of the 1000 million events recorded per year. The ATLAS
computing model is described with more detail in chapter 5.

4.1 Coordinate system

The ATLAS detector and the objects it detects are described using a standardised
coordinate system, as shown in figure 4.2. In the Cartesian x-y-z coordinate system, the
z-axis is defined to be along the beam-pipe with positive z pointing in the anti-clockwise
direction around the ring. As in previous chapters, clockwise and anti-clockwise are
defined as viewed from above. The x-y plane is defined perpendicular to the beam
pipe such that the positive x-direction points towards the centre of the LHC ring and
the positive y-direction points towards the surface. Due to the tilt of the ring relative
to the vertical axis, the y-direction is tilted by 0.704° with respect to vertical. These
conventions are shown in figure 4.2.

In addition to the Cartesian coordinates, the polar coordinates \(\phi\) and \(\theta\) are also used in
both the physical description of the ATLAS detector and in the trajectory of a particle
in the detector. The azimuthal angle \(\phi\) is the angle between the x-axis and the track
and sweeps over the x-y plane, between \(-\pi < \phi < \pi\) and with \(\phi = 0\) on the positive
x-axis. Figure 4.3 shows the \(\phi\) convention used in the ATLAS collaboration. This image
was taken from reference [8]. The polar angle \(\theta\) is measured from the beam axis in the
y-z plane. ATLAS adopts the convention by which positive $\theta$ values refer to the positive $z$-direction.

The transverse momentum, $p_T$, is defined as the momentum perpendicular to the LHC beam axis. The pseudo-rapidity of a track, $\eta$, is a measure of the angle between that track and the $z$-axis. It is defined as:

$$\eta = -\ln \left( \tan \left( \frac{\theta}{2} \right) \right)$$  \hspace{1cm} (4.1)$$

where the polar angle $\theta$ is the angle between the track and the positive $z$-axis. A perfectly transverse particle has zero pseudo-rapidity, whereas a particle travelling along
the beam-pipe has either positive or negative infinite pseudo-rapidity, depending on the z-direction of travel.

4.2 Beam-pipe

The ATLAS beam-pipe spans the distance between two collimators at either end of the point 1 cavern and forms an ultra-high vacuum system. The collimators serve to protect the ATLAS experiment and the beam-pipe maintains the same vacuum as the LHC, roughly $10^{-10}$ Torr. The vacuum was created by vacuum pumps and baking-out the beam-pipe. Baking-out is the heating of the beam-pipe to make trapped gases evaporate out of the relatively transparent beam-pipe.

The 38 m long beam-pipe consists of seven pipes bolted together. The central Beryllium beam-pipe section is 7.1 m long, roughly the length of the inner detector, and is centred around the interaction point, with an inner and outer radii of 29 mm and 34.3 mm respectively. The remainder of the beam-pipe in the ATLAS detector is composed of six stainless steel sections, installed symmetrically on either side of the Beryllium beam-pipe section. Each symmetric pair becomes progressively thicker with distance from the interaction point, with diameters of 60 mm, 80 mm and 120 mm.

The amount of material in the inner detector and beam-pipe upstream of calorimetry needs to be minimised in order to reduce multiple scattering, which would otherwise inundate the inner detector with hits. Beryllium was chosen as it is almost transparent to energetic particles because of its low atomic number. This transparency effectively reduces the amount of material.

4.3 Inner detector

After the beam-pipe, the inner most part of the ATLAS detector is appropriately called the inner detector (ID). A schematic of the inner detector is shown in figure 4.4. The ATLAS inner detector is cylinder-shaped with length of 7 m and a radius of 1.15 m and it is held within a the solenoidal magnet such that it bathes in a 2 Tesla magnetic field. As it is extremely close to the interaction point, the inner detector need to be resistant to radiation and magnetic fields.

The aim of the inner detector is to provide high granularity, precise tracking formation, and high quality measurements of momenta. With this information, tracking can be used to form primary and secondary vertices. Secondary vertices are crucial for performing
B-physics measurements, and can also be used to understand the influence of alignment. The inner detector alignment process is described in chapters 8 and 7.

The inner detector is built of three independent but complementary sub-detectors: the pixel detector, the semi-conductor tracker (SCT) and the transition radiation tracker (TRT). These sub-detectors are described in sections 4.3.1, 4.3.2 and 4.3.3 respectively. An overview of the inner detector size is given in table 4.1 and an overview of its properties is given in table 4.2. These tables both originate in reference [25].

Section 8.3 contains a study of the impact of extra material in the inner detector on inclusive $b\bar{b} \rightarrow J/\psi X$ studies. Extra material upstream of the calorimeter systems is dangerous because of multiple scattering, which can increase the amount of late forming tracks in the inner detector. Extra material can also reduce the energy resolution of the calorimeters. The size of the inner detector in terms of radiation lengths are shown in table 4.3. This table was calculated from simulated inner detector as implemented in simulation and the data are averaged over $\phi$. This table originated in the technical design report in reference [25]. The total radiation length of the ATLAS inner detector, as defined in equation 4.2, is $0.469 X_0$ at $|\eta| = 0$ and $1.126 X_0$ at $|\eta| = 1.8$.
Figure 4.5: The layers of the ATLAS inner detector barrel.

Figure 4.6: The layers of the ATLAS inner detector end-cap.
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Table 4.1: Summary of the dimensions of the inner detector.

<table>
<thead>
<tr>
<th>Item</th>
<th>Radial extension (mm)</th>
<th>Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>0 &lt; R &lt; 1150</td>
<td>0 &lt;</td>
</tr>
<tr>
<td>Beam-pipe</td>
<td>29 &lt; R &lt; 36</td>
<td></td>
</tr>
<tr>
<td>Pixel Overall envelope</td>
<td>45.5 &lt; R &lt; 242</td>
<td>0 &lt;</td>
</tr>
<tr>
<td>3 barrel layers Sensitive barrel</td>
<td>50.5 &lt; R &lt; 122.5</td>
<td>0 &lt;</td>
</tr>
<tr>
<td>2x3 disks Sensitive end-cap</td>
<td>88.8 &lt; R &lt; 149.6</td>
<td>495 &lt;</td>
</tr>
<tr>
<td>SCT Overall envelope</td>
<td>255 &lt; R &lt; 549 (barrel)</td>
<td>0 &lt;</td>
</tr>
<tr>
<td>4 barrel layers Sensitive barrel</td>
<td>251 &lt; R &lt; 610 (end-cap)</td>
<td>810 &lt;</td>
</tr>
<tr>
<td>2x9 disks Sensitive end-cap</td>
<td>299 &lt; R &lt; 514</td>
<td>0 &lt;</td>
</tr>
<tr>
<td>TRT Overall envelope</td>
<td>554 &lt; R &lt; 1082 (barrel)</td>
<td>0 &lt;</td>
</tr>
<tr>
<td>73 straw planes Sensitive barrel</td>
<td>617 &lt; R &lt; 1106 (end-cap)</td>
<td>827 &lt;</td>
</tr>
<tr>
<td>160 straw planes Sensitive end-cap</td>
<td>563 &lt; R &lt; 1066</td>
<td>0 &lt;</td>
</tr>
<tr>
<td></td>
<td>644 &lt; R &lt; 1004</td>
<td>848 &lt;</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of the properties of the inner detector.

| System       | Position       | Area m² | Resolution σ(μm) | Channels 10⁶ | | coverage |
|--------------|----------------|---------|------------------|--------------|-----------|
| Pixels       | B layer        | 0.2     | Rφ = 12, z = 66  | 16           | 2.5       |
|              | Other 2 barrel layers | 1.4     | Rφ = 12, z = 66  | 81           | 1.7       |
|              | 4 end-cap disks | 0.7     | Rφ = 12, z = 77  | 43           | 1.7 -2.5  |
| SCT          | 4 barrel layers | 34.4    | Rφ = 16, z = 580 | 3.2          | 1.4       |
|              | 9 end-cap wheels | 26.7    | Rφ = 16, z = 580 | 3.0          | 1.4-2.5   |
| TRT          | Axial barrel straws | 170 (per straw) | 0.1 | 0.7 |
|              | Radial end-cap straws | 170 (per straw) | 0.32 | 0.7 2.5 |

4.3.1 Pixel detector

The pixel detector is the inner most sub-detector and was designed to measure precisely three spatial points per track over an angular acceptance of |η| < 2.5. The pixel detector determines the impact parameter resolution and strongly influences ATLAS’s ability to observe short lived particles such as B-mesons and τ-leptons.

The barrel section of the pixel detector consists of three layers of silicon wafers at radii of 50.5 mm, 88.5 mm and 122.5 mm and extends from |η| < 1.7. The inner most layer is known as the B-layer, it extends slightly further over |η| < 2.5 and is essential for
Table 4.3: Summary of the properties of the inner detector in terms of radiation lengths.

|                | \( |\eta| = 0 \) | \( |\eta| = 1.8 \) |
|----------------|----------------|-----------------|
|                | Radius (mm) | \( X_0 \) | Radius (mm) | \( X_0 \) |
| Exit beam-pipe | 36          | 0.0045 | 36          | 0.014   |
| Exit pixel layer-0 | 57        | 0.037 | 57          | 0.105   |
| Exit pixel layer-2 | 172       | 0.108 | 172         | 0.442   |
| Entry SCT       | 253         | 0.119 | 253         | 0.561   |
| Entry TRT       | 552         | 0.205 | 621         | 0.907   |
| Exit TRT        | 1081        | 0.469 | 907         | 1.126   |

secondary vertex finding. These three barrel layers are made of identical slightly overlapping staves inclined with azimuthal angle of 20 degrees. These overlapping staves can be seen clearly for the \( B \)-layer in the photograph of figure 4.7. The slightly overlapping stave construction ensures that there is some minimal overlap between modules and reduces the number of “cracks” or gaps in coverage. There are 22, 38 and 52 staves in each of these layers respectively and each stave is composed of 13 pixel modules.

Figure 4.7: A photograph of the pixel detectors \( B \)-layer under construction.

The pixel detector end-cap covers the region \( 1.7 < |\eta| < 2.5 \) and is formed of three disks on each side of the barrel at distances \( 11 \text{ cm} < z < 20 \text{ cm} \) from the interaction point. Each disk is made of eight sectors, with six modules in each sector and disk modules are identical to the barrel modules, except for their connecting cables. The end-cap sections of the entire inner detector can be seen in figure 4.6.
The barrel and the end-cap modules all measure 62.4 mm by 22.4 mm and contains 16 chips. Each chip has 24 by 160 pixels. This means that the pixel detector has a total of 80.4 million readout channels spread over a surface area of 2.3 $m^2$. Each individual pixel measures 50 $\mu m$ wide and 300 $\mu m$ long and the sensors are 250 $\mu m$ thick, resulting in very little ambiguity in the position of a hit. The intrinsic accuracy of the pixels are 10 $\mu m$ (R-$\phi$) and 115 $\mu m$ (z) in the barrel region and 10 $\mu m$ (R-$\phi$) and 115 $\mu m$ (R) in the disks.

Both the pixel detector and the semi-conductor tracker rely on the use of semi-conductors to detect traversing charged particles or the absorption of photons. In essence, ionising radiation causes an electron from the semi-conductor valence band to jump into the conduction band, leaving a “hole” in the valence band. The number of electron-hole pairs produced is proportional to the energy transmitted by the ionising radiation to the semi-conductor. Under the influence of an electric field, the electron travels to an electrode. The current on this electrode is then amplified and translated into a digital signal which is later measured as a hit.

The readout of the inner detector varies between sub-detectors, but all three share some common features. A 40.08 MHz clock signal synchronous with the LHC bunch-crossings is used to time-stamp the signal generated in the low noise front-end electronics. As the level 1 trigger latency is 2.5 $\mu s$, the ID electronics signal is stored in binary or digital buffers for 3.2 $\mu s$. Following a L1 trigger, the buffer content associated with the bunch-crossing is transferred to a readout driver (ROD) o the detector. For more on triggers and data acquisition, see section 4.7.

4.3.2 Semi-conductor tracker

The semi-conductor tracker (SCT) uses silicon microchips to ensure that a further eight hits (four spatial points) are made for each track. Like the pixel detector, it is divided into three sections: a barrel and two end-caps. The sensitive detecting is performed by 4088 silicon modules arranged to form four barrel cylinders and eighteen disks with nine disks per end-cap. In total, the SCT has 61 $m^2$ of silicon with 6.3 million readout channels. Each of the silicon layers is double sided, with a stereo pitch between each side. This stereo pitch improves the knowledge of the hit position, by allowing a measurement in the y-direction from pulse timing as well as the x-direction.

The semi-conductor tracker barrel assembly has a coverage of $|\eta| < 1.4$. The barrel’s 2112 modules are shared between four cylindrical layers, and each module has four detector elements. The four barrel layers of the SCT are mounted on carbon fibre cylinders at radii of 30.0, 37.3, 44.7, and 52.0 cm. Like the pixel detector, the SCT modules
have a slightly overlapping stave construction that ensures full coverage. The barrel detector elements are 6.36 cm long by 6.40 cm wide and each element has 768 read out strips. As the sensitive detecting surface is composed of long and thin strips, the stereo pitch between each side allows for an increased precision in the measurement of z-axis coordinate of a hit. The accuracy of a spatial point in the SCT barrel is 17 $\mu$m in the $(R-\phi)$ direction and 580 $\mu$m in the $z$-direction.

The nine disks of each end-cap hold 1976 wedge-shaped modules in total and covers a pseudo-rapidity range of 1.1 < $|\eta|$ < 2.5. The end-cap trapezoidal sensors are built with radial strips of constant azimuth, and a mean pitch of 80 $\mu$m, and a set of stereo strips at an angle of 40 mrad. This results in an end-cap spatial point accuracy 17 $\mu$m in the $(R-\phi)$ direction and 580 $\mu$m the radial direction.

### 4.3.3 Transition radiation tracker

The transition radiation tracker (TRT) is the outermost sub-system of the inner detector. All charged particles with $p_T > 0.5$ GeV and $|\eta| < 2$ will traverse 36 straws and should increase the number of inner detector hits by an average of 36 hits, except in the barrel to end-cap transition region between 0.8 < $|\eta|$ < 1 where they are expected to traverse 22 straws. Relative to a semi-conductor detector, these 36 points are equivalent to a single point with 50 $\mu$m precision. The TRT also allows some identification between electrons and hadrons.

The 50000 TRT barrel straws are 144 cm long, 4 mm in diameter and run parallel to the beam axis. The barrel TRT is divided into three rings of 32 modules with a triangular substructure, as shown in figure 4.5. The number of straws per module is 329 for the
inner layer, then 520, then 793 straws per module in the outer layer. The barrel TRT provides information with an intrinsic accuracy of 170 μm per straw in the axial \((R - \phi)\) direction.

The end-cap of the TRT has 320000 straws arranged radially into 160 layers at \(|z| < 274.4\) cm. Each of the TRT end-caps is divided into two sections, the closest to the interaction point has 12 wheels with eight straw layers at 8 mm apart. This structure can be seen in the illustration in figure 4.6. The second section has eight wheels, also with eight layers but spaced at 15 mm apart. Each layer holds 768 straws of 37 cm in length with uniform azimuthal spacing. The end-cap TRT provides information with an intrinsic accuracy of 170 μm per straw in the radial direction.

The straws themselves contain a 31 μm diameter gold plated tungsten wire that runs the length of the straw. The straw tube wall is composed of two bonded 35 μm thick multi-layer films, and they are filled with a xenon based mixture, operating within an envelope of \(\text{CO}_2\). Unlike the shorted end-cap straws, the barrel straws gold-plated wire is divided into two at \(|\eta| = 0\). Transition radiation is emitted when particles cross boundaries between materials with different dielectric properties, in this case \(\text{CO}_2\), the films and xenon.

### 4.3.4 Inner detector track reconstruction

Approximately 1000 particles will emerge from the collision point every 25 ns resulting in a large track density, but momentum and vertex resolution require high precision measurements with high granularity. Inner detector track reconstruction is divided into three stages:

1. **Pre-processing**: which converts pixel and SCT raw data into clusters and converts TRT raw timing into calibrated drift cycles.

2. **Track finding**: Using hit information to form track candidates.

3. **Post processing**: a dedicated vertex finder is used to reconstruct the primary vertex. Later, offline algorithms build secondary vertices.

Track finding is run in stages, the first of which is the creation of track seeds from space-points in the three pixel layers. Then the seed is extended to the SCT to form a track candidate. Candidates are then fitted and cuts are performed to remove outliers and fakes. Then selected tracks are extended into the TRT, then tracks are refitted to include all three sub-detectors. The combination of the three sub-detectors gives high
precision in R-\(\phi\) and \(z\) coordinates. The probability of charge misidentification for muons and electrons increases with \(p_T\) and \(|\eta|\), as high \(p_T\) tracks are straighter in a magnetic field. Highly transverse muons suffer from a shorter effective trajectory in the magnetic field, which can increase muon misidentification. \(B\)-physics muons are sufficiently low momentum that charge misidentification is not problematic.

![Figure 4.9: The relative inner detector \(p_T\) resolution as a function of \(|\eta|\) for muons with \(p_T = 1, 5\) and 100 GeV.](image)

The resolution of a track parameter increases with the inverse of the transverse momentum. Figure 4.9 shows the transverse momentum resolution against pseudo-rapidity. This plot originates in reference [27]. This figure also shows that low \(p_T\) tracks can be measured with relatively low uncertainty, when compared with high \(p_T\) tracks.

### 4.4 Calorimetry

The aim of the ATLAS calorimetry system is to measure and absorb the energy of charged and neutral particles. Calorimetry is responsible for the accurate measurement of energy and position of electrons, photons, jets, missing energy, missing transverse momentum. The study of calorimetry can also allow the differentiation and identification of each of these particles. Furthermore, the calorimeters should provide good containment for electromagnetic and hadronic showers and limit punch-through and sneak-through into the muon system.

Calorimetry in the ATLAS detector is performed by two systems: the electromagnetic calorimeter and the hadronic calorimeter. The aim of the electromagnetic calorimeter is to provide precision measurement of electrons and photons, whereas the hadronic calorimetry system has coarser granularity which is acceptable for jet reconstruction.
and missing $E_T$ measurements. Energy is deposited into the detector as the particle showers, until the incident energy is absorbed entirely in the calorimeter.

Both the electromagnetic calorimeter and hadronic calorimeter are split into a barrel and two end-caps. The electromagnetic calorimeter, the hadronic calorimeter, and all subsystems are all shown in figure 4.10.

![Figure 4.10: The ATLAS calorimeters.](image)

The characteristic parameter of a electromagnetic calorimeter is its radiation length, $X_0$, usually expressed in g cm$^{-2}$. The radiation length is defined as the “both the mean distance over which a high-energy electron loses all but $1/e$ of its energy by Bremsstrahlung, and $7/9$ of the mean free path for pair production by a high-energy photon. It is also the appropriate scale length for describing high-energy electromagnetic cascades.” [1].

The radiation length, $X_0$, is described below to a good approximation:

$$X_0 = \frac{716.4 \cdot A}{Z(Z + 1) \ln \frac{287}{\sqrt{Z}}} \text{ g cm}^{-2} \quad (4.2)$$

where $Z$ is the atomic number and $A$ is the mass number.

Hadronic calorimeters are described by their interaction length. The interaction length is the mean free path of a particle before undergoing an interaction that is neither elastic nor quasi-elastic. The interaction length is designated by $\lambda$ and is dependant on the medium traversed by the particle.
### 4.4.1 Liquid Argon electromagnetic calorimeter

The electromagnetic calorimeter is an interleaved lead and liquid Argon (LAr) detector with a radial accordion geometry. This allows for complete $\phi$ coverage and easier readout without azimuthal cracks. The lead is an absorber and the liquid Argon is the sampling material. The electromagnetic calorimeter barrel covers the $|\eta| < 1.475$ region while its end-cap covers the region: $1.375 < |\eta| < 3.2$. The barrel electromagnetic calorimeter consists of two identical barrels separated by 4 mm at $z = 0$. The total thickness of the EM calorimeter is $>22 X_0$ in the barrel region and $>24 X_0$ in the end-cap regions. The thickness of the lead absorbers varies with $\eta$ and was optimised for best performance. A summary of the electromagnetic calorimeter granularity is given in table 4.4. This table originates in reference [25].

<table>
<thead>
<tr>
<th></th>
<th>Barrel</th>
<th>End-cap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \eta \times \Delta \phi$</td>
<td>$</td>
</tr>
<tr>
<td>Pre-sampler</td>
<td>0.025 $\times$ 0.1</td>
<td>$</td>
</tr>
<tr>
<td>1$^{st}$ layer</td>
<td>$0.025/8 \times 0.1$</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>$0.025 \times 0.025$</td>
<td>$1.40 &lt;</td>
</tr>
<tr>
<td></td>
<td>$0.025/8 \times 0.1$</td>
<td>$1.5 &lt;</td>
</tr>
<tr>
<td></td>
<td>$0.025/4 \times 0.1$</td>
<td>$2.0 &lt;</td>
</tr>
<tr>
<td></td>
<td>$0.1 \times 0.1$</td>
<td>$2.5 &lt;</td>
</tr>
<tr>
<td>2$^{nd}$ layer</td>
<td>$0.025 \times 0.025$</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>$0.075 \times 0.025$</td>
<td>$1.40 &lt;</td>
</tr>
<tr>
<td></td>
<td>$0.1 \times 0.1$</td>
<td>$2.5 &lt;</td>
</tr>
<tr>
<td>3$^{rd}$ layer</td>
<td>$0.050 \times 0.025$</td>
<td>$</td>
</tr>
</tbody>
</table>

Upstream of the electromagnetic calorimeter, a pre-sampler detector is used in the region $|\eta| < 1.8$ to correct for the energy lost by photons and electrons before they reach the electromagnetic calorimeter. The pre-sampler consists of an active LAr layer of thickness 1.1 cm in the barrel region and 0.5 cm in the end-cap region. Furthermore, in order to reduce the material in front of the calorimeter, the electromagnetic calorimeter shares a vacuum vessel with the central solenoid magnet, which is described below in section 4.6.1.

### 4.4.2 Hadronic calorimeter

The hadronic calorimeter is built of layers of steel and scintillator and it absorbs energy from particles that interact via the strong force. The hadronic calorimeter has three
distinct sections, the tile calorimeter, the liquid Argon end-caps and the liquid Argon forward calorimeter. All three sections can be seen in figure 4.10. It allows for measurements of $E_T^{miss}$ and also stops particles from punching through to the muon chambers. The granularity of all three sub-detectors is shown in table 4.5. In total, the hadronic calorimeters cover the pseudo-rapidity range $|\eta| < 4.9$.

Table 4.5: Summary of the hadronic calorimeter granularity.

| $\Delta \eta \times \Delta \phi$ | $|\eta|$ range  |
|-------------------------------|----------------|
| Scintillator tile calorimeter |
| Barrel                        | $0.1 \times 0.1$ | $|\eta| < 1.0$ |
|                               | $0.2 \times 0.1$ | $|\eta| < 1.0$ |
| Extended Barrel               | $0.1 \times 0.1$ | $0.8 < |\eta| < 1.7$ |
|                               | $0.2 \times 0.1$ | $0.8 < |\eta| < 1.7$ |
| LAr Hadronic End-cap calorimeter |
| Inner Disks                   | $0.1 \times 0.1$ | $1.5 < |\eta| < 2.5$ |
| Outer Disk                    | $0.2 \times 0.1$ | $2.5 < |\eta| < 3.2$ |
| LAr forward calorimeter       |
| FCal1:                        | $3.0 \times 2.6$ | $3.15 < |\eta| < 4.30$ |
|                               | $4 \times$ finer | $3.10 < |\eta| < 3.15$ |
|                               |                  | $4.30 < |\eta| < 4.83$ |
| FCal2:                        | $3.3 \times 4.2$ | $3.24 < |\eta| < 4.50$ |
|                               | $4 \times$ finer | $3.20 < |\eta| < 3.24$ |
|                               |                  | $4.50 < |\eta| < 4.81$ |
| FCal3:                        | $5.4 \times 4.7$ | $3.32 < |\eta| < 4.60$ |
|                               | $4 \times$ finer | $3.29 < |\eta| < 3.32$ |
|                               |                  | $4.60 < |\eta| < 4.75$ |

The tile calorimeter is immediately downstream of the electromagnetic calorimeter and solenoid magnet. It is composed of two sections, a barrel and an extended barrel, as shown in figure 4.10. The barrel section of the tile calorimeter cover the pseudo-rapidity range $|\eta| < 1.0$, and is similar in length to the electromagnetic calorimeter barrel. The tile calorimeter extended barrels cover the pseudo-rapidity range $0.8 < |\eta| < 1.7$. The tile calorimeter is divided into 64 modules azimuthally. The total thickness of the tile calorimeter at $\eta = 0$ is 9.7 $\lambda$. Each scintillator is read out twice by wavelength shifting optical fibres into two photo-multipliers.

The hadronic end-cap calorimeter covers the range $1.5 < |\eta| < 3.2$. In order to avoid cracks in pseudo-rapidity coverage, the hadronic end-cap calorimeter has an overlap with
both the tile calorimeter and the forward calorimeter. The hadronic end-cap calorimeter consists of four independent disks, with two on either side of the interaction point. Each of the disks is built from 32 identical wedge-shaped modules. There are two segments per module, giving each end-cap four layers. The two wheels closest to the interaction point use 25 mm thick disk shaped copper plates, while the outer wheels use 50 mm thick copper plates, with an inner radius of 0.475 m and an outer radius of 2.03 m. The active material is 8.5 mm of liquid Argon between the copper plates.

The forward calorimeter provides high $|\eta|$ coverage ($3.1 < |\eta| < 4.9$) while simultaneously reducing background radiation in the muon spectrometer. It is integrated with the end-cap cryostats and measures close to ten interaction lengths deep. Each forward calorimeter consists of three modules, which are built of a metal matrix, filled with tubes, rods, gaps, cathodes and liquid Argon scintillator. The first module is for electromagnetic measurements and is made of copper. The second and third modules are made of tungsten and measure the energy of hadronic interactions.

### 4.5 Muon system

Muons are light (105.7 MeV), stable and penetrating particles and are not produced in elastic (QCD) background collisions. Muons with a transverse momentum of order 10 GeV are characteristic of $B$ decays.

![Figure 4.11: A cut away view of the muon system.](image)
The muon chambers were built with the goal of maximal coverage. ATLAS can perform muon measurements from $|\eta| < 2.7$ and trigger on muons from $|\eta| < 2.4$. In the barrel region, there are three cylindrical layers at inner radii at 5 m, 7.5 m and 10 m. Each muon chamber end-cap is comprised of four vertical disks at a distance of 7 m, 10 m, 14 m and 21-23 m from the interaction point. The ATLAS muon chambers have in total $6.2 \times 10^6$ channels. The muon systems are located downstream of calorimetry, such that hadrons, electrons and photons all shower before they reach the muon chambers. It is rare for a hadron to “punch-through” into the muon chambers, so particles originating in the interaction region that reach beyond the calorimeters are likely to be muons.

The muon system is comprised of four distinct and complementary subsystems, each of which uses different technology according to pseudo-rapidity and purpose: monitored drift tubes make a precision measurement of muon track hits in the barrel region, cathode strip chambers make a precision measurement of muon track hits in the end-cap region, resistive plate chambers perform barrel region triggering and thin gap chambers perform end-cap region triggering. These four subsystems are shown in figure 4.11. This figure taken from reference [28].

All four types of muon chambers operate using the same underlying principle as the Geiger counter: a wire is held under high voltage down the length of a metal tube filled with some gas. When an ionising particle traverses the tube, the gaseous atoms become ionised, releasing electrons which cascade to the wire, resulting in an electric current. This allows the counting of particles and in some cases the determination of energy.

The monitored drift tubes (MDT) perform a precision tracking measurement of muon track hits in the barrel region for $|\eta| < 2.7$. In total, there are 1150 chambers and 354 000 channels*. They are composed of aluminium tubes of diameter 30 mm with a 50 $\mu$m diameter central Tungsten-Rhenium (W-Re) wire. These tubes vary from 70 cm to 630 cm in length and contain a mixture of 93% Argon and 7% CO$_2$ at a pressure of 3 bar. The MDT chambers consist of 2×4 or 2×3 layers of drift tubes.

The cathode strip chambers make a precision measurement of muon track hits in the end-cap region, where the background conditions and rates are more demanding than in the barrel region. Cathode strip chambers are multi-wire proportional chambers with cathode readout segmented into strips. As a muon penetrates the chamber, an avalanche is formed on the anode wire, which induces a charge on the segmented cathode. The cathode strip chambers cover the region $2.0 < |\eta| < 2.7$ using 32 chambers and 31 000 channels.

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*The number of channels and chambers for MDT and RPC changed between the two technical design reports in 1999 and 2008; the figures shown in this section are taken from the 2008 edition [25].
The resistive plate chambers provide a muon trigger in the barrel region for $|\eta| < 1.05$. There are 606 chambers and 373 000 channels in the RPC system. They also provide the second coordinate measurement. The thin gap chambers or TGC provide end-cap triggering within the range $1.05 < |\eta| < 2.4$, and provide a secondary coordinate within the range $1.05 < |\eta| < 2.7$. In total, there are 3588 thin gap chambers, which provide 318 000 channels. The trigger chambers of the muon spectrometer have three roles: to provide bunch-crossing identification, to provide well-defined $p_T$ thresholds on which trigger menu can be applied and to measure the muon coordinate in the direction orthogonal to that determined by the precision-tracking chambers.

4.6 Magnets

The trajectory of a charged particle bends in the presence of a magnetic field. By measuring the path a particle takes through a magnetic field using the detector, the momentum of a particle can be devised. To this end, the ATLAS detector has two distinct types of magnets, as shown in figure 4.12:

- A solenoid magnet around the inner detector.
- A toroidal magnet system, which is divided into one barrel system and two end-cap systems.

The magnetic system measures 26 m long by 22 m in diameter and uses super conducting technology. In total, it has a stored energy of 1.6 GJ, and provides a magnetic field over a volume of 12 000 m$^3$.

4.6.1 Solenoid magnets

The ATLAS solenoidal magnet provides a 2 Tesla field in the central tracking region, but peaks at 2.6 T at the surface of the solenoid. The enormous magnetic field strength ensures that even very energetic or highly transverse particles are deflected and thus allow their momentum to be measured. The outer and inner radial dimensions of the solenoid magnet are 1.32 m and 1.22 m respectively and it is 5.3 m long. The solenoid weighs 5.7 tonnes, uses a current of 7.5 kA and stored energy of 40 MJ. The solenoid magnet is shorter than the inner detector. As such, the magnetic field drops off to 0.5 T near the inner detector end-cap. The magnet’s conductor is a flat superconducting Niobium Titanium (NbTi) cable located in the centre of an aluminium stabiliser with rectangular cross section which runs at a nominal temperature of 4.5 K. As the solenoid
is upstream of the electromagnetic calorimeter, it was very carefully optimised to reduce the material thickness, resulting in the solenoid assembly contributing 0.66 radiation lengths at $|\eta|=0$. This minimisation of radiation lengths is largely due to the solenoid and the liquid Argon calorimeter sharing a common vacuum.

4.6.2 Toroid magnets

The toroidal magnet system surrounds the calorimeters with an inner diameter of 9.4 m and an outer diameter of 20.1 m. It is divided into three sections, a barrel toroid system and two end-cap toroid systems. The eight barrel toroids are “racetrack-shaped”, stainless-steel vacuum vessels and are built around the outside the solenoid. Each of them measures 25.3 m long and is 5.4 m wide. The conductor in the end-caps is superconducting aluminium stabilised niobium titanium copper (NbTiCu) which runs at a nominal temperature of 4.5 K.

The system weighs 830 tonnes and has a peak field of 3.9 T in the direction perpendicular to the beam and is due to a current of 20.5 kA. The two end-cap toroidal magnets both sit within the barrel toroid assembly, on either side of the hadronic calorimeters. The end-cap toroids are on rails and can be slid out to facilitate access and maintenance. The toroids were among the first constructed section of the atlas detector and are visible in the iconic photograph of figure 4.13.
4.7 Trigger and data acquisition

No particle collider experiment has ever had to deal with the rates that are expected at the LHC. The LHC will produce proton-proton bunch crossing at a rate of 40 MHz. This means that there will be a bunch crossing every 25 ns. The nominal luminosity of $10^{34}$ cm$^{-2}$ s$^{-1}$ will result in on average $\sim 24$ collisions per bunch crossing, and for every bunch crossing, there are 23 minimum bias events, that shatter into thousands of particles. Even with this vast quantity of potential physics to be recorded, events in ATLAS can only be recorded to disk at a rate of 200 Hz. This means that ATLAS requires a rejection factor of $2 \times 10^5$. The process of selecting and saving permanently 200 events out of 40 million per second is known as trigger and data acquisition, or TDAQ.

4.7.1 The trigger system

ATLAS uses a three level trigger system, where each level rejects some of the events and does a more complex online analysis of the event. The decision as to which events are accepted and sent to permanent storage is determined by the trigger menu. Trigger menus are lists of specific thresholds and selection criteria at each of the trigger levels to address the requirements of physics working groups. The menu needs to be balanced,
efficient and fair to the working groups. The trigger menu in use changes during the experiment’s lifetime according to delivered luminosity.

To be permanently recorded to disk, an event must pass first the level I trigger, then the level II trigger and finally the event filter. The level I trigger cuts the rate from the nominal collision rate to 1 GHz and the level II trigger reduces it further to 75 kHz. Finally, the event filter diminishes the rate to the capacity of the storage system, 200 Hz. In this way, each successive trigger level is allowed more time and information per event to make its decision. Figure 4.14 shows a schematic of the trigger process. This figure is taken from reference [8].

![Diagram of the ATLAS trigger system](image)

**Figure 4.14:** An overview of the ATLAS trigger system.

### 4.7.2 Level I trigger

The level I trigger menu is a list of 256 calorimeter and muon chamber signatures that generally vary in terms of $p_T$ and $E_T$. The calorimeters trigger on jets, $E_T^{\text{miss}}$, $\tau$-leptons, electromagnetic clusters and large total transverse energy and the resistive plate and thin gap muon chambers trigger on muons. For each potential signal, the level I trigger defines a region of interest in $\phi$ and $\eta$ around the triggering object and only uses a very small fraction of the detector for each region of interest with coarse granularity. The maximum acceptance rate for level I triggers is 75 kHz and the level I trigger decision takes 2.5 $\mu$s per event. If an event is accepted by the level I trigger, its region of interest
is passed to the level II trigger. The rest of the event is passed to the read out buffers, which are in the detector.

4.7.3 Level II trigger

The level II trigger takes the level I trigger’s region of interest and studies the region at full granularity, precision, using all the sub-detectors including the inner detector. However, the trigger decision is limited to the region of interest defined in level I. The level II menus have been optimised to reduce the trigger rate to 3.5 kHz with an event processing time of 40 ms averaged over many events. If the event passes the trigger criteria defined in the trigger menu, it is passed to the next trigger level, the event builder. The level II trigger systems do not use radiation hardened circuitry and are located on a hardware farm on the other side of a 7 m thick concrete wall. This ensures that they are safe from the harmful radiation generated in the collisions while remaining close enough to the detector to make a quick trigger decision with low latency. The signal travels to the hardware farm using fast Ethernet switches.

4.7.4 Event builder and event filter

Events that pass the level II trigger are passed to the event builder. The event builder performs full reconstruction on the event, using full granularity and precision from the muons chambers, calorimetry and the inner detector. The reconstructed event is then passed to the final stage of the trigger, the event filter.

The event filter is an offline filtering system to reduce the 3.5 kHz events of level II down to 200 Hz. The event filter can perform topological cuts on the event. This stage is more time intensive than the earlier trigger levels but still the event filter makes a decision within 4 seconds of the event occurring. If an event is accepted by the event filter, it is passed to the Tier 0 computing site for further processing and distribution. Data which has passed the event filter will be passed to the Tier 0 processing and storage facility. The onward transmission and processing of events by Tier 0 is described in chapter 5.

4.7.5 Data acquisition

The data acquisition system runs in parallel to the trigger system. It holds the detector read-out until they can be passed to the next trigger level, if necessary. Each sub-detector stores collision information in pipeline memory, which are located very close to or on the detector systems. The readout drivers (ROD) are detector specific front-end
systems that store the initial data signals for long enough to accommodate for level I trigger latency. If the trigger accepts a specific event, dedicated links are used to transmit the data to the next stage of the trigger. The level II trigger checks the information in the buffer specific to the region of interest defined in level I. The result of the level II trigger decision determines if an event is transferred to the event building system and to the event filter where the final decision on permanent storage is made.

### 4.7.6 Di-muon trigger

For the specific case of $B$-physics at ATLAS, the rate of $b\bar{b}$ events is far in excess of the allocated maximum rate. This means that not all beauty events can be recorded. The use of a pre-scale selects some fraction of the candidate events to be passed through the trigger. A pre-scale can be applied at any trigger level.

The principle category of trigger used in this thesis are the di-muon triggers. This trigger requires two muons to be observed above a transverse momentum threshold. There are many di-muon triggers with varying thresholds and they depend on the luminosity and centre of mass collision energy. The trigger menu and pre-scale menu is subject to change and ATLAS physicists should consult the trigger twiki page in reference [29] for up to date trigger menus.

Muon triggers look for coincidences in the layers of muon trigger chambers. Two such hits are required for low $p_T$ triggers ($p_T < 6$ GeV) and three are required for high $p_T$ triggers ($p_T > 20$ GeV). The signal in the muon chambers is much clearer than in other parts of the detector as there are very few non-muonic energetic charged particles reaching the chambers. Muons can be read relatively quickly, with low false rates and high efficiency.

### 4.8 ATLAS first beam events

Although the ATLAS experiment had already performed many months of cosmic data taking, on the 10th of September 2008 beam was circulated completely around the LHC for the first time. Due to its position between to two injection points, ATLAS was the last LHC experiment to observe the beam. One of the first beam splash events on that day is shown using the event visualisation software, Virtual Point 1, in figure 4.15. This same event is also shown in figure 4.16 using an different event visualiser, Atlantis.
4.9 First collisions candidate

On November 23rd 2009, the first collision candidates were recorded in the ATLAS detector with a centre of mass energy of 900 GeV. Figure 4.17 shows the first collision candidate event in the LHC. This image is from reference [30]. Before the winter shutdown period at the end of 2009, ATLAS has recorded roughly 1.1 M minimum bias candidate events.
Figure 4.17: The first collisions candidate event in the ATLAS detector.
Chapter 5

The ATLAS computing model

During the data taking period, ATLAS is expected to write to disk at a rate of 420 MB s\(^{-1}\) for an expected \(10^7\) seconds in a year. This means that there will be significantly more than a petabyte of RAW data every year. The main goals of the ATLAS computing model are to ensure all collaboration members have speedy access to all reconstructed data and to allow appropriate access to raw data for monitoring, calibration and alignment. This model makes use of distributed grid computing to permit democratic use of computing resources and data access from all 37 countries of the collaboration. This chapter uses information and figures from the ATLAS computing technical design report of reference [31].

Distributed computing or “Grid” computing is a system in which computing resources such as processors, disk or tape storage are shared and are physically separated [32]. A description of the characteristics of Grid computing is given in section 5.1.

The event data used in the ATLAS collaboration is stored in a variety of formats, differing in size, purpose and event information stored. These formats are described in section 5.2.

The ATLAS computing hardware resources are divided into tiers, with each tier and each site having a distinct storage and processing role. Section 5.3 describes the roles of each tier within the collaboration.

Once an event has passed all triggers, it is reconstructed and converted into compact data formats. In addition to collision data, simulated events can be produced via the “full-chain”. Both reconstruction and simulation are described in section 5.4.
5.1 Distributed computing

The ATLAS experiment and other LHC experiments have embraced computing resource decentralisation and sharing. The LHC computing grid, the LCG, is composed of globally distributed computing sites and should allow all ATLAS members democratic access to reconstructed data. Real-time information about the LCG is available at their website [33]. Distributed or “Grid” computing, unlike batch systems, are distributed over multiple sites and extended geographical regions. Both the world wide web and the Grid are used to share stored data, but a grid is also used to share computing resources.

The raw data is needed for future reprocessing, and the results of the reprocessing need to be accessible to users around the world for the lifetime of the experiment. Furthermore, multiple copies of this data must be stored at separate sites, in case of accidents, natural disaster or some other system failure. Users also expect reliability, consistency and availability of the resources from around the world. In order to satisfy these requirements, the computing model has been built as a hierarchical and distributed system, with computers and storage spread around the globe.

Furthermore, in order to meet these characteristics and to presumably enhance and facilitate the user experience, a Grid has the following requirements:

- A common user interface. It should not matter to the user which computer they are using or where they are.
- All grid sites should accept common protocols, and use a similar framework.
- The required software for the experiment (in various versions) should be pre-installed on the many distributed sites available to the experiment.

The three grid user interfaces used in the thesis are Ganga, Don Quijote 2 and the ATLAS metadata interface. Ganga is a user interface for distributed computing jobs designed for use in the ATLAS and LHCb collaborations. Its principle use by the ATLAS collaboration is to run distributed Athena jobs. It allows users to send their jobs to their data, no matter where the data is located. Ganga also allows users to split their work into sub-jobs, distributing the work around many processors, and thus completing the work in less time. Further details about Ganga are available in references [34, 35].

Don Quijote 2 (DQ2) [36] is part of the ATLAS Distributed Data management suite, DDM, and provides the basic tools for file location placement, replication and deletion in ATLAS. For the user perspective, it allows them to locate and retrieve files such as user data or analysis output. In addition DQ2 also facilitates uploading files and locating replicas using simple intuitive commands such as dq2-get or dq2-put.
A web interface version of the DQ2 search tools is available as the ATLAS metadata interface, AMI [37]. AMI is a web-based interface to a database which contains information about all the files registered on the Grid, but also many events data-set details, such as the geometry with which the data was reconstructed, the release version, the physics groups who use such data, and information relevant to the Monte Carlo simulation, if present.

### 5.2 Event data types

The ATLAS collaboration uses a variety of types of data; each of which serves a different purpose and varies in size, number of replicas and end users. As such, the data types have differing processing schedules, distribution models and access patterns. The following data types are used by the ATLAS collaboration, these data types are summarised in table 5.1.

**RAW** Raw Data Objects are the output of the detector before any offline processing. It is the output of the Event Filter, the last stage in the ATLAS online trigger system. Unlike derived objects, RAW data will not change with improvements to reconstruction, alignment or material and magnetic field mapping; these optimisations are added during reprocessing of the raw data. The computing model assumes a RAW event size of between 1.6 MB and 2.3 MB and an output rate of 200 Hz. RAW data are grouped into files no smaller than 2 GB and no larger than 5 GB and are grouped by detector run, instead of any other physics selection criteria.

**ESD** Event Summary Data is the full set of reconstructed objects. It contains event data and physics objects, such as tracks, vertices, jets, electrons and muons. The computing model assumes an ESD size of ~ 0.5 MB per event. Like RAW data, ESD files are grouped by detector run as they are mapped one to one with RAW data.

**AOD** Analysis Object Data contains much of the same information as the ESD but is further reduced in size per event. AOD are intended for physics analysis and contain object-oriented physics objects. The size per event was planned to be around 100 kB. Unlike ESD and RAW, AOD are grouped by physics selection criteria.

**TAG** The TAG database contains event level metadata and are created alongside the AOD. TAGs holds a light summary of characteristics features or “thumbnail” of
each event in order to facilitate and speed up event selection. The use of TAG data removes the necessity to open AOD or ESD files that fail initial selection criteria, reducing the resources required of an analysis. The TAG also allows for the direct navigation to a given event within its associated ESD or AOD file without running sequentially through other events in the file. The computing model assumes it has a size of $\sim 0.1$ kB per event.

**DPD** Derived Physics Data is data that has been selected for physics analysis, usually of relevance to a particular physics group or event for a study. The term ‘DPD’ is a class of physics data and as such does not have one specific format, but rather the contents varies between physics groups. There are currently between 4 and 10 different DPD formats. Derived physics data serves to facilitate histogramming, fitting and visualisation for end user analysis. As many physics analyses only require a fraction of the total event data, DPD data has been skimmed, thinned and slimmed to the satisfaction of the group that uses it.

**Simulated Data** Simulated data or Monte Carlo data is the term to describe data that has been produced in the simulation process, as described in section 5.4. Simulated data can be of any type between simulated RAW data to simulated DPD data. As simulated data also contains all the truth information, the size per event is larger than its real data counterpart.

Table 5.1: The types of data used in the ATLAS collaboration, their acronym, the size per event and purpose.

<table>
<thead>
<tr>
<th>Data type</th>
<th>Acronym</th>
<th>Size per event</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Data Object</td>
<td>RDO</td>
<td>1.6 - 2 MB</td>
<td>RAW data from the detector.</td>
</tr>
<tr>
<td>Event Summary Data</td>
<td>ESD</td>
<td>0.5 MB</td>
<td>First pass reconstructed data.</td>
</tr>
<tr>
<td>Analysis Object Data</td>
<td>AOD</td>
<td>0.1 MB</td>
<td>Physics analysis.</td>
</tr>
<tr>
<td>TAG</td>
<td>TAG</td>
<td>1. kB</td>
<td>Event selection database.</td>
</tr>
<tr>
<td>Derived Physics Data</td>
<td>DPD</td>
<td>10 - 100 kB</td>
<td>Compact end user analysis.</td>
</tr>
</tbody>
</table>

5.3 **The tier system**

The ATLAS computing model uses a 4 level tier system, with Tier 0 at CERN as a main hub and the source of the data. The wide variety of tasks required by the collaboration allows for the role of a site to be tailored to its available resources.
5.3.1 Tier 0

Tier 0 is the experiment’s primary computing element, it is closest to the ATLAS cavern at CERN and it performs the three initial roles of processing, storage and distribution of the raw data and the first pass processed data. Tier 0 performs three separate types of operation: processing, storage and distribution. In terms of processing, Tier 0 produces and reconstructs the initial event summary data and analysis object data of the calibration and express stream data. Tier 0 stores a complete and permanent copy of all RAW data and is also the source of the ATLAS data distribution to the Tier 1 sites. Tier 0 also archives the RAW, ESD, AOD and TAG data that it produces.

5.3.2 Tier 1

The ten Tier 1 sites are globally distributed and each leads a group of Tier 2 sites known as a “cloud”. Each Tier 1 site holds a subset of the raw data and are responsible for some reprocessing, with the exception of the Tier 1 site at Brookhaven National Laboratory (BNL) which holds a complete replica of the RAW data. The other Tier 1 sites each hold a partial copy (28%) such that there are 2.72 complete replicas of the RAW data.

The Tier 1 sites hold a short latency up to date copy of the ESD, AOD and TAG data, and hold a longer latency copy of the previous versions. The ESD are also copied to two sites in a Tier-1’s cloud. In the UK, the Tier 1 site is Rutherford Appleton Laboratory (RAL).

5.3.3 Tier 2

The Tier 2 sites are smaller and more abundant than the Tier 1 sites. Each Tier 2 is a member of a Tier 1 cloud. The cloud membership of a site is determined geographically or politically. Tier 2 sites play a variety of roles in the computing model, reflecting the range of resources available at a Tier 2 site. The roles include simulation, user analysis, calibration constants, data hosting and other services depending on the resources available at the site. Each the Tier 2 sites will hold roughly a third of the current AOD and the full TAG sample. The Tier 2 sites also vary in size, but are required to all have the same ratio of tape, disk and CPU resources. An example of a Tier 2 site would be the Lancaster Grid cluster.
5.3.4 Tier 3

A Tier 3 site is a local resource, it may be a local group cluster, or a collection of desktop computers. The resources and structure of a Tier 3 is driven by the requirements and limitations of its local group. Tier 3 sites can access the Grid via a user interface, such as Ganga or DQ2 described in section 5.1. However, they are for specific use of a university of laboratory group and are not considered to be “on the grid”.

5.4 Full-chain simulation

Another aspect of the particle physics research using ATLAS is the continual production of simulated Monte Carlo data. This thesis uses simulated Monte Carlo data exclusively. The generation of Monte-Carlo data is a many step process and each step is performed independently. The process of producing Monte Carlo data is known as the “full-chain” and is illustrated in Figure 5.1. In this diagram, the ovals represent data formats and the rectangles represent computing processes. This image taken from reference [10].

Many of the steps of the full-chain are performed using the Athena framework, which is fully described in references [31, 38]. Athena is an object oriented C++ framework for data processing in the ATLAS experiment and is based on Gaudi [39]. It provides many tools, toolboxes, and services to facilitate and lighten the burden of code preparation for detector studies and final physics analysis. It is highly modular, and component based, but uses a common framework in order to achieve greater flexibility. The ATLAS offline reprocessing, production and simulation and many other ATLAS computing tasks are all performed using the Athena framework.

5.4.1 Generation

The generation stage is the creation of the physics event. In the case of proton-proton collisions at the LHC, the events is modelled as the interaction of two partons. The generator models their hadronization, decay and all subsequent interactions, but does not take into account any effects of their interaction with matter in the beam pipe or detector.

There are many generators available for use in high energy physics. The ATLAS B-physics group Monte Carlo generation is performed using a specialised B-physics interface to the PYTHIA generator [40] called PYTHIAB. PYTHIAB generates data first by simulating the interaction of a pair of partons. Roughly 1 % of these interactions will produce a $b\bar{b}$ pair. As this is inefficient for generation of a specific exclusive data-set, the
yield is increased in the following way: Firstly, if the event does not contain a $b\bar{b}$ pair, then this parton-parton interaction is rejected immediately. If the random generation does produce a $b\bar{b}$ pair, the parton parton interaction is cloned many times, and each clone is allowed to decay freely. Users can apply kinematic cuts, such as minimum $p_T$ or request a specific decay. The number of clones is set such that roughly one clone per partonic interaction passes the requested criteria. For this reason, generation can be time consuming, especially in the case of very rare events.

As shown in figure 5.1, programs such as AtlFast can be used to skip out many stages of the full chain. AtlFast data was not used in the thesis. For further reading, consult reference [41].

Figure 5.1: A flow chart of the ATLAS data production for simulated data and real data.
5.4.2 Simulation and digitisation.

After the appropriate events have been generated, the GEANT4 software toolkit (G4) is used to model the interaction of the decay products with the detector. GEANT4 is a toolkit for the simulation of the passage of particles through matter [42, 43].

GEANT4 breaks this process down into two steps. The first of these steps is called simulation. Using a detailed model of the ATLAS detector, GEANT4 models the effect that the detector has on the high energy particles of an interaction, taking into account energy loss, ionisation, Bremsstrahlung and multiple scattering. Once the interaction of the high energy particles and the detector has been modelled, GEANT4 compares the trajectories of the high energy particles with the position of sensitive volumes in ATLAS to create “hits”.

The second step performed by GEANT4 is digitisation. Given the hits created during simulation, the digitisation process converts these hits into detector responses such as voltages and pre-amplifier times, which should look very similar to the byte stream output of the running ATLAS detector.

5.4.3 Reconstruction

The reconstruction stage is the conversion of detector responses into high level physics objects such as muons, tracks, energy deposits, jets, vertices and missing energy. As shown in figure 5.1, the reconstruction phase can take as input either GEANT4 digitisation or the byte-stream data from the detector. The output of the reconstruction stage is event summary data, ESD. In addition to physics-objects, ESD also contains hit-information, as well as information of the algorithms used for object reconstruction.

In general, the reconstruction phase is performed using the same detector model and conditions that were used in the simulation phase. However, it is possible to use a different detector model in reconstruction than in simulation. In this way, extra material, a different magnetic mapping or alignment can be introduced into simulated data-sets during the reconstruction phase. This was how the misaligned data-sets were produced for the studies of chapters 7 and 8.

5.4.4 AOD and DPD preparation

Analysis object data (AOD) is created alongside event summary data (ESD) but can also be derived from ESD. This is because AOD contains a subset of the objects in ESD data. As opposed to ESD, AOD are grouped by their physics properties and
trigger information, instead of by the run-number. As described in section 5.2, AOD are intended for physics analysis.

Derived physics data (DPD) are compact and derived from reconstructed data. There are between three and ten types of DPD and each type is produced for a specific physics group or study. Their properties and production method depend on their purpose. In general, DPD data should be grouped as a subset of the data, and should be very light and compact.

The derived physics data used in this thesis was prepared using AAna [44] and was produced directly from AOD. AAna is a light-weight framework of classes and scripts that allow for fast and flexible physics analysis. It was built for use in ATLAS B-physics working group. AAna is fully compatible with Athena but can also operate in a stand-alone mode. AAna uses its own DPD format, AAnaDPD, which is created from AOD or ESD files using the Athena framework.

5.4.5 Histogramming and fitting

The production of histograms in this thesis was performed in the ROOT framework [45]. ROOT is an object-oriented physics analysis program written in C++ and based on the previous generator of high-energy physics histogramming software, PAW. ROOT uses input “ntuples” or “.root” format files and is used to make plots and perform fits. ROOT provided the tools required for histogramming, graphing, fitting, statistical studies, four-vector computation, 3D visualisations, and graphical file manipulation.

In addition to ROOT, MINUIT was used to perform some fits. MINUIT is minimisation program that uses a range of algorithms to find a local minimum of a given function [46, 47]. Although it is available in standalone format, MINUIT was used in this thesis as a toolkit for ROOT.
Chapter 6

Jet charge flavour tagging

Flavour tagging is the study of hadrons with the aim to determine the flavour of their constituent quarks. Flavour taggers attempt to determine the flavour at production or at decay. As shown in section 2.4 of chapter 2, flavour tagging is a crucial part of the study of $CP$ violation in neutral $B_s^0$ meson lifetime measurements.

In order to minimise the uncertainty of the $CP$ violation measurement, the tagging must be as effective as possible. A good tagger is applicable to as many events as possible and wrong as rarely as possible. The values used to describe the effectiveness of a flavour tagger are shown in section 6.1. The principle tagging algorithm used in this study is the jet charge tagger and is described in section 6.2.

To build an effective tagger, the jet charge tagging algorithm needs to be calibrated and optimised. This chapter contains the results of an optimisation study of the flavour tagger when applied to three distinct simulated exclusive $B$-meson decays. The data and reconstruction methods are described in section 6.3. Flavour tagging in $B^+ \rightarrow J/\psi K^+$ is described in section 6.4, flavour tagging in $B_d^0 \rightarrow J/\psi K^{0*}$ is described in section 6.5 and flavour tagging in $B_s^0 \rightarrow J/\psi \phi$ is described in section 6.6. The results and methods described in this chapter were also published in part in reference [27].

It should be noted that the word “tag” has many uses in ATLAS nomenclature. The word can be used to describe a $b$-jet tag, the TAG database or a flavour tag. However, for the course of this chapter, it can be assumed that the use of the word “tag” refers exclusively to flavour tagging.
6.1 Flavour tagger effectiveness

The effectiveness of the discrimination between a $B$-hadron and its anti-particle is characterised by two quantities: its efficiency and the wrong tag fraction. In general, the flavour tagger cannot make a tagging decision for all reconstructed events and some events are not tagged. The efficiency of the tagger, $\varepsilon$, is the fraction of $B$-hadrons for which a tag decision can be made, correctly or otherwise.

\[
\varepsilon = \frac{N_r + N_w}{N_t} = \frac{N_r + N_w}{N_r + N_w + N_n}
\]

(6.1)

where $N_r$ and $N_w$ are the numbers of correctly and incorrectly tagged $B$-hadrons respectively, $N_n$ is the number of untagged $B$-hadrons and $N_t$ is the total number of reconstructed $B$-hadrons. $N_r$, $N_w$ and $N_n$ includes all reconstructed $B$-hadrons such that $N_r + N_w + N_n = N_t$.

The second significant parameter of a flavour tagger is the wrong tag fraction $\omega$. The wrong tag fraction arises as the tagger can occasionally incorrectly tag a $B$-hadron as its anti-particle. The wrong tag fraction is the fraction of all tagged events that are incorrectly tagged.

\[
\omega = \frac{N_w}{N_r + N_w}
\]

(6.2)

The dilution, $D$, is related to the wrong tag fraction. The dilution is defined:

\[
D = \frac{N_r - N_w}{N_r + N_w} = 1 - 2\omega
\]

(6.3)

In a typical $CP$ violation study, the aim is to identify a difference in some property between a particle and its anti-particle. In this case, the relationship between the true asymmetry of this property, $A_{true}$, and the asymmetry as measured in the data, $A_{meas}$, is

\[
A_{true} = \frac{1}{D} A_{meas}
\]

(6.4)

which is derived in Appendix B.

The statistical uncertainty on $A_{true}$ is:

\[
\sigma_A \approx \sqrt{\frac{A^2 D^2}{\varepsilon D^2 N_t}}
\]

(6.5)
From the denominator of the right hand side of equation 6.5, the “quality factor” or tagging power, $Q$, is defined:

$$Q = \varepsilon D^2$$  \hfill (6.6)

The uncertainty on the asymmetry in a $CP$ violation measurement decreases with an increasing tagging quality and with increasing number of events. For this reason, the study performed in this chapter maximises the quality factor in order to optimise the flavour tagger. This maximisation is performed by varying the input parameters of the jet charge tagger over a range of values. In the case that the dilution were negative, the square root of the quality factor could be used: $\sqrt{Q} = \sqrt{D}$. This would allow for the case that a tagger was incorrect more often than correct. However, this was not performed in the following study, as the jet charge tagger was found to have $\omega < 0.5$, as shown in figure 6.3.

### 6.2 The jet charge flavour tagger

The jet charge flavour is a same side flavour tagging technique that is used to infer the flavour of a $B$-hadron at production. The term same side means the flavour of the reconstructed $B$-hadron is determined. Opposite side flavour tagging is also possible, these techniques study the properties on the non-reconstructed $B$-hadron of a given event. An opposite side flavour tagger is discussed briefly in section 6.6.1.

The jet charge flavour tagger looks at charge and momentum corrections to estimate the flavour at production of the $B$-hadron. According to fragmentation models, the jets particles are ordered in the momentum component parallel to the original quark direction, while charge conservation also imposes charge ordering [48]. This means that the particles that hadronised alongside the $B$-hadron are expected to be near the $B$-hadron in terms of $\eta$ and $\phi$. Furthermore, the overall electric charge of a $b$-quark jet is influenced by the flavour of the initial $b$-quark. The jet charge flavour tagger uses these correlations to identify the flavour of the signal $B$-hadron at production.

In a small number of cases, the flavour at decay of a $B$-hadron can be inferred from the charge of the highest $p_T$ lepton unassociated with the signal decay, under the assumption that this tagging lepton originates from a semi-leptonic decay of the other $B$-hadron in the event. This method is known as opposite side lepton tagging. However, the majority of the events lack the unassociated high $p_T$ lepton. Fortunately, the jet charge tagging method can be applied to almost all events.
6.2.1 Jet charge tagging methods

This section describes the process used to perform the jet charge flavour tagging method. The first step in the flavour tagging process is the reconstruction of the B-hadron. This is channel dependent. The reconstruction process is described for the three B-meson decays in section 6.3. In all cases, the B-meson is not observed directly but its decay products are reconstructed as tracks. The decay products of the B-meson are also called its “descendants” or “children”. The B-meson decay channels studied here have either three descendants ($B^+ \rightarrow J/\psi K^+$) or four descendants ($B^0 \rightarrow J/\psi K^0*$ where $K^0* \rightarrow K^\pm \pi^\mp$ and $B^0_d \rightarrow J/\psi \phi$ where $\phi \rightarrow K^+ K^-$). In all three channels, the $J/\psi$ decays to two oppositely charged muons. The cross sections of these channels are shown in table 6.2.

Once the B-meson has been successfully reconstructed, the difference in $\phi$ and $\eta$ between each track and the B-meson is calculated:

$$\Delta \eta = \eta_B - \eta_{track}$$
$$\Delta \phi = \phi_B - \phi_{track}$$  \hspace{1cm} (6.7)  \hspace{1cm} (6.8)

where the $\eta_B$ is the pseudo-rapidity of the B-meson and the $\eta_{track}$ is the pseudo-rapidity of the track. Similarly the $\phi_B$ is the azimuthal angle of the B-meson and the $\phi_{track}$ is the azimuthal angle of the track. The values $\Delta \eta$ and $\Delta \phi$ are combined as in equation 6.9 to form $\Delta R$.

$$\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2}$$  \hspace{1cm} (6.9)

A track is considered to be a candidate member of the jet if it has a value of $\Delta R$ less than the maximum cut off, $\Delta R_{max}$. In this way, a “cone” containing jet tracks candidates is formed in the $\phi$ and $\eta$ space around the B-meson. Furthermore, the jet should not contain any descendants of the signal B-meson or any leptons. These tracks are rejected from the list of jet track candidates. Finally, as the average decay length of a B-meson is approximately 0.4 mm, each jet track is checked to ensure that it has a stereo and axial impact parameter that is less than 1 cm. In the case that the jet contains no tracks at all, the flavour tagging method can not be applied to this event.

Once a suitable set of tracks has been created, the average charge of the jet is calculating using the three-momentum and charge of the tracks. The jet charge, $Q_{jet}$, is defined in equation 6.10.

$$Q_{jet} = \frac{\sum_i q_i p_i^\perp}{\sum_i |p_i^\perp|}$$  \hspace{1cm} (6.10)
where the index \( i \) runs over all the tracks in the cone, \( q_i \) is the track charge, \( \kappa \) is a weighting factor and \( p_i \) is a measure of the track momentum.

The jet charge \( Q_{\text{jet}} \) tends to be positive for \( \bar{b} \)-jets and negative for \( b \)-jets. In the case that a \( B \)-meson has a jet charge near \( Q_{\text{jet}} = 0 \), the tag is considered ambiguous. The edge of this ambiguous region is called the exclusion cut, \( E_{\text{cut}} \). An event is considered ambiguous and remains untagged when the following condition is met:

\[
-E_{\text{cut}} < Q_{\text{jet}} < E_{\text{cut}}
\]  

The exclusion cut is a free parameter and can be set to any value between 0 and 1.

Equation 6.10 uses \( p_i \) as some measure of the track momentum. Three ways to include the jet track momentum were studied in this thesis: the \( p_t \) method, the \( p_l \) method and the \( p_T \) method. Each method uses a different momentum component of the track.

1. \( p_t \) Method: \( p_i \) is the momentum component of the jet track perpendicular to the signal \( B \)-meson.

2. \( p_l \) Method: \( p_i \) is the momentum component of the jet track parallel to the signal \( B \)-meson.

3. \( p_T \) method: \( p_i \) is the momentum of the track particle in the plane transverse to the beam-pipe.

A plot of \( Q_{\text{jet}} \) is shown in figure 6.1 for the \( B^0_d \rightarrow J/\psi K^{0*} \). It should be noted that the positive-negative asymmetry is present because the sample contains no \( B^0_d \rightarrow J/\psi K^{0*} \) events. This figure shows the correct, incorrect and untagged regions of the jet charge, \( Q_{\text{jet}} \). It also shows peaks at \( Q_{\text{jet}} = 0 \) and \( Q_{\text{jet}} = \pm 1 \). The peak at zero occurs when no tracks survive the jet cuts. The peaks at \( \pm 1 \) occur when all the tracks in the jet are either positively charged or negatively charged.

The three input parameters that are varied in this study are the maximum size of the cone, \( \Delta R_{\text{max}} \), the weighting factor of the momentum, \( \kappa \) and the exclusion cut, \( E_{\text{cut}} \). This study looks at how these three parameters and the choice of track momentum component can impact the efficiency, dilution, wrong tag fraction and quality factor for the decays \( B^0_d \rightarrow J/\psi K^{0*} \), \( B^0_s \rightarrow J/\psi \phi \) and \( B^+ \rightarrow J/\psi K^+ \).

The jet charge taggers’ three input parameters were varied over a range of input values and the effectiveness parameters were calculated for all combinations. The range of the three input parameters is shown in table 6.1. It should be noted that other sweeps were performed with values beyond those in this table, but the wider parameter space did not result in improved tagging performance.
Chapter 6: Jet charge flavour tagging

6.3 Description of the samples and reconstruction methods

This study uses three samples of exclusive hadronic $B$-meson decays. The decays are $B^0_d \to J/\psi K^{0*}$, $B_s^0 \to J/\psi \phi$ and $B^+ \to J/\psi K^+$. The cross section of each channel and the number of events available is shown in table 6.2. These results are taken from reference [27].

These events were generated using ‘PYTHIA and the full chain production method’ described in section 5.4. The initial cuts on transverse momentum and pseudo-rapidity were made at generator level. These samples were prepared such that the signal decay is a $B$-meson, not a $\bar{B}$-meson events. As such, the wrong tag fraction is the fraction of events tagged as a $\bar{B}$-meson.

*For completeness, PYTHIAB 6.4, Athena 12.0.6 using the ATLAS-CSC-01-02 geometry were used here.

Table 6.1: The range of values used for the input parameters in the parameter sweep.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>-2</td>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\Delta R$</td>
<td>0.1</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>$E_{cut}$</td>
<td>0</td>
<td>0.5</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Figure 6.1: The jet charge, $Q_{jet}$, in the channel $B^0_d \to J/\psi K^{0*}$ using the $p_t$ method.
During the reconstruction, \( J/\psi \rightarrow \mu^+ \mu^- \) candidates were formed from all pairs of oppositely charged muon tracks that passed the \( p_T \) criteria. Pairs were assumed to originate from a common vertex if the vertex fit resulted in \( \chi^2/n.d.f. < 6 \).

Once the \( B \)-mesons had been reconstructed, they were matched up with their Monte Carlo truth. This ensured that the tagging optimisation is performed using correctly reconstructed events. The number of events that passed these cuts are shown in table 6.2.

It should be noted in the \( B_d^0 \rightarrow J/\psi K^{0*} \) channel, the \( K^{0*} \) is not forced to decay to a charged final state. As such, approximately two thirds of the \( K^{0*} \) decay to \( K^+ \pi^- \). The remaining \( K^{0*} \) decay to \( K^0 \pi^0 \). The events that decay to neutrally charged final state are not reconstructed. As a result of the presence of other final states in this data-set, the truth matching cuts were required to be more stringent than in the other two channels. This is reflected in a smaller fraction of correctly reconstructed events in table 6.2.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Cross section</th>
<th>Total</th>
<th>Correctly reconstructed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^+ \rightarrow J/\psi K^+ )</td>
<td>0.55 nb</td>
<td>49250</td>
<td>34431</td>
</tr>
<tr>
<td>( B_d^0 \rightarrow J/\psi K^{0*} )</td>
<td>0.24 nb</td>
<td>27435</td>
<td>6816</td>
</tr>
<tr>
<td>( B_s^0 \rightarrow J/\psi \phi )</td>
<td>0.02 nb</td>
<td>47967</td>
<td>24149</td>
</tr>
</tbody>
</table>

### 6.4 Flavour tagging in \( B^+ \rightarrow J/\psi K^+ \)

In the case of the \( B^+ \rightarrow J/\psi K^+ \) decay, charge conservation allows the flavour of the \( B \)-meson at the time of decay to be completely specified using the charge of the \( K^\pm \). This means that the flavour of the \( B \)-meson at the time of decay can be determined in the real data and the jet charge flavour can be cross-checked directly.

In addition, mixing is not possible between the \( B^+ \) meson and its anti-particle, \( B^- \). As such, the flavour of the \( b \)-quark at production is the same of the flavour at decay. Furthermore, of the three available channels, \( B^+ \rightarrow J/\psi K^+ \) has a relatively high cross section and clear event topology.

Once the signal decay had been successfully matched to the Monte Carlo truth, the jet charge tagging mechanism was applied to the track particles. Using the sample of 34k correctly reconstructed events, the input parameters of the jet charge tagger were varied between the values in table 6.1.
Table 6.3: The jet charge input parameters and effectiveness parameters that maximise the quality factor in $B^+ \rightarrow J/\psi K^+$ for all three momentum methods.

<table>
<thead>
<tr>
<th>Method:</th>
<th>$p_t$</th>
<th>$p_t$</th>
<th>$p_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$\Delta R$</td>
<td>0.8</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$E_{cut}$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.73±0.002</td>
<td>0.73±0.003</td>
<td>0.716±0.002</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.162±0.002</td>
<td>0.174±0.003</td>
<td>0.154±0.002</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.335±0.005</td>
<td>0.31±0.005</td>
<td>0.342±0.004</td>
</tr>
</tbody>
</table>

Before this study was performed, it was expected that the method with the highest quality factor would be the $p_t$ method, which uses the momentum component of the track parallel to the $B$-meson momentum. However, all three methods maximise the quality factor with similar input parameters. Figures 6.2, 6.3 and 6.4 shows the efficiency, wrong tag fraction and quality factor, respectively in terms of $\kappa$ and $\Delta R$ for $B^+ \rightarrow J/\psi K^+$ for all three methods. The behaviour of efficiency, wrong tag fraction and quality factor are roughly the same between the three methods, for a given set of input parameters.

The combination of three input parameters that resulting in maximal quality factor for each momentum component method is shown in table 6.3. The difference in the maximum quality factor between the $p_T$ and $p_t$ methods is $0.007\pm0.009$. At these input values the $p_t$ method has a tagging efficiency of $73.1\pm0.2\%$. At maximum quality, the $p_T$ method has a tagging efficiency of $71.6\pm0.2\%$ and a wrong tag fraction of $15.4\pm0.2\%$.

6.5 Flavour tagging the decay $B_d^0 \rightarrow J/\psi K^{0*}$

Once the signal decay had been successfully matched to the Monte Carlo truth, the jet charge tagging mechanism was applied to the particle tracks. Using the sample of $7K$ correctly reconstructed events, the input parameters of the jet charge tagger were varied between the values in table 6.1. The point with the maximal quality factor for each momentum component method is shown in table 6.4.

All three methods maximise the quality factor with similar input parameters. Furthermore, the difference between the methods maximum quality is very small at $0.002\pm0.01$. This is equivalent to no difference between methods.
Table 6.4: The jet charge input parameters and effectiveness parameters that maximise the quality factor in $B_d^0 \rightarrow J/\psi K^{0*}$ for all three momentum methods.

<table>
<thead>
<tr>
<th>Method: $B_d^0 \rightarrow J/\psi K^{0*}$</th>
<th>$p_t$</th>
<th>$p_t$</th>
<th>$p_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\Delta R$</td>
<td>0.6</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$E_{\text{cut}}$</td>
<td>0.075</td>
<td>0.1</td>
<td>0.075</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.672±0.006</td>
<td>0.574±0.007</td>
<td>0.664±0.006</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.365±0.007</td>
<td>0.357±0.009</td>
<td>0.367±0.007</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.049±0.005</td>
<td>0.047±0.006</td>
<td>0.047±0.005</td>
</tr>
</tbody>
</table>

6.5.1 Self tagging in $B_d^0 \rightarrow J/\psi K^{0*}$

The $B_d^0 \rightarrow J/\psi K^{0*}$ channel contains an asymmetry which can be used as a self-tagging mechanism. In the $B_d^0 \rightarrow J/\psi K^{0*}$ sample, the $K^{0*}$ decays into a positive Kaon and negative Pion. In the charge conjugate of this sample, $\overline{B_d^0} \rightarrow J/\psi \overline{K^{0*}}$, the $\overline{K^{0*}}$ decays to a negative Kaon and a positive Pion. The self tagging uses this asymmetry to differentiate between $K^{0*}$ and $\overline{K^{0*}}$. Although the ATLAS experiment does not have the capacity to distinguish individual Kaons from Pions, it is possible to use the difference between them to tag the flavour $B$-meson in this decay.

The tagging method is applied when calculating the mass of the $K^{0*}$. Calculating the mass of a combined particle requires an assumption of the mass of its constituents. For instance, when calculating the mass of a $J/\psi$ in $J/\psi \rightarrow \mu^+\mu^-$, it was assumed that both muons had a mass of 105.65 MeV. This assumption is called a mass hypothesis. In general, when a particle decays to two oppositely charged particles that are not each others anti-particles, the mass of the combined particle can be calculated using two mass hypotheses. The two hypotheses here are $K^{0*} \rightarrow K^+\pi^-$ and $\overline{K^{0*}} \rightarrow K^-\pi^+$. The mass of the combined particle here was calculated for both hypotheses for each event. These masses are shown in figure 6.5.

Once the mass has been calculated using both hypotheses, two methods of mass hypothesis flavour tagging were investigated. The first method places a cut-off region in the mass around the simulated $K^{0*}$ mass. The mass of the $K^{0*}$ was simulated to be at $892$ MeV [1]. To optimise this method, the accepted region was varied between $892\pm5$ MeV and $892\pm100$ MeV in increments of 5 MeV. If only the $K^+\pi^-$ mass hypothesis falls within the accepted region, the event is tagged as a $B_d^0$ event and if only the $K^-\pi^+$ mass hypothesis falls within the accepted region, the event is tagged as a $\overline{B_d^0}$ event. If both or neither hypotheses fall within the accepted region, the event is not flavour tagged. In table 6.5, this is labelled “method 1”.

The second $K^{0*}$ mass hypothesis method tags the $B$-mesons according to which ever of the two mass hypothesis falls closer to the simulated mass of the $K^{0*}$. If both mass hypotheses fall at approximately the same distance from the known $K^{0*}$ mass, then the tag is considered ambiguous and the event remains untagged. In other words, if the difference in mass between the two mass hypotheses is too small, then the event is not tagged. The minimum difference in mass between the two hypothesis was varied from 5 MeV to 100 MeV in increments of 5 MeV. In table 6.5, this is labelled “method 2”.

### Table 6.5: Effectiveness of self-tagging in $B_s^0 \rightarrow J/\psi K^{0*}$ using the two mass hypothesis methods.

<table>
<thead>
<tr>
<th></th>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut off</td>
<td>892±50 MeV</td>
<td>$</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.725 ± 0.005</td>
<td>0.611 ± 0.006</td>
</tr>
<tr>
<td>Wrong tag fraction</td>
<td>0.060 ± 0.003</td>
<td>0.095 ± 0.005</td>
</tr>
<tr>
<td>Quality</td>
<td>0.562 ± 0.009</td>
<td>0.401 ± 0.009</td>
</tr>
</tbody>
</table>

The values that maximise the quality factor for both methods are shown in table 6.5. At their respective maximum quality factors, method 1 has a higher efficiency and lower wrong tag fraction than method 2.

#### 6.6 Flavour tagging $B_s^0 \rightarrow J/\psi \phi$

The $B_s^0 \rightarrow J/\psi \phi$, where $J/\psi \rightarrow \mu^+\mu^-$ and $\phi \rightarrow K^+K^-$, is reconstructed using the two muons from the $J/\psi$ and two oppositely charged tracks. A vertex is created with those two muons and then, another vertex is created using those muons and two other oppositely charged tracks. In this study, the signal decay is then matched to the Monte Carlo truth and the jet charge tagging mechanism is applied to all correctly reconstructed events.

As for the previous two channels, the input parameters of the jet charge tagger were varied between the values in table 6.1. The point with the maximum quality factor for each momentum component method is shown in table 6.6.

All three methods maximise the quality factor to the same value. There is no significant difference between the maximum quality obtained in any of the methods. The $p_t$ and $p_T$ methods have similar input parameters. However, the $p_t$ uses a value of kappa of zero. This means that the maximum quality factor for the $p_t$ method occurs when the jet charge tagger is independent of the track momentum and is uniquely the charge of the track. Furthermore, this results in a similar quality factor to the other two methods.
When comparing the optimised input parameters in table 6.6 with the other channels in tables 6.3 and 6.4, it is clear that the optimised input values for $B_s^0 \rightarrow J/\psi \phi$ are very different to the optimised input values for $B_d^0 \rightarrow J/\psi K^0$ or $B^+ \rightarrow J/\psi K^+$. For this reason, it is not possible to optimise the jet charge flavour tagger for one $B$-meson using a different $B$-meson. This behaviour is thought to be due to differences in the hadronization process.

### Table 6.6: The jet charge input parameters and effectiveness parameters that maximise the quality factor in $B_s^0 \rightarrow J/\psi \phi$ for all three momentum methods.

<table>
<thead>
<tr>
<th>Method:</th>
<th>$p_t$</th>
<th>$p_t$</th>
<th>$p_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.2</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>$\Delta R$</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.644±0.003</td>
<td>0.625±0.003</td>
<td>0.629±0.003</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.396±0.004</td>
<td>0.396±0.004</td>
<td>0.395±0.004</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.028±0.002</td>
<td>0.027±0.002</td>
<td>0.028±0.002</td>
</tr>
</tbody>
</table>

### 6.6.1 Alternative flavour tagging methods in $B_s^0 \rightarrow J/\psi \phi$.

There is no readily available and clean self-tagging mode in the decay $B_s^0 \rightarrow J/\psi \phi$. Furthermore, the optimal input parameters of the $B_s^0$ meson flavour tagger are very different to the optimal parameters for the other $B$-mesons. For this reason, these channels cannot be used to calibrate the jet charge flavour tagger for $B_s^0$ mesons.

Two other tagging methods, $B^*$ tagging and opposite side lepton tags, were studied with the goal of finding a method to inter-calibrate the jet charge flavour tagging method. The $B^*$ tagging method looks at the highest $p_T$ track in the jet unassociated with the $B_s^0$ decay. It assumed that this track originates from a $B^{*\pm} \rightarrow B_s^0 \pi^\pm$ decay and that the charge of the Pion indicates the flavour of the $B$-meson. This method had a very high wrong tag fraction and was little better than flipping a coin.

The opposite side lepton tag looks for a high $p_T$ lepton that is unassociated with the signal decay. It assumes that this lepton originates from an opposite side semi-leptonic decay such as $B_s^0 \rightarrow D_s^+ \mu^+ \nu$, where the charge of the lepton indicates the flavour of the $B_s^0$ at decay. It was found that roughly 6% of events had a high $p_T$ opposite side lepton that could be used for tagging. However, as the expected number of $B_s^0 \rightarrow J/\psi \phi$ events is already low, it was decided that 6% of these events would not be sufficient to use in optimising the jet charge flavour tagging until much later in the lifetime of ATLAS.
As both other channels and self tagging methods are unavailable, the Monte Carlo dependent calibration will be used for early papers. However, the agreement of the Monte Carlo with real data will be tested indirectly though the $B_d^0 \to J/\psi K^{0*}$ and $B^+ \to J/\psi K^+$ channels.

6.7 Flavour tagging conclusions

In this chapter, a study was performed to maximise the quality factor of the jet charge tagger for three samples. As the quality factor increases, the statistical uncertainty on $A_{\text{true}}$ decreases. Using events that were successfully reconstructed and matched with Monte Carlo truth, the effectiveness parameters were calculated for a range of input variables with the goal of maximising the quality factor.

For the $B^+ \to J/\psi K^+$ and $B_d^0 \to J/\psi K^{0*}$ decay channels, a self tagging mechanism is already present and can be used in real data. Using the truth matched events, it was found that the highest quality factor for the jet charge tagger occurred with an efficiency of $71.2\pm0.2\%$ and a wrong tag fraction of $15.4\pm0.2\%$ for $B^+ \to J/\psi K^+$. In the case of $B_d^0 \to J/\psi K^{0*}$, the highest quality factor for the jet charge tagger occurred with an efficiency of $67.2\pm0.6\%$ and a wrong tag fraction of $36.5\pm0.7\%$.

Calibrating with real data for the $B_s^0$ meson is more challenging as there is no readily available and clean self-tagging mode. In this case, the Monte Carlo dependent calibration will be used, but the agreement of the Monte Carlo with real data will be tested indirectly though the $B_d^0 \to J/\psi K^{0*}$ and $B^+ \to J/\psi K^+$ channel.

It should be noted that the bulk of this study was performed in 2007. In 2007, roughly $20k B_s^0 \to J/\psi \phi$ events were expected to be delivered before this thesis’ deadline. Once it became apparent that flavour tagging would not be performed during the course of this project, the focus of this work shifted to the early data studies of the following chapters 7 and 8. Event generator parameter tuning and generator dependence would have been natural choices for further investigation, because these can explicitly change the momentum spectrum of a jet.
Figure 6.2: Efficiency in terms of $\kappa$ and $\Delta R$ for $B^+ \to J/\psi K^+$ for all three methods. Top row: $p_t$ method, mid row: $p_t$ method and lower row: $p_T$ method. The x-axis shows $\kappa$ and the y-axis shows $\Delta R$. All plots use the exclusion cut that maximises the quality factor as shown in table 6.3.
Figure 6.3: Wrong tag fraction in terms of \( \kappa \) and \( \Delta R \) for \( B^+ \rightarrow J/\psi K^+ \) for all three methods. Top row: \( p_l \) method, mid row: \( p_t \) method and lower row: \( p_T \) method. The x-axis shows \( \kappa \) and the y-axis shows \( \Delta R \). All plots use the exclusion cut that maximises the quality factor as shown in table 6.3.
Figure 6.4: Quality factor in terms of $\kappa$ and $\Delta R$ for $B^+ \to J/\psi K^+$ for all three methods. 
*top row:* $p_l$ method, *mid row:* $p_t$ method and *lower row:* $p_T$ method. The x-axis shows $\kappa$ and the y-axis shows $\Delta R$. All plots use the exclusion cut that maximises the quality factor as shown in table 6.3.
Figure 6.5: The $K^{0*}$ mass using the two mass hypotheses: $K^{0*} \rightarrow K^+ \pi^-$ and $K^{0*} \rightarrow K^- \pi^+$. 
Chapter 7

The impact of inner detector alignment on $B^0_d \rightarrow J/\psi K^{0*}$ measurements

The ATLAS inner detector was described in section 4.3. As the inner detector is a real world object, the possibility exists that the design position of its modules and the physical location of those modules do not coincide exactly. The map of the module positions used in reconstruction can be changed when new understanding of the module positions arises. These changes should ensure that the map more accurately reflects the physical positions of modules, instead of their design positions. Nevertheless, the actual positions of a module may still differ from our understanding of it position. This is known as misalignment.

Inner detector alignment is the procedure of deducing the physical position of modules. Both direct survey mapping and track-based techniques are used. The methods used to align the ATLAS inner detector are described in section 7.1.

In general, the precision of the track based alignment improves with the number of tracks available. However, in some cases, a global systematic deformation can be introduced to the inner detector without changing the quality of the fit used in the alignment procedure. These misalignments are known as ‘weak modes’ and can be difficult to detect and correct. In order to gain a better understanding of some possible effects of weak modes on the inner detector, some global deformations were artificially added to the reconstruction geometry. The new misaligned geometries are described in section 7.2.

The data that was created using these misaligned geometries is described in section 7.3. The initial sample contained 20k $B^0_d \rightarrow J/\psi K^{0*}$ events. These events were reconstructed.
first using an alignment map corresponding exactly to the geometry used in their simulation. This is referred to as the ideal alignment. The events were also reconstructed using ten different alignment maps which did not correspond to the simulated geometry. These samples are ‘misaligned’ or ‘deformed’. Eight of the deformed alignments were reconstructed using an inner detector map that contained global systematic deformations relative to the ideal case. The other two were alignments in which the inner detector modules were randomly displaced in the detectors $x$-$y$ plane relative to the ideal. These two samples should resemble the alignment precision after approximately one day of data-taking and after one hundred days of data-taking.

A detailed analysis of the change in performance characteristics due to alignment for some $B$-physics observables was performed using each of the samples. Unbinned maximum likelihood fits to lifetime and mass were performed with these data-sets. The unbinned maximum likelihood method and the fits are described in section 7.4 and the results of the fits are shown in section 7.4. In order to achieve a clearer image of the impact of the inner detector misalignment, a technique known as ‘event-matching’ was introduced. This method is described and the results are shown in section 7.5.

The study contained in this chapter was published in part in reference [49]. This chapter also contains figures that were produced for that publication.

### 7.1 Alignment and the alignment procedure

In particle physics, a good understanding of the detector is essential for any measurement. As mentioned in the introduction to this chapter, the possibility exists that the physical location of a module and our understanding of its position do not coincide exactly. This phenomenon is known as misalignment.

The presence of misalignment can have many far reaching impacts on physical studies including a reduction in the number of reconstructed tracks, secondary vertex displacements, mass shifts, resolution degradation and misalignment propagation. Furthermore, errors in the alignment of the silicon detectors can propagate downstream into the TRT and other ATLAS subsystems, as these are aligned relative to the inner detector.

The process of alignment creates a new description of the inner detector module positions that more closely resembles the physical detector. Each successive version of the description should (hopefully) come closer to the real inner detector. The long term alignment goal is to have an accurate detector description to within $1 \mu m$ in the $R - \phi$ plane.
When treated as a rigid body, each module has six degrees of freedom (x-y-z position and rotation). The pixel detector and semi-conductor tracker are composed of 1744 and 4088 modules, respectively. This means that the alignment must deal with 35 000 degrees of freedom. During early running, the alignment accuracy is expected to be between 7 and 12 μm in the R − φ plane.

### 7.1.1 Inner detector alignment methods

The first stage of the alignment procedure was an X-ray survey, which was applied to the SCT before it was installed into the inner detector. In this process, the SCT’s modules were scanned with a pair of X-ray beams, then the response of each module is monitored in order to determine their positions. More details about the survey can be found in reference [50]. In addition to the X-ray survey, two run-time techniques are used:

**Frequency Scanning Interferometry (FSI)** uses interferometers arranged in the inner detector to obtain a series of length measurements. This allows the determination of the relative positions of the interferometry nodes.

**Chi-squared (∇2) minimisation** uses particle tracks and performs a global fit of the alignment parameters using a chi-squared minimisation process.

Frequency scanning interferometry measures the distance between two points that have clear line of sight. The technique measures lengths by tuning two lasers and comparing the resulting phase shifts. The inner detector contains 842 interferometers which range in length from 30 mm to 1440 mm. This system must resolve changes to the inner detector of order 10 μm. In order to do so, each interferometer must measure length with a precision of 1 μm. Further details about the FSI in ATLAS can be found in reference [51].

The track-based alignment is centred around the minimisation of the ∇2 shown in equation 7.1. This equation contains information for a large number of extrapolated tracks and detector hits. As such, it is very computationally intensive to produce new alignments. The details related to the chi-squared minimisation are taken from reference [52]. The chi-squared is formed using the formula:

\[
∇^2 = \sum_{\text{tracks}} r^T V^{-1} r
\]  

where

\[
r_i = (\overline{m}_i - \overline{e}_{i, \pi}(\pi, \overline{\alpha})) \hat{k}
\]
and where $V$ is the covariance matrix of $r_i$, $\hat{\kappa}$ is the unit vector defining the sensor plane and $\vec{m}_i$ is the position of the associated detector hit and $\vec{e}_i$ is the intersection point of the extrapolated track with a sensor plane. This intersection point is a function of the track parameters $\mathbf{\pi}$ and the alignment parameters of the intersected module, $\mathbf{\pi}$.

In order to simplify the chi-squared alignment procedure, the process has been divided into several steps. The first step of the alignment is to align the large inner detector structures such as the barrel and end-caps of each sub-detector relative to each other. The second stage of alignment aligns the layers of the barrel and end-caps with each other, resulting in a system with several hundred degrees of freedom. The third stage of alignment is the complete alignment of all inner detector modules relative to each other.

The inner detector alignment will be derived from a dedicated stream of tracks selected at 10 Hz. High momentum tracks are preferred because they have fewer distortions introduced by multiple scattering. The inner detector alignment conditions can be updated every 24 hours, if required. Approximately one million tracks per 24 hours will be required to attain a precision of 10 $\mu$m in the silicon modules positions. Corrections for imperfections within modules will also be necessary. For more details, references [53] and [1] both describe the alignment procedure in more detail.

Certain global distortions in the inner detector might be only weakly constrained by these high momentum tracks. These global distortions are known as “weak modes”. They can introduce biases on the measured track parameters while preserving helical trajectory of the track. Furthermore, the chi-square can remain unchanged in the presence of such a mode. To reduce the impact of weak modes on the inner detector tracking performance, alignment can be performed using other track topographies such as:

- Primary vertex tracks.
- Cosmic ray tracks.
- Beam halo tracks.
- Tracks in overlap region of adjacent modules.
- Track pairs from $Z$ and $J/\psi$ decays.

Nevertheless, some global deformations could remain in the inner detector description. In such cases, detecting and removing the misalignment becomes a challenge, even with a large number of tracks. It was postulated that a well understood process with a large cross section and acceptance such as $B_d^0 \to J/\psi K^{0*}$ or $B^+ \to J/\psi K^+$, could be influenced by the presence of the weak modes.
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7.2 Types of Misalignments

Three different levels of misalignment can occur in the inner detector.

Level 1 is the misalignment of each sub-detector system (pixels, SCT, TRT) relative to each other.

Level 2 is the misalignments of individual disks and barrel layers relative to each other.

Level 3 misalignment occurs when individual modules in a sub detector are displaced relative to the design position, or relative to our understanding of their position.

Two categories of level 3 misalignment have been considered in this thesis:

- Expected alignment after a certain number of days of running.
- Global systematic misalignments.

The first type of misalignment is a random displacement of individual modules in the reconstruction stage by some distance in the x-y direction. The second type of misalignment are global systematic misalignments, which are an approximation of the expected influence of inner detector weak modes. These alignments are applied to the data during the reconstruction phase of the full chain. The full chain was described in section 5.4. To be precise, this is a study of the impact of a difference between the model of detector used in simulation and the module of the detector used in reconstruction and thereafter.

7.2.1 Expected alignment after 1 and 100 days of running.

The “Day 1” and “Day 100” alignments contain random displacement of individual inner detector modules in the x and y directions. The magnitude of the displacements is distributed according to a random Gaussian distribution with a mean at zero and a width inferred from the residuals of hits to tracks in cosmic ray data. The width of the Gaussians are shown in table 7.1 and vary with the duration of the alignment period, the sub-detector and the pseudo-rapidity. They were ascertained by subtracting quadratically the width determined with cosmic data from the expected width determined with Monte Carlo events. The magnitude of the alignment constants are in agreement with the cosmic data module displacements. These constants were produced by the ID alignment group and published in reference [54].

After one day, the alignment is expected to be better in the barrel than the end-cap. This is because cosmic muons predominately travel in the negative y direction. The
position uncertainty of the Day 1 alignment constants for the pixel detector and the SCT are 20 μm in the barrel and 50 μm in the end-caps. After 100 days of alignment, the alignment no longer relies on the cosmic muons alignment. Thus, the silicon barrel and end-cap alignments are expected to converge to alignment uncertainties of 10 μm.

### 7.2.2 Global systematic misalignments

There are nine known weak modes, as shown in figure 7.1, of which four have been presented here. These are the four highlighted modes of figure 7.1 and are called curl, elliptical, telescope and twist. They were chosen by the inner detector alignment group because they retain helical trajectories for particles coming from the interaction point, such that the interaction point is a fixed point under any of those transformations. Furthermore, not all nine modes were allowed by the physical and geometrical construction of the inner detector. For instance, certain deformations are unlikely to occur, because mechanical supports do not allow certain groups of modules to move with respect to each other. There are also practical problems; some modes can not be represented in the Athena geometry model as modules can not be bent internally in this computational description. The final and perhaps most significant reason for the global systematic modes selected for study is that they are the most likely weak modes to occur.

It should be noted that there is a difference between a weak mode and a global systematic misalignment. A weak mode is a mathematically describable shift in module positions that arises because of the χ² minimisation process. It does not imply that the modules are in an elliptical shape for instance, only that the computer description of them is different between where they physically sit and where the model places them. These weak modes arise naturally out of the alignment procedure and can be very difficult to identity and remove. On the other hand, a global systematic misalignment is a shift in module position that is artificially applied to the detector modules when reconstructing
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Figure 7.1: Nine possible weak modes. The four highlighted modes are used in this study.

data-sets. Global systematic deformation shift modules according to some function of $\phi$, $z$, radius or $\eta$.

Each type of global systematic misalignment has been produced for a large misalignment and for a small misalignment. A summary of all eight types of misalignment follows this chapter in table 7.2.

7.2.2.1 Curl mode

The curl global systematic deformation is a rotation of the inner detector modules in the $\phi$ direction such that the size of the rotation changes with radius.

$$\Delta \Phi = c_1 \cdot R + \frac{c_2}{R^2}$$ (7.3)

In this case, it a shift in the positive $\phi$ direction, where the change in $\phi$ decreases with radius. The outer section of the SCT barrel has a smaller shift in $\phi$ than the first layer of the pixel detector. The parameters were chosen for the large curl misalignment such that the outermost SCT barrel layer rotated by approximately 200 $\mu$m and the innermost pixel detector layer rotated by approximately 50 $\mu$m. The parameter values used are $c_1 = 7.6 \cdot 10^{-4} \text{ mrad} \cdot \text{mm}$ and $c_2 = 5. \cdot 10^{-5} \text{ mrad} \cdot \text{mm}$. The small curl alignment was produced by running the alignment procedure on the large curl deformations.
7.2.2.2 Elliptical mode

The elliptical global systematic deformation is an elliptical shift according to equation 7.4. It describes a shift from a circular detector in the x-y plane towards an elliptical detector, with the long axis of the ellipse along the x-axis.

\[ \Delta R = \frac{1}{2} c \cdot \cos (2\phi) R \]  
(7.4)

The value of \( c = 3.9 \times 10^{-3}\text{mm}^{-1} \) was chosen for the large misalignment such that the outermost SCT modules shift by \( \pm 1 \text{ mm} \). The value of \( c = 9.8 \times 10^{-4}\text{mm}^{-1} \) was chosen for the small misalignment such that the outermost SCT modules shift by \( \pm 250 \text{ \mu m} \).

7.2.2.3 Telescope mode

Telescope global systematic deformation shifts modules in the z-direction where the magnitude of the shift is proportional to the module’s radial coordinate, as described in equation 7.5.

\[ \Delta Z = cR \]  
(7.5)

For the large telescope deformation, \( c = 9.7 \cdot 10^{-4} \) was chosen such that SCT modules in the outermost barrel layer move by approximately 3 mm in the z-direction. A value of \( c = 5.8 \cdot 10^{-3} \) was chosen such that the small telescope alignment shifts the outermost SCT modules by 500 \( \mu \text{m} \) in the z-direction.

7.2.2.4 Twist mode

The twist deformation rotates modules in the \( \phi \) direction proportion to their Z coordinate, as in equation 7.6.

\[ \Delta \phi = cZ \]  
(7.6)

To produce the large twist geometry, \( c = 2.5 \cdot 10^{-4}\text{mrad/mm} \) was chosen such that the SCT modules in the outermost \( \eta \) rings shift by 200 \( \mu \text{m} \). The residual twist alignment was produced by running the ID alignment procedure on the large twist geometry.

The TRT barrel modules span the entire length of the Barrel and hence have no individual slices that can be rotated. Instead, they are rotated by an angle \( \alpha \) in the R\( \phi \) plane.

\[ \alpha = -\frac{2\phi_{\text{max}}}{l} R \]  
(7.7)
where $\phi_{\text{max}}$ is the required maximum $\Delta \phi$ shift in the highest pseudo-rapidity region of the SCT barrel, and $l$ is the total length of the barrel.

Table 7.2: A summary table of the four global systematic deformations.

<table>
<thead>
<tr>
<th>Deformation</th>
<th>Description</th>
<th>$c$ value</th>
<th>SCT shift (direction)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large curl</td>
<td>$\Delta \phi = c_1 \cdot R + \frac{c_2}{R}$</td>
<td>$c_1 = 7.6 \times 10^{-4} \text{ mrad mm}^{-1}$, $c_2 = 5.0 \times 10^{-5} \text{ mrad mm}$</td>
<td>250 $\mu$m ($\phi$)</td>
</tr>
<tr>
<td>Small curl</td>
<td></td>
<td></td>
<td>Aligned</td>
</tr>
<tr>
<td>Large elliptical</td>
<td>$\Delta R = \frac{1}{2}c \cos (2\phi) R$</td>
<td>$c_L = 3.9 \times 10^{-4} \text{ mm}^{-1}$, $c_S = 9.8 \times 10^{-4} \text{ mm}^{-1}$</td>
<td>1000 $\mu$m (R)</td>
</tr>
<tr>
<td>Small elliptical</td>
<td></td>
<td></td>
<td>250 $\mu$m (R)</td>
</tr>
<tr>
<td>Large telescope</td>
<td>$\Delta Z = cR$</td>
<td>$c_L = 5.8 \times 10^{-3}$, $c_S = 9.7 \times 10^{-4}$</td>
<td>3000 $\mu$m (Z)</td>
</tr>
<tr>
<td>Small telescope</td>
<td></td>
<td></td>
<td>500 $\mu$m (Z)</td>
</tr>
<tr>
<td>Large twist</td>
<td>$\Delta \phi = cZ$</td>
<td>$c = 2.5 \times 10^{-4} \text{ mrad mm}$</td>
<td>200 $\mu$m ($\phi$)</td>
</tr>
<tr>
<td>Small twist</td>
<td></td>
<td></td>
<td>Aligned</td>
</tr>
</tbody>
</table>

7.3 Data preparation

The sample consisted of 20k $B_d^0 \to J/\psi K^{0*}$ events where $J/\psi \to \mu^+\mu^-$ and $K^{0*} \to \pi^+0.5K^-0.5$. The trailing numbers denote the minimal $p_T$ in GeV requested in MC generation. This sample corresponds roughly to 100 pb$^{-1}$. The events were simulated using the standardised job transformations as described in section 5.4. The definition of the geometry as well as the details about trigger were published in reference [27]. This sample was selected because its simulation geometry matched the geometry required for the inclusion of inner detector deformations.

The events were reconstructed using Athena\* in such a way that the reconstruction alignment was different to the simulation alignment. This sample of exclusive 20k $B_d^0 \to J/\psi K^{0*}$ was repeatedly reconstructed with each of the alignments described in section 7.2.2 and in reference [55]. The sample was also reconstructed in the ‘ideal’ alignment. This is the situation where the simulated and reconstruction alignment fully coincide.

7.3.1 Selection criteria

The $J/\psi$ in this sample were reconstructed by forming a vertex using two tracks that were identified as muons during reconstruction. No restrictions were made on the $J/\psi$.

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\*Athena version 14.2.20
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invariant mass or on the $\chi^2$ of the vertex. The $B$-mesons used in this study were found by forming a vertex using the $J/\psi$’s muons and all other pairs of oppositely charged tracks. When reconstructing using real data, cuts are usually applied to the $B^0_d$ and $J/\psi$ mass, $p_T$ and vertex $\chi^2$. The study in reference [27] (pages 1122 and 1123) contains some analysis of the effectiveness of the cuts. However, more up to date studies are currently being prepared by members of the ATLAS $B$-physics working group.

In the case of simulated data, a truth cut was performed to select the signal. This cut checked that each set of four tracks matched the decay particles of the $B^0_d$ meson. Using truth cuts instead of a physics motivated cut was justified here for of two reasons. Firstly, the effects of weak modes would be overwhelmed by the effects of Day 1 and Day 100 -like distortions. As such, weak modes could be a problem in the long run, but are expected to be less worrying in the short term. The short term study was performed using 5 pb$^{-1}$ sample in chapter 8. Secondly, given the long time frame for weak modes to become dangerous, it is expected that the detailed angular analysis of this exclusive channel will have been performed. This study is expected to have optimised signal selection criteria such that background effects can be minimised.

7.4 The impact of misalignment on unbinned maximum likelihood fits to the $J/\psi$, $B^0_d$ mass and $B^0_d$ lifetime

The first step of this study was to perform an unbinned maximum likelihood fit to the $B^0_d$ mass, the $B^0_d$ Lifetime and the $J/\psi$ mass for each misaligned data-set. The unbinned maximum likelihood fits were performed using the following methods.

7.4.1 Introduction to maximum likelihood theory

The maximum likelihood technique is a technique for finding the best description of a set of data points in terms of a parametric model. It is also used to provide estimates for the model’s parameters. This section holds a brief description of maximum likelihood techniques, but for more details, many references are available, such as [1, 56].

Maximum likelihood techniques work in the following way: there is some observable quantity $x$, which is distributed according a probability density function or PDF $f (x; \lambda)$. The set of parameters to describe the model are $\lambda$, but $\lambda$ is as of yet unknown. The measurement of $x$ is performed $n$ times, resulting in $n$ data points or in our case, $n$ events. The theory uses the $\lambda$ parameters to describe the observed values for $x$. The goal of the study is to extract the values in $\lambda$ that are the estimators of the true $\lambda$.
that give the best description of the data points \( x \). Assuming a negligible systematic error, the probability for the \( i^{th} \) measurement to be in an interval \( x_i + dx_i \) is given by \( f (x_i; \lambda) \, dx_i \).

The probability, \( P \), that this holds for all \( n \) measurements is

\[
P = \prod_{i=1}^{n} f (x_i; \lambda) \, dx_i \tag{7.8}
\]

If the model is correct and the values of \( \lambda \) are close to their physical values, the set of measured data-points should yield a high value of \( P \). So maximising \( P \) brings the set of \( \lambda \) closer to their physical values. The interval \( dx_i \) is arbitrary, so equation 7.8 can be written:

\[
\mathcal{L} (\lambda) = \prod_{i=1}^{n} f (x_i; \lambda) \tag{7.9}
\]

where \( \mathcal{L} (\lambda) \) is the likelihood function. When \( \mathcal{L} \) is maximised, the set of values \( \hat{\lambda} \) are known as the maximum likelihood estimator, and they represent the physical values which make the measured set of data points most probable.

In practice, the logarithm of \( \mathcal{L} \) is taken to reduce in computing time required. This converts the product into a sum. Furthermore, the likelihood function must be normalised such that

\[
\int_0^\infty \mathcal{L} \, dx = 1 \tag{7.10}
\]

The limiting factor is that the Maximum Likelihood technique is only as good as the theory that produced the probability density function, and is limited by the measurement technique. Also, it can be quite a challenge is to find the set of parameters \( \hat{\lambda} \) that maximise \( \mathcal{L} \) or \( P \).

As described in section 5.4.5, Minuit is a numerical minimisation program. The package used here is an implementation of Minuit in the root framework\(^\text{[45, 46]}\). When using Minuit, there is the possibility that the solution is a local minimum. Furthermore, the PDF must accurately describe the data and there must be as few free parameters as possible. Large correlations between parameters can lead to problems finding a fit and unreliable errors. Finally, large numbers of parameters and high statistics can require considerable computing resources to converge to a fit.

It should be noted that there is no widely accepted measure of goodness of fit for unbinned maximum likelihood fits. This problem is described in references \(^\text{[57, 58]}\). The fits shown in this thesis were required to be considered “converged” by the fitting program Minuit and to have a relatively low estimated distance to minimum and positive error matrices.
7.4.2 Unbinned maximum likelihood fits in exclusive $B_d^0 \rightarrow J/\psi K^{0*}$ alignment studies

Three fits have been performed for each alignment. They are the $J/\psi$ mass, the $B_d^0$ mass and the $B_d^0$ lifetime fits. Each fit is performed independently.

The reconstructed $J/\psi$ mass was approximated by applying an unbinned maximum likelihood Gaussian fit to the reconstructed events. The $J/\psi$ mass maximum likelihood function is defined by:

$$ L = \prod_{i=1}^{N} \frac{1}{\sigma m \sqrt{2\pi}} e^{-\frac{(m_i - m_{J/\psi})^2}{2\sigma m^2}} $$

where the index $i$ runs over the total number of reconstructed events, and $m_i$ is the measured mass in each event. The mass of $J/\psi$ ($m_{J/\psi}$) and the mass resolution ($\sigma m$) are both free parameters determined in the fit.

The $B_d^0$ mass is fitted to two Gaussian functions. One Gaussian has a mean fixed at zero and the others mean is allowed to vary. The $B_d^0$ mass likelihood function is defined by:

$$ L = \prod_{i=1}^{N} \frac{1}{\sigma m_1 \sqrt{2\pi}} e^{-\frac{(m_i - m_{B_d^0})^2}{2\sigma m_1^2}} + (1 - r) \frac{1}{\sigma m_2 \sqrt{2\pi}} e^{-\frac{m_i^2}{2\sigma m_2^2}} $$

where the index $i$ runs over the total number of reconstructed events, and $m_i$ is the measured mass in each event. The mass of $B_d^0$ ($m_{B_d^0}$), the Gaussian widths ($\sigma m_1$ and $\sigma m_2$) and the ratio between the two Gaussians ($r$) are free parameters determined in the fit. A double Gaussian is a reasonable fit because it includes the single Gaussian effects of the $J/\psi$ mass and also includes the smearing introduced by the Kaon Pion pair.

The $B_d^0$ lifetime was fitted to an exponential function convoluted with a Gaussian function. The $B_d^0$ lifetime maximum likelihood function is defined as:

$$ L = \prod_{i=1}^{N} \frac{1}{\Gamma} e^{-\frac{t_i}{\Gamma}} \frac{1}{\sqrt{2\pi}} \left(1 + \text{Erf}\left(\frac{t_i}{\sqrt{2}\sigma}\right)\right) $$

where $t$ is the measured lifetime of the $B_d^0$ in this event, $\Gamma$ is the inverse of the fitted lifetime, $\sigma$ is the width, and where $\text{Erf}(z)$ is the error function:

$$ \text{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_{0}^{z} e^{-t^2} dt $$

During the fit, the width $\sigma$ varies from event to event, according to the uncertainty of each event.

$$ \sigma = S \Delta \tau_{\text{incl.}} $$
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This $S$ value is a free parameter to be fitted by Minuit and the uncertainty on an event’s lifetime $\Delta \tau_{incl.}$ is a measured quantity. When quoted in tables or on the plots, the width $\sigma$ is the overall width. This is calculated by multiplying the fitted value of $S$ with the measured uncertainty on the fitted lifetime averaged over all events.

### 7.4.3 The impact of Day 1 and Day 100 alignment on unbinned maximum likelihood fits.

The $J/\psi$ mass, $B_d^0$ mass and $B_0^0$ lifetime were fitted for the Day 1, Day 100 and ideal alignments using the methods described in section 7.4.2. This fits are shown in figures 7.2, 7.3 and 7.4 respectively. The results of the Gaussian fits are given in table 7.3.

#### Table 7.3

<table>
<thead>
<tr>
<th>Mass (Rec-PDG) [MeV]</th>
<th>$\sigma$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day Ideal</td>
<td>57.4</td>
</tr>
<tr>
<td>Day 100</td>
<td>58.9</td>
</tr>
<tr>
<td>Day 1</td>
<td>61.5</td>
</tr>
</tbody>
</table>

**Figure 7.2:** The reconstructed $J/\psi$ mass fitted to a single Gaussian function in the ideal, Day 1 and Day 100 alignments.

**Figure 7.3:** The reconstructed $B_d^0$ mass fitted to a double Gaussian function in the ideal, Day 1 and Day 100 alignments.
The fits of the $J/\psi$ mass are shown in figure 7.2. In this study, no $J/\psi$ mass shifts beyond the statistical precision of this test were observed due to Day 1 or Day 100 misalignments. The $J/\psi$ mass resolution for an ideally aligned detector is 57.4 MeV. An increase in the $J/\psi$ mass resolution was observed for the misaligned cases relative to the ideal case by 1.5 MeV for the Day 100 case and by 4.1 MeV after one day of alignment data. The study of the $J/\psi$ mass was performed with higher statistics and including a background sample in chapter 8.

Figure 7.3 shows a comparison for the $B_0^d$ mass fit between the ideal alignment and the Day 1 and Day 100 alignments. The width of the Gaussian centred at zero increases by 0.6 MeV for the Day 100 and by 10.5 MeV for Day 1 relative to the ideal case. The Day 100 fit to $B_0^d$ mass gives the fixed Gaussian more importance (0.97) than the floating Gaussian relative to the ideal and the Day 100 cases (0.92 and 0.90). This means that the floating Gaussian describes fewer events but is wider and further from zero.

The fits of the $B_0^d$ lifetime are shown in figure 7.4. With the statistical uncertainty of this study, no shift in $B_0^d$ lifetime is introduced by the Day 1 or Day 100 misalignments relative to the ideal case. An increase in the $B_0^d$ lifetime resolution was observed for both misaligned cases relative to the ideal case by 0.03 ps$^{-1}$ for the Day 100 case and by 0.028 ps$^{-1}$ after one day of alignment data.
Table 7.3: The results of the unbinned maximum likelihood fits to $J/\psi$ and $B^0_d$ mass and $B^0_d$ lifetime for the ideal, Day 1 and Day 100 alignments.

<table>
<thead>
<tr>
<th></th>
<th>Ideal</th>
<th>Day 100</th>
<th>Day 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi$ mass (Rec-MC) [MeV]</td>
<td>1.0±1.0</td>
<td>0.0±1.0</td>
<td>-0.1±1.0</td>
</tr>
<tr>
<td>$J/\psi$ mass resolution [MeV]</td>
<td>57.4±0.7</td>
<td>58.9±0.7</td>
<td>61.5±0.7</td>
</tr>
<tr>
<td>$B^0_d$ lifetime [ps]</td>
<td>1.54±0.02</td>
<td>1.54±0.02</td>
<td>1.55±0.03</td>
</tr>
<tr>
<td>$B^0_d$ lifetime resolution [ps]</td>
<td>0.109±0.006</td>
<td>0.139±0.007</td>
<td>0.137±0.007</td>
</tr>
<tr>
<td>$B^0_d$ mass (Rec-MC) [MeV]</td>
<td>29±14</td>
<td>22±12</td>
<td>58±37</td>
</tr>
<tr>
<td>$B^0_d$ mass resolution (float) [MeV]</td>
<td>222±17</td>
<td>201±15</td>
<td>330±38</td>
</tr>
<tr>
<td>$B^0_d$ mass resolution (fix) [MeV]</td>
<td>77.8±1.8</td>
<td>78.4±2.1</td>
<td>88.3±1.5</td>
</tr>
<tr>
<td>Float fraction:</td>
<td>0.08±0.02</td>
<td>0.10±0.02</td>
<td>0.03±0.01</td>
</tr>
</tbody>
</table>

7.4.4 The impact of global systematic misalignment on unbinned maximum likelihood fits.

Fits to the $J/\psi$ and $B^0_d$ masses and the $B^0_d$ lifetime were performed to compare each of the global systematic misalignment samples against the ideal sample. These fits are also shown in figures 7.5, 7.6 and 7.7. and the fitted values are shown in tables 7.4, 7.5 and 7.6 respectively. In these tables, the mass is shown as (Rec - MC). This means that the value quoted is the reconstructed mass subtracted by the simulated mass.

Figure 7.5: The reconstructed $J/\psi$ mass fitted to a single Gaussian for the ideal alignment, and the large and small curl (top left), elliptical (top right), telescope (bottom left) and twist (bottom right) global systematic distortions.
Table 7.4: The impact of global systematic deformations on unbinned maximum likelihood $J/\psi$ mass fit.

<table>
<thead>
<tr>
<th>Deformation</th>
<th>Mass (Rec-MC)</th>
<th>Ideal, [MeV]</th>
<th>Small, [MeV]</th>
<th>Large, [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curl</td>
<td>Mass (Rec-MC)</td>
<td>1.0±0.9</td>
<td>1.2±0.9</td>
<td>1.1±1.0</td>
</tr>
<tr>
<td></td>
<td>Mass resolution</td>
<td>57.4±0.7</td>
<td>58.3±0.7</td>
<td>62.5±0.7</td>
</tr>
<tr>
<td>Elliptical</td>
<td>Mass (Rec-MC)</td>
<td>1.0±0.9</td>
<td>0.1±0.9</td>
<td>0.4±1.0</td>
</tr>
<tr>
<td></td>
<td>Mass resolution</td>
<td>57.4±0.7</td>
<td>57.2±0.7</td>
<td>59.3±0.7</td>
</tr>
<tr>
<td>Telescope</td>
<td>Mass (Rec-MC)</td>
<td>1.0±0.9</td>
<td>0.7±0.9</td>
<td>0.4±1.0</td>
</tr>
<tr>
<td></td>
<td>Mass resolution</td>
<td>57.4±0.7</td>
<td>57.3±0.7</td>
<td>57.9±0.7</td>
</tr>
<tr>
<td>Twist</td>
<td>Mass (Rec-MC)</td>
<td>1.0±0.9</td>
<td>1.3±0.9</td>
<td>1.0±0.9</td>
</tr>
<tr>
<td></td>
<td>Mass resolution</td>
<td>57.4±0.7</td>
<td>59.3±0.7</td>
<td>58.6±0.7</td>
</tr>
</tbody>
</table>

Under these global systematic deformations, the $J/\psi$ mass is not observed to shift by more than the statistical uncertainty. Even the largest shift away from the ideal (large elliptical) shifts by less than the statistical uncertainty. The mass resolution is consistently wider in the large deformation samples, the widest of which is in the large curl sample, where the $J/\psi$ mass resolution is 5.1 MeV wider than the ideal case. The small deformations show either a small increase in resolution or are consistent with no change in resolution.

Figure 7.6: The reconstructed $B_d^{0}$ mass fitted to a double Gaussian for the ideal alignment, and the large and small curl (top left), elliptical (top right), telescope (bottom left) and twist (bottom right) global systematic distortions.

In the case that the fit has a vastly different ratio of the fixed Gaussian to the floating
Table 7.5: The impact of global systematic deformations on unbinned maximum likelihood \(B_d^0\) mass fit.

<table>
<thead>
<tr>
<th>(B_d^0) Mass (Rec-MC)</th>
<th>Ideal, [MeV]</th>
<th>Small, [MeV]</th>
<th>Large, [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curl</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (float)</td>
<td>29±14</td>
<td>50±16</td>
<td>37±13</td>
</tr>
<tr>
<td>Width (float)</td>
<td>222±17</td>
<td>224±18</td>
<td>195±19</td>
</tr>
<tr>
<td>Width (fix)</td>
<td>78±2</td>
<td>79±2</td>
<td>88±3</td>
</tr>
<tr>
<td>Gaussian ratio</td>
<td>0.08±0.02</td>
<td>0.08±0.02</td>
<td>0.10±0.04</td>
</tr>
<tr>
<td>Elliptical</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (float)</td>
<td>29±14</td>
<td>23±10</td>
<td>34±23</td>
</tr>
<tr>
<td>Width (float)</td>
<td>222±17</td>
<td>185±13</td>
<td>263±27</td>
</tr>
<tr>
<td>Width (fix)</td>
<td>78±2</td>
<td>74±2</td>
<td>84±2</td>
</tr>
<tr>
<td>Gaussian ratio</td>
<td>0.08±0.02</td>
<td>0.13±0.03</td>
<td>0.05±0.01</td>
</tr>
<tr>
<td>Telescope</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (float)</td>
<td>29±14</td>
<td>24±13</td>
<td>19±15</td>
</tr>
<tr>
<td>Width (float)</td>
<td>222±17</td>
<td>209±17</td>
<td>214±20</td>
</tr>
<tr>
<td>Width (fix)</td>
<td>78±2</td>
<td>77±2</td>
<td>80±2</td>
</tr>
<tr>
<td>Gaussian ratio</td>
<td>0.08±0.02</td>
<td>0.09±0.02</td>
<td>0.08±0.02</td>
</tr>
<tr>
<td>Twist</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (float)</td>
<td>29±14</td>
<td>30±13</td>
<td>17±6</td>
</tr>
<tr>
<td>Width (float)</td>
<td>222±17</td>
<td>215±17</td>
<td>155±8</td>
</tr>
<tr>
<td>Width (fix)</td>
<td>78±2</td>
<td>80±2</td>
<td>69±3</td>
</tr>
<tr>
<td>Gaussian ratio</td>
<td>0.08±0.02</td>
<td>0.10±0.02</td>
<td>0.26±0.04</td>
</tr>
</tbody>
</table>

Gaussian, the behaviour of the other fitted parameters is no longer comparable to other alignments. This occurs in the large twist alignment, where the Gaussian ratio rises to 0.26±0.04. The width of its floating Gaussian is much narrower than in the other samples. The key thing to notice is that many effects are hidden here. In section 7.5, a method is introduced that allows these differences to be studied in greater detail.

Table 7.6: The impact of global systematic deformations on unbinned maximum likelihood \(B_d^0\) lifetime fit.

<table>
<thead>
<tr>
<th></th>
<th>(B_d^0) lifetime</th>
<th>Ideal, [ps]</th>
<th>Small, [ps]</th>
<th>Large, [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curl</td>
<td>(B_d^0) lifetime</td>
<td>1.54±0.02</td>
<td>1.53±0.02</td>
<td>1.55±0.03</td>
</tr>
<tr>
<td></td>
<td>(B_d^0) lifetime resolution</td>
<td>0.109±0.006</td>
<td>0.108±0.006</td>
<td>0.109±0.006</td>
</tr>
<tr>
<td>Elliptical</td>
<td>(B_d^0) lifetime</td>
<td>1.54±0.02</td>
<td>1.54±0.03</td>
<td>1.54±0.03</td>
</tr>
<tr>
<td></td>
<td>(B_d^0) lifetime resolution</td>
<td>0.109±0.006</td>
<td>0.112±0.006</td>
<td>0.113±0.006</td>
</tr>
<tr>
<td>Telescope</td>
<td>(B_d^0) lifetime</td>
<td>1.54±0.02</td>
<td>1.55±0.03</td>
<td>1.54±0.03</td>
</tr>
<tr>
<td></td>
<td>(B_d^0) lifetime resolution</td>
<td>0.109±0.006</td>
<td>0.109±0.006</td>
<td>0.109±0.006</td>
</tr>
<tr>
<td>Twist</td>
<td>(B_d^0) lifetime</td>
<td>1.54±0.02</td>
<td>1.55±0.03</td>
<td>1.54±0.02</td>
</tr>
<tr>
<td></td>
<td>(B_d^0) lifetime resolution</td>
<td>0.109±0.006</td>
<td>0.109±0.006</td>
<td>0.109±0.006</td>
</tr>
</tbody>
</table>
The measurement of the $B_d^0$ lifetime is shown in table 7.6 and in figure 7.7. The table shows that no overall lifetime shift is observed for any alignment. This is echoed in figure 7.7, where the data and the fits coincide for all alignments. Furthermore, the lifetime width changes less than the statistical precision of the measurement between alignments.

The unbinned maximum likelihood fits to the $J/\psi$ mass, $B_d^0$ mass and $B_d^0$ lifetime are not very sensitive to changes in alignment. It is clear that other smearing effects, such as multiple scattering overwhelm the effects of misalignment. With more events, it should be possible to perform cuts in terms of $\phi$, $\eta$ or $p_T$ that could more clearly identify the differences between the global systematic distortions. However, at the time that this study was performed, it was not possible to access more events. For this reason, the following event-matching method was developed to compare alignments.

Figure 7.7: The reconstructed $B_d^0$ lifetime fitted to an exponential convoluted with a Gaussian function for the ideal alignment, and the large and small curl (top left), elliptical (top right), telescope (bottom left) and twist (bottom right) global systematic distortions.
7.5 The event by event method for alignment-sensitive analysis

In order to disentangle the alignment from other factors, an ‘event-matching’ technique was introduced. The ‘event-matching’ method takes advantage of the fact that the simulation step (the modelling of interactions with material of inner detector and trajectories of charged particles in magnetic field) is separate from the reconstruction step, where the misalignments are introduced as described above. Once the event has been simulated, the mass of \( J/\psi \) or \( B_d^0 \) meson was reconstructed several times using different alignments, giving a corresponding set of measurements, denoted: \( m_{\text{ideal}}, m_{\text{twist}} \) or \( m_{\text{curl}} \). Any difference in these observables for a given event must arise uniquely due to differences in detector geometry. Finally, the variables \( \Delta m \), \( \Delta \tau \) and \( \Delta V_B(x,y) \) are defined in equations (7.16), (7.17) and (7.18) respectively:

\[
\Delta m = m_{\text{Misaligned}} - m_{\text{Ideal}} \quad (7.16)
\]

\[
\Delta \tau = \tau_{\text{Misaligned}} - \tau_{\text{Ideal}} \quad (7.17)
\]

\[
\Delta V_B(x,y) = \sqrt{(x_{\text{Misaligned}} - x_{\text{Ideal}})^2 + (y_{\text{Misaligned}} - y_{\text{Ideal}})^2} \quad (7.18)
\]

where \( m \) represents either \( J/\psi \) or \( B_d^0 \) mass, \( \tau \) denotes the \( B_d^0 \) lifetime and the variables \( x_{\text{Misaligned}}, x_{\text{Ideal}}, y_{\text{Misaligned}} \) and \( y_{\text{Ideal}} \) are the \( x \) and \( y \) coordinates of the \( B \)-vertex reconstructed with misaligned and ideal geometry. Plotting the differences in physical variables, \( \Delta m \), \( \Delta \tau \) and \( \Delta V_B(x,y) \) allows the direct observation of the impact of misalignment on \( B \)-physics observables.

This concept is demonstrated in figure 7.8, where a plot of event-matched \( \Delta m(J/\psi) \) is compared with the \( J/\psi \) mass unbinned maximum likelihood plot. The core of the event-matched distributions were fitted to a Gaussian function. In the event-matched (lower) plot, the dotted line shows \( \Delta m(J/\psi)_{1} = m(J/\psi)_{\text{Day 1}} - m(J/\psi)_{\text{Ideal}} \) and the full line shows \( \Delta m(J/\psi)_{100} = m(J/\psi)_{\text{Day 100}} - m(J/\psi)_{\text{Ideal}} \). The \( \Delta m(J/\psi)_{1} \) fit is roughly twice as wide as the \( \Delta m(J/\psi)_{100} \) plot, which is similar to the inner detector module displacement in each alignment considered. This large difference is not obvious in the fitted maximum likelihood plot.

The matching of events was performed using the run number, event number and the exact Monte Carlo \( \phi \) position of each event. Matching events in real data can be performed using run number, event number and near-matching of muon \( \phi \) and \( \eta \). A study of event-matching information is performed in section 8.2.4.
7.5.1 Expected $B$-physics performance after one day and 100 days of alignment using event-matching.

After one day, the module positions are expected to be known with precisions of order 20 $\mu$m precision for pixel and SCT modules in the barrel region. After 100 days, this uncertainty is expected to be reduced to 10 $\mu$m in the same region.

Histograms were prepared for the variables $\Delta m(B_{\psi}^0)$, $\Delta m(J/\psi)$, $\Delta \tau$ and $\Delta V_B(x,y)$ comparing the ideal alignment to Day 1 and Day 100 misalignments. These plots are shown in figure 7.9. The core of the event-matched distributions were fitted to a Gaussian function, except for the variable $\Delta V_B(x,y)$, which displayed non Gaussian behaviour. The results of these fits are shown in table 7.7.

The mass and lifetime plots are consistent with no shift away from zero. This agrees with the observation made in the unbinned maximum likelihood fit of section 7.4.3. For
Chapter 7: Impact of inner detector alignment on $B_d^0 \rightarrow J/\psi K^{*0}$ measurements

The results of the Gaussian fit to the event-matched variables for Day 1 and Day 100. Top left: $J/\psi$ mass. Top right: $B_d^0$ lifetime. Bottom left: $B_d^0$ mass. Bottom right: change in $B_d^0$ vertex position. In all plots, the full line is a plot of $\Delta_{100} = J/\psi_{Day 100} - J/\psi_{Ideal}$ and the dotted line is $\Delta_1 = J/\psi_{Day 1} - J/\psi_{Ideal}$.

Table 7.7: The results of the Gaussian fit for the event-matched variables, $\Delta m(B_d^0)$, $\Delta m(J/\psi)$ and $\Delta \tau$ mass for the Day 1 - ideal and Day 100 - ideal alignments.

<table>
<thead>
<tr>
<th></th>
<th>Day 1 - Ideal</th>
<th>Day 100 - Ideal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi$ mass [MeV]</td>
<td>0.2 ± 0.5</td>
<td>0.135 ± 0.2</td>
</tr>
<tr>
<td>$J/\psi$ mass resolution [MeV]</td>
<td>23.3 ± 0.7</td>
<td>12.3 ± 0.2</td>
</tr>
<tr>
<td>$B_d^0$ mass [MeV]</td>
<td>-0.7 ± 0.8</td>
<td>-0.06 ± 0.3</td>
</tr>
<tr>
<td>$B_d^0$ mass resolution [MeV]</td>
<td>32 ± 1</td>
<td>15.9 ± 0.3</td>
</tr>
<tr>
<td>$B_d^0$ lifetime [ps]</td>
<td>-1 ± 2</td>
<td>0. ± 0.9</td>
</tr>
<tr>
<td>$B_d^0$ lifetime resolution [ps]</td>
<td>68 ± 3</td>
<td>36 ± 2</td>
</tr>
</tbody>
</table>

the $J/\psi$ mass, (top left), the width of the Gaussian using Day 1 misalignments was $\sigma_1 = 23.3 \pm 0.7$ MeV. This width was reduced to $\sigma_{100} = 12.3 \pm 0.2$ MeV using the Day 100 misalignments, a factor of two that corresponds roughly to the relative size of the misalignments in the Day 1 and Day 100 cases.

For the $B_d^0$ mass, $\Delta m(B_d^0)$, (bottom left) width is 32 MeV for the Day 1 - ideal and is 16 MeV for the Day 100 - ideal sample. The $B_d^0$ lifetime width event-matched variable, $\Delta \tau(B_d^0)$ (top right), is 68 ps for the Day 1 - ideal and is 36 ps for the Day 100 - ideal sample.
A smaller mean value of $\Delta V_B(x, y)$ (bottom right) for Day 100 in comparison to Day 1 confirms that the alignment after 100 days is expected to be closer to the ideal alignment than after one day. The average shift in $B$-vertex position is $42.2 \pm 0.3 \text{ \(\mu\)m}$ for $\Delta V_B(x, y)_1$ and is reduced to $34.2 \pm 0.3 \text{ \(\mu\)m}$ for $\Delta V_B(x, y)_{100}$ compared to the ideal case.

7.5.2 The Impact of Global Systematic Misalignments on $B$-physics Observables using event-matching

A weak mode is defined as a global distortion in an otherwise perfectly aligned detector that leaves the global $\chi^2$ per degree of freedom fit unchanged. Thus, weak modes can be difficult to detect and correct by direct alignment methods. In the early data taking period weak modes effects are expected to be smaller than other effects, therefore they will have to be resolved for later using high precision measurements. A goal of this study is to identify which $B$-physics variables are most visibly affected, as well as identify the impact of each weak mode. The event-matching technique was applied to the global systematic deformation samples and the results are shown and discussed here.

Figure 7.10: The impact of the four global systematically deformed alignments on the event-matched $J/\psi$ mass.

Figure 7.10 shows a plot of $\Delta m(J/\psi)$ for all four systematic misalignments. The dotted line shows $\Delta m(J/\psi)_{\text{Large}}$ and the full line shows $\Delta m(J/\psi)_{\text{Small}}$. There is a marked difference in the impact of the large and small misalignments in all deformations except in the case of twist. This is because the impact of the twist deformation occurs at high pseudo-rapidity and these $J/\psi$ events are highly transverse. As such, changes in alignment in the end-caps such as the twist deformation are less visible here.
Table 7.8: The results of the Gaussian fit to the event-matched $J/\psi$ mass for the global systematically deformed alignments.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical</td>
<td>0.2 ± 0.2</td>
<td>14. ± 0.2</td>
<td>0.01 ± 0.06</td>
<td>4.16 ± 0.05</td>
</tr>
<tr>
<td>Curl</td>
<td>-0.2 ± 0.6</td>
<td>25.7 ± 0.9</td>
<td>0. ± 0.1</td>
<td>9.2 ± 0.1</td>
</tr>
<tr>
<td>Twist</td>
<td>0.1 ± 0.1</td>
<td>7.6 ± 0.1</td>
<td>0. ± 0.1</td>
<td>8.3 ± 0.2</td>
</tr>
<tr>
<td>Telescope</td>
<td>-0.3 ± 0.2</td>
<td>11.2 ± 0.2</td>
<td>-0.05 ± 0.03</td>
<td>2.03 ± 0.02</td>
</tr>
</tbody>
</table>

Figure 7.11: The results of the Gaussian fit to the event-matched $B^0_d$ mass for the global systematically deformed alignments.

Figure 7.11 shows a plot of $\Delta m(B^0_d)$ for all systematic misalignments, the results of the fit are shown in table 7.9. The dotted line shows $\Delta m(B^0_d)_{\text{large}}$ and the full line shows $\Delta m(B^0_d)_{\text{small}}$. The widths of $\Delta m(B^0_d)$ distributions are larger than those in the $J/\psi$ case, $\Delta m(J/\psi)$. The widths of these fits are roughly 25% to 35% larger than in the $J/\psi$ case. Similarly to the $J/\psi$ mass, the twist alignment has little effect on the difference between the deformed and the ideal $B^0_d$ mass.

The event-matched plots for the $B^0_d$ lifetime are shown in figure 7.12 and the results of the fits are shown in table 7.10. The largest effect on the $B^0_d$ lifetime is produced by the elliptical misalignments relative to the ideal case. In both the large and the small cases, the width of the elliptical distribution is larger than the width of the three other global
Table 7.9: The results of the Gaussian fit to the event-matched $B^0_d$ mass for the global systematically deformed alignments.

<table>
<thead>
<tr>
<th></th>
<th>$B^0_d$ Mass</th>
<th>Large - Ideal, [MeV]</th>
<th>Small - Ideal, [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical</td>
<td>Mean</td>
<td>$0.3 \pm 0.3$</td>
<td>$0.15 \pm 0.08$</td>
</tr>
<tr>
<td></td>
<td>Width</td>
<td>$17.2 \pm 0.3$</td>
<td>$5.15 \pm 0.09$</td>
</tr>
<tr>
<td>Curl</td>
<td>Mean</td>
<td>$2.1 \pm 0.1$</td>
<td>$0.4 \pm 0.2$</td>
</tr>
<tr>
<td></td>
<td>Width</td>
<td>$35.9 \pm 0.3$</td>
<td>$12.1 \pm 0.3$</td>
</tr>
<tr>
<td>Twist</td>
<td>Mean</td>
<td>$0.1 \pm 0.2$</td>
<td>$0.5 \pm 0.2$</td>
</tr>
<tr>
<td></td>
<td>Width</td>
<td>$9.0 \pm 0.2$</td>
<td>$9.9 \pm 0.3$</td>
</tr>
<tr>
<td>Telescope</td>
<td>Mean</td>
<td>$-0.4 \pm 0.4$</td>
<td>$-0.02 \pm 0.04$</td>
</tr>
<tr>
<td></td>
<td>Width</td>
<td>$17.5 \pm 0.7$</td>
<td>$2.68 \pm 0.05$</td>
</tr>
</tbody>
</table>

Figure 7.12: The impact of the four global systematically deformed alignments on the event-matched $B^0_d$ lifetime.

systematic misalignments. A dedicated study of systematic errors arising from elliptical misalignments should be a part of any $B^0_d$ lifetime measurements in ATLAS. For this reason, section 8.5 contains a detailed study of the impact of elliptical deformation on some $B$-physics observables.

Figure 7.13 shows the change in position of the $B$-vertex, as described in equation 7.18. In all four cases, the misalignments manifest themselves differently. In the case of the twist systematic deformation, no difference is observed here between the large twist alignment the small twist alignment. In comparison, the small elliptical and telescope modes approach the ideal case. The most pronounced effect on the $B$-vertex is produced by the large curl misalignment. Due to the introduction of the $1/R$ term in this specific
Table 7.10: The results of the Gaussian fit to the event-matched variables \( B^0_d \) lifetime for the global systematically deformed alignments.

<table>
<thead>
<tr>
<th>( B^0_d ) lifetime</th>
<th>Large - Ideal, [ps]</th>
<th>Small - Ideal, [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical</td>
<td>Mean: -0.9 ± 0.4</td>
<td>Mean: -0.3 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>Width: 31.7 ± 0.4</td>
<td>Width: 8.2 ± 0.1</td>
</tr>
<tr>
<td>Curl</td>
<td>Mean: -0.7 ± 0.2</td>
<td>Mean: -0.07 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>Width: 13.7 ± 0.2</td>
<td>Width: 4.23 ± 0.06</td>
</tr>
<tr>
<td>Twist</td>
<td>Mean: 0.00 ± 0.06</td>
<td>Mean: -0.17 ± 0.07</td>
</tr>
<tr>
<td></td>
<td>Width: 4.34 ± 0.06</td>
<td>Width: 4.71 ± 0.07</td>
</tr>
<tr>
<td>Telescope</td>
<td>Mean: 0.04 ± 0.07</td>
<td>Mean: 0.06 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>Width: 4.68 ± 0.07</td>
<td>Width: 2.07 ± 0.02</td>
</tr>
</tbody>
</table>

**Figure 7.13:** The impact of the four global systematically deformed alignments on the event-matched \( B^0_d \) vertex position.

The influence of these global systematic misalignments is expected to be most dangerous in the long term. For instance, in the case of the \( B^0_s \) double lifetime measurement, the lifetime resolution is expected to be 85 fs in reference [10]. The presence of a global
systematic misalignment could cause between 2 fs and 30 fs increase in the lifetime resolution.

In the worse case scenario, the presence of multiple weak modes and some distortion similar to the Day 1 deformation could potentially increase the uncertainty in the lifetime width beyond 30 fs. This would dramatically increase the statistics required to match the world’s best measurement. However, it should be remembered that Day 1 and the large global systematic deformations are unrealistically large misalignments.

7.6 Conclusions

Within the framework of this simulation, it was shown that the available statistics were not sufficient to identify the impact of weak modes using an unbinned maximum likelihood fit. It was also shown that the impact of material effects was much larger than the impact of alignment. On the other hand, this means that the impact of global systematic distortions in the inner detector on $B$-physics observable are small.

By applying an ‘event-matching’ method, it became possible to separately study consequences and implications of various types of misalignment. This event-matching allowed the direct comparison on an event by event basis between two alignments. It was found that the consequences of alignment on mass measurements are minimal, but significant broadening of the resolution may occur in the lifetime determinations.
Chapter 8

The impact of additional material and inner detector global deformations on an inclusive $\bar{b}b \rightarrow J/\psi X$ study

In addition to being a physics measurement in its own right, the inclusive lifetime measurement can be used to monitor unaccounted for materials and misalignment in the inner detector. Furthermore, it allows a test of the precision of an exclusive lifetime measurement with high statistics.

This chapter is divided into five sections. The first section, 8.1, is a description of the data used in this chapter. This section describes the data-sets, the production mechanism and the relative proportions of each type of $B$-hadron in the data. It also contains information about cross sections, kinematic cuts and reconstruction efficiencies.

The second section, 8.2, contains a description of the methods used to calculate the $B$-hadron inclusive lifetime. This includes the use of $J/\psi$ to estimate the $B$-hadron properties, a description of the fits used in this chapter and a study of the previously introduced event-matching techniques.

The final three sections of this chapter contain the results of the inclusive lifetime and mass fits under specific circumstances. Section 8.3 is a description of the impact of additional material on an otherwise ideally aligned inner detector on the inclusive lifetime measurement.
Section 8.4 looks at the impact of a small and a large random Gaussian displacement of inner detector module positions. The modules were moved in the x-y plane away from their position during simulation. They were moved according to a random Gaussian distribution. The width reflects the ‘as-built’ detector alignment accuracy, as determined by the ATLAS inner detector alignment group, based on cosmic ray data. These alignment conditions are named “Day 1” and “Day 100”, reflecting the alignment accuracy expected after the corresponding number of days of data taking.

The fifth and final analysis section of this chapter, 8.5, is a study of the impact of an elliptical global deformation in the inner detector on an inclusive $B$-physics data-set. In addition to the ideally aligned sample, this study made use of two misaligned samples. These two samples were made with a large and a small systematic elliptical global distortion in the inner detector.

### 8.1 Inclusive data preparation

The initial inclusive RAW data used for these studies was two RDO data-sets, each of which initially held 150K events*. To simulate the expected signal and background, two data sets were required for this study: an indirect $b\bar{b} \rightarrow J/\psi \rightarrow \mu^+\mu^-$ signal sample, and a direct $pp \rightarrow J/\psi \rightarrow \mu^+\mu^-$ background sample. These two data-sets were simulated using PYTHIA† that are available online to ATLAS members in reference [59].

When re-reconstructing data under a new alignment, the geometry-tag of the raw data is required to match the geometry-tag of the alignment conditions‡. At the time of this study, the alignment conditions had only been produced in two geometry-tags, but simulated $b\bar{b} \rightarrow J/\psi X$ and $pp \rightarrow J/\psi X$ was only available for one of these geometry-tags. This limited the data-sets available for this study to those produced using an older version of Athena§. A description of the alignment conditions is available to members of the ATLAS collaboration in [55]. Fortunately, the older detector description also contained additional material in the $\phi > 0$ half of the detector, allowing for extra material studies in addition to alignment studies. This extra material is described in more detail is section 8.3, below.

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*The RAW data sets were `misal1_mc12.017503.Pythia_directJpsimu6mu4.digit.RDO.v12000604_tid010077` and `misal1_mc12.017516.PythiaB_bbJpsimu6mu4X.digit.RDO.v12003106_tid004366`

†They were made using the PYTHIAB scripts `CSC.017503.Pythia_directJpsimu6mu4.py` and `CSC.017516.PythiaB_bbJpsimu6mu4X.py`

‡The misalignment conditions were produced with the detector geometry-tag `ATLAS-CSC-01-02-00`

§Athena 12.0.3 and 12.0.6
The raw RDO data were converted into Analysis Object Data (AOD) files using Athena and the inner detector reconstruction software\footnote{This package is called RecExCommon, and can be found at \cite{60}. The version used here was RecExCommon-00-10-65.}. The misaligned AOD files were then converted into Atlas Analysis Derived Physics Data (AAnaDPD) files using Athena and AAna.

After the production of the AAnaDPD, the data-sets were combined in the ratio of their expected cross sections. Table 8.1. shows the cross sections reported by the generator after the kinematic cuts are imposed. This table contains the cross sections reported by the generator after the kinematic cuts are imposed. The generator PYTHIA\footnote{PYTHIA 6.403\cite{40}, which used CTEQ6L\cite{61}.} had sharp cuts at $p_T^{J/\psi} > 6$ GeV, $p_T^{\mu} > 4$ GeV and $|\eta| < 2.5$. This means that 150K $pp \rightarrow J/\psi \rightarrow \mu^+\mu^-$ events equates to 7pb$^{-1}$ and 150K $b\bar{b} \rightarrow J/\psi \rightarrow \mu^+\mu^-$ events is equivalent to 13.5pb$^{-1}$. These events were simulated with a centre of mass collision energy of 14 TeV. These cross sections were produced by the ATLAS B-physics working group and were published in reference \cite{62}.

<table>
<thead>
<tr>
<th>$bb \rightarrow J/\psi \rightarrow \mu^+\mu^-$</th>
<th>$pp \rightarrow J/\psi \rightarrow \mu^+\mu^-$</th>
<th>Cross section</th>
<th>Integrated luminosity for 150K events</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.06 nb</td>
<td>21.75 nb</td>
<td>13.5 pb$^{-1}$</td>
<td>7 pb$^{-1}$</td>
</tr>
</tbody>
</table>

Although the initial sample contained 150k events, the efficiency of the data reconstruction on the grid, the reconstruction efficiency and vertex finding, the fraction of events that do not pass trigger, kinematic and mass cuts all need to taken into account. The kinematic cuts are the requirement that $p_T^{J/\psi} > 9$ GeV and the mass cuts are $|M_J - M_{PDG}^{J/\psi}| < 100$ MeV. Table 8.2 shows the number of events available and their fractional relevance for the ideal alignment. Although approximately equal numbers of $b\bar{b} \rightarrow J/\psi X$ and $pp \rightarrow J/\psi X$ events were produced, the lifetime study requires that they are combined here in the ratio of their cross sections of in table 8.1. This table reflects those cross sections. The difference between the last two rows show that 99.8% of events that passed the kinematic cuts had a correctly reconstructed $J/\psi$.

This $b\bar{b} \rightarrow J/\psi X$ sample was created to resemble a natural mixture of $b\bar{b} \rightarrow J/\psi X$ decays. Table 8.3 shows the relative frequency of each kind of $B$-hadron in the sample and their lifetimes. The simulated values of the mass and lifetime differ from the world’s best measurement and the world average measurement. The world’s best measurement is taken from the Particle Data Group in reference [1].
Table 8.2: The number of successfully reconstructed events for the $b\bar{b} \rightarrow J/\psi X$ and $pp \rightarrow J/\psi X$ samples, and their fraction of the inclusive sample.

<table>
<thead>
<tr>
<th></th>
<th>$b\bar{b} \rightarrow J/\psi X$</th>
<th>$pp \rightarrow J/\psi X$</th>
<th>Total Events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Events</td>
<td>Fraction</td>
<td>Events</td>
</tr>
<tr>
<td>Reconstructed</td>
<td>64425</td>
<td>0.337</td>
<td>126694</td>
</tr>
<tr>
<td>Passed kinematic cuts</td>
<td>54215</td>
<td>0.335</td>
<td>107797</td>
</tr>
<tr>
<td>True $J/\psi$ and passed cuts</td>
<td>54076</td>
<td>0.335</td>
<td>107576</td>
</tr>
</tbody>
</table>

Table 8.3: The relative frequency of each kind of $B$-hadron in the $b\bar{b} \rightarrow J/\psi X$ sample, the values that were used for their lifetime in the simulation and the best average lifetime measurement.

<table>
<thead>
<tr>
<th>Object</th>
<th>ID</th>
<th>Count</th>
<th>Fraction</th>
<th>Simulated lifetime, [ps]</th>
<th>Average measured lifetime, [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0$:</td>
<td>511</td>
<td>48697</td>
<td>0.417</td>
<td>1.56108</td>
<td>1.525 ± 0.009</td>
</tr>
<tr>
<td>$B^+$:</td>
<td>521</td>
<td>48811</td>
<td>0.418</td>
<td>1.54107</td>
<td>1.638 ± 0.011</td>
</tr>
<tr>
<td>$B^0_s$:</td>
<td>531</td>
<td>9860</td>
<td>0.085</td>
<td>1.61111</td>
<td>1.472 ± 0.025</td>
</tr>
<tr>
<td>$B^-$:</td>
<td>-511</td>
<td>731</td>
<td>6.26×10^{-3}</td>
<td>1.56108</td>
<td>1.525 ± 0.009</td>
</tr>
<tr>
<td>$\bar{B}^0_s$:</td>
<td>-531</td>
<td>162</td>
<td>1.39×10^{-3}</td>
<td>1.61111</td>
<td>1.472 ± 0.025</td>
</tr>
<tr>
<td>$B^0_c$:</td>
<td>-541</td>
<td>4</td>
<td>3.42×10^{-5}</td>
<td>0.50035</td>
<td>0.453 ± 0.041</td>
</tr>
<tr>
<td>$\Lambda_b^0$:</td>
<td>-5122</td>
<td>7554</td>
<td>0.065</td>
<td>1.14079</td>
<td>1.383 ± 0.048</td>
</tr>
<tr>
<td>$\Lambda_b^0$:</td>
<td>5122</td>
<td>78</td>
<td>6.68×10^{-4}</td>
<td>1.14079</td>
<td>1.383 ± 0.048</td>
</tr>
<tr>
<td>$\Xi_b^-$:</td>
<td>5132</td>
<td>4</td>
<td>3.42×10^{-5}</td>
<td>1.29089</td>
<td>1.42 ± 0.24</td>
</tr>
<tr>
<td>$\Xi_b^0$:</td>
<td>5232</td>
<td>3</td>
<td>2.57×10^{-5}</td>
<td>1.29089</td>
<td>1.42 ± 0.24</td>
</tr>
<tr>
<td>Overall</td>
<td>116643</td>
<td>1</td>
<td>1</td>
<td>1.52923</td>
<td>1.55902</td>
</tr>
</tbody>
</table>

8.2 Inclusive $B$-hadron lifetime measurement methods

There are three steps in measuring the average $B$-hadron lifetime. The first step is to measure the transverse momentum, $p_T^{J/\psi}$, and apparent decay-length, $\bar{L}_{xy}^{J/\psi}$, of the $J/\psi$. This is described in section 8.2.1. The second step is to apply corrections to the $\bar{L}_{xy}^{J/\psi}$ and $p_T^{J/\psi}$ to estimate the $B$-hadron properties. This is described in section 8.2.2. The third step of the measurement is to fit $B$-hadron lifetime and $J/\psi$ mass using an unbinned maximum likelihood fit and is described in section 8.2.3. The unbinned maximum likelihood methods were introduced in section 7.4.1. Once the measurements have been performed for each alignment, they can be compared between alignments using the event-matching technique described in section 7.5. The inclusive event-matching study comparing matching using Monte Carlo to matching using reconstructed data is shown in section 8.2.4.
8.2.1 The measurement of \( J/\psi \) properties

The first step is to measure the \( J/\psi \) transverse momentum, \( p_T^{J/\psi} \), and its apparent decay length, \( \tilde{L}_{xy}^{J/\psi} \). These measurements are used later to estimate the decay properties of the \( B \)-hadron. In order to measure \( \tilde{L}_{xy}^{J/\psi} \), a primary and a \( J/\psi \) vertex are required. The primary vertex position is determined during first pass processing at Tier 0, as described in section 4.7. The \( J/\psi \) vertex is made with the AAAn vertexing fitter. This fitter forms a vertex for all possible muon pairs of an event, and keeps the vertex if it has a \( \chi^2/DOF < 5 \).

It is important to note that the term “reconstructed” can have two distinct meanings. The first meaning of “reconstructed” is the reconstruction of the data-set, which was previously defined as the conversion from raw data to the analysis format. The second definition of “reconstructed” is the vertexing of tracks to produce a physics object such as a “reconstructed \( J/\psi \)”.

Once the primary and the reconstructed \( J/\psi \) vertices have been found, \( \vec{x} \) is defined as the vector that points from the primary vertex position, \( \vec{x}_{primary} \), to the secondary vertex, \( \vec{x}_{J/\psi} \):

\[
\vec{x} = \vec{x}_{primary} - \vec{x}_{J/\psi}
\]  
(8.1)

The apparent decay length in the x-y plane of the \( J/\psi \), \( \tilde{L}_{xy}^{J/\psi} \), is defined in equation 8.2.

\[
\tilde{L}_{xy}^{J/\psi} = \frac{\vec{x} \cdot p_T^{J/\psi}}{p_T^{J/\psi}}
\]  
(8.2)

This is called “apparent” transverse decay length because the \( J/\psi \) lifetime is extremely small. It can be assumed that it decays at the same position that it was created. However, when the \( J/\psi \) is a decay product of a long lived particle such as a \( B \)-hadron, it decays some distance away from the primary vertex. This makes it appear to have some transverse decay length and hence lifetime. This means that if the \( J/\psi \) is the decay product of a \( B \)-hadron, then the position of the \( J/\psi \) decay vertex gives, to a good approximation, the position of the \( B \)-hadron decay vertex.

The apparent lifetime of the \( J/\psi \), \( \tilde{\tau}_{J/\psi} \), can also be determined, but is not used for estimation of \( B \)-hadron properties:

\[
\tilde{\tau}_{J/\psi} = \tilde{L}_{xy}^{J/\psi} \frac{M^{J/\psi}}{p_T^{J/\psi}}
\]  
(8.3)
where \( M^{J/\psi} \) is the mass of the \( J/\psi \). The apparent lifetime and transverse decay length are denoted by an over-line tilde, \( \sim \).

Transverse decay length and lifetime are signed values, meaning that they can be negative. For instance, in the case of a particle with an invisibly short lifetime, such as a direct \( J/\psi \), a Gaussian distribution centred around zero would be observed for its \( \tilde{L}_{xy}^{J/\psi} \) and \( \tilde{\tau}^{J/\psi} \). The negative part of these distributions have little genuine lifetime content but can be used to infer the resolution for similar measurements. The exception is when there is a signing error for an object with a significant decay length.

### 8.2.2 The estimation of the \( B \)-hadron decay properties from the \( J/\psi \)

In an inclusive measurement using \( bb \rightarrow J/\psi X \), the \( B \)-hadrons can decay to many different final states. All these final states necessarily contain a \( J/\psi \) which decayed to two muons. The \( J/\psi \) is fully reconstructed but the other decay products of the \( B \)-hadron are not. As such, the \( B \)-hadron decay properties are not measured directly. Fortunately, the properties of the \( J/\psi \) can be used to estimate the \( B \)-hadron’s properties such as the vertex position \( \vec{x}_B \), the transverse decay length \( L_{xy}^B \), the transverse momentum \( p_T^B \), and the lifetime \( \tau^B \).

The position of the \( B \)-vertex can be assumed to be the position of the \( J/\psi \) vertex, due to the vanishingly small \( J/\psi \) lifetime.

\[
\vec{x}_B \simeq \vec{x}_{J/\psi} \tag{8.4}
\]

As such, the inclusive vertex displacement is:

\[
\vec{x}_{incl.} = \vec{x}_{J/\psi} - \vec{x}_{primary} \tag{8.5}
\]

where \( \vec{x}_{incl.} \) is the vector that points from the primary to the secondary vertex and \( \vec{x}_{J/\psi} \) is the \( J/\psi \) vertex position.

The assumption that the \( B \)-hadron shares a vertex with the \( J/\psi \) also implies that the apparent transverse decay length of the \( J/\psi \) matches the transverse decay length of the \( B \)-hadron.

\[
L_{xy}^B \simeq \tilde{L}_{xy}^{J/\psi} \tag{8.6}
\]

The \( B \)-hadron transverse momentum is estimated from the \( J/\psi \) transverse momentum.

The methods for this estimation are optimised using a well understood exclusive \( B \)-meson sample. In this case, the ratio of \( p_T^B \) to \( p_T^{J/\psi} \) was calculated using the sample of \( B_0^+ \rightarrow J/\psi K^{0*} \) described in chapters 7 and 6. This sample has no overlap with the sample
used for the inclusive lifetime measurements, and as such, it is statistically independent. In the future, this factor would be measured from the high statistics \( B^+ \rightarrow J/\psi K^+ \) sample. These techniques have been applied successfully in the CDF experiment in [63].

\[
\mathcal{B}_d \text{ and } J/\psi p_T \text{ ratio}
\]

**Figure 8.1:** The relationship between \( p_T^{J/\psi} \) on the x-axis and \( p_T^{B_d} \) on the y-axis for the sample \( B^0_d \rightarrow J/\psi K^{0*} \) using Monte Carlo truth. The scale can be interpreted such that larger squares mean higher frequency.

**Figure 8.2:** The fit to the relationship between \( p_T^{J/\psi} \) and \( p_T^{B_d} \).

Figure 8.1 is a two dimensional histogram showing the relationship between the \( p_T^B \) to the \( p_T^{J/\psi} \). The relationship between the Monte Carlo truth \( p_T^B \) and \( p_T^{J/\psi} \) was fitted to the following equations: (8.7 and 8.8). Figure 8.2 shows the fit in a binned histogram.

\[
p_T^{B_{incl.}} \approx p_T^{J/\psi} F\left(p_T^{J/\psi}\right) \tag{8.7}
\]
where

\[ F \left( p_T^{J/\psi} \right) = A + Be^{-Cp_T^{J/\psi}} \]  

(8.8)

This parametrised relationship is then used to estimate the \( p_T^B \) given some value of \( p_T^{J/\psi} \). It requires the assumption that the same relation between the \( B_d^0 \) and \( J/\psi \) applies for all \( B \)-hadrons.

In figure 8.2, the horizontal axis is the \( p_T^{J/\psi} \) and the vertical axis is the quotient of the \( p_B^{J/\psi} \) over the \( p_T^{J/\psi} \). Each bin contains the mean \( B \)-meson ratio per bin and this is what the fit is made to. Formula 8.7 was used to approximate to this ratio, and is known as the correction factor.

This correction requires the assumption that the \( B \)-hadron and the \( J/\psi \) have no difference in \( \phi \). It was noted during the MC study, that 95% of \( B \)-mesons and \( J/\psi \) mesons are within the boundaries \( |\Delta \phi| < 0.122 \) where \( \Delta \phi = \phi^B - \phi^{J/\psi} \). Thus, to a good approximation, \( p_T^{B_{incl.}} \approx p_T^{J/\psi} F \left( p_T^{J/\psi} \right) \) and \( L_{xy}^B = I_{xy}^{J/\psi} \). This fit could be improved using some multivariate analysis techniques. This is a task that is beyond the scope of this work.

The previous equations are combined together to form a method of estimating the \( B \)-hadron lifetime based on the properties of the reconstructed \( J/\psi \). This inclusive lifetime is written in equation 8.9.

\[ \tau_{incl.}^B = \frac{L_{xy}^{J/\psi}}{M_B} \frac{M_B}{F \left( p_T^{J/\psi} \right) p_T^{J/\psi}} \]  

(8.9)

where \( M_B \) is the known average \( B \)-hadron mass of the sample. For the simulated data-sets studied here, \( M_B \) is the average of the simulated value of the \( B \)-hadron masses, 5310.25 MeV. The uncertainty on the inclusive lifetime, \( \Delta \tau_{incl.}^B \), is constructed using the propagation of errors method.

\[ \Delta \tau_{incl.}^B = |\tau| \sqrt{ \left( \frac{\Delta L_{xy}^{J/\psi}}{L_{xy}^{J/\psi}} \right)^2 + \left( \frac{\Delta p_T^{B_{incl.}}}{p_T^{B_{incl.}}} \right)^2 } \]  

(8.10)

where \( \Delta L_{xy}^{J/\psi} \) is the uncertainty of \( L_{xy}^{J/\psi} \), and \( \Delta p_T^{B_{incl.}} \) is the uncertainty of \( B \)-meson \( p_T \).

### 8.2.3 Inclusive lifetime fit methods

Once the inclusive \( B \)-hadron lifetime has been obtained for each event, the fit is performed using the maximum likelihood method described in section 7.4.1. There are two type of \( J/\psi \) decays here, the direct \( pp \to J/\psi \to \mu^+\mu^- \) and the indirect \( bb \to J/\psi \to
\[ F_{bb}(t, \Gamma, \sigma) = \frac{\Gamma}{2} e^{-\left(\frac{(t+\frac{t^2}{\sigma^2})}{\Gamma} \right)} \left( 1 + Erf \left( \frac{t}{\sqrt{\frac{\sigma^2}{2}}} - \frac{\Gamma}{\sqrt{\frac{\sigma^2}{2}}} \right) \right) \] (8.11)

where \( t \) is the measured estimated lifetime of the \( B \)-hadron in this event, \( \Gamma \) is the inverse of the fitted lifetime, \( \sigma \) is the width of the convolution Gaussian, and where \( Erf(z) \) is the error function:

\[ Erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt \] (8.12)

During the fit, the width \( \sigma \) varies from candidate to candidate, according to the uncertainty of each case.

\[ \sigma = S\Delta\tau_{incl}^B \] (8.13)

This \( S \) value is a free parameter to be fitted by Minuit, whereas \( \Delta\tau_{incl}^B \) is a measured quantity, as described in equation 8.9. When quoted in tables or on the plots, the width \( \sigma \) is calculated by multiplying the fitted value of \( S \) with the average uncertainty on the fitted lifetime.

The \( pp \rightarrow J/\psi X \) (prompt) lifetime is fitted with a Gaussian:

\[ F_{pp}(t, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} \] (8.14)

where \( \sigma \) is the width of the Gaussian, \( \mu \) is the mean of the Gaussian. This \( \sigma \) is identical to that described in equation 8.13.

Equations 8.11 and 8.14 describe 99% of events. The remaining 1% of events are highly visible and at the edge of the negative scale, so the following fit was included.

\[ F_{extra}(t, \Gamma_{neg}, \sigma) = \frac{\Gamma_{neg}}{2} e^{-\left(\frac{(t+\frac{t^2}{\sigma^2})}{\Gamma_{neg}} \right)} \left( 1 - Erf \left( \frac{t}{\sqrt{\frac{\sigma^2}{2}}} + \frac{\Gamma_{neg} + \sigma}{\sqrt{\frac{\sigma^2}{2}}} \right) \right) \] (8.15)

Although there was no physical motivation for the inclusion of function 8.15 in \( L \), fits that were simply a combination of an exponential convoluted with a Gaussian added to a Gaussian were not successful at describing the negative tail of the lifetime distribution for high statistics. The origin of this tail is not fully understood and could be combinatorial, but it could originate from non-prompt decay, or from discrepancies in the primary and secondary vertex position. Furthermore, a similar method has been applied elsewhere in reference [63]. A cut on lifetime or \( L_{xy}^B \) would simplify the fit and could entirely remove the prompt and the extra sections of the lifetime. However, such a cut would
also remove the negative tail in the lifetime and remove the possibility of measuring the lifetime resolution.

![Lifetime, Ideal](image)

**Figure 8.3**: Inclusive lifetime plot for ideal alignment. Each of the elements of equation 8.16 are shown separately. The continuous (black) line is $F_{\text{total}}(t, \sigma)$, the large dashed (blue) line is $f_{bb}F_{bb}(t, \sigma)$, the dotted (green) line is $f_{pp}F_{pp}(t, \sigma)$, and the dash-dot-dot-dot (red) line is $(1 - f_{bb} - f_{pp})F_{\text{extra}}(t, \sigma)$.

Each of these three functions (equations 8.11, 8.14 and 8.15) are normalised to have a total area of unity. A requirement of maximum likelihood fits is that the function be normalised to one. They are combined together in the following way:

$$ F_{\text{total}}(t, \sigma) = f_{bb}F_{bb}(t, \Gamma, \sigma) + f_{pp}F_{pp}(t, \sigma) + (1 - f_{bb} - f_{pp})F_{\text{extra}}(t, \Gamma_{\text{neg}}, \sigma) \quad (8.16) $$

The weighting factors $f_{bb}$ and $f_{pp}$ are fitted by Minuit. They relate to the proportion of events that behave like a direct event, like an indirect event or like a negative tail event. The extra events are described with the fraction $(1 - f_{bb} - f_{pp})$. For the unbinned maximum likelihood inclusive lifetime fit, there are five simultaneously fitted parameters: $\Gamma$, $\sigma$, $\Gamma_{\text{neg}}$, $f_{bb}$ and $f_{pp}$.

There is also the possibility of studying the lifetime using two Gaussian functions to describe the prompt $J/\psi$ events and using two separate exponential functions convoluted two Gaussians. This reduces the dependence on the negative lifetime-like function. This method is illustrated in figure 8.4.

In the case of the $J/\psi$ mass measurement, there are no behavioural differences between the prompt $J/\psi$ mass and the non-prompt $J/\psi$ mass. As such, it can be modelled as a
**Figure 8.4:** Inclusive lifetime plot for ideal alignment using a double Gaussian. Each of the elements of equation 8.16 are shown separately. The continuous (black) line is $F_{total}(t, \sigma)$, the large dashed (blue) line is $f_{bb}F_{bb}(t, \sigma)$, the dotted (green) line is $(1 - f_{bb})F_{pp}(t, \sigma)$.

**Figure 8.5:** The inclusive $J/\psi$ (reconstructed - simulated) mass plot under the ideal alignment.

Gaussian, as described in the follow equation:

$$F_{mass}(t, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$  \hspace{1cm} (8.17)

where $\mu$ is the mean mass and $\sigma$ is the mass resolution. In the case of the $J/\psi$ mass measurements, it is often useful to study the difference between the reconstructed $J/\psi$ mass and the simulated mass. In this way, a mass shift due to alignment or extra material becomes more visible. The Gaussian fit to the $J/\psi$ mass is shown in figure 8.5.
8.2.4 The ‘event-matching’ techniques

The event-matching methods have been described in section 7.5. However, this section contains a study comparing the performance of the technique between when the event-matching is done using Monte Carlo to when it is done using reconstructed information.

In event-matching, the same event is matched-up between different alignments using its event and run number. It works this way both in simulated and recorded data. However, it is possible to have two distinct $J/\psi$’s in the same event. This means that further matching is required to uniquely identify each one. When using simulated data, the Monte Carlo true $p_T^{J/\psi}$ is used as a unique identifier to match $J/\psi$’s in an event, ensuring that the entire sample is correctly matched.

![Plot of $\Delta \eta$ (top) and $\Delta \phi$ (bottom).](image)

Figure 8.6: Plot of $\Delta \eta$ (top) and $\Delta \phi$ (bottom). In the absence of MC truth, $\Delta \eta$ and $\Delta \phi$ were selected to match events, due to the relatively small impact that alignment has on them. For both $\phi$ and $\eta$, the Gaussian describing the smearing due to alignment has a particularly tiny width of order 0.001.

In the real data, decays are to be matched using $\eta^{J/\psi}$ and $\phi^{J/\psi}$. $\eta^{J/\psi}$ and $\phi^{J/\psi}$ were selected as matching variables because of the relatively small impact that alignment deformations can have on them. The change in $\phi$ (or $\eta$) between the ideal and a misaligned
event can be described using a Gaussian fit. This fit of the difference is shown for the Day 1 and Day 100 samples in figure 8.6.

The Gaussian describing the smearing due to alignment has a particularly tiny width of order 0.001 radians in $\phi$ and $\eta$, as shown in figure 8.6. If an event in one alignment has similar $\eta^{J/\psi}$ and $\phi^{J/\psi}$ to an event in another sample, then they are matched together. Otherwise, if the difference in $\eta^{J/\psi}$ and $\phi^{J/\psi}$ is greater than some cut-off the two events do not match. This width of acceptance in $\phi$ or $\eta$ can be varied, which in turn affects the quality of the event-matching. Two measures of the quality of event-matching are the efficiency, $\varepsilon$, and the incorrectly matched fraction, $\omega$. The efficiency is:

$$\varepsilon = \frac{N_{\text{Correct}} + N_{\text{Incorrect}}}{N_{\text{Total}}}$$  

(8.18)

where $N_{\text{Total}} = N_{\text{Correct}} + N_{\text{Incorrect}} + N_{\text{unmatched}}$ and the incorrectly matched fraction is

$$\omega = \frac{N_{\text{Incorrect}}}{N_{\text{Correct}} + N_{\text{Incorrect}}}$$  

(8.19)

where $N_{\text{Correct}}$ and $N_{\text{Incorrect}}$ are the number of correctly matched and incorrectly matched events, $N_{\text{unmatched}}$ is the number of events that are not matched and $N_{\text{Total}}$ is the total number of events. Table 8.4 shows the efficiency, $\varepsilon$, and incorrectly matched fraction, $\omega$, of the event-matching in terms of the width of the allowed $\Delta \eta$ and $\Delta \phi$ region.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \phi, \Delta \eta &lt; 0.002$</th>
<th>$\Delta \phi, \Delta \eta &lt; 0.003$</th>
<th>$\Delta \phi, \Delta \eta &lt; 0.006$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Events</td>
<td>$\varepsilon$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$pp \rightarrow J/\psi X$</td>
<td>127007</td>
<td>0.702±0.001</td>
<td>0.115±0.001</td>
</tr>
<tr>
<td></td>
<td>Events</td>
<td>$\varepsilon$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$bb \rightarrow J/\psi X$</td>
<td>117266</td>
<td>0.717±0.001</td>
<td>0.0077±0.0003</td>
</tr>
<tr>
<td></td>
<td>Events</td>
<td>$\varepsilon$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$pp \rightarrow J/\psi X$</td>
<td>127007</td>
<td>0.9795±0.0004</td>
<td>0.1181±0.0009</td>
</tr>
<tr>
<td></td>
<td>Events</td>
<td>$\varepsilon$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$bb \rightarrow J/\psi X$</td>
<td>117266</td>
<td>0.9950±0.0002</td>
<td>0.0089±0.0003</td>
</tr>
</tbody>
</table>

Table 8.4 shows that the event-matching technique has higher efficiency and lower incorrect fraction with improvements in alignment. When looking at the same sample with
the same difference in $\phi$ and $\eta$, the small sample has higher efficiency and lower incorrect fraction than its large counterpart. As one would expect, with an increase in allowed $\phi$ and $\eta$ difference, both $\varepsilon$ and $\omega$ increase. The incorrectly matched fraction, $\omega$, is found to be of order ten times smaller in the $bb \to J/\psi X$ sample than in the $pp \to J/\psi X$ sample.

### 8.3 The impact of additional material on inclusive measurements

The majority of accepted ATLAS $B$-physics events contain relatively low transverse momentum tracks. As such, $B$-physics performance is dominated by material effects, while inner detector alignment and magnetic field effects are expected to be less influential.

The $bb \to J/\psi X$ and $pp \to J/\psi X$ samples described in section 8.1 were both made using a detector description which had the extra materials. A full description is available in [54]. Extra material or additional material is material added into the detector description during the simulation phase. It makes parts of inner detector fractionally thicker.

![Figure 8.7: Distorted material in the inner detector. The figures show the percentage increase in material for the $+z$ A side (left) and for the $-z$ C side (right).](image)

This extra material increases the radiation length of a sub-detector and increases the effects of multiple scattering. Additional materials were added to the $\phi > 0$ half of the detector but were not added symmetrically in $z$. The added materials are shown in figures 8.7 and 8.8. These figures were taken from reference [54]. The amount of material in the inner detector was increased up by up to 7% of one radiation length.

#### 8.3.0.1 Mass and lifetime fits for nominal material and extra material

It is possible to compare $B$-physics observables between the nominal material and extra material regions of the detector. The fits described in section 8.2.3 were performed for
both regions. Fortunately, this channel's large cross section should allow future analyses to look at the changes between relatively small slices in $\phi$ and $\eta$. However, it is not possible to study the effects of extra material using the event-matching methods. The fit to the $J/\psi$ mass is shown in figure 8.9 and the fit to the inclusive lifetime is shown in figure 8.10. The results of these fits are shown in table 8.5.
Figure 8.9 shows a fitted $J/\psi$ mass plot for both the nominal material and the extra material regions of the detector. The difference between the mean $J/\psi$ and the simulated $J/\psi$ mass is 0.2±0.2 MeV in the nominal material region. This is not a significant mass shift for these sample sizes. In the extra material region, difference between the mean $J/\psi$ and the simulated $J/\psi$ mass is -0.8±0.2 MeV. A shift in $J/\psi$ mass of 1.0±0.4 MeV is observed relative to the nominal material region. The mass width also changes with the additional material. The nominal material $J/\psi$ mass width is 49.2±0.2 MeV, and
it increases to 51.8±0.2 MeV in the additional material region. This is an increase of 2.6 ±0.4 MeV. The influence of nominal material is visible in this $J/\psi$ mass fit.

Figure 8.10 shows the unbinned maximum likelihood lifetime fit for each region. The results of these plot are shown in table 8.5. The additional material does not affect all of the fitted quantities. The fitted lifetime and the relative fractions of $b\bar{b} \rightarrow J/\psi X$ and $pp \rightarrow J/\psi$, $f_{bb}$ and $f_{pp}$, all remain unchanged with the extra materials. However, there is an increase in the resolution, the $S$-value and a decrease in $\Gamma_{neg}$. The lifetime resolution increases from 15.5±0.1 fs in the nominal material region to 16.2±0.1 fs in the additional material region. This is an increase of 0.7±0.2 fs. The fitted value of $\Gamma_{neg}$, the negative exponential tail, decreases by 0.9±0.3 with the addition of extra material. This is reflected in the fits shown in figure 8.10, where the fit in the nominal region contains fewer negative events.

### 8.4 The impact of the expected alignment after 1 day and 100 days on the inclusive $B$-physics observables

This section contains a study of the impact of a random displacement in the transverse plane of individual inner detector modules relative to their simulated positions on inclusive $B$-physics observables. The magnitude of the displacements is distributed according to a random Gaussian distribution with a mean at zero and a width of order ten microns. The width of the Gaussian distribution is expected to vary with the duration of the alignment period, the sub-detector and the pseudo-rapidity. This type of misalignment was produced after cosmic data studies had been made. The magnitude of the alignment constants are in agreement with the cosmic data module position uncertainty. These alignments were described previously in section 7.2.1, and the samples used are described in section 8.1. It should be mentioned that the alignment titles “Day 1” and “Day 100” are assuming a set energy and assured luminosity.

The two RAW data-sets were re-reconstructed under the ideal, Day 1 and Day 100 alignments. The production reconstruction is a process with multiple steps, each of which has a certain efficiency. These efficiencies are shown in table 8.6. The first step takes RAW data and converts into Analysis object data with some new alignment. This is shown in the “Aligned” column of table 8.6. The “Reconstructed” column of this table shows the number of events in which a $J/\psi$ was successfully reconstructed and passed kinematic cuts. The “True” column show the number of reconstructed events in which the reconstructed $J/\psi$ matches a true $J/\psi$. It should be noted that the number of events for the $b\bar{b} \rightarrow J/\psi X$ sample shown here are before any subsequent scaling due to cross section.
Table 8.6: Inclusive data production, reconstruction and truth finding efficiency for the ideal, Day 1 and Day 100 alignments.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Aligned</th>
<th>Reconstructed</th>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bb \to J/\psi X$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ideal</td>
<td>145750</td>
<td>121851</td>
<td>116643</td>
</tr>
<tr>
<td>Day 1</td>
<td>140150</td>
<td>116158</td>
<td>109848</td>
</tr>
<tr>
<td>Day 100</td>
<td>145750</td>
<td>121699</td>
<td>116469</td>
</tr>
<tr>
<td>$pp \to J/\psi X$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ideal</td>
<td>147550</td>
<td>128112</td>
<td>126694</td>
</tr>
<tr>
<td>Day 1</td>
<td>147550</td>
<td>126834</td>
<td>124079</td>
</tr>
<tr>
<td>Day 100</td>
<td>148250</td>
<td>128554</td>
<td>127148</td>
</tr>
</tbody>
</table>

The Day 1 $bb \to J/\psi X$ sample had a larger number of failed sub-jobs than the other samples. This happens occasionally when using the Grid, and resubmitting the sub-jobs consistently resulted in the same success rates. This is not a significant problem because approximately half the $bb \to J/\psi X$ events were not used, due to cross section restrictions described in section 8.1.

8.4.1 Unbinned maximum likelihood fit to mass and lifetime for inclusive Day 1 and Day 100 sample

Using the techniques described in section 8.2.3, a fit was performed to the inclusive lifetime and $J/\psi$ mass using Day 1 and Day 100 alignments. Figure 8.11 shows the plots for the inclusive lifetime for the Day 1 and Day 100 samples. Figure 8.12 shows the plots for the inclusive mass fits for these samples. The results of these fits and the ideal fit are shown in table 8.7.

Table 8.7: The results of the fits of the lifetime and mass for the ideal, Day 1 and Day 100 alignment inclusive samples.

<table>
<thead>
<tr>
<th></th>
<th>Ideal</th>
<th>Day 1</th>
<th>Day 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entries</td>
<td>162012</td>
<td>152106</td>
<td>161632</td>
</tr>
<tr>
<td>$M^{J/\psi} - M^{J/\psi}_{PDG}$, [MeV]</td>
<td>-0.3 ± 0.1</td>
<td>-0.5 ± 0.2</td>
<td>-0.4 ± 0.1</td>
</tr>
<tr>
<td>Mass resolution, [MeV]</td>
<td>50.5 ± 0.1</td>
<td>53.8 ± 0.2</td>
<td>51.4 ± 0.1</td>
</tr>
<tr>
<td>Lifetime, [ps]</td>
<td>1.51 ± 0.01</td>
<td>1.53 ± 0.01</td>
<td>1.53 ± 0.01</td>
</tr>
<tr>
<td>Resolution, [fs]</td>
<td>11.46 ± 0.05</td>
<td>21.28 ± 0.11</td>
<td>14.09 ± 0.06</td>
</tr>
<tr>
<td>$\Gamma_{reg}$[ps$^{-1}$]</td>
<td>1.7 ± 0.1</td>
<td>1.6 ± 0.2</td>
<td>1.6 ± 0.1</td>
</tr>
<tr>
<td>$f_{bb}$</td>
<td>0.341 ± 0.003</td>
<td>0.339 ± 0.006</td>
<td>0.339 ± 0.004</td>
</tr>
<tr>
<td>$f_{pp}$</td>
<td>0.65 ± 0.003</td>
<td>0.656 ± 0.006</td>
<td>0.654 ± 0.004</td>
</tr>
</tbody>
</table>
Figure 8.11: Inclusive lifetime fits for the Day 1 and Day 100 alignments samples.

The difference between the mean $J/\psi$ and the simulated $J/\psi$ mass is $-0.3 \pm 0.1$ MeV in the ideal alignment, $-0.5 \pm 0.2$ MeV in the Day 1 alignment and $-0.4 \pm 0.1$ in the Day 100 alignment. The shifts from the ideal alignment to the Day 1 and Day 100 samples are both smaller than the statistical uncertainty of the measurement. However, there are changes in the $J/\psi$ mass resolution. Relative to the ideal alignment, the $J/\psi$ mass resolution increases by $3.3 \pm 0.3$ MeV in the Day 1 alignment and by $0.9 \pm 0.2$ MeV in the Day 100 alignment.

The fitted inclusive lifetime in the Day 1 and Day 100 samples are not significantly shifted relative to the ideal case, However, there is an increase in the lifetime resolution of $9.8 \pm 0.2$ fs between ideal and the Day 1 alignments and an increase of $2.6 \pm 0.1$ fs between ideal and Day 100 alignments. The $\Gamma_{neg}$, $f_{bb}$ and $f_{pp}$ all remain unchanged between alignments.

No mass or lifetime shift was observed but the mass resolution and lifetime resolution both degrade with unaccounted shifts in the inner detector module positions. This decrease in the measurement clarity is exactly what is expected from shifts of inner
detector modules in a random direction. However, with improved alignment, as in the Day 100 sample, there is less degradation in the resolution.

8.4.2 Event-matching study for Day 1 and Day 100 alignments

The event-matching method described in section 7.5 was applied to the inclusive Day 1 and Day 100 inclusive samples. The plots shown in figures 8.13 and 8.14 were made by matching up specific events between each alignment. Figure 8.13 has plots of the differences in the $J/\psi$ mass and transverse momentum between the ideal and the Day 1 and Day 100 samples. Figure 8.14 shows the event-matched plots for the $J/\psi$ apparent transverse decay length and pseudo-lifetime. These event-matched difference plots were fitted to a Gaussian and the results of these fits are shown in table 8.8.

In table 8.8, there are no significant shifts between the ideal and misaligned samples in any of these observables. These Gaussian fits all have a mean that is compatible with no shift. As in the unbinned maximum likelihood fit of section 8.4, the $J/\psi$ mass is not
shifted away from zero. When comparing the width of the ideal - Day 1 to the width of the ideal - Day 100 fits, the large deformation sample (Day 1) are roughly twice as wide. The factor of two in the resolution was also present in the difference in alignment conditions, shown in table 7.1.

However, when comparing the event-matched difference fits for $J/\psi$ mass to the unbinned
maximum likelihood fits for $J/\psi$ mass, the value of the event-matching technique becomes clear. The increase in the unbinned width of the unbinned maximum likelihood fit between Day 1 and the ideal alignments was $3.3\pm0.3$ MeV. In the event-matched plots, this difference is much more visible at $22.3\pm0.1$ MeV. This is because it cancels out the effects of multiple scatterings.

8.5 The impact of an elliptical global geometrical distortion on an inclusive $b\bar{b} \rightarrow J/\psi X$ study

Global systematic deformations are shifts in the inner detector module positions where the modules are displaced according to some mathematical distribution relative to our understanding of their position. The shift moves the modules from the position in simulation to the position they occupy for re-reconstruction. In the elliptical deformation, the re-reconstruction inner detector module position were shifted from the nominal circular cylinder to form an elliptical cylinder.
As in the case of the Day 1 and Day 100 samples, the $b\bar{b} \to J/\psi X$ and $pp \to J/\psi X$ samples were separately reconstructed from the raw RDO data to AOD files under both the ideal alignment and under large and small elliptical deformation. The “large” sample describes how this deformation could look with an unrealistically large deformation. The “small” sample describes how the inner detector could look after the alignment procedure has removed most of misalignment.

In both elliptical samples, modules between $4 < |\phi| < 3$ are shifted away from the centre. Modules with $\frac{\pi}{4} > |\phi| > \frac{3\pi}{4}$ are shifted towards the centre. For the purpose of this study, the terms “stretched” and “squashed” are introduced to describe the regions shifted away and shifted towards the centre, respectively. Figure 8.15 shows four plots of the difference in module positions between the ideal case and the elliptical case.

The two RAW data-sets were reconstructed under the ideal, large elliptical and small elliptical alignments. The production reconstruction is a process with multiple steps,

**The conditions InDetSi_CSCMisaligned_PhiDeltaR.03.D2 and InDetTRT_CSC_PhiDeltaR.03.D2**

††The conditions InDetSi_CSCMisaligned_PhiDeltaR.03 and InDetTRT_CSCMisaligned_PhiDeltaR.03

†††These plots were produced with the CompareGeometries.C root macro by John Alison.
Table 8.9: Inclusive data production, reconstruction and truth finding efficiency for the ideal, large and small elliptical alignments.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Total</th>
<th>Reconstructed</th>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b \bar{b} \rightarrow J/\psi X )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ideal</td>
<td>145750</td>
<td>121851</td>
<td>116643</td>
</tr>
<tr>
<td>Large</td>
<td>140850</td>
<td>117638</td>
<td>112524</td>
</tr>
<tr>
<td>Small</td>
<td>145750</td>
<td>121850</td>
<td>116603</td>
</tr>
</tbody>
</table>

| \( pp \rightarrow J/\psi X \) |       |               |      |
| Ideal  | 147550| 128112        | 126694| 98.9% |
| Large  | 148250| 128687        | 127286| 98.9% |
| Small  | 145100| 125778        | 124323| 98.8% |

Each of which has a certain efficiency. These production, reconstruction and truth finding efficiencies are shown in table 8.9. It should be noted that the number of events for the \( b \bar{b} \rightarrow J/\psi X \) sample shown here are before any subsequent scaling due to cross section.

8.5.1 Unbinned maximum likelihood fit to mass and lifetime under elliptical deformations

This section contains a study of the influence of these elliptical global deformations on the unbinned maximum likelihood lifetime measurement and the \( J/\psi \) mass fit. The fits used here are described in section 8.2.3. The fit to the lifetime is shown in figure 8.16. The fit to the \( J/\psi \) mass is shown in figure 8.17. The results of these fits are compared to the results of the fit in an ideally aligned detector in table 8.10.

Table 8.10: Inclusive lifetime and mass fit results for the ideal, large elliptical and small elliptical alignments.

<table>
<thead>
<tr>
<th>Large</th>
<th>Ideal</th>
<th>Small</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entries</td>
<td>162012</td>
<td>163318</td>
<td>154955</td>
</tr>
<tr>
<td>Lifetime, [ps]</td>
<td>1.51 ± 0.01</td>
<td>1.51 ± 0.01</td>
<td>1.52 ± 0.01</td>
</tr>
<tr>
<td>Resolution, [fs]</td>
<td>11.12 ± 0.05</td>
<td>11.05 ± 0.05</td>
<td>12.21 ± 0.05</td>
</tr>
<tr>
<td>( \Gamma_{neg} ) [ps(^{-1})]</td>
<td>1.7 ± 0.1</td>
<td>1.8 ± 0.1</td>
<td>1.6 ± 0.1</td>
</tr>
<tr>
<td>( f_{bb} )</td>
<td>0.341 ± 0.002</td>
<td>0.342 ± 0.002</td>
<td>0.339 ± 0.002</td>
</tr>
<tr>
<td>( f_{pp} )</td>
<td>0.650 ± 0.002</td>
<td>0.649 ± 0.002</td>
<td>0.654 ± 0.002</td>
</tr>
<tr>
<td>( 1- f_{bb} - f_{pp} )</td>
<td>0.009 ± 0.004</td>
<td>0.009 ± 0.004</td>
<td>0.007 ± 0.004</td>
</tr>
<tr>
<td>( M_{J/\psi} - M_{PDG, J/\psi} ) [MeV]</td>
<td>-0.3 ± 0.1</td>
<td>-0.3 ± 0.1</td>
<td>-0.8 ± 0.2</td>
</tr>
<tr>
<td>Mass width, [MeV]</td>
<td>50.5 ± 0.1</td>
<td>50.8 ± 0.1</td>
<td>54.6 ± 0.2</td>
</tr>
</tbody>
</table>

Table 8.10 has the fitted parameters for the ideal, large elliptical and small elliptical alignments. There is no significant shift in lifetime between the three samples. However,
there is an increase in the lifetime resolution. The lifetime resolution is larger for the large case relative to the ideal by 1.1±0.1 fs. There are no significant changes in the relative proportions of $\Gamma_{neg}$ $f_{bb}$ and $f_{pp}$.

In terms of the mass fit, all three samples contain a small shift in $J/\psi$ mass relative to the simulated mass. It should be noted that the simulated mass is 3096.88 MeV and the world average mass is 30969.16 MeV [1], a difference of 0.72 MeV. Nevertheless, the large elliptical sample has a negative mass shift of -0.8±0.2 MeV, whereas the small elliptical and the ideal alignments have similar and small mass shifts of 0.3±0.1 MeV. The ideal sample $J/\psi$ mass fit has the smaller width, and the small elliptical sample has a slightly larger mass width by 0.3±0.2 MeV. The large elliptical sample has the largest mass width width, which is 4.1±0.3 MeV larger than the ideal case.
In order to maximise the visibility of the impact of an elliptical deformation, these fits can be performed with two $\phi$ regions to separate the effect in the stretched section from the effect in the squashed region. In the case of the elliptical distortion used here, the detector is divided into two regions. The two regions are:

- The stretched region with $\frac{\pi}{4} < |\phi| < \frac{3\pi}{4}$
- The squashed region with $|\phi| < \frac{\pi}{4}$ or $|\phi| > \frac{3\pi}{4}$.

Table 8.11 holds the results of the mass fits for each alignment and region. In the ideal case, there is no significant difference in mass between the stretched and the squashed sections of the detector. In the small elliptical case, the difference between the stretched and the squashed halves of the detector is $0.9 \pm 0.4$ MeV. In the large elliptical alignment, the difference between the two halves increases to $4.3 \pm 0.4$ MeV. As such, elliptical...
Table 8.11: Inclusive $J/\psi$ mass fit results for the ideal, large and small elliptical samples, in the entire, the stretched and squashed regions.

|            | no $\phi$ cut | $|\phi| < \frac{\pi}{4}$ or $|\phi| > \frac{5\pi}{4}$ (Squashed) | $\frac{\pi}{4} < |\phi| < \frac{3\pi}{4}$ (Stretched) |
|------------|---------------|--------------------------------------------------|-----------------------------------------------|
| **Ideal**  |               |                                                  |                                               |
| Entries    | 162012        | 80973                                           | 81039                                         |
| $M_{J/\psi} - M_{PDG}$, [MeV] | -0.3 ± 0.1    | -0.3 ± 0.2                                      | -0.2 ± 0.2                                    |
| Mass Width, [MeV] | 50.5 ± 0.1  | 50.6 ± 0.2                                      | 50.4 ± 0.2                                    |
| **Small**  |               |                                                  |                                               |
| Entries    | 163318        | 81608                                           | 81710                                         |
| $M_{J/\psi} - M_{PDG}$, [MeV] | -0.3 ± 0.1    | 0.2 ± 0.2                                       | -0.7 ± 0.2                                    |
| Mass Width, [MeV] | 50.8 ± 0.1  | 50.9 ± 0.2                                      | 50.6 ± 0.2                                    |
| **Large**  |               |                                                  |                                               |
| Entries    | 154955        | 77574                                           | 77381                                         |
| $M_{J/\psi} - M_{PDG}$, [MeV] | -0.8 ± 0.2    | 1.3 ± 0.2                                       | -3 ± 0.2                                      |
| Mass width, [MeV] | 54.6 ± 0.2  | 54.5 ± 0.2                                      | 54.7 ± 0.2                                    |

deformations cause a mass shift depending on whether the modules move towards or away from the centre of the detector. No significant differences were found in the lifetime fit between the stretched and the squashed regions of the detector. For this reason, the plots are not shown here.

8.5.2 Event-matching study under elliptical deformations

This section looks at the impact of the elliptical misalignments relative to the ideal case using the event-matching techniques described in section 7.5. In all plots, the solid line shows a Gaussian fit to the difference between the ideal and the small elliptically misaligned case on an event by event basis. The dotted line shows the same same fit to the difference between the ideal and the large elliptically misaligned case. The Gaussian is fitted to the central region of the event-matched difference plots.

Figure 8.18 contains the event-matched plots for the $J/\psi$ mass and $p_T$ and figure 8.19 are the event-matched plots for the $J/\psi$ apparent transverse decay length and pseudo-lifetime. These four plots show the impact of the large and small elliptical deformation relative to the same event in an ideal alignment.

The results of these fits are shown in table 8.12. This table shows that shifts are observed in the $J/\psi$ mass and $p_T$ for the large elliptically misaligned sample relative to the ideal sample. These shifts are not observed in the small elliptically misaligned sample relative to the ideal sample. These shifts are not observed in the apparent transverse decay...
length of the pseudo-lifetime. For all four observables shown here, the width of the Gaussian fit is dramatically smaller in small - ideal than in the large - ideal sample.

The asymmetry between the stretched and squashed regions can be studied by comparing the event-matched plots and fits between those regions. Figures 8.20, 8.21, 8.22 and 8.23 show plots of the $J/\psi$ mass, $p_T$, apparent transverse decay length and pseudo-lifetime divided into the two $\phi$-regions for squashed and stretched regions. The result of these
fits are shown in tables 8.13 and 8.14.

In table 8.13, a shift in the mean $J/\psi$ mass and $p_T$ is observed between the stretched and squashed regions in the event-matched plots. For both variables, there is a positive shift in the squashed region and a negative shift in the stretched region. In the squashed region, the $J/\psi$ mass is shifted up by $+0.50\pm0.01$ MeV in the small - ideal case and by $1.8\pm0.1$ MeV in the large - ideal case. In the stretched region, the $J/\psi$ mass is shifted
Figure 8.21: Event-matched plots for the $J/\psi$ $p_T$ in the left: squashed region, right: stretched region.

Table 8.13: Results of the Gaussian fits for the event-matched $J/\psi$ mass and $p_T$ under an elliptical deformation for the squashed region and the stretched region.

|                      | $|\phi| < \frac{\pi}{4}$ (Squashed) | $|\phi| > \frac{3\pi}{4}$ (Squashed) | $|\phi| < \frac{\pi}{4}$ (Stretched) |
|----------------------|-------------------------------------|--------------------------------------|-------------------------------------|
| **Small - Ideal**    |                                     |                                      |                                     |
| Entries              | 101978                              | 101665                               |                                     |
| $M^{J/\psi}$, [MeV ] | 0.50 ± 0.01                         | -0.50 ± 0.01                         |                                     |
| $M^{J/\psi}$ resolution, [MeV ] | 3.75 ± 0.01                        | 3.74 ± 0.01                          |                                     |
| $p_T^{J/\psi}$, [MeV ] | 2.50 ± 0.06                         | -2.60 ± 0.07                         |                                     |
| $p_T^{J/\psi}$ resolution, [MeV ] | 13.5 ± 0.07                        | 13.6 ± 0.08                          |                                     |
| **Large - Ideal**    |                                     |                                      |                                     |
| Entries              | 95968                                | 95588                                |                                     |
| $M^{J/\psi}$, [MeV ] | 1.8 ± 0.1                            | -2.5 ± 0.1                           |                                     |
| $M^{J/\psi}$ resolution, [MeV ] | 18.2 ± 0.1                         | 18.1 ± 0.1                           |                                     |
| $p_T^{J/\psi}$, [MeV ] | 9.3 ± 0.4                            | -12.0 ± 0.4                          |                                     |
| $p_T^{J/\psi}$ resolution, [MeV ] | 73.3 ± 0.6                           | 73.0 ± 0.5                           |                                     |

down by -0.50±0.01 MeV in the small - ideal case and by -2.5±0.1 MeV in the large - ideal case.

The shifts in the $J/\psi$ mass are reflected in the shifts in $p_T$. For both the small - ideal and large - ideal cases, the mean $p_T^{J/\psi}$ is negatively shifted in the stretched region and positively shifted in the squashed region. In the small - ideal case the shift is +2.50±0.06 MeV and -2.6 ± 0.06 MeV. In the large - ideal case, this shift is enhanced such that it is 9.3±0.4 MeV and -12.0 ± 0.4 MeV. These shifts in $p_T^{J/\psi}$ are visible in figure 8.21, especially in the small - ideal case.

The width of the event-matched $J/\psi$ mass and $p_T$ fits do not display any variation between the stretched and squashed regions. However, both the stretched and squashed
regions have a smaller width than the fit using the entire detector.

Figures 8.22 and 8.23 shows the event-matched plots of the apparent transverse decay length and the pseudo-lifetime of the $J/\psi$ in the stretched and squashed regions of the detector. The results of the Gaussian fits are shown in table 8.14.

Unlike in the case of the shifts observed for $J/\psi$ mass and $p_T$, the mean of the Gaussian fits of the apparent transverse decay length, $L_{xy}$, in table 8.14 are not coherently shifted between the stretched and the squashed regions. The mean is shifted upwards in the small - ideal stretched region, yet shifted downwards in the same region for the large - ideal case. The squashed region of the detector has a mean consistent with zero for the small - ideal sample. However, for the large - ideal sample, the mean is shifted upwards to $1.1\pm0.2\mu m$.

The mean in the Gaussian fits of the pseudo-lifetime, $\bar{\tau}^{J/\psi}$, in table 8.14 are consistent with no shift away from the ideal case. Similarly to the $J/\psi$ mass and $p_T$, the width of the event-matched $L_{xy}$ and $\bar{\tau}^{J/\psi}$ fits do not vary between the stretched and squashed regions. Furthermore, the fits in the stretched and squashed regions have a smaller width than the fit for the entire detector.
Table 8.14: Results of the Gaussian fits for the event-matched \( \tilde{L}_{xy}^J \) and \( \tilde{\tau}^J \) under an elliptical deformation for the squashed region and the stretched region.

|                | \( |\phi| < \frac{\pi}{4} \) or \( |\phi| > \frac{3\pi}{4} \) (Squashed) | \( \frac{\pi}{4} < |\phi| < \frac{3\pi}{4} \) (Stretched) |
|----------------|-------------------------------------------------------------------|-------------------------------------------------|
| Entries        | 79955                                                             | 80023                                           |
| \( \tilde{L}_{xy}^J \), [\( \mu m \)] | -0.02 \( \pm \) 0.05                                             | 0.20 \( \pm \) 0.05                             |
| \( \tilde{L}_{xy}^J \) resolution, [\( \mu m \)] | 13.7 \( \pm \) 0.05                                             | 13.7 \( \pm \) 0.05                             |
| \( \tilde{\tau}^J \), [fs]                | -0.09 \( \pm \) 0.04                                             | 0.05 \( \pm \) 0.04                             |
| \( \tilde{\tau}^J \) resolution, [fs]      | 8.62 \( \pm \) 0.04                                             | 8.6 \( \pm \) 0.04                              |

|                | \( |\phi| < \frac{\pi}{4} \) or \( |\phi| > \frac{3\pi}{4} \) (Squashed) | \( \frac{\pi}{4} < |\phi| < \frac{3\pi}{4} \) (Stretched) |
|----------------|-------------------------------------------------------------------|-------------------------------------------------|
| Entries        | 75379                                                             | 75344                                           |
| \( \tilde{L}_{xy}^J \), [\( \mu m \)] | 1.1 \( \pm \) 0.2                                                | -1.0 \( \pm \) 0.2                              |
| \( \tilde{L}_{xy}^J \) resolution, [\( \mu m \)] | 48.6 \( \pm \) 0.2                                              | 48.4 \( \pm \) 0.2                              |
| \( \tilde{\tau}^J \), [fs]                | 0.2 \( \pm \) 0.2                                                | -0.3 \( \pm \) 0.2                              |
| \( \tilde{\tau}^J \) resolution, [fs]      | 33 \( \pm \) 0.3                                                 | 32.7 \( \pm \) 0.3                              |

8.6 Inclusive \( b\bar{b} \rightarrow J/\psi X \) study conclusions

This study looked at the impact of alignment and extra material on an inclusive \( B \)-physics study. The study consisted of comparing the results of unbinned maximum likelihood fits to the inclusive lifetime and \( J/\psi \) mass between different alignments. The second method matched up the same event between alignments and compared the difference.

The region of the detector with additional material saw an increase of \( 2.6 \pm 0.4 \) MeV in the \( J/\psi \) mass width in the unbinned maximum likelihood fit relative to the nominal material region. Similarly, an increase of \( 0.7 \pm 0.2 \) fs in the inclusive lifetime resolution was observed between the nominal material region and the extra material region.

The second part of the study looked at the influence of a small random difference between the simulated inner detector positions and their positions during reconstructed. This kind of misalignment mimics the effects of alignment after 1 day and 100 days. It was found that the Day 1 and Day 100 alignments did not cause a shift in the \( B \)-physics observables. However, in the event-matched plots, it was found that width of the difference increased by approximately a factor of two between the Day 1 - ideal and the Day 100 - ideal event-matched plots for \( J/\psi \) mass, \( J/\psi p_T \), \( \tilde{L}_{xy}^J \) or \( \tilde{\tau}^J \). The widths of the changes between two alignments can be identified and quantified using the event-matched techniques.
The third part of this study looked at the influence of an elliptical deformation in the reconstruction geometry. This deformation moved some sections of the detector towards to beam-pipe and some sections away. It was shown that the regions that move towards the beam-pipe have increases in $J/\psi$ mass and $p_T$, whereas the regions that move away from the beam-pipe display decreased $J/\psi$ mass and $p_T$.

These studies have looked at the impact of alignment and extra materials on $B$-physics observables. The methods shown here will be applied to real data and will be instrumental to furthering the understanding of the inner detector in the $B$-physics working group.
Chapter 9

Conclusions

ATLAS is a general purpose detector designed to study the high energy proton-proton collisions produced at the Large Hadron Collider. The ATLAS collaboration’s B-physics working group is planning to measure Standard Model CP violating observables. The measurement of CP violation in the $B_s^0 \rightarrow J/\psi\phi$ channel at ATLAS is determined by three factors: statistics, lifetime resolution and flavour tagging. This thesis concentrated on the flavour tagging and lifetime measurements.

The jet charge flavour tagging method was optimised to increase its effectiveness. It was shown that each kind of $B$-meson requires its own jet charge flavour tagger optimisation. It is possible to perform this task with real data for the self tagging channels $B^+ \rightarrow J/\psi K^+$ and $B_d^0 \rightarrow J/\psi K^{0*}$. Calibrating with real data for the $B_s^0$ meson is more challenging as there is no readily available and clean self-tagging mode. In this case, the Monte Carlo dependent calibration will be used, but the agreement of the Monte Carlo with real data will be tested indirectly through the $B_d^0 \rightarrow J/\psi K^{0*}$ and $B^+ \rightarrow J/\psi K^+$ channels.

The lifetime of a $B$-hadron is calculated using the transverse momentum, mass and vertexing. The quality of the measurement of these variables is dependent on the tracking and the tracking is dependent on our knowledge of the presence of extra material, the inner detector alignment and the magnetic field.

The exclusive channel $B_d^0 \rightarrow J/\psi K^{0*}$ was studied with 100 pb$^{-1}$. The $J/\psi$ mass, $B_d^0$ mass and $B_d^0$ lifetime were fitted using unbinned maximum likelihood methods. It was found that the fits were not very sensitive to changes in alignment with these statistics. It became clear that other smearing effects, such as multiple scattering overwhelmed the effects of misalignment. Through the use of an event-matching technique, the same event can be compared between different alignments. This helped clarify the impact of
global systematic deformations on $B$-physics observables through completely removing multiple scattering effects. It was found that increased misalignment lead to increases in the mass and lifetime width, relative to the ideal. In some specific cases, the type of global systematic deformation can be identified using this method.

The third section was a study of the inclusive $B$-hadron lifetime. In this section, greater statistics were available. Using the Day 1 alignment, it was found that poor alignment can lead to a dramatic increase in the lifetime resolution. The ideal lifetime resolution increased from $11.46\pm0.05$ fs in the ideal case to $21.28\pm0.11$ fs under the Day 1 alignment. It was also found that the presence of extra material and misalignment can degrade the mass and lifetime resolution.

There are many possible ways that this work could be extended. The most obvious change would be to apply these techniques to real data. All of the studies outlined here play a part in the systematic studies for real data measurements, which are the ultimate motivation for the work. However, other improvements are possible in addition. For instance, in flavour tagging optimisation, it should be possible to check the influence of background samples. The method to maximise the quality factor could be changed from a parameter sweep to a multi-variate analysis. It should also be possible to include generator level studies to understand the impact of each momentum method.

The study of inner detector alignment on the exclusive $B_{d}^{0} \rightarrow J/\psi K^{0*}$ channel could be supplemented by a similar study comparing the impact of deformations in the magnetic field. Also, it could be helpful to apply the event-matching methods to study the impact of extra material on tracking in this exclusive channel.

In terms on the inclusive lifetime study, a significant next step would be to test the methods used to estimate the $B$-hadron momentum. The first step would be to raise the statistics of the Monte Carlo study. Later, it should be possible to tailor the estimation techniques to real data. Alternatively, the Monte Carlo study could be improved to take into account other $J/\psi$ properties, such as it’s pseudo-rapidity or the angle between the two muons.
Appendix A

Abbreviations

AOD Analysis Object Data
ATLAS A Toroidal LHC ApparatuS
BS Byte Stream
BSM Beyond the Standard Model
CKM Cabibbo Kobayashi Maskawa (matrix).
COM Centre of Mass.
COOL Conditions Database.
CSC Cathode Strip Chambers
DDM Distributed Data Management
DPD Derived Physics Data
DQ Don Quixote
DQA Data Quality Assessment
EDM Event Data Model
EF Event Filter
ESD Event Summary Data
HLT High Level Trigger
ID Inner Detector
L1 Level 1
**LAr**  Liquid Argon  
**LCG**  LHC Computing Grid  
**LEP**  Large Electron Positron Collider  
**LHC**  Large Hadron Collider  
**MDT**  Monitored Drift Tube  
**NbTi**  Niobium Titanium  
**NP**  New Physics  
**PSB**  Proton Synchrotron Booster  
**PS**  Proton Synchrotron  
**RDO**  Raw Data Object  
**RF**  Radio Frequency (Cavity)  
**ROI**  Region Of Interest  
**ROS**  Read Out System  
**RPC**  Resistive Plate Chambers  
**SCT**  Semi-Conductor Tracker  
**SPS**  Super Proton Synchrotron  
**TAG**  TAG, (data type) not an acronym.  
**TDAQ**  Trigger and Data Acquisition  
**TGC**  Thin Gap Chambers  
**TRT**  Transition Radiation Tracker
Appendix B

True and measured asymmetry

A system is considered in which a tagger is applied to some events which can be of type $a$ or type $b$. The tagger can apply an $a$ tag or a $b$ tag to these events, but can also tag them incorrectly. This derivation was also shown in reference [64]. The measured asymmetry is defined as

$$A_{\text{meas}} = \frac{N_a - N_b}{N_a + N_b}$$  \hspace{1cm} (B.1)

where $N_a$ and $N_b$ are the number of events tagged as $a$ and $b$, respectively. Similarly, the true asymmetry is defined as

$$A_{\text{true}} = \frac{N_a^0 - N_b^0}{N_a^0 + N_b^0}$$  \hspace{1cm} (B.2)

where $N_a^0$ and $N_b^0$ are the true number of events that are $a$ and $b$, respectively. The efficiency, $\varepsilon$, of the tagging is

$$\varepsilon = \frac{N_a + N_b}{N_a^0 + N_b^0}$$  \hspace{1cm} (B.3)

The number of events that are tagged as $a$ or $b$ can be written in terms of the true number of events:

$$N_a = \varepsilon N_a^0 P_R + \varepsilon N_b^0 P_W$$

$$N_b = \varepsilon N_a^0 P_W + \varepsilon N_b^0 P_R$$  \hspace{1cm} (B.4)

where $P_R$ is the probability that the tagger is right and $P_W$ is the probability that the tagger is wrong. Equations B.4 assume the tagging efficiency and the probability of being correct or wrong are identical for $a$ and $b$ type events.
As this case does not consider untagged events. An event is either correct or wrong, as such:

\[ P_R + P_W = 1 \]  \hspace{1cm} (B.5)

\( P_R \) and \( P_W \) relate to the dilution defined in equation 6.3 in the following way:

\[ P_R - P_W = D \]  \hspace{1cm} (B.6)

Equations B.4 can be substituted into equation B.1:

\[
A_{\text{meas}} = \frac{\varepsilon N^0_a P_R + \varepsilon N^0_b P_W - \varepsilon N^0_b P_W - \varepsilon N^0_a P_R}{\varepsilon N^0_a P_R + \varepsilon N^0_b P_W + \varepsilon N^0_a P_W + \varepsilon N^0_b P_R}
\]

\[ = \frac{(N^0_a - N^0_b) P_R - (N^0_a - N^0_b) P_W}{(N^0_a + N^0_b) P_R + (N^0_a + N^0_b) P_W}
\]

\[ = \frac{(N^0_a - N^0_b) (P_R - P_W)}{(N^0_a + N^0_b) (P_R + P_W)} \]

Substituting in the identities of B.5 and B.6, the previous equality becomes:

\[ A_{\text{meas}} = \frac{(N^0_a - N^0_b)}{(N^0_a + N^0_b)} D = A_{\text{true}} D \]  \hspace{1cm} (B.7)

or

\[ A_{\text{true}} = \frac{1}{D} A_{\text{meas}} \]  \hspace{1cm} (B.8)
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