AXION DEPENDING ON THE HIGGS SECTOR IN

\[ \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1) \]

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ABSTRACT

Using Higgses with quantum numbers of fermion bilinears we discuss the axion in four different Higgs sectors in \( \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1) \). Three of the cases are similar to the "standard axion" in the Salam-Weinberg model and in one case the axion can be made invisible.
Some years ago the Peccei-Quinn (PQ) symmetry was invented to solve the strong CP violation problem in QCD\(^1\). This leads to the prediction of a very light pseudoscalar meson called the axion\(^2\). In the Salam-Weinberg (SW) model of weak and electromagnetic interactions, the properties of the so-called "standard axion" have been widely discussed in the literature. However, it has turned out that the experimental situation is not in favour of the "standard axion"\(^3\) and additional mechanisms had to be invented, such as the "invisible axion"\(^4\) and the "grand unified axion"\(^5\), to avoid contradiction with experiment. Nevertheless, it seems useful to us to consider the axion also in the context of \(SU(2)_L \times SU(2)_R \times U(1)\) left-right symmetric (LRS) models, because its properties depend very much on the Higgs sector of a theory. We choose the Higgses to have the quantum numbers of the fermion bilinears of the theory, not only because they can possibly be replaced by a dynamical symmetry breaking mechanism, but also because one can easily give a heavy mass to the nearly right-handed gauge bosons \(W_2, Z_2\) by a Higgs triplet to obtain the same low-energy phenomenology as in the SW model and there is a nice connection between the smallness of the mass of the light neutrinos and the heavy mass of \(W_2\) and \(Z_2\)\(^6\). On the other hand, there exist left-right symmetric models with horizontal symmetries without couplings of the charge conjugate Higgses\(^7\). The introduction of the PQ symmetry excludes these couplings automatically\(^8\) as in the SW model and leads to a real determinant of the mass matrix of the quarks, whereas otherwise the imaginary part of this determinant is not naturally small.

Our hope was to suppress the coupling of the axion to matter by an axion decay constant \(f_a = O(M_{W_2})\), which would happen if the axion consisted mainly of the neutral member of the Higgs triplet giving the big vacuum expectation value, but unfortunately this cannot be achieved by only one triplet coupled to right-handed leptons. So it turns out that the situation concerning the axion in LRS models is very similar to the SW model, except that one uses more triplet Higgses, which disturbs the connection between very light neutrinos and heavy \(W_2\).

According to our assumption, we have the Yukawa interaction\(^6\)

\[
-\mathcal{L}_Y = \sum_i \bar{\psi}_i \mathcal{G}_i \phi_i \psi_R + \sum_j \bar{\psi}_L \mathcal{P}_j \phi_j q_R + \\
+i \left( \psi_L^T \tilde{c} \tau_2 \mathcal{G}_L \psi_L + \psi_R^T \tilde{c} \tau_2 \mathcal{G}_R \psi_R \right) + h.c.
\]
\[ \begin{align*}
q &= \begin{pmatrix} q_1 \\ \vdots \\ q_{N_u} \end{pmatrix},
q_i &= \begin{pmatrix} u_i \\ \vdots \\ d_i \end{pmatrix},
\psi &= \begin{pmatrix} l_1 \\ \vdots \\ l_{N_d} \end{pmatrix},
L_i &= \begin{pmatrix} \nu_i \\ \vdots \\ e_i \end{pmatrix} \\
\phi_i &= \begin{pmatrix} \phi_{1i}^* \\ \phi_{12}^* \\ \phi_{21}^* \\ \phi_{22}^* \end{pmatrix},
\Delta_{L,R} &= \begin{pmatrix} \frac{1}{\sqrt{2}} & \delta^+ & \delta'^+ \\
\delta^- & \frac{1}{\sqrt{2}} & \delta'^+ \\
\delta'^+ & -\frac{1}{\sqrt{2}} & \delta^+ \end{pmatrix}_{L,R}
\end{align*} \] cont. 

where \( u(d) \) up(down) are quarks, e charged leptons, \( \nu \) neutrinos, \( N_G = \) number of generations, \( C \) = charge conjugation matrix and \( \tau_a \) are Pauli matrices. We have no \( \Delta \) corresponding to \( q \) because of \( SU(3)_c \) invariance. The \( \phi \)'s and \( \Delta_{L,R} \) transform under \( SU(2)_L \times SU(2)_R \times U(1) \) according to \((1/2,1/2,0), (1,0,2), (0,1,2)\), respectively. We have the following vacuum expectation values:

\[ \langle \phi_i \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \nu_i \\ 0 \\ 0 \end{pmatrix}, \langle \Delta_{L,R} \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ U_{L,R} \end{pmatrix} \] (2)

The masses of the vector bosons are (neglecting terms of order \( V^*/|U_R|^2 \))

\[ \begin{align*}
M_{W_1}^2 &= \frac{g^2}{4} (V^2 + 2 |U_L|^2) \\
M_{W_2}^2 &= \frac{g^2}{4} (V^2 + 2 |U_R|^2) \\
M_{Z_1}^2 &= \frac{g^2}{4} (V^2 + 4 |U_L|^2) / \cos^2 \theta_w \\
M_{Z_2}^2 &= g^2 |U_R|^2 \frac{\cos^2 \theta_w}{\cos^2 \theta_w} + \frac{1}{4} \frac{V^2 \cos^2 \theta_w}{\cos^2 \theta_w} \\
V^2 &= \sum_i (|\nu_i|^2 + 1 \omega_i)^2, \quad |U_R|^2 \gg V^2
\end{align*} \] (3)

and

\[ \sin \theta_w = \frac{g'}{\sqrt{g^2 + 2 g'^2}} \]

(4)
\( s \) is the SU(2)_L \times SU(2)_R coupling constant and \( g' \) belongs to U(1). To preserve the SW relation \( M^2_{W_1} = M^2_{Z_1} \cos^2 \theta_W \), we have to assume that \( |U_L|^2 \ll V^2 \). Therefore we neglect \( U_L \) in the following because it gives only a small contribution to the axion, and we call \( U_R = U \). We now consider four different Higgs sectors and look for the properties of the corresponding axions.

**Case 1.** This is the simplest one. To reproduce the known mass ratios of the current current quarks\(^9\) we have to introduce at least two \( \phi \)'s\(^8\) because the charge conjugate \( \phi \)'s do not couple. So we arrive at \( \phi_1, \phi_2, \Delta_L, \Delta_R \). The transformation properties under PQ symmetry are

\[
\begin{align*}
\psi_L, q_L &\rightarrow e^{i \alpha} \psi_L, q_L; \\
\psi_R, q_R &\rightarrow e^{-i \alpha} \psi_R, q_R \\
\phi_i &\rightarrow e^{i \alpha} \phi_i \\
\Delta_L &\rightarrow e^{-i \alpha} \Delta_L; \\
\Delta_R &\rightarrow e^{i \alpha} \Delta_R
\end{align*}
\]  

(5)

**Case 2.** Here we want to decouple the leptons from PQ symmetry and introduce two different kinds of \( \phi \)'s; \( \phi_i^l \) (\( i=1,2 \)) couples only to the leptons and \( \phi_i^q \) (\( i=1,2 \)) only to the quarks. PQ symmetry:

\[
\begin{align*}
q_L &\rightarrow e^{i \alpha} q_L; \\
q_R &\rightarrow e^{-i \alpha} q_R \\
\phi_i^q &\rightarrow e^{i \alpha} \phi_i^q
\end{align*}
\]  

(6)

\( \psi, \Delta, \phi^l \) do not transform under PQ symmetry.

**Case 3.** The fermion masses and \( M_{W_1}, M_{Z_1} \) could arise from two different scales. This is achieved by simply introducing an additional \( \Delta_W^{\nu} \) for the gauge bosons\(^6\), which does not couple to fermions and does not transform under PQ symmetry. The other part of the Higgs sector and PQ symmetry is as in case 1.

**Case 4.** In this case we want the axion to couple very weakly to matter. This is done by introducing an additional \( \Delta_{L,R}^{\nu} \) which does not couple to the fermions and does not transform under PQ symmetry, but which has a vacuum expectation value \( \langle \delta_{WR}^{\nu} \rangle = \mathcal{O}(1/U) \).
In the neutral Higgs sector there are three Goldstone bosons. Because of the Higgs mechanism, two of them are eaten by $Z_1$ and $Z_2$ and the remaining one is the axion. We now discuss the different cases.

Case 1). We obtain the axion (see Appendix)

$$\begin{align*}
A &= \sqrt{2} \Im \left\{ \frac{x}{\sqrt{V}} \left( V_1^* \phi_1^* + V_2^* \phi_2^* \right) + \frac{1}{x} \left( V_1^* \phi_1 + V_2^* \phi_2 \right) \right\} \\
\phi' &= \phi - \langle \phi \rangle_0, \quad x = \sqrt{\frac{|V_1|^2 + |V_2|^2}{|V_1|^2 + |V_2|^2}}
\end{align*}$$

(7)

It has no $\delta^e_R$ component because the corresponding one is eaten by the Higgs mechanism. $x$ is not restricted by the fermion masses. To construct the axion current, which has to contain the part $\delta^IA$, we have three currents at our disposal: $j^H_L$, coming from the PQ symmetry and $j^H_L$, $j^H_R$, the currents of the third components of left- and right-handed weak isospin. Because of the relation

$$j_{em}^\mu = j_{3L}^\mu + j_{3R}^\mu + \frac{1}{2} j^\mu_Y$$

(8)

and the fact that the electromagnetic current $j_{em}^H$ does not contain neutral Higgses, the hypercharge current $j_Y^H$ does not contribute to the construction of the linear combination of $\delta^IA$. We obtain the result

$$\begin{align*}
j_{FF}^\mu &= V \partial^\mu A + \frac{x}{2} \left( \bar{d} y^\mu y^*_5 \nu + \bar{u} y^\mu y^*_5 u \right)
+ \frac{1}{2x} \left( \bar{e} y^\mu y^*_5 e + \bar{d} y^\mu y^*_5 d \right) - \frac{x}{4} \left( \bar{\nu} \gamma^\mu \tau_3 \nu + \bar{q} \gamma^\mu \tau_3 q \right)
\end{align*}$$

(9)

$\nu = \nu_L + \nu_R$ (is not a mass eigenstate); $e, u, d$ can be chosen to be mass eigenstates. We have neglected all quadratic boson terms and all terms containing gauge fields. The current (9) is conserved except for the strong interaction anomaly. Still

$$V = f_a = \left( \frac{1}{2} G_F \right)^{\frac{1}{2}}$$

(10)

remains valid as an approximate relation, whereas it is exact in the SW model. $f_a$ is the analogue of the pion decay constant $f_\pi$. Because of (10) and the fact that the axial part of (9) is exactly the same as in the "standard axion" case except for its neutrino term, we have the same properties of the axion here as well, except for its coupling to neutrinos. From $f_Y$ we can derive
\[ \mathcal{L}_{\nu_{V}A} = A \bar{N} \left( K_1 i \tilde{\nu}_5^C + K_2 \right) N \]

\[ K = \frac{X}{V} \ U^T \begin{pmatrix} 0 & \frac{1}{2} M^*_{LR} \\ \frac{1}{2} M^*_{LR} & 0 \end{pmatrix} U = K_1 + i K_2, \quad M_{LR} = \sum_{i=1}^{2} \frac{V_i}{\sqrt{2}} G_i \]

\[ X_i (i=1,2) \] are real and symmetric. \( N \) is the vector of the \( 2N \) Majorana neutrinos and \( U \) is the unitary diagonalization matrix of the symmetric Majorana mass matrix

\[ M_{\nu} = \begin{pmatrix} 0 & M^*_{LR} \\ M^*_{LR} & -V^T U^* G^* R \end{pmatrix} \]

\[ \mathcal{L}_{\nu_{\text{mass}}} = \frac{1}{2} \begin{pmatrix} \nu_L^T & \nu_R^C^T \end{pmatrix} M_{\nu} \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} + \text{h.c.}, \quad \nu_R^C = C \nu_R^T \]

\[ \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = U \begin{pmatrix} 1 - \frac{X}{2} \\ \frac{X}{2} \end{pmatrix} N \]

For the decay \( A \to N_k N_{k^*} \) we obtain

\[ \Gamma_{kl} = \frac{1}{\pi m_A^2} W(m_A^2, m_k^2, m_{k^*}^2) \left\{ \frac{1}{2}(m_A^2 - m_k^2 - m_{k^*}^2) f + m_k m_{k^*} g \right\} \]

\[ W(a, b, c) = \left[ a^2 + b^2 + c^2 - 2(ab + bc + ca) \right]^{\frac{1}{2}} \]

\[ f = (K_{1kl})^2 + (K_{2kl})^2, \quad g = (K_{1kl})^2 - (K_{2kl})^2 \]

\( m_A \) = axion mass, \( m_k, m_{k^*} \) = neutrino masses. Because the mass of the electron neutrino is at most of the order of \( 50 \text{ eV} \), we assume that \( m_{\nu_e} \ll m_A \) and estimate

\[ \Gamma_{\nu_e \nu_e} \approx \frac{m_A^2}{2\pi} \frac{x^2}{2V^2} m_{\nu_e}^2 \]
which can be of the order of $\Gamma(A+\gamma\gamma)$ taking $m_{\nu_e} \approx 10$ eV without violating the bound $m_A \geq 0.2$ MeV \(^{11}\)) coming from stellar evolution. So the lifetime of the axion can be shorter than in the standard case, but not short enough to be invisible in beam dump experiments. Anyhow, it is ruled out like the standard axion. Introducing some additional $\Delta$'s with the same types of couplings does not change anything in our conclusions.

Case 2). Because of the coupling matrix $\mathcal{G}_1$, it is possibly sufficient to take only one $\phi_1^l$ to adjust the lepton masses. However, this question does not affect the axion.

\[
A = \sqrt{2} \text{ Im} \frac{\sum_l \left( -\frac{W^2}{V^2} (V^{*}_{l1} \phi_{3l}^* - W^{*}_{l1} \phi_{32}^*) + \left( 1 - \frac{W^2}{V^2} \right) V_{l1}^* \phi_{21}^* + \left( 1 + \frac{W^2}{V^2} \right) W_{l1}^* \phi_{22}^* \right)^2}{\sqrt{\sum_l \left( \frac{W^2}{V^2} \right)^2 (|V_{l1}|^2 + |W_{l1}|^2) + \left( 1 - \frac{W^2}{V^2} \right)^2 |V_{l1}|^2 + \left( 1 + \frac{W^2}{V^2} \right)^2 |W_{l1}|^2}} \quad (15)
\]

\[
W_q^2 = |V_q|^2 + |V_q|^2 - |W_q|^2 - |W_q|^2 \quad \text{(can be positive or negative)}
\]

The indices $l$, $q$ refer to the lepton and quark, respectively. There is no essential difference from Case 1). Only the coupling strengths to the different fermions have changed a little. If $|V_q|, |W_q| \ll |V^3|, |V^3|$, the couplings to leptons would be suppressed, but the axion would couple to the quarks as in the standard case. If $W_q^2 = 0$, it does not couple to leptons at all, but now the coupling strengths to up and down quarks are equal and a factor $V^2/V_q^2$ larger than before, which makes the situation worse ($V^2/V_q^2$ should not be much larger than two, comparing lepton and quark masses). All other choices of $W_q^2$ bring Case 2) nearer to 1), so it is also excluded.

Case 3). In this case we obtain

\[
A = \sqrt{2} \text{ Im} \frac{-\frac{W^2}{V^2} (V_{l1}^* \phi_{3l}^* - W_{l1}^* \phi_{32}^*) + \left( 1 - \frac{W^2}{V^2} \right) (V_{l1}^* \phi_{21}^* + V_{l1}^* \phi_{22}^*) + \left( 1 + \frac{W^2}{V^2} \right) (V_{l1}^* \phi_{21}^* + V_{l1}^* \phi_{22}^*)}{\sqrt{\left( \frac{W^2}{V^2} \right)^2 (|V_{l1}|^2 + |W_{l1}|^2) + \left( 1 - \frac{W^2}{V^2} \right)^2 (|V_{l1}|^2 + |V_{l1}|^2) + \left( 1 + \frac{W^2}{V^2} \right)^2 (|V_{l1}|^2 + |V_{l1}|^2)}} \quad (16)
\]

\[
W_f^2 = |V_f|^2 + |V_f|^2 - |W_f|^2 - |W_f|^2 \quad \text{(can be positive or negative)}
\]
Let us assume that $|v_W|^2 + |w_W|^2 \gg v_f^2 = |v_1|^2 + |v_2|^2 + |w_1|^2 + |w_2|^2$. This might be the case because the fermion masses are much lighter than $W_1$, $Z_1$ with the possible exception of the top quark. In this limiting case we get

$$A = \frac{1}{V_f^2} \Sigma_i (v_i^* \phi_i + w_i^* \phi_i^{''})$$

and

$$j'_{\mu} = j_{\mu}$$

This corresponds to Case 1) with $x = 1$, $f_a = V_f$, which means that the axion is a factor $V/V_f$ heavier than the "standard axion". This can considerably reduce its lifetime. It mixes with $\pi^0$ and $\eta$ with mixing angles$^{12}$

$$\phi_{\pi} = -\frac{\pi}{V_f^2} N_G \frac{m_s (m_d - m_u)}{m_u m_d + m_d m_s + m_s m_u}$$

$$\phi_{\eta} = -\frac{1}{V_f^2} \frac{\pi}{V_f^2} N_G \frac{(m_u + m_d) m_s - 2 m_u m_d}{m_u m_d + m_d m_s + m_s m_u}$$

The $m$'s are the quark masses. Now, $\phi_{\pi}$ cannot become zero, in contrast to the "standard axion" and Case 1). If the axion is heavy enough$^{13}$, it is energetically impossible to produce it in low-energy experiments such as nuclear transitions and reactor experiments, and because of its short lifetime it would not be seen in beam dump experiments. But the production rate would be more substantial if the process is allowed. Comparing the order of magnitude of fermion masses and $M_W$, we would guess that $V/V_f < 0(10^3)$. So, $K^+ \to \pi^+$ should still be allowed, but now the theoretical value is even enhanced by the factor $V/V_f$ compared to the standard case$^{12}$, which is already ruled out by experiment.

Case 4). In all the previous cases the axion had no $\Delta$ component. This is because of the assumption of one $\Delta_L R$ (see Appendix) and $U_L = 0$. The correction to $A$ according to $U_L \neq 0$ is of the order $\ln(\Lambda_L^2 \delta L)$, which we can neglect. We must have at least two $\Delta$'s with different behaviour under PQ symmetry. Taking the assumptions of Case 4), we obtain in the limit $|U|, |U_W| >> V$ ($\sqrt{2} U_W = \delta W^0_0$)
\[ A = \sqrt{2} \text{ Im} \frac{U^* \delta^{\sigma'}_R \delta^{\sigma'}_{\nu R} - U_{\mu w}^* U_{\nu w}}{\sqrt{|U|^2 + |U_{\mu w}|^2}}. \]

\[ j_{\mu PQ}^{\nu} = \sqrt{|U|^2 + |U_{\mu w}|^2} \, \mathcal{A}_{\mu} A + \frac{1}{4} \left( \frac{U_{\mu w}}{|U_{\mu w}|} + 2 \frac{U_{\nu w}}{|U_{\nu w}|} \right) (\bar{\nu}_S^{\nu} y_{\nu S}^S + \bar{\nu}_S^{\nu} (3 \frac{U_{\mu w}}{|U_{\mu w}|} + 2 \frac{U_{\nu w}}{|U_{\nu w}|}) (\bar{e} y_{\nu e}^S e + \bar{d} y_{\nu d}^S d) + \right. \\
\left. + \frac{1}{4} \bar{\nu}_S^{\nu} y_{\nu S}^S \bar{\nu}_S^{\nu} + \frac{1}{4} \bar{q} y_{\nu q}^S \bar{q} \right) \]

Now \( f_a = \sqrt{|U|^2 + |U_{\mu w}|^2} \approx 0(M_{\nu}) \gg (\sqrt{2}G_F)^{-1/2} \) and the smallness of the couplings of the axion are connected with the masses of the nearly right-handed gauge bosons. Of course, to make the axion "invisible" and to be below the bound \( m_a \leq 6 \cdot 10^{-8} \text{ eV} \), coming from stellar evolution, \( f_a \) or \( M_{W_2} \) must be of the order of \( 10^9 \text{ GeV} \), which is actually obtained by SO(10) grand unification\(^{14}\), but this order of magnitude is far away from the present bounds \( M_{W_2} \geq 200-300 \text{ GeV} \) using experiments as input\(^{15}\). In addition, half of the one-to-one correspondence between large \( M_{W_2} \) and the small masses of the light neutrinos is destroyed because now we cannot conclude from \( M_{W_2} \) being small that the light neutrinos have small masses.

To conclude, it seems difficult to include the axion in LRS theories without facing the same problems as the "standard axion". The only way out, we found, destroys some of the economical features of the model.
APPENDIX

Here we want to sketch how to obtain Eq. (7). To calculate the linear combination of Higgses, i.e., the axion, we only need consider the neutral Higgses

\[ \bar{\Phi}_o = (\phi^1_{\eta}, \phi^2_{\eta}, \phi^2_{\eta}, \phi^2_{\eta}, \phi^2_R)^T \]

\[ \bar{\Phi}_o = \sqrt{2} Re \bar{\Phi}_o, \bar{\Phi}_o = \sqrt{2} Im \bar{\Phi}_o, \phi_o = \left( \bar{\Phi}_o \right)^T \]

\((\cdot)^T\) denotes transposition.

We must now calculate the action of the third component of the left- and right-handed isospin, the hypercharge and the generator of the PQ symmetry on the vacuum expectation value

\[ V = \sqrt{2} \left< \bar{\Phi}_o \right> \]

\[ T_{3L} V = \frac{1}{2} g (V_1, -W_1, V_2, -W_2, 0)^T \]

\[ T_{3R} V = \frac{1}{2} g (-V_1, W_1, V_2, W_2, -2U)^T \]

\[ Y_V = g' (0, 0, 0, 0, U)^T \]

\[ T_{PQ} V = (V_1, W_1, V_2, W_2, U)^T \]

From the first three vectors, only two are linear independent, corresponding to the two would-be Goldstone bosons eaten by \( Z_1 \) and \( Z_2 \). We take, for instance,

\[ (V_1, -W_1, V_2, -W_2, 0)^T / V \] and \[ (0, 0, 0, 0, U/1U)^T \]

We now change to the corresponding real representation of the Higgses. To do this we just have to change a vector \( X \) from (A.3) or (A.4) into \( (\text{Im}X, -\text{Re}X)^T \)
\[ T_{PQ} \nu \rightarrow \sigma^j \rightarrow (A4) \rightarrow Y_1, Y_2 \quad (A.5) \]

Then the axion belongs to the linear combination of the vectors of (A.5) orthogonal to \( Y_1, Y_2 \):

\[ z = (\sigma^j - \sigma^j \cdot Y_1 - \sigma^j \cdot Y_2) / |\sigma^j - \sigma^j \cdot Y_1 - \sigma^j \cdot Y_2| \]

\[ A = z \cdot \phi_0 \quad (A.6) \]

One can easily see that \( A \) has no \( \sigma^j \) component when there is only one \( \Delta_R \).

The same procedure is of course valid for all kinds of Higgs sectors.
REFERENCES


3) See, for example: G. Girardi, LAPP preprint TH-58 (1982).


