Optics Design of the Delay Loop in the CLIC Damping Rings Complex

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Abstract

For the recombination of the two trains coming from the Compact Linear Collider (CLIC) Damping rings (DR), a delay loop (DL) will be used in order to obtain the nominal 0.5 ns bunch spacing. The optics design of the loop is based upon an isochronous ring, in order to preserve the longitudinal beam distribution. Analytical expressions for achieving isochronous conditions for various optics cells are obtained and compared with optics simulations. In particular, the Theoretical Minimum Emittance (TME) cell similar to the one of CLIC DR is studied in detail and an optics design based on this cell is presented.
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INTRODUCTION

An isochronous delay loop requires the parameters of the cell to be tuned for the elimination of the momentum compaction factor of the cell. The TME cell, which is compact and has been a full study [1] on it, will be used in the DL. Elimination or minimization of the zero order momentum compaction factor is inadequate if higher order terms are not considered. The procedure followed to acquire the equations necessary for the calculation of higher order momentum compaction factor integrals is described. Zero order momentum compaction factor and its dependence from the dipole parameters are also studied. An analytic calculation of the momentum compaction factor up to second order are being presented. Finally, the accordance of the parameters fulfilling the isochronicity condition with the occurrence of beam effects, such as horizontal emittance spread and synchrotron radiation is being studied.

CALCULATION OF HIGHER ORDER DISPERSION FUNCTIONS

In order to analytically calculate the momentum compaction factor of the entire TME cell, it is essential to calculate the dispersion function up to second order. This is done by propagating and solving the equations of the dispersion for every element of the cell. The momentum compaction factor cannot be estimated for a thin element. Therefore, approximations can be applied after the thick lens calculation, in order for the results to become less complicated. In order to study the behaviour of the momentum compaction factor in a TME cell its conditions are applied.

In Figure 1 the layout of the cell is shown, with the dipole D of length $l_{dip}$, drift spaces $s_1$, $s_2$, $s_3$ and two quadrupoles Q1 and Q2, of focal distances $f_1$ and $f_2$ respectively. The propagation of the dispersion function starts from the center of the dipole, where the derivative of the dispersion is zero [1] until the point of symmetry at the end of the drift space $s_3$, where also the derivative is zero. Under these considerations, a set of solutions for the dispersion equations up to second order for every element of the TME cell can be obtained.

The analytic form of the differential equations of the dispersion can be calculated [2] from the Hamiltonian of the off momentum particle with a momentum deviation $\delta = \frac{p}{p_0}$ where $p_0$ is the nominal momentum. The Hamiltonian is given by:

$$
H = -(1 + \frac{1}{\rho_x} x + \frac{1}{\rho_y} y) \left[ \sqrt{(1 + \delta)^2 - (p_x - \frac{x}{\rho_x} A_x)^2} - (p_y - \frac{y}{\rho_y} A_y)^2 \right] - \frac{e}{p_0} A_x
$$

(1)

Assuming that the delay loop has no vertical bending gradient, $\frac{1}{\rho_y} = 0$, and considering motion along the x axis only, the canonical Hamilton equations $\left( x' = \frac{\partial H}{\partial p_x} \right.$ and $\left. p_x' = -\frac{\partial H}{\partial x} \right)$ can be used to derive the equations of motion for the off momentum particle. Furthermore, the hard-edge approximation for the magnetic field of the elements is being used and the magnets up to quadrupole are considered to be of separated function. In addition, the components $A_x$ and $A_y$ of the vector potential of the magnetic field are considered to be zero and the longitudinal component $\frac{e}{p_0} A_x$ becomes:

$$
\frac{e}{p_0} A_x = -\frac{1}{2} \left( 1 + \frac{x}{\rho_x} \right)
$$

(2)

while for the quadrupole:

$$
\frac{e}{p_0} A_s = \frac{1}{2} g_0 x^2
$$

(3)

where $g_0 = \frac{e}{p_0} \left( \frac{\partial H}{\partial x} \right) \bigg|_{x=y=0}$ is the quadrupole strength.

Using the expansions up to third order of the x position of the particle with respect to $\delta$:

$$
x = \eta_0 \delta + \eta_1 \delta^2 + \eta_2 \delta^3
$$

(4)

and its derivative with $s$

$$
x' = \eta_0' \delta + \eta_1' \delta^2 + \eta_2' \delta^3
$$

(5)

with $\eta_0$ being the zero order dispersion function, $\eta_1, \eta_2$ the first and second order dispersion function respectively, and
The zero order momentum compaction factor can be calculated using the solution of the dispersion inside the dipole:

\[ \eta_0 = \rho_x + (\eta_c - \rho_x) \cos \left( \frac{s}{\rho_x} \right) \]  

(9)

Using the relation for the zero order momentum compaction factor

\[ \alpha_0 = \frac{1}{L} \int_0^L \frac{\eta_0}{\rho_x} ds \]

where \( L \) is the length of the dipole, Eq.(9) can be used to calculate \( \alpha_0 \). This result can be simplified if thin lens approximation is applied, resulting to:

\[ \alpha_0 = \frac{24\eta_c - (\eta_c - \rho_x)\theta^2}{12\rho_x} \]  

(10)

Solving Eq.(10) with respect to \( \eta_c \), the dispersion at the center of the dipole which eliminates the zero order momentum compaction factor becomes negative:

\[ \eta_c = -\frac{7\theta^2\rho}{24} \]  

(11)

The behaviour of the zero order momentum compaction factor of the TME cell is shown in Figure 2 with respect to the bending radius \( \rho_x \) and the bending angle \( \theta \) (left) and with respect to the magnetic field \( B \) of the dipole and the number of dipoles \( N_d \) (right), using a colour scale. For higher values of \( \theta \) there is a small dependence of \( \alpha_0 \) on \( \rho_x \), meaning that the choice of \( \theta \), resulting to a small value of \( \alpha_0 \), doesn’t affect the choice of the bending radius of the dipole. In addition, there is no dependence of \( \alpha_0 \) on the number of dipoles \( N_d \) (for high values). Lower values of \( \alpha_0 \) can be achieved for \( B < 0.5T \). In both figures, the initial dispersion \( \eta_0 \) is considered to be negative. The light coloured areas represent higher absolute values of \( \alpha_0 \).

**High Order Momentum Compaction Factor**

Expanding the momentum compaction factor function up to second order with respect to \( \delta \), yields:

\[ \alpha_x = \alpha_0 + \alpha_1 \delta + \alpha_2 \delta^2 \]  

(12)

The general equation of the momentum compaction factor is:

\[ \alpha_0 \delta = \frac{1}{C_0} \oint_C (dl - dl_0) \]  

(13)

where \( C_0 \) the nominal closed orbit of the synchronous particle, \( C \) is the closed orbit of the off-momentum particle, \( dl \) the infinitesimal path length of an off-momentum particle which is given by:

\[ dl = \sqrt{(1 + \frac{x}{\rho_x})^2 + (x')^2 ds} \]  

(14)

and \( dl_0 \) the nominal one, which can be calculated by setting \( x = 0 \) in Eq.(14). Using the expansions (4) and (5) in Eq.(14), the difference \( dl - dl_0 \) can be calculated. Inserting this in Eq.(13) and equating with Eq.(12), the analytic formulas for high orders terms can be derived:

\[ \alpha_1 = \frac{1}{L} \int_0^L \left( \frac{\eta_0}{2\rho_x^2} + \frac{\eta_1}{\rho_x} + \frac{1}{2} \eta_0^2 \right) ds \]  

(15)

\[ \alpha_2 = \frac{1}{L} \int_0^L \left( \frac{\eta_0 \eta_1}{\rho_x^2} + \frac{\eta_2}{\rho_x} + \frac{\eta_0 \eta_1}{2} \right) ds \]  

(16)

where \( \alpha_1 \) and \( \alpha_2 \) are the first and second order momentum compaction factor respectively and \( L \) is the length of the
element. Inserting the dispersion solutions for the TME cell in the above equations $\alpha_1$, $\alpha_2$ and $\alpha_c$ can be calculated from (12).

Figure 3 shows $\alpha_1$ and $\alpha_2$ with respect to $f_1$, $f_2$. Larger absolute values of $f_1$, $f_2$ indicate lower values for $\alpha_1$ and $\alpha_2$.

From Eq. (12), the momentum compaction factor of the entire cell, $\alpha_c$, can be calculated. On the left of Figure 4 $\alpha_c$ of the TME cell with respect to $f_1$ and $f_2$ is shown. A combination of large negative $f_2$ and a relatively low $f_1$, could lead to small values of $\alpha_c$. The same colordode is used as in the previous plots. The plot on the right shows values for positive $f_1$ and negative $f_2$ for which $\alpha_c$ is zero.

Figure 4: Left: $\alpha_c$ of the TME cell with respect to $f_1$, $f_2$. Right: Curve for sets of values of $f_1$, $f_2$ that are solutions for $\alpha_c = 0$.

ENERGY LOSS PER TURN AND EMITTANCE SPREAD

Due to synchrotron radiation, the particles inside the bends lose energy, inversely proportional to the bending radius. On the left plot of Figure 5 the energy loss $\delta \epsilon$ dependence on $\rho$ is shown. However, in the delay loop, the perturbation of the energy and the horizontal emittance of the beam should be minimum. For these reasons, only the values of the parameters which minimize these values are taken into account. The TME cell, has a specific function for the horizontal emittance which is [1]:

$$\epsilon_x = \frac{C_q \gamma_r^2}{J_x \rho_x} \left[ \frac{1}{\beta_c} \left( \frac{\eta_c^2}{12} - \frac{\theta \eta_c l_{dip}}{12} + \frac{\theta^2 \eta_c^2 l_{dip}^2}{320} \right) + \frac{\theta^2 \beta_c}{12} \right]$$

where $C_q = 3.84 \cdot 10^{-13} m$, $\gamma_r$ is the relativistic gamma factor, $J_x$ the damping partition number, $\rho_x$ the bending radius, $\beta_c$ the beta function at the center of the dipole, $\eta_c$ the dispersion at the center of the dipole, $\theta$ the bending angle, and $l_{dip}$ the length of the dipole. It can be shown that the horizontal emittance spread, $\delta \epsilon_x$, is inversely proportional to $\theta$. The right plot of Figure 5 shows that a large $\theta$, would lead to a smaller horizontal emittance spread while a large $\rho$ (small B) leads to smaller energy loss per turn.

Figure 5: $\delta \epsilon$ with respect to $\rho$ (left) and $\frac{\delta \epsilon_x}{\epsilon_x}$ with respect to $\theta$ (right).

CONCLUSIONS

It is obvious that higher order momentum compaction factor terms (up to second order), are non negligible in the non bend elements of the TME cell. In order to achieve the isochronicity condition it is essential to tune the cell parameters so that $\alpha_c$ is zero. Two families of sextupoles can be used to achieve this goal, one for the horizontal and one for the vertical plane. The cell parameters can be chosen so that $\alpha_c$ is minimized or eliminated and at the same time to be in accordance with the energy loss minimization after every particles revolution and the emittance conservation. A relatively low magnetic field combined with a choice of number of dipoles satisfying the minimization of emittance spread could be chosen, since $\alpha_0$ and consequently $\alpha_c$ is independent of the number of dipoles. A large bending radius is needed for a minimum value of $\alpha_0$. Using the beam rigidity $(B \rho_x = \frac{\gamma_r}{\rho})$ for $\rho_0 = 2.86 GeV c$ in the delay loop and $B = 0.5 T$, the minimum value of $\rho_x$ is calculated to be $\rho = 19 m$ and therefore this requirement is fulfilled. In addition a choice of small quadrupoles strengths can lead to minimization of $\alpha_1$ and $\alpha_2$, and therefore $\alpha_c$.

REFERENCES
