DESIGN OF Nb$_3$Sn MAGNETIC DEVICES TO STUDY THE SUPERCONDUCTOR DEGRADATION UNDER VARIABLE MECHANICAL LOAD

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2009
The theory is when you know everything and nothing works.
The practice is when everything works and nobody knows why.
We have put together the theory and practice: there is nothing that
works and nobody knows why.

Albert Einstein
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Part I

Introduction
Chapter 1

Introduction

1.1 Problem definition: the LHC upgrade

The Large Hadron Collider (LHC) is a two-ring, superconducting synchrotron accelerator and collider installed in a 27 km long tunnel aiming at the discovery of the Higgs particle and the study of rare events with center mass collision energies of up to 14 TeV \[1\]. The particle beam will be pre-accelerated by three different machines (LINAC, PS and SPS) to 450GeV and sent to the LHC (injection). The beam energy will be then increased up to the 7TeV in the LHC itself (acceleration). The collisions will finally occur in four interaction points where four experiments (ALICE, CMS, LHC-b and ATLAS) are placed. The number of collisions per unit of area and time are evaluated through the Luminosity function:

$$ L = F \frac{n_b N_b^2 f_{\text{rev}}}{4\pi \sigma^*^2} $$ (1.1)

where \(n_b\) is the number of bunches circulating in the machine, \(N_b\) is the number of protons per bunch, \(f_{\text{rev}}\) is the revolution frequency of the machine and \(\sigma^*\) is the transverse RMS beam size at the collision point. The factor \(F\) is called geometrical factor, depending on the bunches crossing angle. The nominal value of the LHC luminosity is \(10^{34} \text{ cm}^{-2} \text{s}^{-1}\).

Inside the LHC, superconducting magnets aligned with a precision of a few tenth of millimeters are used to bend and focus the particle trajectories. Namely:

- dipoles produce the main vertical magnetic field, perpendicular to the particle direction, used to deflect particle motions to circular trajectories. The nominal field for these magnets is 8.33 T (7 TeV beam energy), while the ultimate field is 9 T. The beam energy is directly dependent on the magnetic field produced by the main dipoles, and on the accelerator curvature:
\[
\frac{mv}{e} = B\rho
\]

where: \(mv\) is the particle momentum, \(B\) the dipole field, and \(\rho\) the accelerator curvature.

- Quadrupoles produce a field that is null in the center of the vacuum chamber and linearly dependent from the distance to the center, whose purpose is to focus the beam. The nominal gradient is 223 T/m, with a peak field in conductor of 6.85 T.

In the LHC tunnel, 1232 main dipoles (MB: main bending) and 386 main quadrupoles (MQ: main quadrupoles) are installed. In July 2001, a task force has been set up to discuss about possible scenarios to increase the luminosity to \(10^{35} \text{ cm}^{-2}\text{s}^{-1}\) and to double the proton beam energy \([2]\). Two main solutions were proposed, presenting LHC hardware changes:

- LHC upgrade Phase 1: in order to increase the luminosity, the target is mainly to reduce the transversal beam size \(\sigma^*\). This could be achieved by modifying the insertion quadrupoles, i.e. the quadrupoles close to the interactions regions, and eventually their layout. This solution will imply quadrupoles with bigger aperture (120 mm).

- LHC upgrade Phase 2: in order to significantly extend the capabilities of LHC in enabling precision measurements of rare processes, an upgrade in the accelerator beam energy is required. Being the energy directly dependent on the dipole performances, doubling the beam energy means the design and construction of high field dipole magnets (peak field beyond 15 T). Furthermore, the interaction regions will be redesigned, requiring quadrupoles able to provide field gradient higher than 200 T/m.

The LHC can be considered as the state of the art for superconducting magnets using the Nb-Ti technology. Higher fields and gradients could be only reached by using a different superconductor technology. If for the Phase 1 the Nb-Ti technology can be further exploited to upgrade the quadrupoles close to the interaction region, for the Phase 2 the use of the Nb₃Sn has to be envisaged. The latter is considered as the most suitable superconductor to be used in high field magnets.
1.2 The Nb$_3$Sn

The Nb$_3$Sn is an intermetallic compound of niobium and tin belonging to the A15 crystallographic family. More precisely, it consists of a body centered lattice of Sn atoms, with two Nb atoms on each face of the cube. Between the A15 compounds, the Nb$_3$Sn is the easiest to produce in a form suitable for large magnets. From the mechanical point of view, Nb$_3$Sn is not easy to handle as the Nb-Ti, because of his brittleness. Nb$_3$Sn for magnets fabrication are made of superconducting subelements embedded in a copper matrix. The copper matrix is for magnet protection and stabilization. Subelements are composed by Nb$_3$Sn fine filaments embedded in a bronze matrix. Before being heat treated, the subelements do not contain Nb$_3$Sn. At this stage, the strand is still ductile and can be used to produce cables. Cables composing the magnet coil are usually of Rutherford type: such a cable is made up of few tens of strands, twisted together, and shaped into a flat, two layer cable. The Nb$_3$Sn is then formed during a heat treatment at $\sim 650 \, ^\circ C$, by solid diffusion of Sn into Nb, through the bronze matrix. The copper in the subelements works as a catalytic agent for the preferential growth of the Nb$_3$Sn phase reducing the heat treatment time.

![Figure 1.1: Critical current density of superconducting materials. Courtesy of the Applied Superconductivity Center.](image_url)

The Nb$_3$Sn has much better superconducting properties compared to Nb-Ti: for a $j_c$ equal to 1000 A/mm$^2$, a peak field of 11 T at 1.9 K is reached with a
1. Introduction

Nb-Ti wire, whereas 17 T at 4.2 K can be achieved with Nb$_3$Sn. Moreover, above 11 T the $j_c$ of Nb-Ti drops drastically, whilst above 15-17 T the $j_c$ reduction for Nb$_3$Sn is much more gradual (see fig. 1.1).

From the early 60’s, variations of the critical temperature and of $j_c$ for a Nb$_3$Sn wire have been found as strongly dependent on the mechanical deformation provided. Such a phenomenon has been largely studied in order to understand the effects of the manufacturing and operative processes on a Nb$_3$Sn cable, that obviously influence the magnet performances. Analysis have been carried out on the single wire and on the cable as a whole, trying to define general laws which describe the magneto-mechanical behavior of a such a conductor.

Studies performed by [3] outlined how the critical current degradation can be triggered if the axial tensile strain on the wires is higher than 0.3%, leading to irreversible current degradation of $\sim 60\%$ for axial strain of the order of 0.8 %. The relation between axial strain and current has been reviewed by [4], who proposed a scaling law describing the linear dependence of the peak field $B^*_{c2}$ on the deviatoric strain. Nevertheless this approach can not describe the relation between the field reduction and the applied transverse pressure.

Separate studies have been performed on the effect of the transverse pressure on Nb$_3$Sn cables, showing how complex is the stress-critical current relationship [5]-[6]-[7]-[8]. Several are the parameters influencing the superconductor behavior, such as:

- Rutherford cable angle and structure
- cable impregnation
- manufacturing process
- cable filling factor

Nowadays, the superconductor research community addresses the irreversible current degradation to a level of stress of about 150-200 MPa, referring to evidences of experimental tests on single cables rather than on Nb$_3$Sn magnets. Nevertheless, this limit has not yet well understood, being the analysis results largely spread due to the influence of the cable features and of the test devices.

Due to the high inductions achievable with Nb$_3$Sn magnets, the Lorentz forces produced notably increase with respect to Nb-Ti magnets, thus leading to stress levels inside the coils that could be of the order of the intrinsic limit of the superconductor. That is why a large number of magnets have been realized since the 90’s, above all in the US.
1.3 State of the art of Nb$_3$Sn magnets

At present, research and development of Nb$_3$Sn accelerator magnets is largely carried out by three government laboratories [9] and by a European Joint Research Activity named NED (Next European Dipole) [10].

The USA laboratories are: Fermi National Accelerator Laboratory (FNAL), Lawrence Berkeley National Laboratory (LBNL) and Brookhaven National Laboratory (BNL); these labs beyond having a base High Field Magnet (HFM) program, are also collaborating within the framework of the US LHC Accelerator Research Program (LARP) on the development of Nb$_3$Sn quadrupoles for the LHC luminosity upgrade.

NED is supported by a collaboration of eight partners: CCLRC/RAL (UK), CEA (France), CERN, CIEMAT (Spain), INFN-Genova and INFN-Milano (Italy), Twente University (The Netherlands), and Wroclaw University of Technology (Poland). The goal of the NED project is to build a large aperture dipole (88 mm bore) able to reach 15 T peak field on conductor for the LHC upgrade. Till now, great efforts have been spent on the conductor optimization, whereas the dipole is still at design stage.

The two laboratories among the others who spent the largest efforts since the past decade are FNAL and LBNL. At FNAL, different dipole magnets have been developed since 1999 [11], with the aim of being cost effective and robust as well as acceptable in the future for industrialization of full-scale magnets. They differentiate on the base of: (1) study purposes, (2) cable type and features, (3) coil design ($cos\theta$ type, and racetrack layout), and (4) winding technique (Wind-and-React, and React-and-Wind).

![Figure 1.2: Two examples of Nb$_3$Sn magnets tested at FNAL: (a) $cos\theta$ dipole, and (b) common coil dipole.](image)
1. Introduction

The peak field reached by those magnets are of the order of 10-12 T, with a coil length of about 1 m. Tests performed on these magnets (HFDA and HFDM series) shown premature quenches at 60-70 % of the short sample current, to be addressed to the non-optimized mechanical structure, to some manufacturing problems, and to magneto-electrical instability of the conductor. Small racetracks have been consequently realized in order to further study and develop the conductor.

At LBNL, different magnets have been designed and tested since 1997. They can be grouped into: Racetrack dipoles (RD) [12], Subscale Model (SM) [13], and high field dipole magnets (HD series) [14]. LBNL has developed an original magnet structure, which allows to reach the required coil pre-stress by using a double step technique: part of the pre-compression is transmitted at room temperature, by using the keys and bladders technology, whereas the total pre-load is achieved at cryogenic temperature due to the differential thermal contraction between components [15]. Large efforts have been spent in understanding the correlation between the magnet performances and the mechanical structure; indeed, the coil was initially affected by the so-called *slip-stick* motion. As at FNAL, the SM magnet series is composed by small racetrack coils, connected in common coil layout, able to achieve fields of 10-12 T. They were built to allow study on cable performance in a magnet-like environment. They were easy to assemble and disassemble and allowed to vary the pre-stress on conductor. The HD (HD01 and HD02) series aimed at reaching high fields, with a block-type coil layout. The HD01 reached 16 T maximum field at 4.5 K. As for the magnets developed at FNAL, the latter magnets have a maximum length of about 1 m.

![TQS cross section](a) ![TQC cross section](b)

Figure 1.3: TQS cross section (a), TQC cross section (b).

In the frame of the LARP program for the upgrade of the LHC IR, a magnet featuring an aperture of 90 mm, field gradient higher than 205 T/m, excellent field
quality, and high radiation loads has been developed. Two different structures have
been pursued: the TQS01 (LBNL) [16], based on keys and bladders technology,
and the TQC01 (FNAL), based on stainless steel collars supported by an iron yoke
and thick SS skins [17].

Tests performed separately shown quite low magnet training and reached the
plateau at 87% (TQS) and 70% (TQC) of the short sample current. The main
difference between the two is due to the mechanical structure, which has to be
further optimized (central pole materials and pre-stress set-up) More recent tests,
performed in collaboration with CERN, showed magneto-thermal instability issue
[18]. The main goal of realizing a prototype for a IR upgrade magnet was success-
fully completed; nevertheless the overall length is still far reduced with respect to
a full scale quadrupole, used in LHC. The next step will be the test of a longer
quadrupole (LQ) based on the experience of the TQ magnets. This will be 4 m
long, sharing the same cross section and magnetic parameters as the TQ. The
main challenge will be the coil heat treatment, the assembly procedure and the
development of the quench protection system, due to the high energy stored during
powering. Tests on the LQ are scheduled for summer 2009.

1.4 Aim of the work and tools

As it has been previously outlined, the open questions about the Nb$_3$Sn as the
next generation superconductor for high field magnets are still a lot, most of them
concerning its magneto-mechanical properties. Aim of this work is to design an-
alytical and experimental tools that will allow studies on the correlation between
applied mechanical stress and magnetic performances.

New generation magnets will lead to higher inductions, although at the same
time they will imply higher Lorentz forces that will stress the coils and magnet
structure more severely. The magnets composing the LHC are all of the so-called
cos$^2$ layout, which is the closest layout to an ideal current density distribution
to get pure dipole or quadrupole fields [19]. In order to understand if this layout
could be held for Nb$_3$Sn magnets, a preliminary evaluation and understanding of
the coil stress distribution is needed. We will provide analytical tools to define
the peak stress inside the winding at short sample current, as a function of the
geometrical coil layout and superconductor material. This tool will precede the FE
analysis, aimed at giving first assessment values on the stress produced at magnet
powering. The equations have been implemented and solved for the different cases
in Mathematica®, a fully integrated environment for technical computing [20].

Due to the uncertainty in the current degradation of Nb$_3$Sn impregnated cables
phenomenon under variable transverse load, a new cable sample holder has to be
designed. The challenge will be the integration of this sample holder, specified
to reach up to 200 MPa pressure, into the test facility FRESCA (Facility for the REception of Superconducting CAbles) at CERN. The design will make use of the keys and bladders technique for magnet assembly, developed at LBNL. The numerical design of the sample holder will be carried out in the FE code ANSYS\textsuperscript{\textregistered} [21]. A test of a sample holder mockup will be done in order to validate the structure.

Although the sample holder will allow a systematic approach to cable test, a dipole magnet will be designed in order to analyze the cable performance in a magnet-like environment. The dipole magnet takes over the working principles of a previous magnet, called the SD01. Though the SD01 was successfully built and tested at LBNL, some improvements were advised. These concern the structure and coil layout as well as the instrumentation set up. The SD01 optimization will be carried out by a joint venture between European laboratories (RAL, CEA, CERN) and will be set in the frame of the NED project. The magnetic optimization will be done in ANSYS\textsuperscript{\textregistered} and ROXIE [22], whereas the mechanical design will be done in ANSYS\textsuperscript{\textregistered}. As for the sample holder, the design will require 2D and 3D modeling, as well as the creation of ad-hoc routines for model pre and post-processing.

### 1.5 Content of the thesis

The analytical approach presented in the first part of the thesis is intended in a preliminary understanding of the stress field distribution in a $\cos \vartheta$ coil. A simple sector coil layout with constant current density will be analyzed, as a representative of real coil cross sections. We will focus in particular on the peak stress on coil mid-plane at short sample conditions, being the main responsible for cable current degradation. The peak stress will be correlated to the main magnetic design parameters, i.e. the bore field for dipoles, and the field gradient for quadrupoles. This parametric analysis will be carried out considering firstly a simple coil in air, and then accounting for a magnetic screen which encloses the coil. The equations for the critical current density, as well as for the mechanical stress will be consequently modified for the iron yoke cases. The results will be compared to values of real cross sections, implemented in ANSYS\textsuperscript{\textregistered} and Roxie.

The value of 150 MPa as limit of Nb$_3$Sn cables is nowadays assumed on the base of experimental evidence on magnets tests. Nevertheless, the lack of reproducible measurements and of a dedicated test facility led to the idea of developing a new sample holder. This will be used in the cable test facility FRESCA at CERN, capable of providing a 10 T dipole-like background field. It will be used to test cables up to widths of 16 mm, with a pressure variable from 50 to 200 MPa. The challenge has been to develop a device to be integrated in the FRESCA magnet.
1.6 Structure of the thesis

The thesis is divided into four parts. In the first one, Chapter 1 introduces the subject of the thesis, describing the main features and issues concerning the Nb$_3$Sn. Part II includes the dissertation on the analytical tools describing the stress field inside $cos\theta$ coils. Parametric analysis of Lorentz forces and stress for quadrupoles is presented in Chapter 2, whereas dipole magnets are analyzed in Chapter 3. Quadrupole type magnets are analyzed as first because they likely represent the first application of Nb$_3$Sn for the LHC upgrade.

In Part III the design of the new cable sample holder is presented. The different solutions analyzed and the proposed configuration are reported in Chapter 4. Chapter 5 includes the test results on a scaled sample holder model. The results are then compared with the numerical model and finally discussed.

The design and optimization of the SD01 is described in Part IV. In Chapter 6 is presented the magnetic optimization process, whereas in Chapter 7 the mechanical analysis is reported, on the base of the optimized magnetic configuration. Chapter 8 is dedicated to conclusions.

In Appendix A the equations of the electro-magnetic forces are listed, whereas in Appendix B the related mechanical stress equations are described, for both quadrupoles and dipoles. In Appendix C the equation of the peak stress on a quadrupole coil mid-plane is derived. In Appendix D, the expressions of magnetic field, Lorentz forces, and mechanical stress are modified taking into account the presence of an iron yoke surrounding the magnet coil.
1. Introduction
Part II

Analysis of forces and stresses in \( \cos \theta \) type magnets
Chapter 2

Forces and stresses in superconducting quadrupoles

2.1 Introduction

In superconducting magnets, electromagnetic forces and associated stresses are generated by the interaction of the cable current with the magnetic field. The stress produced are generally of the order of 10-100 MPa, for Nb-Ti magnets. Two main issues have to be analyzed: first, a mechanical structure must be envisaged to contain these forces and to limit the cable movements during the magnet ramp; secondly, the stresses inside the coil must not exceed the limits beyond which insulation can start creeping and the superconductor properties are degraded. This last aspect is critical for the Nb$_3$Sn which, according to measurements carried out on cables, cannot tolerate compressive stresses larger than 150 MPa [23]. In this chapter we aim at analyzing how forces and stresses depend on the quadrupole aperture, on the width of the coil, and on the superconducting material. We used a simplified coil layout made up of a sector of inner radius $r_i$, radial width $w$ and angular extension $\alpha_0 = 30^\circ$ (thus canceling the sixth order field harmonic). A uniform current density $j$ is applied (see fig. 2.1-b). This simple geometry has the advantage of being closer to a real coil than the classical cos$2\theta$ current distribution, whilst still allowing an analytical approach. In [24] it has been shown that a similar lay-out well represents, from the electromagnetic point of view, several quadrupoles based on the shell geometry that have been built in the last 30 years. The main steps carried out in this analysis have been the following ones:

- an analytical estimation by using the formalism developed in [25]-[26] of the electromagnetic forces and the induced stresses in the sector coil as a function of $w$ and $r_i$ for a given $j$. In particular, we will focus on the evolution of the
2. Forces and stresses in superconducting quadrupoles

position and of the value of the maximum compressive stress on the coil mid plane. In this section, no iron yoke is introduced.

- The evaluation of the critical current density relative to a given coil lay-out and to the specific superconductor (either Nb-Ti or Nb$_3$Sn). The relationship between the peak compressive stress and the obtained gradient will be studied.

- The development of the formulae for the critical current density in presence of an iron yoke, revising the peak stress behavior in this new condition.

- The analytical formulae are cross-checked with a two-dimensional finite element model coded in ANSYS™ of one quadrupole octant. In the first part of the work the magnetic model of the winding built in ANSYS™ is completely surrounded by air, and the "width" of the air section has been optimized in ANSYS™ to get a good convergence for the values of the magnetic field. In the last part, the iron yoke has been implemented in the model.

![Figure 2.1: LHC quadrupole cross section (a) and sector coil model adopted (b).](image)

2.2 Analytical formulae of magnetic field and Lorentz force components

The expressions for the magnetic field in a sector coil with constant current density distribution can be derived starting from the definition of the magnetic vector
2.2 Analytical formulae of magnetic field and Lorentz force components

potential \[26\]. Consequently the expressions for the magnetic field components within the aperture \((0 \leq r \leq r_i)\) are as follows:

\[
\begin{align*}
\{ B_r \} &= -\frac{j\mu_0}{2\pi} \left\{ 4r \ln \left( \frac{r + w}{r_i} \right) \sin(2\alpha_0) \right\} \left\{ \frac{\sin2\varphi}{\cos2\varphi} \right\} + \\
&\quad + \sum_{m=1}^{\infty} \frac{2r^{4m+1}}{-m(4m+2)} \left( (r_i + w)^{-4m} - r_i^{-4m} \right) \sin(4m + 2) \alpha_0 \left\{ \frac{\sin(4m + 2)\varphi}{\cos(4m + 2)\varphi} \right\} \\
\{ B_\varphi \} &= \sum_{m=1}^{\infty} \frac{2r^{4m+1}}{-m(4m+2)} \left( (r_i + w)^{-4m} - r_i^{-4m} \right) \sin(4m + 2) \alpha_0 \left\{ \frac{\sin(4m + 2)\varphi}{\cos(4m + 2)\varphi} \right\}
\end{align*}
\]

(2.1)

where \(\alpha_0\) is the angle at the pole \((\pi/6)\).

The expressions for the field components outside the winding \((r \geq r_i + w)\) are as follows:

\[
\begin{align*}
\{ B_r \} &= -\frac{j\mu_0}{2\pi} \left\{ r^{-3} \left( (r_i + w)^4 - r_i^4 \right) \sin(2\alpha_0) \right\} \left\{ \frac{\sin2\varphi}{\cos2\varphi} \right\} + \\
&\quad + \sum_{m=1}^{\infty} \frac{2r^{4m-3}}{(m+1)(4m+2)} \left( (r_i + w)^{4m+4} - r_i^{4m+4} \right) \sin(4m + 2) \alpha_0 \left\{ \frac{\sin(4m + 2)\varphi}{\cos(4m + 2)\varphi} \right\} \\
\{ B_\varphi \} &= \sum_{m=1}^{\infty} \frac{2r^{4m-3}}{(m+1)(4m+2)} \left( (r_i + w)^{4m+4} - r_i^{4m+4} \right) \sin(4m + 2) \alpha_0 \left\{ \frac{\sin(4m + 2)\varphi}{\cos(4m + 2)\varphi} \right\}
\end{align*}
\]

(2.2)

Summing up these two components by imposing that the first contribution is 0 at \(r = r_i\) and the second is 0 at \(r = (r_i + w)\), we get the magnetic field components inside the coil:

\[
\begin{align*}
\{ B_r \} &= -\frac{j\mu_0}{2\pi} \left\{ 4r \ln \left( \frac{r + w}{r_i} \right) + r^{-3} \left( r^4 - r_i^4 \right) \right\} \sin(2\alpha_0) \left\{ \frac{\sin2\varphi}{\cos2\varphi} \right\} + \\
&\quad + \sum_{m=1}^{\infty} \frac{2r^{4m+1}}{-m(4m+2)} \left( (r_i + w)^{-4m} - r_i^{-4m} \right) + \frac{2r^{4m-3}}{(m+1)(4m+2)} \left( r_i^{4m+4} - r_i^{4m+4} \right) \\
&\quad \sin(4m + 2) \alpha_0 \left\{ \frac{\sin(4m + 2)\varphi}{\cos(4m + 2)\varphi} \right\}
\end{align*}
\]

(2.3)

The equations of the magnetic forces along the coordinate cartesian system can be obtained from the integration of the magnetic field inside the coil (see Appendix A):

\[
F_x = -\frac{j^2\mu_0\cos\alpha_0\sin^2\alpha_0}{9\pi(r_i + w)} f_x(r_i^4, w^4)
\]

(2.4)

\[
F_y = \frac{j^2\mu_0\sin(2\alpha_0)}{18\pi(r_i + w)} f_y(r_i^4, w^4)
\]

(2.5)
2. Forces and stresses in superconducting quadrupoles

2.2.1 Comparison with the Finite Element model

A comparison with a numerical model (aperture \( r_i = 84 \text{ mm} \), coil width \( w = 20 \text{ mm} \)) has been carried out in order to validate the formulae described in 2.2. The results obtained can be summarized as follows:

- the formulae for a sector coil have been evaluated considering firstly the main term of the series expansion of the magnetic field, and then adding the second one (i.e. \( m = 1 \)). The first term only provides a good description of the magnetic field

- the sector coil approach gives reliable results for the field produced within the aperture and outside the coil.

- the agreement is worse inside the winding, the difference increasing at the pole (see fig. 2.2). Consequently, the equations derived are not suitable for peak field computations inside the coil.

Nevertheless, an analysis on the magnetic energy associated to the field produced inside the coil revealed a difference between the analytical and the numerical approach below 5%. Since the magnetic forces derive from the magnetic field integral, a good agreement is expected between the two approaches. The resultants forces have been analytically computed for different geometrical layouts, and then compared to the results given by the FE model in air. The model has been studied for the following sets of radial dimensions and coil widths:

- \( r_i : [14, 28, 56, 84, 112, 140, 168, 196] \text{ mm} \)

- \( w : [5, 10, 15, 20, 25, 30, 35, 40] \text{ mm} \)

All the 64 possible combinations have been explored, setting a constant current density \( j \) equal to 1000 \( \text{ A/mm}^2 \), neglecting the real possibility of using such a current with the layout proposed.

Notwithstanding the approximation shown for the magnetic field, the magnetic forces show a good match with numerical data as expected (see fig. 2.3). Both \( F_x \) and \( F_y \) follow a linear trend with the coil aperture and with the square of the coil width. In the first case, the numerical results are underestimated of about 4%, in the second of less then 3%.
2.2 Analytical formulae of magnetic field and Lorentz force components

![Graph](image1)

![Graph](image2)

Figure 2.2: Field distribution inside the coil for a sector winding of $r_i=84\text{mm}$, $w=20\text{mm}$: (a) $B_r$, and, (b) $B_\varphi$. 
2. Forces and stresses in superconducting quadrupoles

Figure 2.3: Magnetic forces varying the geometrical layout parameters. Fig. (a): the coil width has been set to 30 mm, varying the aperture from 14 to 196 mm. Fig. (b): the aperture $r_i = 28$ mm, the coil width varies from 5 to 40 mm.
2.3 Mechanical stresses

The magnetic forces distribution produce a squeeze of the coil in the azimuthal direction as well as they push the coil outward, leading to a compressive state both inside the coil and on the mechanical confinement structure (i.e. collars). By considering the stress balance in a sector winding element and neglecting the effect of shear components (see Appendix B) [27], one can derive the expression of the azimuthal compressive stress (negative value if considering tension as a positive) for a 30° sector coil as follows:

\[
\sigma_\varphi(r) = -\frac{j^2 \mu_0 \sqrt{3}}{16\pi r^2} \left[ r^4 - r_i^4 + 4r_i^4 \ln\left( \frac{r_i + w}{r} \right) \right]
\]

(2.6)

The expression of the radial stress produced at the interface with the mechanical structure is as follows:

\[
\sigma_r(\varphi) = -\frac{j^2 \mu_0 \sin(2\alpha_0)}{36\pi (r_i + w)^2} f_{pr}(r_i^4, w^4, \varphi)
\]

(2.7)

where \( f_{pr}(r_i^4, w^4, \varphi) \) is defined in Appendix B. The numerical model has been modified in order to perform a coupled analysis (magnetic and mechanical).

A constraint along the azimuthal direction has been applied on the coil mid plane, thus reproducing the structural symmetry with the lower coil block. The winding has been constrained along the radial direction, thus simulating the contact with the retaining structure, i.e. the collar, of infinite stiffness.

By comparing the numerical results to the analytical ones (see fig. 2.4), it can be observed that:

- the analytical formulae give a good approximation on the maximum compressive stress overestimating the absolute value of about 5%. The position of the maximum is evaluated with a maximum error of about -10% (small apertures and big widths). A larger error is committed in estimating the value of the stress at the inner radius. This effect is mainly due to the fact that neglecting the shear stress in balance equations, we do not take into account the role that the material plays in the distribution of the compressive stress.

- the maximum radial stress value is at the pole and is overestimated of about +10% in the worst case. This occurs for small apertures and large coil widths. For larger apertures, the stress profile diverges from the numerical one, overestimating the stress towards the coil mid-plane, underestimating it at the pole.
2. Forces and stresses in superconducting quadrupoles

Figure 2.4: $\sigma_\varphi$ distribution on the coil mid plane. (a) $r_i=28$ mm, $w=30$ mm; (b) $r_i=14$ mm, $w=40$ mm.
2.3 Mechanical stresses

2.3.1 Influence on the stress distribution induced by an anisotropic material

The formulae derived before are applicable in case of isotropic material and are not affected by change of the Young’s modulus $E$. In this section, the mechanical stress in case of an orthotropic material ($E_\varphi = E_z \neq E_r$) are studied.

Figure 2.5: $\sigma_\varphi$ distribution on the coil mid plane for different anisotropy ratios.

This approach is more representative of a superconducting cable. The reference Young’s modulus is about 13 GPa, which typical of a Nb-Ti superconductor; then different ratios $E_r/E_\varphi$ – respectively: 0.5, 1, 2, 4, 6, 8 – have been imposed to study the stress distribution on the coil mid plane, i.e. the azimuthal internal stress, and the radial stress on the contour line, i.e. on the contact area between the coil and the collar.

The study has been carried out analyzing the maximum value assumed by both the distributions, since the maximum should be kept low to avoid any degradation of the conductor. It has been found out that the shift between the isotropic case and the orthotropic one in terms of $|\sigma_\varphi|$ depends on the $w/r_i$ ratio; nevertheless it is always less than 2.5%. Concerning the position of the maximum compressive stress $r(\sigma_{\varphi,\text{max}})$, the difference with respect to the isotropic case depends on the aspect ratio $w/r_i$, being in any case less than 10%. In general, the larger is $E_r/E_\varphi$, the larger is the error committed.
2. Forces and stresses in superconducting quadrupoles

2.4 Forces and related stresses at short sample

Here we introduce the critical current density expressions for both Nb-Ti and Nb$_3$Sn conductors into the equations for the e.m. forces and mechanical stress. The aim of this section is to address the mechanical stress produced at the short sample condition for a given layout (sector coil approach).

2.4.1 Nb-Ti

In a superconducting quadrupole $j$ is limited by the critical current density $j_c$, i.e. the current corresponding to a peak field on the critical surface. In order to estimate the critical current, following the approach of [24] we define the lay-out parameters:

\begin{align}
\text{critical gradient} : & \quad G = \gamma(r_i, w) j \\
\text{peak field} : & \quad B_p = \beta(r_i, w) j
\end{align}

For a sector coil of 30° one has:

\begin{align}
\gamma(r_i, w) &= \gamma_0 \ln \left( 1 + \frac{w}{r_i} \right) \\
\beta(r_i, w) &= r_i \gamma_0 \ln \left( 1 + \frac{w}{r_i} \right) \lambda(r_i, w) = r_i \gamma_0 \ln \left( 1 + \frac{w}{r_i} \right) \left( a_{-1} \frac{r_i}{w} + 1 + a_1 \frac{w}{r_i} \right)
\end{align}

where $a_{-1}, a_1, \gamma_0$ are constants related to the 30° sector layout. In this study they are set to:

\begin{align}
a_{-1} &= 0.06 \quad a_1 = 0.1 \quad \gamma_0 = 0.693
\end{align}

The Nb-Ti critical surface is fit by a linear relation:

\begin{align}
j_{sc,c} = \kappa c (B_{c2}^* - B) \quad B < B_{c2}^*
\end{align}

The equation is written for the overall current density $j$, i.e. the current divided by the area of the insulated conductor. The filling ratio $\kappa$ is a dilution factor taking into account the presence of copper, voids and insulation in the coil; $\kappa$ is equal to 1 for a coil made only of superconductor and in the real case is in a range between 0.23 and 0.35 [24]. The factor $c$ is the slope of the fitting function. Introducing the critical surface fit in (2.8)-(2.12), we obtain the expression for the critical current of a sector winding of inner radius $r_i$ and width $w$ made of Nb-Ti conductor:
2.4 Forces and related stresses at short sample

\[ j_{c,Nb-Ti} = \frac{\kappa cB_{c2}^*}{1 + \kappa c r_i \lambda (r_i, w) \gamma_0 \ln \left(1 + \frac{w}{r_i}\right)} \quad (2.14) \]

The critical gradient \( G_c = \gamma(r_i, w)j_c \) can be obtained by multiplying (2.14) by (2.8):

\[ G_c(r, w) = \frac{\kappa cB_{c2}^* \gamma_0 \ln \left(1 + \frac{w}{r_i}\right)}{1 + \kappa c r_i \lambda (r_i, w) \gamma_0 \ln \left(1 + \frac{w}{r_i}\right)} \quad (2.15) \]

<table>
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<th>Temp (K)</th>
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<th>4.4</th>
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</thead>
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<tr>
<td>( c ) (A/Tmm(^2))</td>
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<td>600</td>
</tr>
<tr>
<td>( B_{c2} ) (T)</td>
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<td>10</td>
</tr>
</tbody>
</table>

Table 2.1: Nb-Ti: \( j_c \) vs. \( B \) characteristics.

The parameters of a Nb-Ti superconducting cable are listed in table 2.1 [24].

2.4.2 Nb\(_3\)Sn

The critical surface for the Nb\(_3\)Sn can be approximated by an hyperbolic law as follows:

\[ j_c(r, w) = \kappa c \left( B_{c2}^* - 1 \right) \quad B < B_{c2}^* \quad (2.16) \]

which is accurate within 5% with respect to the usual Summer law [28] between 11 and 17 T at 1.9 K, and has the advantage of allowing an explicit solution for the critical current:

\[ j_c(r, w) = \frac{k_c}{2} \left[ \sqrt{\frac{4B_{c2}^*}{k_c \beta}} + 1 - 1 \right] \quad (2.17) \]

<table>
<thead>
<tr>
<th>Temp (K)</th>
<th>1.9</th>
<th>4.4</th>
</tr>
</thead>
<tbody>
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<td>3900</td>
</tr>
<tr>
<td>( B_{c2} ) (T)</td>
<td>23.1</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 2.2: Nb\(_3\)Sn: \( j_c \) vs. \( B \) characteristics.

The parameters of a Nb\(_3\)Sn superconducting cable are listed in table 2.2.
2. Forces and stresses in superconducting quadrupoles

2.4.3 Magnetic forces

The distribution of the e.m. forces has been analyzed by setting different aspect ratios $w/r_i$, varying the aperture radius and coil width independently as well as the dilution factor $\kappa$.

Figure 2.6: $F_{x,y}$ as a function of the coil width assuming $\kappa$ equal to 0.25.
The same analysis performed for a Nb-Ti cable has been done for Nb$_3$Sn varying only the cable features, using the data in table 2.2, at 4.2K. As a general remark, the magnetic forces increase almost linearly with the coil dimension, which rules over the decreasing current density. For a given geometrical layout, the e.m. forces for a Nb$_3$Sn cable are proportional to $(j_{c,Nb_3Sn}/j_{c,Nb-Ti})^2$. One can also verify that the variation in the dilution factor $\kappa$ for a given configuration is accompanied by a change in the net magnet e.m. force as follows:

$$\frac{F_{x,y}(k_1)}{F_{x,y}(k_0)} \sim \left( \frac{\kappa_1}{1 + C(r_i, w)\kappa_1} \right)^2 \left( \frac{\kappa_0}{1 + C(r_i, w)\kappa_0} \right)^{-2}$$

(2.18)

### 2.4.4 Azimuthal stress

As for the magnetic forces, we can compute the value of the azimuthal stress on the coil mid-plane as a function of the critical current density.

![Figure 2.7: $|\sigma_{\varphi,max}|$ as a function of the coil aperture $r_i$.](image)

The maximum stress inside the coil can be computed from the analytic expression (2.6) as follows:

$$\sigma_{\varphi}(j_c) = -\frac{\sqrt{3}J_c\mu_0}{16\pi r^2} \left[ r^4 - r_i^4 + 4r^4 \ln \left( \frac{r_i + w}{r} \right) \right]$$

(2.19)

One has to solve a transcendental function, recurring to the Lambert W-Function (see Appendix C):
2. Forces and stresses in superconducting quadrupoles

\[ k = 0.25 \]

\[ 0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100 \quad 120 \quad 140 \]

\[ 40 \quad 50 \quad 60 \quad 70 \quad 80 \quad 90 \quad 100 \]

\[ r_i = 70 \text{ mm} \]
\[ r_i = 60 \text{ mm} \]
\[ r_i = 50 \text{ mm} \]
\[ r_i = 40 \text{ mm} \]
\[ r_i = 30 \text{ mm} \]

\[ Figure 2.8: |\sigma_{\phi,max}| \text{ as a function of the coil width } w. \]

\[ r = \exp \left[ -\frac{1}{4} + \frac{1}{4} \text{ProductLog} \left( \frac{e r_i^4}{(r_i + w)^4} \right) \right] (r_i + w) \quad (2.20) \]

So the maximum azimuthal stress on the coil mid plane:

\[ \sigma_{\phi,max} = -\frac{\sqrt{3} j_c^2 \mu_0}{16\pi (r_i + w)^2} \left[ -r_i^4 + e^{-1+\text{ProductLog} \left( \frac{e r_i^4}{(r_i + w)^4} \right)} (r_i + w)^4 + 4e^{-1+\text{ProductLog} \left( \frac{e r_i^4}{(r_i + w)^4} \right)} \right] \quad (2.21) \]

In fig. 2.7-2.8 we plot the maximum stress (\( \kappa = 0.25 \), LHC-MQ cable), varying the aperture radius for different values of coil width: 20, 30, 40, 60 mm respectively and viceversa, setting the aperture equal to 30, 40, 50, 60 and 70 mm. Since the most important design parameter for a quadrupole is the gradient, an analysis of \( \sigma_{\phi,max} \) vs. the critical gradient \( G_c \) has been done. Quadrupole apertures between \( r_i = 30 \text{ mm} \) and \( r_i = 70 \text{ mm} \) have been analyzed; coil widths \( w \) have chosen between 5 mm up to the value corresponding to a saturation of the critical gradient, which in our case is one-two times the inner radius. Each of the curves in fig. 2.10 correspond to a quadrupoles family with the same aperture and different coil widths. For larger coil widths one obtains larger \( G_c \) (in the analyzed range).
2.4 Forces and related stresses at short sample

Figure 2.9: Critical gradient $G_c$ as a function of the coil surface at short sample ($r_i = 30$ and 70 mm).

Figure 2.10: $|\sigma_{\varphi,\text{max}}|$ vs. the critical gradient $G_c$ at short sample for Nb-Ti cable and comparison with numerical results.

For small apertures, larger coil widths and larger $G_c$ correspond to a saturation of the stress values. On the other hand, for very large apertures the stress reaches a peak for a given coil width and then decreases; this means that adding more
2. Forces and stresses in superconducting quadrupoles

material and more cable one can reduce stress and still gain in gradient (see fig. 2.9). The results obtained from the analytical approach are then compared to the numerical ones given by a mechanical model of the winding built in ANSYS (see fig. 2.10), revealing a good agreement with the latter ones. In general in the largest analyzed case \((r_i=70 \text{ mm})\), \(\sigma_{\varphi,max}\) is just below 110 MPa. Since the function of the maximum stress is rather complicated, we can make considerations on the stress at the outer radius because it follows an analogous trend as \(\sigma_{\varphi,max}\) (see fig. 2.11). This can help in understanding the behavior of the maximum stress, though without giving indications on the value assumed by the stress at the local maximum point.

![Graph showing comparison between \(\sigma_{\varphi,r_o}\) and \(\sigma_{\varphi,max}\) for two different apertures: 30 and 60 mm respectively.](image)

The stress at the outer radius reads:

\[
|\sigma_{\varphi,r_{\text{ext}}}| = \frac{\sqrt{3} j_c^2 \mu_0}{16 \pi (r_i + w)^2} \left[ (r_i + w)^4 - r_i^4 \right] \approx \phi_1 \phi_2 \tag{2.22}
\]

where: \(\phi_1 \propto j_c^2\) and \(\phi_2 \propto \left[ (r_i + w)^4 - r_i^4 \right]/(r_i + w)^2\). The limits for small and large widths are as follows:

\[
w \to 0 \quad \phi_1 \phi_2 = r_i w \to 0; \quad w \to \infty \quad \phi_1 \phi_2 = 1/(\ln(w)) \to 0 \tag{2.23}
\]

Between these limits, the stress behaves differently according on the aperture, the coil width and the dilution factor \(\kappa\) and depends on how they combine in the
product $\phi_1\phi_2$; for an increasing stress the geometrical parameter $\phi_2$ rules over the decreasing $j_c$. When the stress decreases, it is the decrease of $j_c$ due to the higher $B_p$ ruling over the augmented dimension of the coil.

Figure 2.12: The aspect ratio $w/r_i$ where the maximum value of $\sigma_{x,\text{max}}$ occurs has been determined as a function of the aperture radius.

The stress shows a local maximum depending also on the dilution factor $\kappa$ (see fig. 2.12): the higher is $\kappa$, the higher $j_c$ and so $\phi_1$. Therefore, the higher is the dilution factor, the smaller is the ratio $w/r_i$ where the compressive stress stands at his maximum value.

We can observe that with a dilution factor $\kappa$ equal to 0.3, the maximum stress for an aperture of 60 mm reaches the limit value of 150 MPa for a Nb$_3$Sn cable. The same considerations made on the behavior of $\sigma_{x,\text{max}}$ for a Nb-Ti cable apply for the Nb$_3$Sn, the only difference is that a maximum of $\sigma_{x,\text{max}}$ appears for $r_i > 50$mm (see fig. 2.13).

### 2.4.5 Comparison between Nb-Ti and Nb$_3$Sn

The aim of this section is to compare the performances of a sector winding quadrupole constituted by either Nb-Ti or Nb$_3$Sn superconducting cable. By setting a particular geometrical layout, the two superconductors have been compared at their nominal operating temperatures: 1.9K (Nb-Ti) and 4.2K (Nb$_3$Sn) respectively (see figs. 2.14-2.15).

For instance, cooling a Nb$_3$Sn from 4.2K to 1.9K leads to a low increase in $j_c$ of about 7%. At 1.9K, being constant the coil layout and the dilution factor as
2. Forces and stresses in superconducting quadrupoles

Figure 2.13: $|\sigma_{\varphi,max}|$ vs. the critical gradient $G_c$ at short sample for Nb$_3$Sn cable and comparison with numerical results.

Figure 2.14: $|\sigma_{\varphi,max}|$ vs. the critical gradient $G_c$: comparison between Nb-Ti and Nb$_3$Sn coil at 1.9K.

well, a Nb$_3$Sn cable produces a $j_c$ higher than a Nb-Ti cable one of about 40%, on the other hand the peak stress doubles.
Nevertheless, the same gradient can be achieved with a thinner cable: e.g. to obtain 280 T/m with an aperture of 30mm, a coil width of 14 mm is needed (Nb$_3$Sn cable) instead of 40 mm (Nb-Ti cable, $\kappa$=0.25).

![Figure 2.15](image)

Figure 2.15: $|\sigma_{\phi,\text{max}}|$ vs. the critical gradient $G_c$: comparison between Nb-Ti and Nb$_3$Sn coil at 4.2K.

### 2.5 Iron effect

In this section the formulae for the magnetic field due to a magnetic coil with an iron yoke placed at a distance $R_s$ are evaluated. The effect of the iron yoke can be estimated by using the "Image current" approach (see Appendix D): the additional field is produced by an imaginary coil enclosed between radii $r'_i = R_s^2/r_i$ and $r'_o = R_s^2/r_o$, where $R_s = r_o + w_{\text{coll}}$ (see fig. 2.16).

The results obtained by the analytical approximations have been compared to the ones given by a FE model, where the iron yoke has been implemented. The condition of perpendicularity of the field lines has been imposed on the model outer boundary. In the analytical approximations, $\mu_r$ has been set equal to 3000, thus describing a not saturated iron yoke. As for a coil in air, the difference of the analytical approach from the numerical results in terms of magnetic field is below few percentage inside the aperture, but it is not so accurate if evaluated inside the winding.

An iron yoke has the main function of closing the magnetic circuit, thus increasing the magnetic field produced for the same current density. This means that
also the magnet critical current \( j_c \) is reduced because the load line hits the critical surface at higher field. Since the analytical approximation of the field inside the coil is not reliable, we cannot analytically compute the peak field, neither in the case of coil in air, nor when an iron yoke is introduced. As shown, in the first case the peak field can be computed by using the formulae proposed [24]. When an iron screen is present, a new formulation of the peak field and current density must be found.

The gradient and the peak field are assumed to be linear functions of the current \( j \), i.e. we neglect saturation. At first order one can expect that the gradient and the peak field have a similar relative increase when an iron yoke is added.

In fig. 2.17 we plot the difference between the relative increase in the peak field \( \Delta B_p / B_p \) and the relative increase of the gradient in the aperture \( \Delta G / G \), normalized to \( \Delta G / G \), for different ratios \( w/r_i \) and collar thickness \( w_{coll} \). The increment of the gradient in the aperture \( \Delta G \) has been analytically derived using the formulae of a sector coil with constant current density and iron yoke, whereas the increment of the field \( \Delta B_p \) has been computed through the numerical code ANSYS™.

It can be stated that:

- the relative difference mainly depends on the ratio \( w/r_i \), and is practically independent on the distance between the coil and the iron (i.e. the collar width);
- the 1\(^{st}\) order approximation \( \Delta B_p / B_p = \Delta G / G \) is correct within 10\% in a large part of the domain of interest \( 0.5 < w/r_i < 1 \). Since this difference has to be applied on an increment, it represents a small error.

Figure 2.16: LHC-MQ cross section.
2.5 Iron effect

The critical current density formula has to be reviewed in order to take into account the effect of the iron yoke. Since the magnetic field reached is higher than the one obtained from a simple coil in air, the current density in the winding has to be lower in order to respect the limit imposed by the superconductor critical curve. This means that by keeping the peak field, we will have an increased factor $\beta$ (magnetic field per unit current density).

Figure 2.17: Difference between the relative increase in the peak field $\Delta B_p/B_p$ and the relative increase of the gradient in the aperture $\Delta G/G$, normalized to $\Delta G/G$. The relation with the aspect ratio $w/r_i$ is independent of the yoke radius $R_s$.

The curve in fig. 2.17, obtained by averaging the relative increment at a given aspect ratio, has been empirically fit with:

$$\frac{\Delta B_p}{B_p} - \frac{\Delta G}{G} = \frac{\Delta G}{G} \left[ p_0 + p_1 \left( \frac{w}{r_i} \right)^q \right] 10^{-2} \tag{2.24}$$

where:

$$p_0 = 30 \quad p_1 = 37.4 \quad q = 0.71 \tag{2.25}$$

Though the peak field is not well described by the analytical approach, the field in the center of the aperture is in good agreement with the numerical data, due to the reliability of the field expression inside the aperture. The field gradient can be defined as:
2. Forces and stresses in superconducting quadrupoles

\[ G_{\text{iron}} = \frac{j\mu 10^6}{2\pi} \left[ \left( \frac{\mu_r - 1}{\mu_r + 1} \right) \left( \frac{(r_i + w)^4 - r_i^4}{R_s^4} \right) + 4\ln\left(\frac{r_i + w}{r_i}\right) \right] \sin(2\alpha_0) \] (2.26)

It is now possible to derive the expression of \( \beta_{\text{iron}} \) introducing (2.9) and (2.26) into (2.24) as follows (lengths are expressed in (mm)):

\[ \beta_{\text{iron}} = 10^{-5} r_i \lambda(r_i, w) \left\{ \gamma_0 \left( p_0 + p_1 \left( \frac{w}{r_i} \right)^q \right) \ln \left(1 + \frac{w}{r_i}\right) + \frac{10^7 \mu_0 \sin(2\alpha_0)}{\left( \frac{1}{\mu_r + 1} \right) R_s^4} \left( 4.456 + 2.382 \left( \frac{w}{r_i} \right)^q \right) \right\} \left( \mu_r - 1 \right) w(r_i^3 + 1.5 r_i^2 w + r_i w^2 + 0.25 w^3) + (\mu_r + 1) R_s^4 \ln \left(1 + \frac{w}{r_i}\right) \right\} \] (2.27)

Now the new expression of \( j_c \) can be derived introducing \( \beta_{\text{iron}} \) in (2.14). This new current density can be used to define the e.m. forces as well as the peak stress acting on the coil mid-plane, both for a Nb-Ti and Nb\(_3\)Sn sector coil.

![Figure 2.18](image)

Figure 2.18: \( F_x \) for a Nb-Ti sector winding (\( \kappa=0.254 \) - LHC MQ) with infinitely permeable iron yoke. The coil width is equal to 15.4mm.

By increasing the collar width the contribution of the iron yoke decreases, whilst the behavior of the magnetic forces is antithetic, due to the different field
2.5 Iron effect

Figure 2.19: \( F_y \) for a Nb-Ti sector winding (\( \kappa=0.254 \) - LHC MQ) with infinitely permeable iron yoke. The coil width is equal to 15.4mm.

![Graph showing \( F_y \) vs. \( w_{coll} \) for two different \( r_i \) values.

Figure 2.20: \(|\sigma_{\varphi,\text{max}}| \) vs. \( G_c \) at short sample for a Nb-Ti cable and comparison with numerical results (iron yoke: \( w_{coll} = 20 \) mm).

![Graph showing \(|\sigma_{\varphi,\text{max}}| \) vs. \( G_c \) for different \( r_i \) values.

distribution in the coil (see fig. 2.18-2.19). The maximum stress was computed for two different apertures, scanning finely the coil width, for an ironless case and
for a case with iron having a collar thickness of 20mm. Results are shown in figs. 2.20-2.21 for Nb-Ti and Nb₃Sn: the iron acts as a larger coil width, but the relation stress/gradient remains the same.

Figure 2.21: $|\sigma_{\phi,max}|$ vs. $G_\epsilon$ at short sample for a Nb₃Sn cable and comparison with numerical results (iron yoke: $w_{coll} = 20$ mm).

### 2.6 Analysis of accelerator quadrupoles

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<th>$w_{eq}$ (mm)</th>
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<th>$T$ (K)</th>
<th>$R_s$</th>
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</table>

Table 2.3: Characteristics for eight Nb-Ti quadrupoles cross sections; the last three have current grading.

The equations of e.m. forces have been applied to different real Nb-Ti quadrupoles layouts at short sample current, both considering a coil in air and including the
2.6 Analysis of accelerator quadrupoles

effect of an infinite permeable iron yoke. The quadrupole sections have been implemented in ROXIE. For the case of coils in air, a general good agreement within 10% is shown, exception made for the ISR quadrupole, where the current density law is less accurate. The iron yoke has been introduced for every cross section by setting the proper collar thickness, as indicated in table 2.3. The results confirm the good agreement outlined for the same coils in air, revealing a worst difference for the graded quadrupoles LHC-MQXA and MQXB.

<table>
<thead>
<tr>
<th></th>
<th>$F_x$</th>
<th>$F_y$</th>
<th>$F_{x,\text{An}}$</th>
<th>$F_{y,\text{An}}$</th>
<th>%Diff,$F_x$</th>
<th>%Diff,$F_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHC-MQ</td>
<td>0.69</td>
<td>-1.22</td>
<td>0.63</td>
<td>-1.17</td>
<td>-8.9</td>
<td>-4.1</td>
</tr>
<tr>
<td>LHC-MQM</td>
<td>0.38</td>
<td>-0.73</td>
<td>0.34</td>
<td>-0.70</td>
<td>-10.2</td>
<td>-4.4</td>
</tr>
<tr>
<td>RHIC MQ-ARC</td>
<td>0.09</td>
<td>-0.21</td>
<td>0.08</td>
<td>-0.20</td>
<td>-8.5</td>
<td>-5.9</td>
</tr>
<tr>
<td>HERA MQ</td>
<td>0.30</td>
<td>-0.61</td>
<td>0.27</td>
<td>-0.58</td>
<td>-9.7</td>
<td>-4.6</td>
</tr>
<tr>
<td>ISR MQ</td>
<td>1.22</td>
<td>-2.53</td>
<td>0.93</td>
<td>-2.17</td>
<td>-23.4</td>
<td>-14.1</td>
</tr>
<tr>
<td>Tevatron MQ</td>
<td>0.17</td>
<td>-0.35</td>
<td>0.15</td>
<td>-0.33</td>
<td>-9.7</td>
<td>-5.4</td>
</tr>
<tr>
<td>LHC-MQXA</td>
<td>1.10</td>
<td>-2.04</td>
<td>1.04</td>
<td>-1.93</td>
<td>-5.1</td>
<td>-5.2</td>
</tr>
<tr>
<td>LHC-MQXB</td>
<td>0.76</td>
<td>-1.49</td>
<td>0.72</td>
<td>-1.41</td>
<td>-5.4</td>
<td>-5.4</td>
</tr>
</tbody>
</table>

Table 2.4: Analytical force estimation and comparison with eight Nb-Ti quadrupoles; the last three quadrupoles have current grading (no iron; computations made at short sample). Forces are expressed in (MN/m).

<table>
<thead>
<tr>
<th></th>
<th>$F_x$</th>
<th>$F_y$</th>
<th>$F_{x,\text{An}}$</th>
<th>$F_{y,\text{An}}$</th>
<th>%Diff,$F_x$</th>
<th>%Diff,$F_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHC-MQ</td>
<td>0.537</td>
<td>-0.732</td>
<td>0.515</td>
<td>-0.731</td>
<td>-4.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>LHC-MQM</td>
<td>0.309</td>
<td>-0.446</td>
<td>0.300</td>
<td>-0.436</td>
<td>-2.9</td>
<td>-2.3</td>
</tr>
<tr>
<td>RHIC MQ-ARC</td>
<td>0.099</td>
<td>-0.0842</td>
<td>0.092</td>
<td>-0.077</td>
<td>-6.7</td>
<td>-8.3</td>
</tr>
<tr>
<td>HERA MQ</td>
<td>0.148</td>
<td>-0.187</td>
<td>0.134</td>
<td>-0.180</td>
<td>-9.5</td>
<td>-3.8</td>
</tr>
<tr>
<td>ISR MQ</td>
<td>0.911</td>
<td>-0.838</td>
<td>0.754</td>
<td>-0.685</td>
<td>-17.2</td>
<td>-18.2</td>
</tr>
<tr>
<td>Tevatron MQ</td>
<td>0.137</td>
<td>-0.209</td>
<td>0.121</td>
<td>-0.201</td>
<td>-11.4</td>
<td>-4.0</td>
</tr>
<tr>
<td>LHC-MQXA</td>
<td>1.635</td>
<td>-1.573</td>
<td>1.356</td>
<td>-1.343</td>
<td>-17.1</td>
<td>-14.6</td>
</tr>
<tr>
<td>LHC-MQXB</td>
<td>0.868</td>
<td>-1.13</td>
<td>0.704</td>
<td>-0.925</td>
<td>-18.9</td>
<td>-18.1</td>
</tr>
</tbody>
</table>

Table 2.5: Magnetic forces computations for some superconducting magnets and comparison with analytical values. The data are derived at nominal operating conditions, assuming an infinitely permeable iron (lengths are in (mm), forces in (MN/m)).
2.7 Summary

A simplified model of superconducting quadrupole has been analyzed, namely a 30 sector coil with uniform current density $j$. We outlined an analytical approach that allows to predict the stress distribution along the mid-plane with good precision, and that holds also for more realistic non-isotropic materials. We computed the stress at the short sample condition, showing that it increases for larger quadrupole apertures, whereas the dependence on the coil thickness is more involved and for large apertures larger coils can give larger gradients and smaller stress. For the Nb-Ti the stress is below 100 MPa for apertures radii smaller than 60 mm ($\kappa=0.3$). For the Nb$_3$Sn the stress for aperture radii smaller than 60 mm is below 150 MPa with the same cable features, which is taken as guideline for $\sigma_{\phi,max}$ before degradation of the superconducting properties of the material. Introducing an iron screen, both the critical gradient and the peak stress increases to reach the same level as it would have in an ironless larger coil creating the same gradient. So further studies can be accomplished on coil in air (easier to treat analytically) since the peak stress depends only on $G_c$. 
Chapter 3

Forces and stresses in superconducting dipoles

3.1 Introduction

As it has been introduced in Chapter 2, Nb-Ti is nowadays used in superconducting magnets of particle accelerators, leading to a maximum field of about 10 T at short sample condition and stress of the order of 100 MPa \([29], [30], [31], [32], [33]\). In order to push the dipole central field beyond the threshold of 10 T, Nb\(_3\)Sn is at present the only practical superconductor that can be used \([34], [35] and [36]\). Following the main guidelines adopted in Chapter 2, aim of this chapter is to provide a set of equations that give an estimate of the electromagnetic forces and stresses of a dipole sector coil as a function of the aperture \(r_i\), of the coil width \(w\) and of the superconducting material. A simplified sector coil layout has been used with angular extension \(\alpha_0 = 60^\circ\) (thus canceling the sextupole coefficient in the field series expansion) with constant current density \(j\). A similar layout can well represent from the electromagnetic point of view several dipoles built in the past decades \([37]\).

Following the same approach adopted for quadrupoles, the main steps carried out are as follows:

- estimate analytically the magnetic field, the electromagnetic forces and the induced stresses in the sector coil as a function of \(w\) and \(r_i\) for a given current density \(j\). A similar approach has been developed in \([34]-[35]\) using a \(\cos \varphi\) model, and will not be treated in this chapter.

- Introduce the expression of the critical current density \(j_c\), thus evaluating the peak compressive stress on the coil mid plane as a function of the bore
3. Forces and stresses in superconducting dipoles

field $B_0$. The results have been cross-checked with a simple finite element model built in ANSYS”. No iron yoke has been considered in this analysis.

- Modify the expression of $j_c$ and revise the formulae of forces and stresses when an infinite permeable iron yoke surrounds the coil.
- Compare the results given by a sector coil with some dipoles cross-sections, both with and without iron yoke at short sample condition. The cross sections have been implemented in ROXIE and ANSYS.”

![Figure 3.1: LHC MB dipole cross section (a) and sector coil model adopted (b).](image)

### 3.2 Analytical formulae of magnetic field and Lorentz force components

The expressions for the magnetic field can be derived from the Magnetic Vector Potential for a sector coil with constant current density, following the formalism developed in [38]. The expression for the magnetic field components within the coil aperture ($0 \leq r \leq r_i$) are as follows:

\[
\begin{align*}
\{ B_r \} &= -\frac{j\mu_0}{\pi} \left\{ (r_i + w) - r_i \right\} 2\sin\alpha_0 \left\{ \sin\varphi \cos\varphi \right\} + \\
&\quad + \sum_{m=1}^{\infty} \frac{r_i^{2m}}{(2m+1)(1-2m)} \frac{r_i^{1-2m}}{2\sin(2m+1)\alpha_0} \left\{ \sin\varphi \cos\varphi \right\} \\
\{ B_\varphi \} &= -\frac{j\mu_0}{\pi} \left\{ (r_i + w) - r_i \right\} 2\sin\alpha_0 \left\{ \sin\varphi \cos\varphi \right\}
\end{align*}
\]

(3.1)
3.2 Analytical formulae of magnetic field and Lorentz force components

Whilst the expression for the field components outside the winding are as follows:

\[
\begin{align*}
\begin{cases}
B_r \\
B_\phi
\end{cases} &= -\frac{j\mu_0}{\pi} \left\{ \frac{((r_i + w)^3 - r_i^3)}{3r^2} \right\} 2\sin\alpha_0 \left\{ \frac{\sin\varphi}{\cos\varphi} \right\} + \\
+ \sum_{m=1}^{\infty} \frac{(r_i + w)^{2m+3} - r_i^{2m+3}}{r^{2m+2}(2m+3)(2m+1)} \frac{2\sin(2m+1)\alpha_0}{2\sin\varphi \cos\varphi} \right\}
\end{align*}
\]

By combining them as in Sec. 2.2 and considering only the first term of the series expansion, one can get the expressions for the fields inside the coil:

\[
\begin{align*}
\begin{cases}
B_r \\
B_\phi
\end{cases} &= -\frac{j\mu_0}{\pi} \left\{ ((r_i + w) - r) + \frac{r_i^3 - r_i^3}{3r^2} \right\} 2\sin\alpha_0 \left\{ \frac{\sin\varphi}{\cos\varphi} \right\} 
\end{align*}
\]

By integrating the field components over the coil surface (see Appendix A), one can obtain the equations for the magnetic forces acting inside the coil as follows:

\[
\begin{align*}
F_x &= \frac{j^2\mu_0\sin\alpha_0\sin\alpha_0}{9\pi} \left[ 3\alpha_0((r_i + w) - r_i)^2((r_i + w) + 2r_i) + r_i^3 - (r_i + w)^3 + 3r_i^3\ln\left(\frac{r_i + w}{r_i}\right) \right] \\
F_y &= \frac{2j^2\mu_0\sin^3\alpha_0}{9\pi} \left[ (r_i + w)^3 - r_i^2 - 3r_i^3\ln\left(\frac{r_i + w}{r_i}\right) \right]
\end{align*}
\]

3.2.1 Comparison with the Finite Element model

The equations for the magnetic field given in Sec. 2.2 has been numerically verified by means of a finite element model of a sector coil with: \( r_i=30 \text{ mm}, \ w=30 \text{ mm}, \) and with constant current density \( j=1000 \text{ A/mm}^2. \) No assumption has been done on the superconductor, so the current density value is just as a reference to validate the analytical model. The results obtained can be summarized as follows:

- the analytical formulae have been evaluated considering the first term only of the series expansion, since they give a good description of the field;
- as for a quadrupole coil, the analytical expressions describes the field inside the aperture and outside the coil within a few percentage of difference with the numerical model;
- the field inside the coil is in worst agreement with the numerical results, leading to a difference of about 20% at the coil pole. As a matter of fact, the analytical equations are not reliable for peak field calculation.
3. Forces and stresses in superconducting dipoles

Figure 3.2: $B_r$ and $B_\phi$ distribution within the coil for a sector winding with $r_i=30$ mm, $w=30$ mm, and constant current density $j=1000$ A/mm$^2$. 
3.2 Analytical formulae of magnetic field and Lorentz force components

Figure 3.3: Magnetic forces varying the geometrical lay out parameters. Fig. (a): the coil width has been set to 30 mm, varying the aperture from 20 to 50 mm. Fig. (b): the aperture $r_i = 30$ mm, the coil width varies from 15 to 50 mm.
3. Forces and stresses in superconducting dipoles

Though the magnetic field is not well described within the coil by the analytical approach, the integration of the magnetic energy component over the coil surface differs from the numerical one of few percentage. Consequently, the magnetic forces are expected to be in good agreement with the numerical results. A parametric analysis has been set up, realizing a model with the following sets of radial apertures and coil widths:

- \( r_i \): [20, 30, 40, 50] mm
- \( w \): [15, 20, 30, 40, 50] mm

all the possible combinations have been studied for a constant current density of 1000 A/mm\(^2\). The analysis reveal that both \( F_x \) and \( F_y \) have a linear trend with the square of the coil width, as well as with the aperture. In particular, \( F_x \) underestimates the numerical value of about 3%, whilst \( F_y \) overestimates it of about 6%, both regardless of the coil geometrical layout.

3.3 Mechanical stresses

By balancing the forces acting on an infinitesimally small coil element, it is possible to derive the equations of the azimuthal compressive stress on the coil mid plane \( \sigma_\varphi(r) \), and of the radial compressive stress on the external profile of the coil \( \sigma_r(\varphi) \) (see Appendix B). Since the shear effect has been neglected, it is not possible to analytically quantify how the coil material affects the stress distribution; nevertheless the numerical results show this is a second order effect (see sec. 2.3.1), so that the provided equations can represent the stress profile on a real anisotropic coil within a few percent of difference. By solving the differential balance equations one can get:

\[
\sigma_\varphi(r) = \frac{j^2 \mu_0 \sqrt{3}}{6\pi r} \left[ 2r^3 + r_i^3 - 3r^2(r_i + w) \right]
\]  
(3.6)

\[
\sigma_r(\varphi) = \frac{j^2 \mu_0 \sqrt{3}}{18\pi (r_i + w)} f_{pr}(r_i^3, w^3, \varphi)
\]  
(3.7)

where:

\[
f_{pr}(r_i^3, w^3, \varphi) = 6w^2(3r_i + w)\cos\varphi - \frac{1}{2} \left[ w(6r_i^2 + 15rw + 5w^2) + 6r_i^3\ln\left(\frac{r_i}{r_i + w}\right) \right]
\]  
(3.8)
3.3 Mechanical stresses

A numerical study has been carried out to compute both the maximum stress values and their profiles on the coil, as a function of the aperture radius - [20, 30, 40, 50] mm - and of the coil radial width: [10, 20, 30, 40, 50] mm.

Figure 3.4: Azimuthal stress distribution on coil mid-plane $\sigma_r(\varphi)$ for two different aspect ratios $w/r_i$ (0.4 for $r_i=50$ mm and 1 for $r_i=30$ mm) with a current density $j=1000$ A/mm$^2$.

The azimuthal peak stress given by (3.6) generally differs from the numerical one by about 3%. A first order guess of the location of the peak azimuthal stress is $\sim 2/3$ of the coil width [35]; the FE model shows that this location moves towards the outer part of the coil for large coil widths. The difference in the peak stress position spans from $<5\%(w/r_i \sim 0.4)$ to $\sim 30\%$ for $w/r_i = 2$.

The radial peak stress, placed on the midplane at $\varphi = 0$, is in very good agreement with the numerical model (difference below 1%); nevertheless the two distributions differ towards the coil pole in a range of $\pm 30\%$ with the coil width varying from 10 to 50 mm.

The expressions for the peak azimuthal and radial stress for a 60° sector coil are as follows:

$$\sigma_{\varphi,max} = \frac{j^2\mu_0\sqrt{3}}{6\pi r} \max_{r\in[r_i,r_i+w]} \left[2r^3 + r_i^3 - 3r^2(r_i + w)\right]$$

$$\sigma_{r,max} = \frac{j^2\mu_0\sqrt{3}}{36\pi(r_i + w)} \left[w(6r_i^2 + 15rw + 5w^2) + 6r_i^3\ln\left(\frac{r_i}{r_i + w}\right)\right]$$

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3. Forces and stresses in superconducting dipoles

Figure 3.5: Radial stress distribution $\sigma_r(\varphi)$ on coil radial edge for two aspect ratios $w/r_i$ (0.3 for $w=10$ mm and 1.6 for $w=50$ mm) with an aperture radius $r_i=30$ mm and a current density $j=1000$ A/mm$^2$.

3.4 Forces and stresses at short sample

As it has been introduced in section 2.4, in a superconducting magnet the current density $j$ is limited by the critical current density $j_c$ at peak field. In the following paragraphs the magnetic forces and the peak stress on the coil mid-plane, responsible of the conductor degradation, will be evaluated for different coil layouts, computing the critical current density for each configuration using the approximation based on sector coils given in [37].

3.4.1 Nb-Ti

In order to define the critical current density $j_c$ for a dipole sector coil, we introduce the following parameters:

central field : $B = j \gamma(w)$ \hspace{1cm} (3.11)

peak field : $B_p = j \lambda(r_i, w) \gamma(w)$ \hspace{1cm} (3.12)

where $\gamma$ (Tm$^2$/A) is the central field per unit of current density and $\lambda$ (adim) is the ratio between the peak field and the central field.
3.4 Forces and stresses at short sample

In the case of one sector layer at 60° one has:

\[ \gamma(w) = \gamma_0 w \]  \hspace{1cm} (3.13)

and \( \lambda \) is well fit by an empirical expression:

\[ \lambda(r_i, w) = 1 + \frac{ar_i}{w} \]  \hspace{1cm} (3.14)

where \( a = 0.04 \) and \( \gamma_0 = 6.93 \times 10^{-7} \) are constants related to the 60° sector layout.

In Nb-Ti superconducting dipoles, the filling factor \( \kappa \) is in a range between 0.23 and 0.3 [37], whereas the slope of the fitting function \( c \) can be defined as in table 2.1. By introducing (3.12) into (2.13) we can get the expression of \( j_c \) corresponding to the peak field \( B_{p,c} \) on the critical surface:

\[ j_{c,Nb-Ti} = \frac{\kappa c B_{c2}^*}{1 + \kappa c \lambda(r_i, w) \gamma_0 w} \]  \hspace{1cm} (3.15)

3.4.2 Nb\(_3\)Sn

The equation of the critical current density for a Nb\(_3\)Sn cable is given by (2.16). By introducing (3.12) into (2.16), one can easily get the expression for \( j_c \) as a function of the peak field:

\[ j_c(r, w) = \frac{k c}{2} \left[ \frac{4B_{c2}^*}{k c \lambda \gamma} + 1 - 1 \right] \]  \hspace{1cm} (3.16)

The values of \( B_{c2}^* \) and \( c \) are given in table 2.2, whereas typical values for the dilution factor \( \kappa \) range from 0.26 to 0.34.

3.4.3 Magnetic forces

The forces have been obtained at 1.9 K for Nb-Ti and at 4.2 K for Nb\(_3\)Sn; in order to have comparable results, the dilution factor has been set to 0.35 for both superconductors.

The coil radius has been varied from 20 to 60 mm, whilst the coil widths from 5 to 80 mm. For a given geometrical layout, the increase in current density, and so in central field, is about 30 to 40%, depending on the aspect ratio \( w/r_i \); nevertheless the magnetic forces, being functions of \( j^2 \), almost double (see fig. 3.6). The trend followed is directly related to the gain in central field: the smallest the coil, the highest is the increase of the central field, and the related forces, due to cable add-on. For larger coil widths, the central field tends to saturate as the added material is less and less effective (see fig. 3.7).
3. Forces and stresses in superconducting dipoles

Figure 3.6: $F_y$ varying the coil width, assuming $\kappa=0.35$ for Nb-Ti (a) and Nb$_3$Sn (b) cables.
3.4 Forces and stresses at short sample

3.4.4 Azimuthal stress

By introducing the expression of \( j_c \) into (3.9), and substituting \( r_{\text{max}} \) with \( r_i + 2/3w \), one can obtain the equation for the peak stress on the coil mid plane at short sample condition. The peak stress has been related to the maximum central field achievable for a given coil layout, since the latter can be considered as the main design parameter for a dipole magnet.

In figs. 3.8 and 3.9 we show that if larger coil widths are used, thus obtaining higher fields, one can reduce at the same time the peak stress. This effect becomes important at large apertures. For instance, with Nb-Ti and 80 mm aperture radius, 10 mm coil width provides 7 T and 250 MPa, whereas a 40 mm coil width provides 10.5 T and 180 MPa: higher fields and larger coils imply lower stress. This effect is mainly due to the fact that a larger coil width allows to decrease the current density, and in (3.9) the reduction induced by the current density term is larger than the increase to the geometrical factor, i.e., the term between square brackets in (3.9). In order to respect the mechanical limit of 150 MPa for a Nb\(_3\)Sn dipole, the biggest aperture radius that can be used is 30 mm with a coil width of 45 mm and a central field of 15 T (\( \kappa = 0.35 \)). In order to get lower stress for larger apertures, larger coil widths are needed; nevertheless for smaller apertures, the mechanical limit is no more constraining the coil size, its choice being a compromise between the bore field and the coil performance.

Figure 3.7: Dipole central field \( B_0 \) as a function of the coil surface at short sample \((r_i = 30 \text{ and } 60 \text{ mm})\).
3. Forces and stresses in superconducting dipoles

Figure 3.8: $|\sigma_{\phi,\text{max}}|$ vs. the bore field $B_0$ at short sample for a Nb-Ti cable and comparison with numerical results.

Figure 3.9: $|\sigma_{\phi,\text{max}}|$ vs. the bore field $B_0$ at short sample for a Nb$_3$Sn cable and comparison with numerical results.
3.4 Forces and stresses at short sample

It is important to remind that large dilution factors (i.e. more superconductor in the coil) increase stress even when the comparison is made at the same short sample field. For instance, assuming $r_i=30$ mm and a target field $B_0$ of 15 T, this can be achieved with a 60 mm coil width and $\kappa=0.25$, or with 45 mm coil width and $\kappa=0.35$. In the second case the stress is 25% larger (160 MPa instead of 130 MPa, case of Nb$_3$Sn at 4.2K). Of course, another way to reduce stress is to operate the magnet far from the critical surface.

3.4.5 Comparison between Nb-Ti and Nb$_3$Sn

The aim of this section is to compare the performances of a sector winding dipole constituted by either Nb-Ti or Nb$_3$Sn superconducting cable.
3. Forces and stresses in superconducting dipoles

3.5 Iron effect

The main effects of using an iron yoke in a superconducting magnet have been introduced in section 2.5. They can be summarized as follows: (a) to shield the external side of the magnet from the inner magnetic field, (b) to induce higher peak and bore fields for the same current density and (c) to help increase the structure stiffness under the effect of the mechanical stresses. As we have done with quadrupoles, we use the image current method to analytically account for the effect of an infinitely permeable iron yoke (Appendix D). The same considerations made for a coil in air hold in the case of a ferromagnetic screen: we can very well reproduce the bore field, but not the peak field, unless committing an error of about 20%. Since the magnet load line hits the critical surface at higher fields, the critical current density $j_c$ has to be reduced.

As a first approximation, we assume that the peak field and the bore field are linear functions of the current density $j$, i.e. we neglect saturation induced in the ferromagnetic yoke. In fig. 3.12 we plot the difference between the relative increase in the peak field $\Delta B_p / B_p$ and the relative increase in central field $\Delta B_0 / B_0$, induced by the iron with respect to the ironless case, normalized to $\Delta B_0 / B_0$. Different collar thickness $w_{coll}$ have been considered (see fig. 3.12) ranging from 10 to 60 mm in steps of 10 mm, and different aspect ratios $w/r_i$. The increment in the bore field has been analytically computed using (D-6), whereas the increment in peak field has been numerically evaluated by means of a FE model.
3.5 Iron effect

Figure 3.12: Relative difference (in percentage) between the relative increment in $B_p$ and $B_0$ versus the aspect ratio $w/r_i$. The variation bars define the spread from the average values due to the different collar thickness $w_{coll}$, varying from 10 to 60 mm.

It has been observed that:

- the approximation $\frac{\Delta B_p}{B_p} = \frac{\Delta B_0}{B_0}$ is correct within 16% for aspect ratios $w/r_i=0.5$;
- the relative difference is mainly dependent on the aspect ratio, whereas the dependence on collar width $w_{coll}$ can be neglected, being within 4% for $w/r_i=0.5$.

The curve in fig. 3.12 can be empirically fit with:

$$\frac{\Delta B_p}{B_p} - \frac{\Delta B_0}{B_0} = \frac{\Delta B_0}{B_0} \left[ p \left( \frac{w}{r_i} \right)^{-q} \right]$$

(3.17)

where: $p = 8.72 \times 10^{-2}$ and $q=0.861$. Using this fit one can derive the expression for $(\lambda \gamma)_{iron}$:

$$(\lambda \gamma)_{iron} = \lambda_{iron} \left\{ \left[ p \left( \frac{w}{r_i} \right)^{-q} \right] + 1 \right\}$$

(3.18)

by introducing (3.18) into (3.15)–(3.16) one can get then the value of the critical current density $j_{c,iron}$ for both superconductors. As for the magnetic field and forces, the equations of the mechanical stresses have to be properly modified in
order to take into account the iron effect. The expression of the stress profile on the coil mid plane is given by:

\[
\sigma_{\varphi,\text{iron}} = \frac{j^2 \mu_0 \sqrt{3}}{6\pi r} \left[ 2r - 3(r_i + w) + \frac{r_i^3}{r^2} - \left( \frac{\mu_r - 1}{\mu_r + 1} \right) \frac{(r_i + w)^3 - r_i^3}{((r_i + w) + w_{\text{coll}})^2} \right]
\]  \hspace{1cm} (3.19)

By fitting the results given by the analytical approximation of the azimuthal stress, we can obtain the position of the peak stress along the mid plane in a semi-empirical way:

\[
r(\sigma_{\varphi,\text{iron}}) = ar_i + bw + c
\]  \hspace{1cm} (3.20)

where \(a=1.03\), \(b=0.88\) and \(c=-2.96\). This fit has been verified for an aperture range of 10-60 mm, a coil width range of 10-50 mm and for collar width varying from 20 to 35 mm. By introducing (3.20) in (3.19), one can obtain the expression of the peak stress on the coil mid plane.

![Figure 3.13: Maximum compressive stress on coil mid plane \(\sigma_{\varphi,\text{max}}\) as a function of the coil width \(w\) at short sample for a Nb-Ti coil in air and with iron screen (coil width: 5-80 mm, step of 2.5 mm, \(w_{\text{coll}}=30\) mm).](image-url)
3.5 Iron effect

Figure 3.14: $|\sigma_{\phi,\text{max}}|$ vs. bore field $B_0$ at short sample for a Nb-Ti cable and comparison with numerical results (coil width: 5-80 mm, step of 2.5 mm. Iron yoke: $w_{\text{coll}}=30$ mm).

Figure 3.15: $|\sigma_{\phi,\text{max}}|$ vs. bore field $B_0$ at short sample for a Nb$_3$Sn cable and comparison with numerical results (coil width: 5-80 mm, step of 2.5 mm. Iron yoke: $w_{\text{coll}}=30$ mm).
3. Forces and stresses in superconducting dipoles

If we assume the same geometrical layouts as for the case in air, the analysis of the peak stress vs. the bore field $B_0$ gives the following results:

- for small coil widths ($w < 15 - 20$ mm), the maximum stress is higher than for the same case in air.
- for larger widths the stress decreases almost monotonically with the increase of the central field.

This effect is more evident if we increase the dilution factor $\kappa$, i.e. the quantity of superconductor in a cable, thus obtaining a stress decrease in a range of 4-10%, by comparing the same layout in air and with iron.

3.6 Analysis of accelerator dipoles

In order to verify the validity of the analytical approach, we have compared the results of some real dipole cross-sections [29]-[30]-[31]-[32]-[33] implemented in ANSYS®, both for the case in air and with a circular iron yoke. The input current density has been computed for each configuration by using [39] and then introduced into the numerical model. In order to apply (3.6) and (3.19), one has to use an equivalent coil width so that the sector coil share the same surface as the real cross section:

$$w_{eq} = \left( \sqrt{1 + \frac{3A}{2\pi r_i^2}} - 1 \right) r_i$$  \hspace{1cm} (3.21)

where $A$ is the surface of the insulated coil, taking into account the presence of copper wedges. The magnetic forces are in agreement with the numerical data within 20% in air and 10% considering the iron yoke (table 3.2). The peak stress along the mid plane agrees within 10% for LHC, RHIC and SSC, whereas the analytical $\sigma_\phi$ underestimates the numerical one of about 30% for Tevatron and HERA dipoles (table 3.3).

Nevertheless, the position of the peak stress given by (3.20) approximates the numerical result within 16%. The discrepancy observed in the peak stress for the latter two dipoles has been numerically investigated by means of a double layer sector coil with constant current density ($j = 1000$ A/mm$^2$) and no wedges, as the Tevatron main dipole. By varying the relative angle $\Delta \alpha$ between the first and the second layer pole, thus keeping constant the coil surface, one can observe an increase in the peak azimuthal stress of $\sim 40\%$ mainly due to a redistribution of the magnetic forces inside the inner layer.
3.7 Summary

In this chapter tools to evaluate magnetic field, forces and related stresses have been introduced. By studying the stress distribution at the short sample limit, we outlined how the maximum stress depends on the central field, on the coil aperture and width. In general, for aperture radii larger than 30 mm one finds that larger and larger coil widths provide higher field but lower peak stress. For Nb$_3$Sn, in order to respect the limit of 150 MPa at the short sample limit, apertures radii $<$30 mm are advisable, unless a large coil is adopted ($w>$40 mm). Moreover, a cable with a lower filling ratio, i.e., with more copper, can help reducing the mechanical...
3. Forces and stresses in superconducting dipoles

Figure 3.16: Azimuthal stress on LHC MB and Tevatron MB dipoles with iron yoke and comparison with the sector coil approach at short sample.

...stress, having fixed a required field. These considerations hold at short sample limit; it is always possible to reduce the peak stress by operating far from the critical surface, by setting $j < j_c$. The use of an iron screen helps to reduce the coil width for a given field $B_0$, leading to an increase of the peak stress over the coil mid-plane within few percent. A comparison with several dipole cross-sections reveals an agreement between the numerical and the analytical approach within 30%. The agreement is within 10% for coils which look closer to a simple 60°sector coil. Cases with two layers and very different pole angles give a less favorable pre-stress, up to 30% larger than the corresponding sector coil. It is important to remark that in quadrupoles the peak stress increases as a function of the coil width for small apertures (<30 mm), whereas it always decreases for dipoles.

By comparing dipoles and quadrupoles sharing the same aperture, superconductor...
type and coil width, it appears that the difference in peak stress is actually constant for a given aspect ratio $w/r_i$. A dipole with aspect ratio $w/r_i=2$ will always present peak stresses 20% higher than a quadrupole with the same aspect ratio. For a $w/r_i=0.5$ the difference increases to $\sim 55\%$. 
3. Forces and stresses in superconducting dipoles
Part III

Conceptual design of a sample holder for Nb$_3$Sn cable tests
Chapter 4

The Cable Sample Holder

4.1 Introduction

So far Nb$_3$Sn appears to be the most suitable superconductor for new generation magnets for future accelerators, leading to peak fields above 10 T. Nevertheless, high fields means large Lorentz forces in the coil which load the cable and could eventually lead to the degradation of its capacity to carry high transport current. Though several studies have been done on the axial strain-current dependence of Nb$_3$Sn wires ([3]-[4] among the others), the behavior of such a conductor under applied transverse load is nowadays still not well described. Several studies have been carried out in the past [5]-[6]-[7]-[8], where different cable structures have been analyzed and current degradation phenomena have been outlined for a given transverse load applied. These studies outlined a reduction of the cable transport current due to the applied stress, and above all they highlighted permanent current degradation phenomena, once the cable is unloaded. Different can be the causes of this behavior; the main parameters affecting the current degradation can be summarized as follows:

- strand production techniques: Modified Jelly Roll (MJR), Powder-In-Tube (PIT), Rod Restack Process (RRP), and Bronze processes;

- strand structure: strand diameter, number of subelements, and ratio Cu/non-Cu. The latter is defined as the ratio between the amount of stabilizing Cu over the superconductor volume in a strand;

- cable structure: number of strands, filling ration and twist pitch;

- test apparatus, i.e. how the transverse pressure is applied on the cable and the effective pressure length;
4. The Cable Sample Holder

- cable cross section, namely keystoneed cross section, used in cosθ magnets, and rectangular cross section used in racetrack and common coil magnets [41];

According to [6], PIT strand cables show a current reduction of about 4% at 100 MPa up to 8% at 200 MPa, whereas VAC type bronze route cable experiences a reduction ranging within 7-22% with pressure of 100-200 MPa. Nevertheless cables sharing the same strand production procedure but with different structure behave differently. TWA-MJR cables tested at National High Magnetic Field Laboratory (NHMFL) [8] show a current reduction of about 15% under 185 MPa, with no relevant current permanent degradation. For higher transverse loads (~ 200 MPa) the current slightly degrades to 96 % of the virgin cable’s.

A similar MJR strand cable tested at the University of Twente [7] revealed a more severe reduction of 40-60% at 200 MPa, depending on the number of strand subelements. Moreover, the reversibility of the phenomenon appears somehow contradictory: these tests outlined a permanent endamaging of the cable, limiting the current up to 30% of \( j_c \) for a 48 strand cable.

The main differences between the results could be reasonably addressed to the test apparatus used, and not only on the cable features, even though an influence of the latter cannot be neglected. The need of a more systematic approach to cable testing to have a reproducible set of data appears to be nowadays fundamental. The aim of this chapter is to describe a new possible solution for a cable sample holder to be tested in the FRESCA facility at CERN.

In the first part, we will briefly describe the different test facilities and existing cable sample holders. The conceptual mechanical design for a cable sample holder will be then introduced and discussed, comparing different solutions. These will discriminate on the base of the overall structure strength; the influence of the magnetic field and related e.m. forces will be analyzed for the proposed design only.

4.2 Test facilities and existing cable sample holders

4.2.1 The test facility at Twente University

The test device developed at Twente is represented in fig. 4.1. It consists of a 16 T/80 mm bore magnet in combination with a current supply and a cryogenic press. The current supply is a superconducting transformer system operated in a feedback mode to generate a truly stationary current in the sample of a 50 kA maximum.
4.2 Test facilities and existing cable sample holders

Figure 4.1: Schematic view of the test facility at University of Twente, the Netherlands.

The press is capable of producing 250 kN which is equivalent to about 300 MPa transverse pressure onto a cable area of $20 \times 42 \text{ mm}^2$. It consists of a superconducting coil system by which the repulsing force between the coils is transferred to the pressure blocks which impresses a prepared section of the cable. The force acting on the cable is adjusted by control of the current in the coil system. The cable to be investigated is formed into a U-shape and inserted in the background field magnet; since the magnet bore is 80 mm, the sample section that is perpendicular to the applied field is limited to about 55 mm. By changing the shape of the pressure block, both rectangular and keystoned cables can be tested. The test reported in literature have been performed at 11 T maximum of background field. Due to the reduced section where the transverse pressure is applied, the cable twist pitch plays a fundamental role in the results. Lower twist pitch means that a strand can pass from the inner to the outer face of the cable along the pressure length, whereas with a bigger twist pitch one strand can remain straight above or below the cable broad face. Both strands experience the compression of the cross-over points with strands in the other layer, but the first type of strand is subject to a complete different strain pattern. This influence in length has been shown to be especially important if the edge effects are the dominant source of current reduction in the test device.
4. The Cable Sample Holder

4.2.2 The test facility at NHMFL

The loading fixture developed at NHMFL [8] is substantially different from the one developed at Twente. The test facility consists of a pair of split solenoid magnet able to produce a field of 13 T with the solenoid axis in the horizontal orientation. The cable samples are inserted into a radial access port, 30 mm by 70 mm, which is perpendicular to the solenoid axes, so that the cable are tested with a background field perpendicular to the cable current direction.

Figure 4.2: Schematic view of the test facility at NHMFL (National High Magnetic Field Laboratory). The sample holder is placed within a split solenoid and then subject to transverse pressure exerted by a He gas piston.

The transverse pressure is transmitted to the sample by means of a system integrated in the magnet: it consists of a steel piston pushing the sample holder against a rigid Anvil back-plate. The piston assembly has been designed to work with He gas pressures up to 10 MPa, which is just below the solidification point of liquid helium at 4.2 K, set as test temperature. The sample holder [42] consists of a stainless steel "U" channel base accommodating a stack of four cables in the loading region, two active cables which carry the current and two dummies. The active cables carry the current in opposite directions and the dummy cables on either side of the active cables simulate a magnet environment and also reduce any
stress concentration that may arise from non uniform loading. The active cables are spliced together in a copper case at the lower end to provide a continuous current path, and are soldered to Cu bas plates at the upper end where the connections to the test system current supply are made.

Figure 4.3: Sample holder developed by [42] for testing cable under transverse pressure at NHMFL. The pressure is transmitted to the cable over a length of 970 mm, by means of a pressure bar moved by the steel piston.

The cable samples are vacuum impregnated with epoxy in situ in order to provide additional cable support and again reproduce the magnet coil mechanical environment. The sample holder is 1220 mm long overall, with an active sample length of 970 mm; the cross section dimensions are fixed by the inlet port dimensions to 60.5x26 mm. The pressure is exerted on the cable stack by means of rectangular bar, held in place by the cover plate. This rectangular bar lies along the cable stack and features a rectangular protrusion, called ”pressure-dog”, that contacts the loading piston. The sample holder area where the pressure is exerted is so defined by the width of the pressure dog, i.e. of the cable, and by the diameter of the active piston, equal to 122 mm. It is then fixed to a maximum value of 122x15 = 1830 mm$^2$. This gives a maximum available applied transverse pressure of about 290 MPa. Different pressure dog and bar dimensions can be set to test
different cable dimensions; this fixture also allows to perform both "face loaded" and "edge loaded" tests, by simply rotating the cables by 90° in the holder.

### 4.2.3 The FRESCA facility at CERN

The FRESCA facility (Facility for the REception of Superconducting CAbles) at CERN has been originally realized around 1999 for $I_c$ measurements of the LHC NB-Ti cable at 1.9K [43].

![Figure 4.4: The FRESCA test facility [43].](image)

1) Outer cryostat, 2) Inner cryostat, 3) Magnet, 4) λ-plate outer cryostat, 5) λ-plate inner cryostat, 6) λ-plate sample holder, 7) Sample holder, 8) Current leads magnet 18 kA, 9) Current leads sample 32 kA, 10) Rotating system for the sample holder.

It consists of a 1.7 m long dipole magnet with an aperture of 88 mm used as the background magnet, having a maximum central field of 10 T, for tests both at 1.9 K and 4.5 K, and a field uniformity over a length of 600 mm. Due to the
magnet dimensions, a double cryostat configuration is chosen: the background magnet is put in a separate cryostat which is cooled down independently; the sample holder is then housed in a second cryostat, contained in the aperture of the main magnet. This solution allows to test different cables by keeping the main magnet cold. The magnet is connected through a pair of 18 kA current leads to a 16 kA current supply system, whereas the cables tested are powered by a 32 kA current supply through self cooled current leads. The main disadvantage of the double cryostat system is that a diameter of 72 mm is available in the inner cryostat to accommodate the sample holder, which was designed with a diameter 69 mm. As for the system at NHFML, a system has been integrated in the facility to rotate the holder of 90° inside the magnet, to perform test with both parallel and transverse background field to the broad face of the cable.

Figure 4.5: The Nb₃Sn sample holder to be used at FRESCA facility. (a) The sample holder, (b) bottom section with the sample holder inserted in the "collars", the bolts holes are highlighted in green, (c) top section with the current leads, (d) schematic of the cross section. The holes right below the top plate house the O-rings to seal the sample holder during impregnation.
A specific sample holder for Nb$_3$Sn cable for critical current and instability measurements at FRESCA has been developed by [44]. This sample holder is based on the same concept as the one developed by [42] for tests at BNL and NHFML: two active cables are surrounded by two dummy cables, assembled and impregnated in the same stainless steel case. It has been designed to fit the external part of the existing Nb-Ti ”collars” that house the holder for LHC cables testing, having channels for liquid helium, pins for centering and M16 bolts to apply pre-stress on the samples.

The sample holder consists of a U-shaped base housing the impregnated cable stack, 1725 mm long, closed by a top plate; the pre-stress is applied by means of interference between the cable stack and the top plate of the holder. This interference will be created by inserting a 150 $\mu$m thick Kapton film on the top of the cable stack after impregnation. The maximum pressure applied at warm, is about 45 MPa (around 53kN of bolts tightening force), reducing to 30 MPa at cold, due to the differential thermal contraction of the cables with respect to the sample holder [45]. The sample can be oriented such to have parallel (P), anti-parallel (AP) or transverse (T) background field with respect to the self-field: in parallel configuration, the stress on cables is in the range of 13 to 35 MPa, after powering. The transverse field configuration has been avoided since a shear stress of around 20 MPa arises at maximum pre-load. This level of stress could affect the cable at the boundary between the copper surface of the strands and the epoxy insulation, easily subjected to peeling. Therefore, this configuration may cause a long training.

### 4.3 A new sample holder design

The level of residual stress achieved at cold with the sample holder used in FRESCA is not enough to perform studies on current degradation under transverse pressure. A new solution of a cable sample holder should be studied. The aim of the new design is to provide an adequate pre-stress at warm so to have about 200 MPa at cold on a cable stack of 10 mm width, which is the dimension used for SD01 racetrack magnet and for its upgraded version, the SMC. The sample holder used now at FRESCA is not able to provide the required pre-stress level, both for strength limits imposed by the bolts material and for the loss in pre-stress due to the differential thermal contraction between the cable stack and the stainless steel structure. In order to have about 200 MPa at 4.2K, M36 bolts could be theoretically used. Nevertheless the space available on the collars does not allow the use of bolts larger than M20: at warm it would be possible to get 92 MPa, reducing to 60 MPa at 4.2 K. For the new design, the *keys and bladders* technology will be used.
4.3 A new sample holder design

Figure 4.6: Schematic of keys and bladders in the RD3 Nb$_3$Sn magnet [15]. The bladders are inflated with pressurized water, creating the required clearance between the pad and the yoke to insert an interference key which will be left in place during magnet operation. The forces applied to the coil at point B are counterbalanced by the forces acting on the shell at point A.

Figure 4.7: Schematic of the new sample holder to be used at FRESCA.
This technique has been successfully developed and applied at LBNL in Nb$_3$Sn high field magnets construction [15]. The bladder, placed between the coil pack and the iron yoke, can provide the required pre-stress at warm by compressing the coil and tensioning the Aluminum shell (see fig. 4.6). Interference keys then replace the bladders after they are deflated. By cooling the magnet down to cryogenic temperature, the target pre-stress to counterbalance the Lorentz stress at powering is achieved.

The new sample holder (fig. 4.7) consists basically of a U-shaped cable stack holder, called cage, placed into an Aluminum shell. The pressure is transmitted to the stack by means of two intermediate pieces called top and lower cap, respectively. At warm, interference keys are placed between the lower cap and the cage, to provide part of the target pressure. At cold, the remaining part of the force is transferred from the shell to the stack by means of the top and lower caps. The cage has no structural role in the holder, but containing the stack and providing lateral support through calibrated shims. The clearance between the cage and the shell allows the He-II to flow and cool the cables down to 4.2 K. The main challenge of this new design has been the integration of the structure able to carry large forces in the small FRESCA inner cryostat. A sensitivity analysis of materials has been carried out to avoid yielding of the device components under the most severe test conditions. Different shell shape solutions have been analyzed in order to have the highest $\Delta \sigma$ between room and cryogenic temperature, together with a uniformly distributed pressure over the cable width at 4.2K.

4.3.1 Materials analysis

By considering the sample holder as an infinitely rigid structure, the force transmitted to the cable stack after cool down can be defined as:

\[
F_{\text{cable}} = \frac{k_{\text{Nb}_3\text{Sn},C}}{k_{\text{Nb}_3\text{Sn},\text{warm}}} F_0 + k_{\text{Nb}_3\text{Sn},\text{warm}} (\alpha_{\text{shell}} - \alpha_{\text{holder}} - \alpha_{\text{Nb}_3\text{Sn}}) \Delta T
\]  

(4.1)

where: $F_0$ [N/m] is the force after keys assembly, $\alpha_i$ is the thermal contraction coefficient, and $k_{\text{Nb}_3\text{Sn},i}$ is the stiffness of the cable at room (warm) and cryogenic temperature (C). The higher is the differential in thermal contraction between the shell and the cable holder, the higher is the pressure exerted on the cable. On the other hand, it is advisable to have a similar thermal contraction between the Nb$_3$Sn filament within the strand and the cage in order to minimize the risk of endamaging the superconductor. Finally we opted for the Titanium alloy Ti6Al4V, largely used in cryogenics applications and in Nb$_3$Sn magnets. For the shell, an high strength Aluminum alloy has been selected, namely the Al7075 used for aeronautic
4.3 A new sample holder design

applications. The materials have been considered as isotropic (see tables 4.1-4.2 [46]-[47]-[48]-[49]).

<table>
<thead>
<tr>
<th>Material</th>
<th>E (GPa)</th>
<th>$R_{p0.2}$ (MPa)</th>
<th>$\alpha\Delta T$ (mm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell Al7075</td>
<td>70</td>
<td>480</td>
<td>4.19</td>
</tr>
<tr>
<td>Cage and caps Ti6Al4V</td>
<td>110</td>
<td>700</td>
<td>1.71</td>
</tr>
<tr>
<td>Cable stack Nb$_3$Sn</td>
<td>30</td>
<td>-</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table 4.1: Materials properties of sample holder components at 293 K.

<table>
<thead>
<tr>
<th>Material</th>
<th>E (GPa)</th>
<th>$R_{p0.2}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell Al7075</td>
<td>79</td>
<td>690</td>
</tr>
<tr>
<td>Cage and caps Ti6Al4V</td>
<td>130</td>
<td>1000</td>
</tr>
<tr>
<td>Cable stack Nb$_3$Sn</td>
<td>42</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.2: Materials properties of sample holder components at 4.2 K.

4.3.2 Different shell solutions

The aim of the sample holder is to apply a uniformly distributed pressure along the cable width, in order to avoid stress concentrations that could mislead the understanding of the experimental results. The shape of the caps has so to be defined in a such a way the pressure gradient over the active cables is reduced to few percentage.

![Figure 4.8](image)

Figure 4.8: Three different solutions for the new cable sample holder to be used in FRESCA facility at CERN. (a) solution no.1, (b) solution no.2, (c) solution no.3.
4. The Cable Sample Holder

On the other hand, the VonMises failure criterion for the outer shell and the Titanium components has to be satisfied by imposing a safety factor $s.f. = 1.5$. Three different 2D FE models have been built in ANSYS, by using higher order elements, with quadratic shape functions. This choice allows a more precise evaluation of the stress concentration effect on the structure. The contact elements have been considered frictionless. The analysis is divided into two steps:

1. assembly procedure: a proper interference gap between the keys and the cage has been set, thus simulating an infinitely rigid shim to provide the required pre-load on cable stack.

2. cool down of the structure at 4.2 K.

For this assessment study, no magnetic forces have been computed. The comparison between the three solutions have been done by imposing about 200 MPa equivalent pressure on a 10 mm width cable; at the same time a shell shape optimization has been done for each configuration, reiterating the computation to minimize the peak equivalent stress. Results are shown in table 4.3.

<table>
<thead>
<tr>
<th>Material</th>
<th>Stress</th>
<th>Solution</th>
<th>Assembly</th>
<th>Cool Down @ 4.2K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb$_3$Sn</td>
<td>$\sigma_y$ (MPa)</td>
<td>1</td>
<td>125 ($i_y = 490\mu$m)</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>130 ($i_y = 440\mu$m)</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>115 ($i_y = 320\mu$m)</td>
<td>190</td>
</tr>
<tr>
<td>Shell</td>
<td>$\sigma_{eq,max}$ (MPa)</td>
<td>1</td>
<td>530</td>
<td>790</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>480</td>
<td>744</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>275</td>
<td>460</td>
</tr>
<tr>
<td>Ti6Al4V parts</td>
<td>$\sigma_{eq,max}$ (MPa)</td>
<td>1</td>
<td>450</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>580</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>400</td>
<td>670</td>
</tr>
</tbody>
</table>

Table 4.3: FE results for three different sample holder solutions. In the first part, the average pressure on 10 mm cable stack is outlined after assembly and cool down (the calculated shim thickness is reported).

The maximum stress in the aluminum shell is beyond the design limit for Sol. 1-2, whereas it satisfies the failure criteria for Sol. 3. For the Titanium components, all of the three solutions observe the maximum stress limit, exception made for a small region around the edge between the top cap and the cable stack, where plasticization may initiate. In table 4.4, the calculated equivalent pressure on a 25 mm-width bladder is defined; since it is the smallest bladder ever realized, a test campaign will be carried out to verify its performances. The shell structure in Sol. 3 appears to be the most effective since leads the maximum $\Delta \sigma$ from 293
to 4.2 K, due to better repartition of stress induced by the top cap (higher contact area between the top cap and the shell). This means that the pressure required during assembly can reduce up to 12%, with respect to the other configurations. At 293 K, the stress gradient over the active cables is within few MPa. At 4.2 K, a gradient of about 20 MPa is outlined due to the cylinder shrinkage: the stress increases from the cable vertical mid-plane to the outer edge. From this first calculations, Solution 3 seems to be the best candidate for the sample holder. A coupled analysis has then been realized, computing the magnetic forces to be transferred to the mechanical model in order to evaluate the stress distribution inside the active cables at critical current.

<table>
<thead>
<tr>
<th></th>
<th>Sol. 1</th>
<th>505</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{\sigma}_{\text{bladder}}) (MPa)</td>
<td>Sol. 2</td>
<td>525</td>
</tr>
<tr>
<td></td>
<td>Sol. 3</td>
<td>465</td>
</tr>
</tbody>
</table>

Table 4.4: Average pressure on 25 mm bladder for three different sample holder configurations.

### 4.3.3 Proposed configuration

A magnetic model of Sol. 3 has been realized in ANSYS™ in order to compute the magnetic forces on the active cables, to be transferred to the mechanical model. Two different orientations of the sample field with respect to the 10 T background field have been considered: parallel (P) and anti-parallel (AP). The background field was simulated by setting boundary conditions on the vector potential. From the definition of magnetic flux density, we have:

\[ \vec{B} = \nabla \times \vec{A} \]  \hspace{1cm} (4.2)

Because of the 2D symmetry, only the vector potential Z-component exists. The background uniform field value is then given by:

\[ B_x = \frac{\partial A_z}{\partial y} \]  \hspace{1cm} (4.3)

being \(X\) and \(Y\) horizontal and vertical directions with respect to the cable broad face. By solving (4.3), the expression of the vector potential to have a uniform field is obtained:

\[ A_z = y B_x \]  \hspace{1cm} (4.4)
4. The Cable Sample Holder

We assumed a strand with \( j_c = 3000 \text{ A/mm}^2 \) at 12.5 T, being the highest critical current density for a commercial Nb\(_3\)Sn strand (see table 4.5). The cable features are the same as for the SMC magnet, with a filling factor of 1.53.

<table>
<thead>
<tr>
<th>Strand ( \varnothing ) (mm)</th>
<th>1.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j_c ) (A/mm(^2)) @ 12.5 T</td>
<td>3000</td>
</tr>
<tr>
<td>( B_{c2} ) (T)</td>
<td>24</td>
</tr>
<tr>
<td>( c ) (A/Tmm(^2))</td>
<td>44220</td>
</tr>
<tr>
<td>Cu/non-Cu</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.5: Nb\(_3\)Sn strand characteristics at 4.2 K.

Figure 4.9: Field and forces distribution in the active cables, with 10 T background field in parallel (a)-(b) and anti-parallel (c)-(d) configuration.
4.3 A new sample holder design

We set the \( A_z \) boundary conditions so to reduce the error in the field distribution of the cables within few percents. 4-nodes elements mesh the model, since the Ansys-MVP (Magnetic Vector Potential) formulation does not support high order elements. The mesh has been optimized in order to get consistent results with the ones in sec. 4.3.2. The e.m. forces have been computed at \( j_c \), derived for each of the two background field configurations by intersecting the sample load line (numerically obtained) with the critical surface. The results for the top active cables at 10 T of background field are shown in table 4.6.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Field} & j_c (\text{A/mm}^2) & F_{\text{horiz.}} (\text{kN/m}) & F_{\text{vert.}} (\text{kN/m}) \\
\hline
\text{Parallel} & 989 & 0 & 284 \\
\text{Anti-Parallel} & 1480.8 & 0 & -334.3 \\
\hline
\end{array}
\]

Table 4.6: Magnetic forces on top active cable at 10 T background field.

In fig. 4.9, the magnetic field and forces distribution is represented. In the P configuration, the magnetic forces tend to separate the two active cables, whereas in the other one the cables are pressed against each other. In fig. 4.10-4.11, the stress profile along the cables width is represented. The stress behavior can be summarized as follows (see figs. 4.12-4.13):

- P-configuration: as the magnetic forces are applied, the stress tends to redistribute in the cable, decreasing from 190 MPa at the upper edge down to 170 MPa at the boundary between the two active cables (fig. 4.12-(a)).

- AP-configuration: during powering the stress spans from 190 MPa at the outer edge to 220 MPa between the active cables (fig. 4.12-(b)).

In both configurations, the stress the cables stack is imposed by the structure pre-load and equal to about 200 MPa. The P-configuration leads to the highest field between the cables, equal to 12.4 T; nevertheless in that region the stress decreases, as outlined before. On the other hand, the AP-configuration features higher stress at the cable common edge, where a lower peak field occurs, equal to 6.42 T. This means that it is not possible a direct correlation between the region of high stress and high field, unless we are able to provide highest stress after cool down in P-configuration, therefore increasing the risk of shell yielding.

By analyzing the \( \sigma_y \) distribution along the cable width, the lower active cable is mostly affected by the cage bending after bladder release. The \( \Delta \sigma_y \) along the width is about 15% on the lower edge (boundary between the lower dummy and the active cable), decreasing to 6% (AP), and 9% (P) at the active cables boundary respectively. The same effect is observable on the top cable, though reduced by the lower dummy and active cables; the stress thus varies in a range of 6% (AP)-9% (P) to 7% on the upper face, at the boundary with the top dummy cable.
Figure 4.10: Stress distribution inside the upper active cable at lower, middle and upper face. The stress has been represented at (i) assembly, (ii) cool down, and (iii) powering. (a) Parallel configuration, (b) anti-parallel configuration.
Figure 4.11: Stress distribution inside the lower active cable at lower, middle and upper face. The stress has been represented at (i) assembly, (ii) cool down, and (iii) powering. (a) Parallel configuration, (b) anti-parallel configuration.
Figure 4.12: Average transverse stress $|\bar{\sigma}_y|$ on the upper active cable at low, mid, and top edge. The two magnetic configurations, P and AP, are shown, during the: (1) assembly, (2) cool down, and (3) powering stages.
4.4 Summary

A new design for a cable sample holder has been proposed, after evaluating different possible scenarios. It has been shown how it is possible to reach about 200 MPa on a 10 mm width cable after cool down at 4.2 K.

The VonMises failure criterion imposed for the components has been fulfilled, by imposing a safety factor equal to 1.5. Nevertheless the assembly procedure will require a bladder overpressure in order to slide the shimmed keys inside the sample holder. This additional clearance between the cage and the lower cap is evaluated in the order of 100 µm. Preliminary computations showed that this will lead the
outer shell closer to the yield stress, but avoiding plasticity anyway.

The results after powering simulation showed that either the test is performed at 12 T with a reduction in stress at the active cables boundary, or a lower field has to be accounted for, in order to keep the stress high enough. This choice will concern the operators, once the tests will be done. The transverse background field configuration has been initially rejected, in order to avoid torque to be transmitted to the assembly, with the risk of endamaging the test facility.

This results have been defined via a 2D frictionless model. Nevertheless, it is mandatory to test a mockup of the sample holder in order to firstly validate the structure, but also to compare with a full 3D numerical model, including friction elements.
Chapter 5

The short scale model

A short scale model of the sample holder has been realized in order to validate the design described in Chapter 4. The length of the model was constrained by the manufacturer’s tooling to 40 cm for the titanium components, in order to respect the tight geometric tolerances required, aimed at reducing the effect on the dummy cable stack, and assure stress uniformity. The dummy stack will reproduce the four cable stack, and is manufactured in stainless steel for cryogenic applications (AISI 316L) [50]. A test campaign on the 25 mm width bladder has been preliminary carried out, evaluating the maximum pressure achievable, and simulating semi-operative conditions. The SHSM (Sample Holder Short Model) has then been tested with different values of assembly interferences and then cooled down in liquid nitrogen at 77 K. The experimental results has been compared to the numerical ones given by a ANSYS 3D model.

5.1 Bladder tests

The bladder intended to be used in the sample holder is the smallest ever built (see fig. 5.1), being the width \( w = 25 \) mm much smaller than the conventional bladders used in Nb₃Sn magnets \( (w \approx 60 \) mm). The active width is a relevant factor in order to get the required assembly pressure: the largest it is, the higher the force provided to the stack for a given inlet pressure. This force can be simply obtained as follows:

\[
F_{\text{bladder}} = P_{\text{bladder}}wl
\]

(5.1)

where \( P_{\text{bladder}} \) is the inlet pressure, \( w \) is the width and \( l \) the bladder length. The active width is anyway dependent on the welding edge thickness, which has consequently to be small enough. Some preliminary tests made with electron beam
welding showed an active width of about 20 mm, furthermore being extremely weak. These bladder prototypes experienced premature leaks, after low pressures tests. Since the laser welding shows a higher reliability and allows an increased active width, it has been finally selected to realize the bladders pre-series to be used in the mockup model.

Figure 5.1: Bladder for sample holder. At the left hand side, the water inlet tube.

A previous study carried out at LBNL [15] outlines how the maximum pressure sustained is a function of the gap the bladder is inserted in: this can be explained by considering that the failure usually occurs along the welding. For higher gaps, the net opening force on the gap is intuitively higher and could lead to break for
5.1 Bladder tests

Leaks. We aimed at adopting the same test procedure by using a modular test bench in order to characterize the sample holder bladder. The test bench (see fig. 5.2) consists of two stainless steel plates tightened together by means of three pairs of steel rods. The initial gap is set by means of calibrated spacers put in between the plates. The bladder is inflated by means of a manual water pump providing up to 700 bars. The rods are instrumented with strain gages, whereas two LVDTs are mounted on the upper plate. In this way it will be possible not only to determine the maximum working pressure of the bladder, but to have an indication of the exerted force and consequently of the effective surface. The first tests aimed at cycling each bladder to a maximum pressure of 700 bar and complete release. Results in fig. 5.3 show that for \( P_{\text{max}} = 700 \) bar the bladder can be used for a maximum of two runs if the gap is lower than 2 mm.

![Figure 5.3: Number of cycles at 700 bar as a function of the initial gap. Measurements with 2 mm gaps have been repeated in order to test the slip shim.](image)

This test also suggests that for lower working pressures the bladder could be used for more than once. This behavior has been verified by applying a more severe test cycle: a ramp-and-release procedure is applied on the bladder, inflating up to 600 bar in steps of 100 bar, and deflating each time to zero. The results in fig. 5.4 show that one complete cycle is done at the maximum pressure, while during the second last ramp a leakage occur around 400 bar for both of the two bladders used. In the most part of the cases analyzed, the leakage occurred along the lateral welding. Only in one case the leak was due a welding failure at the
water inlet. Measurements taken on the rods revealed that the actual force exerted by the bladders is about 10% lower than expected, underlining the effectiveness of the welding technique.

![Bladder working cycle test](image)

**Figure 5.4: Bladder working cycle test.**

The test on the bladders revealed the possibility of exerting high pressures with small geometrical dimensions, beyond the required level estimated by the FE model. Furthermore, this suggest the possibility of using the bladder for more than one cycle, without the risk of premature failure inside the sample holder, provided a gap lower than 2 mm. It is also important to point out that once the bladder is inflated to pressure higher than 100 bar, it goes into sticky contact with the plates. In order to slide it out from the test bench a slip-shim is mandatory. This is simply a steel bar, which is hooked to a winch and pulled by hand. The shim will be taken out exploiting friction between parts.

### 5.2 Experimental set-up

Both the outer shell and the dummy stack bar have been instrumented with strain gages, following the pattern shown in fig. 5.5-5.6. Since in the definitive model the Nb$_3$Sn cable will not be instrumented, but the outer shell, the aim of the scale model is to analyze the correlation between the stress on the shell and on the dummy cable stack. The results will confirm the possibility on relying on the shell measurement to deduce the stress provided to the cable stack. The gages are not
thermally compensated (quarter bridge electrical scheme), so a preliminary thermal cycle of every single component has been done in order to set the thermal zero, to be subtracted from the total strain recorded at 77 K under loading conditions.

Figure 5.5: Strain gages distribution on the shell. A cylindrical coordinate system is assumed: $\vartheta$ is the azimuthal direction, and $Z$ is the longitudinal one.

![Figure 5.5: Strain gages distribution on the shell. A cylindrical coordinate system is assumed: $\vartheta$ is the azimuthal direction, and $Z$ is the longitudinal one.](image)

The azimuthal stress on the shell is obtained as follows:

\[ \text{Shell : } \sigma_\vartheta = E \frac{(\varepsilon_\vartheta + \nu \varepsilon_z)}{1 - \nu^2} \]  

(5.2)
5. The short scale model

whereas the vertical stress on the dummy cable reads:

\[
\sigma_y = \frac{E(\varepsilon_y + \nu\varepsilon_z)}{1 - \nu^2}
\]  \hspace{1cm} (5.3)

According to the gages layout (see fig. 5.6), we will rely on gages 3-4-5-6 for
stress measurements after assembly and during cool down, when the longitudinal
strain is much more relevant, due to friction effects. Gages 1-2 are used during
bladder inflation, since the dummy stack can be considered under pure uniaxial
load (the longitudinal strain is negligible).

![Image](image1.png)  \hspace{1cm} ![Image](image2.png)

Figure 5.7: Short model of the sample holder. (a) Front view: the system is turned
upside down in order to allow the insertion and removal of the bladder. (b) Dewar
for cryogenic test.

5.3 3D Finite Element model

A 3D model has been developed in ANSYS™, using higher order brick elements
(SOLID95) and friction contact elements. The model reproduces 1/4\textsuperscript{th}
of the sample holder, for symmetry reasons. The final geometry implemented slightly
differs from the one proposed in Chapter 3, mainly along the inner radii, according
to manufacturing constraints. From some preliminary computations this lay-out
increases the stiffness of the outer shell and allows higher stress on the dummy
stack. All the three tests phases have been modeled: assembly, release and cool
down.
The assembly phase has been simulated for two reasons: (1) to check the stress distribution in the shell and dummy stack at maximum bladder pressure, before inserting the keys; (2) to better understand how the friction between the components could eventually influence the stress distribution inside the dummy stack. For this purpose, the birth and death option has been invoked: ANSYS assigns a very small stiffness to the not-active elements, i.e. the keys during bladder inflation. These are then raised up once the release phase starts. The all structure is finally cooled down at 77 K, according to the procedure described in Chapter 4.

The phenomenon of friction plays a fundamental role in the system. Therefore a parametric analysis has been carried out on the friction coefficients between: (i) the outer shell and the caps, (ii) between the dummy stack items and the u-cage, and (iii) between the stainless steel keys and the titanium items. First trial values have been selected from data in [51]. We aim at defining these coefficients so to reproduce the variation in stress $\Delta \sigma_y$ on the dummy stack between low and high transverse pressure cases. At room temperature (RT), $\Delta \sigma_y \approx 65$ MPa (see fig. 5.9), whereas it almost doubles at 77 K, being $\approx 120$ MPa (see fig. 5.10).

The shim thickness is usually not the same for the two keys, and is tuned after some cycles, as far as the stress difference is reduced between few tens of microstrains. The difference between the two is anyway lower than 50 µm. The shim thickness we will refer to in the coming sections is then the average value. As it is shown in fig. 5.9, a thickness value of 100 µm leads to a negligible stress along the stack at warm. Nevertheless, the interpolation of the three remaining values
5. The short scale model

Figure 5.9: Transverse stress $\sigma_y$ on dummy stack as a function of the shim size (Room Temperature). The average pressure has been computed between gages 3-4.

Figure 5.10: Transverse stress $\sigma_y$ on dummy stack as a function of the shim size (77 K). The average pressure has been computed between gages 3-4.
indicates 125 µm as the real zero reference value of our system. This value is mainly related to the geometrical tolerances of the components. The shim thickness used in the tests will be normalized to the reference value and then input in the FE model.

At room temperature, friction values found in literature allows a very good agreement between numerical and experimental results, as it will be reported in the coming sections. We set $\mu_{Al} = 0.3$ and $\mu_{steel} = 0.2$ respectively for: the static friction coefficient between the shell and the caps, and between the steel parts (dummy stack and keys) and titanium items. During cool down, the friction coefficients can considerably change as outlined in [52].

![Figure 5.11: Analysis on friction coefficients. Longitudinal deformation $\varepsilon_z$ on dummy stack as a function of the shim size at cryogenic temperature.](image)

A numerical parametric analysis of the influence of the latter coefficients has been carried out at cold, in order to study the the effects on the strain distribution in the dummy stack. In fig. 5.11 the longitudinal deformation of the sample is shown. The longitudinal coefficient $\mu_{Al,C}$ strongly influence the strain pattern on the dummy stack, and as a consequence on the u-cage. The lower is the coefficient ($\mu_{Al,C} \approx 0.1$), the less the thermal contraction of the shell impacts the sample, and consequently the u-cage $\varepsilon_z$. In this configuration, the longitudinal strain of the bar is mainly due to its own contraction. Furthermore, the $\Delta \varepsilon_z$ between 50 and 190 µm shim thickness does not notably change, compared to the experimental data. By increasing $\mu_{Al,C}$ a reduction of $\varepsilon_z$ is induced, increasing at the same time $\Delta \varepsilon_z$.  

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5. The short scale model

A higher $\mu_{\text{bar},C}$ further reduces the longitudinal strain on the bar at higher transverse pressures, slightly increasing the transverse strain $\varepsilon_y$. The latter is indeed less affected by the effect of friction, as it is shown in fig. 5.12. A value of 0.5 will be finally implemented. A further test has been done by implementing an orthotropic friction effect between the shell and the caps: the longitudinal $\mu_{\text{Al},C}(\text{long.})$ has been set to 0.2, according to the previous considerations, whereas the azimuthal $\mu_{\text{Al},C}(\text{azim.})$ has been set to 0.4. This choice has been done consistently with the surface roughness due to the shell manufacturing process.

![Figure 5.12: Analysis on friction coefficients. Transverse deformation $\varepsilon_y$ on dummy stack as a function of the shim size at cryogenic temperature.](image)

The difference in absolute values between the numerical and the experimental strain data can be reduced by varying the thermal contraction of the outer shell and of the titanium items within $\pm10\%$ of their nominal value (values on graphs depicted with *). The friction coefficients used are summarized in table 5.1.

<table>
<thead>
<tr>
<th></th>
<th>293 K</th>
<th>77 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{Al}}$</td>
<td>0.3 (Z. dir)</td>
<td>0.2 (Z. dir)</td>
</tr>
<tr>
<td></td>
<td>0.3 ((\vartheta) dir.)</td>
<td>0.4 ((\vartheta) dir.)</td>
</tr>
<tr>
<td>$\mu_{\text{stack}}$</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>$\mu_{\text{keys}}$</td>
<td>0.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 5.1: Friction coefficients used in the FE model.
5.4 Comparison between experimental and numerical results

Four tests have been carried out, starting from the minimum pressure achievable, due to the thermal contraction only, up to the maximum pressure on the stack before shell yielding.

5.4.1 Assembly

During assembly, the bladder is inflated to a maximum pressure which is set in accordance to the shim thickness to insert in. Usually, it has been verified that an additional clearance of 100 µm is enough to slide the shimmed key inside the shell. During the ramp in pressure, it can be observed a good agreement between the numerical and experimental results along the cable stack (see fig. 5.13).

![Figure 5.13: Transverse stress $\sigma_y$ on dummy stack as a function of the bladder pressure.](image)

The vertical stress as been computed considering the vertical strain only, being negligible the longitudinal one. The average pressure has been computed averaging the stress along the cable length; whereas the estimated pressure derives from the bladder pressure, scaling on the ratio between the stack to bladder width. A maximum difference of $\approx 20\%$ is measured between the sides of the dummy stack, especially on the gages on the tails of the stack.
5. The short scale model

Figure 5.14: Azimuthal stress $\sigma_\theta$ on shell (mid-plane side) as a function of the bladder pressure.

Figure 5.15: Azimuthal stress $\sigma_\theta$ on shell (top cap side) as a function of the bladder pressure.
The difference in the center of the bar is far reduced, being below some percentages. These values are affected by the geometrical tolerances of the system and by the bladder position. The last one has not a centering groove, and so allowed to tilt horizontally thus unbalancing the system. The outer shell undergoes azimuthal compression along the mid-plane, whereas a tension state is produced in correspondence of the titanium caps. The difference along the mid-plane (gages: 1-3-5-7-9-11) increases at higher pressure, being anyway below 13% of difference. The difference along the vertical symmetry plane is about 10% at 500 bar for gages 2-10 (top cap side, see fig. 5.14), whereas it is higher for gages 4-8-12 (bladder side, see fig. 5.15).

Figure 5.16: Azimuthal stress $\sigma_\theta$ on shell (low cap side) as a function of the bladder pressure.

### 5.4.2 Release of bladder pressure

The pressure in the bladder is now released, and the assembly interference leads to the first stage of stress on the structure. Fig. 5.17-5.18 show the correlation between the stress on the stainless steel bar and on the aluminum shell; the values are averaged in the center of the system: gages 3-4 on the dummy cable, and gages 5-7 on the shell. The numerical model is in very good agreement with the experimental data, showing a maximum difference of about 10%.
Figure 5.17: Transverse stress $\sigma_y$ on dummy stack: comparison between numerical and experimental results (Room Temperature).

Figure 5.18: $\sigma_y$ on dummy stack vs. $\sigma_\theta$ on shell mid-plane at Room Temperature.
In particular, fig. 5.18 reproduces the stress correlation values both during the assembly phase and after bladder release. The measurements reveal a perfectly elastic behavior of the structure and suggesting the need of about 40% overpressure in the system in order to slide the shimmed keys inside the sample holder.

### 5.4.3 Cool Down

The assembly is cooled down at 77 K in liquid nitrogen. A maximum stress of about 250 MPa is achieved, beyond the design target initially imposed. As we mentioned before, this is mainly due to the revised shell structure with respect to the conceptual numerical design. Numerically results are in very good agreement with the experimental ones, as shown in figs. 5.19-5.20. In the data reported the experimental zero value is not taken into account, being the uncertainty higher for the system intrinsic tolerances. As in the previous case, the average stress along the bar is computed taking into account gages 3-4 only, where the strain is measured along both vertical and longitudinal directions. The other gages have not been considered for different reasons:

- gages 1-2 can be used during the assembly phase only, since the bar undergoes pure compression. Once the bladder is taken out and the structure is consequently cooled down, the longitudinal strain becomes more relevant.

- the stress profile along the dummy cable at cold is not flat, due to a stress reduction towards the tails of about 17%. This effect has been evaluated and depicted in fig. 5.21: gages 3-4 at z=0 mm, and gage 5 at z=100 mm (gage 6 showed problems in the longitudinal direction). This effect is less relevant during assembly and after keys insertion.

We will have to account for the tail effect once the final model will be implemented; the active-stress length will be shorter than the applied-stress length, in order to flat the stress profile down to some percentages. In fig. 5.22 we report the relationship between the stress provided at warm, after keys insertion and the correspondent at 77 K.
5. The short scale model

Figure 5.19: Transverse stress $\sigma_y$ on dummy stack: comparison between numerical and experimental results (77 K).

Figure 5.20: $\sigma_y$ on dummy stack vs. $\sigma_\theta$ on shell mid-plane at 77 K.
5.4 Comparison between experimental and numerical results

Figure 5.21: Transverse stress $\sigma_y$ along the dummy stack length and comparison with the experimental data. Markers in yellow represent the experimental data. The assembly interference is equal to 125 $\mu$m.

Figure 5.22: Transverse stress $\sigma_y$ after Cool Down vs. $\sigma_y$ at warm temperature.
5. The short scale model

5.5 Summary

The test of the sample holder short scale model has been successfully concluded. The possibility of achieving $\sim 250$ MPa on a 10 mm width dummy cable has been verified, fulfilling the failure criteria for the assembly components. The minimum transverse pressure of about 60 MPa is comparable with the stress imposed in the previous sample holder, used for transport current measurements only. Moreover, the results also suggest the possibility of measuring larger cables, up to $w \approx 16$ mm, where a maximum pressure of about 150 MPa could be exerted.

The $\Delta \sigma_y$ achieved at cryogenic temperature is a function of the initial stress at warm: the highest the latter, the highest the final stress on the cable. In the case of no shims used, the minimum $\Delta \sigma_y \approx 60$ MPa, becoming 160 MPa, when the thicker shims are used.

![Figure 5.23: Transverse stress $\sigma_y$ along the dummy stack length. The transverse pressure has to be applied along 80 cm in order to get a uniform pressurized length equal to 30 cm (data from FE half length model).](image)

A good agreement with the numerical model has been outlined, both at room and at cryogenic temperature. This has been achieved by tuning properly the friction coefficients between parts, according to experimental data and reference values found in literature. We have shown that it is possible to rely on the strain measurements on the shell to derive the applied stress on the cable. Nevertheless, during the assembly phase both of the signals (dummy stack and shell) appeared
to be fundamental for a good system set-up. Due to the difficulty in having reliable strain values on the Nb$_3$Sn cable stack, we will probably use strain gages glued on the keys.

We outlined the so-called "tail effect": the stress distribution lowers down longitudinally towards the ends of the dummy stack with respect to the maximum values achieved in the center. We have to account for this effect in the design of the final sample holder in order to assure a uniform compression over the required measured length. For instance, in order to have a measuring length of about 30 cm, we will have to provide transverse pressure over a length equal to 80 cm, as depicted in fig. 5.23.

Finally, we remarked that the position of the bladder and keys influence the stress distribution along the dummy stack during the assembly phase. In this scaled model, the alignment of the bladder, the keys and the u-cage with respect to the shell is not perfectly centered, but has to be adjusted by the user in order to get a better transverse stress distribution. Aim of the final model is to redesign the lower cap area, thus assuring a self centering system in order to achieve the best stress distribution during the assembly phase and reduce the set up time.
5. The short scale model
Part IV

The SMC project: design of a \( \text{Nb}_3\text{Sn} \) racetrack dipole coil
Chapter 6

Magnetic Design

6.1 Goal of the project

The Short Model Coil (SMC) working group was set in February 2007 within the NED program, in order to develop a short-scale model of a Nb$_3$Sn dipole magnet. NED is an acronym for Next European Dipole. This Joint Research Activity (JRA) was launched on January 2004 to promote the development of high performance Nb$_3$Sn conductors in collaboration with the European industries and to assess the suitability of Nb$_3$Sn technology to the next generation of accelerator magnets.

The SMC group comprises four laboratories: CERN/AT-MCS group (CH), CEA/IRFU/SIS with support from CEA/IRFU/SACM (FR), RAL (UK) and LBNL (US). The group will supervise the design and manufacture of the coils to be tested, of their support and testing structure and of the associate tooling.

The SMC magnet has been conceived to reach a peak field of about 13 T on conductor, by using a 2500 A/mm$^2$ PIT strand. The aim of this magnet device is to study the degradation of the magnetic properties of the Nb$_3$Sn cable, by applying different level of pre-stress. The same structure could be eventually used to test different cable types, as well as to investigate different cable insulations, namely ceramic and conventional (glass fiber + organic matrix). To fully satisfy this purpose, a versatile and easy-to-assemble structure has to be realized. The design of the first SMC program magnet has been developed from an existing dipole magnet, the SD01 [49], designed, built and tested at LBNL. In this chapter, we will firstly introduce the SD01 magnet, describing the structure and its main features. The magnetic design steps of the SMC will be then split into two parts: in the first one, preliminary computations are described, whereas in the second part a more detailed analysis is presented, leading to the final configuration proposed.
6. Magnetic Design

6.2 A precedent study: the SD01

The SD01 is a short scale magnet aimed at performing studies on Nb$_3$Sn cable degradation under variable coil pre-stress and eventually, under different field patterns (either dipole or common coil configuration). The need for an accurate understanding of the superconductor behavior led to the idea of a modular structure, easy to assemble and disassemble thus allowing an independent variation of the pre-stress levels required. The coils, SC01 (Short Coil) and SC02, are in the so called *racetrack* shape, obtained by winding the cable along its broad face around a stainless steel pole. The strand and cable properties are described in tables 6.1 and 6.2. The two coils could be superposed and electrically connected so to have two different magnetic field patterns: (i) Subscale Dipole (SD) configuration, the field vector being parallel to the cable broad face, and (ii) Subscale Common Coil (SCC) configuration, where the field vector is perpendicular to the broad cable side.

![Figure 6.1: Schematic of the SD01 cross section. The stars represent the position of the bladders in the structure. Please note the reference system; the longitudinal direction is defined as Z.](image)

In SD01, the dipole configuration has been chosen. The two superposed coils are inserted into a magnet pack, formed by four pads, having both magnetic and
6.2 A precedent study: the SD01

Figure 6.2: Schematic of the SD01: the system for longitudinal pre-tensioning is represented. Two aluminum rods are tightened by an hydraulic jack.

<table>
<thead>
<tr>
<th>Strand Ø (mm)</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>RRR</td>
<td>37</td>
</tr>
<tr>
<td>Cu/non-Cu</td>
<td>0.8</td>
</tr>
<tr>
<td>( j_c ) (A/mm²) @ 11.882 T</td>
<td>2200</td>
</tr>
</tbody>
</table>

Table 6.1: Nb₃Sn strand characteristics of the SD01 dipole.

<table>
<thead>
<tr>
<th>No. of strands</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cable width</td>
<td>7.793 mm</td>
</tr>
<tr>
<td>Cable thickness at 20 MPa</td>
<td>1.275 mm</td>
</tr>
<tr>
<td>Insulation thickness</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>Short Sample current</td>
<td>9871 A</td>
</tr>
<tr>
<td>B field at Short Sample</td>
<td>11.882 T</td>
</tr>
<tr>
<td>Twist pitch</td>
<td>54.88 mm</td>
</tr>
</tbody>
</table>

Table 6.2: SD01 cable properties.

mechanical purposes. The magnet pack is then accommodated into two half iron yokes, surrounded by an aluminum shell. Both the pads and the yoke are partly composed by magnetic iron that complete the magnetic circuit together with the coil, and by amagnetic stainless steel. The lateral pre-stress at warm is provided by using the _bladders and keys_ technology, as described in sec. 4.3, inserting calibrated keys in between the yoke and the magnet pack. The longitudinal pre-stress at warm is provided by a couple of aluminum rods, tightened by an hydraulic piston. The complementary part of the total pre-stress is then reached at cryogenic
temperature, by the differential thermal contraction of the aluminum shell and rods with respect to the enclosed structure. The nominal magnetic features of the SD01 are listed in table 6.3.

The dipole were successfully assembled and tested at LBNL. The first magnet quench occurred at 95% of the maximum current, suggesting a very good agreement between the mechanical behavior of the structure and the numerical models. Nevertheless, some improvements have been suggested:

- to re-design the coil in order to match the peak field region with the high stress one. This has not been possible in SD01 since the peak field occurred on the coil heads, whereas the maximum stress was applied along the straight section. It was advised to introduce an end spacer on the coil thus reducing the induction on the heads.

- to improve the coil instrumentation, both for mechanical and magnetic purposes. It is advisable to insert strain gages closer to the coils, in order to have a more precise information on the stress provided to the cable. In SD01, the values of stress were indirectly derived relying on measurements of gauges placed on the outer shell and longitudinal rods. Furthermore, the use of voltage taps on the coil could also help to understand the origin of quenches in the superconductor.

In this work, we will focus on the magnetic optimization (this Chapter), and on the mechanical design, reported in Chapter 7.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Sample current</td>
<td>$I_{ss}$</td>
</tr>
<tr>
<td>Peak field position</td>
<td>8750 A</td>
</tr>
<tr>
<td>Peak field</td>
<td>$B_{max}$ 12.45 T</td>
</tr>
<tr>
<td>Maximum field on straight section</td>
<td>$B_{ss,max}$ 11.8 T</td>
</tr>
<tr>
<td>Maximum field at straight section center</td>
<td>$B_{ss,c}$ 11.45 T</td>
</tr>
<tr>
<td>$\Delta B_{max} = B_{max} - B_{ss,c}$</td>
<td>1 T</td>
</tr>
<tr>
<td>$\Delta B_{min} = B_{max} - B_{ss,max}$</td>
<td>0.65 T</td>
</tr>
<tr>
<td>Bore field</td>
<td>$B_0$ 11.68 T</td>
</tr>
<tr>
<td>Transversal force on 1/8th magnet</td>
<td>$F_x$ 145.5 kN</td>
</tr>
<tr>
<td>Vertical force on 1/8th magnet</td>
<td>$F_y$ -119 kN</td>
</tr>
<tr>
<td>Longitudinal force on coil head</td>
<td>$F_z$ 84.8 kN</td>
</tr>
</tbody>
</table>

Table 6.3: SD01 magnetic features (VF Opera results).
6.3 Design parameters

The main magnetic conception constraint for the SMC dipole is to get the peak field $B_{\text{max}}$ around 13 T in the center of the straight section of the racetrack, with a margin of about 0.5 T on the end field. The uniformity of the field should be satisfied along the cable twist pitch length (60 mm) within approximately 1% of tolerance. The cable candidate for the SMC dipole differs from the SD01’s for performance and dimensions, as shown in table 6.4. The insulation as well is thicker and equal to 0.2 mm for cable side.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of strands</td>
<td>$N_{\text{str}}$</td>
<td>/</td>
<td>14</td>
</tr>
<tr>
<td>Strand diameter</td>
<td>$\varnothing_{\text{str}}$</td>
<td>mm</td>
<td>1.25</td>
</tr>
<tr>
<td>Cu/non-Cu ratio</td>
<td>Cu/\text{non-Cu}</td>
<td>/</td>
<td>1.25</td>
</tr>
<tr>
<td>Cable width</td>
<td>$w_{\text{cbl}}$</td>
<td>mm</td>
<td>60</td>
</tr>
<tr>
<td>Cable thickness at 20 MPa</td>
<td>$t_{\text{cbl}}$</td>
<td>mm</td>
<td>9.7</td>
</tr>
<tr>
<td>Insulation thickness</td>
<td>$i_{\text{cbl}}$</td>
<td>mm</td>
<td>2.2</td>
</tr>
<tr>
<td>Critical strand current density</td>
<td>$J_{\text{str}}$</td>
<td>A/mm$^2$</td>
<td>2500</td>
</tr>
<tr>
<td>Cable current density</td>
<td>$J_{\text{eng}}$</td>
<td>A/mm$^2$</td>
<td>654.23</td>
</tr>
</tbody>
</table>

Table 6.4: SMC cable features.

In addition to these fundamental magnetic specifications there are further constraints: the working current is limited to 20 kA by the power supply, whereas the overall coil dimensions are driven by the reaction furnace length (originally equal to 220 mm), and by the cryostat available. The parameters to be defined are summarized in table 6.5.

The first phase of the magnetic optimization has been dedicated to set up the main features, like the overall number of turns, the island dimensions, the straight section length and the dimensions of the head spacer. As it will be outlined in the second phase, the original constraint imposed by the oven length for thermal treatment led to an optimized configuration that could not match the requirements. Since it is anyway important to outline and justify the main steps leading to the final version, a description of the first assessment computations will be introduced. Three different teams have worked on the magnetic optimization, using dedicated programs: at CERN, ANSYS and ROXIE have been used; at RAL, Vector Fields (VF) Opera, and CAST3M at CEA. The duplicated calculations served two functions. Primarily, they aimed at validating the computations, and secondly they allowed the different institutions involved to interact and check the validity of their modeling solutions. Details on the different simulation techniques are described in Appendix F, where a comparison on a benchmark configuration is introduced.
Table 6.5: Optimization parameters for SMC dipole.

6.4 2D Finite Element model

A 2D model has been developed in ANSYS™ to define the minimum number of cables to have \( B_{\text{max}} \geq 13 \text{T} \), to perform sensitivity analysis on the iron parts dimensions, and to define the minimum coil bending radius. Only \( 1/4 \) th of the model has been realized (see fig. 6.3), by imposing the vector potential component \( A_z = 0 \) along the vertical \( Y \) direction for symmetry reasons. Planar 4-nodes magnetic elements (PLANE13) has been used, with a relevant dimension of 2 mm, which has been set after some mesh refinement study. The far-field decay in magnetic field has been reproduced by infinite boundary elements, called INFIN110, allowing a reduced dimension of the model and so diminishing the simulation time required.
All components, exception made for the iron parts, have relative permeability $\mu_r = 1$ equal to vacuum’s one. For the same reason, the outer aluminum tube has not been modeled. The magnetization curve for the ferromagnetic materials is represented in fig. 6.4.

Figure 6.4: Magnetization curve for iron components.

Figure 6.5: SMC load line and cable critical surface.
For peak field computations, the short sample current is required. It has been calculated for every geometrical configuration by interpolation of the load line numerical data close to the critical surface, which has been linearized around 12 T (see fig. 6.5). The equations for the critical current and peak field at short sample are as follows: [53]

\[
I_{ss} = \frac{I_2 - I_1}{B_2 - B_1} (B_{ss} - B_1) + I_1 = m_{ll}(B_{ss} - B_1) + I_1 \quad (6.1)
\]

\[
B_{ss} = \frac{1}{m_{ll} + m_{cs}} (m_{cs}B_c^* - I_c + m_{ll}B_1 - I_1) \quad (6.2)
\]

where: \(m_{ll}\) and \(m_{cs}\) are respectively the slope of the load line and of the critical surface. The results obtained at short sample have been numerically verified.

### 6.5 Magnetic design: phase I

By imposing a pole radius equal to SD01’s, a parametric analysis of the number of turns has been carried out at short sample condition by means of the 2D model, to satisfy the constraint on peak field \(B_{max}\). The dimensions of every magnet parts have been proportionally varied accordingly with the coil encumbrance. The results on coil turns optimization are listed in table 6.6. By taking a margin of about 2%, 21 turns has been set as the reference value. A larger coil dimension could imply winding issues as well as costs rising. Further computations have been carried out on the insulation thickness between the vertical pad and in between pancakes, revealing a slight dependency < 1% on the peak field.

<table>
<thead>
<tr>
<th>Turns</th>
<th>(I_{ss}) (kA)</th>
<th>(B_{max}) (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>14.434</td>
<td>12.828</td>
</tr>
<tr>
<td>19</td>
<td>13.39</td>
<td>13.106</td>
</tr>
<tr>
<td>20</td>
<td>13.1</td>
<td>13.182</td>
</tr>
<tr>
<td>21</td>
<td>12.849</td>
<td>13.266</td>
</tr>
<tr>
<td>25</td>
<td>11.986</td>
<td>13.518</td>
</tr>
</tbody>
</table>

Table 6.6: Coil turns optimization.

Once the coil dimension has been set, a sensitivity study on the yoke lamination width has been carried out. By increasing the width up to 200 mm, the gain in \(B_{max}\) is \(\sim 1.45\%\), then the yoke dimension is no more effective on the magnet induction (see fig. 6.6). A simple 3D model of the coil pack (coils, main post, and horse-shoe) then allowed to analyze the effect of the straight section length on the
field uniformity along the dipole Y-Z plane. A study of the head spacer dimension aimed at define its influence on the peak field has been carried out, too.

Preliminary computations showed that a minimum spacer length $L_{ss}$ of 10 mm after two cable turns led to a peak field reduction on the heads of about 8%. This was assumed as the reference configuration for our studies. The short sample current has been calculated for each run, by means of (6.1)-(6.2). Different straight section lengths have been chosen, starting from SD01’s.

By increasing $L_{ss}$, the mutual induction of the heads on the straight section reduces, so the peak field lowers down as well as $B_{ss}$ and $B_0$. By keeping the SD01 straight section length, one can observe a field uniformity extended over the twist pitch length. If $L_{ss}$ is set to about 200 mm, the field uniformity has a margin of about 30% on the twist pitch constraint. The peak field on the heads can be diminished by using a thicker spacer, so that the mutual induction of the outer pack (19 turns) on the inner pack (2 turns) is less effective. The increase in $L_{ss}$ causes a decrease in peak field, moving from the inner pack to the outer pack if $L_{ss} \geq 20\, mm$.

After the first iteration results on the coil pack properties, the iron circuit components have been implemented into the model, aiming at moving effectively the peak field from the heads to central part of the dipole. Since the MVP magnetic formulation requires a long time to solve the model accounting for the iron non

Figure 6.6: Influence of the yoke width on the peak field. The first point on the curve has been set by imposing the same shell outer radius as SD01, whereas the second represents the same yoke thickness as SD01’s.
6. Magnetic Design

Figure 6.7: Results of the analysis on field uniformity length.

Figure 6.8: $B_{\text{max}}$ as a function of the head spacer length $L_s$.

linearity, parallel computations have been performed by RAL using VF Opera. In ANSYS\textsuperscript{®}, firstly a simplified iron tube has been designed to surround the coil pack in order to optimize the ratio $L_{\text{iron}}/L$, i.e. the length of the magnetic part over the straight section length. The detailed design of the yoke and pads has been later realized, in order to study the influence of the lateral pad length on the peak field.
The main results can be summarized as follows:

- the difference between the peak field on the head and the field on the straight section $\Delta B_{ss}$ can be reduced by using an iron length $L_{iron}$ between 60% and 80% of the straight section length (ANSYS simplified model). Detailed computations in VF Opera revealed that the optimum is about 70% of $L_{ss}$.

- by decreasing the horizontal pad length with respect to the yoke and vertical pad length is not effective in terms of $\Delta B_{ss}$ (still $>0$), being the decrease in central field much relevant than the peak field decrease on the heads.

The oven length initially available is so that $L_{ss} + 2L_s = 110$ mm: due to this constraint and adopting the preliminary results obtained in the first phase of optimization, we are not able to move the peak field from the head to the central section, with a single head spacer (see table 6.7). Moreover, some computations performed by [54] highlight how the increase in $L_{ss}$ is still not enough to reach our design goals.

<table>
<thead>
<tr>
<th>$L_s$ (mm)</th>
<th>$L + 2L_s$ (mm)</th>
<th>$L$ (mm)</th>
<th>%Iron</th>
<th>$L_{yoke}$ (mm)</th>
<th>$B_0$ (T)</th>
<th>$B_{ss,max}$ (T)</th>
<th>$B_{max,ip}$ (T)</th>
<th>$B_{max,op}$ (T)</th>
<th>$\Delta B_{max}$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>110</td>
<td>90</td>
<td>0.5</td>
<td>45</td>
<td>14.91</td>
<td>16.48</td>
<td>16.95</td>
<td>16.28</td>
<td>0.46</td>
</tr>
<tr>
<td>15</td>
<td>110</td>
<td>80</td>
<td>0.5</td>
<td>40</td>
<td>14.83</td>
<td>16.35</td>
<td>16.6</td>
<td>16.33</td>
<td>0.25</td>
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<tr>
<td>20</td>
<td>110</td>
<td>70</td>
<td>0.5</td>
<td>35</td>
<td>14.73</td>
<td>16.31</td>
<td>16.4</td>
<td>16.4</td>
<td>0.09</td>
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<tr>
<td>25</td>
<td>110</td>
<td>60</td>
<td>0.5</td>
<td>30</td>
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<td>16.25</td>
<td>16.22</td>
<td>16.41</td>
<td>0.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L_s$ (mm)</th>
<th>$L + 2L_s$ (mm)</th>
<th>$L$ (mm)</th>
<th>%Iron</th>
<th>$L_{yoke}$ (mm)</th>
<th>$B_0$ (T)</th>
<th>$B_{ss,max}$ (T)</th>
<th>$B_{max,ip}$ (T)</th>
<th>$B_{max,op}$ (T)</th>
<th>$\Delta B_{max}$ (T)</th>
</tr>
</thead>
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<tr>
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<td>54</td>
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<td>16.61</td>
<td>17.08</td>
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<td>80</td>
<td>0.6</td>
<td>48</td>
<td>14.99</td>
<td>16.57</td>
<td>16.69</td>
<td>16.36</td>
<td>0.12</td>
</tr>
<tr>
<td>20</td>
<td>110</td>
<td>70</td>
<td>0.6</td>
<td>42</td>
<td>14.89</td>
<td>16.46</td>
<td>16.46</td>
<td>16.4</td>
<td>0</td>
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<tr>
<td>25</td>
<td>110</td>
<td>60</td>
<td>0.6</td>
<td>36</td>
<td>14.79</td>
<td>16.35</td>
<td>16.36</td>
<td>16.43</td>
<td>0.08</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$L_s$ (mm)</th>
<th>$L + 2L_s$ (mm)</th>
<th>$L$ (mm)</th>
<th>%Iron</th>
<th>$L_{yoke}$ (mm)</th>
<th>$B_0$ (T)</th>
<th>$B_{ss,max}$ (T)</th>
<th>$B_{max,ip}$ (T)</th>
<th>$B_{max,op}$ (T)</th>
<th>$\Delta B_{max}$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>110</td>
<td>90</td>
<td>0.7</td>
<td>63</td>
<td>15.22</td>
<td>16.73</td>
<td>17.14</td>
<td>16.35</td>
<td>0.4</td>
</tr>
<tr>
<td>15</td>
<td>110</td>
<td>80</td>
<td>0.7</td>
<td>56</td>
<td>15.13</td>
<td>16.72</td>
<td>16.84</td>
<td>16.41</td>
<td>0.12</td>
</tr>
<tr>
<td>20</td>
<td>110</td>
<td>70</td>
<td>0.7</td>
<td>49</td>
<td>15.03</td>
<td>16.63</td>
<td>16.59</td>
<td>16.45</td>
<td>0.04</td>
</tr>
<tr>
<td>25</td>
<td>110</td>
<td>60</td>
<td>0.7</td>
<td>42</td>
<td>14.92</td>
<td>16.46</td>
<td>16.49</td>
<td>16.45</td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L_s$ (mm)</th>
<th>$L + 2L_s$ (mm)</th>
<th>$L$ (mm)</th>
<th>%Iron</th>
<th>$L_{yoke}$ (mm)</th>
<th>$B_0$ (T)</th>
<th>$B_{ss,max}$ (T)</th>
<th>$B_{max,ip}$ (T)</th>
<th>$B_{max,op}$ (T)</th>
<th>$\Delta B_{max}$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>110</td>
<td>90</td>
<td>0.8</td>
<td>72</td>
<td>15.34</td>
<td>16.87</td>
<td>17.22</td>
<td>16.38</td>
<td>0.35</td>
</tr>
<tr>
<td>15</td>
<td>110</td>
<td>80</td>
<td>0.8</td>
<td>64</td>
<td>15.25</td>
<td>16.81</td>
<td>16.96</td>
<td>16.43</td>
<td>0.15</td>
</tr>
<tr>
<td>20</td>
<td>110</td>
<td>70</td>
<td>0.8</td>
<td>56</td>
<td>15.16</td>
<td>16.69</td>
<td>16.71</td>
<td>16.47</td>
<td>0.02</td>
</tr>
<tr>
<td>25</td>
<td>110</td>
<td>60</td>
<td>0.8</td>
<td>48</td>
<td>15.05</td>
<td>16.56</td>
<td>16.6</td>
<td>16.47</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 6.7: Optimization parameters for SMC dipole [54]. $B_{max,ip}$ and $B_{max,op}$ are peak fields on inner and outer packs respectively.
6.6 Magnetic design: Phase II

The second phase design developed along two different paths: a longer oven has to be selected for the thermal treatment, and at the same time new design techniques have to be planned in order to set peak field in the cable straight section. At RAL, it has been proposed to adopt a longer iron section of the lateral pad so reduce the field in the heads. The best results have been achieved by doubling the lateral iron pad length with respect to the vertical and yoke lengths. By keeping the present oven constraints, a $\Delta B_{\text{max}} = 0.02 \text{T}$ is obtained, with $L_{ss} = 60 \text{ mm}$; by increasing it to 150 mm, we can get $\Delta B_{\text{max}} = -0.09 \text{T}$. The peak field is now on the straight section, but still too low and likely within the uncertainty given by the FE model.

At CERN, two alternative configurations have been proposed to lower the peak field in the head; both of them have been validated for a coil in air, implemented in ROXIE. They can be summarized as follows:

- to use an asymmetric pancake layout, either by increasing the spacer length of the upper layer only or by keeping the same spacers length and varying the layers $L_{ss}$;
- to introduce two spacers in the heads, and keep a symmetrical structure between layers.

6.6.1 Asymmetric coil configuration

Figure 6.9: SMC coil heads optimization. Asymmetric spacers are adopted, keeping the overall dimensions as a constant. The layer jump will be placed along the straight section.
By varying the length of the upper spacer (see fig. 6.9), it is possible to reduce the mutual induction between layers and consequently the peak field on the lower one. The computations have been done assuming: \( L_s = 10 \) mm, \( L = 120 \) mm, and \( I = 13.3 \) kA. The upper spacer length \( L_{s,up} \) has been varied as shown in table 6.8.

<table>
<thead>
<tr>
<th>( L_{s,up} ) (mm)</th>
<th>( B_{max} ) (T)</th>
<th>( B_{ss} ) (T)</th>
<th>( \Delta B_{max} ) (T)</th>
<th>( \Delta max ) (T)</th>
<th>( \Delta ss ) (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>13.49</td>
<td>12.04</td>
<td>1.45</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>13.39</td>
<td>12.01</td>
<td>1.38</td>
<td>-0.11</td>
<td>-0.03</td>
</tr>
<tr>
<td>16</td>
<td>13.28</td>
<td>12.02</td>
<td>1.26</td>
<td>-0.21</td>
<td>-0.02</td>
</tr>
<tr>
<td>20</td>
<td>13.16</td>
<td>12.02</td>
<td>1.14</td>
<td>-0.33</td>
<td>-0.02</td>
</tr>
<tr>
<td>25</td>
<td>13.09</td>
<td>12.03</td>
<td>1.06</td>
<td>-0.40</td>
<td>-0.01</td>
</tr>
<tr>
<td>30</td>
<td>13.10</td>
<td>12.03</td>
<td>1.07</td>
<td>-0.39</td>
<td>-0.01</td>
</tr>
<tr>
<td>40</td>
<td>13.12</td>
<td>12.03</td>
<td>1.09</td>
<td>-0.38</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

Table 6.8: Asymmetric spacer configuration. The first computation refers to a symmetric spacer lay-out, with \( L_s = 10 \) mm, \( L = 120 \) mm, and \( I = 13.3 \) kA. The last three lines (Italic font) indicate that the peak field moved to the outer pack.

A reduction of the peak field on the first pack \( \Delta max = 0.33 \) T is achieved, raising to 0.4 T with the peak field on the outer coil turns. The variation of the straight section field \( \Delta ss \) is negligible (see fig. 6.10).

Figure 6.10: SMC coil heads optimization. Field profile along the magnet midplane for different upper spacer thickness.
6. Magnetic Design

The drawbacks of this solution are mainly two: for a given overall encumbrance, the layer jump, i.e. the transition between the two layers, will be placed on the straight section. This means that a mechanical non-uniformity could be induced in the high field region and consequently affect the magnet performance. Furthermore a larger upper spacer means a different $L_{ss}$ between layers, thus reducing the field uniformity.

![Figure 6.11: SMC coil heads optimization. The upper layers are shifted so to have two different $L_{ss}$: $L_{ss,\text{low}} = 120 \text{ mm}$ and $L_{ss,\text{up}} = 100 \text{ mm}$. Spacer length $L_s = 20 \text{ mm}$.](image)

Another case has been analyzed: two layers having the same spacer length, but different $L_{ss}$, are superposed (see fig. 6.11). The drawbacks are mainly the same as those described for the asymmetric lay-out; nevertheless this solution will also imply a more complicated design of the coil containing structure.

<table>
<thead>
<tr>
<th>$L_{ss,\text{low}}$ (mm)</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{ss,\text{up}}$ (mm)</td>
<td>100</td>
</tr>
<tr>
<td>$L_s$ (mm)</td>
<td>20</td>
</tr>
<tr>
<td>I (kA)</td>
<td>13.3</td>
</tr>
<tr>
<td>$B_{ss}$ (T)</td>
<td>11.97</td>
</tr>
<tr>
<td>$B_{\text{max}}$ (T)</td>
<td>12.85</td>
</tr>
<tr>
<td>$\Delta\text{max}$ (T)</td>
<td>0.64</td>
</tr>
<tr>
<td>$\Delta ss$ (T)</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 6.9: SMC coil heads optimization: results for the configuration with single identical spacers and different straight section lengths between layers.
Due to these issues, only one case has been analyzed in order to validate this option. The maximum reduction in peak field $\Delta max = 0.64$ T (see table 6.9) is achieved with respect to the reference case shown in table 6.8.

### 6.6.2 Double spacer configuration

In order to further decrease the peak field on the heads, the outer coil pack can be split by means of an additional spacer, as it has been already done for some of the superconducting magnets built in the last decades, such as LHC dipoles. The second spacer has to be put as close as possible to the first pack, in order to be effective. All the computations have been done by placing the second spacer in between the fourth and fifth cable (see fig. 6.12), assuming a fixed thickness of 5 mm.

![Figure 6.12: SMC coil heads optimization. Double spacer configuration: in between the second and third blocks the spacer length has been set to 5 mm.](image)

The length of the first spacer has been varied from 5 to 20 mm in step of 5 mm. The results in term of field reduction have been compared to a reference model, having the following features:

<table>
<thead>
<tr>
<th></th>
<th>$L_{ss}$ (mm)</th>
<th>$L_s$ (mm)</th>
<th>$I$ (kA)</th>
<th>Coil turns</th>
</tr>
</thead>
<tbody>
<tr>
<td>block 1</td>
<td>120</td>
<td>15</td>
<td>13.3</td>
<td>block 1</td>
</tr>
<tr>
<td>block 2</td>
<td>2</td>
<td></td>
<td></td>
<td>block 2</td>
</tr>
<tr>
<td>block 3</td>
<td>17</td>
<td></td>
<td></td>
<td>block 3</td>
</tr>
</tbody>
</table>

Table 6.10: SMC coil heads optimization: reference model for double spacer layout computations.
The results exposed in fig. 6.13 reveal how the head splitting up is as effective as the asymmetric solution shown before, leading to a maximum reduction in peak field of $\simeq 0.4$ T; nevertheless this solution involves less manufacturing and assembly issue than the former one.

![Figure 6.13: SMC coil heads optimization. The peak field occurs on the outer pack for all the cases analyzed.](image)

Please note that the results obtained can be easily extrapolated for different straight section lengths and coil dimensions. The combination of one of the solutions presented up to now with the iron assembly can reasonably lead to the required field configuration. A first computation made in ANSYS reveals how this solution can be effective in terms of field reduction. The peak field, now moved on the straight section, is 0.22 T higher than the maximum field on the coil head. This preliminary result suggests to orient the optimization in this direction.

### 6.6.3 Bending radius analysis

The SD01 bending radius, or main post radius, was set to 18.6 mm and kept as a constant for the SMC. Nevertheless, the SMC cable is considerably bigger than SD01’s; the winding of such a larger cable on the same main post could be problematic and lead to cable pop-out on the heads. This implies probable endamaging of the superconductor strand with obvious consequences on the cable performances. According to the experience in cable winding at LBNL, a larger main post was advised, at least linearly scalable on the cable dimensions with
6.6 Magnetic design: Phase II

respect to SD01. A study on the main post radius has been carried out in ANSYS™ by means of a 2D model with iron circuit; all the others geometric parameters has been linearly scaled.

<table>
<thead>
<tr>
<th>$R_i$ (mm)</th>
<th>$B_{ss}$ (T)</th>
<th>$B_0$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>13.66</td>
<td>13.32</td>
</tr>
<tr>
<td>21</td>
<td>13.29</td>
<td>11.92</td>
</tr>
<tr>
<td>30</td>
<td>13.09</td>
<td>10.89</td>
</tr>
<tr>
<td>40</td>
<td>12.92</td>
<td>9.93</td>
</tr>
<tr>
<td>52</td>
<td>12.77</td>
<td>9.01</td>
</tr>
</tbody>
</table>

Table 6.11: Winding radius $R_i$ study. The increase in pole radius leads to a decrease in peak and bore field.

![Figure 6.14: Net magnetic forces as a function of the main post radius.](image)

Figure 6.14: Net magnetic forces as a function of the main post radius.

The results show that the increase of $R_i$ leads to a lower straight section field $B_{ss}$, related to a higher short sample current $I_{ss}$. This means that higher forces and consequently higher mechanical stress could be expected. This is what occurs in terms of vertical force $F_y$, parallel to the broad cable side, due to the increase in the field horizontal component $B_x$. The perpendicular component $B_y$ lowers down, together with the horizontal force $F_x$, which is usually highest than $F_y$ and responsible for cable current degradation, if not properly counterbalanced by the mechanical pre-stress. This means that an increase in pole radius is actually
6. Magnetic Design

beneficial in terms of reduction of mechanical stress on the cable pack. The augmented dimensions and characteristics of SMC cable lead to higher forces than SD01: \( \sim 1.6 \) for \( F_x \), and \( \sim 2.1 \) for \( F_y \) with a pole radius equal to 40 mm. By scaling the pole radius on the cable thickness, we would get a minimum \( R_i \) of 32 mm, which will be set to 40 mm by taking a safety factor equal to 1.25.

6.7 3D Finite Element model

Different 3D models have been realized in ANSYS\textsuperscript{™} for the magnetic optimization. These model exploit two main solution procedures which are implemented in the ANSYS\textsuperscript{™} solver: the MVP (Magnetic Vector Potential) and the MSP (Magnetic Scalar Potential) formulations. The first one has been used for modeling respectively: (1) the coil block, (2) the coil block with simplified iron circuit, (3) and the entire magnet assembly.

With the MVP formulation, the current sources can be modeled as an integral part of the finite element model; it so possible to compute directly the magnetic forces as a combination of the current density \( j_{eng} \) and of the flux density \( B \). Only \( 1/8^{th} \) of the assembly has been realized for symmetry reasons; 8-node brick elements have been selected (SOLID97), by setting a relevant element dimension of 2 mm. The open model boundary has been modeled with INFIN111 elements.

Nevertheless, since the results obtained with the MVP formulation could be incorrect if different permeable materials are used, and due to the larger simulation time required (3 magnetic d.o.f. per node), a second magnetic model has been realized to overcome these problems, using the MSP formulation. This second model allowed faster simulations runs at the same time to check the accuracy of the results obtained by the MVP model. Only one scalar d.o.f. per node exists, and the current sources are modeled as primitives rather then elements, therefore the current sources are not part of the finite element mesh. Even if only \( 1/8^{th} \) of the coil is modeled for symmetry reason, the current sources system has to be completely defined. The only drawback of this formulation is that no direct computation of the magnetic forces can be done without associating each coil element to a current density value. This problem can be overcome by solving a double step model. The first solves for an electric step, applying a voltage drop on the coil mesh to get the current density distribution; the second one deals with the magnetic solution. The coil has been realized as a cable stack coil instead of a solid coil as for MVP approach.

The bare cable and its insulation have been modeled, associating a different virtual electric resistance proportional to each cable length. This choice has been done with the purpose of implementing the mechanical properties of the insulation and of the bare cable separately. At the end of the magnetic step, the Lorentz
forces can be obtained at the centroid of each coil element.

The simulation time saving by using the MSP formulation is about 80% compared to a MVP model. A cross check of these two models with CAST3M and VF Opera models have been performed, revealing a good general agreement [53].

Figure 6.15: SMC coil pack. Note that, $X$ direction is perpendicular to the cable broad face, $Y$ is parallel to that and $Z$ is the longitudinal direction. (a) entire assembly; (b) coil pack and magnetic circuit components.
6. Magnetic Design

6.7.1 Final configuration

The results presented up to now show that a bigger oven is mandatory to match the design requirements. The choice of using the oven available at RAL was finally dismissed. A bigger vacuum furnace provided by the Culham Special Techniques group, close to RAL was selected. Being the geometrical dimensions no more a constraint, exception made for the overall magnet diameter, final computations have been made by combining the design solutions introduced before. The double spacer configuration has been kept as the most effective in terms of optimum $\Delta B_{\text{max}}$, together with the overextended lateral iron pad. Computations made in VF Opera (see table 6.12) show that to get $\Delta B_{\text{max}} \geq 0.5$ T, we should set $L_{ss} \geq 100$ mm, $L_{s,1} \geq 30$ mm and $L_{s,2} = 10$ mm.

<table>
<thead>
<tr>
<th>$L$ (mm)</th>
<th>$L_{s,1}$ (mm)</th>
<th>$L_{s,2}$ (mm)</th>
<th>$B_0$ (T)</th>
<th>$B_{ss,\text{max}}$ (T)</th>
<th>$B_{ip,\text{max}}$ (T)</th>
<th>$\Delta B_{\text{max}}$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10</td>
<td>15</td>
<td>11.64</td>
<td>15.71</td>
<td>15.50</td>
<td>-0.21</td>
</tr>
<tr>
<td>100</td>
<td>15</td>
<td>10</td>
<td>11.63</td>
<td>15.70</td>
<td>15.47</td>
<td>-0.23</td>
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<td>20</td>
<td>11.58</td>
<td>15.63</td>
<td>15.37</td>
<td>-0.26</td>
</tr>
<tr>
<td>100</td>
<td>15</td>
<td>15</td>
<td>11.57</td>
<td>15.62</td>
<td>15.33</td>
<td>-0.29</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>10</td>
<td>11.56</td>
<td>15.61</td>
<td>15.31</td>
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<tr>
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<td>11.53</td>
<td>15.60</td>
<td>15.26</td>
<td>-0.34</td>
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<tr>
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<td>15</td>
<td>20</td>
<td>11.52</td>
<td>15.59</td>
<td>15.23</td>
<td>-0.36</td>
</tr>
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<td>11.51</td>
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<td>-0.38</td>
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<td>15.57</td>
<td>15.18</td>
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<tr>
<td>100</td>
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<td>30</td>
<td>11.48</td>
<td>15.52</td>
<td>15.17</td>
<td>-0.35</td>
</tr>
<tr>
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<td>25</td>
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<td>15.51</td>
<td>15.13</td>
<td>-0.38</td>
</tr>
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<tr>
<td>100</td>
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<td>15</td>
<td>11.45</td>
<td>15.49</td>
<td>15.08</td>
<td>-0.41</td>
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<tr>
<td>100</td>
<td>30</td>
<td>10</td>
<td>11.44</td>
<td>15.49</td>
<td>15.06</td>
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</tr>
<tr>
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<td>30</td>
<td>20</td>
<td>11.36</td>
<td>15.44</td>
<td>14.97</td>
<td>-0.47</td>
</tr>
<tr>
<td>100</td>
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<td>20</td>
<td>11.45</td>
<td>15.48</td>
<td>14.80</td>
<td>-0.68</td>
</tr>
<tr>
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<td>30</td>
<td>20</td>
<td>11.49</td>
<td>15.53</td>
<td>14.68</td>
<td>-0.85</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>20</td>
<td>11.51</td>
<td>15.47</td>
<td>14.64</td>
<td>-0.83</td>
</tr>
</tbody>
</table>

Table 6.12: SMC magnetic design, results of semi-defined configuration [55].

Before selecting the final parameters, a last analysis has been carried out on the yoke width thickness influence over the peak field and the outer field. The latter has to be lower enough not to interact with the cryostat components. The results are summarized in fig. 6.16: a thicker yoke leads to higher central induction allowing at the same time a reduction of the outer field to 0.6 T for $w_{\text{yoke}} = 90$ mm. We will finally select this value, in order to match both magnetic, mechanical
requirements (as it will be outlined in the next chapter), and cost limitation (the yoke laminations will be obtained via fine blanking from stock plates at CERN).

Figure 6.16: Influence of the yoke width on the peak field $B_{\text{max}}$ and on the outer maximum field $B_{\text{out, max}}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turns number</td>
<td>$N_{\text{tot}}$</td>
<td>/</td>
<td>21</td>
</tr>
<tr>
<td>Inner turns number</td>
<td>$N_{\text{int}}$</td>
<td>/</td>
<td>2</td>
</tr>
<tr>
<td>Mid-pack turns number</td>
<td>$N_{\text{mid}}$</td>
<td>/</td>
<td>2</td>
</tr>
<tr>
<td>Outer turns number</td>
<td>$N_{\text{out}}$</td>
<td>/</td>
<td>17</td>
</tr>
<tr>
<td>Island half-width</td>
<td>$R_{i}$</td>
<td>mm</td>
<td>40</td>
</tr>
<tr>
<td>Outer radius</td>
<td>$R_{o}$</td>
<td>mm</td>
<td>94.6</td>
</tr>
<tr>
<td>Straight section length</td>
<td>$L$</td>
<td>mm</td>
<td>150</td>
</tr>
<tr>
<td>Interlayer thickness</td>
<td>$i^{t}_{\text{int}}$</td>
<td>mm</td>
<td>0.2</td>
</tr>
<tr>
<td>Mid-plane insulation thickness</td>
<td>$i^{t}_{\text{mid}}$</td>
<td>mm</td>
<td>1.6</td>
</tr>
<tr>
<td>Inner spacer length</td>
<td>$L_{s,1}$</td>
<td>mm</td>
<td>30</td>
</tr>
<tr>
<td>Outer spacer length</td>
<td>$L_{s,2}$</td>
<td>mm</td>
<td>10</td>
</tr>
<tr>
<td>Magnetic vertical pad length</td>
<td>$L_{y-\text{pad, iron}}$</td>
<td>mm</td>
<td>105 (0.7$L_{ss}$)</td>
</tr>
<tr>
<td>Magnetic horizontal pad length</td>
<td>$L_{x-\text{pad, iron}}$</td>
<td>mm</td>
<td>210 (1.4$L_{ss}$)</td>
</tr>
<tr>
<td>Magnetic yoke length</td>
<td>$L_{yoke, \text{iron}}$</td>
<td>mm</td>
<td>105 (0.7$L_{ss}$)</td>
</tr>
<tr>
<td>Yoke width</td>
<td>$w_{\text{yoke}}$</td>
<td>mm</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 6.13: Optimized parameters for SMC dipole.
6. Magnetic Design

The parameters for the proposed configuration are listed in table 6.13. The results obtained with ANSYS™ have been compared to the ones obtained via CAST3M and VF Opera, and listed in table 6.14. This configuration assures an efficient field distribution along the coil, with a difference between the peak field, in the straight section, and the field in the heads of 0.7 T.

Figure 6.17: Field distribution in SMC coil: (a) MVP model results, (b) MSP model results.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Unit</th>
<th>ANSYS</th>
<th>MVP</th>
<th>ANSYS</th>
<th>MSP</th>
<th>VF Opera</th>
<th>CAST3M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak field</td>
<td>$B_{\text{max}}$</td>
<td>T</td>
<td>12.85</td>
<td>12.94</td>
<td>12.96</td>
<td>12.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak field on heads</td>
<td>$B_{\text{ip,max}}$</td>
<td>T</td>
<td>12.2</td>
<td>12.22</td>
<td>12.23</td>
<td>12.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform field length</td>
<td>$L_{u,1%}$</td>
<td>mm</td>
<td>55</td>
<td>55</td>
<td>60</td>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_{\text{max}} - B_{\text{ip,max}}$</td>
<td>$\Delta B_{\text{max}}$</td>
<td>T</td>
<td>0.71</td>
<td>0.71</td>
<td>0.73</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_{\text{ip,max}} - B_{\text{op,max}}$</td>
<td>$\Delta B_{\text{end}}$</td>
<td>T</td>
<td>1</td>
<td>1.3</td>
<td>0.9</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central field</td>
<td>$B_0$</td>
<td>T</td>
<td>9.85</td>
<td>9.65</td>
<td>-</td>
<td>9.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X-Force on 1/8th coil</td>
<td>$F_x$</td>
<td>kN</td>
<td>338</td>
<td>326</td>
<td>333</td>
<td>328</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y-Force on 1/8th coil</td>
<td>$F_y$</td>
<td>kN</td>
<td>-406</td>
<td>-388</td>
<td>-396</td>
<td>-393</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z-Force on 1/8th coil</td>
<td>$F_z$</td>
<td>kN</td>
<td>136</td>
<td>131</td>
<td>132</td>
<td>131</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X-Force on straight section</td>
<td>$F_{x,ss}$</td>
<td>MN/m</td>
<td>2.06</td>
<td>1.98</td>
<td>2</td>
<td>1.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y-Force on straight section</td>
<td>$F_{y,ss}$</td>
<td>MN/m</td>
<td>-1.95</td>
<td>-1.86</td>
<td>-1.9</td>
<td>-1.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total stored energy</td>
<td>$E_{\text{mag}}$</td>
<td>kJ</td>
<td>198</td>
<td>190</td>
<td>209</td>
<td>211</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnetic inductance</td>
<td>$L$</td>
<td>mH</td>
<td>2</td>
<td>1.9</td>
<td>2.1</td>
<td>2.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.14: SMC dipole field and forces results and comparison between programs.
6. Magnetic Design

The differential field between the coil heads is equal to 1 T. The choice of increasing the straight section length to 150 mm allowed to match a field uniformity length equal to the cable twist pitch, within 1% of tolerance. The computed magnetic forces are considerably higher compared to SD01’s, due to a more performing layout as well as to a different cable used: $F_{x,SMC} \sim 2F_{x,SD01}$, $F_{y,SMC} \sim 3.2F_{y,SD01}$, whereas $F_{z,SMC} \sim 1.5F_{z,SD01}$. The maximum stress $\sigma_x$ along the peak field area, i.e. the coil straight section, can be roughly evaluated by considering the lateral force $F_x$ and the straight section surface:

$$\sigma_x = \frac{F_x}{L_{ss}h_{pack}} \simeq 100\text{MPa} \quad (6.3)$$

where $h_{pack}$ is the height if the insulated coil pack. The value obtained is in principle low enough to avoid cable degradation, but this is only a reference result revealing the feasibility of the SMC, opening the way to the mechanical design which will be described in the next chapter. The values of the stored magnetic energy and of the magnetic inductance have been computed, since the design of a quench protection system has to be envisaged. This system will preserve the coil from irreversibly endamaging during tests quenches, and it will basically consist of quench heaters.

6.8 Summary

The SMC magnetic optimization procedure described has been split in two parts. This choice chronologically reflects the design path, driven by the availability of a bigger furnace for the cable heat treatment.

It has been shown how the length of the magnet is an essential parameter to effectively move the peak field on the straight section. In the first part, being the furnace too short, the design goal could not be reached, even if adopting an asymmetric iron circuit layout. In the second part a more refined solution has been introduced: a longer furnace and a more complicated coil heads layout was essential to match the requirements. Two head spacers will be used to reduce the mutual induction between cables in the head. The choice of an overextended lateral pad completes the SD01 optimization, confining the peak field along the straight section, 0.7 T higher than the field along the heads.

The peak field is slightly below the target value ($\sim 12.94$ T). This is mainly related to the main post (also called central island) width. Nevertheless, being the mechanical features of such a cable still an unknown, a larger main post radius has been set in order to avoid any possible coil endamaging, especially during winding phase.
Due to the peak field value and to the operative temperature (4.2 K), the superconductor should not undergo any magneto-thermal instability problem [56]. This phenomenon affected some of the Nb\(_3\)Sn magnets tested in the past [18].

The magnetic forces produced at short sample are largely higher than SD01’s, but leading to a maximum stress which is well below the estimated limit for this type of superconductor. The mechanical design will take over the configuration now proposed and it will described in the next chapter.
6. Magnetic Design
Chapter 7

Mechanical Design

The aim of the SMC mechanical study is firstly to validate the structure as it comes from the optimized magnetic analysis. The stress distribution on the coil pack has to be fully understood in order to avoid stress levels higher than 150 MPa, causing the superconductor degradation. At the same time, we would like to reduce the possibility of magnet premature quenches by applying a sufficient pre-stress to the coil in order to avoid conductor movements. This is set as the starting point for the nominal operating condition of the SMC. Since the structure has been designed to be as much modular as possible, variation of the pre-stress could be possibly done in order to perform current degradation study of the Nb$_3$Sn cable. All the mechanical analysis have been done by considering three main steps: (i) assembly, (ii) cool down at 4.2 K, and (iii) powering at short sample current.

Specifically, a 2D model has been developed in order to have a general feeling on the structure behavior and to set up the on-plane pre-stress on the coil. In particular, it will be described how the pre-stress can be managed between warm and cryogenic conditions by changing the keys assembly interference and the tube thickness. The stress distribution inside the coil has been evaluated, whereas all the magnet components have been verified and validated by means of mechanical failure criteria.

The 3D model is based on the MSP magnetic model. It consists of a cable coil, with refined cable mesh; the outer aluminum shell as well as the whole structure providing the longitudinal pre-stress have been modeled. The aim of this model has been firstly to validate the results obtained with the simplified 2D; the longitudinal pre-stress has then been set-up in order to counterbalance the magnetic forces which tend to open up the coil. The results of the stress analysis for the final configuration will be finally outlined.
7. Mechanical Design

7.1 2D Finite Element model

The 2D model consists of planar 4 nodes elements (plane 42) with plane stress option (see fig. 7.1). The mesh used for the coil pack is the same as the magnetic model, so that the magnetic forces could be transferred using the ANSYS internal routine LDREAD. The mesh of the magnet pack and magnetic circuit has been slightly revised, so to decrease the simulation time. The materials have been considered fully isotropic (see table 7.1).

Figure 7.1: SMC 2D mechanical model scheme.

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>E (GPa)</th>
<th>( R_p^{0.2} ) (MPa)</th>
<th>( \alpha \Delta T ) (mm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>293 K</td>
<td>4.2 K</td>
<td>293 K</td>
</tr>
<tr>
<td>Coil</td>
<td>Nb(_3)Sn</td>
<td>30</td>
<td>42</td>
<td>-</td>
</tr>
<tr>
<td>Central pole</td>
<td>Ti6Al4V</td>
<td>110</td>
<td>130</td>
<td>827</td>
</tr>
<tr>
<td>Horseshoe</td>
<td>Ti6Al4V</td>
<td>110</td>
<td>130</td>
<td>827</td>
</tr>
<tr>
<td>Magnetic items</td>
<td>MAGNETIL</td>
<td>205</td>
<td>200</td>
<td>180</td>
</tr>
<tr>
<td>Shell</td>
<td>Al 2014 T651</td>
<td>70</td>
<td>80</td>
<td>415</td>
</tr>
</tbody>
</table>

Table 7.1: SMC material properties.

The pre-compression at warm is given by the assembly interference of the keys inserted in between the pads and the yoke. The stress component which is mainly responsible of cable movement within the coil and possible superconductor degradation is the horizontal one. The vertical key plays so an active role in coil pre-stress, whereas the horizontal key is only responsible for magnetic pack positioning.
within the yoke. In the 2D model, the assembly interference is modeled by means of contact elements interference, which will be called $i_x$ and expressed in ($\mu$m). In reality, the interference will be set by means of steel shims inserted with the keys. These have been placed in correspondence of the coil pack, in order to provide a uniform stress distribution after cool-down. An arbitrary shell thickness of 20 mm has been set; all the other geometrical parameters are the optimized ones coming from the magnetic design. The consistency of the model has been preliminarily verified at warm, by investigating the net compressive force between the outer coil profile and the related net tension force on the shell thickness. Since all the contact elements have been considered frictionless, the two corresponding forces have to vary linearly as follows:

$$\bar{\sigma}_{x,\text{coil}} h_{\text{coil}} = \bar{\sigma}_{r,\text{shell}} t_{\text{shell}}$$

(7.1)

The results are shown in fig 7.2.

![Figure 7.2: Net forces on shell and coil pack as functions of the assembly interference.](image)

From the net force exerted on the coil pack, it is also possible to evaluate the equivalent pressure provided by the bladders:

$$P_{\text{bladder}} = \frac{\bar{\sigma}_{\theta,\text{shell}} t_{\text{shell}}}{w_{\text{bladder}}} = 127 \text{ bar}$$

(7.2)
Please note that this is not the pressure needed to insert the shimmed keys since the clearance we need is obviously bigger, but it can be considered as a reference value of the minimum pressure we have to asset.

### 7.1.1 Assembly interference analysis

An analysis on the coil pre-stress during assembly, cool down, and powering has been carried out, by varying the lateral interference from 100 to 1000 µm, in steps of 200 µm. The aim of the analysis is to verify which is the maximum pre-load we can provide at warm with a maximum stress on the coil around 150 MPa at powering. According to fig. 7.3, the limit value for the assembly interference is about 400 µm, with $\sigma_{eqv,max} = 50$ MPa at warm, and $\sim 155$ MPa at short sample condition. The stress profile on the inner coil edge has been studied, in order to understand the stress transmitted to the G10 composite (glass fiber reinforced epoxy resin) that impregnate the coil. This material can sustain a maximum traction load estimated around 20 MPa, limit beyond which the resin cracks and the coil separates from the titanium components. The analysis focuses on the powering phase, when the magnetic forces tend to separate the coil inner edge from the main post, loading at the same time the outer edge (horseshoe side). As shown in fig. 7.4, the residual stress at powering is $\leq 20$ MPa.

![Figure 7.3: $\sigma_{eqv,max}$ as a function of the assembly interference $i_x$.](image-url)

The difference in stress distribution between lower and upper coil is due to the magnetic force pattern, leading to higher stress gradient in the upper layer.
Figure 7.4: Stress $\sigma_x$ at the boundary between coil and main post: (a) lower coil, (b) upper coil.
7. Mechanical Design

7.1.2 Pre-stress parameters analysis

The limit value of 400 $\mu$m outlined before is referred to a shell thickness of 20 mm. Nevertheless, the coil pre-load derives from the combination of the assembly interference and the shell thickness, as we have pointed out before. We want now to analyze how these parameters can combine to provide a given pre-stress, and how the yoke thickness can influence it.

![Figure 7.5: Influence of the yoke width on the shell thickness $t_{\text{shell}}$.](image)

The influence of the yoke width $w_{\text{yoke}}$ on $t_{\text{shell}}$ has been investigated for a given set of assembly interferences, by imposing the following design criteria:

- the residual stress between the coil and the main post $\bar{\sigma}_{x,\text{cp}}$ has to be lower than 20 MPa;
- the maximum stress $\sigma_{\text{eqv,max}}$ on the coil must be $\leq$ 150 MPa;
- the failure criteria for the dipole components have to be observed by means of a safety factor equal to 1.5. The VonMises criterion was selected for ductile materials both at warm and at cold, exception made for the magnetic items, made in low carbon steel experiencing brittle fracture at cryogenic temperature [57]-[58]-[59]. The failure criterion which has been used is the Rankine’s criterion: the first principle stress is the responsible of component collapse, and has to be compared to the UTS (Ultimate Tensile Strength) value (see table 7.1).
The analysis (see fig. 7.5) shows that the increase in yoke thickness up to 55 mm leads to higher pre-stress at warm, due to the augmented system rigidity, so that a thinner shell can be used. For thicker yoke widths, the pre-load is then completely determined by the shell and the keys’ dimensions. For a given pre-stress, a bigger interference increases the pre-load at warm thus implying a thinner aluminum shell. The curves in fig. 7.5 represent the boundary between: (i) an insufficiently loaded system, involving the risk of coil detachment from the main post, and (ii) an over-loaded system, where the equivalent stress provided to the coil is higher than 150 MPa. For a given magnet configuration, it will be anyway possible to play around the curve with the shims, in order to decrease or increase the coil pre-stress, avoiding premature quenches due to coil movement.

### 7.1.3 Stress analysis

We present here the results obtained from the stress analysis. The three phases of the magnet life cycle are described, reporting the maximum stress reached in every magnet component according to the design criteria assumed. It is also shown the average stress at the coil lateral edges: pole side, and horseshoe side. The assembly interference has been set to 400 μm, whilst the shell thickness is equal to 17 mm.

<table>
<thead>
<tr>
<th></th>
<th>Assembly</th>
<th>Cool down</th>
<th>Powering</th>
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<tbody>
<tr>
<td>Coil (Nb₃Sn)</td>
<td>$\sigma_{\text{eqv, max}}$</td>
<td>40</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{lim}}$</td>
<td>150</td>
<td>150</td>
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<tr>
<td>Shell (Al2014 T651)</td>
<td>$\sigma_{\text{eqv, max}}$</td>
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<tr>
<td></td>
<td>$\sigma_{\text{lim}}$</td>
<td>415</td>
<td>545</td>
</tr>
<tr>
<td>Yoke (MAGNETIL)</td>
<td>$\sigma_{\text{max}}$</td>
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<td>90</td>
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<tr>
<td></td>
<td>$\sigma_{\text{lim}}$</td>
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<td>723</td>
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<td>Pads (MAGNETIL)</td>
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<td></td>
<td>$\sigma_{\text{lim}}$</td>
<td>180</td>
<td>723</td>
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<tr>
<td>Coil-pole contact</td>
<td>$\sigma_{x}$</td>
<td>-40</td>
<td>-116</td>
</tr>
<tr>
<td>Coil-horseshoe contact</td>
<td>$\sigma_{x}$</td>
<td>-40</td>
<td>-115</td>
</tr>
</tbody>
</table>

Table 7.2: SMC stress analysis results. The maximum stress for the items in MAGNETIL after cool down and powering is the first principal.

At warm, the equivalent bladder pressure exerted to have 40 MPa along the coil is about 140 bar. Nevertheless, the maximum pressure will be higher than that, due to the need of enough clearance to slide the keys inside the magnet. The safety margin on the shell and coil stress limit is anyway large enough not to be a concern in the assembly phase. This consideration holds for the bladder maximum pressure as well, which have been tested up to 600 bar.
7. Mechanical Design

As it is shown in table 7.2, the stress along the coil is uniform both after assembly and cool down (see fig. 7.6-7.7); after powering the stress profile is notably unbalanced, due to the magnetic forces distribution. The residual stress at the pole edge anyway assures a stable contact in between the coil and the main post. During the assembly phase, all the materials are ductile, so we can...
use the VonMises criterion to evaluate the stress state inside the components. At cryogenic temperature, the magnetic steel components show brittle fracture: the failure criterion adopted is Rankine’s (maximum first principle stress). All the items observe the stress limits imposed, as stated in the design criteria before mentioned.

Figure 7.7: $\sigma_{eqv}$ in the coil after powering.
7. Mechanical Design

7.2 3D Finite Element model

The aim of the 3D model is firstly to set up the longitudinal pre-load in order to keep the cables in contact with the Titanium components (main post and spacers), under the action of the magnetic forces. The pre-load system will be designed and consequently dimensioned. Secondly, a complete analysis on the coil stress distribution as well as on the magnet items will be performed, according to the same procedure and criteria followed for the 2D model.

The 3D model has been realized in ANSYS\textsuperscript{TM} on the base of the magnetic one, implementing the outer shell and the longitudinal pre-load system (fig. 7.8). Only $\frac{1}{8}$th of the model have been defined for symmetry reason in the space domain ($Ox \geq 0, Oy \geq 0, Oz \geq 0$). The materials properties used are listed in table 7.1.

The coil is the only part affected by major mesh changes; numerical imprecisions due to the highest aspect ratio between the insulation and bare cable elements have been observed adopting the same mesh of the magnetic model. After some refinements, the bar cable width has been finally divided into three elements (0.7 mm width) instead of one (2.1 mm) (fig. 7.9).

![Figure 7.8: 3D mechanical model.](image)

As we have outlined before, the coil is impregnated by a glass fiber epoxy reinforced composite, and substantially glued to the containing structure after impregnation. For stress higher than 20 MPa (tension) the impregnation can break allowing cable movements. There are two ways of defining the characteristics of the contact elements between the winding and the coil items: either (1) considering the contact area as fully bonded (linear analysis) and checking the local stress with
7.2 3D Finite Element model

Figure 7.9: Cable mesh for: (a)-(b) the magnetic model, (c)-(d) the mechanical model.

respect to the assumed limit, or (2) considering friction contact with sliding and separation allowed, since the coil could actually separate from the inner pole and spacers, if the stress is too high. None of the two approach can be considered fully exact, since it should be introduced a control on the maximum stress on the G10 at every simulation step. We have anyway opted for the first solution, since it can be considered as the closest to the real configuration of glued assembly. The contact elements used elsewhere in the model have been considered frictionless, in order to have a general picture of the system behavior. This can be considered as a preliminary model set-up: later modifications and improvements on contact status will be introduced once the magnet will be tested. In particular, the friction coefficients will be tuned on the base of the state of the art on material tribology and on the magnet experimental results. Since the mesh of the models used for
the magnetic and mechanical optimization do not coincide, it will not be possible to use the LDREAD command to transfer the magnetic forces to the mechanical model. We will use a *ad-hoc* routine realized in ANSYS™ in order to store the forces from the magnetic model and to transfer them to the mechanical one using nodal coordinates.

**7.2.1 Parametric analysis of the longitudinal pre-load**

The system used in SD01 to exert the longitudinal pre-load is represented in fig. 7.10. This system has been successfully used in magnets previously realized at LBNL. It consists of two aluminum rods running through the magnet pack from one extremity to the other one, and bolted on two stainless steel end-plates which transfer the load to the coil pack. At warm, the rods are put under tension by means of an hydraulic jack placed at one end, and then fixed in place at the jack side by means of a couple of nuts. By acting on the jack pressure and on the nuts is so possible to set-up the required pre-load level, accordingly to the user needs. The total coil pre-stress is then achieved at cryogenic temperature by exploiting the differential thermal contraction between the rods and the coil pack. For this analysis, we assume a rod diameter of 28 mm. The Aluminum alloy is the same as for the outer shell (Al 2014 T651).

![Figure 7.10: 3D model of the longitudinal pre-load system.](image)

The coil longitudinal pre-stress has to be set up in relation to the maximum stress at the boundary between the coil and the Titanium components, namely the radial stress $\sigma_r$ accordingly to the local cylindrical coordinate system. Since
the highest stress level occurs between the first cable and the main post, this will be assumed as our reference value in order to define the correct longitudinal load. This parameter will be expressed in terms of rod displacement $dz$ ($\mu$m) at $z=0$. The results of the parametric analysis are represented in fig. 7.11. Three different lateral interferences have been set ($i_x = 200, 400, 600 \mu$m) whereas the longitudinal displacement ranges from 1000 to 2500 $\mu$m. It can be seen that the minimum displacement we have to provide the structure is about 1.5 mm, to have $\sigma_{r,max} \leq 20$ MPa, independently from the lateral interference. This is due to the invariance of the lateral pre-load achieved, since the shell thickness has been changed for a given assembly interference according to the results shown in fig. 6.16.

Figure 7.11: $\sigma_{r,max}$ on the inner coil head as a function of the longitudinal pre-load factor $dz$ for different lateral assembly interferences.

The net magnetic longitudinal force in SD01 was about 85 kN for each pancake; the longitudinal rod displacement $dz_{SD01}$ was set to 380 $\mu$m. The total magnetic force has been estimated around 260 kN for the SMC; by scaling on the magnetic forces, the expected $dz_{SMC} \simeq 1200 \mu$m. This first guess value is so consistent with the results described before.

7.2.2 Final configuration

As it was explained at the beginning of this chapter, the SMC has been developed by a joint venture between laboratories under the consultancy of LBNL. The aim of
the SMC is not only to apply variable pre-stress values on the coil, but to allow the
test of cables of different type and size. The design parameters we have presented
up to now are tailored on the SMC cable, with conventional glass fiber insulation.
Nevertheless, the CEA lab will perform tests on the same coil but using a ceramic
insulation, thicker than the conventional one. A larger magnet pack aperture is
consequently required to accommodate both coil types, implying a larger vertical
pad and thinner lateral pads. Since the magnetic computation had to be reviewed,
a different proposition has been made, consisting in the use of a massive lateral
pads without magnetic insertions, in order to grant the uniformity of the stress
distribution on the coil sides. The results presented by [60] show that the peak
field is slightly influenced by the lateral magnetic pad, about 5% lower (12.91 T)
than the optimized value of 12.94 T obtained before.

The assembly parameters used in the final configuration are summarized in
table 7.3.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell thickness</td>
<td>$t_{shell}$</td>
<td>mm</td>
</tr>
<tr>
<td>Assembly interference</td>
<td>$i_x$</td>
<td>µm</td>
</tr>
<tr>
<td>Rod diameter</td>
<td>$\phi_{rod}$</td>
<td>mm</td>
</tr>
<tr>
<td>Rod displacement</td>
<td>$dz$</td>
<td>µm</td>
</tr>
</tbody>
</table>

Table 7.3: SMC: assembly and pre-load parameters.

As we have done for the 2D model, the failure criteria will checked for the
whole magnet, according to the material behavior at the given temperature (Von-
Mises/Rankine). The results of the whole analysis are listed in table 7.4.

The global reference system used is the same as for the magnetic analysis; for
the coil heads we will use a local cylindrical system, as follows: $r$ is the radial
direction, $\vartheta$ is the azimuthal one, following the cable arc, and $z$ is the vertical one,
corresponding to the global Y direction.

**Assembly: longitudinal pre-load**

The first step of the assembly consists of inserting the two yoke halves into the
aluminum shell. Two nominal keys are placed in between the halves; they have a
small interference with respect to the yoke vertical gap clearance such as to assure
the contact with the shell and allow the magnet pack insertion. Before applying
the longitudinal pre-load, both the vertical and the horizontal keys are placed
with a reduced interference so to asset the stability of the magnet assembly. In
particular, the lateral interference aims at avoiding the coil detachment from the
pole along the straight section during the rod loading. Though all these steps are
not modeled in ANSYS”, we will not be anyway too much concerned about the stress in between the coil and the pole straight section, in case it will undergo under tension after applying the axial pre-load. The longitudinal pre-load produces a rod tension which can be evaluated as follows:

\[
\sigma_{z,rod} = E_{rod} \frac{dz_{rod}}{L_{rod}} = \frac{70000 \cdot 1.5}{302.3} \approx 347 \text{ MPa (7.3)}
\]

The value obtained in the FE simulation is about 260 MPa, \( \sim 75\% \) of the estimated value. This effect is mainly due to finite rigidity of the coil pack and of the G10 block which is interposed in between the end-plate and the coil pack in order to better distribute the axial load. Moreover the end plate undergoes bending during the application of the axial load, further reducing the overall stiffness of the pre-tension system. The value obtained is consistent with the SD01 results, where the numerical value was about 62 MPa, equal to \( \sim 70\% \) of the estimated 87 MPa.

The longitudinal compressive stress on the coil \( \sigma_{z,coil} \) increases from the coil straight section towards the heads due to the rod effect. By looking at the coil heads locally (see fig. 7.12), this aspect is more evident: the maximum radial stress occurs at the Ox symmetry plane, whilst decreasing towards the straight section. This trend is more evident in the outer pack, where \( \sigma_r \) decreases from 60 MPa (compression) to 8 MPa (tension). The residual stress between the winding straight section and the main post is negligible.

The longitudinal rod shows a localized maximum in correspondence of the nut equal to 400 MPa; nevertheless this point can be considered as a singularity due to the contact element behavior between two different materials. Due to the uniformity of the stress distribution along the rod body, we can conclude by neglecting this singularity, and consider the VonMises criterion as fulfilled with \( s.f. = 1.5 \). The end-plate shows a similar region of localized stress (\( \simeq 306 \text{ MPa} \)) in correspondence of the nut. We can draw the same conclusions as for the rod, considering the failure criterion as fulfilled.

**Assembly: lateral pre-load**

The lateral pre-load is given by the contact interference between the vertical key and the X-Pad (Horizontal pad), as it was done in the 2D model. The force per unit length on the straight section of the coil pack can be related to the net force on the shell according to the following simplified formula:

\[
\bar{\sigma}_{x,coil} \approx \frac{\bar{\sigma}_{\theta,shell} t_{shell}}{h_{\text{pack}}} \tag{7.4}
\]
The numerical value obtained on the shell is 38 MPa; according to (7.4), the expected value on the coil pack is about 36 MPa, consistent with the numerical result of 30 MPa. The minimum inflating pressure of the lateral bladder is about 127 bar, obtained with (7.2). By applying the lateral pressure, the radial stress along the cable heads is now under uniform compression, comprise in a range of [14,60] MPa, moving from the straight section to the Ox symmetry plane. The azimuthal stress on the heads follows an identical trend, now spanning in a range of [16,30] MPa in compression, from a range of [0,20] MPa after longitudinal pre-load. The longitudinal stress in the rod $\sigma_{z,\text{rod}}$ increase for the Poisson’s effect from 265 MPa to 274 MPa. The equivalent stress distribution has an analogous pattern as before: a localized maximum of 415 MPa, which can be neglected, and
7.2 3D Finite Element model

a uniform stress distribution along the rod body below 280 MPa, fulfilling the failure criterion (see fig. 7.13-(a)).

Figure 7.13: $\sigma_{eq}$ for the aluminum rod after: (a) complete assembly, and (b) powering.
Cool Down
The gain in stress in the outer shell $|\Delta \sigma_{\theta,\text{shell}}|$ after cool down can be roughly evaluated considering the differential of thermal contraction between aluminum ($\sim 4.2 \text{ mm/m}$) and steel ($\sim 2.3 \text{ mm/m}$), obtained from averaging the stainless steel and magnetic iron contraction coefficients. We can estimate $|\Delta \sigma_{\theta,\text{shell}}|$ according to the following simplified expression:

$$|\Delta \sigma_{\theta,\text{shell}}| \approx E_{\text{shell},4.2K} \alpha_{\text{shell}} - E_{\text{steel},4.2K} \alpha_{\text{steel}} = 164 \text{ MPa} \quad (7.5)$$

By not taking into account the thermal effect of the coil and the containing structure, we are overestimating the actual value of about 100 MPa which have been measured during tests on LBNL magnets. Nevertheless, the numerical results confirm the latter value, revealing an average gain in azimuthal stress on the shell of about 80 MPa. By scaling the incremental stress at cool down on the coil pack thickness, we expect a gain in the lateral stress on the coil $|\Delta \sigma_{x,\text{coil}}| \approx 75 \text{ MPa}$, leading to an overall stress of about 100 MPa, which is confirmed by the numerical results.

The rod maximum stress is about 430 MPa, below the Yield stress limit, but with a safety factor of 1.3. The magnetic iron components are now verified with Rankine’s criteria, without showing any relevant issue. As for the 2D model, the highest stress occurs at the bending radius in the yoke.

The lower thermal contraction of the titanium components with respect to the cables lead to an azimuthal tension status along the coil heads, in a range of $[-26,70]$ MPa. On the other hand, the effect of the aluminum items lead to an increase of the radial compression to a maximum value of 150 MPa on the intermediate coil head.

Powering
After cool down the average stress $\bar{\sigma}_{x,cp}$ along the straight section is about -110 MPa, reducing to -30 MPa after powering, under effect of the magnetic forces. Being the net magnetic force on the coil straight section about 144 kN, we can simply evaluate the effect at the coil-pole contact as follows:

$$|\Delta \sigma_{x,cp}| \approx \frac{F_{x,ss}}{L_{ss} h_{\text{coil}}} = 100 \text{ MPa} \quad (7.6)$$

which is consistent with the numerical results, indicating a $|\Delta \sigma_{x,cp}|$ of 80 MPa. After cool down, the coil as a whole undergoes a maximum equivalent stress of about 190 MPa (see fig. 7.14-(a)), at the boundary between the first lower cable
head and the main post. Nevertheless, the magnetic forces arising during powering help redistributing the stress, lowering down to a maximum of 150 MPa (see fig. 7.14-(b)), as our design criteria impose. The equivalent stress in the aluminum components does not change, revealing that the pre-load parameters can perfectly balance the magnetic forces preventing from separation at the coil-pole boundary.
7. Mechanical Design

All the steel components fulfill the related failure criteria; no particular issues on the titanium components are observable.

The minimum radial compressive stress on the first coil head is about 0 MPa (see fig. 7.12-(d)), raising to a maximum of 100 MPa on the outer head. Moreover, the stress pattern on the outer shell as well as on the rod does not change from cool down to powering, revealing that the action of the magnetic forces is counter-balanced by a proper system pre-load. This configuration could reasonably lead to the test of the SMC without leading to quenches due to the cable movements. It will be anyway mandatory to perform preliminary test on the SMC structure to verify the FEM model by means of a dummy coil (e.g. aluminum alloy), and eventually to feed back into the coil model to set up properly the contact elements behavior.

<table>
<thead>
<tr>
<th></th>
<th>σ (MPa)</th>
<th>Axial pre-load</th>
<th>Lateral pre-load</th>
<th>Cool down</th>
<th>Powering</th>
</tr>
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<tbody>
<tr>
<td><strong>Coil</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>(Nb3Sn)</td>
<td>σeq,max</td>
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<td>(Al 2014)</td>
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<td><strong>End plate</strong></td>
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<td>(Ti6Al4V)</td>
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</table>

Table 7.4: SMC analysis results of 3D model. The maximum stress for the items in MAGNETIL after cool down and powering is the **first principal**.

The stress profile along the first cable, where the peak field occurs, is analyzed more in detail in figs. 7.15-7.16. Two positions have been taken into account: in the center of the first cable in the lower and upper layer respectively. The stress component acts transversally to the cable broad face; it is defined as $\sigma_x$ along the cable straight section, and $\sigma_r$ along the head. This is related to the coordinate systems used to analyze the stress: global cartesian for the straight
section part, and local cylindrical for the head. As it appears from the plots, the stress distribution is homogeneous in the two layers, the difference being less than some percentages. The longitudinal pre-load affects the stress distribution on the heads only, to a maximum compressive stress of about 50 MPa.

As soon as the lateral pre-load is applied, the difference in stress is smothered over a value of about 40 MPa in compression, all along the contact surface with the main post. The stress level increases at cryogenic temperature to 100 MPa, keeping the same homogeneous distribution as after assembly. The raise in e.m. forces leads to a decrease in the transverse stress along the pack: a minimum value of 15-20 MPa of uniform compression occurs along the straight section, decreasing to some MPa’s in correspondence of the magnet symmetry plane.

7.3 Summary

The mechanical design of the SMC magnet has been done, by setting the required pre-load in order to avoid both cable movements during powering, and observing the foreseen mechanical limit of the Nb$_3$Sn. A sensitivity analysis on the lateral pre-stress parameters has been performed, outlining how the shell thickness and the assembly key interference interact. It has been shown that the yoke thickness plays a mechanical role up to 55 mm thickness, then its importance is related mainly to magnetic purposes.

The 3D model allowed a complete structural study of the assembly. The results obtained with the 2D model were confirmed, and the longitudinal pre-stress has been finally fixed. The rod pretension at warm are evaluated in terms of end-plate displacement: this was set to 1.5 mm.

The choice of using fully bonded elements for the coil and frictionless contact elsewhere in the model will require a cross check with the experimental data. It will be performed an assembly test by means of a dummy coil pack in Aluminum, in order to validate both the structure and the magnet instrumentation. At present, the first coil has been wound and impregnated at RAL (UK). The second double pancake will be soon prepared, on the base of the former results. Then the dipole will be ready for tests in summer 2009.
Figure 7.15: $\sigma_x$ along the straight section of the first pack: (a) lower layer, and (b) upper layer. The stress is taken at half the height of the each cable.
Figure 7.16: $\sigma_r$ along the head of the first pack: (a) lower layer, and (b) upper layer. The stress is taken at half the height of the each cable.
7. Mechanical Design
Chapter 8

Conclusions

In this work we have presented the development of devices for the study of the Nb$_3$Sn magneto-mechanical behavior, in the research frame for the LHC luminosity and energy upgrade. This superconductor is the main candidate to substitute the Nb-Ti in superconducting magnets, nowadays at the top of its performance, in order to achieve higher magnetic inductions and field gradients.

Due to the observed correlation between mechanical stress and cable current degradation, and to the intrinsic brittleness of the compound, the design of Nb$_3$Sn magnets appeared since the very beginning as really challenging. Higher induction means higher electro-magnetic forces, and so higher stress on the coil and magnet structure. The structure of the magnets used in LHC is of $cos\vartheta$ type, being the coil disposed as to reproduce an ideal current distribution. Assuming the limit of 150 MPa in order to avoid irreversible current degradation, a feasibility study on $cos\vartheta$ type magnets was required.

We started from a formalism found in literature, approaching real magnets cross sections with a simplified sector coil. Equations of the magnetic forces and of the mechanical stress have been derived, as functions of the geometrical coil layout, namely of: aperture $r_i$, and coil equivalent width $w$. We focused on the maximum azimuthal stress at short sample current, occurring the coil mid-plane, since it is the main responsible of current degradation. We have then put in relation the peak stress $\sigma_\varphi$ either with the field gradient, for quadrupoles, or with the bore field, for dipoles. The analysis has been carried out not only for a simple coil in air, but also accounting for a magnetic yoke enclosing the coil. This required a modification of the stress equations, and a re-parametrization of the short sample current, carried out in an original and semi-analytic way. The results have been compared to numerical results of real cross sections, showing a general good agreement.

The design of a new cable sample holder have been treated in the second part. This sample holder will allow tests on cable transport current in static stress conditions, aimed at defining the mechanical effects on cable through a systematic
8. Conclusions

The test on a reduced scale model of the proposed configuration showed very good results in term of stress distribution along the dummy cable stack and reproducibility with the 3D numerical model. A maximum stress of 250 MPa has been reached on the dummy stack, fulfilling the design requirements. These results will open the way of a full scale model design; tests on real cables will be hopefully performed before summer.

In the last part, the optimization of a small racetrack dipole, called SMC (Short Model Coil), has been reported. The optimization takes over a precedent dipole magnet, tested in the US. The design has been carried out in collaboration between European laboratories, in the frame of the NED project. At CERN, we aimed at set-up an integrated design procedure in the FE code ANSYS”. Both 2D and 3D models have been implemented. The largest efforts have been spent for the 3D analysis procedure. The 3D magnetic model was fully developed in ANSYS”, according to different magnetic formulations, finally compared and cross checked with the results coming from the collaboration labs, using specific programs for magnetic computations. Ad-hoc routines have been specifically created and optimized in order to transfer the magnetic forces from the magnetic model to the mechanical one.

The dipole is expected to reach 12.9 T at a short sample current of 13.96 kA. The mechanical optimization of the dipole led to the dimensioning of the preload system, fulfilling the failure criteria imposed at the different magnet working phases. An extensive analysis of the stress distribution on the coil has been done, checking for high stress regions that could eventually lead to cable degradation. The analysis revealed a peak stress below 200 MPa, after the cool down at 4.2 K, which can be considered as acceptable for preventing degrading. Tests of the SMC are scheduled in the first half of this year.
Appendix A

Electro Magnetic Forces

The equations of the electro-magnetic forces produced by the magnetic field over a conductor of infinite area $rdrd\phi$ are as follows:

\[ dF_r(r, \varphi) = -jB_\varphi rdrd\varphi \quad (A-1) \]
\[ dF_\varphi(r, \varphi) = jB_r rdrd\varphi \quad (A-2) \]

The forces $F_x$ and $F_y$ along the Cartesian Reference System, center in the aperture, can be evaluated as follows:

\[ dF_x(r, \varphi) = dF_r \cos \varphi - dF_\varphi \sin \varphi \quad (A-3) \]
\[ dF_y(r, \varphi) = dF_r \sin \varphi + dF_\varphi \cos \varphi \quad (A-4) \]

A.1 Forces in quadrupoles

In order to define the resultant of the magnetic forces for a quadrupole coil, one have to integrate magnetic field equations (2.3) between the geometrical limits of the winding. For the $\cos \varphi$ we obtain:

\[
F_x = -\frac{j^2 \mu_0 \sin \alpha_0}{72(r_i + w)} \left[ 2((r_i + w)^4 - 4(r_i + w)r_i^4 + 3r_i^4) \cos(2\alpha_0) + 3 \left( - (r_i + w)^4 + r_i^4 + 4(r_i + w)r_i^3 \ln \left( \frac{r_i + w}{r_i} \right) \right) \right] \quad (A-5)
\]
whereas, for the sector coil:

\[ F_y = \frac{J^2 \mu_0}{63(r_i + w)} \left[ -5(r_i + w)^4 + 8(r_i + w)r_i^3 - 3r_i^4 + \right. \\
\left. + ((r_i + w)^4 - 4(r_i + w)r_i^3 + 3r_i^4)\cos 3\alpha_0 + 12(r_i + w)r_i^3 \ln \left( \frac{r_i + w}{r_i} \right) + \right. \\
\left. + 4(r_i + w)\cos \alpha_0 \left( (r_i + w)^3 - r_i^3 - 3r_i^3 \ln \left( \frac{r_i + w}{r_i} \right) \right) \right] \quad (A-6) \]

whereas, for the sector coil:

\[ F_x = -\frac{J^2 \mu_0 \cos \alpha_0 \sin^2 \alpha_0}{9\pi (r_i + w)} \left[ 2((r_i + w)^4 - 4(r_i + w)r_i^3 + 3r_i^4)\cos(2\alpha_0) + \right. \\
\left. + 3 \left( (r_i + w)^4 - r_i^4 + 4(r_i + w)r_i^3 \ln \left( \frac{r_i + w}{r_i} \right) \right) \right] \quad (A-7) \]

\[ F_y = \frac{J^2 \mu_0 \sin(2\alpha_0)}{18\pi (r_i + w)} \left[ -5(r_i + w)^4 + 8(r_i + w)r_i^3 - 3r_i^4 + \right. \\
\left. + ((r_i + w)^4 - 4(r_i + w)r_i^3 + 3r_i^4)\cos 3\alpha_0 + 12(r_i + w)r_i^3 \ln \left( \frac{r_i + w}{r_i} \right) + \right. \\
\left. + 4(r_i + w)\cos \alpha_0 \left( (r_i + w)^3 - r_i^3 - 3r_i^3 \ln \left( \frac{r_i + w}{r_i} \right) \right) \right] \quad (A-8) \]
Appendix B

Mechanical Stress

By considering the stress balance in a sector winding element and neglecting the effect of shear components, one can define the relations between the stresses and volume e.m. forces \((F_r, F_\phi)\) as follows:

\[
\sigma_\phi dr - d(r\sigma_r) + F_r rdr = 0 \quad (B-1)
\]

\[
- d\sigma_\phi + F_\phi r d\phi = 0 \quad (B-2)
\]

By integrating eq. B-2 between the coil angular limits \((0, \alpha_0)\), we can get the stress resultant along the mid plane as a function of the radius \(r\). The stress on the collar contact profile can be computed by introducing the expression of \(\sigma_\phi\) in B-1, considering that in this case \(\sigma_\phi\) does not have to be computed as a resultant, but as a point value:

\[
\sigma_\phi = \int F_\phi rd\phi + \text{const.} \quad (B-3)
\]

where \(\text{const}\) is set to have \(\sigma_\phi = 0\) at the coil pole pole.

B.1 Quadrupole coil

By integrating eq. B-2 for a quadrupole coil, one gets:

\[
\sigma_\phi = -\frac{j^2 \mu_0 \cos(\alpha_0) \sin^3(\alpha_0)}{\pi r^2} \left[ r^4 - r_i^4 + 4r^4\ln\left(\frac{r_i + w}{r}\right) \right] \quad (B-4)
\]

By replacing \(\alpha_0\) with \(\pi/6\), we can get the azimuthal stress for a 30° sector coil as follows:

\[
\sigma_\phi = -\frac{j^2 \mu_0 \sqrt{3}}{16 \pi r^2} \left[ r^4 - r_i^4 + 4r^4\ln\left(\frac{r_i + w}{r}\right) \right] \quad (B-5)
\]
B. Mechanical Stress

The stress on the collar contact profile can be computed by considering $\sigma_\varphi=0$ at $\alpha = \alpha_0$ at the pole, thus obtaining:

$$\sigma_\varphi = -\frac{j^2 \mu_0}{4\pi r^2} (\cos(2\alpha_0) + \cos 2\varphi) \left[ r^4 - r_i^4 + 4r^4 \ln \left( \frac{r_i + w}{r} \right) \right] \sin(2\alpha_0) \quad (B-6)$$

By introducing the expression for the azimuthal stress on a sector winding element into $B-1$, we can derive the expression for the radial reaction stress. By integrating between the inner and outer radius, we finally obtain:

$$\sigma_r = -\frac{j^2 \mu_0 \sin(2\alpha_0)}{36\pi (r_i + w)^2} f_{pr}(r_i^4, w^4, \varphi) =$$

$$-\frac{j^2 \mu_0 \sin(2\alpha_0)}{36\pi (r_i + w)^2} \left\{ \left[ 12r_i^3(r_i + w) \ln \left( \frac{r_i + w}{r_i} \right) - w(12r_i^3 + 42r_i^2w + 28r_iw^2 + 7w^3) \right] \cdot \cos(2\alpha_0) + \left[ 9w(2r_i + w)(2r_i^2 + 2r_iw + w^2) - 36r_i^3(r_i + w)\ln \left( \frac{r_i + w}{r_i} \right) \right] \cos 2\varphi \right\} \quad (B-7)$$

B.2 Dipole coil

The expression for the azimuthal stress distribution on a dipole mid-plane is as follows:

$$\sigma_\varphi(r) = \frac{j^2 \mu_0}{3\pi r} \left[ 2r^3 + r_i^3 - 3r^2(r_i + w) \right] \sin \alpha_0 \quad (B-8)$$

By setting $B-8=0$ at $\alpha = \alpha_0$, we get:

$$\sigma_\varphi(r) = \frac{2j^2 \mu_0}{3\pi r} \left[ 2r^3 + r_i^3 - 3r^2(r_i + w) \right] (\cos \alpha_0 - \cos \varphi) \sin \alpha_0 \quad (B-9)$$

Then the expression of the radial stress is straightforward:

$$\sigma_r(\varphi) = \frac{j^2 \mu_0 \sin \alpha_0}{9\pi (r_i + w)} f_{pr}(r_i^3, w^3, \varphi) \quad (B-10)$$

where:

$$f_{pr}(r_i^3, w^3, \varphi) = 6w^2(3r_i + w) \cos \varphi - \frac{1}{2} \left[ w(6r_i^2 + 15rw + 5w^2) + 6r_i^3 \ln \left( \frac{r_i}{r_i + w} \right) \right] \quad (B-11)$$
Appendix C

Maximum Azimuthal Stress in a quadrupole sector coil

The equation of the compressive stress on the coil mid plane is as follows:

\[
\sigma_\varphi(j_c) = -\frac{\sqrt{3} j_c^2 \mu_0}{16\pi r^2} \left[ r^4 - r_i^4 + 4r^4 \cdot \ln\left(\frac{r_i + w}{r}\right) \right] \quad (C-1)
\]

By deriving the eq. (C-1), we obtain:

\[
\frac{d\sigma_\varphi(j_c)}{dr} = -\frac{\sqrt{3} j_c^2 \mu_0}{8\pi r^3} \left[ -r^4 + r_i^4 + 4r^4 \cdot \ln\left(\frac{r_i + w}{r}\right) \right] \quad (C-2)
\]

By imposing eq. (C-2) equal to 0 and \( r \neq 0 \), we get the transcendental equation:

\[
 r^4 - r_i^4 - 4r^4 \cdot \ln\left(\frac{r_i + w}{r}\right) = 0 \quad (C-3)
\]

that can be solved by mean of the Lambert W-Function (also called \textit{Product log} function), i.e. the inverse function of:

\[
f(W) = We^W \quad (C-4)
\]

Rearranging eq. C-3, we get:

\[
r^4 \cdot \left[ 1 + 4\ln\left(\frac{r}{r_i + w}\right) \right] = r_i^4 \quad (C-5)
\]

Multiplying both members times \( e/(r_i + w)^4 \), we get:

\[
\frac{e}{(r_i + w)^4} \cdot r^4 \cdot \left[ 1 + 4\ln\left(\frac{r}{r_i + w}\right) \right] = r_i^4 \cdot \frac{e}{(r_i + w)^4} \quad (C-6)
\]

that is equal to:
C. Maximum Azimuthal Stress in a quadrupole sector coil

\[ e^{1+4\ln\left(\frac{r}{r_i+w}\right)} \cdot \left[ 1 + 4\ln\left(\frac{r}{r_i+w}\right) \right] = \frac{e \cdot r_i^4}{(r_i + w)^4} \]  \hspace{1cm} (C-7)

The radius where the azimuthal stress is maximized is then given by:

\[ r = \exp \left[ -\frac{1}{4} + \frac{1}{4} \cdot \text{ProductLog}\left( \frac{e \cdot r_i^4}{(r_i + w)^4} \right) \right] \cdot (r_i + w) \]  \hspace{1cm} (C-8)

by introducing this expression into eq. (C-1), we can get the maximum azimuthal stress on the coil mid plane:

\[ \sigma_{\varphi, \text{max}} = -\frac{\sqrt{3} j_z^2 \mu_0}{16\pi (r_i + w)^2} \left[ -r_i^4 + e^{-1+\text{ProductLog}\left( \frac{\sigma_f^4}{(r_i + w)^4} \right)} \right] \]  
\[ + e^{-1+\text{ProductLog}\left( \frac{\sigma_f^4}{(r_i + w)^4} \right)} \cdot (r_i + w)^4 \ln \left( e^{\frac{3}{4} - \frac{1}{2} \text{ProductLog}\left( \frac{\sigma_f^4}{(r_i + w)^4} \right)} \right) \]  \hspace{1cm} (C-9)
Appendix D

Iron Yoke Effect

The effect of the iron yoke can be evaluated by using the "Image current" approach: the additional field is produced by an imaginary coil enclosed between radii \( r'_1 = R_s^2/r_i \) and \( r'_o = R_s^2/r_o \), where \( R_s = r_o + w_{coll} \) (see fig. 2.16). By summing up the field contribution due to the iron yoke to the field components inside the coil in air, one can evaluate the field distribution as well as the magnetic forces and the mechanical stress, by applying the same analytical approach introduced for coils in air.

D.1 Quadrupole coil

For a sector coil approach, one has:

\[
\begin{align*}
\begin{bmatrix} B_{r,iron} \\ B_{\phi,iron} \end{bmatrix} &= \begin{bmatrix} B_{r,air} \\ B_{\phi,air} \end{bmatrix} - \frac{1}{\mu_r + 1} \left\{ \frac{1}{\pi} \sum_{m=0}^{\infty} \frac{r^{4m+1}(r_0^{4(m+1)} - r_i^{4(m+1)})}{(m + 1)(4m + 2)R^4_s} \sin(4m + 2)\alpha_0 \right\} \\
&\cdot \begin{bmatrix} \sin(4m + 2)\phi \\ \cos(4m + 2)\phi \end{bmatrix}
\end{align*}
\]

For a \( \cos \phi \) sector coil, one has:

\[
\begin{align*}
\begin{bmatrix} B_{r,iron} \\ B_{\phi,iron} \end{bmatrix} &= \begin{bmatrix} B_{r,air} \\ B_{\phi,air} \end{bmatrix} - \frac{1}{\mu_r + 1} \left\{ \frac{1}{2} \ln\left(\frac{r_o^2 - r_i^2}{4R_s^4}\right) \right\} \begin{bmatrix} \sin2\phi \\ \cos2\phi \end{bmatrix}
\end{align*}
\]
The magnetic forces are as follows:

$$F_x = \frac{j^2 \mu_0 \sin^2 \alpha_0}{9\pi} \left[ \frac{(\mu_r - 1)}{(\mu_r + 1)} \frac{6w_r}{R_o^4} \left(2r_i^2 + 2r_i w + w^2\right)(r_o^3 - r_i^3) - \frac{2w^2}{r_o} \left(6r_i^2 + 4r_i w + w^2\right) \cos 2\alpha_0 + 3(r_i^4 - r_o^4 + 4r_i^3 r_o \ln\left(\frac{r_o}{r_i}\right)) \right]$$

$$F_y = \frac{j^2 \mu_0}{9\pi} \left[ -\frac{2}{r_o} \left(\cos \alpha_0/2 + \cos \frac{3\alpha_0}{2}\right) \left(9r_i^4 - 16r_i^2 r_o + 7r_o^2 + 2w^2 \left(6r_i^2 + 4r_i w + w^2\right) \cos 2\alpha_0 + 3r_i (\mu_r - 1) (2r_i + w) \left(2r_i^2 + 2r_i w + w^2\right) \left(r_o^3 - r_i^3\right) (\cos \alpha_0 - 1) \frac{\cos \alpha_0 \sin \alpha_0}{(1 + \mu_r) R_o^4} \right]$$

The azimuthal stress along the coil mid-plane is finally given by:

$$\sigma_{\varphi, \text{iron}} = \frac{j^2 \mu_0 \sin \alpha_0}{8\pi R_o^2} \left[r_i^4 + r_o^4 - \frac{(\mu_r - 1)}{\mu_r + 1} \frac{r_o^4 - r_i^4}{R_o^4} - 4r_i^4 \ln\left(\frac{r_o}{r_i}\right) \right]$$

### D.2 Dipole coil

The equations for the magnetic field inside the coil are as follows:

$$\{B_{r, \text{iron}}\} = \{B_{r, \text{air}}\} - \frac{j\mu_0}{\pi} \frac{2 \sin \alpha_0}{\mu_r + 1} \frac{r_o^4 - r_i^4}{3R_o^2} \left\{ \sin \varphi \right\}$$

where the expressions for $B_{r, \varphi}$ for a coil in air are given by (3.3).

The magnetic forces components then are:

$$F_x = \frac{j^2 \mu_0 \sin \alpha_0 \sin 2\alpha_0}{9\pi (\mu_r + 1) R_i^2} \left[ 3\alpha_0 (r_o - r_i)^2 \left(1 - \mu_r\right) (r_o + r_i) + (r_o^2 + r_i^2 + r_o r_i) + (\mu_r + 1) (r_o + 2r_i) R_i^2 \left[ r_o^3 - r_i^3 - 3r_i^3 \ln\left(\frac{r_o}{r_i}\right) \right] \right]$$

$$F_y = \frac{2j^2 \mu_0 \sin^3 \alpha_0}{9\pi} \left[ (r_o)^3 - r_i^3 - 3r_i^3 \ln\left(\frac{r_o}{r_i}\right) \right]$$
Note that the expression for $F_{y,\text{iron}}$ is an invariant of the magnetic coil layout, being the same as (3.5) even with iron yoke. The expression for the azimuthal stress on the coil mid-plane can be defined as:

$$
\sigma_{\varphi,\text{iron}} = \frac{j^2 \mu_0 \sin \alpha_0}{3\pi r} \left[ 2r - 3r_o + \frac{r_o^3}{r^2} - \left( \frac{\mu_r - 1}{\mu_r + 1} \right) \frac{r_o^3 - r_i^3}{R_s^2} \right] \quad (D-9)
$$
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