SOFTWARE FOR CO ACQUISITION AND CORRECTION
IN A VARIABLE CONFIGURATION

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ABSTRACT

The transformation of the SPS into a proton-antiproton collider significantly altered its original periodicity of 108 identical FODO sections. In particular the installation of low beta insertions in L554 and L555 changed the betatron amplitude and phase values around the ring and modified the physical layout of beam position detectors and closed orbit correction dipoles. A closed orbit correction package designed to be independent of the machine configuration has been implemented. By exploiting the concept of a 'configuration defining' data base a high degree of flexibility, speed and operational convenience has been achieved. Operational experience with the correction package has shown that the orbit can be acquired and corrected rapidly for a variety of working points and machine configurations.
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1. **Introduction**

The transformation of the SPS into a proton-antiproton collider significantly altered its original periodicity of 108 identical FODO sections. In particular the installation of low beta insertions in LSS4 and LSS5 changed the betatron amplitude (\(\beta\)) and phase (\(\phi\)) values around the ring and modified the physical layout of beam position detectors (BP) and closed orbit correction dipoles (COD). In fixed target operation the SPS ring is completely filled with proton bunches, equidistant by 1.50 m (200 MHz acceleration frequency) while only a few proton and antiproton bunches circulate in collider operation (e.g. 3 proton and 3 antiproton bunches). New detectors of the directional coupler type (BPC) were installed at specific locations around the ring to allow independent position measurements of both proton and antiproton bunches\(^1\). Due to the strong directivity and high sensitivity of the couplers and through the use of special control electronics, it is possible to select and measure the position of single bunches of either type. Eight types of detectors are employed to satisfy aperture requirements and for similar reasons there are eight types of correction dipole, some of which are pulsed for correcting the high energy closed orbit mainly in the low beta regions, the rest are d.c.\(^2,3,4\).

The closed orbit (CO) correction package previously used at the SPS exploited the regularity of the lattice, i.e. the same values of \(\beta\) and \(\phi\) between BP's and COD's, to implement the simple beam bump method. Unfortunately this algorithm does not work in an irregular configuration\(^5\). An acceptable alternative must take into account the properties of different machine configurations and at the same time retain the power, speed and convenience of the simple method. To achieve this degree of flexibility a CO correction package designed to be independent of the machine configuration has been implemented. The configuration dependent aspects of the problem are taken care of by a CO data base which defines the characteristics of a desired configuration. The CO data base is generated from two sources: the first is a hardware data base which describes all CO correction elements
currently installed in the machine and the second is a lattice parameter data base which is produced by the AGS computer program. The AGS calculations are done off-line at the CERN Computer Centre and the results transferred via a data link to the SPS Control System\textsuperscript{6,7,8}).

In the remainder of this paper the philosophy and structure of the new CO correction package is described in detail (see fig.1). The three main software units which make up the package, namely a set of off-line programs for generating the CO data base, low level real time routines for controlling the CO hardware and applications programs which are used in the SPS Control Room for acquiring and correcting the CO, are discussed individually.

2. Data bases and file handling

In order to achieve flexibility for the closed orbit acquisition and correction software, programs were written which are independent of machine configuration but use data bases which contain the parameters needed for the current status of the accelerator.

These parameters can be modified if the physical layout (insertion of new detectors, updated calibration factor, etc..) or lattice parameters (working points, low beta insertion values, etc...) change.

All elements (detectors or correctors) are then defined by their hardware parameters stored in a 'hardware data base' and lattice parameters stored in a 'lattice data base'.

The link between the two data bases is obtained by a keyword which is an abbreviated name of the element (4 alphabetical and 3 numerical characters such as MDVA 217) which is a short form of the physical layout name (MDVA 21703) as indicated in all SPS drawings. The physical layout name is also used in AGS.
To limit the number of elements specified in AGS data files, a detector physically linked to a corrector is generally considered as a single element whose keyword is the abbreviated name of the corrector and $\beta$ and $\mu$ values are given at the mid-point between the two elements. It can be shown that the resulting error in closed orbit correction is negligible for most of the circumference, but at some parts of the low beta insertions where $\beta$ and $\mu$ values vary rapidly, detectors and correctors are considered separately with $\beta$ and $\mu$ values given at the centre of each element.

As closed orbit acquisition and correction are made by plane (horizontal or vertical), the elements in the two data bases are sorted by plane. They are logically ordered following the increasing absolute value of the betatronic phase advance ($\mu$) which varies in the same order as the physical layout.

2.1 'Hardware data bases'

In instrumentation where hardware is often modified to comply with new beam requirements, a unique data base is the only source for all required files.

This data base contains the current list of elements needed for CO acquisition and correction in fixed target and collider modes and is stored on a floppy disk.

Each element is defined by the following parameters:

- the keyword, described above,
- the names of the detector and corrector (given by the SPS layout) associated to the keyword,
- the name of the data module subroutine (DMS) monitoring the element. For the detector, DMS 'DETP' deals with the electrostatic pick-ups (BPH/V) and the directional couplers (BPC) when used in fixed target mode, while DMS 'PAPOS' deals with the BPC when in collider mode.
For the corrector, DMS 'COD' controls the d.c. dipole magnets (MDH/V), while DMS 'PAGEF' controls the pulsed dipole magnets (MDPH/V and PODH/V).

- the number of the element in the DMS (N value) and the name of the computer dealing with it,
- the calibration factors.

All types of position detector consist of two electrodes where the difference of two signals (Δ) and their sum (Σ) are transmitted for acquisition. The calibration factor is the number by which the ratio Δ/Σ is multiplied to obtain the position (in 0.01 mm units).

For the corrector, the calibration factor is the number by which the product of the angular kick (in rad) and momentum (in GeV/c) is multiplied to obtain the dipole current (in Amperes).

The floppy disk is updated only when a new element is physically added to or removed from the SPS or the AGS input file. As a large amount of data is involved and it cannot be manipulated on-line, it is condensed and coded by an OFF-LINE program which produces master files stored in the library computer. Each file contains parameter arrays which are indexed in increasing order of η.

2.2 'Lattice parameter data bases'

The lattice parameter for each COD element (set of detector-corrector and in some cases individual detectors or correctors) consists of:

- the betatronic amplitude (β) value, and
- the betatronic phase advance (μ) value.

For a given SPS configuration (working point 'QH,QV' and value of β at the crossing points), AGS computes off-line the β and μ values at the COD elements and outputs them to files stored on the Service computer linked to the SPS network. For each plane the β and μ values are sorted in increasing order of μ.
2.3 Active file generation

The 'hardware' and 'lattice' data bases contain all 20 elements installed along the SPS. For a given mode only a subset is used. For example, amongst the 114 vertical and 116 horizontal elements, 108 elements are used in 'fixed target' in each plane. Therefore, for a given SPS mode or configuration the active files to be used for CD acquisition and correction have to be extracted from the 'hardware' and 'lattice' files.

Each element is defined by an index number which is its position in the arrays according to its \( w \) value. For each element a program compares the corresponding keywords in the 'hardware' and 'lattice' files. If there are no errors the active 'hardware' and 'lattice' files are generated and stored onto the library computer prior to the start of the run.

3. Low-level real time control software

3.1 Acquisition

Two types of detector are used in the SPS to measure beam position, the first is a simple electrostatic pick-up (BP) while the second is a directional coupler (BPC). Detailed descriptions of these devices are given elsewhere\(^1,3\). This section will deal with the data acquisition system of each detector. Both detectors provide two signals: \( \Sigma \) (corresponding to the sum of the signals induced by the beam on the electrodes) and \( \Delta \) (corresponding to their difference). BP signals are fed directly through \( V/F \) converters into CAMAC scalers, whose gate is programmable. BPC signals are routed through two parallel acquisition channels. The first is the same channel as for BP signals, the second includes a sample and hold device, triggered by a timing system precisely synchronized to the bunch in front of the \( V/F \) converter.
Thus the beam position measured by BPC's can be acquired either by using software developed for BP's (i.e. the data module DETP) or by new software designed explicitly for proton-antiproton operation (i.e. the data module PAPOS).

The data module DETP \(^9\) (200 MHz structure) acquires the beam position averaged over many bunches for a given time. (The scaler's gate is open for 1 ms for CO measurement, hence the SPS master timing, 1 ms clock, is fully adequate.) Acquisitions are made at three different times during the SPS cycle: the first always coincides with the first injection while the other two are asynchronous and are software programmable for different times during the machine cycle.

Since no parameters should be changed during a machine cycle a simple buffer containing the three acquisitions is sufficient.

The situation is more complicated in the case of PAPOS due to stronger requirements. A full description of its properties is given in ref. 12; only its timing sequence will be considered here (fig. 2).

The first requirement is to acquire the position of an individual bunch at several detectors at the same time. This time could be the injection of a given bunch or any time during the SPS cycle. The acquisitions are triggered by external events such as injection pre-pulses or time programmable events, via a sophisticated hardware which consists of a bunch selector, a mode selector (\(p\) or \(\bar{p}\)), several programmable delays (global for a sextant of the SPS and individual for each detector) and gated integrator modules. The external event also opens the gate of the scalers, via a 'gater' module, and at the end of the gate sends an interrupt to an autonomous CAMAC processor (ACC module) where the data module PAPOS resides. At that time all data are available in the scalers in digital form to be picked up by PAPOS. The data, consisting of \(\Delta\) and \(\Xi\) of both planes from all BPC's of one SPS sextant, constitute one 'acquisition'.
The second requirement is to initiate a 'sequence' of acquisitions at a given time. It could be interesting, for example, to compare the position of different bunches at about the same instant or for correcting the position of the bunch at injection from the injection oscillations (acquired over one turn) relative to the closed orbit (averaged over 1 ms). A sequence is defined by the order and the mode in which the bunches will be acquired. Therefore, once the interrupt of the first acquisition has been sent to the ACC module, a PAPOS real-time program is activated which reads the micro-processor clock, acquires the data stored in the scalers, looks to the next acquisition in the sequence and sets the hardware accordingly.

The real-time program ends by triggering the gater module by a CAMAC command for starting the new acquisition. It should be noted that the gater will not accept an external trigger unless it is again enabled by software. Hence a series of up to 6 acquisitions can be carried out automatically. The time interval between acquisitions depends on the execution speed of the real-time program and is of the order of a few tens of ms. At the end of a sequence the gater is enabled to react to a new external event as it will arrive.

Finally, it is important for PAPOS, as it is for DETP, to acquire several sequences (up to 4) during the SPS cycle; for example, one at the injection of each bunch and another at high energy. These sequences are initiated by external events and follow their order of arrival. Thus, for example, sequence number 2 associated with a pre-pulse in a given machine cycle may be associated with the following timing pulse if the pre-pulse is missing in the next machine cycle. For this reason the time intervals between reset and each activation of the real-time program are recorded by PAPOS and may be examined by the operator.

The various acquisition possibilities at three levels of timing during the SPS cycle leads to considerable flexibility, but it soon became evident that it would be necessary to fix the measurement settings during one machine cycle in order to safeguard operation and avoid conflicts when several
consoles have access to the same program. A double buffer ensures that all data displayed belong to the same machine cycle and correspond to well defined settings.

At the end of each SPS cycle a second real-time program runs to load the parameters which define the acquisitions for the next machine cycle and to swap the acquisition buffers. The gain, calibration switch, proton-anti-proton switch are all set at that time. The ACC internal time is read and stored. Finally the hardware is set to accept the first timing pulse.

The ACC module, referred to in the above description, is a CAMAC module containing a microcomputer system based on the TMS 9900[14]. The ACC software system was developed to accept compiled NODAL[15] and this strongly influenced the decision to use it. In fact PAPOS and PAGEF and the associated real-time programs for the former are written in NODAL. This feature reduces software development time and permits rapid software maintenance to suit changing operational requirements. Moreover the decision to use a dedicated control processor solved all possible priority conflicts with other tasks competing for the general purpose NORD-100 computer (GP). Finally with this approach software generation and loading becomes independent of the other data modules resident in the GP.

3.2 Correction

The closed orbit can be corrected by a maximum of 110 dipole magnets in each plane. The data module COD controls 93 dipoles in the horizontal plane and 87 in the vertical plane. The remaining dipoles are controlled by the new data module PAGEF.

The data module COD sets the dipole current to a demanded value which remains constant during the machine cycle. To remove the hysteresis effect when setting a new value, real-time programs running during the SPS dead-time send to the correction dipoles the maximum current followed by the requested value one second later. Dipoles controlled by this data module can correct
the closed orbit only at injection. However, to compensate for perturbations in the insertion regions caused by low-beta configurations set at different energies, it is necessary to pulse the power supplies of some new dipoles which require larger currents. This is achieved through a function generator identical to those already used to pulse other magnets in the machine \(10\), with the exception of main bending and focusing magnets. The main differences are that (i) real-time loading of the function is carried out individually by a microprocessor and (ii) the function may be stopped and restarted on any vector by an external trigger (start storage, stop-storage \(11\)). A facility was added for sampling the output from the function generator and the power supply in order to check for deviations from the nominal function. New software had to be developed and it was decided to use compiled NODAL. The new data module (PAGEF) is described in ref. \(13\). It should be noted that PAGEF does not have an associated real-time program which is further distributed inside the function generator itself. However the object code generated by the NODAL compiler exceeds the standard size of a GP coreload and therefore cannot be installed in a GP. The ACC module was also chosen to house the data module PAGEF because it has sufficient memory.

The compiled NODAL provided the same ease of writing PAGEF as that experienced with PAPDS. Unfortunately the execution speed of parts of the data module proved to be inadequate and it was necessary to rewrite some of the coding in TMS 9900 assembler. The resulting data module is large because it deals with many hardware facilities. In addition, the application programs must contain some checks which, if incorporated into the data module, would slow down the execution time.
4. **CO acquisition and correction**

4.1 **Fixed target physics mode**

The CO acquisition and correction program currently in use at the SPS was written to be configuration independent. By exploiting the concept of a 'configuration defining' data base a high degree of flexibility has been achieved. Operational experience has shown that the low energy orbit (injection plateau) can be acquired and corrected rapidly for a variety of working points and machine configurations.

The functional structure of the program is illustrated in fig. 3. The main code is divided into two separate units which run as sequential but mutually coupled tasks in any one of the console computers of the SPS control system. The first unit controls the acquisition and display of orbit data, i.e. beam position in both planes, beam loss around the ring and correction dipole strength in both planes, while the second uses the acquired data to compute and apply the required correction. The correction may be applied in one of several ways, either to the whole orbit, to one location or to change the mean orbit in the horizontal plane. Communication between the two main program units is organised via three buffers.

The first and largest of these is the so-called 'Defined Function Area'. In the present application this contains defined functions*) written in NODAL and FORTRAN and data (simple variables*), arrays, strings, etc.), 95% of which is 'static' and includes for example the CO data base. The remaining 5% is 'dynamic', i.e. variables whose 'value' can and does change. The defined functions are used for acquisition and correction calculations and in the interest of speed, several are written in FORTRAN. The types of calculation programmed in FORTRAN include global orbit correction, mean, standard deviation and the conversion of measurements using calibration factors.

*) A 'defined function' is the NODAL equivalent of a FORTRAN subroutine.

'Simple variables' are NODAL variables which may or may not be constants.
The second buffer is really a small (2k word) local scratch file called BEEF which resides in each console computer. Large amounts of 'dynamic' orbit acquisition data are transferred via BEEF to the correction program. The data consists of the orbit measurement itself and the correction dipole currents. Clearly this data changes from one acquisition to another, hence the use of the term 'dynamic'.

The third and last buffer is in fact an array called ARG which is permanently resident in each console computer. ARG has 16 elements of which 7 are used by the program. The data stored in these 7 elements are 'dynamic' and include the energy at which the current acquisition was made, the orbit mean, the orbit standard deviation, the plane (H or V) under consideration and Q of the machine.

The CO hardware in each sextant of the SPS is controlled locally by a corresponding general purpose (GP) computer in the control system. The appropriate data modules, i.e. DETP, PAPOS, DETL, COD and PAGEF, are used to acquire simultaneously from the GP's the beam position, beam loss and dipole currents in each sextant. The orbit measurement for the selected plane is converted to mm using calibration factors from the CO data base (held in the Defined Function Area), its mean and standard deviation are computed and it is displayed as a bar diagram on the console colour screen (see fig.4a). The beam loss measurements are simply scaled and displayed below the orbit as a histogram. Finally the COD currents are converted to equivalent kicks in mrad using the appropriate calibration factors from the CO data base and the energy of the beam. The kicks are displayed as a bar diagram below the loss histogram.

To correct the orbit several options are available. If either a global or a local correction is desired, the correction is calculated using a version of the beam bump method which takes into account the precise values of $\beta$ and $\gamma$ at each BP/COD location. The method is to correct the orbit error at a given position by applying a 'compensated' bump to the beam using three correction dipoles, one at or near the position where the error is
measured and the other two adjacent. The kick strengths are obtained by solving three simultaneous linear equations:

\[- y_i = \sum_{j=1}^{3} k_j \frac{\delta_j}{2 \sin \pi Q} \cos \left[ \mu_i - \mu_j + (\text{sign}(i - j) \pi Q) \right] \]  

(1)

\[i = m-1, m, m+1, \hspace{1cm} 1 \leq m \leq n, \hspace{1cm} n = \text{total number of detectors used}\]

which relate kicks of strength \( k_j \) at positions \( j \) in the ring to the orbit displacement \( y_i \) at position \( i \) in the ring. The coefficients of these three equations contain the betatron amplitudes at the kick and displacement locations as well as the betatron phase between the two. The fact that the bump is compensated means that the rest of the orbit is unperturbed by the local correction (i.e. \( y_{m-1} = y_{m+1} = 0 \), \( y_m \) being the measured orbit error).

The principle of global orbit correction by the beam bump method is described in the Annex.

This algorithm is programmed in FORTRAN and takes the betatron parameters from the CO data base (see fig. 4b). In the case of a local correction (compensated bump) the location is selected on the colour screen display by a cursor programmed to follow the console tracker-ball. The CO acquisition data at the cursor location is displayed on the colour screen and is continually updated as the cursor moves. If the orbit mean is to be corrected (H plane only) the radial displacement is obtained either by RF radial steering or by trimming the current in the main bending magnets, whichever is the more appropriate.

The initialisation, calibration and setting up of the CO hardware is carried out by several small programs which loosely interact with the main CO program. The position monitor gains and acquisition times/acquisition sequence are established in this manner and the calibration performed automatically. In addition there are facilities for manipulating the CO data base files and archiving acquired CO data.
4.2 CO with low beta insertions

4.2.1 At injection

A low beta configuration in the intersection regions is obtained by changing at both sides the sign and strength of some machine quadrupoles and by adding some others. In particular the beta values are strongly enhanced in the two doublets of QWL quadrupoles. For example, for a typical low beta configuration at injection of $\beta_y^* = 3.5$ m and $\beta_x^* = 7$ m, $\beta_H$ reaches a maximum of 410 m between the QWL's. Any misalignment of such a quadrupole or dipole error will have a significant effect on the closed orbit. In addition the betatronic phase $\mu$ between consecutive correcting dipoles changes rapidly with low beta insertions. For example, for $Q_H = 26.62$ without low beta, the horizontal phase advance $\Delta \mu$ between correcting dipoles 4-16, 4-18 and between dipoles 4-18, 4-20 is nearly 90° while with the '3.5 x 7' low beta it becomes 26° and 142° respectively. The beam bump method requires excessively large currents when $\Delta \mu$ is near 0° or 180°. In this case it is very sensitive to errors in beam position, $\beta$ and $\mu$ values, which would perturb the whole orbit.

For these reasons CO correction at injection is achieved in two steps:

(a) The correcting dipoles in each insertion are set to zero and the corresponding pick-ups software disconnected. Then the normal beam bump method is applied outside the insertions using the lattice parameters associated with the '3.5 x 7' configuration. Typical results are given in fig. 5.

(b) Having reconnected the position monitors, a local correction is then applied in each insertion using a version of the beam bump method in which the three dipoles phase advance are chosen to be as close as possible to an odd number of $\lambda/4$ ($\lambda$ = betatronic wavelength). This method minimizes the currents in the dipoles as well as the influence of errors.
4.2.2 At high energy

(a) For a local correction in each insertion the method described in 4.2.1 (b) is also applied. The pulsed correction dipole functions are built up from vectors whose duration is calculated from events which occur during the SPS machine cycle. A typical function is shown in fig. 6. The threshold corresponds to the injection value, three vectors interpolate the correction during the acceleration ramp, one vector starting from the 'beginning of flat top' lasts the duration of low beta squeezing and at the 'end of flat top' the demagnetisation curve starts. It is possible to insert new vectors when necessary but the number of vectors has to be the same for all dipoles to simplify correction programs.

All correction programs compute corrections in kicks (angular units in mrad) which are energy dependent.

As the time when the position measurement is made is known from the acquisition program, the beam energy is calculated from the main bending field and is used to compute currents from kicks.

(b) For a global correction it is possible to make an efficient use of the limited number of pulsed dipoles (Vertical: 5 in sextant 3, 9 in sextant 4, 9 in sextant 5; Horizontal: 5 in sextant 3, 6 in sextant 4, 6 in sextant 5).

A program simulates the closed orbit produced by a kick from one of these dipoles and adds or subtracts it from the measured high energy closed orbit stored on file. The program finds the best dipole and optimum kick strength required to reduce the peak amplitude of the measured orbit.

The kick is converted to an equivalent current in Amperes which is then applied to the dipole. The resulting orbit is measured and further corrections can be made if necessary.
Fig. 7 shows the effect of this correction on high energy closed orbit with two low beta insertions '1.5 x 0.75'. The resulting orbit is better than the orbit without insertion and without correction at the same energy.

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OVERALL STRUCTURE OF CO. CORRECTION PACKAGE

FUNCTION LANGUAGE

MAIN CONTROL ROOM
- Acq. Display Correction Off line data base generation
- Nodal Fortran Ass.

MULTIPLEX DMS Database
- Nodal Ass.

LOCAL AUXILIARY BUILDING
- Data base Surv Ass

TUNNEL
- Electr
- Electr
- Timing
- Pow. S.
- Pow. S.

CERN COMPUTER CENTER
AGS

CONSOLE LIBRARY SERVICE

MTS

GP DETP COD

µP PAPOS µP PAGERF

µP

BP Directional Coupler

BEAM Pulsed Dipole

FIG. 1
FUNCTIONAL DIAGRAM OF THE SPS CLOSED ORBIT CORRECTION SOFTWARE

FIG. 3
Fig. 4a: Typical orbit acquisition before correction

Fig. 4b: Orbit after global correction
Fig. 5 : CO at 26 GeV with two insertions '7 x 3.5 m'

a) before correction

b) after global correction outside of the two insertions
SPS TYPICAL CYCLES

a) SPS main bending magnet

by Pulsed correcting dipole

FIG. 6
Fig. 7: CO at 270 GeV with two insertions '1.5 x 0.75 m'

a) before correction

b) After correction with 1 horizontal dipole and 3 vertical dipoles
By writing a global orbit acquisition as an \( n \)-tuple called \( \bar{y} \), say, the relationship between the measured orbit and the dipole strengths required to correct it can be written as

\[
\bar{y} = A \bar{k}
\]  

(2)

where \( \bar{k} \) is an \( n \)-tuple of dipole strengths. This follows from eq. (1). It is convenient to treat \( y \) and \( k \) as vector quantities in an \( n \)-dimensional space and to recall that the \( n \) components of any vector can be specified independently if the vector is generated by a set of \( n \) linearly independent vectors which span the space. These \( n \) linearly independent vectors can be used to form a basis in the space whence any vector in the space is a unique linear combination of the basis vectors. Thus

\[
\bar{y} = \sum_{i=1}^{n} \alpha_i \bar{x}_i
\]  

(3)

where the \( n \) linearly independent vectors \( \bar{x}_i \) are the basis vectors.

The formal solution to eq. (2) is

\[
\bar{k} = -A^{-1} \bar{y}
\]  

(4)

\[
= -A^{-1} \sum_{i=1}^{n} \alpha_i \bar{x}_i
\]

\[
= -\sum_{i=1}^{n} \alpha_i A^{-1} \bar{x}_i
\]

\[
= -\sum_{i=1}^{n} \alpha_i A^{-1} \bar{x}_i
\]
It is appropriate to choose a set of orthonormal vectors as a basis for \( y \). Then the problem of solving eq.(2) for a global orbit correction is reduced to one of calculating the correction for each basis vector. Since each basis vector contains only one non-trivial component (a unit component which represents, say, 1 mm orbit displacement) the three dipole-compensated bump strategy as described above can be imposed. In general

\[
\begin{pmatrix}
0 \\
0 \\
\vdots \\
0 \\
0 \\
\end{pmatrix}
= \begin{pmatrix}
A_{11} & 0 & 0 \\
0 & A_{22} & 0 \\
0 & 0 & A_{33} \\
\end{pmatrix}
\begin{pmatrix}
k_1 \\
k_2 \\
k \\
\vdots \\
k_n \\
\end{pmatrix}
\]

The first \( m-2 \) and last \( n-m-1 \) equations form a homogeneous set. For total betatron phase shifts \( \lambda = 0^\circ \) or \( 180^\circ \) the submatrices \( A_{11} \) and \( A_{33} \) have full rank and the set of equations has the trivial solution \( k_1 = 0, \ldots, k_{m-2} = 0, k_{m+2} = 0, \ldots, k_n = 0 \) as was expected. The set

\[
\begin{pmatrix}
0 \\
0 \\
\vdots \\
0 \\
0 \\
\end{pmatrix}
= \begin{pmatrix}
0 & k_{m-1} & k_m & k_{m+1} \\
1 & k_{m-1} & k_m & k_{m+1} \\
\vdots & \vdots & \vdots & \vdots \\
0 & k_{n-1} & k_n & k_{n+1} \\
\end{pmatrix}
\]

is solved by Cramer's rule and the solution is augmented to give

\[
\begin{pmatrix}
k_b \end{pmatrix} = \begin{pmatrix}
k_{m-1} \\
k_m \\
k_{m+1} \\
0 \\
\end{pmatrix} = -A^{-1}k
\]

where \( k_b \) is the basis vector for \( k \).
Hence

\[ k = \sum_{i=1}^{n} a_i \bar{k}_{bi} \]  

(8)

The choice of an orthonormal basis for \( \bar{y} \) is obvious; the \( a_i \)'s are simply the measured orbit displacements in the appropriate units (mm). The treatment of the overlap interaction between the 1st and nth BP/COD has not been included. It requires that the same argument be applied in an \( n+2 \) dimensional space which is the original \( n \) dimensional space augmented by the overlap.