NEW SCALING PROPERTIES OF THE STRUCTURE FUNCTIONS IN THE
SINGLE TIME FORMULATION OF THE QUANTUM FIELD THEORY

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ABSTRACT

The structure functions of hadrons in deep inelastic scattering are calculated in the framework of the single
time formulation of QFT. The initial hadron is considered as a set of bound states of the various numbers of
quarks (partons) and the structure functions are expressed through the relativistic wave functions of these bound
states. A new scaling variable, taking into account the target mass effect, is introduced and the new scaling
properties of the structure functions in the deep inelastic region as well as their behaviour on the exclusive
threshold are investigated.

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INTRODUCTION

In our previous paper \(^1\) we considered the structure functions of deep inelastic electron-hadron scattering in the framework of the single time formulation of QFT using a two-intermediate particle approximation. In particular, it was shown that a new scaling variable appeared in this approach and it turned out to be more natural to describe the structure functions in terms of this scaling variable and the final hadronic system mass squared \(W^2 = (P+q)^2\) rather than in terms of the usual Bjorken variables \(x\) and \(Q^2\) \(^2\).

In recent years a lot of papers have been devoted to the quest for a proper scaling variable in the framework of the parton model \(^3\). For example, in Refs. 4,5) the scaling variable \(\xi\), which took into account non-zero quark masses, was introduced in the usual quark-parton QFT calculations under the assumption that the struck quark is laid up on the mass shell.

In the single time formulation of QFT all the constituents lie on their mass shells from the very beginning, and as a price for that one has no usual energy-momentum conservation law in vertices. As shown in Ref. 1), this approach leads to the appearance of the new scaling variable \(\xi\):

\[
M \xi = \nu + M - \sqrt{\nu^2 + Q^2} = \nu + M - \sqrt{(\nu + M)^2 - W^2} 
\]  

(1)

and the structure functions are expressed by the single time relativistic wave function of the initial hadron, which obeys the covariant three-dimensional (quasipotential) equation.

The variable \(\xi\) is a decreasing function of \(\nu\), so at fixed \(W^2\) we have

\[
\xi_{\text{min}} = \frac{W^2}{\alpha M (\nu + M)} \to 0; \quad M \xi_{\text{max}} = \nu_{\text{min}} + M - \sqrt{(\nu_{\text{min}} + M)^2 - W^2}
\]  

(2)

and since

\[
\nu_{\text{min}} = \frac{W^2 - M^2}{\alpha M}, \quad \xi_{\text{max}} = 1.
\]  

(3)

Thus, at fixed \(W^2\), the variable \(\xi\) can change in the interval \([0, 1]\).
It is easy to calculate that
\[ \nu + \mu = \frac{W^2 + \eta^2 M^2}{2M \eta} \]  
(4)

and
\[ Q^2 = \frac{(1-\eta)(W^2 - \eta M^2)}{\eta} \]
(5)

Hence it follows that
\[ x = \frac{Q^2}{2M \eta} = (1-\eta) \frac{W^2 - \eta M^2}{W^2 - \eta (\rho - \eta) M^2} \]

and we see that at \( M^2 = 0 \), \( \eta = (1-x) \) and \( x \bar{w}^2 = (1-x)Q^2 \), i.e., we arrive at the usual Bjorken variables. Thus we conclude that the new scaling variable \( \eta \) explicitly takes into account the influence of the target mass \( M \).

In the deep inelastic region, when \( W^2 \to \infty \) and \( \eta \) is fixed, we have the following limits:

\[ 2M \eta \equiv \frac{W^2}{\eta} \to \infty ; \]

\[ Q^2 \equiv \frac{(1-\eta)W^2}{\eta} \to \infty , \]

so that \( x \to 1-\eta \) is fixed and we are actually in the usual Bjorken deep inelastic region, but in terms of the other variables \( \eta \) and \( W^2 \).

Our present paper will be devoted to the derivation of the new scaling behaviour of the structure functions in the case of many-body intermediate states in the framework of the single time formulation of QFT.

**STRUCTURE TENSOR IN THE IMPULSE APPROXIMATION**

We start with the usual expression for the structure tensor describing the hadronic interactions with the virtual photon in deep inelastic electron-hadron scattering.
\[ W_{\mu \nu}(P, q) = \frac{i}{\sqrt{2}} \sum_{N} (2\pi)^{4} \delta(P + q - P_{N}) \times 
abla_{\mu} (0) / N \left\{ \begin{array}{c} N \end{array} \right\} \langle N | \phi_{\mu}(0) | P \rangle, \]

where the "summation" goes over all intermediate states, \( P_{N} \) is the momentum of the \( N \)th intermediate state, \( q \) is the momentum of the virtual photon \( (q^2 = -Q^2) \) and \( J_{\mu}(0) \) is the hadronic current operator. As an intermediate state vector, we shall choose the vector in Fock's space \( |N\rangle = |k_{1}, \ldots, k_{N}\rangle \) where \( k_{i} \) are the momenta of the intermediate particles (quarks or partons) which are considered here to be identical. Therefore, the expression (7) takes the following form:

\[ W_{\mu \nu}(P, q) = \frac{i}{\sqrt{2}} \sum_{N=2}^{N} \frac{1}{N!} \int d^{4}k_{1} \cdots d^{4}k_{N} \langle \sum_{i=1}^{N} k_{i} \rangle \delta(k_{i}^2 - m^2) \times (2\pi)^{4} \delta(P + q - P_{N}) \langle k_{1}, \ldots, k_{N} | J_{\mu}(0) | P \rangle, \]

where \( P_{N} = \sum_{i=1}^{N} k_{i} \) and the hadronic current matrix element \( \langle k_{1}, \ldots, k_{N} | J_{\mu}(0) | P \rangle \) is graphically presented in Fig. 1.

In order to express this matrix element through the single time wave function of the initial hadron consisting of \( N \) quarks \(^{(7)}-^{(10)}\), we shall neglect the interaction between quarks in the final state, that is to say, we shall consider here the impulse approximation for the virtual photon hadronic interaction. In this approximation, as in the two-particle case \(^{(1)}\), we can write, for example, the contribution of the diagram presented in Fig. 2 in the following form:

\[ \langle k_{1}, \ldots, k_{N} | J_{\mu}(0) | P \rangle = \int d^{4}k_{1} \cdots d^{4}k_{N} \overline{v}_{BP} (k_{1}, \ldots, k_{N}) \left\{ \begin{array}{c} N \end{array} \right\} \left\{ \begin{array}{c} N \end{array} \right\} . \]

where \( J_{\mu} \) is the quark current and \( \overline{v}_{BP} \) is the above-mentioned wave function of the bound state of \( N \) quarks. Taking into account the summation over \( N \) in Eq. (8), we conclude that in our approach the initial hadron is described by a set of single time wave functions corresponding to various possible numbers of quarks being inside it.
In the usual parton model, due to the energy-momentum conservation

\[ P_N = k_N - q = p - p_{N-1}, \]  

(10)

where \( P_{N-1} = \sum_{i=1}^{N-1} k_i \) and, generally speaking, the momenta of the quarks are not laid up on the mass shells. In the single time approach, the struck quark, as well as the rest constituents, lies a priori on the mass shell \( (p_N^2 = m^2) \). However, instead of the conservation law (10), we have the following constraint:

\[ P_{N-1} + q = P \cdot \frac{p_{N-1} + p_N}{m^2}. \]  

(11)

Hence it follows that

\[ k_N = P \cdot \frac{PP_{N-1} + \sqrt{(PP_{N-1})^2 - m^2 (P_{N-1} - m^2)}}{m^2} - P_{N-1}. \]  

(12)

Now we substitute the expression (9) into Eq. (8) and, taking into account that all \( N \) diagrams of the type presented in Fig. 2 give incoherently the same contributions to the structure tensor, we obtain the following expression

\( W_{\mu\nu}(p, q) = \frac{1}{N^2} \sum_{N^2} \left[ \frac{1}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3} \Theta(k^+) \Theta(k^-) \delta(k^2 - m^2) \right] \times \int \frac{d^4 k}{(2\pi)^4} j_{\mu}(k_N; p_N) j_{\nu}(k_N; p_N) \delta(p + q - p_N) \left\{ \frac{1}{N_{\mu\nu}} \left[ k_{\mu\nu} \right] \right\}^2. \)  

(13)

Here we introduce the new variable of integration \( k \) instead of \( k_N \) with the help of the following replacement:

\[ P + q - k_N = k. \]  

(14)

The structure tensor then takes the form:
\[ W_{\mu\nu}(P,q) = \int \frac{d^4 k}{(2\pi)^4} \frac{\delta \left[ (P' + q - k')^2 - M^2 \right]}{k'^2} \times I(k^2, P_k/M), \]
where
\[ I(k^2, P_k/M) = \frac{1}{2} \sum_{N=0}^{\infty} \frac{1}{(N-1)!} \times \frac{1}{\sqrt{(2\pi)^3 2k^0}} \frac{\delta \left( k - P_{N,w} \right)}{\psi_{np}(\vec{F}_w, \vec{F}_{N,w})} \cdot \]

and, due to Eqs. (12) and (14)
\[ \bar{p}_N = P \cdot \frac{P_k + \sqrt{(P_k)^2 - M^2}}{M^2} - k ; \]
\[ \vec{F}_w = P + q - k. \]

A similar representation for the inclusive cross-section in terms of single time wave functions obeying the many-particle quasipotential equation was proposed in Refs. 11) and 12).

As in Ref. 1), we shall define the vector current of a scalar quark in the following way:
\[ j_\mu (\vec{F}_w; \bar{p}_N) = (\vec{F}_w + \bar{p}_N)_\mu ; \]
\[ j_\mu (\vec{F}_w; \bar{p}_N) = - (\vec{F}_w + \bar{p}_N)_\mu , \]
and introduce two invariant functions:
\[ V_1(Q^2, \nu) = Q^{\mu} W_{\mu\nu}(P,q) ; \]
\[ V_2(Q^2, \nu) = P^{\mu} P^{\nu} W_{\mu\nu}(P,q) / M^4 , \]
where \( \nu = P_9 / M \). From Eqs. (15), (16), (18) and (17), we obtain the following expressions for these two functions:
\[ V_i(Q_i^0, \nu) = -\int d\kappa \Theta(P_i^0, q_i^0 - \kappa^0) \delta(W^2 - 2P \kappa - \nu \kappa + \kappa^2 \cdot m^2) \times \]

\[ A_i(W_i, \nu + M, \kappa^0, P_i \cdot M) I(\kappa^2, \nu \cdot M), \]

where

\[ A_i(W_i, \nu + M, \kappa^0, P_i \cdot M) = 2 \left( \nu + M - \frac{P_i \cdot M}{M} \right) \left( \frac{P_i \cdot M}{M} + \frac{1}{2} \left( \frac{P_i \cdot M}{M} \right)^2 - \kappa^2 \cdot m^2 \right) + \]

\[ + \kappa^2 \cdot m^2 - (W_i - \kappa^2 \cdot m^2); \]

\[ A_2(W_i, \nu + M, \kappa^0, P_i \cdot M) = \left[ \nu + M - \frac{P_i \cdot M}{M} + \frac{1}{2} \left( \frac{P_i \cdot M}{M} \right)^2 - \kappa^2 \cdot m^2 \right]^2. \]

We shall now make the replacement of the variable \( k \rightarrow \Delta = \frac{1}{P} k \) under the integral (20), which is equivalent to the transition to the hadron rest system \( \Theta = 0 \):

\[ V_i(Q_i^0, \nu) = -\int d\Delta^0 \int d\Delta^o \Theta(\nu + M - \Delta^0) \times \]

\[ \times \delta\left[ W^2 - 2(\nu + M)\Delta^0 + \Delta^2 - m^2 + \Delta^0 \cdot \Delta^o \right] A_i(W_i, \nu + M, \Delta^0, \Delta^0) I(\Delta^0, \Delta^0) \]

where

\[ A_i(W_i, \nu + M, \Delta^0, \Delta^0) = 2(\nu + M - \Delta^0) \left( \Delta^0 + \sqrt{\Delta^0^2 - \Delta^2 + m^2} \right) + \]

\[ + \kappa^2 \cdot m^2 - (W_i - \Delta^2 \cdot m^2); \]

\[ A_2(W_i, \nu + M, \Delta^0, \Delta^0) = (\nu + M - \Delta^0 + \sqrt{\Delta^0^2 - \Delta^2 + m^2})^2 \]

and \( q_i = \frac{1}{P} q, \) i.e.,

\[ q_i^0 = \nu; \]

\[ |q_i^1| = \sqrt{\Delta_0^2 + \Delta^2} \equiv \sqrt{(\nu + M)^2 - W_i^2}. \]
After integrating over angles in Eq. (23) we obtain:

\[ V_i(Q^o, \nu) = -\frac{e^2}{2\sqrt{\nu^2+Q^o}} \int_{-\infty}^{\infty} d\Delta^0 \int d\Delta^2 \Theta (\nu + M - \Delta^0) \]

\[ \times \Theta \left[ \nu (\nu^2 + Q^o)(\Delta^2 - \Delta^0) - (\Delta^2 - \Delta^0 (\nu + M) \Delta^0 + \Delta^2 - m^2)^2 \right] \]

\[ \times A_z (\nu^2, \nu + M, \Delta^2, \Delta^0) I(\Delta^2, \Delta^0). \]  

(27)

The second \( \Theta \) function under the integral gives the following region of integration over \( \Delta^0 \):

\[ \Delta^- \leq \Delta^0 \leq \Delta^+ \]  

(28)

where

\[ \Delta^\pm = \frac{N-1}{2} (N \pm 1) \sqrt{\nu^2 + Q^o} \sqrt{(\nu^2 - m^2)^2 - 4 \nu^2 \nu + \nu^2 - m^2} \]  

(29)

Furthermore, from the expression (16) for \( I(\Delta^2, \Delta^0) \), it follows that \( \Delta^2 = \frac{1}{N-1} \) and it is easy to show that the minimal value of \( \nu (\nu^2 + Q^o) \Theta (\nu + M - \Delta^0) \) at \( N = 2 \) is equal to \( (\nu^2 - m^2)^2 \), so that the function \( I(\Delta^2, \Delta^0) \) vanishes outside the interval

\[ m^2 < \Delta^0 < (\sqrt{\nu^2 - m^2})^2 \]  

(30)

In this case it is not difficult to show that

\[ \Delta^+ < \nu + M, \]  

(31)

that is to say, the first \( \Theta \) function in Eq. (27) does not matter at all. Besides, one can calculate that

\[ \sqrt{\Delta^2} \leq \Delta^- , \]  

(32)

and after the permutation of the integrals in Eq. (27), taking account of Eqs. (28), (30) and (32), we get the following expression:
We notice that here it is very easy to make the transition to the two intermediate quark case discussed in Ref. 1. From Eq. (16) at $N=2$ and $P_{N-1}=k_1$ we get

$$I(\Delta^0, \Delta^0) = \frac{1}{2(\Delta^0)^2} \delta(\Delta^0 - \Delta^0) |\langle 2 | \delta(1\Delta^1) | 0 \rangle|^2$$

and substituting this expression into Eq. (33) we come to the formulae of Ref. 1, except for the factor 1/2 which arises due to the identity of quarks.

**SCALING PROPERTIES OF THE STRUCTURE FUNCTIONS**

Since $\Delta^0 = \sqrt{\Delta^2}$ under the integral (33), we can introduce the rapidity of the quark system in the intermediate state $y = \eta_n(\Delta^0 + \sqrt{\Delta^2 - \Delta^2})/\sqrt{\Delta^2}$ ($\Delta^0 = \sqrt{\Delta^2 \cosh y}$):

$$V_i(Q^2, \nu) = -\frac{\sqrt{W^2 - m^2}}{\sqrt{W^2 + m^2}} \int \frac{d^2A}{m} \int dy \sinh y \cdot$$

$$\times A_i(W^2, \nu + M, \Delta^2, \sqrt{\Delta^2 \cosh y}) I(\Delta^0, \sqrt{\Delta^2 \cosh y}),$$

where

$$y_\pm = \pm \ln \frac{W^2 + \Delta^2 + m^2 \pm \sqrt{(W^2 - \Delta^2)^2 - 4W^2 \Delta^2}}{2W \sqrt{\Delta^2}},$$

(36)
and $\zeta$ is defined by Eq. (1). We now introduce the function

$$\gamma(W^o, \sqrt{\Delta^2}) = \frac{2m \sqrt{\Delta^2}}{W^o + \Delta^2 - m^2 - \sqrt{(W^o - \Delta^2 + m^2)^2 - 4m^2 W^o}},$$

(37)

which decreases with $\sqrt{\Delta^2}$ and in the interval $m \leq \sqrt{\Delta^2} \leq \sqrt{W^o - m}$ varies in the following limits:

$$\frac{M}{2m} \left( 1 + \sqrt{1 - \frac{4m^2}{W^o}} \right) \geq \gamma(W^o, \sqrt{\Delta^2}) \geq \frac{M}{\sqrt{W^o}}.$$  

(38)

Hence expression (35) takes the form

$$V_i(Q^o, \nu) = \frac{\delta^o}{\sqrt{W^o + Q^2}} \ln \frac{W^o + Q^2}{m} \int \frac{\Delta^o d\Delta^2}{\Delta^2 - m^2} \int dy \ln y \times$$

$$\times A_i(W^o, \nu + M, \Delta^2, \sqrt{\Delta^2} \cosh y) I(\Delta^2, \sqrt{\Delta^2} \cosh y).$$

(39)

Using the relationships between $V_i$ and the usual structure functions of deep inelastic scattering $W_1$:

$$V_M W_1(Q^o, \nu) = -V_i(Q^o, \nu) + \frac{Q^o}{W^o + Q^2} V_2(Q^o, \nu);$$

(40)

$$\frac{V_M(W^o + Q^2)}{Q^2} W_2(Q^o, \nu) = -V_i(Q^o, \nu) + \frac{Q^o}{W^o + Q^2} V_2(Q^o, \nu),$$

(41)

we obtain, from Eq. (39):

$$F_i(\nu, W^o) = 2m W_1(Q^o, \nu) = \frac{\delta^o}{W^o - \Delta^2 + m^2} \int \frac{\Delta^o d\Delta^2}{\Delta^2 - m^2} \int dy \ln y \times$$

$$\times I(\Delta^2, \sqrt{\Delta^2} \cosh y) \int A_i(W^o, \frac{W^o + \Delta^2 + m^2}{2m^2}, \Delta^2, \sqrt{\Delta^2} \cosh y) -$$

(42)
\[- \frac{\sqrt{\omega^2 + \delta^2 N^2}}{(\omega^2 - \delta^2 N^2)^2} A_2 \left( \frac{\omega^2 + \delta^2 N^2}{2\alpha}, \Delta^2, \sqrt{\Delta^2 \text{cash}^2} \right) \] 

\[ F_2 (\omega, \omega^2) \equiv \sqrt{\omega^2 + \delta^2 N^2} W_2 \left( \frac{\omega^2}{\Delta^2}, \nu \right) = \frac{\delta^2 N^2 (\omega^2 - \delta^2 N^2)}{(\omega^2 - \delta^2 N^2)^2} \times \]

\[ \frac{\sqrt{\omega^2 - \delta^2 N^2}}{\Delta^2 N^2} \int \left( \Delta^2 \text{cash} \right)^{\frac{1}{2}} \int m \Delta^2 \text{cash} ^{1} \]

\[ \times A_2 \left( \frac{\omega^2}{\Delta^2}, \frac{\omega^2 + \delta^2 N^2}{2\alpha}, \Delta^2, \sqrt{\Delta^2 \text{cash}^2} \right) - \frac{\delta^2 N^2 (\omega^2 - \delta^2 N^2)}{(\omega^2 - \delta^2 N^2)^2} A_2 \left( \frac{\omega^2}{\Delta^2}, \frac{\omega^2 + \delta^2 N^2}{2\alpha}, \Delta^2, \sqrt{\Delta^2 \text{cash}^2} \right) \]

where we have used Eqs. (4) and (5) in order to express \( \nu \) and \( \delta^2 \) in terms of \( \omega^2 \) and \( \zeta \). Due to Eqs. (24) and (25), we have:

\[ A_1 \left( \frac{\omega^2}{\Delta^2}, \frac{\omega^2 + \delta^2 N^2}{2\alpha}, \Delta^2, \sqrt{\Delta^2 \text{cash}^2} \right) = \]

\[ = 2 \left( \frac{\omega^2 + \delta^2 N^2}{2\alpha} - \sqrt{\Delta^2 \text{cash}^2} \left( \sqrt{\Delta^2 \text{cash}^2} + \sqrt{\Delta^2 \text{min}^2 N^2 + \omega^2} \right) \right) \]

\[ + \nu \omega^2 - (\omega^2 - \Delta^2 + \nu \omega^2) ; \]

\[ A_2 \left( \frac{\omega^2}{\Delta^2}, \frac{\omega^2 + \delta^2 N^2}{2\alpha}, \Delta^2, \sqrt{\Delta^2 \text{cash}^2} \right) = \]

\[ = \left( \frac{\omega^2 + \delta^2 N^2}{2\alpha} - \sqrt{\Delta^2 \text{cash}^2} + \sqrt{\Delta^2 \text{min}^2 N^2 + \omega^2} \right) . \]
It is now easy to calculate that in the deep inelastic region, when $W^2 \to \infty$ and $\zeta$ is fixed \[\gamma(W^2, \sqrt{s}) \to W/\sqrt{s^2},\] the structure functions are

\[
F_1 (Q^2, W^2) = \frac{1}{\sqrt{s}} \int \frac{d^2 \Phi}{2 \pi} \int d\xi \sin \chi \int \frac{d^2 \Phi}{2 \pi} \int d\xi \sin \chi \int \frac{d^2 \Phi}{2 \pi} \int d\xi \sin \chi \left( \frac{1}{\ln \frac{M^2}{\sqrt{s^2}}} \right)
\]

\[
\times \left( \sqrt{Q^2} \sin \chi + \sqrt{Q^2} \sin \chi + \frac{W^2}{\sqrt{s^2}} - M \right) \frac{1}{\sqrt{s}} \int \frac{d^2 \Phi}{2 \pi} \int d\xi \sin \chi \left( \sqrt{Q^2} \sin \chi + \sqrt{Q^2} \sin \chi + \frac{W^2}{\sqrt{s^2}} - (3 - 2\zeta) M \right)
\]

\[\text{cf., Eqs. (68) and (71) of Ref. 1}\] Thus we come to the scaling behaviour of the structure functions which depend only on $\zeta$ in the limit $W^2 \to \infty$.

**BEHAVIOUR OF THE STRUCTURE FUNCTIONS ON THE EXCLUSIVE THRESHOLD**

In this section we shall consider the behaviour of the structure functions in the case when $W^2 = W_0^2$ is fixed and $\zeta \to 0$ (exclusive threshold). In this region, from Eqs. (4) and (5), it follows that

\[
Q^2 \equiv \frac{W_0^2}{\zeta} \to \infty;
\]

\[
Q^2 \equiv \frac{W_0^2}{\zeta} \to \infty.
\]

From Eqs. (42) and (43) we obtain

\[
F_1 (Q^2, W_0^2) \approx \frac{1}{\zeta} \int \frac{d^2 \Phi}{2 \pi} \int d\xi \sin \chi \left( \frac{1}{\ln \frac{M^2}{\sqrt{s^2}}} \right)
\]

\[
\times \left( \sqrt{Q^2} \sin \chi + \sqrt{Q^2} \sin \chi + \frac{W_0^2}{\sqrt{s^2}} \right) \frac{1}{\zeta} \int \frac{d^2 \Phi}{2 \pi} \int d\xi \sin \chi
\]

\[\text{cf., Eqs. (68) and (71) of Ref. 1}\]}
\[ x^I (\Delta^0, \sqrt{\Delta^0 \text{cosh}}) \{ A_1 (W_0^0, \frac{W_0^0}{\Delta^0}, \Delta^0, \sqrt{\Delta^0 \text{cosh}}) - \]

\[ - \frac{\gamma_{\text{elc}}}{W_0^0} A_2 (W_0^0, \frac{W_0^0}{\Delta^0}, \Delta^0, \sqrt{\Delta^0 \text{cosh}}) \}^2 \]

\[ F_2 (\xi, W_0^0) \equiv \frac{\gamma_{\text{elc}}}{W_0^0} \int \frac{\Delta^0 d\Delta^0}{m} \int_{\Delta^0} dy \text{cosh} \]

\[ \left( \frac{\gamma_{\text{elc}}}{W_0^0} \right)^2 \]

\[ x^I (\Delta^0, \sqrt{\Delta^0 \text{cosh}}) \{ A_1 (W_0^0, \frac{W_0^0}{\Delta^0}, \Delta^0, \sqrt{\Delta^0 \text{cosh}}) - \]

\[ - \frac{\gamma_{\text{elc}}}{W_0^0} A_2 (W_0^0, \frac{W_0^0}{\Delta^0}, \Delta^0, \sqrt{\Delta^0 \text{cosh}}) \}^2 \]

where \( \gamma_{\text{elc}} \rightarrow \infty \) and due to Eq. (37):

\[ \gamma_0 = \gamma (W_0^0, \sqrt{\Delta}) = \frac{\Delta^0 \sqrt{\Delta^0}}{W_0^0 + \Delta^0 - \sqrt{(W_0^0 - \Delta^0)^2 - \gamma_{\text{elc}} W_0^0}} \]  \( \text{(51)} \)

Taking into account Eqs. (44) and (45), it is easy to see that the main contribution to the structure functions at \( \xi \rightarrow 0 \) will be

\[ F_2 (\xi, W_0^0) \equiv \frac{\gamma_{\text{elc}}}{W_0^0} \int \frac{\Delta^0 d\Delta^0}{m} \int_{\Delta^0} dy \text{cosh} \]

\[ \left( \frac{\gamma_{\text{elc}}}{W_0^0} \right)^2 \]

\[ x (1 - \frac{\Delta^0 \gamma_{\text{elc}} \sqrt{\Delta}}{W_0^0} \text{cosh}) I (\Delta^0, \sqrt{\Delta^0 \text{cosh}}). \]

We can now put under the integral \( 2 \text{sinh} y = 2 \text{cosh} y = e^y \) and replace the variable \( y = \gamma_{\text{elc}} (1/\gamma_0) + y' \):

\[ \left( \frac{\gamma_{\text{elc}}}{W_0^0} \right)^2 \]
Thus we conclude that the behaviour of the structure functions on the exclusive threshold ($\xi \to 0$) is defined by the asymptotics of the function $I(\Delta^2, \Delta^0)$ at large $\Delta^0$.

Near the exclusive threshold, when $\bar{w}_0^2$ is fixed and sufficiently small, only the two-particle intermediate state will contribute to the structure tensor (13). This means that we are allowed to replace the whole system of quarks $\mathbf{1}, \ldots, \mathbf{k}_{BM-1}$ in the intermediate state (see Fig. 2) by only one particle with varying, but limited mass $\sqrt{\Delta^2}$, $(m < \sqrt{\Delta^2} < \sqrt{\bar{w}_0^2} - m)$. In this case, we obtain from Eq. (16) the following expression for the function $I(\Delta^2, \Delta^0)$:

$$I(\Delta^0, \Delta^0) = \frac{1}{\Delta^2} \left| \frac{\psi_{BM}(\Delta^0, \Delta^0)}{\psi_{BM}(\Delta^0, \Delta^1)} \right|^2,$$

(54)

where $\psi_{BM}(\Delta^2, \Delta^1)$ is the two-particle wave function of the bound state of the quark and the particle with the effective mass $\Delta^2$.

As in Ref. 1), we write down the wave function $\psi_{BM}$ in the following way

$$\psi_{BM}(\Delta^2, \Delta^1) = \frac{\sqrt{\Delta^2}}{\sqrt{\Delta^0 \sinh y}} \Phi_{\Delta^2}(y),$$

(55)

where the rapidity $y = \Delta^0(\Delta^0 + 1)/\sqrt{\Delta^2}$. Then for the structure functions (53) we have [cf., Eq. (75) in Ref. 1]:

$$F_1(d, \bar{w}_0^2) \approx F_2(d, \bar{w}_0^2) \approx \int d\Delta^0 \int_0^{\Delta^0} dy \cdot x \left( 1 - \frac{\Delta^0}{\bar{w}_0^2} e^{2y} \right) \left| \frac{\Phi_{\Delta^2}(\Delta^0 + y)}{\Phi_{\Delta^2}(\Delta^0 + y)} \right|^2,$$

(56)
and the leading term in the limit $\zeta \to 0$ is
\[
F_1 (d, \omega, \omega^0) \approx F_2 (d, \omega, \omega^0) \approx 
\left( \frac{1}{\omega^0 - \omega} \right) \frac{1}{\omega^0} \frac{d}{d \omega} 
\int \frac{d^2 l^2}{\pi^2} \left( \frac{\omega^0}{\omega} \frac{\omega^0 \omega^2}{M^2} + \frac{N \lambda^2}{M} - \frac{\omega^0 \lambda^2}{M} \right) \phi_k (l, \omega, m^2) \left( \frac{l^2}{m^2} \right).
\]

Thus, the behaviour of the structure functions near the exclusive threshold is effectively defined by the asymptotics of the two-particle bound state wave function which were particularly investigated in Ref. 1).

CONCLUSIONS

In the present paper we have managed to express the hadron structure functions of deep inelastic scattering in the impulse approximation through the relativistic wave functions of the hadron in the framework of the single time formulation of QCD. In this approach, the initial hadron consists of an arbitrary number of quarks (partons) which organize the bound states corresponding to this hadron. Thus, the initial hadron is described by a set of single time wave functions corresponding to various possible numbers of quarks being inside it.

In our approach, a new scaling variable $\zeta$ appears which explicitly takes into account the non-zero target mass. It is shown that the structure functions depend on $\zeta$ and $\omega^2 = (p+q)^2$ and in the deep inelastic region ($\omega^2 \to \infty$) the scaling behaviour of the structure functions is derived. The expression for the structure functions near the exclusive threshold ($\zeta \to 0$) is obtained as well.

In order to calculate the explicit form of the structure functions in the whole kinematical region of the variables $\zeta$ and $\omega^2$, we need to know the many-particle relativistic wave functions of the hadron which have to obey the single time dynamical equations (\textsuperscript{7}-\textsuperscript{10}). Up to now, this problem was solved exactly only for the two-particle system \textsuperscript{13},\textsuperscript{14},\textsuperscript{11} because, in the general case, it requires the introduction of the many-particle quasi-potential \textsuperscript{9},\textsuperscript{15}.\textsuperscript{15}.
However, it seems to be more reasonable to describe the composite hadron participating in the inclusive reaction with the help of the density matrix which was introduced in Ref. 16) in the framework of the single time formulation of QFT. In our subsequent papers we shall try to solve this problem, as well as generalize our approach for the spinor quark case.

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