ON LOW ENERGY TESTS OF QCD

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ABSTRACT

Some discrepancies in low energy tests of QCD can be eliminated by introducing at phenomenological level a non-zero effective mass for the gluon of about 0.8 GeV. We discuss the results for $\eta$ decays.
It is now apparent that Quantum Chromodynamics is in impressive qualitative agreement with experimental data. However, if we try to obtain successful quantitative predictions, we face mainly two problems:

a) the scaling violations found in deep inelastic scattering (DIS)\(^1\) and the transverse momentum distributions of muon pairs\(^2\) produced in hadronic collisions suggest a value for the effective running coupling constant

\[ \alpha_s(q^2) \approx 0.4 \div 0.6 \]

\[ \frac{1}{q^2} \div 10 \text{ GeV}^2 \] \hspace{1cm} (1)

On the contrary, if we use the well-known formula for the ratio of leptonic over hadronic widths of \( \psi \)

\[ B_\psi = \frac{\Gamma(\psi \rightarrow e^+e^-)}{\Gamma(\psi \rightarrow \text{hadrons})} = \frac{48\pi}{5(m_\psi^2 - m_e^2)} \frac{\alpha_s^2}{\alpha_s^3} \] \hspace{1cm} (2)

we get \( \alpha_s(10 \text{ GeV}^2) \approx 0.2 \)\(^3\), which is not compatible with the value obtained from deep inelastic scattering analysis. It is well known that in the case of the \( \eta_c \) particle lowest order prediction for the equivalent quantity

\[ \frac{\Gamma(\eta_c \rightarrow \text{hadrons})}{\Gamma(\eta_c \rightarrow \gamma \gamma)} \]

is strongly corrected by the next order of perturbation theory\(^4\). However, the bulk of the corrections may be absorbed (after elimination of a spurious \( \Delta N = 4 \pi \)) using as coupling constant \( \alpha_s(\bar{E}_g) \), where \( \bar{E}_g \) is the mean single gluon energy of the decay: \( E_g^2 \approx \frac{m_{\eta_c}^2}{2} \) for \( \eta_c \) and \( E_g^2 \approx \frac{m_\psi^2}{2} \) for the \( \psi \).

In other words, the mean effect of radiative corrections amounts to computing the effective coupling constant at a smaller energy scale: in the conventional approach this increases the discrepancy between the value of \( \alpha_s \) obtained with Eqs (1) and (2).

b) we expect for the proton form factor a behaviour proportional to\(^5\)\(-7\)

\[ \frac{c}{q^4} \alpha_s(q^2)^{\frac{3}{2} + \beta_0} \left[ \sum_k a_k (\alpha_s \bar{E}_g^2)^{-b_k} \right]^2 \]

where \( \beta \) is the one-loop coefficient of the \( \beta \) function and \( c, b_k \) are positive coefficients; unfortunately, the data for \( 3 \leq q^2 \leq 25 \text{ GeV}^2 \) are well represented by a pure \( 1/q^4 \) behaviour.
A similar situation occurs in small angle proton-proton elastic scattering with $t$ fixed but $t/s \to 0$. Theoretical predictions give

$$\frac{d\sigma}{dt} \sim \frac{C}{t^8} \left[d_s(t)\right]^6 F(t)$$

(3)

where $F(t)$ is a Sudakov type form factor: the process is in fact seen as resulting from the scattering of the three constituent quarks of the two protons. Experimental data are well fitted by $1/t^8$ for $2 \leq t \leq 9$ GeV$^2$.

Also neglecting the Sudakov form factor $F(t)$, one stays in a very bad disagreement since there is no trace in the data of the rapidly varying factor $a_s(t)^6$. The effect of radiative corrections probably consists in substituting for $t$ the effective $t$ exchanged in each of the three elementary subprocesses, which can be estimated of order $t_s/9$. Although the nominal value of $t_s$ is high, we are actually in the intermediate $q^2$ region. In the conventional approach $a_s(q^2/9)$ varies faster than $a_s(q^2)$ and therefore the absence of scaling violations becomes even more dramatic.

Discrepancies happen at low energy where non-perturbative effects may be relevant. A very important consequence of confinement is the existence of a threshold in the two-gluon channel (the mass of the glueball is expected to be of order of 1-2 GeV), so it is not reasonable to neglect these "mass" effects at low energy. It is not clear how to treat them correctly; something has been done in this direction in the dispersive theory of charmonium [cf., Ref. 3]; the aim of this letter is to propose a simple phenomenological recipe to estimate the size of the effect.

Our proposal consists of giving a mass to the gluon. The final theory is not a consistent one unless Higgs' mechanism is used: it is renormalizable but not unitary at higher orders of perturbation theory. Lack of unitarity at high loops does not disturb us, because we work only at tree level to estimate phase space corrections.

It is evident that a non-zero effective mass for the gluon ($m_g$) will freeze the value of the effective coupling constant around $\alpha_s(4m_g^2)$ [i.e., $\alpha_s(q^2) \approx \alpha_s(4m_g^2)$ for $|q^2| \leq 4m_g^2$]. If $m_g^2$ is about 0.7 GeV$^2$ (corresponding to an effective threshold in the two-gluon channel at $M_{2g} \approx 1.6$ GeV), $\alpha$ does not increase when $q^2$ is smaller than $\sim 3$ GeV$^2$, explaining the absence of observed scaling violation in the form factor. However, a careful analysis must be made in order to have a quantitative description of the phenomenon.
As far as normal scaling violations in DIS are concerned, the presence of a gluon mass does not affect significantly the prediction in the region of large $Q^2$ and $Q^2(1-x)$, and the measured value of $\alpha_s(Q^2)$ should therefore remain unchanged. Of course, non-negligible effects may occur at the boundaries of the allowed kinematical regions.

In this letter we address specifically the computation of gluon mass effects on $\psi$-onia decays. The correct formulae are:

$$B_\psi = \frac{18\pi}{5(\pi^2-9)} \frac{\alpha_s^2}{\alpha_s^3} \frac{1}{F_3(2m_g/M_\psi)}$$

$$B_\psi' = \frac{18\pi}{5(\pi^2-9)} \frac{\alpha_s^2}{\alpha_s^3} \frac{1}{F_2(2m_g/M_\psi')}$$

$$\frac{\Gamma(\eta_c \rightarrow \text{hadrons})}{\Gamma(\psi \rightarrow \text{hadrons})} = \frac{27\pi}{5(\pi^2-9)} \frac{1}{\alpha_s} \frac{F_2(2m_g/M_\eta_c)}{F_2(2m_g/M_\psi)}$$

Phase space effects are stronger for the $\psi$ than for $\psi'$ or $\eta_c$, as it can be seen by looking at the functions $f_3$ and $f_2$ which are reported in Fig. 1 as a function of $\eta = 2m_g/M_{\text{res}}$.

We fix the values of $\alpha_s$ and $m_g$ by requiring that $B_\psi$ agrees with the experimental value and $B_\psi' \approx 2B_\psi$ (of course $\Gamma_h$ does not contain cascade decays in other $\psi$-onia). We can use a constant value of $\alpha_s$ for all three processes, because in our approach $\alpha_s$ is frozen below 3 GeV$^2$. We get for $\alpha_s = 0.35$ and $m_g = 800$ MeV

$$B_\psi \approx 0.08$$
$$B_\psi' \approx 2B_\psi$$

$$\frac{\Gamma_{\eta_c}}{\Gamma_{\psi}} = 160 \quad \text{and} \quad \Gamma_{\eta_c} = 9 \text{ MeV}$$

The value of $\alpha_s$ is now consistent with the one extracted from deep inelastic scattering.
The same values of $\alpha_s$ and $m_\pi$ can be used to calculate the centre-of-mass frame energy spectrum of direct photons produced from $\psi$ decays: $\frac{d\sigma}{dx}$, $x = (2p_\gamma)/M_\psi$. In our approach $\frac{d\sigma}{dx}$ should be zero for $x > 1 - (4m_\pi^2)/m_\psi^2 \sim 0.7$. This zero is removed, for example, by higher order corrections like the conversion of the two gluons into light quark pairs.

The experimental data for the quantity $(1/N_{tot}) \frac{dN}{dx}$ are reported in Fig. 2, as well as QCD predictions. $N_{tot}$ represents the total number of produced $\psi$'s: the curve displayed is then normalized to the fraction of radiative decays of total $\psi$ decays:

$$\frac{S \frac{dN}{dx} \frac{dx}{N_{TOT}}}{\Gamma(\psi \rightarrow \gamma + \text{all})} = \frac{\Gamma(\psi \rightarrow \gamma + \text{all})}{\Gamma_{TOT}} \sim 0.12$$

The curve referring to the massive gluon case is uncorrected for the experimental energy resolution; these effects can be qualitatively estimated by comparing the uncorrected and corrected predictions for the massless gluon case. Experimental data favour clearly the massive case.

We finally observe that a more precise measurement of the $x$ spectrum close to the boundary ($x \sim 0.7$) should be one of the best ways for finding glueballs.
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Figure 1 : The functions $f_2(\tau)$ and $f_3(\tau)$ representing the gluon mass corrections to $\bar{\psi} \rightarrow \gamma \psi$ and $\tau^c \rightarrow \gamma \tau^c$ decays, respectively, are plotted versus $\tau = 2m_g / M_{\tau^c}$.

Figure 2 : Experimental data taken from Ref. 11) are confronted with theoretical predictions for massless gluons corrected for experimental energy resolution (dashed curve). Full lines represent the uncorrected predictions for massless (B) or massive (A) gluons.
\eta = \frac{2Mg}{M_{\text{res}}}

fig. 1

\frac{1}{N_{\text{TOT}} \frac{dN}{dx}}

fig. 2